Power Optimized Battery Swap and Recharge Strategies for Electric Aircraft Operations

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Electric propulsion for commuter air transportation is becoming promising because of significant strides in battery specific energy and motor specific power. Energy storage and rapid battery recharge remain nonetheless challenging owing to the significant energy and power requirements of even small aircraft. By modifying algorithms developed in the field of scheduling theory, we propose *power optimized* and *power-investment optimized* strategies for electric aircraft battery swaps and recharges. Several aspects are considered: electric energy expenditures, capital expenditures, and flight schedule integrity. The first strategy optimizes the swaps and recharges to minimize the peak-power draw from the grid and to reduce electric energy expenditures. The second strategy optimizes the swaps and recharges to minimize electricity expenditures and capital expenditures associated with battery and charger procurement. In both cases, the optimization is decomposed into two simpler problems. The first is a recharge schedule feasibility analysis given a number of chargers and batteries, which is based on a network flow representation of the battery swap and recharge. The second is a recharge schedule generation given a number of chargers and batteries. Both strategies are applied to the operations of two commuter airlines and are contrasted with a benchmark non-optimized *power-as-needed* strategy. Promising results are obtained with up to 61% reduction in peak-power draw and up to 25% reduction in electricity costs.

Keywords: Electric Aircraft, Battery Recharge, Battery Swap, Scheduling, Electricity Price

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1 Introduction

At the time of writing, battery technology has advanced sufficiently to motivate serious consideration of electric propulsion for small aircraft. The benefits of electric propulsion include near-zero aircraft emissions, reduced energy costs associated with sourcing energy from the electrical grid, and reduced maintenance costs resulting from simpler mechanical systems (Patterson, Derlaga and Borer 2016). The implications of these benefits on aircraft operating costs are significant and motivate an assessment of their impacts not only on existing aviation business models but also on new aviation markets such as urban air mobility (Holden and Goel 2016).

A major technical challenge associated with electric propulsion for aircraft is the gravimetric energy density of current battery chemistries, which is two orders of magnitude less than that of jet fuel (Winter and Brodd 2004). This places severe constraints on the design and operations of electric aircraft by substantially limiting range. In contrast, the battery specific energy density is not as challenging for electric automobiles for which the range now exceeds the average daily urban trip distance (Santos, et al. 2011).

For high frequency commercial air transportation services such as commuter airlines, the limited range of electric aircraft implies not only the need to restrict operations to shorter routes but also the need to recharge aircraft batteries after each flight. Commercial aircraft need high daily utilization to be profitable and therefore batteries need to be recharged quickly during the ground turnaround time at each airport visited. These operations are unlike personal electric automobiles which average less than an hour of driving per day with morning and late afternoon trips split by a long period of inactivity during which batteries can be recharged at low power (van Haaren 2011).

Given the significant energy requirements and short turnaround times typical of commercial aircraft operations, fast recharges at high power may be required. With high power levels come new challenges such as thermal management, battery health and longevity (Rezvanizaniani, et al. 2014), and electric infrastructure issues. Indeed, the electrical grid may not be able to sustain high power draws associated with fast charging in some remote locations. There is therefore considerable uncertainty regarding if, and how, adequate recharging rates can be achieved to enable commercial operations of electric aircraft.

There are also cost implications when using high power levels for fast battery recharging. The price of electricity not only varies from city to city and from utility to utility but also depends on how the electric energy is drawn from the grid. Electricity pricing is typically based on both the amount of energy used and the peak-power delivered – or likely to be delivered – over a month of operations (Warwick, et al. 2016). The energy part of the bill is related to the cost of producing electricity using natural resources. The peak-power part of the bill is related to the investments needed for oversized electricity generation, transformation, and transmission infrastructures to adequately serve the peak demand of end-users.

Because the price of electricity is sensitive to the peak-power demand, care must be given to how electric energy is drawn from the grid in order to lower energy expenditures. Disregarding time-of-use effects, the lowest price of electricity is achieved by minimizing the peak power drawn, or likely to be drawn, from the grid by chargers. This minimum is reached when batteries are continuously charged at low power levels since this strategy maximizes the amount of energy transferred to batteries while minimizing the peak-power demand. The resulting flat power profile can be achieved using a local energy storage system such as battery banks able to supply extra power in periods of high demand and store energy in periods of low demand. Another way to generate a flat power profile is to consider swappable batteries that are interchanged from the aircraft and recharged in such a manner that power is continuously drawn from the grid at relatively low levels. After an aircraft lands at an airport, discharged batteries are swapped with previously recharged batteries during the ground turnaround time. The removed batteries are recharged and installed later on another aircraft departing the airport.

Swaps have the additional benefit of ensuring that batteries are adequately cooled in-between flights. Indeed, batteries tend to heat-up during fast recharges and discharges which may impact their longevity and capacity (Shim, et al. 2002). In turn, this degradation impacts the aircraft range capability. By recharging at lower powers and ensuring that batteries are as cool as possible upon installation on the aircraft, these thermal challenges can be mitigated.

In this paper, we explore the use of battery swaps and recharges for the operations of a fleet of electric commuter aircraft. We optimize the design and operations of the supporting battery and charger infrastructure to minimize expenditures. In section 2, we present relevant background on battery swaps for electric vehicles. In section 3, we describe the relevant models and algorithms developed in the field of scheduling theory and we formulate the optimization model using a network flow representation of the battery swap and recharge problem. In section 4, we modify the aforementioned algorithms and propose two battery swap and recharge strategies that maintain the integrity of the flight schedule. The first is a *power optimized* strategy which optimizes the battery recharge schedule to minimize the peak-power draw from the grid and therefore the electricity price. This is achieved by minimizing the numbers of chargers required at any given airport. The second is a *power-investment optimized* strategy which minimizes the overall recharge expenditures (capital expenditures and recurring energy expenditures) by determining the optimal numbers of chargers and batteries at any given airport. We also develop a non-optimized *power-as-needed* strategy to benchmark the two aforementioned strategies. In section 5, we describe the operating environment for the study as well as the networks and schedules of two commuter aircraft operators. The electric aircraft used for the analysis is described along with its power consumption in various phases of flight. The rates and schedules for relevant electric utilities are also presented. In section 6, we implement the charging strategies and compare the results in terms of electricity price and capital expenditures. Finally, in section 7, we conclude with the main contributions of our research, and we highlight future improvements.

2 Battery Swaps

Battery swaps have been studied primarily in the context of electric cars and public transportation (Sultana, et al. 2018) (Jing, Kim and An 2018). Swapping has been proposed as an alternative to simple plug-in charging with the aim of alleviating long recharge times, high energy costs, and potential harmful impacts on the grid such as overloads, voltage overages, and losses (Tran-Quoc, et al. 2012) (Sarker, Pandžic and Ortega-Vazquez 2015) (Kang, et al. 2016).

In this context, two main themes may be identified from the literature. The first focuses on the design of the network of battery swap and recharge stations. The objective is to determine the most appropriate locations for battery swap stations and/or the required number of chargers and/or spare batteries to maximize profits or minimize investment costs associated with the deployment of a battery-swap infrastructure. This optimization is achieved while satisfying electricity requirements for various demand profiles such as daily commuting, road trips, public transportation, and package delivery (Zheng, et al. 2014) (Mak, Rong and Shen 2015) (Yang and Sun 2015) (Hof, Schneider and Goeke 2017).

The second theme focuses on the operation of the battery swap and recharge stations, sometimes including the scheduling of recharges. The objective is to develop an optimal charging strategy to reach goals such as: minimizing the infrastructure operating costs with or without battery inventory costs; minimizing loads on microgrids or entire distribution networks; maximizing the use of renewable energy sources to reduce carbon emissions; and maximizing the availability of recharged batteries under various concepts of operation (Schneider, Thonemann and Klabjanb 2017) (Yang, Guo and Zhang 2017) (Wu, et al. 2017) (Li, et al. 2018) (Widrick, Nurre and Robbins 2018) (Mahoor, Hosseini and Khodaei 2019).

The considerations and challenges related to the design and operation of a battery swap and recharge infrastructure to support electric aircraft operations are nevertheless quite different from those encountered in the context of electric ground vehicles owing to the nature of air transportation. Indeed, in the case of airline operations, the battery swap and recharge problem is constrained by the number of electric aircraft, by the locations of charging stations, and by recharge requirements corresponding to pre-determined aircraft routings and schedules. The demand for electric energy is therefore assumed to be known deterministically and the goal is to determine the optimum numbers of chargers and batteries at each location to meet the demand while minimizing peak loads and energy costs to the airline. As a result, many of the studies carried out for electric vehicles are not directly applicable to electric aircraft operations.

3 Model Development

In this paper, we investigate two battery swap and recharge approaches: a *power optimized* strategy which minimizes the price of electricity and a *power-investment optimized* strategy which minimizes the overall recharge cost to the operator. Both approaches involve removing discharged batteries from aircraft as they arrive at their destinations in order to recharge them. Meanwhile, other batteries previously recharged with enough energy to complete the subsequent flights are loaded aboard the aircraft. Assuming no energy tankage between low-electricity-cost airports and high-electricity-cost airports (i.e. batteries are recharged just enough to be able to fly the subsequent flight with adequate reserves), minimizing the cost of electricity requires minimizing the peak-power draw at each airport within the network. For each airport, minimizing the peak power is equivalent to minimizing the number of chargers needed to charge the batteries assuming that each charger operates at the same power level. Given this minimum number of chargers, the minimum number of batteries required to operate all flights at that airport is also determined. This solution describes the *power optimized* strategy.

Because batteries and chargers are expensive (U.S. SEC, Form 10-K, Tesla, Inc. 2016) and there is no reason to tie significant amount of capital in unnecessary inventories of batteries and chargers, the *power-investment optimized* strategy goes a step further and accounts for the cost of chargers and batteries to find a solution that minimizes the combined capital expenditures (associated with battery and charger acquisition) and recurring energy expenditures (associated with electricity consumption).

