# Semi-Continuous Robust Approach for Strategic Infrastructure Planning of Reverse Production Systems

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Semi-Continuous Robust Approach for Strategic
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"*Time is running fast as always, please make the best use of every second in our life.*"

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# SUMMARY

Growing attention is being paid to the problem of efficiently designing and operating reverse supply chain systems to handle the return flows of production wastes, packaging, and end-of-life products. Because uncertainty plays a significant role in all fields of decision-making, solution methodologies for determining the strategic infrastructure of reverse production systems under uncertainty are required. This dissertation presents innovative optimization algorithms for designing a robust network infrastructure when uncertainty affects the outcomes of the decisions. In our context, robustness is defined as minimizing the maximum regret under all realizations of the uncertain parameters.

These new algorithms can be effectively used in designing supply chain network infrastructure when the joint probability distributions of key parameters are unknown. These algorithms only require information on potential ranges and possible discrete values of uncertain parameters, which often are available in practice. These algorithms extend the state of the art in robust optimization, both in the structure of the problems they address and the size of the formulations. An algorithm for dealing with the problem with correlated uncertain parameters is also presented.

Case studies in reverse production system infrastructure design are presented. The approach is generalizable to the robust design of network supply chain systems with reverse production systems as one of their subsystems. The resultant system will tend to be more financially and operationally viable if properly planned, since even with the least favorable realization of the parameters, the system may still perform close to optimal levels.

# CHAPTER I

# INTRODUCTION

## 1.1 Introduction

Growing attention is being given to the problem of efficiently designing and operating reverse supply chain systems to handle the return flows of production wastes, packaging, and end-of-life products. Figure 1.1 is an abstraction of forward and reverse production systems (RPS). The overall cycle shows that in the forward direction the manufactured value increases, but in the reverse direction the manufactured value is reduced, as the value-added operations are undone. The driving forces of recycling are the recovery of manufactured value, in a form in which reuse is possible, and the avoidance of waste disposal costs. These benefits must be balanced against several costs associated with transporting, sorting, inspecting, de-manufacturing, refurbishing, and material recycling.

The motivation for recycling is growing; however the information that exists for these new reverse supply chains is limited. How many units of obsolete computers are in Atlanta and other cities? What will be the quality (broken, reusable, etc) of the resources collected? What is the current selling price of the specific material in the market? Where is the demand point for the specific material? How many units are in demand? Thus uncertainty should definitely be taken into account, but there is no known underlying probability distribution for each uncertain parameter, much less joint probability distributions for the entire set of uncertain parameters. For these reasons, development of an approach to design the strategic infrastructure of reverse production systems under uncertainty, when the information on the uncertainty is limited, is critical to support effective business and government decision making.

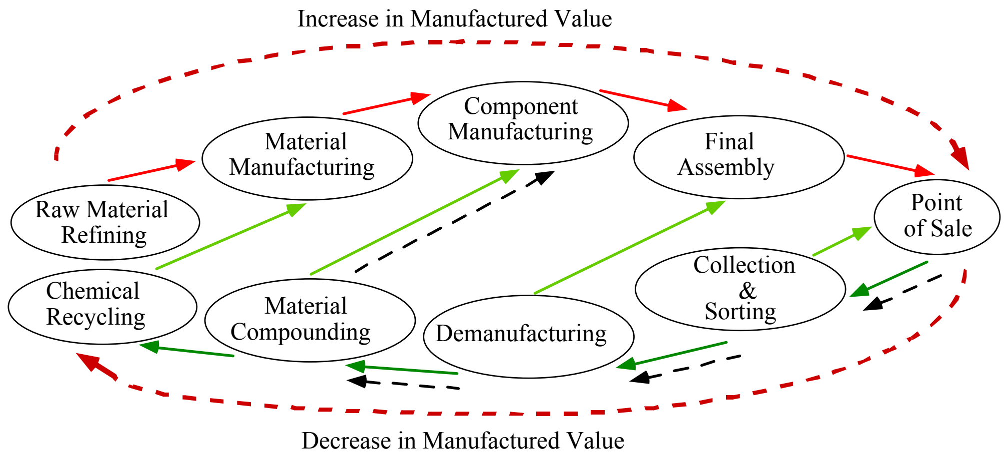


**Figure 1.1 Material Flows in Forward and Reverse Production Systems (from Ammons and Realff, 1999)**

The research develops several analytical approaches and algorithms for determining the robust strategic infrastructure of the supply chain network including the RPS network. Initially, the approach was developed for finding an optimal robust solution that minimizes the maximum regret from optimal objective function value over all considered scenarios (scenario based deviation robustness) by Ammons, Realff, and Newton (2000). Extensions are developed in this dissertation with the purpose of solving the scenario based robust optimization problem when the numbers of scenarios considered are large but finite. These extensions are 1) the development of the scenario-relaxation algorithm and 2) the use of the accelerated Benders' decomposition algorithm introduced by Ahmed (2003).

The approach is further extended to the development of a semi-continuous robust algorithm which solves the robust optimization problem when each parameter takes its value from real compact intervals or some specific discrete values. The assumption of independence among uncertain parameters is required for the initial version of the semi-continuous robust algorithm. The next contribution of this dissertation is the development of parameter-space transformation algorithm. By applying this algorithm together with the semi-continuous robust algorithm, the semi-continuous robust optimization problem can be solved without the parameter independency assumption.

The semi-continuous robust algorithm can be effectively used in designing network infrastructure when the joint probability distributions of key parameters are unknown. The algorithm only requires the information on potential ranges and possible discrete values of uncertain parameters, which often are available in practice. The solution from this algorithm may not be optimal for any given set of potential future conditions, but instead will provide a solution with a predicted objective function value close to the optimal predicted objective function value no matter what values the uncertain parameters take from among their potential values.

## 1.2 Problem Statement

The infrastructure planning is one of the critical strategic decisions for designing effective supply chain systems. Many questions need to be answered when designing the strategic infrastructure of the reverse supply chain systems. Some of these questions include:

- Where are the locations to open the collection centers and processing centers?
- What types of processes to be installed at each specific location?
- How much money should be invested in equipment at each specific location?
- How much money should be invested in labor at each specific location?
- What type of materials should be collected at each specific location?
- What type of transportation modes should be used between each pair of locations?

Given the answers to these strategic questions, many tactical questions need to be answered. Some of these questions include:

- How many units of the specific material should be collected at each collection center in the specific time period?
- What should be done with the collected materials?
- How many units of the specific material should be transported between each pair of locations using which transportation mode?
- How many units of the specific material should be sold to each specific customer?

In designing the reverse production systems infrastructure, the decision makers must deal with additional complications arising from uncertainty such as:

- Uncertainty in the supply of each material type at each source.
- Uncertainty in the demand of each material type for each customer.
- Uncertainty in prices of each material type.

- Uncertainty in the process reliability of each process.
- Uncertainty in the maximum process capacity of each machine type.
- Uncertainty in the buying cost for each specific machine type.
- Uncertainty in the transportation cost rate.

If long-term *perfect* forecasts of all model parameters are provided, both strategic and tactical decisions can be made together by solving a mixed integer liner programming problem. The mixed integer linear programming model assuming *perfect* information will be referred as the **RPS model** in this dissertation.

Unfortunately, most of the uncertain parameters are not known precisely and cannot be accurately predicted. As a result, the decision makers are unable to make a *perfect* decision that would be *best* in all circumstances. They would, therefore, want to assess the benefits and losses associated with each potential decision in each situation. The strategic decisions are then made without perfect information for model parameters' uncertain values. The tactical decisions are made later once the strategic decisions have been made and the values of uncertain parameters are realized. There are many ways to make these decisions, and one such approach is to find a robust approach for planning the strategic infrastructure.

Robustness in solution can be measured in several ways (Kouvelis and Yu, 1997). One approach is to determine a solution that corresponds to the objective function value which is close by percentage or absolute measure to the best objective function value over a wide range of possible uncertain parameter values. This dissertation proposes several ways to make strategic decisions for the reverse production systems under uncertainty in parameters' values. As explained in more detail in Chapter III, the planning is done in the robust manner so as to minimize the maximum regret between the optimal objective function value and the robust objective function value over all possibilities of parameters' values. This definition of robustness will be referred as *deviation robustness* in this dissertation. The next section overviews solution methodologies for finding the robust optimal solution for strategic decisions of reverse production systems that are developed in this dissertation.

### 1.3 Dissertation Overview

This dissertation develops robust approaches for determining an optimal deviation robust solution of a mixed integer linear programming model for supply chain problems where uncertainty exists in parameters' values. The approaches are validated using several case studies and examples. With these methods, decision makers are able to make robust strategic infrastructure decisions for supply chain systems under uncertainty in model parameters' values when the joint probability distributions of key parameters are unknown. The approach is developed throughout this dissertation and presented in the following chapters.

Chapter II is a review of the relevant literature to this dissertation. This literature can be classified into three main areas: recycling literature, robust optimization literature, and the bi-level linear programming literature.

Chapter III covers a basic mixed integer linear programming model for reverse production systems (RPS model) and an optimization approach for finding an optimal deviation robust solution using the scenario based robust optimization method.

Chapter IV covers the development of a scenario relaxation algorithm and the use of an accelerated Benders' decomposition algorithm (Ahmed, 2003) for solving the scenario based robust optimization problem when the number of considered scenarios are large but finite.

Chapter V presents a case study on planning the e-scrap reverse production system under uncertainty in the state of Georgia using the methodologies developed in Chapter IV. This chapter shows the significant reduction of computational solution time using the proposed methods compared to the runtime required by the direct method.

Chapter VI covers the development of the semi-continuous robust algorithm for solving the robust optimization problem when each model parameter can take its value from real compact intervals or some specific discrete values. This algorithm requires an assumption of independence among all model parameters. This chapter also outlines theoretical results and methodologies required to solve the problem effectively. It proves that the optimal solution may not depend only on the endpoints of the range of parameters. Several example problems are presented.

Chapter VII provides several problems demonstrating the design of a robust strategic reverse production system infrastructure using the semi-continuous robust algorithm developed in Chapter VI.

Chapter VIII covers the development of a parameter space transformation algorithm, which can be used together with the semi-continuous robust algorithm for solving the semi-continuous robust RPS problem when correlations exist among model parameters. This approach does significantly rely on available information on parametric variations.

In Chapter IX, a summary and the contributions of the results in this dissertation are documented. Additionally, potential future extensions of the methodologies of this dissertation are discussed.

# CHAPTER II

# REVIEW OF RELEVANT LITERATURE

## 2.1 Introduction

This chapter reviews the literature relevant to the development of the work in this dissertation. While the work in this dissertation is built upon many sources of knowledge, the fundamentals of this work are constructed by the following three main areas: Reverse Production System, Robust Optimization and Bi-level Programming. The literature in all these three areas is reviewed in the following section in this chapter. Section 2.2 reviews the literature in the area of reverse logistics network design. Section 2.3 reviews the literature in the area of robust optimization in supply chain planning and Section 2.4 reviews the literature on bi-level optimization.

## 2.2 Literature Review of Reverse Logistics Network Design Models

The design and analysis of reuse and recycling systems has been a topic of interest for some period of time. Their brief history reflects the growth of interest in environmentally conscious manufacturing and the advent of interest in industrial ecology (Graedel and Allenby, 1995). Logistics network design is one of the areas within the field of reverse logistics for which evidence is available from a relatively wide collection of case studies. In several of these studies dedicated optimization models have been developed that rely on extensions and modifications of traditional facility location models. Flapper (1995 and 1996) and Fleishmann (1997, 2000, 2001) provide overviews of reverse production system models and their application to recycling system analysis. Gungor and Gupta (1999) give a state-of-the-art survey of the academic literature on environmentally conscious manufacturing and product recovery.

Specific product and material recycling systems that have been analyzed include carpet (Newton, 2000; Ammons and Realff, 1999), copying machines (Thierry, Salomom, Nunen and Wassenhove, 1995; Thierry, 1997; and Krikke, 1998), monitors (Krikke, Harten and Schuur, 1999), cameras (Nagel, 1997), paper (Huttunen, 1996), iron (Russell and William, 1974), steel (Spengler *et al.*, 1997), electronics (Fleischmann *et al.*, 2001), cell phone (Jayaraman *et al.*, 1999), reusable packaging (Kroon and Vrijens, 1995) and sand (Barros and Scholten, 1998).

Kroon and Vrijens (1995) address the design of a closed-loop deposit based system for collapsible plastic containers that can be rented as secondary packaging material. The system involves multiple actors, including a central agency who owns a pool of reusable containers and a logistics service provider who is responsible for storing, delivering, and collecting the empty containers. For the latter operations a set of depots needs to be located. The authors document how this issue may be addressed by means of a standard warehouse location model. In addition, they emphasize that the overall network design problem is characterized by the interaction between the various parties involved and their respective roles. Depot location, pool size, and payment structures all have an important impact on the system's performance as a whole and its competitiveness with respect to traditional "one-way" packaging.

Spengler *et al*. (1997) have examined recycling networks for industrial by-products in the German steel industry. Steel production gives rise to a substantial volume of residuals that have to be recycled in order to comply with environmental regulation and to reduce disposal costs. For this purpose, different processing technologies are available. The authors analyze which recycling processes or process chains to install at which locations at which capacity level in order to minimize overall costs. They propose a modified mixed integer linear program warehouse location model. The model formulation allows for an arbitrary number of network levels, corresponding to individual processing steps, and an arbitrary number of end products, linked to alternative processing options. Analyzing multiple scenarios the authors emphasize the need for industry-wide co-operation to achieve sufficient capacity utilizations. Moreover, they conclude that recycling targets and disposal bans may entail severe investment burdens for the industry and should therefore be handled with care.

Barros *et al*. (1998) provide an example of a material recycling network, namely sand recycling from construction waste. In view of a substantial annual volume of sand landfilled on the one hand and the need for sand in large infrastructure projects, such as road construction on the other hand a consortium of waste processing companies in The Netherlands is investigating opportunities for a nation-wide sand-recycling network. Pollution is a major issue in this context. This means that sand needs to be analyzed and possibly cleaned before being reused. Cleaning of polluted sand requires the installation of fairly expensive treatment facilities. In addition, regional depots need to be set up for inspection and storage. The authors develop a tailored multi-level capacitated facility location model for this network design problem. In their analysis, they emphasize the need for a robust network structure since both supply and demand involve significant uncertainties. Therefore, multiple scenarios are evaluated, of which the solution with the best worst-case behavior is selected.

Jayaraman *et al.* (1999) have analyzed the logistics network of an electronic equipment remanufacturing company in the USA. The activities considered include core collection, remanufacturing, and distribution of remanufactured products, where delivery and demand customers do not necessarily coincide. In this setting, the optimal number and locations of remanufacturing facilities and the number of cores collected are sought, considering investment, transportation, processing, and storage costs. The authors show that this network design problem can be modeled as a standard multi-product capacitated warehouse location mixed integer linear program. In this formulation, limited core supply acts as a capacity restriction to the overall level of operation. The authors highlight that managing this "capacity" which is crucial for the system's performance, requires different approaches than in a traditional production distribution network. Rather than considering technical capacity extension options, appropriate marketing instruments are needed to assure a sufficient core supply.

Fleischmann *et al.* (2000 and 2001) focus on the consequences for OEMs of adding product recovery operations to an existing production-distribution network. A fairly general mixed integer linear program facility location model is presented that encompasses both "forward" and "reverse" product flows.

One aspect that is worth considering concerns the issue of uncertainty in the reverse chain. Ammons and Realff (1999) illustrate the discrete robust strategic multi-period network design model for the reverse production system for carpet recycling. They are the first group to provide the step in this direction. They handle uncertainty in the reverse chain by using scenario-based robust optimization to find the solution that minimizes the maximum regret from optimality for each scenario.

Newton (2000) extends the methodology of Ammons and Realff (1999) in strategic infrastructure planning for carpet recycling to generate the solution that minimizes the maximum regret from optimality for each scenario when each random parameter takes value from a real compact interval. There are some limitations for this approach, which are described in the next section.

Listes and Dekker (2001) explicitly take the uncertainty issue into account in their model approach. They propose a multi-stage stochastic programming model where location decisions need to be taken on the basis of imperfect information on supply and demand while subsequent processing and transportation decisions are based on the actual volumes. The model maximizes the expected performance for a set of scenarios with given probabilities. The authors emphasize that the solution needs not to be optimal for any individual scenario and hence that this approach is more powerful than simple scenario analyes.

Similar to the approach presented in Newton (2000), Ammons and Realff (1999), and Spengler (1997), this dissertation defines a location/allocation model to determine the number, size, and location of facilities and demanufacturing plants. Materials to be recycled are generated and can be transformed to different states by processes. These materials may then be further processed or sold. The objective of the model is to maximize profit of recycled and reused materials. The RPS model in Chapter III presents a general framework similar to that of Newton (2000), Ammons and Realff (1999), and

Spengler (1997) and extends the model to suit electronic recycling system and to include planning over multiple periods. Table 2.1 contains a summary of reverse production system literature.

**Table 2.1 Summary of Reverse Production System Literature**

| **Authors** | **Products** | **Philosophy** | **Heuristic Method** | **Mathematical Model** | **Multi-period Model** | **MILP** | **Inventory Model** | **Uncertainty** |
|---|---|---|---|---|---|---|---|---|
| **Russel and Vaughan (1974)** | **Iron** | × | | × | | | | |
| **Gupta and Taleb (1994)** | | | | × | | | | |
| **Kroon and Vrijens (1995)** | **Reusable packaging** | × | | × | | × | | |
| **Thierry *et al.* (1995)** | **Copy Machine** | × | | | | | | |
| **Barros *et al.* (1996)** | **Sand** | × | | × | × | × | | × |
| **Flapper (1996)** | | × | | | | | | |
| **Huttunen (1996)** | | × | × | | | | | |
| **Nagel (1997)** | **Camera** | × | | | | | | |
| **Spengler *et al.* (1997)** | **Steel** | × | | × | × | × | | |
| **Lave *et al.* (1998)** | **Carpet** | × | | | | | | |
| **Ammons and Realff (1999)** | **Carpet** | × | | × | × | × | | × |
| **Jayaraman *et al.* (1999)** | **Electronics** | × | | × | | × | | |
| **Krikke (1999)** | **Monitor** | × | | × | | | | |
| **Louwers *et al.* (1999)** | **Carpet** | × | | × | × | × | | |
| **Newton (2000)** | **Carpet** | × | | × | × | × | | × |
| **Fleishmann (2000 and 2001)** | **Electronic and Paper** | × | | × | | × | × | |
| **Listes and Dekker (2001)** | | | | × | | × | | × |
| **Assavapokee (2004)** | **Electronics** | × | × | × | × | × | | × |

## 2.3 Literature Review of Robust Optimization in Supply Chain Planning

Uncertainty in parameter values is a basic structural feature that decision makers in all fields of study must confront. The way to handle uncertainty, and to make decisions under uncertainty, is to accept uncertainty, make a strong effort to understand it, and finally, make it part of the decision making process.

Deterministic optimization approaches feed one instance of the input data to a decision model and with the use of one or multiple objectives generate the mathematically optimal decisions. This approach either completely ignores uncertainty or uses historical data to forecast the future. The selected instance of the input data represents the most likely estimator of the realization of the data in the future. A major weakness of deterministic optimization can be its inability to account for plausible data instances other than the most likely one used to generate the optimal decision. Even though that decision is optimal for the most likely future scenario, it may lead to poor performance of the design when a future realization is different than the forecasted most likely one.

One of the ways to handle uncertainty is stochastic optimization. The stochastic optimization approach recognizes the presence of multiple data instances that may be potentially realized in the future. However, before feeding the data instances to the decision model, it requires explicit information for the probability values, which may not be available or may be difficult to obtain. Even if all probability data are available, the typical decision model will attempt to generate a decision that maximizes (or minimizes) an expected performance measure, where the expectation is taken over the assumed probability distribution, which may not reflect the decision maker's true utility function that may be risk averse. The requirement for a specified probability distribution makes the use of stochastic optimization a challenge to implement when the knowledge of parameters is not available.

Another way to handle uncertainty is robust optimization. The aim of this approach is to produce decisions that will have a reasonable objective function value under any likely input data scenario to the decision model over a pre-specified planning horizon. Different criteria can be used to select among robust decisions. One possible criterion is the mini-max regret criterion. The first step is to compute the "regret" associated with each combination of decision and input data scenario. "Regret" can be defined as the difference between the optimal objective value and robust objective value for each input data scenario. The mini-max criterion is then applied to the regret values, so as to choose the decision with the least maximum regret. A solution to a mathematical program is *robust* with respect to optimality if it remains close to optimality for any input data scenario to the model.

We divide the robust optimization for the application of the supply chain models into two basic categories: regret models and variability models. The "regret" of a scenario is measured as the closeness between the optimal objective function value for that scenario and the objective function value of the chosen solution for that scenario. Kouvelis and Yu (1997) define "close" to the optimal solution in several different ways. They define two regret criteria for robustness. The *robust deviation decision* is the decision that exhibits the best worst-case deviation from optimality. In other words, the robust deviation solution is one that minimizes the maximum regret over all possible realizations of the parameters in the model. This is the robustness definition used in this dissertation.

The *robust relative decision* is the decision that exhibits the best worst-case percentage deviation from optimality.

There is also a definition of *absolute robustness* presented by Kouvelis and Yu (1997). Absolute robustness evaluates the objective function value in each scenario without reference to the best possible decision that could have been made in that scenario. Absolute robustness defines a solution that minimizes the maximum total costs. This would be appropriate for risk adverse or highly competitive environments where even the worst case must guarantee a certain level of performance.

The robust deviation measure was chosen in this dissertation for two reasons. First, it incorporates more information in the solution than absolute robustness and so is believed to provide a better answer. Second, robust deviation places more of an emphasis on scenarios that tend to produce large objective values than the other two measures. The use of the relative robustness measure will result in more opportunity lost than the robust deviation measure. This is because scenarios that would tend to have very small positive or negative objective functions tend to totally dominate solutions using a relative robustness measure.

The work of Kouvelis and Yu made use of scenarios for determining robustness. The approach of using scenarios to capture uncertainty can also be found in the stochastic optimization literature. Scenarios are decided upon and weights are placed on the realization of the scenarios. The final solution must satisfy each scenario and minimize some objective based on the difference between the proposed solution and optimal solution. In this respect the concept is close to robustness approach used in this dissertation.

Ammons and Realff (1999) apply the definition of deviation robustness to the application of carpet recycling. They introduced a mixed integer linear programming model and solved for the robust infrastructure design for carpet recycling problems.

Newton (2000) introduces a continuous robust approach using the deviation robustness definition. Instead of using discrete scenarios to capture uncertainty, he introduces the innovative idea of using the information from parameter possible ranges for making robust infrastructure decision of the reverse logistic problems. This approach has some limitations when it is applied on some types of uncertain parameters. This approach cannot handle the uncertainty when any coefficient of a continuous variable in the model is random and cannot handle the uncertainty corresponding to the combination of discrete scenarios and continuous range scenarios. This approach also requires the assumption of independent model parameters. This approach also requires the assumption that there always exists a feasible robust infrastructure solution for the problem, which is not always true in general.

Gutierrez, Kouvelis, and Kurawarwala (1996) apply a different robustness approach. Instead of addressing the worst case, they require a robust network design to be within *p*% of the optimal solution for any realizable scenarios. Therefore, they in effect add a constraint to their model to ensure robustness. They solve the model by modifying Benders' decomposition algorithm to use cuts from one master problem on all scenarios.

An alternative definition of robustness is to find a near-optimal solution that is not overly sensitive to any specific realization of the uncertainty (Bai, Carpenter and Mulvey, 1997). The goal is to minimize expected cost (maximize expected profit) and to reduce the variability over all possible scenarios. Thus, these robust optimization models include a measure of variability rather than regret. Variability can be measured by variance (Hodder and Dincer, 1986; Mulvey, Vanderbei and Zenios, 1995; Bok, Lee, and Park, 1998) or by standard deviation (Goetschalckx, *et al*., 2001), both of which make the objective function a nonlinear function. Both methods also assume symmetric risk, so that it is equally bad for costs to be below or above average. Several other measures of variability have been used, including the von Neumann-Morganstern expected utility function (Bai, Carpenter and Mulvey, 1997) and the upper partial mean (Ahmed and Sahinidis, 1998), to allow asymmetry, but these functions are often hard to compute. Additionally, when coefficients in a model are uncertain, the functional constraints may not necessarily be satisfied for all scenarios. In such a situation, it is convenient to introduce additional variables that represent the slack or surplus in the functional constraints. These variables, called recourse variables, are included in the objective function as an infeasibility penalty (Mulvey, Vanderbei and Zenios, 1995; Yu and Li, 2000). We also discuss the variability models in more detail below.

Hodder and Dincer (1986) present a model for international plant location and financing decisions under uncertainty. They model risk aversion via a mean-variance objective function of firm profit and consider fixed cost and net revenue uncertainty. The resulting model is a quadratic mixed integer program. They show that a multifactor approach can transform the problem into one that can be easier to solve.

Mulvey, Vanderbei, and Zenios (1995) were the first to present robust optimization as the integration of goal programming formulations with a scenario-based description of the problem data. They define *solution robustness* as the case when the optimal overall solution is near optimal for every possible demand scenarios. They define *model* *robustness* as the case when the optimal overall solution is almost feasible for all scenarios. They add norms, such as variance or utility functions, to the objective function to encourage solution robustness. They also add a feasibility penalty function to the objective function to encourage model robustness. The feasibility penalty term is a function of the demand slack. A penalty is assessed when the slack holds the positive or negative value, so the penalty applies when the model is infeasible, and when there is excess capacity. Malcolm and Zenios (1994) apply the robust model of Mulvey, Vanderbei, and Zenios (1995) to a power system capacity expansion problem with demand uncertainty.

Bok, Lee, and Park (1998) define a quadratic objective function to maximize the expected net profit with penalties for the expected deviation of profit and excess capacity. The net present value of profit is calculated from sales revenues, material costs, processing costs, and capacity expansion costs. The scenarios consist of different demand levels, each with an associated probability. They use Benders' decomposition to solve their two-stage stochastic programs.

Yu and Li (2000) reformulate the robust optimization model of Mulvey, Vanderbei, and Zenios (1995) into a linear program that requires only half as many variables. They demonstrate their model with four economic scenarios with different demand and production cost. The main limitation to this formulation is that it can only applied to linear models.

Bai, Carpenter, and Mulvey (1997) advocate using the von Neumann-Morganstern expected utility model (Keeney and Raffia, 1976) over mean-variance robust models as it presents a more general approach for handling risk aversion. Additionally, the model captures asymmetries in the random variable distributions and is easier to expand to multi-period planning. The disadvantage of the expected utility model is that the decision makers must decide upon an appropriate level of risk tolerance.

Ahmed and Sahinidis (1998) use the definition of robustness of Mulvey, Vanderbei, and Zenios (1995), but propose alternative formulations to the mean plus variance objective function. They argue against using variance because it penalizes cost below the mean and it also introduces nonlinearities to the formulation. They propose the upper partial mean (UPM) of the recourse costs as the measure of variability. The upper partial mean is the positive deviation of a scenario's cost from the expected cost. The key advantage of UPM is that it does not require the *a priori* specification of a target level for variance and is therefore more flexible. The formulation limits the number of expansions allowed and the total capital investment.