3.1 Relationship with machine scheduling problems

The battery recharge problem under investigation belongs to the family of job-shop problems which have been studied since the beginning of the industrial revolution. In the field of operations research, a job-shop problem is an optimization problem in which ideal jobs are assigned to various resources at particular times. In its most basic version, *n* jobs with varying processing times need to be scheduled on *m* machines with varying processing powers *p*, while trying to minimize the makespan C_{max} defined as the total time to complete the *n* jobs. Over the years, many variants of the original job-shop problem have been studied, including some featuring release times, deadlines, and various objective functions (Jackson 1955), (Horn 1974), (Labetoulle, et al. 1984). A subclass of these problems features a single type of machine and is called a machine scheduling problem. Using the machine scheduling analogy for the battery swap and recharge problem, the *machines* are the chargers, the *shops* are the airports where machines are located, and the *jobs* are the battery recharges. A recharge job indexed *j* has a *release date* denoted r_j, which is the time at which an aircraft lands and its discharged battery becomes available for recharge. A job also has a *processing time* denoted p_j, which is the time required to recharge the battery sufficiently to complete the subsequent flight with adequate energy reserves. Finally, a job has a *deadline* denoted d_j, which is the time by which the recharge needs to be completed for the battery to be ready for a subsequent flight. A scheduling optimization problem typically has an *objective function* to be minimized. Since the goal of this research is to generate a feasible recharge schedule that minimizes the peak-power draw while minimizing the impact on flight operations, the relevant objective function uses the concept of *lateness* denoted *Lj* and defined as the difference between the *completion time* of a job denoted *Cj* and its deadline *dj*. Given a number of chargers and batteries, the goal is to ensure that a battery is recharged and available for each departure in the flight schedule. As a result, the optimization minimizes the *maximum lateness* across all flights denoted L_{max} and defined in equation (1) as the maximum of all departure delays induced by battery recharges:

$$L_{max} = \max_j(L_j) = \max_j(C_j - d_j) \tag{1}$$

Scheduling optimization problems are notoriously difficult to solve and many have been proven to be non-deterministic polynomial-time hard (*NP*-hard) (Lenstra, Kan and Brucker 1977). This is true for many problems featuring machines working in parallel on jobs with both release dates and deadlines (Brucker 2007). In order to efficiently solve the battery swap and recharge problem using aspects of machine scheduling theory, some assumptions are made to facilitate the search for an optimal solution.

3.2 Modeling assumptions

First, no energy tankage is permitted between airports. This means that operators will charge the battery just enough to fly the subsequent flight with appropriate regulatory reserves (FAA 2000). Operators will not recharge batteries in excess of this minimum amount of charge at airports with cheaper electricity prices in order to minimize recharges at airports with higher electricity prices. This means that the battery recharge scheduling optimization is no longer a network-wide problem but rather a collection of smaller-scale airport-based optimizations which can be studied independently.

Second, batteries are always removed from the aircraft upon arrival and are replaced with batteries that have been recharged sufficiently for their next flights. A buffer time is introduced to account for the time spent removing discharged batteries from the aircraft and the time spent loading recharged batteries back onto the aircraft.

Third, batteries are assumed to have identical states of charge upon arriving at a specific airport. The remaining energy corresponds to the regulatory reserves, and all flights arriving at the same destination airport are assumed to share the same diversion airport and the same final reserves. This implies that batteries can be treated as identical and assigned to upcoming flights in a first-in, first-out (FIFO) sequence. The sequence of arriving batteries at an airport thus determines the pairing of batteries with their subsequent flight. This assumption simplifies the optimization because it is no longer necessary to keep track of the arriving battery state of charge. Additionally, there is no need to optimize the battery-to-flight allocation mechanism.

Fourth, preemption is assumed to be possible during the recharge of batteries. This means that a recharge may be interrupted before completion and resumed at a later time in order to accommodate the recharge of another battery. Upon removal from aircraft, batteries are envisioned to be connected to a centralized recharge station which prioritizes battery recharge jobs according to the flight schedule at that airport (i.e. determines which batteries should be recharged first, second, etc.). The recharge process can pause and resume as necessary, adding flexibility during the search for a feasible battery recharge schedule.

Under these assumptions and using the three-field terminology introduced by Graham et al. (Graham, et al. 1979) to classify machine scheduling problems, the battery recharge problem reduces to $P_m \mid pmtn, r_j \mid L_{max}$. P_m is the machine environment field indicating that m identical machines work in parallel, $prmp, r_j$ is the job characteristic field indicating that preemption is allowed and that recharge jobs are subject to release dates r_j, and L_{max} is the optimality criteria field, implying that recharge jobs are subject to deadlines d_j and that the objective is to minimize the maximum lateness. $P_m \mid pmtn, r_j \mid L_{max}$ problems are not NP-hard and algorithms have been developed to find optimal solutions. In this paper, the battery swap and recharge problem is decomposed into two smaller sub-problems following the approach of Horn (Horn 1974) and Martel (Martel 1981): the feasibility of the recharge is investigated first and the actual recharge schedule is generated next.

3.3 Formulation of the battery recharge schedule feasibility problem

To assess the feasibility of the $P_m \mid pmtn, r_j \mid L_{max}$ battery recharge problem, several constraints are defined. The number of chargers and the number of batteries available at each airport in the network define the *infrastructure constraints*, the departure times and the arrival times of flights define the *flight schedule constraints*, and the origins and destinations and associated energy requirements define the *routing constraints.* Given the infrastructure constraints, we check whether all recharges can be processed given the flight schedule and routing constraints. If all recharges cannot be performed, the infrastructure constraints must be relaxed by allowing either more batteries or more chargers at the airport. The existence of at least one solution can be proven: for *n* flights departing from an airport over the course of a day, the presence of *n* different batteries and *n* different chargers at this airport ensures that the recharge schedule is feasible (provided each recharge lasts less than 24 hours). This solution is nonetheless not optimal from a battery and charger utilization standpoint. Tradeoffs between the number of batteries and the number of chargers can be conceived. One objective of the feasibility study is thus to generate a Pareto frontier highlighting, at each airport, the tradeoff between the number of batteries and the number of chargers.

The recharge schedule feasibility is assessed at each airport separately by representing the schedule and its constraints using a network flow model and computing next the maximum flow through the network. This can be done in $O(n^3)$ time (Horn 1974) (Lawler, et al. 1993). The network flow model helps define the amount of *processing* (i.e. recharge) to be performed on each battery within a given time interval. The network consists of nodes and arcs. There are four different sets of nodes: the source node s, the sink node t, the job nodes J, and the time interval nodes I. The set of job nodes represents the set of battery recharges that must be completed in order to fly the schedule. Each job node J_j, indexed by j, corresponds to the recharge job required for one departure. The corresponding job processing time p_j is the amount of time needed to recharge the battery for that departure. The set of interval nodes represents a discretization of time. Time intervals are constructed by listing out all job release dates and deadlines (i.e. aircraft arrival and departure times), and by subsequently ordering them. This ordered list defines a list of adjacent time intervals during which batteries may be recharged. Each time interval node I_i, indexed by i, represents one of these time intervals and has a length ΔT_i. Each recharge job node J_j is connected to the source node using an arc with a capacity representing the processing time p_j required to recharge the battery. By construction, a recharge job is either possible or impossible during a time interval. Therefore, an arc connects a recharge job node J_j to a time interval node I_i if and only if a battery recharge can be performed during that time interval. Because a battery can be charged by no more than one charger at a time, the capacity of this arc is set to the amount of time available to perform a recharge job during that time interval, namely $\Delta \mathrm{T}_i$. Each time interval node is connected to the sink node using an arc with a capacity representing the total charging capacity expressed in units of time. If m identical chargers are considered, then the total processing capability of the m chargers during the time interval ΔT_i is given by $m \cdot \Delta T_i$. A graphical depiction of the network flow representation of the battery recharge scheduling problem is provided in Figure 1.

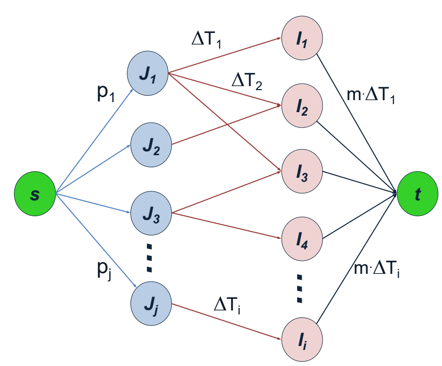


Figure 1: Network flow representation of the battery recharge scheduling problem

Next, we use the Ford-Fulkerson method of augmenting paths (Ford and Fulkerson 1956) to estimate the maximum flow that can be pushed through the network. An augmenting path is a path along the arcs from the source to the sink that has available capacity on all edges. To find augmenting paths, we implement the breadth-first search algorithm of Edmonds-Karp (Dinic 1970) (Edmonds and Karp 1972). It iteratively explores the network looking for the shortest path from the source to the sink with available capacity. The maximum flow is then compared to the total amount of processing power required to complete all the battery recharges. If they match, then the maximum lateness L_{max} is equal to zero and a battery recharge schedule is feasible given the infrastructure and flight schedule constraints. Otherwise, the battery recharge schedule is not feasible and two options can be pursued. Either the infrastructure constraints need to be relaxed by adding more batteries and/or more chargers, or the flight schedule constraints need to be relaxed by allowing flight departure delays (i.e. setting a maximum lateness target L_{target} greater than zero). In this research, disrupting the flight schedule is not allowed since the schedule is assumed to be optimized for (network-wide) revenue-management purposes. As a result, L_{target} is always set to zero and the infrastructure constraints are progressively relaxed until the flight schedule becomes feasible.

Using flow conservation constraints at the different nodes as well as capacity constraints along the arcs, the maximum flow problem can be expressed as a linear programming optimization. This is highlighted in the optimization problem statement of equation (2), where $x_{s,j}$ denotes the flow from the source node to the job node J_j, $x_{j,i}$ denotes the flow from the job node J_j to the time interval node I_i, and $x_{i,t}$ denotes the flow from the time interval node I_i to the sink node in a problem with *n* recharge jobs (*n* batteries), *m* identical chargers, and *2n-1* time intervals (*n* aircraft arrivals and departures at the airport of interest).