Goetschalckx, *et al.* (2001) defines a *flexible* configuration as a "configuration whose profit or total cost does not change much when parameters such as capacities and demand change." Their definition of a *robust* configuration is "a configuration whose objective function value deviates little from the optimal objective function value when the cost parameters change." They use a stochastic decomposition algorithm based on the simulation-based sample average approximation method described in Shapiro and Homm-de-Mello (1998). The algorithm is specialized for designing stochastic supply chain systems. First, a limited number of feasible facility configurations are selected. Then, for each configuration, the parameters are sampled from their respective distributions. The resulting linear network flow problem (with fixed facility variables) is solved for the production and transportation quantities. The expected value and variance is computed over many replications and the “best” configuration is selected based on weighted objective of the mean and standard deviation. The research found that this solution dominates the solution generated using the average values for the parameters.

In addition to regret and variability models, there are several other approached to robust and/or flexible supply chain design. Kouvelis and Yu (1997) minimize the maximum costs of the supply chain, Voudouris (1996) and Sabri and Beamon (2000) address uncertainty by building excess capacity in the supply chain, Applequist, Penky, and Rekalaitis (2000) propose a new metric called *risk premium* for evaluating supply chains, and Vidal and Goetschalckx (2000) use extensive sensitivity analysis.

Voudouris (1996) and Sabri and Beamon (2000) define supply chain flexibility as the ability to respond to unexpected demand. They achieve flexibility by building excess capacity into the system. Both papers use volume flexibility as the capacity slack, similar to what is commonly used in the real industry. Sabri and Beamon also propose that delivery flexibility, the ability to change planned delivery dates, measured by the lead time slack, is important even though it is not normally used in industry.

A different approach to handling uncertainty is measure the risk associated with different supply chain configurations in an uncertain environment. Applequist, Penky, and Rekalaitis (2000) propose a metric called *risk premium* for evaluating supply chains. The risk premium is the increase in expected return in exchange for a given amount of variance. This metric is borrowed from the securities investment domain and provides the basis for a rational balance between expected values and variances of revenue in projects where there is a significant element of uncertainty.

Vidal and Goetschalckx (2000) develop a mixed integer program for international supply chain design. They address uncertainty in exchange rates, demand, supplier reliability, and lead times. The mixed integer programming model can be solved effectively providing fast sensitivity analysis on re-optimization under different conditions.

Ben-Tal and Nemirovski (1998, 1999, 2000) address the over-conservatism of robust solutions (min-max/max-min objective) by allowing the uncertainty sets for the data to be ellipsoids, and propose efficient algorithms to solve convex optimization problems under data uncertainty. However, as the resulting robust formulations involve conic quadratic problems, such methods cannot be directly applied to discrete optimization.

Averbakh (2001) shows that polynomial solvability is preserved for a specific discrete optimization problem (selecting $p$ elements of minimum total weight out of a set of $m$ elements with uncertainty in weights of the elements) when each weight can vary within an interval under the minimax-regret robustness. However, the approach does not seem to generalize to other discrete optimization problems.

Bertsimas and Sim (2003) propose an approach to address data uncertainty for discrete optimization and network flow problems that allows controlling the degree of conservatism of the solution (min-max/max-min objective). When both the cost coefficients and the data in the constraints of an integer programming problem are subjected to uncertainty with the assumption that the random parameter in the functional constraints take values from bounded symmetric distribution, they propose a robust integer programming problem of moderately larger size that allows controlling the degree of conservatism of the solution in terms of probabilistic bounds on constraint violation.

When only the cost coefficients are subject to uncertainty and the problem is a 0-1 discrete optimization problem on $n$ variables, they propose the solution methodology to solve the robust counterpart by solving at most $n+1$ instances of the original problem. They also show that the robust counterpart of an $NP$-hard $\alpha$-approximable 0-1 discrete optimization problem remains $\alpha$-approximable. They also propose an algorithm for robust network flows that solve the robust counterpart by solving a polynomial number of nominal minimum cost flow problems in a modified network.

Butler (2003) proposes a new definition of a robust solution by combining the expected value and the relative robustness definition for an application of supply chain design for new product distribution. Table 2.2 contains a summary of literature in the area of robust optimization in supply chain system design and operations.

**Table 2.2 Summary of Robust Optimization Literature**

| Authors | Robust Approach | Regret Model | Variability | Discrete Scenario | Continuous Scenario | Require PDF | Correlation | Nonlinear | LP or MILP | Simulation | Decomposition |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Hodder and Dincer (1986)** | **Weighted sum of mean and variance** | | × | | | × | | × | | | |
| **Malcolm and Zenios (1994)** | **Weighted sum of mean and variance** | | × | | | × | | × | | | |
| **Mulvey, Vanderbei and Zenios (1995)** | **Weighted sum of mean and variance and keep infeasibility small** | | × | | | × | | × | | | |

**Table 2.2 (Continued) Summary of Robust Optimization Literature**

| Authors | Robust Approach | Regret Model | Variability | Discrete Scenario | Continuous Scenario | Require PDF | Correlation | Nonlinear | LP or MILP | Simulation | Decomposition |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gutierrez, Kouvelis and Kurawarwala (1996) | *p*% from optimal | × | | × | | | × | | × | | × |
| Voudouris (1996) | Use of excess capacity | | | | | × | | | | | |
| Bai, Carpenter and Mulvey (1997) | Use of unsymmetric concave utility function | | × | | | × | | × | | | |
| Kouvelis and Yu (1997) | Absolute Robustness | | | × | | | × | | × | | |
| Kouvelis and Yu (1997) | Relative Robustness | × | | × | | | × | | × | | |
| Kouvelis and Yu (1997) | Deviation Robustness | × | | × | | | × | | × | | |
| Bok, Lee and Park (1998) | Weighted sum of mean and variance and keep infeasibility small | | × | | | × | | × | | | |
| Ahmed and Sahinidis (1998) | Use of UPM instead of symmetric variance measure | | × | | | × | | | × | | |
| Sabri and Beamon (2000) | Use excess capacity and delivery time | | | | | × | | | | | |
| Applequist, Penky and Rekalaitis (2000) | Use security investment to measure the trade off between mean and variance | | | | | × | | | | | |

**Table 2.2 (Continued) Summary of Robust Optimization Literature**

| Authors | Robust Approach | Regret Model | Variability | Discrete Scenario | Continuous Scenario | Require PDF | Correlation | Nonlinear | LP or MILP | Simulation | Decomposition |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Vidal and Goetschalckx (2000) | Use of sensitivity analysis | | | | | × | | | | | |
| Yu and Li (2000) | Reduce number of variables | | × | × | | | | | × | | |
| Ammons and Realff] (1999) | Deviation Robustness | × | | × | | | × | | × | | |
| Ben-Tal and Nemirovski (1998, 1999, 2000) | Absolute Robustness by allowing the uncertainty sets for the data to be ellipsoids. Use conic quadratic problems. | | | × | × | | | × | | | |
| Averbakh (2001) | Illustrate the polynomial solvability of a specific discrete optimization problem when variation only exist in the model objective function (minimax regret) | × | | | × | | | | | | |
| Goetschalckx *et al*. (2001) | Simulation Based Sample Average Approximation Method | | × | | | × | | | × | × | × |
| Newton (2000) | Continuous Deviation Robustness | × | | | × | | | | × | | |
| Bertsimas and Sim (2003) | Allow controlling the degree of conservatism of the solution. Illustrate the polynomial solvability for absolute robustness network flows problems with bounded symmetric distribution | | | | × | | | | × | | |
| Butler (2003) | Combination of Expected Value and Relative Robustness | × | | × | | | | | × | | |
| Assavapokee (2004) | Semi-Continuous Robust (Deviation Robustness) | × | | × | × | | × | | × | | × |

## 2.4 Literature Review of Bi-level Optimization

The bi-level programming problem (BLPP) can be viewed as static version of the noncooperative two-person game with a leader-follower structure. In the basic model, control of decision variables is partitioned among the players who seek to optimize their individual objective function. Perfect information is assumed so that both players know the objective and feasible choices available to the other.

The fact that the game is said to be ‘static’ implies that each player has only one move. The leader goes first and attempts to optimize his objective function. In so doing, he must anticipate all possible responses of his opponent, termed the follower. The follower observes the leader’s decision and reacts in a way that is personally optimal without regard to extramural effects. Because the set of feasible choices available to either player is interdependent, the leader’s decision affects both the follower’s objective value and allowable actions, and vice versa.

The vast majority of research on bi-level programming has centered on the linear version of the problem, alternatively known as the linear Stackelberg game (Bard, 1998).

For $x \in X \subset R^n$, $y \in Y \subset R^m$, $F : X \text{ x } Y \to R^1$, and $f : X \text{ x } Y \to R^1$, the BLPP can be written as follows:

$$\left.\begin{array}{l} \min\limits_{x \in X} F(x,y) = c_1 x + d_1 y \\ \text{subject to} \quad A_1 x + B_1 y \le b_1 \end{array}\right\} \text{Leader's Problem}$$

$$\left.\begin{array}{l} \min\limits_{y \in Y} f(x,y) = c_2 x + d_2 y \\ \qquad\qquad \text{subject to} \quad A_2 x + B_2 y \le b_2 \end{array}\right\} \text{Follower's Problem}$$

where $c_1, c_2 \in R^n$, $d_1, d_2 \in R^m$, $b_1 \in R^p$, $b_2 \in R^q$, $A_1 \in R^{p \text{ x } n}$, $B_1 \in R^{p \text{ x } m}$, $A_2 \in R^{q \text{ x } n}$, and $B_2 \in R^{q \text{ x } m}$.

The set $X$ and $Y$ place additional restrictions on the variables, such as upper and lower bounds or integrality requirements. Note that once the leader selects the $x$ value, the first term in the follower's objective function becomes a constant and can be removed from the problem. In this case we replace $f(x,y)$ with $f(y)$.

The sequential nature of the decisions implies that $y$ can be viewed as function of $x$; i.e., $y = y(x)$. The following definitions are used for solution methodology of BLPP model.

(a) Constraint region of the BLPP:

$$S \overset{\Delta}{=} \{(x,y) \mid x \in X, y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\}.$$

(b) Feasible set for the follower for each fixed $\hat{x} \in X$:

$$S(\hat{x}) \overset{\Delta}{=} \{ y \in Y \mid A_2 \hat{x} + B_2 y \le b_2\}$$

(c) Projection of $S$ onto the leader's decision space:

$$S(X) \overset{\Delta}{=} \{ x \in X \mid \exists y \in Y, A_1 x + B_1 y \le b_1, A_2 x + B_2 y \le b_2\}$$

(d) Follower's rational reaction set for $\hat{x} \in S(X)$:

$$P(\hat{x}) \overset{\Delta}{=} \{y \in Y \mid y \in \arg\min[f(\hat{x},\hat{y}) \mid \hat{y} \in S(\hat{x})]\}$$

(e) Inducible region:

$$IR \overset{\Delta}{=} \{(x,y) \mid (x,y) \in S, y \in P(x)\}$$

To ensure that the BLPP model is well posed, it is common to assume that $S$ is nonempty and compact; i.e., $P(x) \ne \phi$. The rational reaction set $P(x)$ defines the response while the inducible region ($IR$) represents the set over which the leader may optimize. Thus in term of this notation, the BLPP model can be written as $\min\{F(x,y)\,|\,(x,y)\in IR\}$.

In searching for a way to solve the linear BLPP (*F(x,y)* and *f(x,y)* are both linear functions), it would be helpful to have an explicit representation of *IR*. This can be achieved by replacing the follower's problem with Karash-Kuhn-Tucker (KKT) conditions and appending the resultant system to the leader's problem. In another word, the BLPP model can be rewritten as follows:

$$\begin{aligned}
&\min\ F(x,y) = c_1 x + d_1 y \\
&\text{subject to}\quad A_1 x + B_1 y \le b_1 \text{ and } A_2 x + B_2 y \le b_2 \\
&\qquad\qquad\quad uB_2 - v = -d_2 \\
&\qquad\qquad\quad u(b_2 - A_2 x - B_2 y) = 0 \text{ and } vy = 0 \\
&\qquad\qquad\quad x \ge 0, y \ge 0, u \ge 0, v \ge 0
\end{aligned}$$

where $u \in R^q$ and $v \in R^m$.

In theory, nonlinear constraints (complementary slackness conditions) in this model can be handled trivially by using the big *M* technique (Bard, 1998) with binary variables. However, drawbacks of this method came up in real application and will be presented in Chapter VI of this dissertation. This dissertation applies bi-level programming in the second stage and the third stage of the semi-continuous robust algorithm.

Bi-level linear optimization was first proposed since the mid-1960's. The initial work was by Baumol and Fabian (1964). The linear bi-level programming problem was first shown to be NP-hard by Jeroslow (1985) using satisfiability arguments common in computer science. Bard (1991) provided an alternative proof by constructively reducing the problem of maximizing a strictly convex quadratic function over a polyhedron to a linear max-min problem.

In general, there are three different approaches for solving a linear bi-level programming problem that can be considered workable. The first approach makes use of the theorem that the solution of the linear bi-level programming problem occurs at a vertex of $S$ and involves some form of vertex enumeration in the context of the simplex method.

Candler and Townsley (1982) were the first to develop an algorithm that was globally optimal. Their scheme repeatedly solves two linear programs, one for the leader in all of the $x$ variables and a subset of the $y$ variables associated with an optimal basis to the follower's problem, and the other for the follower with all the $x$ variables fixed. In a systematic way they explore optimal bases of the follower's problem for $x$ fixed and then return to the leader's problem with the corresponding basic $y$ variables. By focusing on the reduced cost coefficients of the $y$ variables not in an optimal basis of the follower's problem, they are able to provide a monotonic decrease in the number of follower bases that have to be examined.

Bialas and Karwan (1982) offered a different approach that systematically explores vertices beginning with the basis associated with the optimal solution to the linear program created by removing the follower's objective function. This is known as the *high point problem*; their algorithm is referred as "$K^{th}$*-best*" algorithm.

The second approach for solving the linear bi-level programming problem is known as the "Kuhn-Tucker" approach. The fundamental idea is to use a branch and bound strategy to deal with the complementarity constraints. Omitting or relaxing this constraint leaves a standard linear programming which is easy to solve. The various methods proposed employ different techniques for assuring that complementarity is ultimately satisfied (Bard and Moore, 1990; Fortuny-Amat and McCarl, 1981; Hansen, Jaumard and Savard, 1992; Judice and Faustino, 1992).

The third method is based on some form of penalty approach. Aiyoshi and Shimizu (1984) addressed the general bi-level programming problem by first converting the follower's problem to an unconstrained mathematical program using a barrier method. The corresponding stationarity conditions are then appended to the leader's problem, which is solved repeatedly for decreasing values of the barrier parameter. To guarantee convergence the follower's objective function must be strictly convex. This rules out the linear case, at least in theory.

A different approach using an exterior penalty method was proposed by Shimizu and Lu (1995) that simply requires convexity of all the functions to guarantee global convergence.

Anandalingam and White (1990) used the gap between the primal and dual solution of the follower's problem for $x$ fixed as a penalty term in the leader's problem. Although this results is a nonlinear objective function, it can be decomposed to provide a set of linear programs conditioned on either the decision variables $(x, y)$ or the dual variables $u$ of the follower's problem. They showed that an exact penalty function exists that yields the global solution.

In summary, the commonly used algorithms for solving the linear bi-level programming problem are the $K^{th}$-Best algorithm (Bialas and Karwan, 1982), the Kuhn-Tucker approach (Bard and Moore, 1990), the complementarity approach (Bialas and Karwan, 1984; Judice and Faustino, 1992), the variable elimination algorithm (Hansen, Jaumard and Savard, 1992), and the penalty function approach (Anandalingam and White, 1990).

This dissertation develops a modified version of the original algorithm by Bard and Moore (1990) for solving a bi-level programming problem in the third stage of the semi-continuous robust algorithm. We develop our own methodology based on strong duality theorem and Kuhn-Tucker approach for solving the bi-level programming problem in the second stage of the algorithm.

**Table 2.3 Summary of Linear Bi-Level Programming Literature**

| **Authors** | **K-Best Algorithm** | **KKT Condition** | **Penalty Function** | **Complementarity Approach** | **Variable Elimination** | **Iterative & LP** | **Strong Duality** | **Theoretical Proof** |
|---|---|---|---|---|---|---|---|---|
| **Fortuny-Amat and McCarl (1981)** | | × | | | | | | |
| **Bialas and Karwan<br>(1982)** | × | | | | | | | |
| **Candler and Townsley<br>(1982)** | | | | | | × | | |
| **Bialas and Karwan<br>(1984)** | | | | × | | | | |
| **Aiyoshi and Shimizu<br>(1984)** | | | × | | | | | |
| **Jeroslow<br>(1985)** | | | | | | | | × |
| **Bard and Moore<br>(1990)** | | × | | | | | | |
| **Anandalingam and White<br>(1990)** | | | × | | | | | |
| **Bard<br>(1991)** | | | | | | | | × |
| **Judice and Faustino<br>(1992)** | | | | × | | | | |
| **Hansen, Jaumard and Savard<br>(1992)** | | | | | × | | | |
| **Shimizu and Lu<br>(1995)** | | | × | | | | | |
| **Newton<br>(2000)** | | × | | | | | | |
| **Assavapokee (2004)** | | × | | × | | | × | × |

### 2.5 Summary

The work in this dissertation presents a new min-max regret robust optimization algorithm called semi-continuous robust algorithm for designing a robust supply chain network infrastructure when uncertainty greatly affects the outcomes of the decisions. Unlike continuous and discrete robust approaches reviewed in Section 2.3, the semi-continuous robust algorithm is able to find the min-max regret robust optimal solution when uncertain parameters take their values from real compact intervals and/or some specific discrete real values. The proposed algorithm can also handle uncertainty in coefficients of continuous variables, which cannot be handled by the continuous robust approach. The algorithm is also developed for handling the case when correlation among parameters exists.

This new algorithm can be effectively used in designing robust network infrastructure for the supply chain including reverse production system when the joint probability distributions of key parameters are unknown. The algorithm only requires the information on potential ranges and possible discrete values of uncertain parameters, which often are available in practice. Case studies on reverse production system application of the algorithm are also presented. The mixed integer linear programming model for reverse production system in this dissertation is most closely to the model by Newton (2000) and Pantelides (1996) reviewed in Section 2.2.

The algorithm also involves the uses of the bi-level programming, which represents the game between decision makers and the system, in two of the algorithm's stages. The modified Kuhn-Tucker approach (Bard and Moore, 1990) with priority branching rules and strong duality theory are used for solving the bi-level programming problems in this dissertation.

# CHAPTER III

# BASIC MODEL AND SCENARIO BASED ROBUST OPTIMIZATION

## 3.1 The Reverse Production System (RPS) Model

This chapter will begin by introducing the basic mixed integer linear programming model which represents our reverse production systems problem when the perfect information of model parameters is given. This model will be referred as RPS model in this dissertation. This RPS model was initially developed for the reverse production system planning of carpet recycling presented in Ammons and Realff [1999]. This RPS model has been modified from the original version to include sources of materials and demand points to the system. The objective of this model is to maximize the net profit of the reverse supply chain system: that is the total revenues of the system minus the total operational cost of the system. The RPS model has ability to make the strategic and tactical decisions on the location of collecting centers and processing centers, the type of materials collected at each collecting center, the type of processes installed at each processing center, and amount of materials collected, processed and transported within the reverse supply chain system. A verbal description of the RPS mathematical model is as follows:

**<u>Maximize</u>:** ***Net Profit*** = (Revenues – Operating and Fixed Costs)

Number of units shipped to customer * selling price per unit
+ Number of units collected * collection fee per unit
- Fixed costs for storage, process, collection and transportation
- Fixed costs to open collecting center and processing center
- Fixed costs to close collecting center and processing center
- Variable costs for storage, collection, process and transportation

**<u>Subject to</u>*:***

*1. Flow balance restrictions between sites and between time periods for each material.*

*2. Supply restriction for each source, material and time period*

*3. Demand restriction for each customer, material and time period*

*4. Amount sold definition constraint for each customer, material and time period*

*5. Amount collected definition constraint for each site, material and time period*

*6. Logical constraints consisting of relationship among binary decision variables*

*7. and 8. Upper and lower bound constraints*

*9. Capacity constraints including collection, process, storage and transportation capacity.*

*10 and 11. Non-negativity and Binary constraints*

The model itself is fairly generic and incorporates the features of reverse production system without needing to deviate from the above structure. Transformation tasks in the model allow materials to change to different material types. Tasks also include collection, selling and storing. Tasks are only allowed to occur at sites (both collecting and processing sites), which are physical locations. The model permits materials to flow only along predetermined routes between sites. A single site can accommodate any of the tasks, and each task will have a fixed and a variable cost.

The mathematical representation of the RPS model is presented in Table 3.5 using the following notation for indices, super scripts, parameters and decision variables. Table 3.1 contains the indices and Table 3.2 contains the super scripts used in the RPS model. Table 3.3 contains all parameters and Table 3.4 contains all decision variables in the RPS model.

**Table 3.1 RPS Model Indices**

| | |
|---|---|
| s | Supplier |
| i | Sites |
| c | Customer |
| j | material type |
| m | transportation mode |
| p | process type |
| t | time period |

**Table 3.2 RPS Model Superscripts**

| | |
|---|---|
| Co | Collection |
| Sa | Selling |
| St | Storage |
| Tr | transportation |
| Pr | Process |
| Su | Supplier |
| Si | Site |
| Cu | Customer |

**Table 3.3 RPS Model Parameters**

$S^{(Su)}_{sjt}$ = Amount of material j that is supplied at supplier s at time period t

$D^{(Cu)}_{cjt}$ = Amount of material j that is demanded at customer c at time period t

$P^{(Cu)}_{cjt}$ = Selling Price offered per standard unit of material j from customer c at time period t

$V^{(St)}_{ijt}$ = Storage cost per standard unit of material j per time period at site i at time period t

$V^{(Co)}_{ijt}$ = Collection cost per standard unit of material j at site i at time period t

$V'^{(Co)}_{ijt}$ = Collection fee per standard unit of material j at site i at time period t

$V^{(Pr)}_{ipt}$ = Processing cost per standard unit for process p at site i at time period t

$V^{(Tr)}_{simt}$ = Transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t

$V^{(Tr)}_{ii'mt}$ = Transportation cost per standard unit per distance from site i to i' using transportation mode m at time period t

$V^{(Tr)}_{icmt}$ = Transportation cost per standard unit per distance from site i to customer c using transportation mode m at time period t

$d_{sim}$ = Distance from supplier s to site i by transportation mode m

$d_{ii'm}$ = Distance from site i to i' by transportation mode m

$d_{icm}$ = Distance from site i to customer c by transportation mode m

$F^{(Si)}_{it}$ = Fixed site operating cost if site i is opened at time period t

$F'^{(Si)}_{it}$ = Fixed site opening cost of site i at time period t

$F''^{(Si)}_{it}$ = Fixed site closing cost of site i at time period t

$F^{(St)}_{ijt}$ = Fixed storage cost of material j at site i at time period t

$F^{(Co)}_{ijt}$ = Fixed collecting cost of material j at site i at time period t

$F^{(Pr)}_{ipt}$ = Fixed processing cost for process p at site i at time period t

$F^{(Tr)}_{simt}$ = Fixed cost for transportation from supplier s to site i using transportation mode m at time period t

$F^{(Tr)}_{ii'mt}$ = Fixed cost for transportation from site i to site i' using transportation mode m at time period t

$F^{(Tr)}_{icmt}$ = Fixed cost for transportation from site i to customer c using transportation mode m at time period t

$C^{(Co)}_{ijt}$ = Maximum collection capacity to collect material type j at site i at time period t

$C^{(St)}_{ijt}$ = Maximum amount of material type j that can be stored at site i in at time period t

$C^{(Tr)}_{simt}$ = Maximum amount of material that can be shipped for supplier s to site i using transportation mode m at time period t

$C^{(Tr)}_{ii'mt}$ = Maximum amount of material that can be shipped for site i to i' using transportation mode m at time period t

$C^{(Tr)}_{icmt}$ = Maximum amount of material that can be shipped for site i to customer c using transportation mode m at time period t

$C^{(Pr)}_{ipt}$ = Maximum amount of material that process p can produce at site i at time period t

$a_{it}^{(Si)}$ = 1 if site i is allowed to be opened at time period t

$a_{it}^{(St)}$ = 1 if storage is allowed at site i at time period t, 0 otherwise

$a_{simt}^{(Tr)}$ = 1 if shipment by transportation mode m is allowed between supplier s and site i at time period t, 0 otherwise

$a_{ii'mt}^{(Tr)}$ = 1 if shipment by transportation mode m is allowed between sites i and i' at time period t, 0 otherwise

$a_{icmt}^{(Tr)}$ = 1 if shipment by transportation mode m is allowed between sites i and customer c at time period t, 0 otherwise

$a_{ipt}^{(Pr)}$ = 1 if process p is allowed at site i at time period t, 0 otherwise

$a_{ijt}^{(Co)}$ = 1 if collection of material j is allowed at site i at time period t, 0 otherwise

$m_{it}^{(Si)}$ = 1 if site i must be opened at time period t

$m_{it}^{(St)}$ = 1 if storage at site i must be used at time period t, 0 otherwise

$m_{simt}^{(Tr)}$ = 1 if shipment by transportation mode m must be used between supplier s and site i at time period t, 0 otherwise

$m_{ii'mt}^{(Tr)}$ = 1 if shipment by transportation mode m must be used between sites i and i' at time period t, 0 otherwise

$m_{icmt}^{(Tr)}$ = 1 if shipment by transportation mode m must be used between sites i and customer c at time period t, 0 otherwise

$m_{ipt}^{(Pr)}$ = 1 if process p must be used at site i at time period t, 0 otherwise

$m_{ijt}^{(Co)}$ = 1 if collection of material j must be done at site i at time period t, 0 otherwise

$\rho_{jp}$ = proportion of material type j consumed by process p

$\rho'_{jp}$ = proportion of material type j produced by process p

**Table 3.4 RPS Model Decision Variables**

$x^{(Co)}_{ijt}$ = Amount of material collected of type j at site i at time period t

$x^{(St)}_{ijt}$ = Amount of material stored of type j at site i at time period t

$x^{(Sa)}_{cjt}$ = Amount of material sold of type j to customer c at time period t

$x^{(Tr)}_{sjimt}$ = Amount of material shipped from supplier s to site i of type j using transportation mode m at time period t

$x^{(Tr)}_{iji'mt}$ = Amount of material shipped from site i to site i' of type j using transportation mode m at time period t

$x^{(Tr)}_{ijcmt}$ = Amount of material shipped from site i to customer c of type j using transportation mode m at time period t

$x^{(Pr)}_{ipt}$ = Amount of material processed by process p at site i at time period t

$y^{(Co)}_{ijt}$ = 1 if collection of material type j is to be performed at site i at time period t 0 otherwise

$y^{(Tr)}_{simt}$ = 1 if shipment is to be used between supplier s and site i using transportation mode m at time period t, 0 otherwise

$y^{(Tr)}_{ii'mt}$ = 1 if shipment is to be used between sites i and i' using transportation mode m at time period t, 0 otherwise

$y^{(Tr)}_{icmt}$ = 1 if shipment is to be used between sites i and customer c using transportation mode m at time period t, 0 otherwise

$y^{(Pr)}_{ipt}$ = 1 if process p is to be used at site i at time period t, 0 otherwise

$y^{(St)}_{ijt}$ = 1 if storage is to be used for material type j at site i at time period t

0 otherwise

$y'^{(Si)}_{it}$ = 1 if site i is decided to be opened at period t, 0 otherwise

$y''^{(Si)}_{it}$ = 1 if site i is decided to be closed down at period t, 0 otherwise

$y^{(Si)}_{it}$ = 1 if site i is operated at time period t, 0 otherwise

**Table 3.5 RPS Mathematical Model**

| | |
|---|---|
| *Maximize (Objective)* | *Maximize Net Revenue* |
| $\sum_t \sum_c \sum_j P^{(Cu)}_{cij} x^{(Sa)}_{cjt}$ | - Sales Revenue |
| $-\sum_t \sum_j \sum_i (F^{(Co)}_{ijt} y^{(Co)}_{ijt} + F^{(St)}_{ijt} y^{(St)}_{ijt})$ | |
| $-\sum_t \sum_i (F^{(Si)}_{it} y^{(Si)}_{it} + F'^{(Si)}_{it} y'^{(Si)}_{it} + F''^{(Si)}_{it} y''^{(Si)}_{it})$ | |
| $-\sum_t \sum_p \sum_i F^{(Pr)}_{ipt} y^{(Pr)}_{ipt}$ | |
| $-\sum_t \sum_m \sum_s \sum_i F^{(Tr)}_{simt} y^{(Tr)}_{simt} - \sum_t \sum_m \sum_i \sum_{i' \neq i} F^{(Tr)}_{ii'mt} y^{(Tr)}_{ii'mt}$ | |
| $-\sum_t \sum_m \sum_i \sum_c F^{(Tr)}_{icmt} y^{(Tr)}_{icmt}$ | - Fixed Costs |
| $-\sum_t \sum_j \sum_i V^{(St)}_{ijt} x^{(St)}_{ijt}$ | - Storage Costs |
| $-\sum_t \sum_j \sum_i (V^{(Co)}_{ijt} - V'^{(Co)}_{ijt}) x^{(Co)}_{ijt}$ | - Collection Costs and Fees |
| $-\sum_t \sum_p \sum_i V^{(Pr)}_{ipt} x^{(Pr)}_{ipt}$ | - Processing Costs |
| $-\sum_t \sum_m \sum_i \sum_j \sum_s V^{(Tr)}_{simt} x^{(Tr)}_{sjimt} d_{sim}$ | |

$$-\sum_t \sum_m \sum_i \sum_j \sum_{i' \neq i} V_{ii'mt}^{(Tr)} x_{iji'mt}^{(Tr)} d_{ii'm}$$

$$-\sum_t \sum_m \sum_i \sum_j \sum_c V_{icmt}^{(Tr)} x_{ijcmt}^{(Tr)} d_{icm}$$ - Shipping Costs

**Subject to:**

$$x_{ijt}^{(St)} = x_{ij(t-1)}^{(St)} + \sum_s \sum_m x_{sjimt}^{(Tr)} + \sum_{i' \neq i} \sum_m x_{i'jimt}^{(Tr)}$$
$$- \sum_{i' \neq i} \sum_m x_{iji'mt}^{(Tr)} - \sum_c \sum_m x_{ijcmt}^{(Tr)} + \sum_p \rho'_{jp} x_{ipt}^{(\mathrm{Pr})}$$
$$- \sum_p \rho_{jp} x_{ipt}^{(\mathrm{Pr})} \qquad \forall i, j, t$$

$$S_{sjt}^{(Su)} = \sum_i \sum_m x_{sjimt}^{(Tr)} \qquad \forall s, j, t$$

$$D_{cjt}^{(Cu)} \geq \sum_i \sum_m x_{ijcmt}^{(Tr)} \qquad \forall c, j, t$$

$$x_{cjt}^{(Sa)} = \sum_i \sum_m x_{ijcmt}^{(Tr)} \qquad \forall c, j, t$$

$$x_{ijt}^{(Co)} = \sum_s \sum_m x_{sjimt}^{(Tr)} \qquad \forall i, j, t$$

$$\mathrm{y}_{\mathrm{ijt}}^{(\mathrm{Co})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, j, t$$
$$\mathrm{y}_{\mathrm{ipt}}^{(\mathrm{Pr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, p, t$$
$$\mathrm{y}_{\mathrm{ijt}}^{(\mathrm{St})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, j, t$$
$$\mathrm{y}_{\mathrm{sjimt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ s, i, j, m, t$$
$$\mathrm{y}_{\mathrm{iji'mt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, i', j, m, t$$
$$\mathrm{y}_{\mathrm{i'jimt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, i', j, m, t$$
$$\mathrm{y}_{\mathrm{ijcmt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, c, j, m, t$$
$$\mathrm{y}_{\mathrm{it}}^{(\mathrm{Si})} - y_{i(t-1)}^{(Si)} \leq y'^{(Si)}_{it} \qquad \forall\ i, t$$
$$\mathrm{y}_{\mathrm{i(t\text{-}1)}}^{(\mathrm{Si})} - y_{it}^{(Si)} \leq y''^{(Si)}_{it} \qquad \forall\ i, t$$

$$y_{it}^{(Si)} \le a_{it}^{(Si)} \qquad \forall\, i,t$$

$$y_{ijt}^{(Co)} \le a_{ijt}^{(Co)} \qquad \forall\, i,j,t$$

$$y_{ipt}^{(\mathrm{Pr})} \le a_{ipt}^{(\mathrm{Pr})} \qquad \forall\, i,p,t$$

$$y_{ijt}^{(St)} \le a_{ijt}^{(St)} \qquad \forall\, i,j,t$$

$$y_{sjimt}^{(Tr)} \le a_{simt}^{(Tr)} \qquad \forall\, s,i,j,m,t$$

$$y_{iji'mt}^{(Tr)} \le a_{ii'mt}^{(Tr)} \qquad \forall\, i,i',j,m,t$$

$$y_{ijcmt}^{(Tr)} \le a_{ijmt}^{(Tr)} \qquad \forall\, i,c,j,m,t$$

$$y_{it}^{(Si)} \ge m_{it}^{(Si)} \qquad \forall\, i,t$$

$$y_{ijt}^{(Co)} \ge m_{ijt}^{(Co)} \qquad \forall\, i,j,t$$

$$y_{ipt}^{(\mathrm{Pr})} \ge m_{ipt}^{(\mathrm{Pr})} \qquad \forall\, i,p,t$$

$$y_{ijt}^{(St)} \ge m_{ijt}^{(St)} \qquad \forall\, i,j,t$$

$$y_{sjimt}^{(Tr)} \ge m_{simt}^{(Tr)} \qquad \forall\, s,i,j,m,t$$

$$y_{iji'mt}^{(Tr)} \ge m_{ii'mt}^{(Tr)} \qquad \forall\, i,i',j,m,t$$

$$y_{ijcmt}^{(Tr)} \ge m_{ijmt}^{(Tr)} \qquad \forall\, i,c,j,m,t$$

$$x_{ijt}^{(Co)} \le C_{ijt}^{(Co)} y_{ijt}^{(Co)} \qquad \forall\, i,j,t$$

$$x_{ipt}^{(\mathrm{Pr})} \le C_{ipt}^{(\mathrm{Pr})} y_{ipt}^{(\mathrm{Pr})} \qquad \forall\, i,p,t$$

$$x_{ijt}^{(St)} \le C_{ijt}^{(St)} y_{ijt}^{(St)} \qquad \forall\, i,j,t$$

$$\sum_j x_{sjimt}^{(Tr)} \le C_{simt}^{(Tr)} y_{simt}^{(Tr)} \qquad \forall\, s,i,j,m,t$$

$$\sum_j x_{iji'mt}^{(Tr)} \le C_{ii'mt}^{(Tr)} y_{ii'mt}^{(Tr)} \qquad \forall\, i,i',j,m,t$$

$$\sum_j x_{ijcmt}^{(Tr)} \le C_{icmt}^{(Tr)} y_{icmt}^{(Tr)} \qquad \forall\, i,c,j,m,t$$

$$x_{ijt}^{(Co)}, x_{ijt}^{(St)}, x_{cjt}^{(Sa)}, x_{sjimt}^{(Tr)}, x_{iji'mt}^{(Tr)}, x_{ijcmt}^{(Tr)}, x_{ipt}^{(\mathrm{Pr})} \ge 0 \qquad \forall s,i,c,j,m,p,t,i' \ne i$$

$$y_{ijt}^{(Co)}, y_{ijt}^{(St)}, y_{simt}^{(Tr)}, y_{ii'mt}^{(Tr)}, y_{icmt}^{(Tr)}, y_{ipt}^{(Pr)}, \qquad \forall s,i,c,j,m,p,t,i' \ne i$$

$$y_{it}^{(Si)}, y'^{(Si)}_{it}, y''^{(\mathrm{Si})}_{\mathrm{it}} \in \{0,1\}$$

### 3.2 The Discrete Robust Reverse Production System (DRRPS) Model

This section addresses the scenario based robust approach for solving mixed integer linear programming problem under input data uncertainty when all possible values of all model parameters can be classified into the finite number of scenarios. The general representation of the model can be represented as:

$$\begin{aligned} &\max_{x,y} Z(x,y) = c^T x + f^T y \\ &s.t. \quad A\,x + B\,y \le b \\ &\quad x \ge 0 \ \text{ and } \ y \in \Gamma \end{aligned}$$

where the set $\Gamma$ includes any constraints imposed on *y.*

The basic components of the model's uncertainty are a finite set of all possible scenarios of parameters, $\Omega$, and the given values of parameters $[c_\omega, f_\omega, A_\omega, B_\omega, b_\omega, \Gamma_\omega]$ under each scenario $\omega \in \Omega$. For the specific input data $[c_\omega, f_\omega, A_\omega, B_\omega, b_\omega, \Gamma_\omega]$ for each scenario $\omega \in \Omega$, the problem contains two types of decision variables, one modeling discrete choice design decisions and the other modeling continuous design decisions. Let vector *y* represents choice design decision variables and let vector $x_\omega$ denotes continuous design decision variables under scenario $\omega \in \Omega$. If the parameters' perfect information is given to be a scenario $\omega \in \Omega$, the problem can be formulated and solved as:

$$O_\omega^* = \left\{ \begin{aligned} &\max_{x_\omega, y} Z(x_\omega, y) = c_\omega^T x_\omega + f_\omega^T y \\ &s.t. \quad A_\omega x_\omega + B_\omega y \le b_\omega \\ &\quad x_\omega \ge 0 \ \text{ and } \ y \in \Gamma_\omega \end{aligned} \right\}$$

where the set $\Gamma_\omega$ includes any constraints imposed on *y,* $\left\{ \begin{aligned} &E_\omega y \le d_\omega \\ &y \in \{0,1\}^{|y|} \end{aligned} \right\}$, for scenario $\omega \in \Omega$.

When the uncertainty exists, the search for the *robust solution* is to find discrete design decisions ( $y \in \Gamma_\omega \ \ \forall \omega \in \Omega$ ), such that the function $\max_{\omega \in \Omega}(O_\omega^* - Z_\omega^*(y))$ is minimized

where $Z_\omega^*(y) = \left\{ \begin{array}{ll} \max\limits_{x_\omega} & c_\omega^T x_\omega \\ s.t. & A_\omega x_\omega \le b_\omega - B_\omega y \\ & x_\omega \ge 0 \end{array} \right\} + f_\omega^T y$.

The following algorithm is referred as *scenario based robust optimization* in this dissertation.

<u>*Scenario Based Minimax Robust Optimization*</u>

<u>Step 0:</u> Solve the following problems to optimality

$$O_\omega^* = \left\{ \begin{array}{l} \max\limits_{x_\omega, y} Z(x_\omega, y) = c_\omega^T x_\omega + f_\omega^T y \\ s.t. \quad A_\omega x_\omega + B_\omega y \le b_\omega \\ \quad x_\omega \ge 0 \ \text{ and } \ y \in \Gamma_\omega \end{array} \right\} \quad \forall \omega \in \Omega$$

<u>Step 1:</u> Solve the following mixed integer linear programming problem to optimality.

$$\begin{array}{ll} \min\limits_{\delta, y, x_\omega} & \delta \\ s.t. & \left. \begin{array}{r} \delta \ge O_\omega^* - c_\omega^T x_\omega - f_\omega^T y \\ A_\omega x_\omega + B_\omega y \le b_\omega \\ x_\omega \ge 0 \ \text{ and } \ y \in \Gamma_\omega \end{array} \right\} \ \forall \omega \in \Omega \end{array}$$

Let $\delta^*$ and $y^*$ represent the optimal setting of $\delta$ and $y$ respectively.

<u>Step 2:</u> Solve the following linear programming problems to optimality $\forall \omega \in \Omega$.

$$Z_\omega^*(y^*) = \left\{ \begin{array}{ll} \max\limits_{x_\omega} & c_\omega^T x_\omega \\ s.t. & A_\omega x_\omega \le b_\omega - B_\omega y^* \\ & x_\omega \ge 0 \end{array} \right\} + f_\omega^T y^*$$

Let $x_{\omega}^{*}$ represents the optimal setting of $x_{\omega}$ for each $\omega \in \Omega$.

Step 3: The resulting robust solution is $y^{*}$ and the resulting continuous solution for scenario $\omega \in \Omega$ is $x_{\omega}^{*}$. The difference in objective function value between optimal solution and robust solution for scenario $\omega \in \Omega$ is represented by $O_{\omega}^{*} - Z_{\omega}^{*}(y^{*})$.

When all possible values of all RPS model parameters can be classified into a finite number of scenarios, a mixed integer linear programming model called DRRPS model is developed by applying the idea of scenario based minimax robust optimization to the RPS model. The parameters and continuous variables include a new dimension of scenario, $\omega$. The objective is to minimize the maximum difference over all scenarios between the RPS optimal objective function value and the objective function value for the robust decisions.

The mathematical representation of the DRRPS model is presented in Table 3.10 using the following notation for indices, super scripts, parameters and decision variables. Table 3.6 contains the indices and Table 3.7 contains the super scripts used in the DRRPS model. Table 3.8 contains all parameters and Table 3.9 contains all decision variables in the DRRPS model.

**Table 3.6 DRRPS Model Indices**

| | |
|---|---|
| s | supplier |
| i | sites |
| c | customer |
| j | material type |
| m | transportation mode |
| p | process type |
| t | time period |
| ω | scenario |

**Table 3.7 DRRPS Model Superscripts**

| | |
|---|---|
| Co | collection |
| Sa | selling |
| St | storage |
| Tr | transportation |
| Pr | process |
| Su | supplier |
| Si | site |
| Cu | customer |

**Table 3.8 DRRPS Model Parameters**

$S^{(Su)}_{sjt\omega}$ = Amount of material j that is supplied at supplier s at time period t for scenario ω

$D^{(Cu)}_{cjt\omega}$ = Amount of material j that is demanded at customer c at time period t for scenario ω

$P^{(Cu)}_{cjt\omega}$ = Selling Price offered per standard unit of material j from customer c at time period t for scenario ω

$V^{(St)}_{ijt\omega}$ = Storage cost per standard unit of material j per time period at site i at time period t for scenario ω

$V^{(Co)}_{ijt\omega}$ = Collection cost per standard unit of material j at site i at time period t for scenario ω

$V'^{(Co)}_{ijt\omega}$ = Collection fee per standard unit of material j at site i at time period t for scenario ω

$V^{(Pr)}_{ipt\omega}$ = Processing cost per standard unit for process p at site i at time period t for scenario ω

$V^{(Tr)}_{simt\omega}$ = Transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t for scenario ω

$V^{(Tr)}_{ii'mt\omega}$ = Transportation cost per standard unit per distance from site i to i’ using transportation mode m at time period t for scenario ω

$V^{(Tr)}_{icmt\omega}$ = Transportation cost per standard unit per distance from site i to customer c using transportation mode m at time period t for scenario ω

$d_{sim\omega}$ = Distance from supplier s to site i by transportation mode m for scenario ω

$d_{ii'm\omega}$ = Distance from site i to i’ by transportation mode m for scenario ω

$d_{icm\omega}$ = Distance from site i to customer c by transportation mode m for scenario ω

$F^{(Si)}_{it\omega}$ = Fixed site operating cost if site i is opened at time period t for scenario ω

$F'^{(Si)}_{it\omega}$ = Fixed site opening cost of site i at time period t for scenario ω

$F''^{(Si)}_{it\omega}$ = Fixed site closing cost of site i at time period t for scenario ω

$F^{(St)}_{ijt\omega}$ = Fixed storage cost of material j at site i at time period t for scenario ω

$F^{(Co)}_{ijt\omega}$ = Fixed collecting cost of material j at site i at time period t for scenario ω

$F^{(Pr)}_{ipt\omega}$ = Fixed processing cost for process p at site i at time period t for scenario ω

$F^{(Tr)}_{simt\omega}$ = Fixed cost for transportation from supplier s to site i by transportation mode m at time period t for scenario ω

$F^{(Tr)}_{ii'mt\omega}$ = Fixed cost for transportation from site i to site i’ by transportation mode m at time period t for scenario ω

$F^{(Tr)}_{icmt\omega}$ = Fixed cost for transportation from site i to customer c by transportation mode m at time period t for scenario ω

$C^{(Co)}_{ijt\omega}$ = Maximum collection capacity to collect material type j at site i at time period t for scenario ω

$C^{(St)}_{ijt\omega}$ = Maximum amount of material type j that can be stored at site i in at time period t for scenario ω

$C^{(Tr)}_{simt\omega}$ = Maximum amount of material that can be shipped for supplier s to site i by transportation mode m at time period t for scenario ω

$C^{(Tr)}_{ii'mt\omega}$ = Maximum amount of material that can be shipped for site i to i' by transportation mode m at time period t for scenario ω

$C^{(Tr)}_{icmt\omega}$ = Maximum amount of material that can be shipped for site i to customer c by transportation mode m at time period t for scenario ω

$C^{(Pr)}_{ipt\omega}$ = Maximum amount of material that process p can produce at site i at time period t for scenario ω

$a^{(Si)}_{it\omega}$ = 1 if site i is allowed to be opened at time period t for scenario ω, 0 otherwise

$a^{(St)}_{it\omega}$ = 1 if storage is allowed at site i at time period t for scenario ω, 0 otherwise

$a^{(Tr)}_{simt\omega}$ = 1 if shipment by transportation mode m is allowed between supplier s and site i at time period t for scenario ω, 0 otherwise

$a^{(Tr)}_{ii'mt\omega}$ = 1 if shipment by transportation mode m is allowed between sites i and i' at time period t for scenario ω, 0 otherwise

$a^{(Tr)}_{icmt\omega}$ = 1 if shipment by transportation mode m is allowed between sites i and customer c at time period t for scenario ω, 0 otherwise

$a^{(Pr)}_{ipt\omega}$ = 1 if process p is allowed at site i at time period t for scenario ω, 0 otherwise

$a^{(Co)}_{ijt\omega}$ = 1 if collection of material j is allowed at site i at time period t for scenario ω 0 otherwise

$m^{(Si)}_{it\omega}$ = 1 if site i must be opened at time period t for scenario ω

$m^{(St)}_{it\omega}$ = 1 if storage at site i must be used at time period t for scenario ω, 0 otherwise

$m^{(Tr)}_{simt\omega}$ = 1 if shipment by transportation mode m must be used between supplier s and site i at time period t for scenario ω, 0 otherwise

$m^{(Tr)}_{ii'mt\omega}$ = 1 if shipment by transportation mode m must be used between sites i and i' at time period t for scenario ω, 0 otherwise

$m^{(Tr)}_{icmt\omega}$ = 1 if shipment by transportation mode m must be used between sites i and customer c at time period t for scenario ω, 0 otherwise

$m^{(Pr)}_{ipt\omega}$ = 1 if process p must be used at site i at time period t for scenario ω, 0 otherwise

$m^{(Co)}_{ijt\omega}$ = 1 if collection of material j must be done at site i at time period t for scenario ω, 0 otherwise

$O^{*}_{\omega}$ = Optimal objective value from RPS Model for scenario ω

$\rho_{jp\omega}$ = proportion of material type j consumed by process p for scenario ω

$\rho'_{jp\omega}$ = proportion of material type j produced by process p for scenario ω

**Table 3.9 DRRPS Model Decision Variables**

$x^{(Co)}_{ijt\omega}$ = Amount of material collected of type j at site i at time period t for scenario ω

$x^{(St)}_{ijt\omega}$ = Amount of material stored of type j at site i at time period t for scenario ω

$x^{(Sa)}_{cjt\omega}$ = Amount of material sold of type j to customer c at time period t for scenario ω

$x^{(Tr)}_{sjimt\omega}$ = Amount of material shipped from supplier s to site i of type j using transportation mode m at time period t for scenario ω

$x^{(Tr)}_{iji'mt\omega}$ = Amount of material shipped from site i to site i' of type j using transportation mode m at time period t for scenario ω

$x^{(Tr)}_{ijcmt\omega}$ = Amount of material shipped from site i to customer c of type j using transportation mode m at time period t for scenario ω

$x^{(Pr)}_{ipt\omega}$ = Amount of material processed by process p at site i at time period t for scenario ω

$y^{(Co)}_{ijt}$ = 1 if collection of material type j is to be performed at site i at time period t 0 otherwise

$y^{(Tr)}_{simt}$ = 1 if shipment is to be used between supplier s and site i by transportation mode m at time period t, 0 otherwise

$y^{(Tr)}_{ii'mt}$ = 1 if shipment is to be used between sites i and i' by transportation mode m at time period t, 0 otherwise

$y^{(Tr)}_{icmt}$ = 1 if shipment is to be used between sites i and customer c by transportation mode m at time period t, 0 otherwise

$y^{(Pr)}_{ipt}$ = 1 if process p is to be used at site i at time period t, 0 otherwise

$y^{(St)}_{ijt}$ = 1 if storage is to be used for material type j at site i at time period t, 0 otherwise

$y'^{(Si)}_{it}$ = 1 if site i is decided to be opened at period t, 0 otherwise

$y''^{(Si)}_{it}$ = 1 if site i is decided to be closed down at period t, 0 otherwise

$y^{(Si)}_{it}$ = 1 if site i is operated at time period t, 0 otherwise

**Table 3.10 DRRPS Mathematical Model**

*Minimize* δ (*Minimize maximum regret)*

*Subject to:*

$$\delta \geq O^*_\omega - \sum_t \sum_c \sum_j P^{(Cu)}_{cijt\omega} x^{(Sa)}_{cjt\omega}$$

$$- \sum_t \sum_j \sum_i (F^{(Co)}_{ijt\omega} y^{(Co)}_{ijt} + F^{(St)}_{ijt\omega} y^{(St)}_{ijt})$$

$$- \sum_t \sum_i (F^{(Si)}_{it\omega} y^{(Si)}_{it} + F'^{(Si)}_{it\omega} y'^{(Si)}_{it} + F''^{(Si)}_{it\omega} y''^{(Si)}_{it})$$

$$- \sum_t \sum_p \sum_i F^{(Pr)}_{ipt\omega} y^{(Pr)}_{ipt} - \sum_t \sum_m \sum_i \sum_j \sum_c V^{(Tr)}_{icmt\omega} x^{(Tr)}_{ijcmt\omega} d_{icm\omega}$$

$$- \sum_t \sum_m \sum_s \sum_i F^{(Tr)}_{simt\omega} y^{(Tr)}_{simt} - \sum_t \sum_m \sum_i \sum_{i' \neq i} F^{(Tr)}_{ii'mt\omega} y^{(Tr)}_{ii'mt}$$

$$- \sum_t \sum_m \sum_i \sum_c F^{(Tr)}_{icmt\omega} y^{(Tr)}_{icmt} - \sum_t \sum_m \sum_i \sum_j \sum_s V^{(Tr)}_{simt\omega} x^{(Tr)}_{sjimt\omega} d_{sim\omega}$$

$$- \sum_t \sum_j \sum_i V^{(St)}_{ijt\omega} x^{(St)}_{ijt\omega} - \sum_t \sum_m \sum_i \sum_j \sum_{i' \neq i} V^{(Tr)}_{ii'mt\omega} x^{(Tr)}_{iji'mt\omega} d_{ii'm\omega}$$

$$- \sum_t \sum_j \sum_i (V^{(Co)}_{ijt\omega} - V'^{(Co)}_{ijt\omega}) x^{(Co)}_{ijt\omega} - \sum_t \sum_p \sum_i V^{(\mathrm{Pr})}_{ipt\omega} x^{(\mathrm{Pr})}_{ipt\omega}$$

$$x^{(St)}_{ijt\omega} = x^{(St)}_{ij(t-1)\omega} + \sum_s \sum_m x^{(Tr)}_{sjimt\omega} + \sum_{i' \neq i} \sum_m x^{(Tr)}_{i'jimt\omega}$$

$$- \sum_{i' \neq i} \sum_m x^{(Tr)}_{iji'mt\omega} - \sum_c \sum_m x^{(Tr)}_{ijcmt\omega} + \sum_p \rho'_{jp\omega} x^{(\mathrm{Pr})}_{ipt\omega}$$

$$- \sum_p \rho_{jp\omega} x^{(\mathrm{Pr})}_{ipt\omega} \qquad \forall i, j, t, \omega$$

$$S^{(Su)}_{sjt\omega} = \sum_i \sum_m x^{(Tr)}_{sjimt\omega} \qquad \forall s, j, t, \omega$$

$$D_{cjt\omega}^{(Cu)} \geq \sum_i \sum_m x_{ijcmt\omega}^{(Tr)} \qquad \forall c, j, t, \omega$$

$$x_{cjt\omega}^{(Sa)} = \sum_i \sum_m x_{ijcmt\omega}^{(Tr)} \qquad \forall c, j, t, \omega$$

$$x_{ijt\omega}^{(Co)} = \sum_s \sum_m x_{sjimt\omega}^{(Tr)} \qquad \forall i, j, t, \omega$$

$$\mathrm{y}_{\mathrm{ijt}}^{(\mathrm{Co})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, j, t$$

$$\mathrm{y}_{\mathrm{ipt}}^{(\mathrm{Pr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, p, t$$

$$\mathrm{y}_{\mathrm{ijt}}^{(\mathrm{St})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, j, t$$

$$\mathrm{y}_{\mathrm{sjimt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ s, i, j, m, t$$

$$\mathrm{y}_{\mathrm{iji'mt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, i', j, m, t$$

$$\mathrm{y}_{\mathrm{i'jimt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, i', j, m, t$$

$$\mathrm{y}_{\mathrm{ijcmt}}^{(\mathrm{Tr})} \leq y_{\mathrm{it}}^{(\mathrm{Si})} \qquad \forall\ i, c, j, m, t$$