Maximize:

$$\sum_{j=1}^{n} x_{s,j} = x_{s,1} + x_{s,2} + .. + x_{s,n}$$

Subject to:

(2)

Edge capacity constraints along arcs: $$\begin{cases} \forall j \in [\![1, n]\!];\ x_{s,j} \leq p_j \\ \forall (i,j) \in [\![1, 2n-1]\!] \times [\![1, n]\!];\ x_{j,i} \leq \Delta T_i \\ \forall i \in [\![1, 2n-1]\!];\ x_{i,t} \leq m \cdot \Delta T_i \end{cases}$$

Flow conservation constraints at nodes: $$\begin{cases} \forall \mathrm{j} \in [\![1, n]\!];\ x_{s,j} = \sum_{i=1}^{2n-1} x_{j,i} \\ \forall \mathrm{i} \in [\![1, 2n-1]\!];\ \sum_{j=1}^{n} x_{j,i} = x_{i,t} \end{cases}$$

Flow positivity constraints: $\begin{cases} \forall j \in [\![1, n]\!], x_{s,j} \in \mathbb{R}^+ \\ \forall (i,j) \in [\![1, 2n-1]\!] \times [\![1, n]\!], x_{j,i} \in \mathbb{R}^+ \\ \forall i \in [\![1, 2n-1]\!], x_{i,t} \in \mathbb{R}^+ \end{cases}$

3.4 Generation of a feasible battery recharge schedule

Given a number of batteries and a number of chargers, proving the feasibility of a battery recharge schedule is sufficient to estimate the peak-power demand and therefore the price of electricity. Nevertheless, the actual generation of a battery recharge schedule is also of interest to understand how operators will charge aircraft batteries. A recharge schedule is constructed using the flow values computed during the determination of the maximum flow for the schedule feasibility analysis. Of interest are the flow values $x_{j,i}$ which represent the processing times of the recharge jobs J_j performed during the time intervals I_i. Several recharge jobs may be partially processed during a time interval and building a schedule consists in ordering these partial recharges and allocating them to the various chargers. For each time interval, generating a schedule is equivalent to solving a $P_m \mid pmtn \mid C_{max}$ problem (Horn 1974) with several chargers working in parallel to minimize the total completion time C_{max} of partial recharge jobs with identical release dates coinciding with the beginning of the time interval. By construction of the network, $x_{j,i} \leq \Delta T_i$ and $\sum_{j=1}^{n} x_{j,i} \leq m \cdot \Delta T_i$, which implies that for each time interval the inequality $C_{max}^i \leq \Delta T_i$ holds true (McNaughton 1959). In fact, a lower bound for the total completion time can be attained for each time interval, as provided in equation (3) (Brucker 2007).

$$\forall i \in [\![1, 2n-1]\!];\ C_{max}^i = max\left(\max_j(x_{j,i}), \sum_{j=1}^{n} \frac{x_{j,i}}{m}\right) \quad \textbf{(3)}$$

A schedule is constructed in $O(n)$ time (McNaughton 1959) by filling the chargers successively and by scheduling the partial recharge jobs in any order. When the time bound C_{max}^i is reached, the partial recharge job is split into two parts and the second part is processed by the next available charger starting at the beginning of the time interval. This construct ensures that the recharge schedules generated for each time interval have no more than *m-1* preemptions (Gonzalez and Sahni 1978) and the condition

$\forall(i,j) \in [\![1, 2n-1]\!] \times [\![1, n]\!]$; $x_{j,i} \leq C_{max}^{i}$ ensures that a partial recharge job cannot be processed by two chargers simultaneously.

4 Implementation

4.1 Power optimized battery swap and recharge strategy

The *power optimized* strategy aims at minimizing the peak-power draw from the grid and therefore the price of electricity. This is equivalent to minimizing the number of chargers at each airport in the network and determining the corresponding number of batteries that yield a feasible battery swap and recharge schedule. The search for the minimum number of chargers and the corresponding number of batteries at a given airport is initiated by astute guesses. If chargers were to be used without interruption at maximum power without any release time or deadline constraint, a lower bound for the number of chargers, denoted m, is given by equation (4) where E is the amount of energy supplied to the batteries over a given time period, P is the maximum power output of the chargers, and Δt is the time period length.

$$m = \frac{E}{P.\Delta t} \qquad \textbf{(4)}$$

Similarly, a lower bound for the number of batteries at an airport is given by the number of aircraft, denoted N, that start their operations at that airport (i.e. aircraft that depart from this airport without arriving first). An upper bound for the number of batteries at an airport is given by the number of flights departing that airport, denoted k_{max}. Let m be the initial guess for the minimum number of chargers located at the airport of interest and $k = N$ be the initial guess for the minimum number of batteries. The maximum lateness L_{max} across all flights at that airport may then be computed using equation (1) and compared with the maximum lateness target L_{target}. If $L_{max} > L_{target}$ and $k < k_{max}$, then an additional battery may be necessary to enable operations. This happens when aircraft arrival and departure times are so close that a sufficient battery charge cannot be made during the ground turnaround time. The number of batteries is therefore incremented by one unit and the maximum lateness is recalculated with the same number of chargers. If $L_{max} > L_{target}$ and $k = k_{max}$, then an additional charger may be necessary to enable operations without unacceptable disruptions. The number of chargers is thus incremented by one unit and the maximum lateness is recalculated with the minimum number of batteries N. The process of adding batteries and adding chargers is repeated until $L_{max} < L_{target}$ at which point the flight schedule at that airport is feasible with a minimum number of chargers m^* and a corresponding minimum number of batteries k^*. The algorithm for the *power optimized* battery swap and recharge strategy is illustrated in Figure 2.

The process of adding a battery to the search described previously requires an adjustment to the algorithms and concepts described in section 3. As batteries are added to an airport inventory, the sets of job nodes and time interval nodes are modified. Additional job nodes are created to account for the recharge of these additional batteries and additional time interval nodes are created to account for the availabilities of these new batteries. If one battery is introduced, then a recharge job is added with a release time set at the start of the study period. Using the first-in first-out assumption, all other recharge jobs are shifted by one increment, meaning that their release dates stay the same (i.e. at the time when the aircraft carrying the battery lands) but their deadlines are shifted to the next departure. Consequently, each battery stays longer on the ground and more time is available for charging. The process of introducing one additional battery is illustrated in Figure 3 and this shifting process is repeated each time a new battery is added.

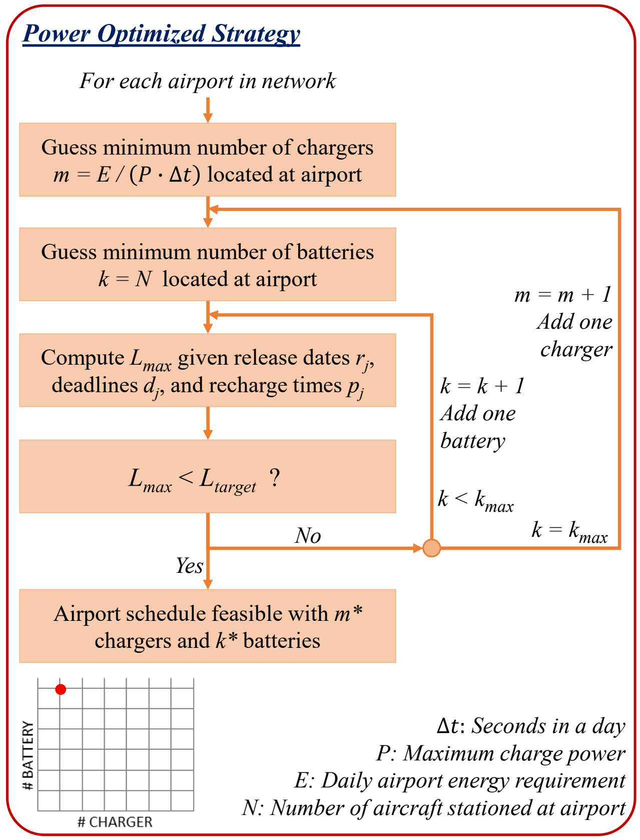


Figure 2: Implementation of the *power optimized* battery swap and recharge strategy by determining the minimum number of chargers

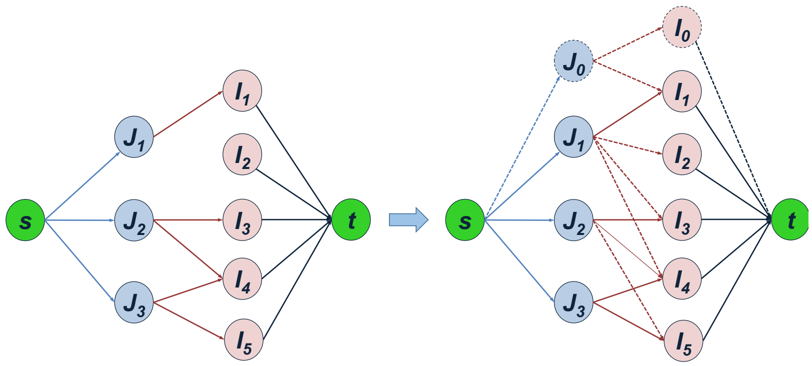


Figure 3: A new job J_0 and a new time interval I_0 are created when a new battery is added. J_0 represents the recharge job for the first departure while I_0 represents the early availability of this new battery. J_1 now represents the recharge job for the second departure. Batteries are now available for longer periods of time on the ground during which they can be recharged.

4.2 Power-investment optimized battery swap and recharge strategy

The *power-investment optimized* strategy aims at minimizing the overall recharge expenditures. Fewer chargers may result in lower electricity prices but may increase the number of batteries necessary, thus increasing capital expenditures. Conversely, additional chargers may increase the price of electricity but may reduce the number of batteries necessary, thus potentially reducing capital expenditures. There is therefore a tradeoff between higher capital expenditures due to the battery and charger procurement costs and lower recurring expenditures due to lower electricity price. Investigating these tradeoffs requires another adjustment to the algorithms and concepts described in section 3. As the number of chargers increases, it is likely that the number of required batteries decreases. This results in a Pareto frontier representing the set of non-dominated solutions to the battery and charger tradeoff problem. To generate this boundary, the battery swap and recharge feasibility analysis is integrated into an algorithm featuring two loops searching respectively for the number of chargers and the number of batteries that satisfy the flight schedule constraints.

At any given airport, the procedure starts with the feasible solution (m^*, k^*) to the *power optimized* strategy which provides an upper bound k^* to the minimum number of batteries required to satisfy the flight schedule. Let $m^* + 1$ be the initial guess for the number of chargers and k^* be the initial guess for the number of batteries. The maximum lateness L_{max} at that airport is computed using equation (1) and compared with the maximum lateness target L_{target}. If $L_{max} < L_{target}$, then the battery recharge schedule is still feasible. The number of batteries is then iteratively decremented by one unit and the maximum lateness is calculated until $L_{max} > L_{target}$ at which point the flight schedule is no longer feasible. Starting at the last feasible solution found, the number of chargers is again incremented by one unit and the process of searching for the minimum number of batteries is repeated. This procedure results in the creation of a Pareto frontier representing the number of chargers and the associated minimum number of batteries that satisfy the flight schedule at the airport.

From this Pareto frontier of non-dominated solutions, an optimal solution is defined as a pair (number of chargers, number of batteries) that minimizes the capital expenditures at that airport. This pair is identified by estimating energy and capital expenditures for each solution along the Pareto frontier. Using the battery lifetime energy throughput, the longevity and replacement date of each battery are approximated. A discounted cash flow analysis is then carried out using the longevity and expected replacement date of batteries, the cost of batteries, the cost of chargers, and the price of electricity. Future cash outflows are discounted to the present time to yield a net present value which enables the identification of the lowest cost solution pair (Fisher 1930) (Williams 1938). The analysis is repeated at each airport in the network to yield the solution to the *power-investment optimized* battery swap and recharge strategy illustrated in Figure 4. This simple analysis is akin to superimposing isopreference curves over the Pareto frontier and selecting the Pareto-optimal solution yielding the highest utility. The *power optimized* strategy solution, the Pareto frontier, and the *power-investment optimized* strategy solution are depicted in Figure 5.