$$\mathrm{y}_{\mathrm{it}}^{(\mathrm{Si})} - y_{i(t-1)}^{(Si)} \leq y'^{(Si)}_{it} \qquad \forall\ i, t$$

$$\mathrm{y}_{\mathrm{i(t-1)}}^{(\mathrm{Si})} - y_{it}^{(Si)} \leq y''^{(Si)}_{it} \qquad \forall\ i, t$$

$$y_{it}^{(Si)} \leq a_{it\omega}^{(Si)} \qquad \forall\ i, t, \omega$$

$$y_{ijt}^{(Co)} \leq a_{ijt\omega}^{(Co)} \qquad \forall\ i, j, t, \omega$$

$$y_{ipt}^{(\mathrm{Pr})} \leq a_{ipt\omega}^{(\mathrm{Pr})} \qquad \forall\ i, p, t, \omega$$

$$y_{ijt}^{(St)} \leq a_{ijt\omega}^{(St)} \qquad \forall\ i, j, t, \omega$$

$$y_{sjimt}^{(Tr)} \leq a_{simt\omega}^{(Tr)} \qquad \forall\ s, i, j, m, t, \omega$$

$$y_{iji'mt}^{(Tr)} \leq a_{ii'mt\omega}^{(Tr)} \qquad \forall\ i, i', j, m, t, \omega$$

$$y_{ijcmt}^{(Tr)} \leq a_{ijmt\omega}^{(Tr)} \qquad \forall\ i, c, j, m, t, \omega$$

$$y_{it}^{(Si)} \geq m_{it\omega}^{(Si)} \qquad \forall\ i, t, \omega$$

$$y_{ijt}^{(Co)} \geq m_{ijt\omega}^{(Co)} \qquad \forall\ i, j, t, \omega$$

$$y_{ipt}^{(\mathrm{Pr})} \geq m_{ipt\omega}^{(\mathrm{Pr})} \qquad \forall\ i, p, t, \omega$$

$$y_{ijt}^{(St)} \geq m_{ijt\omega}^{(St)} \qquad \forall\ i, j, t, \omega$$

$$y_{sjimt}^{(Tr)} \geq m_{simt\omega}^{(Tr)} \qquad \forall\ s, i, j, m, t, \omega$$

$$y_{iji'mt}^{(Tr)} \geq m_{ii'mt\omega}^{(Tr)} \qquad \forall\ i, i', j, m, t, \omega$$

$$y_{ijcmt}^{(Tr)} \geq m_{ijmt\omega}^{(Tr)} \qquad \forall\ i, c, j, m, t, \omega$$

$$x_{ijt\omega}^{(Co)} \le C_{ijt\omega}^{(Co)} y_{ijt}^{(Co)} \qquad \forall\, i,j,t,\omega$$

$$x_{ipt\omega}^{(\mathrm{Pr})} \le C_{ipt\omega}^{(\mathrm{Pr})} y_{ipt}^{(\mathrm{Pr})} \qquad \forall\, i,p,t,\omega$$

$$x_{ijt\omega}^{(St)} \le C_{ijt\omega}^{(St)} y_{ijt}^{(St)} \qquad \forall\, i,j,t,\omega$$

$$\sum_j x_{sjimt\omega}^{(Tr)} \le C_{simt\omega}^{(Tr)} y_{simt}^{(Tr)} \qquad \forall\, s,i,j,m,t,\omega$$

$$\sum_j x_{iji'mt\omega}^{(Tr)} \le C_{ii'mt\omega}^{(Tr)} y_{ii'mt}^{(Tr)} \qquad \forall\, i,i',j,m,t,\omega$$

$$\sum_j x_{ijcmt\omega}^{(Tr)} \le C_{icmt\omega}^{(Tr)} y_{icmt}^{(Tr)} \qquad \forall\, i,c,j,m,t,\omega$$

$$x_{ijt\omega}^{(Co)}, x_{ijt\omega}^{(St)}, x_{cjt\omega}^{(Sa)}, x_{sjimt\omega}^{(Tr)}, x_{iji'mt\omega}^{(Tr)}, x_{ijcmt\omega}^{(Tr)}, x_{ipt\omega}^{(\mathrm{Pr})} \ge 0 \qquad \forall s,i,c,j,$$

$$m,p,t,i' \ne i,\omega$$

$$y_{ijt}^{(Co)}, y_{ijt}^{(St)}, y_{simt}^{(Tr)}, y_{ii'mt}^{(Tr)}, y_{icmt}^{(Tr)}, y_{ipt}^{(Pr)}, \qquad \forall s,i,c,j,m \;\; p,t,i' \ne i$$

$$y_{it}^{(Si)}, y'^{(Si)}_{it}, y''^{(\mathrm{Si})}_{\mathrm{it}} \in \{0,1\}$$

The DRRPS model obviously becomes computationally prohibitive for finding robust solutions for large numbers of scenarios. In the Chapter IV of this dissertation, we concentrate on presenting effective algorithmic procedures to generate such robust design decisions for such problems.

# CHAPTER IV

# SOLUTION METHODOLOGIES FOR SCENARIO BASED ROBUST OPTIMIZATION WITH A FINITELY LARGE NUMBER OF SCENARIOS

## 4.1 Introduction

All decision-making problems are compounded in difficulty by the degree of uncertainty surrounding the key parameters. One strategy is for decision makers to make decisions with performance close to optimal for all future realizations of parameters' values. Thus, instead of finding optimal decisions for one given future scenario, decision makers will search for decisions that are "robust" for a variety of likely future scenarios.

In this chapter, the uncertainty is represented as a finitely large set of scenarios. The mixed integer linear programming formulation is used to represent the decision-making situation for each scenario. Robust decisions for the mixed integer linear programming problem can be obtained by solving the min-max regret robust optimization problem presented in Chapter III. The size of the problem grows substantially for each scenario considered, and consequently the computation time required to find optimal solutions.

In this chapter, we first develop a heuristic algorithm called the scenario relaxation (SR) algorithm for solving the scenario based mini-max regret robust optimization problems when the number of scenarios is large but finite. This heuristic algorithm guarantees the termination at an optimal robust solution but does not guarantee the shorter computation time than solving the problem directly. This heuristic method initially considers subset of all scenarios and solves the relaxation of the full problem. The optimality condition is then checked. The algorithm terminates if the optimal condition is satisfied, otherwise the algorithm will select some subset of scenarios not yet considered and add them to generate a new relaxation problem. The application of this heuristic is demonstrated in the planning of robust e-scrap reverse production systems for the state of Georgia in Chapter V. The results show a significant improvement in computation time over the direct solution method.

Also in this chapter we extend the use of the accelerated Benders' decomposition algorithm as an alternative solution methodology for the scenario based mini-max regret robust optimization problems with finitely large number of scenarios. The idea of accelerated Benders' decomposition algorithm was originally presented in Santoso (2003) for solving two-level stochastic optimization problems. For the accelerated Benders' approach, this dissertation introduces a set of cuts referred as *sub-problem cuts* that carry the information from sub-problems to the master problem. Finally, the use of the SR algorithm within the accelerated Benders' decomposition framework is also introduced in this dissertation as an alternative solution methodology for the problem.

### 4.2 Scenario Relaxation (SR) Algorithm

The key insight upon which the SR algorithm is built is that it is often true that only a small subset of scenarios must be explicitly examined when searching for the optimal robust solution. This subset will be comprised of two types of scenarios. The first type consists of scenarios required to ensure that the resulting solution is feasible for all scenarios. The second type consists of scenarios required to establish the optimal robust solution. Thus, the SR algorithm starts by establishing the first type of scenarios, starting with a guess informed by knowledge of the problem. The algorithm continues constructing this set by adding infeasible scenarios based on the current robust solution, $y$.

The second set of scenario is constructed (after no infeasible scenario exists for the current robust solution) by a very simple procedure. The procedure starts by solving the problem with some scenarios relaxed. The optimal solution of this relaxed problem is then used to calculate the *regrets from optimality* for all relaxed scenarios. If the optimal value of the relaxed problem, δ, is greater than or equal to all of these regrets, the optimal condition can be confirmed and the algorithm terminates at the optimal robust solution. Otherwise, a subset of these relaxed scenarios with their regrets greater than δ are explicitly considered.

The reason that we can expect the number of scenarios required for solving the problem to be small is that the mini-max regret optimal robust solutions typically have a small number of scenarios with $\delta^*$ equal to the max regret $O_\omega^* - Z_\omega^*(y^*)$ and that this constraint will be slack for the rest of scenarios in a finite set of all possible scenarios Ω. If we could identify these defining scenarios and those required for feasibility, they would form the subset of scenarios essentially required for solving the problem. From these insights and observations, the SR algorithm can be summarized as follows.

<u>*Scenario Relaxation Algorithm*</u>

<u>Step 0:</u> Solve the following problems to optimality and let $UB = \infty$ and $LB = -\infty$.

$$O_\omega^* = \begin{Bmatrix} \max\limits_{x_\omega, y} Z(x_\omega, y) = c_\omega^T x_\omega + f_\omega^T y \\ s.t. \quad A_\omega x_\omega + B_\omega y \le b_\omega \\ x_\omega \ge 0 \text{ and } y \in \Gamma_\omega \end{Bmatrix} \quad \forall \omega \in \Omega$$

<u>Step 1:</u> Identify a set of scenarios $C \subseteq \Omega$ (scenarios for feasibility).

<u>Step 2:</u> Solve the following problem to optimality.

$$\begin{aligned} &\min_{\delta, y, x_\omega} \ \delta \\ &s.t. \quad \left.\begin{aligned} \delta \ge O_\omega^* - c_\omega^T x_\omega - f_\omega^T y \\ A_\omega x_\omega + B_\omega y \le b_\omega \\ x_\omega \ge 0 \text{ and } y \in \Gamma_\omega \end{aligned}\right\} \ \forall \omega \in C \end{aligned}$$

If an optimal solution exists, let $\delta_C^*$ and $y_C^*$ represent the optimal setting of $\delta$ and $y$ respectively and update $LB \leftarrow \delta_C^*$ and go to Step 3. Otherwise stop the algorithm with no robust solution for the problem.

<u>Step 3:</u> Solve the following problems to optimality.

$$Z_\omega^*(y_C^*) = \begin{Bmatrix} \max\limits_{x_\omega} \quad c_\omega^T x_\omega \\ s.t. \quad A_\omega x_\omega \le b_\omega - B_\omega y_C^* \\ x_\omega \ge 0 \end{Bmatrix} + f_\omega^T y_C^* \qquad \forall \omega \in \Omega$$

If an optimal solution exists for scenario ω, let $x_\omega^*$ represent the optimal setting of $x_\omega$ for each $\omega \in \Omega$. Let $W_1$ include all scenarios such that the problem is infeasible and let $W = \{\omega \in \Omega \setminus W_1 \mid O^*_\omega - Z^*_\omega(y^*_C) > \delta^*_C\}$.

<u>Step 4:</u> If $W_1 \neq \Phi$, go to Step 5. Otherwise, update $UB \leftarrow \min(UB, \max_{\omega\in\Omega}(O^*_\omega - Z^*_\omega(y^*_C)))$.

If $UB - LB \leq \varepsilon$ for non-negative pre-specified $\varepsilon$, the algorithm is terminated and the resulting $\varepsilon$-optimal robust solution is $y^*$ where $\max_{\omega\in\Omega}(O^*_\omega - Z^*_\omega(y^*)) = UB$. Otherwise, go to Step 6.

<u>Step 5:</u> Select a set $W_1' \subseteq W_1$ and set $C \leftarrow C \cup W_1'$ and go to Step 2.

<u>Step 6:</u> Select a set $W' \subseteq W$ and set $C \leftarrow C \cup W'$ and go to Step 2.

The following proposition shows that by setting $\varepsilon = 0$, the heuristic algorithm will either terminate at an optimal robust solution if one exists or determine that no feasible robust solution exists.

<u>*Proposition* 1</u>: *The scenario relaxation algorithm either terminates at an optimal robust solution or determines that no feasible robust solution exists by setting* $\varepsilon = 0$.

<u>*Proof*:</u> There are two termination rules in the SR algorithm. The first termination rule is in Step 2 when the relaxation problem becomes infeasible. If the relaxed problem has no solution, it can only mean that there exists no feasible robust solution to the full problem. The second termination rule is in Step 4, when $\varepsilon = 0$, the condition is equivalent to $W \cup W_1 = \Phi$. If this is the case, it means that $O^*_\omega - Z^*_\omega(y^*_C) \leq \delta^*_C \;\; \forall \omega \in \Omega$. Because $y^*_C$ is a feasible discrete solution to the problem, it is true that:

$$\delta^* = \max_{\omega\in\Omega}(O^*_\omega - Z^*_\omega(y^*)) \leq \max_{\omega\in\Omega}(O^*_\omega - Z^*_\omega(y^*_C)) = \delta^*_C.$$

On another hand, because $\delta_C^*$ is the optimal objective function value of the relaxation of the original minimization problem, it is true that $\delta_C^* \le \delta^*$. These results show that $y_C^*$ is an optimal robust solution to the problem. □

For some problem structures and some scenario designs, set $C$ in Step 1 of the SR algorithm can be predetermined. Such is the case with the case study on e-scrap reverse production system for state of Georgia presented in the next chapter.

There are no known theoretical results that determine the methodologies for selecting set *W'* and set *W* which will guarantee the improvement in computational time required to solve the problem. In the following section, we present some heuristic algorithms for selecting these sets.

*Selection Methodology for Set W'*

In this section, we present three heuristic selection methods for set *W'* in Step 6 of the SR algorithm. These alternative approaches are the conservative selection method, the fixed size selection method, and the value relation selection method. Each is addressed in turn.

Conservative Selection Method

The conservative selection method sets *W'* to be *W* in Step 6 of the SR algorithm. This selection method requires fewer algorithm iterations than other selection methods with the tradeoff of longer computation times per iteration.

Fixed Size Selection Method

The fixed size selection method selects set $W' \subseteq W$ in Step 6 of the SR algorithm such that *W'* is the scenario set containing the *m* highest $O_\omega^* - Z_\omega^*(y_C^*)$ function values in *W* where $m = \min\{n_i, |W|\}$ and $n_i$ is a pre-specified constant for iteration *i* of the algorithm. This selection method requires fewer algorithm iterations when a large $n_i$ value is used with the trade off of longer computation time per iteration, and vice versa when a small $n_i$ value is used.

Value Relation Selection Method

The value relation selection method selects set $W' \subseteq W$ in Step 6 of the SR algorithm such that

$$W' = \left\{ \omega \in W \mid \frac{\max_{u \in \Omega}(O_u^* - Z_u^*(y_C^*)) - O_\omega^* + Z_\omega^*(y_C^*)}{\max_{u \in \Omega}(O_u^* - Z_u^*(y_C^*))} \le \varepsilon \right\} \text{where } \varepsilon \in [0,1].$$

This selection method requires fewer algorithm iterations when a large ε value is used with the trade off of longer computation time per iteration, and vice versa when a small ε value is used.

Each of these heuristic methods for selecting the set *W'* presented above has advantages and disadvantages relative to computational requirements of the SR algorithm. Decision makers have to select the proper selection method based on the trade off between number of iterations required for the SR algorithm and the time required for each iteration.

*Selection Methodology for Set $W_1$'*

In this section, we present two heuristic selection methods for set $W_1$' in Step 5 of the SR algorithm. These alternative approaches are the conservative selection method and the fixed size selection method. Each is addressed in turn.

Conservative Selection Method

The conservative selection method sets $W_1$' to be $W_1$ in Step 5 of the SR algorithm. This selection method requires fewer algorithm iterations than other selection methods with the trade off of longer computation times per iteration.

Fixed Size Selection Method

This fixed size selection method selects set $W_1' \subseteq W_1$ in Step 5 of the SR algorithm such that $W_1$' is the scenario set containing $m$ scenarios in $W_1$ with the highest objective function value for the phase I problem where $m = \min\{n_i, |W_1|\}$ and $n_i$ is a pre-specified constant for iteration $i$ of the algorithm.

An alternative method is to select $W_1$' such that $W_1$' is the scenario set containing $m$ scenarios in $W_1$ with the lowest objective function value from the following linear programming problem.

$$\begin{aligned} \max \quad & \delta \\ s.t. \quad & \vec{1}\delta \leq \vec{s} \\ & A_\omega x + \vec{s} = b_\omega - B_\omega y_C^* \\ & x \geq 0 \end{aligned}$$

This selection method requires fewer algorithm iterations when a large $n_i$ value is used with the trade off of longer computation time per iteration, and vice versa when a small $n_i$ value is used.

Each of these heuristic methods for selecting the set $W_1'$ presented above has advantages and disadvantages relative to computational requirements of the SR algorithm. Decision makers have to select the proper selection method based on the trade off between number of iterations required for the SR algorithm and the time required for each iteration.

### 4.3 Accelerated Benders' Decomposition Algorithm

It is not unusual for realistically sized mathematical models to produce mixed integer linear programs with many thousands or even millions of rows and columns. To solve such problems, some method must be applied to convert the large problems into one or more appropriately coordinated smaller problems of manageable size. Popular decomposition methodologies include Dantzig-Wolfe decomposition (Dantzig and Wolfe, 1960), Benders' decomposition (Benders, 1962) and Lagrangian relaxation techniques (Falk, 1967).

In general, a decomposition principle is a systematic procedure for solving large-scale general mathematical programs or specific mathematical programs with special structure. The strategy of a decomposition procedure is to iterate between two separate mathematical programs. Information is passed back and forth until a point is reached where the solution to the original problem is achieved.

The decomposition methodology we use for the DRRPS model in this dissertation is an accelerated Benders' decomposition algorithm (Santoso, 2003). The DRRPS model contains only one set of binary decision variables for all scenarios. If their values can be fixed, the problem can be partitioned into several linear programming problems (one for each scenario) that can be solved independently. For this reason, the DRRPS model is an ideal problem structure for applying the Benders' decomposition algorithm.

This section begins by restating a form of the DRRPS model and developing every property required for the application of Benders' decomposition: convexity of the objective function and the subgradient required for support function. This results in a statement of resulting master problem and sub-problems. These structures are used for the accelerated Benders' approach, where several of the cuts developed by Santoso (2003) are extended to the DRRPS model and a new type of cut, the sub-problem cut, is introduced.

As previously introduced in Chapter III, the DRRPS model is a mixed integer linear programming model with the following structure and $\Omega$ is a finite set of scenarios:

$$\min_{x_\omega, y}(\max_{\omega\in\Omega}(O_\omega^* - \left(\vec{V}_\omega\right)^T \vec{x}_\omega - \left(\vec{F}_\omega\right)^T \vec{y}))$$

subject to:

$$\begin{array}{lll}
A_\omega \vec{x}_\omega = \vec{0} & \text{(inventory balancing constraints)} & \forall \omega \in \Omega \\
B_\omega \vec{x}_\omega = \vec{s}_\omega & \text{(supply constraints)} & \forall \omega \in \Omega \\
C_\omega \vec{x}_\omega \le \vec{d}_\omega & \text{(demand constraints)} & \forall \omega \in \Omega \\
D_\omega \vec{y} \le \vec{0} & \text{(logic constraints)} & \forall \omega \in \Omega \\
E_\omega \vec{x}_\omega - G_\omega \vec{y} \le \vec{0} & \text{(capacity constraints)} & \forall \omega \in \Omega \\
\vec{x}_\omega \ge \vec{0} & & \forall \omega \in \Omega \\
\vec{y} \in \{0, 1\}^{|y|} & &
\end{array}$$

This structure can also be rewritten in the following form:

$\min_{\vec{y}}(f(\vec{y}) \mid D_\omega \vec{y} \le \vec{0}\ \forall \omega \in \Omega, \vec{y} \in \{0,1\}^{|y|})$ where $f(\vec{y}) = \max_{\omega\in\Omega}(O^*_\omega - (\vec{F}_\omega)^T \vec{y} - Q_\omega(\vec{y}))$ and

$$Q_\omega(\vec{y}) = \left\{ \begin{array}{lll} & \max_{x_\omega} \vec{V}^T_\omega \vec{x}_\omega & \\ s.t. & A_\omega \vec{x}_\omega = \vec{0} & (\pi^1_{\omega,y}) \\ & B_\omega \vec{x}_\omega = \vec{s}_\omega & (\pi^2_{\omega,y}) \\ & C_\omega \vec{x}_\omega \le \vec{d}_\omega & (\pi^3_{\omega,y}) \\ & E_\omega \vec{x}_\omega \le G_\omega \vec{y} & (\pi^4_{\omega,y}) \\ & \vec{x}_\omega \ge \vec{0} & \end{array} \right\}$$

and $\pi^i_{\omega,y}\ \ \forall i = 1,2,3,4$ represent the dual variables associated with the model constraints. In order to apply Benders' decomposition, it is required that $f(\vec{y})$ is a convex function on $\vec{y}$. The following proposition gives this result.

<u>Proposition</u> <u>2:</u> *$f(\vec{y})$ is a convex function on $\vec{y}$.*

<u>Proof:</u> $f(\vec{y}) = \max_{\omega\in\Omega}(O^*_\omega - (\vec{F}_\omega)^T \vec{y} - Q_\omega(\vec{y}))$ is obviously a convex function on $\vec{y}$ because of the following reasons.

1. $Q_\omega(\vec{y})$ and $(\vec{F}_\omega)^T \vec{y}$ are concave function on $\vec{y}$.
2. (-1)*concave function is a convex function.
3. Summation of convex functions is also a convex function.
4. Maximum function of convex functions is also a convex function. □

The key ideas of Benders' decomposition algorithm, using the convexity of $f(\vec{y})$ on $\vec{y}$, are the use of support functions of function $f(\vec{y})$ to approximate $f(\vec{y})$ and use the minimum value from this approximation as the lower bound on the actual minimum value of the function. Note that the support function of function $f(\bar{y})$ at $\bar{y}^i$ is $f(\bar{y}^i)+s^T(\bar{y}-\bar{y}^i)$ where $s \in \partial f(\bar{y}^i)$ is a *subgradient* of $f$ at $\bar{y}^i$ and $\partial f(\bar{y}^i)$ is the *subdifferential* of *f* at $\bar{y}^i$.

<u>*Definition* 1 (*Nemhauser and Wolsey, 1988*):</u> (Subdifferential and Subgradient) the subdifferential $\partial f(\bar{y}^i)$ of a convex (concave) function f at $\bar{y}^i$ is the set of vectors $s \in \Re^n$ satisfying $f(\bar{y}) \geq (\leq) f(\bar{y}^i)+s^T(\bar{y}-\bar{y}^i) \quad \forall \bar{y}$. A vector $s \in \partial f(\bar{y}^i)$ is called a subgradient of f at $\bar{y}^i$.

The result from the following proposition provides the proper subgradient of *f* at $\bar{y}^i$.

<u>*Proposition* 3:</u> $-F^T_{\omega^i,\bar{y}^i} - (\pi^*_{\omega^i,\bar{y}^i})^T G_{\omega^i} \in \partial f(\bar{y}^i)$ where $\omega^i \in \arg\max_{\omega \in \Omega}(O^*_\omega - F^T_\omega \bar{y}^i - Q_\omega(\bar{y}^i)$.

<u>*Proof*:</u> From the result of strong duality theory,

$$Q_\omega(\bar{y}) = (\pi^{*2}_{\omega,\bar{y}})^T \bar{s}_\omega + (\pi^{*3}_{\omega,\bar{y}})^T \vec{d}_\omega + (\pi^{*4}_{\omega,\bar{y}})^T (G_\omega \bar{y})$$

$$Q_\omega(\bar{y}) - Q_\omega(\bar{y}^i) = (\pi^{*2}_{\omega,\bar{y}})^T \bar{s}_\omega + (\pi^{*3}_{\omega,\bar{y}})^T \vec{d}_\omega + (\pi^{*4}_{\omega,\bar{y}})^T (G_\omega \bar{y}) - (\pi^{*2}_{\omega,\bar{y}^i})^T \bar{s}_\omega - (\pi^{*3}_{\omega,\bar{y}^i})^T \vec{d}_\omega - (\pi^{*4}_{\omega,\bar{y}^i})^T (G_\omega \bar{y}^i)$$

$$Q_\omega(\bar{y}) - Q_\omega(\bar{y}^i) \leq (\pi^{*2}_{\omega,\bar{y}^i})^T \bar{s}_\omega + (\pi^{*3}_{\omega,\bar{y}^i})^T \vec{d}_\omega + (\pi^{*4}_{\omega,\bar{y}^i})^T (G_\omega \bar{y}) - (\pi^{*2}_{\omega,\bar{y}^i})^T \bar{s}_\omega - (\pi^{*3}_{\omega,\bar{y}^i})^T \vec{d}_\omega - (\pi^{*4}_{\omega,\bar{y}^i})^T (G_\omega \bar{y}^i)$$

$$Q_\omega(\bar{y}) - Q_\omega(\bar{y}^i) \leq (\pi^{*4}_{\omega,\bar{y}^i})^T (G_\omega \bar{y}) - (\pi^{*4}_{\omega,\bar{y}^i})^T (G_\omega \bar{y}^i)$$

$$Q_\omega(\bar{y}) - Q_\omega(\bar{y}^i) \leq (\pi^{*4}_{\omega,\bar{y}^i})^T G_\omega (\bar{y} - \bar{y}^i)$$

$$-Q_\omega(\bar{y}) \geq -Q_\omega(\bar{y}^i) - (\pi^{*4}_{\omega,\bar{y}^i})^T G_\omega (\bar{y} - \bar{y}^i) \qquad \forall \omega \in \Omega$$

$$O^*_{\omega^i} - Q_{\omega^i}(\bar{y}) \geq O^*_{\omega^i} - Q_{\omega^i}(\bar{y}^i) - (\pi^{*4}_{\omega^i,\bar{y}^i})^T G_{\omega^i} (\bar{y} - \bar{y}^i)$$

from $-F_{\omega^i}^T \bar{y} = -F_{\omega^i}^T \bar{y}^i - F_{\omega^i}^T(\bar{y}-\bar{y}^i)$ then

$$O_{\omega^i}^* - F_{\omega^i}^T \bar{y} - Q_{\omega^i}(\bar{y}) \geq O_{\omega^i}^* - F_{\omega^i}^T \bar{y}^i - Q_{\omega^i}(\bar{y}^i) - ((\pi_{\omega^i,\bar{y}^i}^{*4})^T G_{\omega^i} + F_{\omega^i}^T)(\bar{y}-\bar{y}^i)$$

$$f(\bar{y}) \geq O_{\omega^i}^* - F_{\omega^i}^T \bar{y} - Q_{\omega^i}(\bar{y}) \geq O_{\omega^i}^* - F_{\omega^i}^T \bar{y}^i - Q_{\omega^i}(\bar{y}^i) - ((\pi_{\omega^i,\bar{y}^i}^{*4})^T G_{\omega^i} + F_{\omega^i}^T)(\bar{y}-\bar{y}^i)$$

$$f(\bar{y}) \geq f(\bar{y}^i) + (-(\pi_{\omega^i,\bar{y}^i}^{*4})^T G_{\omega^i} - F_{\omega^i}^T)(\bar{y}-\bar{y}^i)$$

$$-(\pi_{\omega^i,\bar{y}^i}^{*4})^T G_{\omega^i} - F_{\omega^i}^T \in \partial f(\bar{y}^i)$$ □

From proposition 3, the master problem and sub-problems for Benders' decomposition of the DRRPS model can be defined as follows:

<u>*Master-problem*</u>:

$$\begin{aligned}
\min_{\theta,\bar{y}} \quad & \theta = lower\ bound\ on\ f(\bar{y}) \\
s.t. \quad & \theta \geq f(\bar{y}^i) + (-F_{\omega^i}^T - (\pi_{\omega^i \bar{y}^i}^{4*})^T G_{\omega^i})(\bar{y}-\bar{y}^i) \quad \forall i = 0,1,2,\dots,K \\
& \bar{y} \in Y^K \subseteq \{\bar{z} \mid D_\omega \bar{z} \leq \vec{0} \ \ \forall \omega \in \Omega, \bar{z} \in \{0,1\}^{|y|}\}
\end{aligned}$$

where $\omega^i \in \arg\max_{\omega\in\Omega}(O_\omega^* - F_\omega^T \bar{y}^i - Q_\omega(\bar{y}^i))$, $f(\bar{y}^i) = O_{\omega^i}^* - F_{\omega^i}^T \bar{y}^i - Q_{\omega^i}(\bar{y}^i)$ and $Y^K$ contains all extra cuts together with original constraints and integrality constraints.