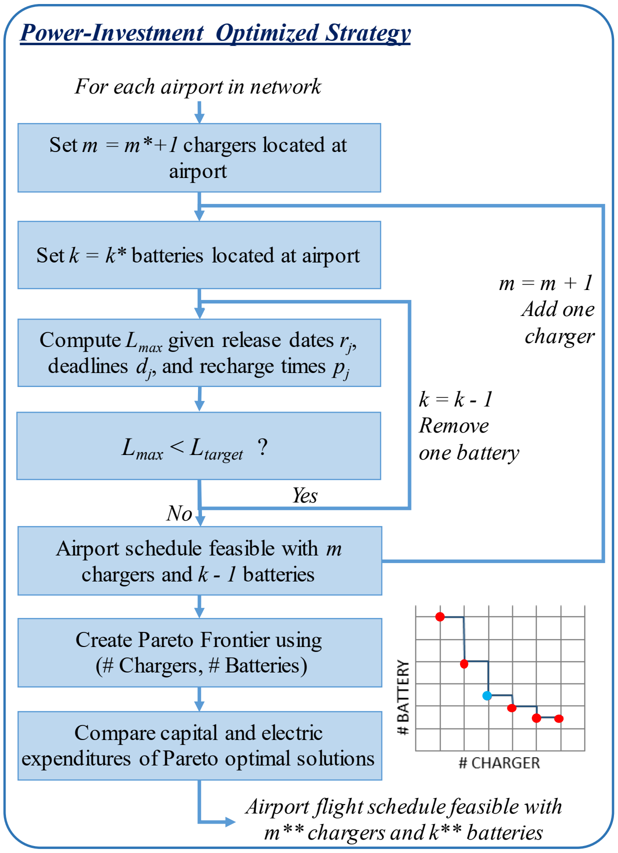


Figure 4: Trading off batteries for additional chargers to generate the Pareto frontier of non-dominated solutions for the *power-investment optimized* strategy

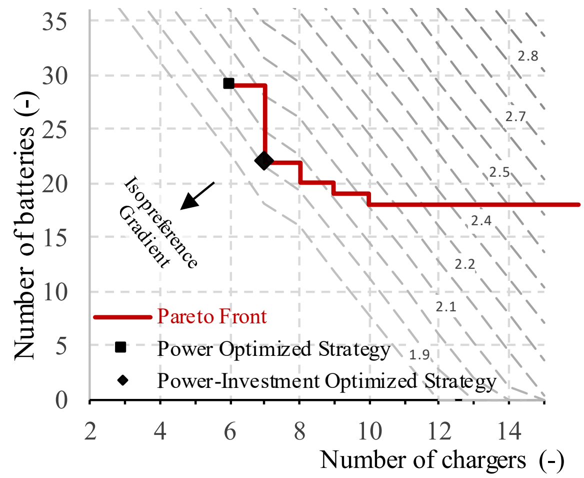


Figure 5: Pareto optimal solutions highlighting the solution to the *power optimized* and *power-investment optimized* strategies

4.3 Charger power selection for battery swap and recharge solutions

A typical battery recharge is composed of three phases: the pre-charge for low battery states of charge, followed by the constant-current fast charge, and finally the constant-voltage charge for high battery states of charge. When searching for solution pairs during the schedule feasibility analysis, we assume that chargers are working at their maximum rated power during the constant-current phase of the charge.

However, this assumption may not be justified at smaller airports characterized by few aircraft movements but short turnaround times. At these airports, a single charger is usually required but the resulting peak power is disproportionately large compared to the amount of energy used. In these cases, chargers may not need to operate at maximum rated power and yet are able to charge batteries sufficiently during the allotted time windows. The main benefit of throttling down the recharge power is to further decrease the peak power drawn from the electrical grid, and thus, to further decrease the price of electricity.

Consequently, a refinement of the battery swap and recharge strategies proposed in sections 4.1 and 4.2 consists in minimizing the power drawn from the grid by throttling down the recharge power once a solution pair (m^*, k^*) is found. This is achieved using a bisection algorithm that iteratively converges to the minimum required charger power yielding a feasible recharge schedule. The feasibility is assessed at each step by computing the maximum lateness L_{max} (which determines if the flight schedule integrity constraint is violated). The minimum charger power is used to determine the peak power and thus the price of electricity at the airport. The implementation of the charger power selection process is highlighted in Figure 6 and its integration within the implementation of the *power optimized* and *power-investment optimized* strategies is depicted in Figure 7. Although this is particularly relevant for smaller airports where fast charging can result in excessive peak powers, this process is applied to all airports in the network as a refinement to the two strategies described previously.

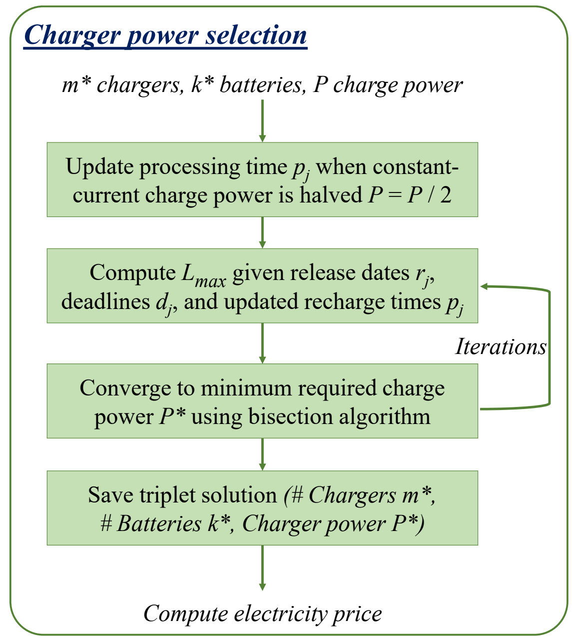


Figure 6: Charger power selection process

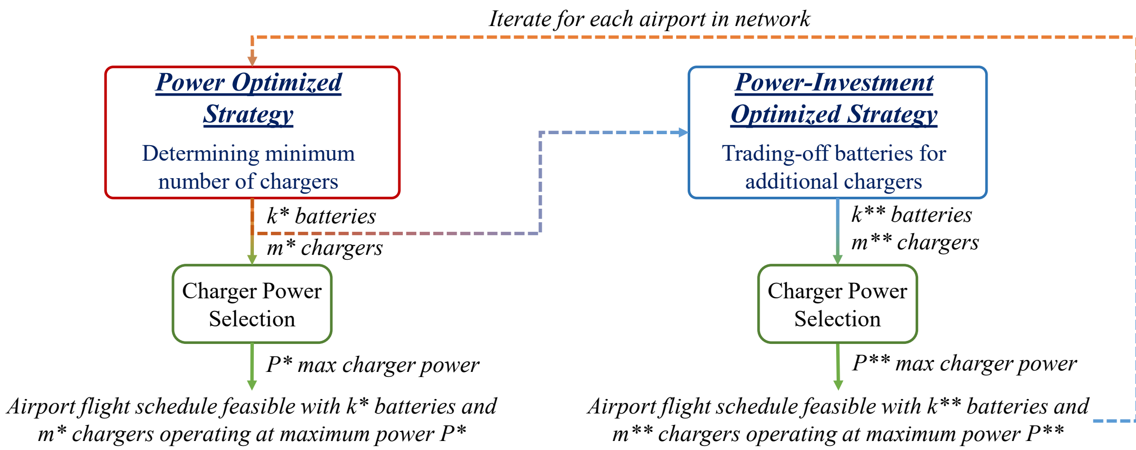


Figure 7: Charger power refinement for *power optimized* and *power-investment optimized* strategies

4.4 Benchmark non-optimized power-as-needed battery recharge strategy

A benchmark strategy is developed to highlight the potential savings using the optimized battery swap and recharge strategies. This *power-as-needed* strategy minimizes neither the peak-power draw from the grid nor the capital expenditures. Instead, it follows a simpler logic in which batteries are fully recharged overnight and partially charged during the ground turnaround time after each flight. Battery swaps are performed if, and only if, the state of charge of the on-board battery at the end of the ground turnaround time is insufficient to complete the subsequent mission with appropriate energy reserves. If a battery swap is necessary, the on-board battery is removed to be fully recharged and is replaced with a fully charged spare battery. A spare battery inventory management plan is implemented so that spare batteries are re-used for subsequent flights to limit the inventory of batteries. Thus, removed batteries are immediately plugged-in and placed in the airport pool of available batteries once fully recharged. This pool of available batteries is used when aircraft land and their on-board batteries need to be swapped. Overall, this unsophisticated strategy represents an approach with little operational complexity, where batteries are recharged whenever possible, and for which no consideration is given to the availability of chargers, the required number of chargers, the price of electricity, or the capital expenditures. A description of the algorithm implementing the simpler *power-as-needed* strategy is proposed in Figure 8.