<u>*Sub-problems*</u>:

$$Q_\omega(\bar{y}^i) = \left\{ \begin{array}{lll}
 & \max_{x_\omega} \vec{V}_\omega^T \bar{x}_\omega & \\
s.t. & A_\omega \bar{x}_\omega = \bar{0} & (\pi_{\omega,y}^1) \\
 & B_\omega \bar{x}_\omega = \bar{s}_\omega & (\pi_{\omega,y}^2) \\
 & C_\omega \bar{x}_\omega \leq \vec{d}_\omega & (\pi_{\omega,y}^3) \\
 & E_\omega \bar{x}_\omega \leq G_\omega \bar{y}^i & (\pi_{\omega,y}^4) \\
 & \bar{x}_\omega \geq \bar{0} &
\end{array} \right\} \forall \omega \in \Omega$$

with $f(\bar{y}^i) = O^*_{\omega^i} - F^T_{\omega^i}\bar{y}^i - Q_{\omega^i}(\bar{y}^i)$ as the upper bound on $f(\bar{y})$ if $Q_\omega(\bar{y}^i)$ exists $\forall \omega \in \Omega$. The accelerated Benders' approach is built on the original Benders' decomposition algorithm with accelerator cuts. The algorithm will terminate when the difference between upper and lower bounds is less than some nonnegative predetermined ε. The following section contains the detailed methodology for the accelerated Bender's decomposition algorithm.

*Accelerated Benders' Decomposition Algorithm*

Step 0: (Initialization) Select $\bar{y}^0 \in \{\bar{z} \mid D_\omega \bar{z} \le \bar{0}\ \forall \omega \in \Omega, \bar{z} \in \{0,1\}^{|y|}\}$ and $Q_\omega(\bar{y}^0)$ exists $\forall \omega \in \Omega$. Set $LB$ = -∞, $UB$ = +∞, $K$ = 0 and $Y^0 = \{\bar{z} \mid D_\omega \bar{z} \le \bar{0}\ \forall \omega \in \Omega, \bar{z} \in \{0,1\}^{|y|}\} \cup$ sub-problem cuts and trust region constraints. Note that $\bar{y}^0$ can be constructed from SR algorithm.

Step 1: (Iteration $K$) Solve sub-problem $\forall \omega \in \Omega$ for $Q_\omega(\bar{y}^K)$.

If the solution is infeasible $\exists \omega \in \Omega$, $K \leftarrow K-1$ and go to Step 5.

Otherwise, $Y^K \leftarrow Y^{K-1} \cup$ Knapsack cuts and go to Step 2.

Step 2: Let $\pi^{4*}_{\omega,\bar{y}^K}$ be the optimal dual solution for sub-problem of scenario $\omega \in \Omega$.

Let $\omega^K \in \arg\max_{\omega\in\Omega}(O^*_\omega - F^T_\omega \bar{y}^K - Q_\omega(\bar{y}^K))$. If $UB > O^*_{\omega^K} - F^T_{\omega^K}\bar{y}^K - Q_{\omega^K}(\bar{y}^K) = f(\bar{y}^K)$,

$UB \leftarrow O^*_{\omega^K} - F^T_{\omega^K}\bar{y}^K - Q_{\omega^K}(\bar{y}^K)$ and $\bar{y}^{Opt} = \bar{y}^K$.

Step 3: If UB – LB ≤ ε (nonnegative predetermined value), stop and $\bar{y}^{Opt}$ is the ε-optimal robust solution. Resolve sub-problems for optimal $\bar{x}_\omega$ $\forall \omega \in \Omega$.

Otherwise go to Step 4.

Step 4: Solve master-problem

$$\min_{\theta,\bar{y}} \theta = \textit{lower bound on } f(\bar{y})$$

$$s.t. \quad \theta \geq f(\bar{y}^i) + (-F_{\omega^i}^T - (\pi_{\omega^i \bar{y}^i}^{4*})^T G_{\omega^i})(\bar{y} - \bar{y}^i) \quad \forall i = 0,1,2,...,K$$

$$\bar{y} \in Y^K$$

Set $K \leftarrow K+1$, $\bar{y}^K \leftarrow \bar{y}^*$, $LB \leftarrow \theta^*$ and go to Step 1.

Step 5: $Y^K \leftarrow Y^K \cup$ Extreme ray cuts and go to Step 4.

To accelerate the Benders' decomposition algorithm, trust regions and additional cuts were proposed by Santoso (2003). These cuts can be extended to the DRRPS model as shown below. Also a new type of cut called the sub-problem cut is introduced. The following subsection provides the detail of trust region cut, knapsack cut, extreme ray cut, and sub-problem cut.

*Trust Region Constraints*

An undesirable feature of Benders' decomposition algorithm is the wild oscillation of solutions from one region of feasible set to another, which causes slow convergence of the algorithm. Santoso (2003) first introduced the use of trust region constraints with Benders' decomposition for two-stage stochastic programming. The trust region constraint in the master problem at iteration $i$+1 can be represented as: $\sum_{j \in \{l | y_l^i = 1\}} (1 - y_j) + \sum_{j \in \{l | y_l^i = 0\}} y_j \leq \Delta^i < |\bar{y}|$. In order to ensure the convergence of the algorithm, the trust region constraints will be imposed in the initial iterations of the algorithm, and will be dropped once the solution has been stabilized.

In this dissertation, we classify trust region constraints into two types. The first type is referred as global trust region constraint. This type of constraint applies the trust region concept to all binary decision variables in the master-problem. The second type of constraints is referred to as the local trust region constraint. This second type of constraint only applies the trust region concept on some of the important binary decision variables. For example, trust region constraints can be applied only to the binary decision variables corresponding to site opening decisions, which have the most effect on the objective function value of the model.

*Knapsack Cuts*

Santoso (2003) first introduced the use of knapsack cuts with Benders' decomposition for two-stage stochastic programming. This type of cuts can improve the quality of the solution from the master problem if the high quality upper bound information is available.

Let $\bar{y}''$ be one of the feasible good robust solutions of the problem attained from any heuristic procedure. The knapsack cut can be constructed by using the following arguments.

$$f(\bar{y}'') \geq \min_{\bar{y}} f(\bar{y}) \geq \theta \geq f(\bar{y}^i) + (-F_{\omega^i}^T - (\pi_{\omega^i,\bar{y}^i}^{4*})^T G_{\omega^i})(\bar{y} - \bar{y}^i) \quad \forall i = 0,1,\ldots,K$$

$$f(\bar{y}'') - f(\bar{y}^i) + (-F_{\omega^i}^T - (\pi_{\omega^i,\bar{y}^i}^{4*})^T G_{\omega^i})\bar{y}^i \geq (-F_{\omega^i}^T - (\pi_{\omega^i,\bar{y}^i}^{4*})^T G_{\omega^i})\bar{y} \quad \forall i = 0,1,\ldots,K$$

Finally, the knapsack is represented in the following form:

$$\left\lfloor f(\bar{y}'') - f(\bar{y}^i) + (-F_{\omega^i}^T - (\pi_{\omega^i,\bar{y}^i}^{4*})^T G_{\omega^i})\bar{y}^i \right\rfloor \geq \left\lfloor (-F_{\omega^i}^T - (\pi_{\omega^i,\bar{y}^i}^{4*})^T G_{\omega^i}) \right\rfloor \bar{y} \quad \forall i = 0,1,\ldots,K$$

Note that $f(\bar{y}'')$ can be replaced with any known good upper bound on the optimal objective function value. If a good upper bound is available, then adding the above knapsack cuts can have a significant impact in generating a high quality solution from the master problem in the iteration $i + 1$.

*Extreme Ray Cuts*

This cut is the classical type of cut for Benders' decomposition algorithm for preventing the master problem from generating the sub-problem infeasible solution.

Let $\omega' \in \Omega$ be a scenario such that the sub-problem associated with this scenario is infeasible under the solution $\bar{y}'$ from the master-problem. The extreme ray cut will be generated with the purpose of eliminating not only the solution $\bar{y}'$ but also some other possible infeasible solutions from the next solution of the master-problem. The extreme ray cut generated for the master-problem will have the following structure:

$$\vec{s}_{\omega'}^{T}\vec{v}_2 + \vec{d}_{\omega'}^{T}\vec{v}_3 + (G_{\omega'}\vec{y})^T\,\vec{v}_4 \geq 0$$

where $\vec{v}_2, \vec{v}_3, \vec{v}_4$ can be calculated from the following linear programming problem where $\vec{1}$ is the vector with all elements equal to one.

$$\begin{aligned} \min_{\vec{v}_1,\vec{v}_2,\vec{v}_3,\vec{v}_4} \; & (\vec{s}_{\omega'}^{T}\vec{v}_2 + \vec{d}_{\omega'}^{T}\vec{v}_3 + (G_{\omega'}\vec{y}')^T\,\vec{v}_4) \\ s.t. \quad & A_{\omega'}^{T}\vec{v}_1 + B_{\omega'}^{T}\vec{v}_2 + C_{\omega'}^{T}\vec{v}_3 + E_{\omega'}^{T}\vec{v}_4 \geq \vec{0} \\ & -\vec{1} \leq \vec{v}_1 \leq \vec{1},\; -\vec{1} \leq \vec{v}_2 \leq \vec{1} \\ & \vec{0} \leq \vec{v}_3 \leq \vec{1}, \quad \vec{0} \leq \vec{v}_4 \leq \vec{1} \end{aligned}$$

*Sub-Problem Cuts*

In the early iterations of the Benders' decomposition, the master-problem contains only logical constraints, $\{D_{\omega}\bar{y} \leq \vec{0} \ \ \forall \omega \in \Omega, \bar{y} \in \{0,1\}^{|y|}\}$ and a few cuts. At these iterations, the master-problem tends to produce the solution $\bar{y}'$ such that it could be sub-problem infeasible for some scenario $\omega' \in \Omega$. In order to improve the quality of the master-problem solutions to be sub-problem feasible for all scenarios, information of the sub-problem should be included in the master-problem constraints. These additional constraints for the master-problem are referred as sub-problem cuts in this dissertation. The sub-problem cuts used in this dissertation are listed below.

$$\mathrm{S}^{(\mathrm{Su})}_{\mathrm{sjt}\omega} \leq \sum_{i}\sum_{m} \mathrm{C}^{(\mathrm{Tr})}_{\mathrm{simt}\omega} y^{(\mathrm{Tr})}_{\mathrm{simt}} \qquad \forall s, j, \omega, t \tag{1}$$

$$\sum_{m}\sum_{s} y^{(Tr)}_{simt} + \sum_{m}\sum_{i'} y^{(Tr)}_{i'imt} + \sum_{j} y^{(St)}_{ijt-1} \leq M\left(\sum_{m}\sum_{i'} y^{(Tr)}_{ii'mt} + \sum_{m}\sum_{c} y^{(Tr)}_{icmt} + \sum_{j} y^{(St)}_{ijt}\right) \quad \forall i,t \tag{2}$$

$$\sum_{m}\sum_{i'} y^{(Tr)}_{ii'mt} + \sum_{m}\sum_{c} y^{(Tr)}_{icmt} + \sum_{j} y^{(St)}_{ijt} \leq M\left(\sum_{m}\sum_{s} y^{(Tr)}_{simt} + \sum_{m}\sum_{i'} y^{(Tr)}_{i'imt} + \sum_{j} y^{(St)}_{ijt-1}\right) \quad \forall i,t \tag{3}$$

$$\sum_{j} y^{(Co)}_{ijt} \geq y^{(Tr)}_{simt} \quad \forall s,i,m,t \tag{4}$$

$$\sum_{p} y^{(\mathrm{Pr})}_{ipt} \geq y^{(Si)}_{it} \quad \forall i,t \tag{5}$$

$$\sum_{i}\sum_{c}\sum_{m} y^{(Tr)}_{icmt} \geq \text{Minimum numbers of satisfied customers in period } t \quad \forall t \tag{6}$$

Constraints (1) ensure that the master-problem solution will always have enough transportation capacity for the supply of each material at each source for each time period for all scenarios. Constraints (2) and (3) ensure that if there are some flows into the specific site at the specific time period, there will always be some flows out of that site at that time period. Constraints (4) ensure that if there are some flows from any source to the specific site, the site will always initiate its collection process. Constraints (5) ensure that if the site is opened, there will always be some activity at that site. Finally constraints (6) ensure that there will always be some flows from some sites to some customers in each time period if the minimum numbers of satisfied customers are positive.

This section illustrates one of many possible extensions of the use of the accelerated Benders' decomposition algorithm (Santoso, 2003) for making the mini-max regret robust decisions with the finitely large number of scenarios. Decision makers can consider this accelerated Benders' decomposition algorithm as one of the good alternative solution methodologies for this type of the problem.

### 4.4 Summary

This chapter presents two alternative solution methodologies for solving the large-scale mini-max regret robust optimization problems caused by finitely large number of possible scenarios when the direct solution methodology fails to solve the problem in reasonable amount of time.

The first alternative algorithm is the SR algorithm, which use the results from the observation that the robust solution can be achieved by solving the problem considering only a smaller subset of all possible scenarios. This subset consists of two types of scenarios. The first type of scenarios consists of scenarios that control the feasibility of the solution over all possible scenarios. The second type of scenarios consists of scenarios that control the minimum maximum regret of the problem. This chapter also provides the proof that the SR algorithm converges to the optimal robust solution if one exists in finite number of iterations. Several heuristics for set selection set in the SR algorithm are presented as the alternative procedures for decision makers. Even though there is no theoretical proof guaranteeing the faster computational time required for solving the problem than the direct method, the result from the next chapter illustrates the significant reduction in computational time required for solving the case study problem.

The second alternative algorithm is the accelerated Benders' decomposition algorithm (Santoso, 2003). This chapter presents one possible extension of the accelerated Benders' decomposition algorithm for solving the mini-max regret robust optimization problem. When applying the algorithm to the DRRPS model, the new type of cuts called sub-problem cut is also presented.

The next chapter of this dissertation presents an application of the SR algorithm on the planning of robust e-scrap reverse production systems for the state of Georgia where the problem cannot be solved using the direct approach.

# CHAPTER V

# CASE STUDY FOR SCENARIO BASED ROBUST OPTIMIZATION

## 5.1 Introduction

Electronic equipment is ubiquitous in current wealthy societies. The variety and volume make it inevitable that we inherit significant reuse/recycling/disposal challenges, including collection, transportation and production costs along with serious hazardous waste concerns. But with this challenge comes an opportunity, and in this case the opportunity is to view the e-scrap as a resource and to capture value from the large stream of used electronics.

The objective of this chapter is to describe a case study for the design of a large scale system for collecting, transporting, and processing used electronics in the state of Georgia. Our objective is to maximize the financial viability of the infrastructure and minimize the maximal risk of capital investment when faced with key uncertainties. Due to their predominance in the waste stream, our primary focus in this case study is a subset of the used electronics stream: televisions, CPUs, and computer monitors.

The recycling of electronic equipment in Georgia is a significant problem. For example, we predict that more than 1,500,000 lbs of used televisions, 2,700,000 lbs of used computer monitors, and 3,300,000 lbs of used CPUs could be collected and processed in the state of Georgia each year if 30% of Georgia state households with recyclable materials participate in the recycle program. The case study considers the configuration of a regional electronics recycling system with local area collecting centers and a host of processing centers ranging from large, moderate and small size commercial firms to non-profit organizations.  An illustration of the physical flows of the reverse production system (RPS) for used electronics in Georgia is shown in Figure 5.1.

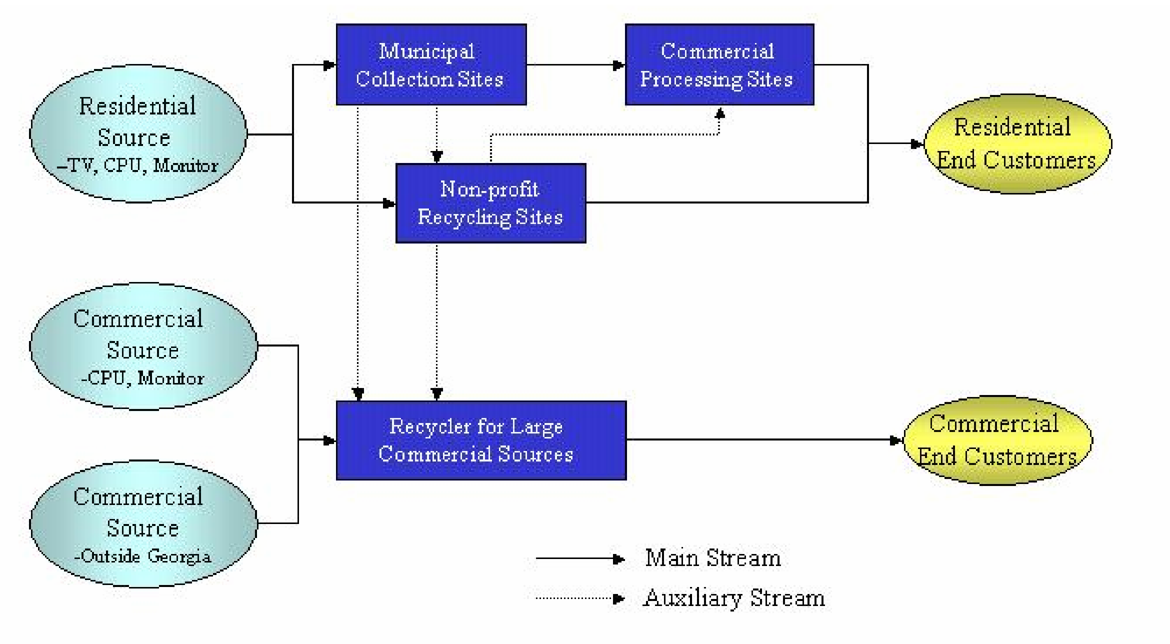


**Figure 5.1  Physical Flow of Used Electronics**

The next section of this chapter will give the detail of the regional case study for the state of Georgia and illustrate the applications of the scenario relaxation algorithm on the case study.

### 5.2 Robust Design for E-Scrap Reverse Production System for the State of Georgia

The case study considers the predominant physical inputs to the system to be used televisions, computer monitors, and CPUs. We assume that no material may go deliberately uncollected, in other words the variables that represent the inflow of the material to the system must equal the amount available for collection. The outputs are in several categories of remanufactured units, component parts, and materials listed in Figure 5.2. The financial flows, depicting profits and costs in different shades are indicated.

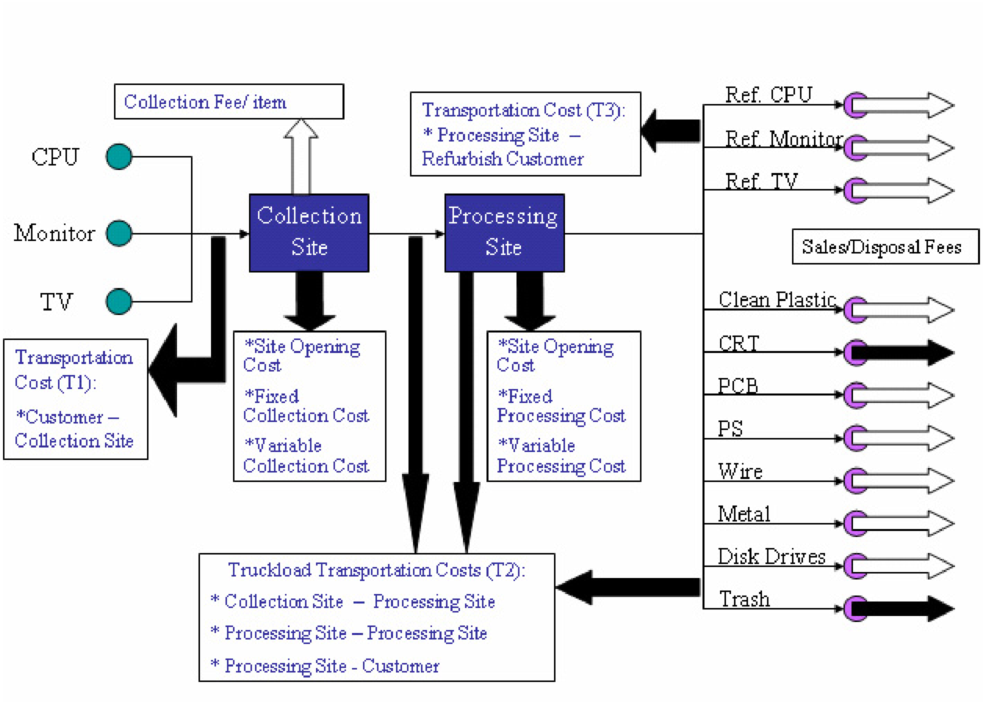


**Figure 5.2 Cash Flow Diagram with Costs (Black) and Profits (White)**

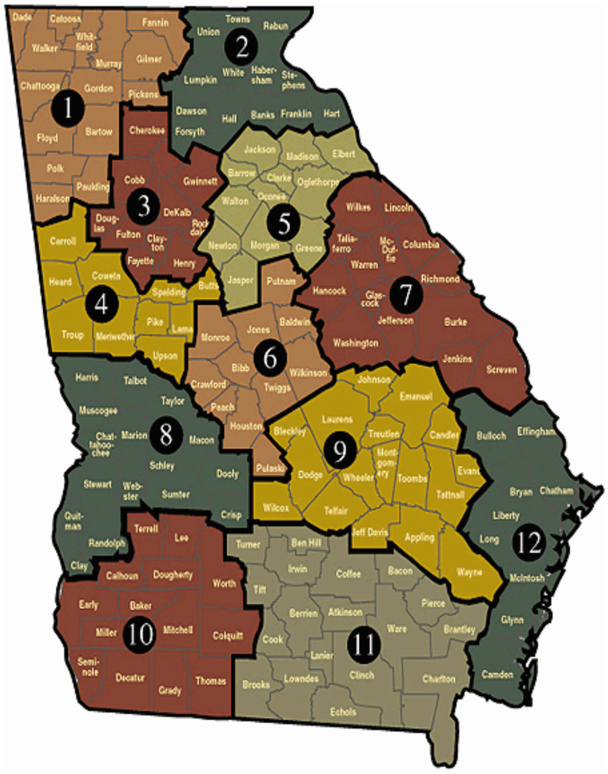


**Figure 5.3 Division of the State of Georgia into 12 DCA Regions**

This case study divides the State of Georgia into 12 disjoint regions as shown in Figure 5.3 based on service delivery regions defined by Georgia's Department of Community Affairs (DCA). Each region represents a source of electronic waste streams, a centralized collection site and also a demand point for the units after refurbishing processes. The amount of used electronic equipments available for collection can be approximated from the population in each region.

For e-scrap originating in the state, the case study considers 12 potential state of Georgia government-collection centers located in the center county of each DCA region. The case study also designates external regions 13 and 14 representing out of state sources of e-scrap. Each collection center is assumed to collect television, monitors, and CPUs supplied by the small business and resident sources located within its 100 miles radius.

Additionally, the case study includes six non-profit collecting centers throughout the state and one large commercial collecting center located in Marietta, Georgia. The large-scale collection center is assumed to collect computer monitors and CPUs supplied by large-scale business sources from both inside and outside of Georgia. Figure 5.4 shows all potential sites considered in the case study.

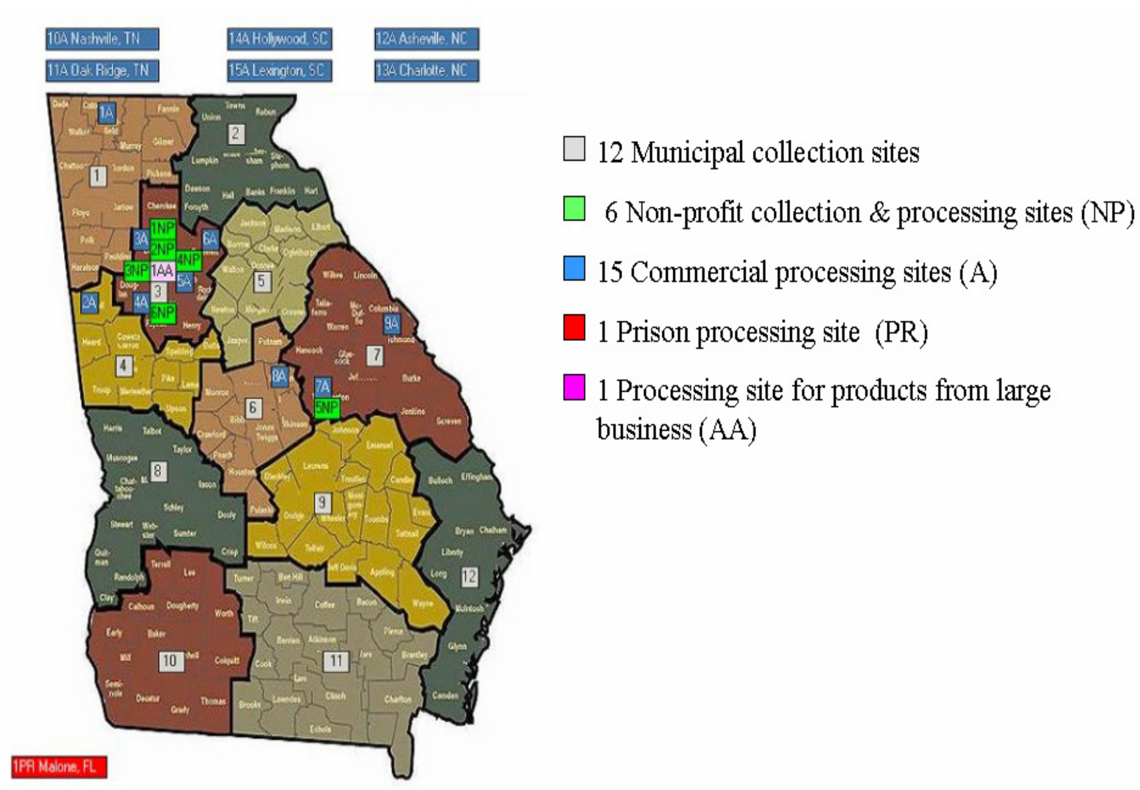


**Figure 5.4 All Potential Sites Considered in the Case Study**

*Supply Information*

Using percentages determined from recent studies in an adjacent state, we estimate the supply of e-scrap by assuming that on average 6.2% of the households have an electronic item ready for recycling (Pasco County, Florida, Pilot Program, April 2000), and 20% to 30% of the total population will participate in the collecting program. This case study assumes that the relative proportions of the amounts collected (in lbs) for televisions, computer monitors, and CPUs are 50:23:27. The case study also assumes that the average weights for televisions, computer monitors, and CPUs are 51.5 lbs, 27.2 lbs, and 29.2 lbs respectively (Alachua County Florida, Summary Report, October 1999). The sources of computer monitors and CPUs are from residential (15%) and business sectors (85%), but the sources for televisions are only from the residential sector.