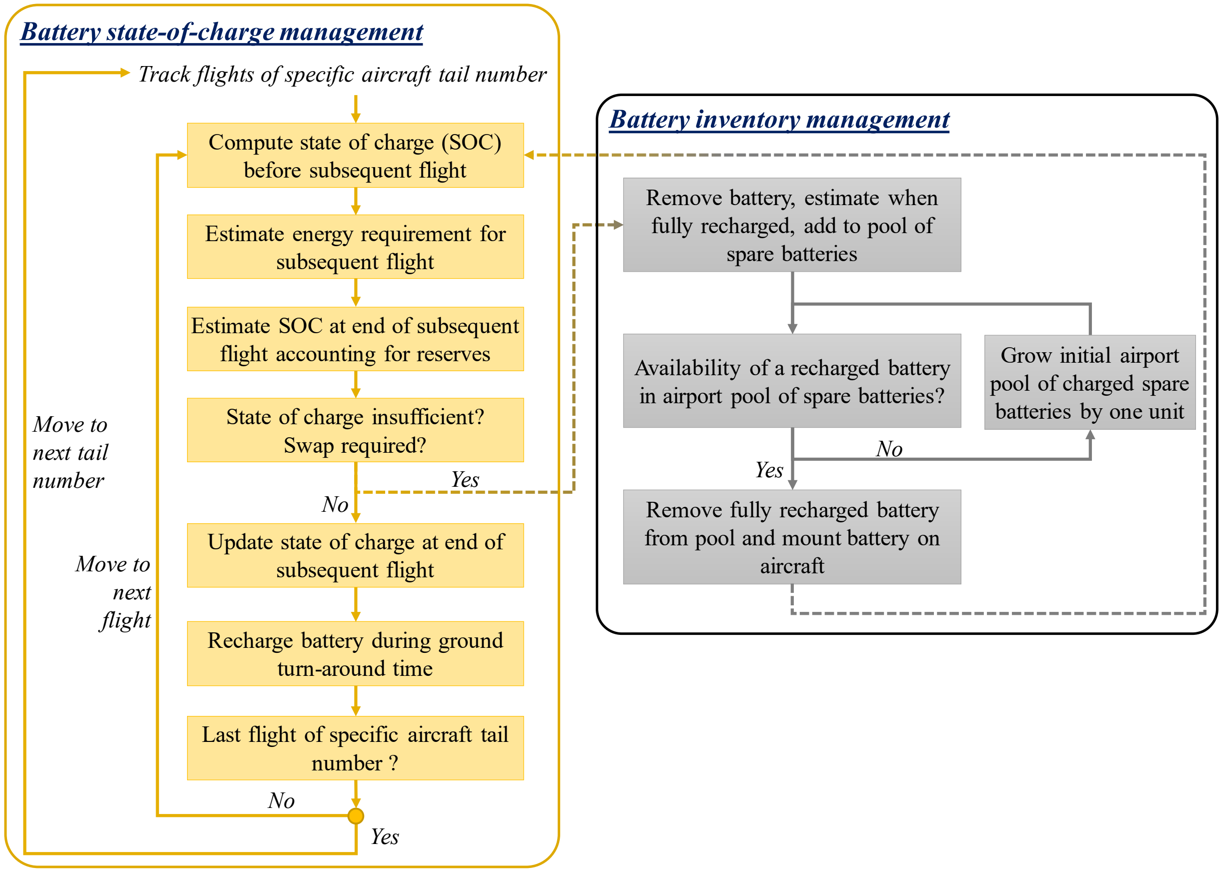


Figure 8: Power-as-needed strategy implementation with battery inventory management plan

5 Applications

The strategies presented in the previous sections are applied to the operations of an electric aircraft by two commuter airlines similar to Cape Air and Mokulele Airlines.

5.1 Description of commuter operators

Cape Air is one of the largest commuter operators in the world and operates a fleet of Cessna 402s, Britten-Norman Islanders, and ATR42 in the New England area, the Caribbean, Illinois, Missouri, Montana, and the Mariana Islands. Mokulele Airlines operates a small fleet of Cessna 208 aircraft in the Hawaiian Islands and California. This study focuses on Cape Air's Cessna 402 operations in New England and on Mokulele's Cessna 208 operations in Hawaii. Some relevant network statistics are presented in Table 1. Based on the analysis of Cape Air's and Mokulele's flight routings, great circle distances are augmented by 6% and 28% respectively to account for operational idiosyncrasies (instrument approaches, traffic patterns) and geographical obstacles (mountains). Reserves for both operators include a diversion to an alternate airport 50nm away and a 45min final reserve (Justin, et al. 2017).

Table 1: Network statistics of two commuter operators

	Cape Air New England Network	**Mokulele Airlines Hawaiian Network**
Week Analyzed	07/31/2015 to 08/07/2015	03/20/2016 to 03/27/2016
Weekly Flights	1,839	732
Cities Served	19	9
Number and Type of Aircraft	48 Cessna 402	8 Cessna 208
Median Day Turnaround Time	35 min	19 min

5.2 Description of the electric aircraft

Cape Air is the launch customer of the Tecnam P2012 Traveller, a twin-engine aircraft expected to replace the Cessna 402 fleet (Hemmerdinger 2017). An electric propulsion retrofit of the Tecnam P2012 based on the design principles of the NASA Maxwell X-57 is envisioned (Justin, et al. 2017). The electric aircraft retains some of the original structure of the Tecnam P2012 design and the nine-passenger cabin. However, it features a new retractable landing gear, a smaller composite wing fitted with single-slotted flaps, and a distributed electric propulsion system similar to the one found on the NASA X-57 (Borer, Derlaga, et al. 2017). The propulsion system consists of twelve high-lift propellers driven by electric motors and distributed along the leading edge of the wing to increase the airflow over the wing at low flight speeds, as well as two larger cruise propellers driven by electric motors and located at the wingtips. This system benefits from several aero-propulsion integration advantages over conventional propulsion systems, including increased cruising speeds, improved cruise lift-to-drag ratios, improved battery-to-shaft conversion efficiency, and zero greenhouse gas emissions (Borer, Patterson, et al. 2016). Some characteristics of the "Electro-Traveller" are summarized in Table 2.

Table 2: "Electro-Traveller" design parameters, power requirements, and operational assumptions

Characteristics and Performance		Power Requirements		Operational Assumptions	
Max. Take-Off Weight	8,730 lb	Taxi	39 kW	Taxi In and Out Phase	10 min
Operational Empty Weight	4,075 lb	Take-off	416 kW	Take-off Phase	2 min
Battery Capacity (x2)	214 kWh	Climb at 4,000 ft and 1,000 ft/min	382 kW	Approach Phase	1 min
Wing Area	175 ft^2	Cruise at 8,000 ft and 65% power	306 kW	Landing Phase	1 min
Max. Lift Coefficient, C_{Lmax}	3.95	Cruise at 8,000 ft and 75% power	353 kW	Min. and Max. Cruise Altitude	3,000 ft 10,000 ft
Carson's Speed at 8,000 ft	184 kt	Descent at 4,000ft and 700 ft/min	50 kW	Distance to Alternate	50 nm
Best Range Speed at 8,000 ft	146 kt	Approach	191 kW	Final Reserve	45 min
Best Endurance Speed at 5,000 ft	113 kt	Landing	191 kW	Battery Swapping Time	5 min

5.3 Electricity schedules and other assumptions

The electricity rates at airports served by Cape Air and Mokulele Airlines were retrieved from the websites of the corresponding utilities and assembled into a database. The database includes 139 rate schedules from 11 different utilities (Hawaiian Electric Company, Nantucket Electric Company, Eversource Energy, NSTAR, Central Maine Power, Emera Maine, Liberty Utilities, Niagara Mohawk Power Corporation, Consolidated Edisson, National Grid, Green Mountain Power). These rate schedules depend on the type of customer, the amount of energy used, the peak power likely to be delivered, and the delivery voltage. When time-of-use schedules are in effect, a time-weighted average electricity price is used since the proposed method does not account for these effects. Additional assumptions related to the recharge process, the cost of batteries, and the cost of chargers are summarized in Table 3.

Table 3: Battery, charger, and electricity rate assumptions for cases studies

Technology Assumptions		Economic Assumptions (2018-US$)	
Charger Power	125 kW	Charger Cost	$100,000
Charger Efficiency	90%	Battery Specific Cost	100 $/kWh
Charger Useful Life	7 years	Battery Inventory Cost	0 $/month
Battery Useful Life	1,000 cycles	Discount Rate	8.1%

6 Results and Discussions

The *power optimized* and *power-investment optimized* strategies are compared and contrasted with the *power-as-needed* strategy using the peak-power demand, the electricity cost, the number of chargers, and the number of batteries required to satisfy the flight schedules of the airlines.

6.1 Results for the power optimized strategy

The first set of results in Figure 9 provides a detailed electricity demand profile over a day of operations for Cape Air's largest station at Boston Logan International Airport (BOS) and for Mokulele Airlines at Molokai Hoolehua Airport (MKK) for the *power-as-needed* strategy and the *power optimized* strategy.

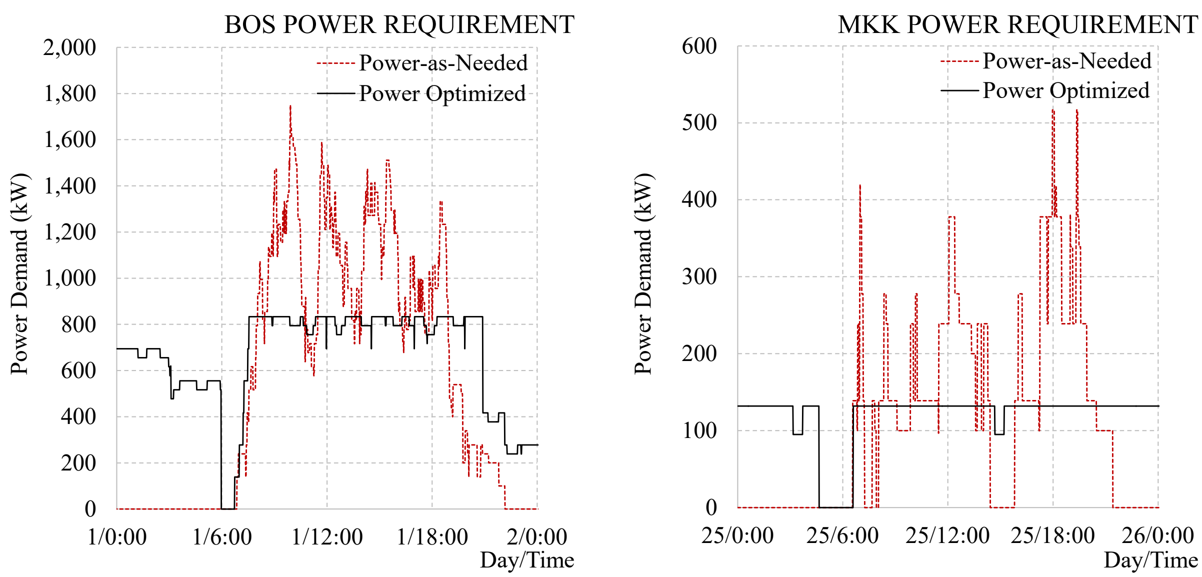


Figure 9: *Power-as-needed* (red) and *power optimized* (black) demand profiles over a day of operations for Cape Air at Boston (left) and for Mokulele Airlines at Molokai (right)

Several salient features can be observed. First, the *power-as-needed* strategy results in a very peaky demand relative to the *power optimized* strategy for the same amount of energy delivered to the aircraft. At Boston airport, the *power-as-needed* strategy leads to four peaks of daily recharge activity at 10am, 12pm, 3.30pm and 6.30pm corresponding to four banks of aircraft arrivals from outer stations. This yields a maximum power demand of 1,789kW. The profile also features a flat zero-power demand from 10pm to 7am when chargers are not used. At Molokai Hoolehua airport, the *power-as-needed* strategy leads to four daily peaks of recharge activity at 7am, 12pm, 6pm, and 7.30pm, with a maximum demand of 517kW. A flat zero-power demand is also observed between 9.30pm and 6.30am. With the *power optimized* strategy, the peak demand is significantly reduced by spreading the 'energy flow' evenly from the grid to the batteries throughout the day and night. At Boston airport, the maximum power demand is reduced by over 53% to 833kW, while it is reduced by 74% to 132kW at Molokai Hoolehua airport.