Table 5.1 shows the estimated supply information for each type of the electronic equipment from each region under the assumption that 30% of the population will participate in the program.

**Table 5.1 Georgia E-Scrap Supply Estimation**

| Region | Supply for TVs (lbs)** | Supply for Monitors (lbs)* | Supply for CPUs (lbs)* |
|---|---|---|---|
| 1 | 133,610 | 216,400 | 272,720 |
| 2 | 87,236 | 141,290 | 178,060 |
| 3 | 657,000 | 1,064,130 | 1,341,040 |
| 4 | 77,388 | 125,340 | 157,960 |
| 5 | 83,970 | 136,000 | 171,400 |
| 6 | 84,318 | 136,570 | 172,110 |
| 7 | 83,339 | 134,980 | 170,110 |
| 8 | 67,680 | 109,620 | 138,150 |
| 9 | 52,283 | 84,680 | 106,720 |
| 10 | 67,605 | 109,500 | 137,990 |
| 11 | 69,912 | 113,240 | 142,700 |
| 12 | 104,024 | 168,480 | 212,330 |
| 13 | 0 | 90,000 | 90,000 |
| 14 | 0 | 90,000 | 90,000 |

* CPUs and Monitors: Amount of supply = participation % × 6.2% × Number of households × Product proportion × (100/15)
** Televisions: Amount of supply = participation % × 6.2% × Number of households × Product proportion

*Collecting Center Information*

The numbers used in the case study for each collecting center are given in Table 5.2.

**Table 5.2 Collecting Center Data**

| Description | Value |
|---|---|
| Fixed collection cost | $16,000 per year per type of material collected* |
| Collection cost | $0.01 per pound |
| Opening cost for government collection sites | $5,000 per year |
| Opening cost for non-profit collection sites | $28,800 per year |
| Opening cost for large commercial-collecting center | $134,500 per year |
| The collection fee charged for small business and residential sources | $5.28 per item |
| The collection fee charged by large business sources | $0.6 per item |

* It is assumed that 1 worker per type of material collected with pay rate of $8 per hour working for 8 hours per day for 250 days per year.
** Assuming subsidies reduce the final cost.

*Processing Center Information*

The case study considers 15 potential commercial processing centers (nine sites located in Georgia, two sites located in Tennessee, two sites located in North Carolina, and two sites located in South Carolina), six nonprofit processing centers, one large commercial processing center, and one prison processing center. Each facility represents an actual refurbishing and/or demanufacturing facility located in Georgia and nearby states. Table 5.3 contains the general information for all 23 potential processing centers considered in the case study.

**Table 5.3 General Information for All 23 Potential Processing Centers**

| Processing Site Designation | State | County/City | Annualized Site Opening Cost | Number of facilities | Type |
|---|---|---|---|---|---|
| 1A | Georgia | Catoosa | $28,800 | 1 | Commercial processing sites |
| 2A | Georgia | Carroll | $28,800 | 1 | |
| 3A | Georgia | Cobb | $28,800 | 2 | |
| 4A | Georgia | Fulton | $28,800 | 5 | |
| 5A | Georgia | DeKalb | $28,800 | 6 | |
| 6A | Georgia | Gwinnett | $28,800 | 1 | |
| 7A | Georgia | Washington | $28,800 | 1 | |
| 8A | Georgia | Baldwin | $28,800 | 1 | |
| 9A | Georgia | Richmond | $28,800 | 1 | |
| 10A | Tennessee | Davidson | $28,800 | 1 | |
| 11A | Tennessee | Anderson | $28,800 | 2 | |
| 12A | North Carolina | Buncombe | $28,800 | 1 | |
| 13A | North Carolina | Mechlenburg | $28,800 | 1 | |
| 14A | South Carolina | Charleston | $28,800 | 1 | |
| 15A | South Carolina | Lexington | $28,800 | 1 | |
| 1NP | Georgia | Marietta | $28,800 | 1 | Nonprofit processing sites |
| 2NP | Georgia | Atlanta | $28,800 | 1 | |
| 3NP | Georgia | Atlanta | $28,800 | 1 | |
| 4NP | Georgia | Tucker | $28,800 | 1 | |
| 5NP | Georgia | Sandersville | $28,800 | 1 | |
| 6NP | Georgia | East Point | $28,800 | 1 | |
| 1PR | Florida | Malone | $19,200 | 1 | Prison processing site |
| 1AA | Georgia | Marietta | $134,500 | 1 | Large commercial processing site |

For each processing center, there are six main potential processes: television refurbishment, monitor refurbishment, CPU refurbishment, television demanufacturing, monitor demanufacturing, and CPU demanufacturing, but not all processing centers can perform all these six processes. The information for these six processes is presented in Tables 5.4 and Table 5.5.

**Table 5.4 Variable Costs for Refurbishing and Demanufacturing Processes**

| Description | Value |
|---|---|
| Variable processing cost for refurbishing TVs | $0.23 per lbs* |
| Variable processing cost for refurbishing monitors | $0.44 per lbs* |
| Variable processing cost for refurbishing CPUs | $0.51 per lbs* |
| Variable processing cost for demanufacturing TVs | $0.05 per lbs** |
| Variable processing cost for demanufacturing monitors | $0.09 per lbs** |
| Variable processing cost for demanufacturing CPUs | $0.08 per lbs** |
| Variable processing cost for demanufacturing process in prison site | $0.00425 per lbs |

* It is estimated by assuming the processing labor cost is $10 per hour and replacing costs are $8, $8, and $10 for TV, monitor, and CPU respectively. The testing process will take on average of 10 minutes and the refurbishing process will take on average of 20 minutes (DAAE30-98-C-1050, 2000)

** This information is the average of the information from Waters (1998), Pepi (1998), and Minnesota Office of Environmental Assistance (2001).

**Table 5.5 Fixed Processing Costs for Each Processing Center**

<table>
<tr><th>Sites</th><th>Description</th><th>Annualized Value</th></tr>
<tr><td rowspan="2">Commercial processing sites</td><td>Fixed processing cost for refurbishing all products</td><td>$8,820 per process (DAAE30-98-C-1050, 2000)</td></tr>
<tr><td>Fixed processing cost for demanufacturing all products</td><td>$8,000 per process</td></tr>
<tr><td>Non-profit processing sites</td><td>Fixed processing cost</td><td>$26,667 per process (Phillips, 2003)</td></tr>
<tr><td rowspan="2">Large commercial processing site</td><td>Fixed processing cost for refurbishing process</td><td>$6,250 per process (Nejad, 2003)</td></tr>
<tr><td>Fixed processing cost for demanufacturing process</td><td>$32,000 per process (Nejad, 2003)</td></tr>
<tr><td>Prison processing site</td><td>Fixed processing cost for demanufacturing process</td><td>$500 per process *</td></tr>
</table>

* Estimated utility fee per year for the process in the prison

*Demand Information*

The processing centers provide an output of remanufactured equipment, parts, and recycled material to a set of demand locations. We consider four types of demand sources and estimate the quantities using the assumption that the demand for refurbished products are greater than or equal to the supply of used products provided by that region. The first type of demand comes from people within Georgia who are interested in buying refurbished electronic equipment. For this type of demand, we use the same 12 DCA regions to designate the demand locations.

The second type of demand source is the group of recycling facilities interested in buying metal, plastic, CRT, and other demanufactured materials. We consider a total of five recyclers located in several states: Georgia (metal recycler), Florida (CRT products and electronics recycler), Texas (plastics recycler), and Ohio (CRT glass recycler).

The third type of demand comes from both resident and commercial sources that are interested in buying refurbished commercial electronic equipments in large batches provided by the large commercial processing site.

The last type of demand describes landfills to which we can send the non-hazardous trash resulting from the demanufacturing. We consider eight landfills located in Georgia and group them into 5 demand points based on the DCA regions. (Landfill location information can be found at http://www.wastebyrail.com/network.html.). Table 5.6 illustrates the price information for each refurbished product and material.

**Table 5.6 Price Information for Refurbished Products and Materials**

| Parameter | Value |
|---|---|
| Selling price for plastic ($ per lb) | 0.175* |
| Selling price for PCB ($ per lb) | 0.9* |
| Selling price for disc drive ($ per lb) | 0.2* |
| Selling price for CRT ($ per lb) | -0.1* |
| Selling price for metal ($ per lb) | 0.0175* |
| Selling price for wire ($ per lb) | 0.165* |
| Selling price for power supply ($ per lb) | 0.06* |
| Selling price for trash ($ per lb) (land fill tipping fee) | -0.028* |
| Selling price for used TV ($ per unit) | 60.00 |
| Selling price for used monitor ($ per unit) | 49.00 *** |
| Selling price for used CPU ($ per unit) | 49.00 *** |
| Selling price for broken CPU ($/lbs) | 0.02** |
| Selling price for usable CPU ($/lbs) | 0.108** |
| Selling price for broken monitor ($/lbs) | -0.257** |
| Selling price for usable monitor ($/lbs) | 0.0184** |
| Selling price for broken television ($/lbs) | -0.25** |
| Selling price for usable television ($/lbs) | -0.25** |

* EPA-901-R-00-002, September 2000
** The data is from http://www.scrapcomputers.com
***The data is from http://www.boxq.net

*Transportation Information*

There are three types of transportation cost considered in this case study. The first type corresponds to the transportation cost of the people who travel to the collecting center and drop off their used electronic equipment. This type of transportation cost is approximated by the gasoline cost ($0.15 per mile) and we assume that on average one trip can carry up to 50 lbs of electronic equipment. With this approximation, the transportation cost per lb per mile is $0.003.

The second type represents the transportation costs for moving material between collection centers and processing centers, the transportation costs for moving material between processing centers and recycler demand points, and the transportation costs for moving material between processing centers and landfill demand points. This type of transportation can be performed by a large truck with the cost of $2 per ton per mile or $0.0009 per lb per mile.

The last type corresponds to the transportation cost charged by United Parcel Service (UPS). This cost is about $0.26 per mile per item. This information can be found on the UPS website (www.ups.com).

The data for the Georgia case study represents a large-scale electronics recycling infrastructure design problem. The objective of the problem is to maximize net profit for the system while determining which collection and processing sites to utilize and then what quantities of each item type to process into what materials at each site.

The key uncertain parameters that we examine are described as follows.

1. *Participation rate.* For one half of the problems, we examined the two situations where 20% or 30% of the households with an item to recycle contributed at least one used electronic item for collection.

2. *CRT recycler.* Currently there are no leaded glass-to-glass recyclers in the state of Georgia, and the closest facility requires expensive transportation of these materials to Ohio. We require either that leaded glass materials be transported to the Ohio processor or we allow the commercial processors pass these materials amongst themselves (as is currently done), even if this may eventually result in the "dumping" of these hazardous wastes.

3. *Televisions usability percentage.* Used televisions that cannot be refurbished and resold incur a high cost. However, many households will hold on to their televisions

until they no longer work. In our study we solve our problems with the condition that either 10% or 30% of the collected televisions are in re-usable condition.

4. *CPU & monitor usability percentage*. Similarly, a key uncertainty is the condition of the CPUs and monitors that are collected. We construct half of our problems assuming usability rates of (CPU 40%, monitor 40%) and the other half with (CPU 20%, monitor 20%).

The four types of uncertainty factors, with two levels specified for each factor, results in $2^4$ or sixteen scenarios to be studied. In other words, each scenario describes a unique electronics recycling infrastructure design problem to be solved. The sixteen problems or scenarios are defined in Figure 5.5.

| ***Useable %:*** TV: 30% CPU: 40% Monitor: 40% | | CRT Recyclers | |
|---|---|---|---|
| | | With all CRT Recyclers | With only CRT recycler in OH |
| Percent Participation | *10%* | ***Sc 1*** | ***Sc 2*** |
| | *30%* | ***Sc 3*** | ***Sc 4*** |

| ***Useable %:*** TV: 10% CPU: 40% Monitor: 40% | | CRT Recyclers | |
|---|---|---|---|
| | | With all CRT Recyclers | With only CRT recycler in OH |
| Percent Participation | *10%* | ***Sc 5*** | ***Sc 6*** |
| | *30%* | ***Sc 7*** | ***Sc 8*** |

| ***Useable %:*** TV: 30% CPU: 20% Monitor: 20% | | CRT Recyclers | |
|---|---|---|---|
| | | With all CRT Recyclers | With only CRT recycler in OH |
| Percent Participation | *10%* | ***Sc 9*** | ***Sc 10*** |
| | *30%* | ***Sc 11*** | ***Sc 12*** |

| ***Useable %:*** TV: 10% CPU: 20% Monitor: 20% | | CRT Recyclers | |
|---|---|---|---|
| | | With all CRT Recyclers | With only CRT recycler in OH |
| Percent Participation | *10%* | ***Sc 13*** | ***Sc 14*** |
| | *30%* | ***Sc 15*** | ***Sc 16*** |

**Figure 5.5 Key Uncertainty Value Settings for Sixteen Scenarios**

There are three observations for this case study problem which will help us determining the set $C$ (subset of scenarios that controls feasibility of the robust solution over all possible scenarios) without any additional calculation. First, the discrete solution, which can handle high supply scenarios, can also handle low supply scenarios in this case study. Second, the discrete solution, which is feasible under the restrictions on specific CRT recyclers, is also feasible for the case without these restrictions. Finally the discrete solution, which can handle the scenarios with extremely high and extremely low percentage of products re-usability, can also handle scenarios with moderate value on percentage of products re-usability. From these three observations, the constraint structure and scenario designs of this case study can determine set $C$ to consist of scenarios 4 and 16.

Our case study problems were solved by a Windows 2000-based Pentium 4 1.80GHz personal computer with 1GB RAM using Visual Express v13D (Dash, 2002) for the optimization software. MS-Access and Visual basic programming languages were used as the case study database and user interface programs. This case study problem consists of two main mixed integer linear programming models (RPS and DRRPS) and one main linear programming model (RPSLP). The RPS model is used to calculate $O_{\omega}^{*}$ function value $\forall \omega \in \Omega$ and the DRRPS model is used to search for a robust optimal solution, $y^{*}$. The RPSLP model is used to find $x_{\omega}^{*}$ and $O_{\omega}^{*} - Z_{\omega}^{*}(y^{*}) \, \forall \omega \in \Omega$ once $y^{*}$ has been calculated. The information on the size of each model is summarized in Table 5.7.

**Table 5.7 Size of Each Model for the Case Study**

| Model Type | Number of Discrete Variables | Number of Continuous Variables | Number of Constraints |
|---|---|---|---|
| RPS | 3,150 | 76,950 | 89,433 |
| DRRPS | 3,150 | 1,231,041 | 1,384,059 |
| RPSLP | N/A | 76,950 | 80,018 |

Using the direct approach for this problem, the robust optimal solution could not be found since the DRRPS model took more than 192 hours of computation time with out returning any feasible solution to the problem. On another hand by using SR algorithm, the problem can be solved to optimality within 3 iterations using less than 50 hours of computational time. The fixed size method of selecting *W'* where $n$ = 2 was used. The information on computational time and detail of the heuristic algorithm are summarized in Table 5.9. Table 5.8 shows the comparison between optimal and robust solution for each scenario. The robust infrastructure design for this case study is shown in Figure 5.6 and Table 5.10. Figure 5.7 shows the bar chart of the objective function values comparison for each scenario from Table 5.9.

**Table 5.8 Comparison between Optimal and Robust Solution for Each Scenario**

| Scenario | Optimal Profit | Robust Profit | Regret | % Comparison |
|---|---|---|---|---|
| 1 | 2,922,602 | 2,825,162 | 97,440 | 96.67% |
| 2 | 2,677,033 | 2,584,692 | 92,341 | 96.55% |
| 3 | 4,371,895 | 4,200,988 | 170,907 | 96.09% |
| 4 | 3,995,216 | 3,837,216 | 158,001 | 96.05% |
| 5 | 2,585,981 | 2,473,352 | 112,629 | 95.64% |
| 6 | 2,310,663 | 2,203,656 | 107,008 | 95.37% |
| 7 | 3,882,715 | 3,652,868 | 229,847 | 94.08% |
| 8 | 3,442,097 | 3,249,933 | 192,164 | 94.42% |
| 9 | 1,375,246 | 1,149,325 | 225,921 | 83.57% |
| 10 | 1,098,493 | 865,521 | 232,972 | 78.79% |
| 11 | 1,956,051 | 1,822,830 | 133,221 | 93.19% |
| 12 | 1,531,504 | 1,390,899 | 140,605 | 90.82% |
| 13 | 1,035,005 | 797,515 | **237,490** | 77.05% |
| 14 | 721,951 | 484,486 | **237,465** | 67.11% |
| 15 | 1,442,492 | 1,274,710 | 167,782 | 88.37% |
| 16 | 981,766 | 803,616 | 178,149 | 81.85% |

**Table 5.9 Computational Time and Detail of the SR Algorithm**

| Iteration | ***C*** | $\delta_C^*$ | ***W***$_1$ | ***W*** | ***W'*** | **CPU time for DRRPS (minutes)** | **CPU time for 16 RPSLP (minutes)** |
|---|---|---|---|---|---|---|---|
| 1 | {4, 16} | 144,454 | *Φ* | {3, 7, 8, 9,10,13, 14} | {13, 14} | 240 | 32 |
| 2 | {4, 13, 14, 16} | 228,245 | *Φ* | {7, 15} | {7, 15} | 780 | 32 |
| 3 | {4, 7, 13, 14, 15, 16} | 237,490 | *Φ* | *Φ* | *Φ* | 1,800 | 32 |

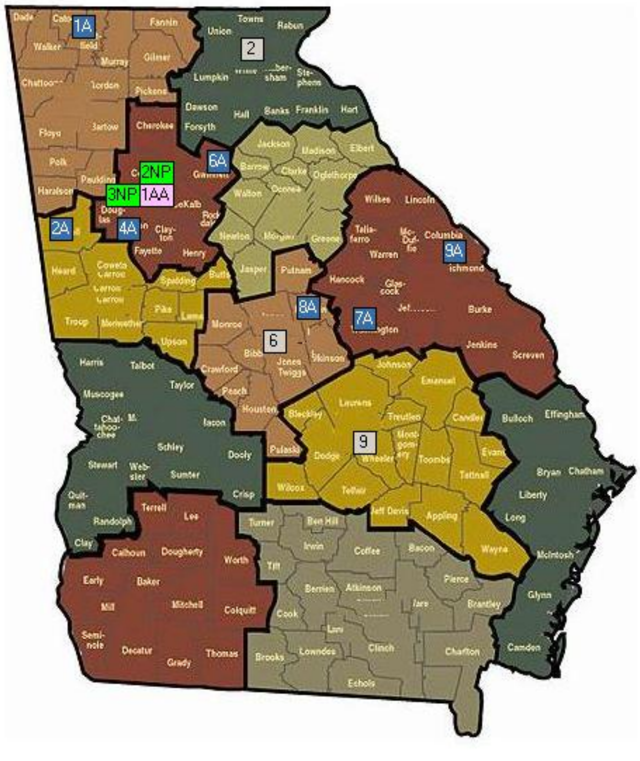


**Figure 5.6 Robust Infrastructure Design for Georgia Case Study**

## Table 5.10 Optimal and Robust Solutions for Georgia E-Scrap RPS Infrastructure

| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Scenario | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | Robust |
| Participation | | | L | L | H | H | L | L | H | H | L | L | H | H | L | L | H | H | |
| TV reusability | | | H | H | H | H | L | L | L | L | H | H | H | H | L | L | L | L | |
| CPU & monitor reusability | | | H | H | H | H | H | H | H | H | L | L | L | L | L | L | L | L | |
| CRT recycler * | | | √ | | √ | | √ | | √ | | √ | | √ | | √ | | √ | | |
| | Site | Location | | | | | | | | | | | | | | | | | |
| Collection Sites | 1 | Gordon Co., GA | | | | | | | | | | | | | | | | | |
| | 2 | White Co., GA | | | | | | | • | • | | | | | | | • | • | • |
| | 3 | DeKalb Co., GA | | | | | | • | | | • | • | | | • | • | | | |
| | 4 | Meriwether Co., GA | | | | | | | | | | | | | | | | | |
| | 5 | Oconee Co., GA | | | | | | | | | | | • | | | | | | |
| | 6 | Bibb Co., GA | • | • | | | • | • | | | • | • | • | | • | • | | • | • |
| | 7 | Richmond Co., GA | | | | | | | | | | | | | | | | | |
| | 8 | Chattahoochee Co., GA | | | • | | | | • | • | • | • | | | | | • | | |
| | 9 | Toombs Co., GA | • | • | • | | • | • | • | | • | • | • | • | • | • | • | • | • |
| | 10 | Dougherty Co., GA | | • | | • | | | | | | | | • | | | | | |
| | 11 | Ware Co., GA | | | | • | | | | | | | | | | | | | |
| | 12 | Chatham Co., GA | | | | • | | | | • | | | | | | | | | |
| Non-profit Sites | 1NP | Marietta, GA, GA | | | • | • | | | | | | | | • | | | | | |
| | 2NP | Atlanta, GA | | • | • | • | • | • | • | • | • | | • | • | | | • | • | • |
| | 3NP | Atlanta, GA | • | • | • | • | • | • | • | • | | • | • | • | • | • | • | • | • |
| | 4NP | Tucker, GA | • | | | | | | | | | | | | | | | | |
| | 5NP | Sandersville, GA | | | • | • | | | • | • | | | | • | | | • | • | |
| | 6NP | East Point, GA | | | | | | | | | | | | | | | | | |
| Commercial Processing Sites | 1A | Catoosa Co., GA | | | • | | | | • | • | | | | | | | | | • |
| | 2A | Carroll Co., GA | | | • | • | | | • | • | | | | | | | • | | • |
| | 3A | Cobb Co., GA | | | | | | | • | • | | | | | | • | | | |
| | 4A | Fulton Co., GA | • | • | | • | • | • | • | | • | • | | | | • | • | | • |
| | 5A | DeKalbCo., GA | | | • | | | | | • | | | • | • | • | | | • | |
| | 6A | Gwinnett Co., GA | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • |
| | 7A | Washington Co., GA | • | • | | | • | • | | | • | • | • | | • | • | • | | • |
| | 8A | Baldwin Co., GA | • | • | • | • | • | • | • | | | | • | | | | | • | • |
| | 9A | Richmond Co., GA | | | • | • | • | | • | • | | | | • | | | | | • |
| | 10A | Davidson Co., TN | | | | | | | | | | | | | | | | | |
| | 11A | Anderson Co., TN | | | | | | | | | | | | | | | | | |
| | 12A | Buncombe Co., NC | | | | | | | | | | | | | | | | | |
| | 13A | Mechlenburg Co., NC | | | | | | | | | | | | | | | | | |
| | 14A | Charleston, SC | | | | • | | | | • | | | | | | | | | |
| | 15A | Lexington Co., SC | • | | | | | | | | | | | | | | | | |
| ** | 1PR | Jackson Co., FL | | | | • | | | | | | | | • | | | • | | |
| *** | 1AA | Marietta, GA | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • | • |

* CRT recycler: "√" denotes the CRT recycler options are with all CRT recyclers, otherwise, the option is only restricted in the CRT recycler in Ohio.

** The prison processing site

*** The large commercial processing site

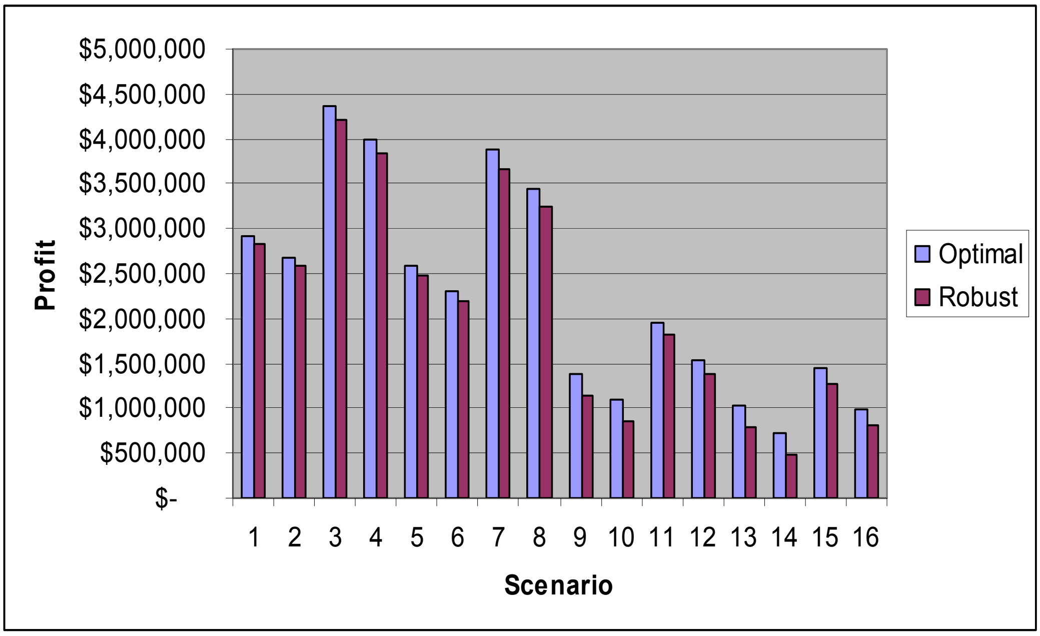


**Figure 5.7 Bar Chart of the Objective Values Comparisons for Each Scenario**

There are several conclusions that can be drawn from Figure 5.7. First, it is clear that while the robust infrastructure solution does not perform as well in any of the scenarios as does the scenario's optimal solution, the robust solution performs very well in all of the possible scenarios (9 scenarios with approximately 95% of the optimal value, 4 scenarios with approximately 90% of the optimal value, 3 scenarios with approximately 80% of the optimal value, 2 scenarios with approximately 77% of the optimal value, and 1 scenarios with 67% of the optimal value). Second, for the given input data values, it appears that economically viable solutions (i.e., solutions that yield a positive net profit) can be found for all of the problem scenarios. Even in financially tough situations like Scenario 14, solutions can be determined that yield an estimated positive net profit.

It is also interesting to analyze how the cost burdens compare between highly favorable economic conditions (like Scenario 3, with a high percentages of households participating, a high number of usable televisions collected, and all CRT processors available) and unfavorable ones (like Scenario 14, with low participation rates, many more unusable televisions, and the restriction of only being able to use the Ohio CRT recycler). Figure 5.8 illustrates this comparison. In Scenario 14, the transportation costs begin to overwhelm processing and other costs. Both cases are consistent with reports from other regions where the transportation costs for electronics recycling compose approximately half of the overall system costs.

Similarly, the relative sources of revenues can be compared when economic conditions are highly favorable (Scenario 3) or when they are not as good (Scenario 14). Figure 5.9 illustrates this comparison. Under less favorable economic conditions the revenue stream is more highly dependent on collection fees as a source of revenue.

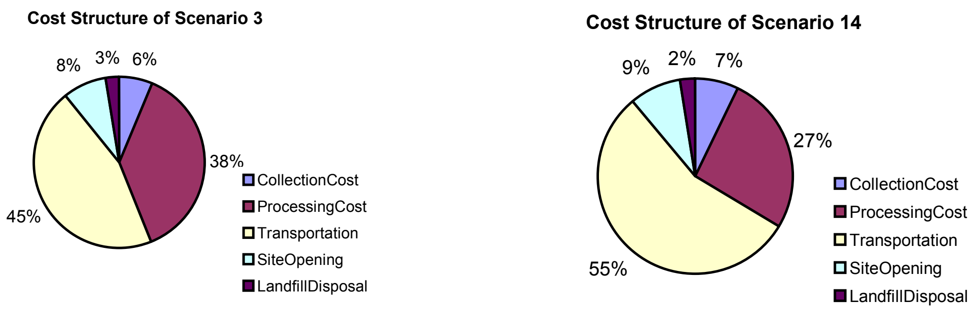


**Figure 5.8 Comparisons of Relative Costs for Highly Favorable Conditions (Scenario 3) and Unfavorable Conditions (Scenario 14)**

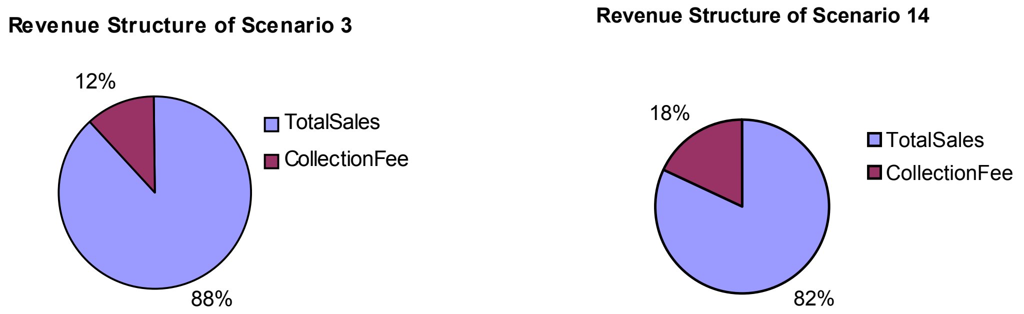


**Figure 5.9 Comparisons of Revenue Sources for Highly Favorable Conditions (Scenario 3) and Unfavorable Conditions (Scenario 14)**

## 5.3 Summary

In this chapter, the heuristic algorithm for the scenario based min-max regret robust optimization has been applied to the case study of designing the robust infrastructure for the realistic size reverse production system problem. The algorithm successfully creates a design for used electronics RPS infrastructure in the state of Georgia. It is distinguished by the novel way that it captures uncertainty and produces robust solutions. Data based on a variety of sources has been used to approximate the regional electronics recycling infrastructure design problem for Georgia.

Sixteen alternative problem scenarios have been analyzed to understand how the infrastructure design solutions are affected by key uncertainties in the household participation rates, the percentage of used electronics collected that are reusable, and the access to glass CRT recyclers. From these solutions we have learned that the resulting net profits and corresponding material flows vary greatly depending on the predicted conditions.

A robust infrastructure design has been found that performs well in all of the scenarios. The resulting solutions suggest that an economically viable electronics-recycling infrastructure is possible for the state of Georgia. This analysis is now being utilized by the Georgia Computer Equipment Disposal and Recycling Council (Georgia Code 12-8-33.1) and state agencies as the region's e-scrap reuse/recycling problem is being addressed.

# CHAPTER VI

# A SEMI-CONTINUOUS ROBUST METHODOLOGY

## 6.1 Introduction

Growing attention is being given to the problem of efficiently designing and operating reverse supply chain systems to handle the return flows of production wastes, packaging, and end-of-life products. Because the information that exists for these new reverse supply chains is limited, solution methodologies for solving strategic infrastructure of reverse production systems under uncertainty are critical to support effective business and government decision making. This chapter presents a new robust optimization algorithm for designing network infrastructure when uncertainty affects the outcomes of the decisions and decision makers are adverse to risk.

This new algorithm for reverse production system planning can be effectively used in designing network infrastructure when the joint probability distributions of key parameters are unknown. The algorithm only requires the information on potential ranges and possible discrete values of uncertain parameters, which often are available in practice. The algorithm involves the use of bi-level programming, which coordinates a "game" between decision makers and the decision environment. The environment is allowed to choose its perturbations and the optimal solution for the set of parameters. Simultaneously, the current candidate robust solution is then allowed to respond by changing certain continuous decision values, such as its flows. This game is played for each iteration of the algorithm.

This chapter also discusses many pre-processing and problem transformation procedures for improving the computational ability of the algorithm. The proof that the algorithm always terminates at an optimal robust solution in finite number of iterations is also provided.

The approach can be generalized to the robust design of network supply chain systems with reverse production systems as one of their subsystems. The resultant system will tend to be more financially and operationally viable if properly planned, since even with the least favorable realization of the parameters, the system may still perform close to optimal levels. Several problems have been solved in Chapter VII to illustrate the application of this new algorithm in designing the robust reverse production system infrastructures.

### 6.2 Outline of the Semi-Continuous Robust Algorithm

The semi-continuous robust algorithm presented here is a newly developed robust optimization algorithm able to handle almost all possibilities in the model uncertain parameters' values for both discrete type parameters and continuous type parameters. The discrete type parameter is the parameter that takes its values from a finite set of discrete values. The continuous type parameter is the parameter that takes its values from a real compact interval. The semi-continuous robust approach is the combination of a three-stage algorithm and several pre-processing algorithms. The three-stage algorithm is structured upon the convergence of an upper and a lower bound to the problem. The information is sent back and forth between the three stages until the optimality condition is satisfied.

The first stage of the algorithm generates a robust decision based on a considered set of scenarios. After the first stage has been solved to optimality using the RPS and DRRPS models, the candidate robust decision is then passed to the second stage.

The second stage of the algorithm performs the feasibility check on this candidate robust decision over all possible scenarios. If there exists a scenario that is infeasible under the current candidate robust decision, the information will be sent back to the first stage requesting a new candidate robust decision. On another hand, if all scenarios are feasible under the current candidate robust decision, the information will be forward to the third stage of the algorithm.

The third stage of the algorithm performs several pre-processing steps and determines new scenarios to make the maximum regret possible for the current candidate robust decision. The scenarios generated by this stage are then passed back to the first stage. Using the scenarios supplied by the third stage, the first stage either confirms the globally optimal robust solution or generates a new candidate robust decision. Figure 6.1 shows a schematic of this approach.

There is one important assumption for applying the semi-continuous robust algorithm presented in this chapter: all model parameters must be independent. Chapter VIII of this dissertation presents a solution methodology for the problem with correlated parameters.

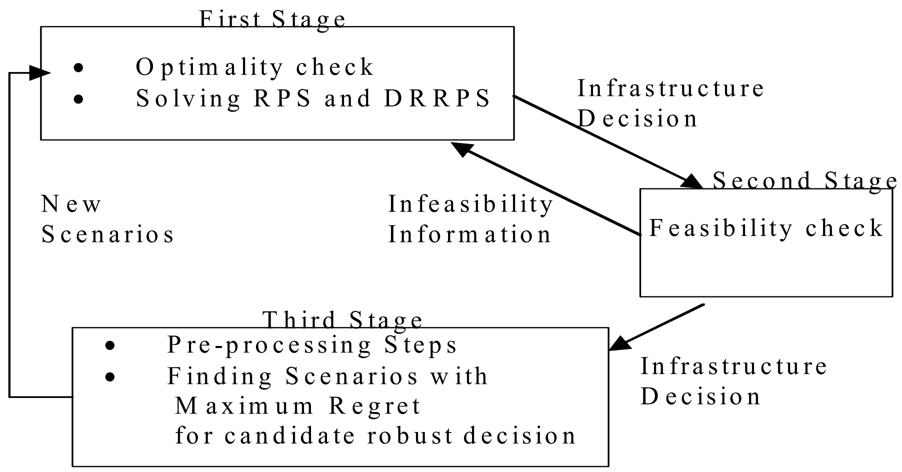


**Figure 6.1 Semi-Continuous Robust Algorithm**

In this chapter, we classify the parameters in RPS model into five major types of parameters, $p_i$ $\forall\, i = 1, 2, \ldots, 5$. The parameters of type $p_1$ represent the parameters corresponding to coefficient of binary decision variables in the RPS objective function. This type of parameter represents site opening costs, site closing costs, fixed site operating costs, fixed storage costs, fixed collecting costs, fixed processing costs and fixed transportation costs parameters.

The parameters of type $p_2$ represent the coefficients of the binary decision variables located in the functional constraints of the RPS model. This type of parameters represents maximum collection capacity, maximum storage capacity, maximum transportation capacity, and maximum process capacity parameters.

The parameters of type $p_3$ represent all right hand side parameters in the functional constraints of the RPS model. This type of parameters represents the maximum supply and maximum demand parameters.

The parameters of type $p_4$ represent the coefficient parameters of the continuous decision variables in the objective function of the RPS model. This type of parameters represents the selling price per unit, storage cost per unit, collection cost per unit, collection fee per unit, processing cost per unit, and transportation cost per unit parameters.

The parameters of type $p_5$ represent the coefficients of continuous decision variables in the functional constraints of the RPS model. This type of parameters represents the proportion of material consumed by the process and the proportion of material produced by the process parameters.

The detailed methodologies of all three stages of the algorithm are presented in the following sections.

### 6.3 The First Stage Methodology

The purposes of the first stage are (1) to find the robust solution for all scenarios considered initially from the previous iteration including the new scenarios from the second stage and the third stage, (2) to find the lower bound for the global robust optimal solution, and (3) to determine if the robust solution is global robust optimal solution for the problem. Let $\Delta^L$ denote the lower bound for the global robust optimal solution.

The first stage of this algorithm consists of two main mathematical models (RPS and DRRPS models). The RPS model is used to find the optimal objective function value for all available scenarios. Each scenario may have been identified in the initial scenario set or from the second stage or the third stage of the algorithm. If the RPS problem is infeasible for any scenario, there exists no robust solution to the problem. Otherwise the

RPS optimal objective function values for all considered scenarios are used as the required parameters in the DRRPS model.

After the optimal value for the RPS model has been calculated for each scenario, the scenario is incorporated into the set of scenarios considered in the DRRPS model. The DRRPS model is used to find the robust solution that achieves the minimum value of the maximum regret from the optimal objective function value in all scenarios considered. If the DRRPS problem is infeasible, there exists no robust solution to the problem. Otherwise the robust solution generated in this step is sent to the second stage of the algorithm if the optimality condition is not satisfied. The optimal condition will be satisfied when the difference between the upper bound and the lower bound is sufficiently close. If this is the case, the algorithm will be stopped with the robust optimal solution, which is the robust solution attaining the best upper bound ($\Delta^U$). In other words, the optimal condition will be satisfied when ($\Delta^U$ - $\Delta^L$) $\leq \varepsilon$ for some positive predetermined $\varepsilon$.

## 6.4 The Second Stage Methodology

The purposes of the second stage are to find scenarios that will make the candidate robust solution from the first stage infeasible in the RPS model. This stage of the algorithm consists of two main steps. The first step consists of the pre-processing procedure for parameters. The second step consists of solving bi-level linear programming problems if the procedure in the first step cannot pre-process all model parameters.

For a feasibility check after the $Y_\Omega$ solution is given to the RPS model, it is clear that the values of parameters in the class of $p_1$ and $p_4$ do not have any effect on feasibility of the RPS model. Also after $Y_\Omega$ is passed to the RPS model, the parameters in the class of $p_2$ and $p_3$ can both be considered as right-hand side parameters.

From these observations, one can find the scenarios that make $Y_\Omega$ RPS infeasible by solving bi-level linear programming problems based on the BLLP model. The leader objective function of the BLLP model is to minimize the minimum value of slack variables in the RPS model by controlling all $p_2$ and $p_3$ as the leader variables with restrictions on the upper and lower bounds for each parameter. The follower objective function of the BLLP model is to maximize the minimum value of slack variables in the RPS model by controlling all $\vec{x}$ and slack variables in the RPS model with the original RPS constraints. Figure 6.2 states the BLLP model using mathematical notation.

$$
\begin{array}{lll}
\underset{\vec{p}}{\text{minimize}} & \delta & \\
\text{s.t.} & \vec{p}_L \le \vec{p} \le \vec{p}_U & \\
& \underset{\vec{x},\vec{s},\delta}{\text{maximize}} & \delta \\
& \text{s.t.} & A\vec{x} \pm \vec{s} = \vec{p} \\
& & \delta\vec{1} \le \vec{s} \\
& & \vec{x} \ge \vec{0}
\end{array}
$$

**Figure 6.2 General BLLP Model**

In the general BLLP model, $\vec{0}$ and $\vec{1}$ represent the vectors with the value of 0 and 1 respectively for all elements in the vectors. The sign "+" will be used in the model for the less than or equal inequality constraints in the RPS model; otherwise the sign "–" will be used. If the optimal objective function value of the BLLP model is greater than or equal to zero, the current $Y_\Omega$ is identified to be feasible over all possible scenarios. Otherwise, the resulting optimal setting of $\bar{p}$ will represent one scenario which is RPS infeasible under $Y_\Omega$. The algorithm needs to solve one BLLP model for each initial discrete scenario.

In general, most of the parameters can be pre-processed to either of their bounds even before solving the BLLP model. The following subsections describe the methodology for the parameter pre-processing step and the solution methodology of the BLLP model.

*Parameter Pre-Processing Step for the BLLP Model*

For any right hand side parameter *b*, there are two cases that *b* can be pre-processed.

Case 1: Parameter *b* appears only in less than or equal inequality constraints and all coefficients of all variables on the left hand side are nonnegative.

In this case, it is obvious that parameter *b* can be set to its lower bound at the optimal solution of the BLLP model.

Case 2: Parameter *b* appears only in greater than or equal inequality constraints and all coefficients of all variables in the left hand side are nonnegative.

In this case, it is also obvious that parameter *b* can be set to its upper bound at the optimal solution of the BLLP model.

*Solution Methodology of the BLLP Model*

After applying the pre-processing step, if there still exist some variables in the BLLP model whose value cannot be fixed, the BLLP model can be transformed into an easier problem using the results of the following lemma.

*Lemma* 1: The BLLP model has at least one optimal solution $\vec{p}^*$ in which each element of $\vec{p}$ takes value at its bounds.

*Proof*: Let $\vec{p}^*$ be an optimal solution of the BLLP model such that an element *i* does not take the value form its bounds or $p_i^L < p_i^* < p_i^U$. There are only two possible cases to be considered.

Case 1: $\delta^* < s_i^*$ where $(A\vec{x})_i \pm s_i^* = p_i^*$.

In this case, the value of $p_i^*$ can be adjusted to either of its bound without any effect on the optimality and feasibility of the problem. This statement is quite obvious from the optimality of $\vec{p}^*$ and the structure of the BLLP model.

Case 2: $\delta^* = s_i^*$ where $\left(A\vec{x}^*\right)_i \pm s_i^* = p_i^*$.

In this case, we can easily show that $p_i^*$ has already taken the value from its bounds. There are two sub-cases to be considered.

Sign is +: If $p_i^* > p_i^L$, $\exists \varepsilon > 0$ such that $p_i^* - \varepsilon \geq p_i^L$ and $\left(A\vec{x}^*\right)_i + s_i^* > p_i^* - \varepsilon$.

The value of $\left(A\vec{x}^*\right)_i$ cannot be decreased because of the optimality of $\vec{p}^*$ and $\vec{x}^*$. For this reason the value of $s_i^*$ can be decreased to $s_i^* - \varepsilon$. This contradicts the optimality of $\delta^*$.

Sign is –: If $p_i^* < p_i^U$, $\exists \varepsilon > 0$ such that $p_i^* + \varepsilon \leq p_i^U$ and $\left(A\vec{x}^*\right)_i - s_i^* < p_i^* + \varepsilon$.

The value of $\left(A\vec{x}^*\right)_i$ cannot be increased because of the optimality of $\vec{p}^*$ and $\vec{x}^*$. For this reason the value of $s_i^*$ can be decreased to $s_i^* - \varepsilon$. This also contradicts the optimality of $\delta^*$. □

The results from Lemma 1 greatly simplify the solution methodology of the BLLP model. By adding dual constraints and a strong duality constraint for the follower problem into the BLLP model, the problem is transformed from a bi-level linear programming problem to a single level mixed integer linear programming problem as shown in Figure 6.3.

$$
\begin{aligned}
\text{Minimize} \quad & \delta \\
\text{s.t.} \quad & \vec{p}_L \le \vec{p} \le \vec{p}_U \\
& A\vec{x} \pm \vec{s} = \vec{p} \\
& \delta\vec{1} \le \vec{s} \\
& A^T \vec{w}_1 \ge \vec{0} \\
& \vec{1}^T \vec{w}_2 = 1 \\
& \pm \vec{w}_1 - \vec{w}_2 = \vec{0} \\
& \delta = \vec{p}^T \vec{w}_1 = \vec{p}^T (\pm \vec{w}_2) \\
& \vec{w}_2,\ \vec{x} \ge \vec{0}
\end{aligned}
$$

**Figure 6.3 The Modified BLLP Model**

The nonlinear term in the constraint $\delta = \vec{p}^T \vec{w}_1$ can be transformed into mixed integer linear constraints by using the results of Lemma 1 as shown in Figure 6.4 where *M* is one significantly large number.

$$\delta = \bar{p}^T(\pm\bar{w}_2) \leftrightarrow \delta = \sum_i \left(\pm PW_{2i}\right)$$

$$\left.\begin{aligned} PW_{2i} &\le p_i^U w_{2i} \\ -PW_{2i} &\le -p_i^U w_{2i} + M(1-bi_i) \\ PW_{2i} &\le p_i^L w_{2i} + Mbi_i \\ PW_{2i} &\ge p_i^L w_{2i} \\ bi_i &\in \{0,1\} \end{aligned}\right\} \forall i \text{ such that } p_i \text{ can not be preprocessed}$$

If $p_i$ can be preprocessed at $\tilde{p}_i$ or have the constant value of $\tilde{p}_i, PW_{2i} = \tilde{p}_i w_{2i}$

**Figure 6.4 Transformation of the Strong Duality Constraint in the BLLP Model**

These constraints will only be applied on the terms where parameters cannot be preprocessed, usually a small portion of all parameters. By using the preprocessing step together with this solution methodology, the BLLP model can be solved effectively. The solution of the BLLP model is used as information for the next step of the algorithm to either add scenarios to the first stage or forward a candidate robust decision to the third stage.

### 6.5 The Third Stage Methodology

The purpose of the third stage is to find scenarios that make the robust decisions from the first stage as bad as possible for each of the initial discrete scenarios. This stage of the algorithm will generate the scenarios with the objective of maximizing the regret between optimal objective function value and the objective function value resulting from the candidate robust solution from the first stage for each of the initial discrete scenarios. These scenarios will then be transferred to the first stage along with the best (minimum) upper bound value on global optimal robust objective function value. Let $\Delta^U$ denote this best upper bound value.

The mathematical model used by this stage is the bi-level programming problem, BLPP, with mixed integer variables for the leader problem. In the third stage, the algorithm needs to solve the BLPP models, one model for each scenario from the initial discrete scenarios. Let $Y_{\Omega}$ denote the candidate robust solution from the second stage, and let $\vec{p}^{\,L}$ denote the vector of the lower bound values for all parameters, and let $\vec{p}^{\,U}$ denote the vector of the upper bound values for all parameters. The BLPP model can be generally written as shown in Figure 6.5.

$$
\begin{aligned}
&\max_{x_1, y_1, p}\{\vec{p}_4^{\,T}\vec{x}_1 + \vec{p}_1^{\,T}\vec{y}_1 - \max_{x_2}(\vec{p}_4^{\,T}\vec{x}_2 + \vec{p}_1^{\,T}Y_{\Omega})\} \\
&\quad s.t. \;\; (\vec{p}_5\vec{x}_1)_i + \vec{p}_{2i}\vec{y}_{1\mathrm{i}} \le \vec{p}_{3i} \quad \forall i \\
&\qquad\quad (\vec{p}_5\vec{x}_2)_i + \vec{p}_{2i}Y_{\Omega i} \le \vec{p}_{3i} \quad \forall i \\
&\qquad\quad \vec{p}_k^{\,L} \le \vec{p}_k \le \vec{p}_k^{\,U} \qquad \forall k = 1,2,3, \text{and } 4 \\
&\qquad\quad \forall \vec{x}_1, \vec{x}_2 \ge \vec{0} \quad \vec{y}_1 \in \text{vector of } \{0,1\}
\end{aligned}
$$

**Figure 6.5 General BLPP Model**

The solution methodology for the BLPP model can be classified into four important steps. These four steps are (1) parameter pre-processing step, (2) variable and constraint elimination step, (3) problem transformation step and (4) solution methodology step. Each of these steps is described in the following sections.

*Parameter Pre-Processing Step*

After studying the structure of the BLPP model, we have found that some of the uncertain parameters can be fixed at their bounds at the optimal solution. In some cases, there are some simple rules to identify the optimal values of these parameters when the information on $Y_\Omega$ is given from the second stage. The pre-processing step allows the values for many of these parameters to be fixed even before solving for the BLPP model. The pre-processing step for each of the five parameter types is now described.

Pre-Processing Step for Parameter of Type $p_1$

The parameters of type $p_1$ represent the parameters corresponding to coefficient of binary decision variables in the RPS objective function. Each element of this type of parameter is represented in the objective function of the BLPP model as $Max \quad \pm(p_{1i}y_{1i} - p_{1i}Y_{\Omega i})$.

*Proposition* 4: Given the value of one specific element of $Y_\Omega$ called $Y_{\Omega i}$ from the second stage and the signs in the objective function are adjusted so that all $p_{1i}^U$ are greater than or equal to zero, an optimal value of the specific element of $p_1$ called $p_{1i}$ can be predetermined by the following rules:

Case 1: If sign is + and $Y_{\Omega i} = 1$, set $p_{1i}$ at $p_{1i}^L$.

Case 2: If sign is + and $Y_{\Omega i} = 0$, set $p_{1i}$ at $p_{1i}^U$.

Case 3: If sign is – and $Y_{\Omega i} = 1$, set $p_{1i}$ at $p_{1i}^U$.

Case 4: If sign is – and $Y_{\Omega i} = 0$, set $p_{1i}$ at $p_{1i}^L$.

*<u>Proof</u>*: There are only four possible combinations of the optimal values of $y_{1i}$ and $Y_{\Omega i}$.

Case $y_{1i} = 1, Y_{\Omega i} = 1$:

It is obvious that there is no different result in the BLPP objective function value by setting $p_{1i}$ to any value in interval $[p_{1i}^L, p_{1i}^U]$, no matter what the sign is (all values are optimal) so by setting $p_{1i}$ at $p_{1i}^L$ when the sign is + and by setting $p_{1i}$ at $p_{1i}^U$ when the sign is – gives an optimal value for $p_{1i}$.

Case $y_1 = 1, Y_\Omega = 0$:

If the sign is +, it is obvious that $p_{1i}$ will be set at its upper bound value $p_{1i}^U$ at the optimal solution. If the sign is –, it is obvious that $p_{1i}$ will be set at its lower bound value $p_{1i}^L$ at the optimal solution.

Case $y_1 = 0, Y_\Omega = 1$:

If the sign is +, it is obvious that $p_{1i}$ will be set at its lower bound value $p_{1i}^L$ at the optimal solution. If the sign is –, it is obvious that $p_{1i}$ will be set at its upper bound value $p_{1i}^U$ at the optimal solution.

Case $y_1 = 0, Y_\Omega = 0$:

It is obvious that there is no difference found in the BLPP objective function value by setting $p_{1i}$ to any value in interval $[p_{1i}^L, p_{1i}^U]$, no matter what the sign is (all values are optimal), so by setting $p_{1\mathrm{i}}$ at $p_{1\mathrm{i}}^{U}$ when sign is + and by setting $p_{1\mathrm{i}}$ at $p_{1i}^{L}$ when sign is – will give an optimal solution for $p_{1\mathrm{i}}$. □

<u>Pre-Processing Step for Parameter of Type $p_2$</u>

The parameters of type $p_2$ represent the coefficients of the binary decision variables located in the functional constraints of the RPS model. Each element of this type of parameters is presented in the functional constraint of the BLPP model as: $\sum_{\exists \mathrm{j}} x_{1j} \leq p_{2\,i} y_{1\,i}$ and $\sum_{\exists \mathrm{j}} x_{2j} \leq p_{2\,i} Y_{\Omega\,i}$ where $p_{2i} \geq 0$.

*<u>Proposition 5</u>*<u>:</u> Given the value of one specific element of $Y_{\Omega}$ called $Y_{\Omega i}$ from the second stage, the optimal solution of $p_{2\mathrm{i}}$ satisfies the following set of constraints if $Y_{\Omega i}$ is equal to one.

$$PY_{21\,\mathrm{i}} - p_{2\,\mathrm{i}} - |\min(0, p_{2\,\mathrm{i}}^{L})|(1 - y_{1\,\mathrm{i}}) \leq 0 \text{ and } -PY_{21\,\mathrm{i}} + p_{2\,\mathrm{i}} - p_{2\,\mathrm{i}}^{U}(1 - y_{1\,\mathrm{i}}) \leq 0$$
$$PY_{21\,\mathrm{i}} \leq p_{2\,\mathrm{i}}^{U} y_{1\,\mathrm{i}} \text{ and } p_{2\,\mathrm{i}}^{L} \leq p_{2\,\mathrm{i}} \leq p_{2\,\mathrm{i}}^{L} + y_{1i}(p_{2\,\mathrm{i}}^{U} - p_{2\,\mathrm{i}}^{L})$$

where the new variable $PY_{21\mathrm{i}}$ will replace the term $p_{2i} y_{1i}$ in the BLPP model.

If $Y_{\Omega i}$ is equal to zero, an optimal solution of $p_{2\mathrm{i}}$ can be attained by fixing the value of $p_{2\mathrm{i}}$ at its upper bound, $p_{2\,\mathrm{i}}^{U}$.

*<u>Proof</u>*<u>:</u> There are only four possible combinations of the optimal values of $y_{1\mathrm{i}}$ and $Y_{\Omega i}$.