Next, the busiest airports of Cape Air and Mokulele Airlines are considered. The total amount of energy required over a week of operations and the peak power are recorded at several busy airports and summarized in Table 4 for the *power-as-needed* and *power optimized* strategies. With the *power optimized* strategy, all airports experience significant reductions in peak-power demand, averaging 57% and 61% for Cape Air and Mokulele Airlines respectively. The largest reductions exceed 80% and typically occur at quieter airports where fewer chargers are required and fast charging is not necessary. This is either due to a lack of traffic, which is the case at Hana and Waimea airports, or to extended ground times, which is the case at Hyannis airport (used primarily as a maintenance facility by Cape Air).

Table 4: Peak powers at several busy airports for the *power-as-needed* and *power optimized* strategies

	Location		Energy (kWh)	Peak Demand (kW)		
				Power as Needed	*Power Optimized*	*Change*
C CAPE AIR – NEW ENGLAND NETWORK	ACK	Nantucket, MA	44,970	1,033	417	-60%
	ALB	Albany, NY	14,550	471	132	-68%
	AUG	Augusta, ME	6,623	139	83	-40%
	BOS	Boston, MA	82,812	1,789	833	-53%
	EWB	New Bedford, MA	7,086	417	132	-68%
	HPN	White Plains, NY	3,367	339	118	-65%
	HYA	Hyannis, MA	14,087	756	139	-82%
	LEB	Lebanon, NY	5,032	139	97	-30%
	MSS	Massena, NY	4,947	139	56	-60%
	MVY	Martha's Vineyard, MA	22,639	694	250	-64%
	OGS	Ogdensburg, NY	5,053	139	56	-60%
	PVC	Provincetown, MA	8,507	417	139	-67%
	PVD	Providence, RI	5,653	278	139	-50%
	RKD	Rockland, ME	9,991	239	125	-48%
	RUT	Rutland, VT	4,296	139	42	-70%
					Network Weighted Average	**-57%**

	Location		Energy (kWh)	Peak Demand (kW)		
				Power as Needed	*Power Optimized*	*Change*
MOKULELE – HAWAII NETWORK	HNL	Honolulu, HI	18,263	478	139	-71%
	HNM	Hana, HI	1,103	139	21	-85%
	JHM	Kapalua, HI	19,930	517	264	-49%
	JRF	Kalaeloa, HI	7,021	239	90	-62%
	KOA	Kona, HI	22,518	517	278	-46%
	LUP	Kalaupapa, HI	551	139	35	-75%
	MKK	Hoolehua, HI	19,860	517	132	-74%
	MUE	Waimea-Kohala, HI	2,223	139	21	-85%
	OGG	Kahului, HI	22,841	656	222	-66%
					Network Weighted Average	**-61%**

Considering the peak-power demand and energy used, the electricity prices are computed at each airport and displayed in Table 5. The electricity prices range between 6.7c/kWh (Albany) and 24.5c/kWh (White Plains) in New England, and between 19.4c/kWh (Honolulu) and 54.6c/kWh (Kalaupapa) in Hawaii. This yields average electricity prices of 13.9c/kWh for Cape Air and 29.6c/kWh for Mokulele Airlines, leading to average energy-related operating costs of \$0.28/nm and \$0.75/nm respectively. These costs compare favorably to the energy-related operating costs of conventional fuel-burning commuter aircraft, which are typically between \$1.50/nm and \$2.50/nm (Justin, et al. 2017). Overall, the electricity price reductions obtained with the *power optimized* strategy average 25% for Cape Air and 23% for Mokulele Airlines. This strategy requires a total of 120 batteries and 23 chargers (corresponding to a capital expenditure of \$8.63M) for Cape Air, and a total of 39 batteries and 12 chargers (corresponding to a capital expenditure of \$3.26M) for Mokulele Airlines. Additional details about electricity costs, number of chargers, and number of batteries are provided in Appendix A.

Table 5: Electricity prices at several airports for the *power-as-needed* and *power optimized* strategies

CAPE AIR – NEW ENGLAND NETWORK

Location		Energy (kWh)	Electricity Price ($/kWh)		
			Power as Needed	*Power Optimized*	*Change*
ACK	Nantucket, MA	44,970	0.141	0.128	-9%
ALB	Albany, NY	14,550	0.103	0.067	-35%
AUG	Augusta, ME	6,623	0.098	0.091	-7%
BOS	Boston, MA	82,812	0.208	0.152	-27%
EWB	New Bedford, MA	7,086	0.244	0.173	-29%
HPN	White Plains, NY	3,367	0.504	0.245	-51%
HYA	Hyannis, MA	14,087	0.211	0.155	-27%
LEB	Lebanon, NY	5,032	0.189	0.179	-5%
MSS	Massena, NY	4,947	0.111	0.068	-39%
MVY	Martha's Vineyard, MA	22,639	0.203	0.155	-24%
OGS	Ogdensburg, NY	5,053	0.11	0.068	-38%
PVC	Provincetown, MA	8,507	0.256	0.172	-33%
PVD	Providence, RI	5,653	0.194	0.171	-12%
RKD	Rockland, ME	9,991	0.1	0.089	-11%
RUT	Rutland, VT	4,296	0.246	0.179	-27%
	Network Weighted Average		**0.186**	**0.139**	**-25%**

MOKULELE – HAWAII NETWORK

Location		Energy (kWh)	Electricity Price ($/kWh)		
			Power as Needed	*Power Optimized*	*Change*
HNL	Honolulu, HI	18,263	0.342	0.194	-43%
HNM	Hana, HI	1,103	1.088	0.370	-66%
JHM	Kapalua, HI	19,930	0.402	0.349	-13%
JRF	Kalaeloa, HI	7,021	0.322	0.210	-35%
KOA	Kona, HI	22,518	0.327	0.284	-13%
LUP	Kalaupapa, HI	551	1.316	0.546	-59%
MKK	Hoolehua, HI	19,860	0.428	0.332	-22%
MUE	Waimea-Kohala, HI	2,223	0.367	0.281	-23%
OGG	Kahului, HI	22,841	0.417	0.332	-20%
	Network Weighted Average		**0.387**	**0.296**	**-23%**

6.2 Results for the power-investment optimized strategy

The *power optimized* strategy yields promising electricity cost reductions but fails to account for the significant amount of capital tied-up in the inventory of batteries and chargers. The next set of results concerns the *power-investment optimized* strategy which accounts for the peak power demand, the number of batteries, and the number of chargers when minimizing overall recharge costs. Figure 10 indicates that the *power-investment optimized* strategy yields a peak-power reduction of 46% at Boston airport (from 1,789kW to 972kW) and 57% at Molokai Hoolehua airport (from 517kW to 222kW) compared to the *power-as-needed* strategy. Even though these reductions are less pronounced than with the *power optimized* strategy, significant reductions are still observed compared to the *power-as-needed* strategy.

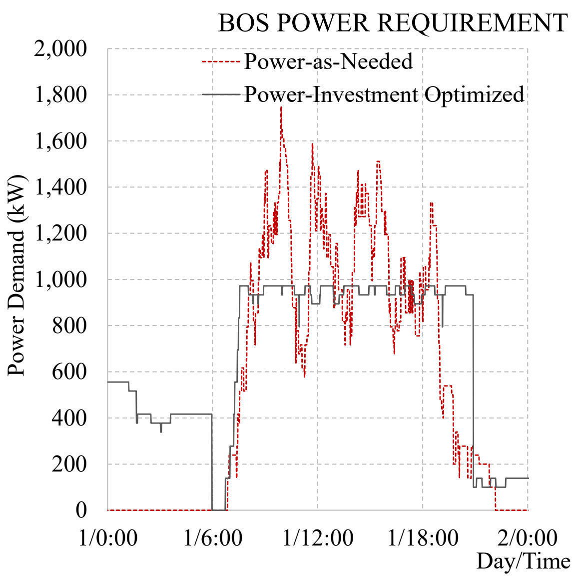


Figure 10: *Power-as-needed* (red) and *power-investment optimized* (dark grey) demand profiles over a day of operations for Cape Air at Boston (left) and for Mokulele Airlines at Molokai (right)

The total amount of energy required and the peak power recorded at the busiest airports during a week of operations are summarized in Table 6 for the *power-as-needed* and *power-investment optimized* strategies. Results indicate an average peak-power reduction of 49% for Cape Air and 54% for Mokulele Airlines. The largest peak power reductions reach 82% at Hyannis airport for Cape Air and 85% at Hana and Waimea-Kohala airports for Mokulele Airlines.

Table 6: Peak powers at several airports served by the two commuter operators for the *power-as-needed* and *power-investment optimized* strategies

CAPE AIR – NEW ENGLAND NETWORK

Location		Energy (kWh)	Peak Demand (kW)		
			Power as Needed	*Power-Investment Optimized*	*Change*
ACK	Nantucket, MA	44,970	1,033	694	-33%
ALB	Albany, NY	14,550	471	132	-68%
AUG	Augusta, ME	6,623	139	83	-40%
BOS	Boston, MA	82,812	1,789	972	-46%
EWB	New Bedford, MA	7,086	417	132	-68%
HPN	White Plains, NY	3,367	339	118	-65%
HYA	Hyannis, MA	14,087	756	139	-82%
LEB	Lebanon, NY	5,032	139	97	-30%
MSS	Massena, NY	4,947	139	56	-60%
MVY	Martha's Vineyard, MA	22,639	694	396	-43%
OGS	Ogdensburg, NY	5,053	139	56	-60%
PVC	Provincetown, MA	8,507	417	139	-67%
PVD	Providence, RI	5,653	278	139	-50%
RKD	Rockland, ME	9,991	239	125	-48%
RUT	Rutland, VT	4,296	139	42	-70%
Network Weighted Average					**-49%**

MOKULELE – HAWAII NETWORK

Location		Energy (kWh)	Peak Demand (kW)		
			Power as Needed	*Power-Investment Optimized*	*Change*
HNL	Honolulu, HI	18,263	478	278	-42%
HNM	Hana, HI	1,103	139	21	-85%
JHM	Kapalua, HI	19,930	517	264	-49%
JRF	Kalaeloa, HI	7,021	239	90	-62%
KOA	Kona, HI	22,518	517	278	-46%
LUP	Kalaupapa, HI	551	139	35	-75%
MKK	Hoolehua, HI	19,860	517	222	-57%
MUE	Waimea-Kohala, HI	2,223	139	21	-85%
OGG	Kahului, HI	22,841	656	222	-66%
Network Weighted Average					**-54%**

The electricity prices for the *power-as-needed* and *power-investment optimized* strategies are compiled in Table 7. The prices range between 6.7c/kWh (Albany) and 24.5c/kWh (White Plains) in New England, and between 21c/kWh (Kalaeloa) and 54.6c/kWh (Kalaupapa) in Hawaii. This results in average electricity prices of 14.5c/kWh for Cape Air and 30.8c/kWh for Mokulele Airlines. In turn, these translate into energy-related operating costs of \$0.29/nm and \$0.78/nm respectively. Again, this compares very favorably to the energy-related operating costs of conventional fuel-powered commuters. Overall, the electricity price reductions obtained with the *power-investment optimized* strategy average 20% for both Cape Air and Mokulele Airlines. This strategy requires a total of 98 batteries and 27 chargers (corresponding to a capital expenditure of \$7.87M) for Cape Air, and a total of 30 batteries and 14 chargers (corresponding to a capital expenditure of \$2.98M) for Mokulele Airlines. This corresponds to a capital expenditure reduction of 8.8% for Cape Air and 8.4% for Mokulele Airlines compared to the *power optimized* strategy. Additional details about electricity costs, number of chargers, and number of batteries are provided in Appendix A.

Table 7: Electricity costs at several busy airports served by the two commuter operators for the *power-as-needed* and *power-investment optimized* strategies

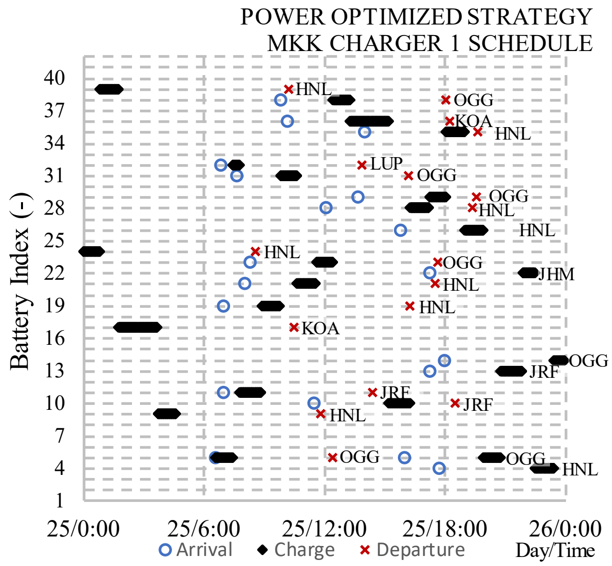
Network	Code	Location	Energy (kWh)	Electricity Price ($/kWh) *Power as Needed*	*Power-Investment Optimized*	*Change*
CAPE AIR – NEW ENGLAND NETWORK	ACK	Nantucket, MA	44,970	0.141	0.133	-6%
	ALB	Albany, NY	14,550	0.103	0.067	-35%
	AUG	Augusta, ME	6,623	0.098	0.091	-7%
	BOS	Boston, MA	82,812	0.208	0.161	-23%
	EWB	New Bedford, MA	7,086	0.244	0.173	-29%
	HPN	White Plains, NY	3,367	0.504	0.245	-51%
	HYA	Hyannis, MA	14,087	0.211	0.155	-27%
	LEB	Lebanon, NY	5,032	0.189	0.179	-6%
	MSS	Massena, NY	4,947	0.111	0.068	-39%
	MVY	Martha's Vineyard, MA	22,639	0.203	0.169	-17%
	OGS	Ogdensburg, NY	5,053	0.11	0.068	-39%
	PVC	Provincetown, MA	8,507	0.256	0.172	-33%
	PVD	Providence, RI	5,653	0.194	0.171	-12%
	RKD	Rockland, ME	9,991	0.1	0.089	-11%
	RUT	Rutland, VT	4,296	0.246	0.179	-27%
	Network Weighted Average			**0.186**	**0.145**	**-20%**

Network	Code	Location	Energy (kWh)	Electricity Price ($/kWh) *Power as Needed*	*Power-Investment Optimized*	*Change*
MOKULELE – HAWAII NETWORK	HNL	Honolulu, HI	18,263	0.342	0.246	-28%
	HNM	Hana, HI	1,103	1.088	0.370	-66%
	JHM	Kapalua, HI	19,930	0.402	0.349	-13%
	JRF	Kalaeloa, HI	7,021	0.322	0.210	-35%
	KOA	Kona, HI	22,518	0.327	0.284	-13%
	LUP	Kalaupapa, HI	551	1.316	0.546	-59%
	MKK	Hoolehua, HI	19,860	0.428	0.351	-18%
	MUE	Waimea-Kohala, HI	2,223	0.367	0.281	-23%
	OGG	Kahului, HI	22,841	0.417	0.332	-20%
	Network Weighted Average			**0.387**	**0.308**	**-20%**

6.3 Recharge schedule generation

At each airport, detailed schedules can be generated for each charger and each battery to help visualize when chargers are used and when batteries are actively recharged. The charts in Figure 11 describe the activities of chargers at the Molokai Hoolehua airport over a day of operations for the *power optimized* and *power-investment optimized* strategies. The *power optimized* strategy requires a single charger (left chart) while the *power-investment optimized* strategy requires two chargers (right charts). The vertical axes list the battery identification indices and the horizontal axes represent time. Each chart indicates when batteries arrive at the airport, when batteries are recharged, when batteries depart the airport, and where batteries are heading next. It is noteworthy that preemption is allowed and thus some battery recharges are split between the two chargers in the *power-investment optimized* strategy. This is for instance the case of the third battery which arrives at Molokai Hoolehua at 7:00am, gets partially recharged on the second charger between

7:36am and 7:59am, completes its recharge on the first charger between 8:00am and 8:42am, and finally departs to Honolulu at 10:15am.



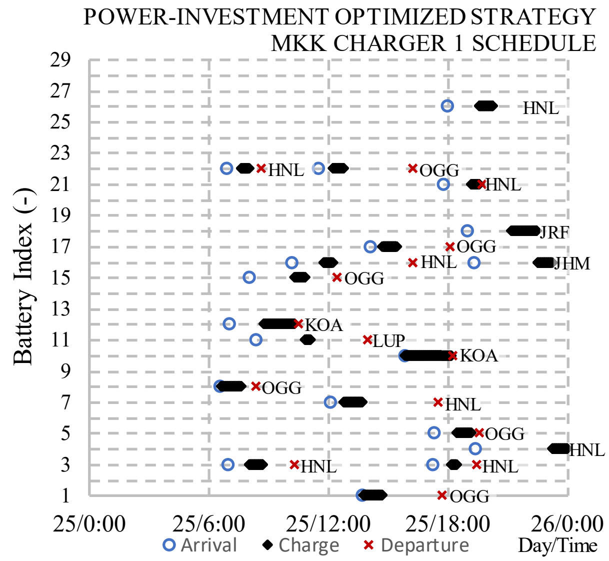
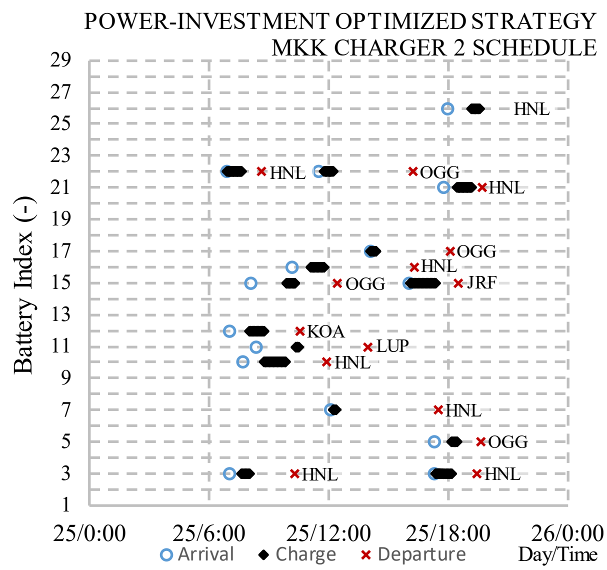


Figure 11: Charger schedules at Molokai airport for the *power optimized* strategy (left, one charger) and for the *power-investment optimized* strategy (right, two chargers). Battery arrivals are indicated with blue circles, recharges are indicated with black diamonds, departure times are denoted by red crosses, and departure destinations are labelled.



The *power-investment optimized* strategy trades some batteries for additional chargers. The average utilization of chargers at each airport therefore decreases, which may provide additional flexibility during periods of irregular operations (i.e. additional recharge capacity is available). In the case of Molokai Hoolehua airport, the *power optimized* strategy requires a single charger used on average 90% of the time, while the *power-investment optimized* strategy requires two chargers used 68% and 39% respectively for an average utilization of 53%. Table 8 summarizes the charger utilization at the busiest airports in the network of Cape Air and Mokulele Airlines. The average utilization reaches 56% (Cape Air) and 62% (Mokulele) for the *power optimized* strategy while it reaches 46% (Cape Air) and 49% (Mokulele) for the *power-investment optimized* strategy.

Table 8: Average charger utilization for the *power-optimized* and the *power-investment optimized* strategies

Network	Location		Average Charger Utilization (%) Power Optimized	Average Charger Utilization (%) Power-Investment Optimized
CAPE AIR – NEW ENGLAND NETWORK	ACK	Nantucket, MA	64%	39%
	ALB	Albany, NY	65%	65%
	AUG	Augusta, ME	47%	47%
	BOS	Boston, MA	59%	51%
	EWB	New Bedford, MA	32%	32%
	HPN	White Plains, NY	17%	17%
	HYA	Hyannis, MA	60%	60%
	LEB	Lebanon, NY	31%	31%
	MSS	Massena, NY	53%	53%
	MVY	Martha's Vineyard, MA	58%	34%
	OGS	Ogdensburg, NY	54%	54%
	PVC	Provincetown, MA	37%	37%
	PVD	Providence, RI	24%	24%
	RKD	Rockland, ME	47%	47%
	RUT	Rutland, VT	61%	61%
	Network Weighted Average		**56%**	**46%**

Network	Location		Average Charger Utilization (%) Power Optimized	Average Charger Utilization (%) Power-Investment Optimized
MOKULELE – HAWAII NETWORK	HNL	Honolulu, HI	78%	39%
	HNM	Hana, HI	32%	32%
	JHM	Kapalua, HI	45%	45%
	JRF	Kalaeloa, HI	46%	46%
	KOA	Kona, HI	48%	48%
	LUP	Kalaupapa, HI	9%	9%
	MKK	Hoolehua, HI	90%	53%
	MUE	Waimea-Kohala, HI	63%	63%
	OGG	Kahului, HI	61%	61%
	Network Weighted Average		**62%**	**49%**

7 Conclusions and Future Work

This paper describes and attempts to address some of the challenges that airlines will face when introducing electric aircraft into their operations. In the quest for lower operating costs, we propose two battery swap and recharge strategies that aim at minimizing energy and capital expenditures. The *power-optimized* and *power-investment optimized* strategies are implemented for the operations of two representative commuter airlines in New England and Hawaii. The *power-optimized* strategy yields peak-power reductions of 57% and 61% respectively, while the *power-investment optimized* strategy yields peak-power reductions of 40% and 54% respectively. In turn, electricity price reductions of 25% and 23% are observed for the *power-optimized* strategy, while 22% and 20% reductions are observed for *the power-investment optimized* strategy.

The proposed approach relies on a network flow representation of the battery swap and recharge problem and leverages algorithms previously developed for machine scheduling problems. The main contributions of this paper are twofold. First, we formulate an optimization problem to address the battery swap and recharge for electric commuter aircraft while preserving routing and schedule integrity. Second, we decompose a large-scale network-wide optimization problem into a set of smaller-scale airport-centric optimization problems enabling an efficient search for feasible recharge schedule solutions. Overall, this research provides significant insights into the economic and operational challenges that will be faced by airlines as well as into the design and operations of the supporting recharge infrastructure.

Subsequent research will introduce uncertainties in arrival times and energy usage in order to better represent typical commercial airline operations and to assess the robustness of the proposed solutions. Future improvements to the current method would include the introduction of variable electricity prices following time-of-use pricing schemes, the possibility to tank electric energy between airports of differing electricity costs, and the ability to optimize the charger power as part of the main optimization problem.

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Appendix A: Airport activity, energy use, electricity price (decomposed into energy cost, power-demand cost, and fixed cost), and infrastructure requirements for the *power-as-needed*, *power optimized* and *power-investment optimized* strategies

Network	Location		Weekly Flights	Power-as-Needed Strategy								Power Optimized Strategy								Power-Investment Optimized Strategy							
				Energy	*Peak Demand*	*Electricity Price*	*Energy Cost*	*Demand Cost*	*Fixed Cost*	*Battery Number*	*Charger Number*	*Energy*	*Peak Demand*	*Electricity Price*	*Energy Cost*	*Demand Cost*	*Fixed Cost*	*Battery Number*	*Charger Number*	*Energy*	*Peak Demand*	*Electricity Price*	*Energy Cost*	*Demand Cost*	*Fixed Cost*	*Battery Number*	*Charger Number*
	(IATA)	*(City, State)*	*(-)*	*(kWh)*	*(kW)*	*($/kWh)*	*($/kWh)*	*($/kWh)*	*($/kWh)*	*(-)*	*(-)*	*(kWh)*	*(kW)*	*($/kWh)*	*($/kWh)*	*($/kWh)*	*($/kWh)*	*(-)*	*(-)*	*(kWh)*	*(kW)*	*($/kWh)*	*($/kWh)*	*($/kWh)*	*($/kWh)*	*(-)*	*(-)*
CAPE AIR – NEW ENGLAND NETWORK	ACK	Nantucket MA	1702	40,347	1,033	$0.141	$0.120	$0.020	$0.001	7	8	44,970	417	$0.128	$0.12	$0.007	$0.001	20	3	44,970	694	$0.133	$0.120	$0.012	$0.001	8	5
	ALB	Albany NY	256	14,810	417	$0.103	$0.043	$0.054	$0.007	5	3	14,550	132	$0.067	$0.043	$0.017	$0.007	5	1	14,550	132	$0.067	$0.043	$0.017	$0.007	5	1
	AUG	Augusta ME	122	6,653	139	$0.098	$0.075	$0.018	$0.004	3	1	6,623	83	$0.091	$0.075	$0.011	$0.005	2	1	6,623	83	$0.091	$0.075	$0.011	$0.005	2	1
	BOS	Boston MA	2397	80,800	1,789	$0.208	$0.095	$0.113	$0.001	17	14	82,812	833	$0.152	$0.097	$0.055	$0.000	29	6	82,812	972	$0.161	$0.097	$0.064	$0.000	22	7
	EWB	New Bedford MA	317	7,922	417	$0.244	$0.122	$0.111	$0.010	8	3	7,086	132	$0.173	$0.122	$0.039	$0.012	4	1	7,086	132	$0.173	$0.122	$0.039	$0.012	4	1
	HPN	White Plains NY	252	4,118	339	$0.504	$0.041	$0.454	$0.009	1	3	3,367	118	$0.245	$0.041	$0.194	$0.010	3	1	3,367	118	$0.245	$0.041	$0.194	$0.010	3	1
	HYA	Hyannis MA	829	19,603	756	$0.211	$0.117	$0.084	$0.010	41	6	14,087	139	$0.155	$0.128	$0.021	$0.006	26	1	14,087	139	$0.155	$0.128	$0.021	$0.006	26	1
	LEB	Lebanon NY	174	5,309	139	$0.189	$0.146	$0.041	$0.002	5	1	5,032	97	$0.179	$0.146	$0.03	$0.003	3	1	5,032	97	$0.179	$0.146	$0.030	$0.003	3	1
	MSS	Massena NY	96	4,896	139	$0.111	$0.037	$0.054	$0.021	3	1	4,947	56	$0.068	$0.039	$0.027	$0.002	2	1	4,947	56	$0.068	$0.039	$0.027	$0.002	2	1
	MVY	Martha's Vineyard, MA	834	19,905	694	$0.203	$0.117	$0.076	$0.010	5	5	22,639	250	$0.155	$0.128	$0.023	$0.004	10	2	22,639	396	$0.169	$0.128	$0.037	$0.004	7	3
	OGS	Ogdensburg NY	87	4,974	139	$0.110	$0.037	$0.053	$0.020	3	1	5,053	56	$0.068	$0.039	$0.026	$0.002	2	1	5,053	56	$0.068	$0.039	$0.026	$0.002	2	1
	PVC	Provincetown MA	417	7,515	417	$0.256	$0.128	$0.117	$0.011	5	3	8,507	139	$0.172	$0.128	$0.034	$0.010	5	1	8,507	139	$0.172	$0.128	$0.034	$0.010	5	1
	PVD	Providence RI	230	6,949	278	$0.194	$0.093	$0.074	$0.028	6	2	5,653	139	$0.171	$0.116	$0.05	$0.006	4	1	5,653	139	$0.171	$0.116	$0.050	$0.006	4	1
	RKD	Rockland ME	182	9,635	239	$0.1	$0.075	$0.021	$0.003	6	2	9,991	125	$0.089	$0.075	$0.011	$0.003	3	1	9,991	125	$0.089	$0.075	$0.011	$0.003	3	1
	RUT	Rutland VT	91	4,583	139	$0.246	$0.145	$0.099	$0.002	3	1	4,296	42	$0.179	$0.145	$0.032	$0.002	2	1	4,296	42	$0.179	$0.145	$0.032	$0.002	2	1
	Network-Wide Total / Weighted Average		***7,986***	***238,018***	***995***	***$0.186***	***$0.099***	***$0.082***	***$0.005***	***118***	***54***	***239,613***	***432***	***$0.139***	***$0.102***	***$0.034***	***$0.003***	***120***	***23***	***239,613***	***546***	***$0.145***	***$0.102***	***$0.040***	***$0.003***	***98***	***27***
	(Variation)												***(-57%)***	***(-25%)***	***(+3%)***	***(-59%)***	***(-46%)***	***(+2%)***	***(-57%)***		***(-49%)***	***(-20%)***	***(+3%)***	***(-52%)***	***(-50%)***	***(-17%)***	***(-50%)***
MOKULELE – HAWAIIAN NETWORK	HNL	Honolulu HI	525	14,937	478	$0.342	$0.151	$0.179	$0.012	7	4	18,263	139	$0.194	$0.172	$0.020	$0.001	8	1	18,263	278	$0.246	$0.151	$0.085	$0.009	4	2
	HNM	Hana HI	61	438	139	$1.088	$0.307	$0.731	$0.050	0	1	1,103	21	$0.370	$0.307	$0.044	$0.020	1	1	1,103	21	$0.370	$0.307	$0.044	$0.020	1	1
	JHM	Kapalua HI	486	20,893	517	$0.402	$0.280	$0.114	$0.008	4	4	19,930	264	$0.349	$0.280	$0.061	$0.008	4	2	19,930	264	$0.349	$0.280	$0.061	$0.008	4	2
	JRF	Kalaeloa HI	178	8,876	239	$0.322	$0.151	$0.151	$0.019	3	2	7,021	90	$0.210	$0.172	$0.035	$0.003	2	1	7,021	90	$0.210	$0.172	$0.035	$0.003	2	1
	KOA	Kona HI	512	23,442	517	$0.327	$0.221	$0.099	$0.008	7	4	22,518	278	$0.284	$0.221	$0.055	$0.008	5	2	22,518	278	$0.284	$0.221	$0.055	$0.008	5	2
	LUP	Kalaupapa HI	26	690	139	$1.316	$0.298	$0.835	$0.184	1	1	551	35	$0.546	$0.372	$0.145	$0.028	1	1	551	35	$0.546	$0.372	$0.145	$0.028	1	1
	MKK	Hoolehua HI	669	17,436	517	$0.428	$0.298	$0.123	$0.007	3	4	19,860	132	$0.332	$0.298	$0.028	$0.006	10	1	19,860	222	$0.351	$0.298	$0.046	$0.006	5	2
	MUE	Waimea-Kohala, HI	61	2,985	139	$0.367	$0.250	$0.110	$0.007	3	1	2,223	21	$0.281	$0.250	$0.022	$0.009	2	1	2,223	21	$0.281	$0.250	$0.022	$0.009	2	1
	OGG	Kahului HI	660	23,180	656	$0.417	$0.280	$0.130	$0.007	8	5	22,841	222	$0.332	$0.280	$0.045	$0.007	6	2	22,841	222	$0.332	$0.280	$0.045	$0.007	6	2
	Network-Wide Total / Weighted Average		***3,178***	***112,875***	***504***	***$0.387***	***$0.243***	***$0.134***	***$0.010***	***36***	***26***	***114,310***	***197***	***$0.296***	***$0.248***	***$0.042***	***$0.006***	***39***	***12***	***114,310***	***234***	***$0.308***	***$0.244***	***$0.056***	***$0.008***	***30***	***14***
	(Variation)												***(-61%)***	***(-23%)***	***(+2%)***	***(-68%)***	***(-36%)***	***(+8%)***	***(-54%)***		***(-54%)***	***(-20%)***	***(+1%)***	***(-58%)***	***(-24%)***	***(-17%)***	***(-46%)***