Case $y_{1i} = 1, Y_{\Omega i} = 1$:

Because there is no obvious choice of optimal solution of $p_{2\mathrm{i}}$ in this case, the algorithm has to search for optimal solution of $p_{2\mathrm{i}}$ in entire interval [ $p_{2\,\mathrm{i}}^{L}, p_{2\,\mathrm{i}}^{U}$ ].

Case $y_{1i} = 1, Y_{\Omega i} = 0$:

It is obvious that by setting the value of $p_{2i}$ at $p_{2i}^{U}$ results in the largest feasible region for the leader problem and is the optimal setting for this parameter.

Case $y_{1i} = 0, Y_{\Omega i} = 1$:

It is obvious that by setting the value of $p_{2i}$ at $p_{2i}^{L}$ results in the smallest feasible region for the follower problem and is the optimal setting for this parameter.

Case $y_{1i} = 0, Y_{\Omega i} = 0$:

It is obvious that no difference results in BLPP objective function value for setting $p_{2i}$ to any value in interval $[p_{2i}^{L}, p_{2i}^{U}]$ (all values are optimal). Therefore setting the value of $p_{2i}$ at $p_{2i}^{U}$ will result in the optimal setting for this parameter. □

Pre-Processing Step for Parameter of Type $p_3$

The parameters of type $p_3$ represent all right hand side parameters in the functional constraints of the RPS model. There are two distinct groups of this type of parameters. The first group represents all maximum supply and maximum demand parameters. Because these parameters' values are continuous, they can be handled in the model by treating them as continuous variables.

The second group represents must-logic parameters and allowance-logic parameters, which are binary parameters. Because these parameters' values are binary, it is not wise to handle them as additional binary variables in the model. The next proposition will define the rules of setting these parameters' values to their optimal values. Let $y_{m,i}$ define $i^{th}$ must-logic parameter and let $y_{ai}$ define $i^{th}$ allowance-logic parameter associated with $y_{1i}$ and $Y_{\Omega i}$. Each element of this type of parameter is present in the functional constraint of the BLPP model as $y_{mi} \le y_{1i} \le y_{ai}$ and $y_{mi} \le Y_{\Omega i} \le y_{ai}$.

<u>*Proposition*</u> <u>6:</u> Given the value of one specific element of $Y_{\Omega}$ called $Y_{\Omega i}$ from the second stage, an optimal solution of $y_{m,i,}$ is zero and an optimal solution of $y_{a,i}$ is one and the associated logical constraints can be removed from the BLPP model.

<u>*Proof*</u>: There are only two possible values of $Y_{\Omega i}$.

Case $Y_{\Omega i} = 1$:

It is obvious that the optimal setting of $y_{ai}$ has to be one. Setting the value of $y_{mi}$ at zero will result in a bigger feasible region for the leader problem, and is thus the optimal setting for this parameter.

Case $Y_{\Omega i} = 0$:

It is obvious that the optimal setting of $y_{mi}$ has to be zero. Setting the value of $y_{ai}$ at one will result in a bigger feasible region for the leader problem, and is thus the optimal setting for this parameter.

For these reasons, an optimal setting of $y_{mi,}$ is zero and an optimal setting of $y_{ai}$ is one. Because $y_{1i}$ is a binary variable, the setting method of $y_{ai}$ and $y_{mi}$ as proposed will result in redundancy of the constraints. From this reason, these type constraints can be removed from the BLPP model. □

Pre-Processing Step for Parameter of Type $p_4$

The parameters of type $p_4$ represent the coefficient parameters of the continuous decision variables in the objective function of the RPS model. Each element of this type of parameters is presented in the objective function of the BLPP model as:

$$Max \pm (p_{4i}x_{1i} - p_{4i}x_{2i}).$$

*Proposition* 7: Given the value of one specific element of $Y_\Omega$ called $Y_{\Omega k}$ from the second stage, where there exists a constraint in the BLPP model as $x_{2i} + \sum_{\exists j} x_{2j} \le C\ Y_{\Omega k}$, and the signs in the objective function are adjusted so that all $p_{4i}^U$ is greater than or equal to zero, the following rules can narrow the search for an optimal setting of $p_{4i}$. If $Y_{\Omega k}$ is equal to one, an optimal setting of $p_{4i}$ is either at $p_{4i}^L$ or $p_{4i}^U$. If $Y_{\Omega k}$ is equal to zero and the sign is +, an optimal setting of $p_{4i}$ is $p_{4i}^U$. If $Y_{\Omega k}$ is equal to zero and the sign is –, an optimal setting of $p_{4i}$ is $p_{4i}^L$.

*Proof*: There are only five possible cases of the optimal values of $x_{1i}$ and $x_{2i}$.

*Case* $x_{1i} = x_{2i}$:

It is obvious that there is no difference in the objective function value of the BLPP model by setting $p_{4i}$ to any value in the interval [$p_{4i}^L, p_{4i}^U$], no matter what the sign is (all values are optimal). By setting $p_{4i}$ at either $p_{4i}^L$ or $p_{4i}^U$, this setting is also an optimal setting.

Case where the sign is + and $x_{1i} > x_{2i}$:

It is obvious that by setting $p_{4i}$ at its upper bound value $p_{4i}^{U}$, an optimal setting of $p_{4i}$ is attained.

Case where the sign is – and $x_{1i} > x_{2i}$:

It is obvious that by setting $p_{4i}$ at its lower bound value $p_{4i}^{L}$, an optimal setting of $p_{4i}$ is attained.

Case where the sign is + and $x_{1i} < x_{2i}$:

It is obvious that by setting $p_{4i}$ at its lower bound value $p_{4i}^{L}$, an optimal setting of $p_{4i}$ is attained.

Case where the sign is – and $x_{1i} < x_{2i}$:

It is obvious that by setting $p_{4i}$ at its upper bound value $p_{4i}^{U}$, an optimal setting of $p_{4i}$ is attained.

These reasons prove the first claim when $Y_{\Omega k}$ is equal to one. In the case where $Y_{\Omega k}$ is equal to zero, the results in first three cases with the fact that $x_{1i}$ is non-negative prove the claim. □

The results from proposition 7 lead to an important method that can be used to find an optimal value of $p_{4i}$ without solving a nonlinear bi-level programming problem. After setting the values of associated $p_{4i}$ to their optimal settings for all $Y_{\Omega k}$ with zero value, we can successfully handle the variation in the rest of $p_{4i}$ parameters by adding the following constraints into the BLPP model.

Let $bi_i$ be a binary variable in the model that will take the value of one when $p_{4i} = p_{4i}^U$ or zero when $p_{4i} = p_{4i}^L$ and let $PX_{41i}$ represents the term $p_{4i}x_{1i}$ and $PX_{42i}$ represents the term $p_{4i}x_{2i}$. Figure 6.6 illustrates these required constraints.

$$PX_{41i} - p_{4i}^U x_{1i} \leq 0$$

$$-PX_{41i} + p_{4i}^U x_{1i} - (p_{4i}^U x_{1i}^U + |\min(0, p_{4i}^L)|\, x_{1i}^U)(1 - bi_i) \leq 0$$

$$PX_{41i} - p_{4i}^L x_{1i} - (p_{4i}^U x_{1i}^U + |\min(0, p_{4i}^L)|\, x_{1i}^U) bi_i \leq 0$$

$$-PX_{41i} + p_{4i}^L x_{1i} \leq 0$$

$$PX_{42i} - p_{4i}^U x_{2i} \leq 0$$

$$-PX_{42i} + p_{4i}^U x_{2i} - (p_{4i}^U x_{2i}^U + |\min(0, p_{4i}^L)|\, x_{2i}^U)(1 - bi_i) \leq 0$$

$$PX_{42i} - p_{4i}^L x_{2i} - (p_{4i}^U x_{2i}^U + |\min(0, p_{4i}^L)|\, x_{2i}^U)\, bi_i \leq 0$$

$$-PX_{42i} + p_{4i}^L x_{2i} \leq 0$$

$$-p_{4i} + p_{4i}^U - (p_{4i}^U - p_{4i}^L)(1 - bi_i) \leq 0$$

$$p_{4i} - p_{4i}^L - (p_{4i}^U - p_{4i}^L)\, bi_i \leq 0$$

**Figure 6.6 Required Constraints for Parameters of Type $p_4$**

Pre-Processing Step for Parameter of Type $p_5$

The parameters of type $p_5$ represent the coefficients of continuous decision variables in the functional constraints of the RPS model. This type of parameters represents the proportion of material consumed by the process and the proportion of material produced by the process. Because of the restriction that summation of all proportions consumed by each specific process must be equal to one and the restriction that the summation of all proportions produced by each specific process must be equal to one, these parameters would best be modeled as discrete parameters and can be included in initial discrete scenarios.

From the results of these pre-processing steps, one might be misled that all parameters will take the value from either of their bounds at the optimal solution of the BLPP model. This statement can be shown to be not true by the counter example shown in Figure 6.7.

$$
\begin{aligned}
\text{maximize}\quad & 2x_{11} + x_{12} + x_{13} - 2x_{21} - x_{22} - x_{23} \\
\text{s.t.}\quad & x_{11} + x_{12} \le 10y_1 \\
& x_{11} \le 5y_2 \\
& x_{12} \le p_3 \\
& x_{11} \le x_{12} \\
& x_{11} + x_{13} \le 5y_3 \\
& x_{11}, x_{12}, x_{13} \ge 0 \\
& 0 \le p_3 \le 10 \\
& \underset{x_{21}, x_{22}, x_{23}}{\text{maximize}}\ 2x_{21} + x_{22} + x_{23} \\
& x_{21} + x_{22} \le 10 \\
& x_{21} \le 5(0) \\
& x_{22} \le p_3 \\
& x_{21} \le x_{22} \\
& x_{21} + x_{23} \le 5 \\
& x_{21}, x_{22}, x_{23} \ge 0
\end{aligned}
$$

**Figure 6.7 Counter Example of Fixing Parameters at their Bounds**

For the counter example, the only optimal solution of $p_3$ is 5 with the optimal leader's objective function value of 5. On another hand, the optimal leader's objective function value is zero when $p_3$ is fixed at either of its bounds. This example shows that considering all parameters at their bounds is not enough to solve the problem.

*<u>Variable and Constraint Elimination Step</u>*

In the next section, Karash-Kuhn-Tucker (KKT) conditions are applied in order to solve the BLPP model. One important concern on the effectiveness of solving the BLPP model is the size of the complementarily slackness constraints which are part of the KKT conditions. The size of these complementarily slackness constraints are determined by the number of variables and the number of constraints in the inner problem of the BLPP model. The smaller the number of variables and number of constraints, the more efficiently the BLPP can be solved.

In this section, we propose some elimination steps in order to eliminate unnecessary variables and constraints of the inner problem of the BLPP model before applying the KKT conditions to the problem. The ideas of these elimination steps are very important and can determine success or failure of the algorithm to solve realistically sized problems. The effectiveness of this elimination step is illustrated in case studies presented in Chapter VII. Three main ideas of these elimination rules are presented as follows.

<u>Elimination by the Information from $Y_\Omega$</u>

After the information from $Y_\Omega$ is given from the first and second stages of the algorithm, some variables of the inner problem can be predetermined and some inner constraints become redundant. These variables and constraints can be eliminated from the BLPP model by setting those variables to their predetermined values and by ignoring those redundant constraints. The simplest example of this case is if there exists any constraint in the inner problem of the BLPP model with the structure, $\sum x_j \leq C_i Y_{\Omega i}$ , and $Y_{\Omega i} = 0$, this constraint and all its variables can be eliminated from the model by setting all $x_j$ to zero and ignoring this constraint.

Elimination by the Information from Model Parameters

After the parameter information is given either from the original problem statement or from the results of the preprocessing steps, some variables of the inner problem can be predetermined and some inner constraints become redundant. These variables and constraints can be eliminated from the BLPP model by setting those variables to their predetermined values and by ignoring those redundant constraints. The simplest example of this case is if there exists any constraint in the inner problem of the BLPP model with the structure, $\sum x_j \leq C_i$ , and $C_i = 0$ (from the original problem or from the results of preprocessing steps), this constraint and all its variables can be eliminated from the model by setting all $x_j$ to zero and ignoring this constraint.

Elimination by the Results of the First Two Rules

After performing the previous two elimination steps, some variables of the inner problem can be further predetermined and some inner constraints become redundant. These variables and constraints can be eliminated from the BLPP model by setting those variables to their predetermined values and by ignoring those redundant constraints. The simplest example of this case is if there exists any constraint in the inner problem of the

BLPP model with the structure, $\sum x_j + x_k \leq C_i$, where all $x_j$ are previously eliminated from the problem and $C_i$ is constant, this constraint and $x_k$ variable can be eliminated from the model by setting the value of $x_k$ to its appropriate value and ignoring this constraint.

*<u>Problem Transformation Step</u>*

In searching for a way to solve the BLPP model, it would be helpful to have an explicit representation of Inducible Region (*IR*) of the linear bi-level programming. This can be achieved by replacing the follower's problem with Karash-Kuhn-Tucker (KKT) conditions and append the resultant system to the leader's problem. In another word, the BLPP model can be rewritten as a single level mixed integer nonlinear programming problem with complementary slackness constraints as shown in Figure 6.8.

We also would like to point out that the complementary slackness constraints could be equivalently replaced by strong duality constraint (as shown in Section 6.4) for the problem with no uncertainty in parameters of type $p_2$ and $p_3$. In this case, the problem becomes much easier to handle (no branching on complementary slackness is required).

There is one final transformation to convert the BLPP model to a single level mixed integer linear programming problem with complementary slackness constraints. This is performed by adding all necessary constraints and applying our pre-processing steps as shown in Figure 6.9.

Note that because large portion of $Y_\Omega$ will be zero, the pre-processing and elimination algorithms will be able to eliminate a large number of variables and constraints and fix the values for a large number of uncertain parameters to their appropriate bounds. This means that the pre-processing and elimination algorithms will significantly reduce the size of the BLPP model. For this reason, this proposed algorithm is able to solve the large-scale BLPP model effectively, and warrants computational investigation for realistic problems.

Even though the elimination algorithm can significantly reduce the number of complementary slackness constraints, it often cannot eliminate all of them. In order to solve the BLPP model effectively, the next question is "How are we going to handle the rest of the complementary slackness constraints?" One of the most direct approaches for dealing with the complementary slackness constraints is the use of big *M* method. The constraints $w_i g_i(x,y) = 0 \quad \forall i$ can be converted into two mixed integer linear constraints by replacing them with the following two sets of inequalities constraints with binary variables $binary_i$ and a sufficiently large number *M*:

$w_i \le M\, binary_i$ and $g_i(x,y) \le M\,(1 - binary_i) \quad \forall i$.

$$
\begin{aligned}
&\max_{x_1, y_1, p_k} \{p_4^T x_1 + p_1^T y_1 - p_4^T x_2 - p_1^T Y_\Omega)\} \\
&s.t. \quad \left(p_5 x_1\right)_i + p_{2i} y_{1\mathrm{i}} \le p_{3i} \qquad \forall i \\
&\left(p_5 x_2\right)_i + p_{2i} Y_{\Omega i} + s_{2i} = p_{3i} \qquad \forall i \\
&\left(p_5{}^T w_2\right)_j - a_{2j} = p_{4j}^T \qquad \forall j \\
&w_{2i} s_{2i} = 0 \qquad \forall i \\
&a_{2j} x_{2j} = 0 \qquad \forall j \\
&p_k^L \le p_k \le p_k^U \qquad \forall k = 1,2,3, \text{ and } 4 \\
&x_1, x_2, s_2, a_2, w_2 \ge \vec{0} \quad y_1 \in \text{ vector of } \{0,1\}
\end{aligned}
$$

**Figure 6.8 The BLPP Model with KKT Conditions**

$$\max_{x_1, y_1, p_k} \{(Ind_4)^T PX_{41} + (p_4^*)^T x_1 + (p_1^*)^T y_1 - (Ind_4)^T PX_{42} - (p_4^*)^T x_2 - (p_1^*)^T Y_\Omega)\}$$

$$s.t.\quad (p_5 x_1)_i + Ind_{2i} PY_{21i} + p_{2i}^* y_{1i} \le p_{3i} \qquad \forall i$$

$$(p_5 x_2)_i + Ind_{2i} p_{2i} Y_{\Omega i} + p_{2i}^* Y_{\Omega i} + s_{2i} = p_{3i} \qquad \forall i$$

$$(p_5{}^T w_2)_j - a_{2j} = (Ind_{4j} p_{4j} + p_{4j}^*) \qquad \forall j$$

$$\left.\left(\begin{array}{l} PY_{21i} - p_{2i} - |\min(0, p_{2i}^L)|(1 - y_{1i}) \le 0 \\ -PY_{21i} + p_{2i} - p_{2i}^U(1 - y_{1i}) \le 0 \\ PY_{21i} \le p_{2i}^U y_{1i} \\ p_{2i} \le p_{2i}^L + y_{1i}(p_{2i}^U - p_{2i}^L) \end{array}\right) Ind_{2i} \right\} \qquad \forall i$$

$$\left.\left(\begin{array}{l} PX_{41j} - p_{4j}^U x_{1j} \le 0 \\ -PX_{41j} + p_{4j}^U x_{1j} - (p_{4j}^U x_{1j}^U + |\min(0, p_{4j}^L)|\, x_{1j}^U)(1 - bi_j) \le 0 \\ PX_{41j} - p_{4j}^L x_{1j} - (p_{4j}^U x_{1j}^U + |\min(0, p_{4j}^L)|\, x_{1j}^U) bi_j \le 0 \\ -PX_{41j} + p_{4j}^L x_{1j} \le 0 \\ PX_{42j} - p_{4j}^U x_{2j} \le 0 \\ -PX_{42j} + p_{4j}^U x_{2j} - (p_{4j}^U x_{2j}^U + |\min(0, p_{4j}^L)|\, x_{2j}^U)(1 - bi_j) \le 0 \\ PX_{42j} - p_{4j}^L x_{2j} - (p_{4j}^U x_{2j}^U + |\min(0, p_{4j}^L)|\, x_{2j}^U)\ bi_j \le 0 \\ -PX_{42j} + p_{4j}^L x_{2j} \le 0 \\ -p_{4j} + p_{4j}^U - (p_{4j}^U - p_{4j}^L)(1 - bi_j) \le 0 \\ p_{4j} - p_{4j}^L - (p_{4j}^U - p_{4j}^L)\ bi_j \le 0 \end{array}\right) Ind_{4j} \right\} \qquad \forall j$$

$$w_{2i} s_{2i} = 0 \qquad \forall i$$

$$a_{2j} x_{2j} = 0 \qquad \forall j$$

$$p_k^L \le p_k \le p_k^U \qquad \forall k = 1, 2, 3, \text{ and } 4$$

$$x_1, x_2, s_2, a_2, w_2, PY_{21}, PX_{41}, PX_{42} \ge \vec{0} \qquad y_1, bi \in vector\ of\ \{0,1\}$$

$$Ind_{2i} = \begin{cases} 1 & \text{if } p_{2i} \text{ value cannot be predetermined.} \\ 0 & \text{otherwise} \end{cases}$$

$$Ind_{4j} = \begin{cases} 1 & \text{if } p_{4j} \text{ value cannot be predetermined.} \\ 0 & \text{otherwise} \end{cases}$$

**Figure 6.9 Final Version of the BLPP Model**

Our computational results show that by using the big $M$ method for handling complementary slackness constraints, even a small numerical imprecision in representing the binary variable, $binary_i$, value can cause the problem to terminate at the wrong solution of the bi-level programming problem. For this reason, the big $M$ method is strongly **not** recommended for handling the complementary slackness constraints in the BLPP model.

*Drawback of Big-M Method for Handling Complementary Slackness Constraints*

The following results show that by using the big $M$ method for handling complementary slackness constraints, even a small numerical imprecision in representing the binary variable, $binary_i$, value can cause the problem to terminate at the wrong solution of the bi-level programming problem. The following small example illustrates this claim. Consider the following bi-level programming problem.

$$
\begin{aligned}
&\max_{x_1, y_1} \; z = x_1 + y_1 - x_2 - 1 \\
&s.t. \qquad x_1 \le 3y_1 \\
&\qquad \max_{x_2} \; x_2 \\
&\qquad\quad s.t. \quad x_2 \le 3 \\
&\qquad\quad x_1, x_2 \ge 0 \quad \text{and} \quad y_1 \in \{0, 1\}
\end{aligned}
$$

By adding all dual constraints for the follower problem and complementary slackness constraints represented by big-M constraints, the following mixed integer linear programming problem is equivalent to the previous bi-level programming problem.

$$
\begin{aligned}
\max_{x_q, y_q} \quad & x_1 + y_1 - x_2 - 1 \\
s.t. \quad & x_1 \le 3y_1 \\
& x_2 + s = 3 \\
& w - a = 1 \\
& x_2 \le M_1 binary_1 \\
& a \le M_1 (1 - binary_1) \\
& w \le M_2 binary_2 \\
& s \le M_2 (1 - binary_2) \\
& x_1, x_2, s, w, a \ge 0 \qquad y_1, binary_1, binary_2 \in \{0, 1\}
\end{aligned}
$$

Figure 6.10 represents the geometric structure and the path from the initial basic feasible solution to the optimal basic feasible solution of this problem.

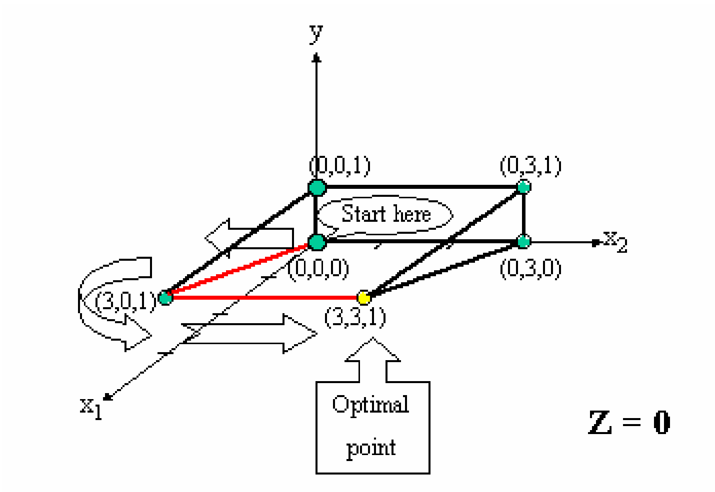


**Figure 6.10 Geometric Structure of the Example**

If the values of all binary variables (*binary*$_1$ and *binary*$_2$) are precisely 0 or 1, this problem can be readily solved to optimality. Unfortunately, most current optimization software often cannot provide the perfect value of 0 or 1 for binary decision variables for all computations. Numerical estimations are used to make the value like 0.999999 as 1 and the value like 0.0000001 as 0. These numerical estimations can cause the serious problems in the bi-level programming with big-*M* complementary slackness constraints.

The optimal solution of this example is: $x_1 = 3$, $x_2 = 3$, $y_1 = 1$, $s = 0$, $w = 1$, $a = 0$, $binary_1 = 1$, $binary_2 = 1$, $z = 0$. Now consider the case where the value of $binary_2$ is 0.99999 instead of 1 and the $M_{i2}$ value is 1000, 10000, 100000, and 1000000 for cases 1, 2, 3, and 4 respectively. The optimal solutions generated from the optimization software for each case are as follows:

<u>Case 1</u>: $x_1 = 3$, $x_2 = 2.99$, $y_1 = 1$, $s = 0.01$, $w = 1$, $a = 0$, $bi_1 = 1$, $bi_2 = 0.99999$, $z = 0.01$

<u>Case 2:</u> $x_1 = 3$, $x_2 = 2.90$, $y_1 = 1$, $s = 0.10$, $w = 1$, $a = 0$, $bi_1 = 1$, $bi_2 = 0.99999$, $z = 0.1$

<u>Case 3:</u> $x_1 = 3$, $x_2 = 2$, $y_1 = 1$, $s = 1$, $w = 1$, $a = 0$, $bi_1 = 1$, $bi_2 = 0.99999$, $z = 1$

<u>Case 4:</u> $x_1 = 3$, $x_2 = 0$, $y_1 = 1$, $s = 3$, $w = 1$, $a = 0$, $bi_1 = 1$, $bi_2 = 0.99999$, $z = 3$

The following four figures demonstrate the geometric structure for the numerical error in each case.

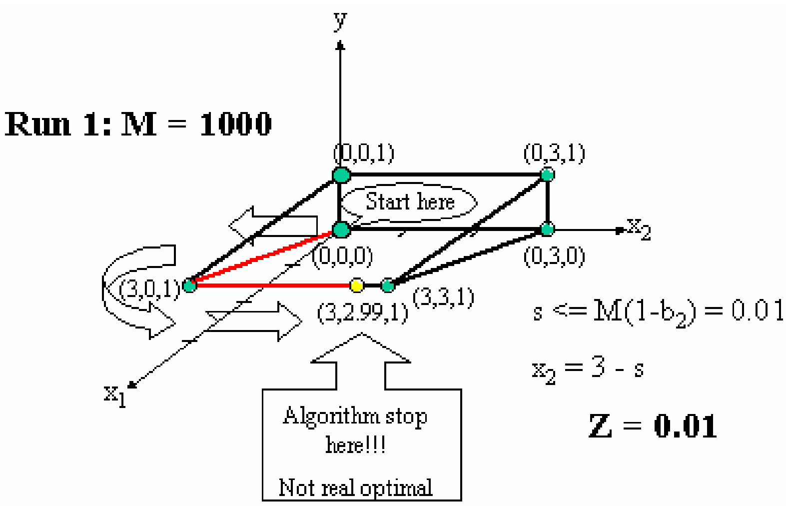


**Figure 6.10 Geometric Structure for the Numerical Error in Case (Run) 1**

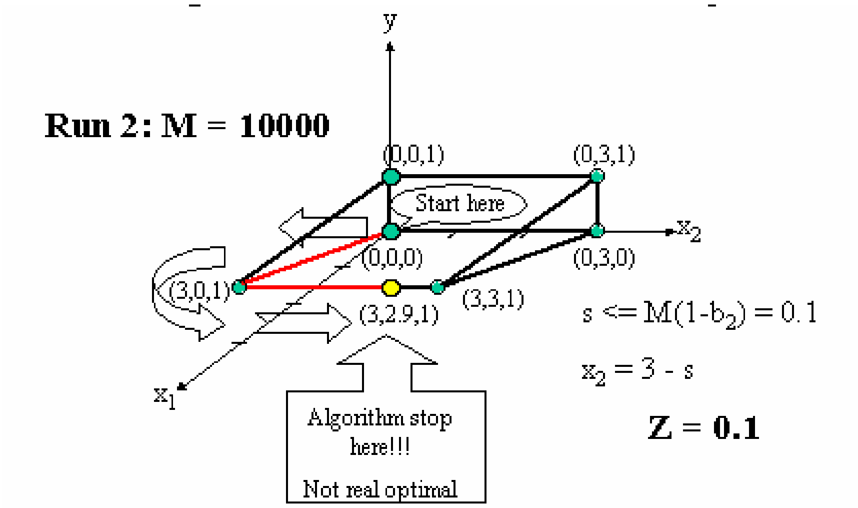


**Figure 6.11 Geometric Structure for the Numerical Error in Case (Run) 2**

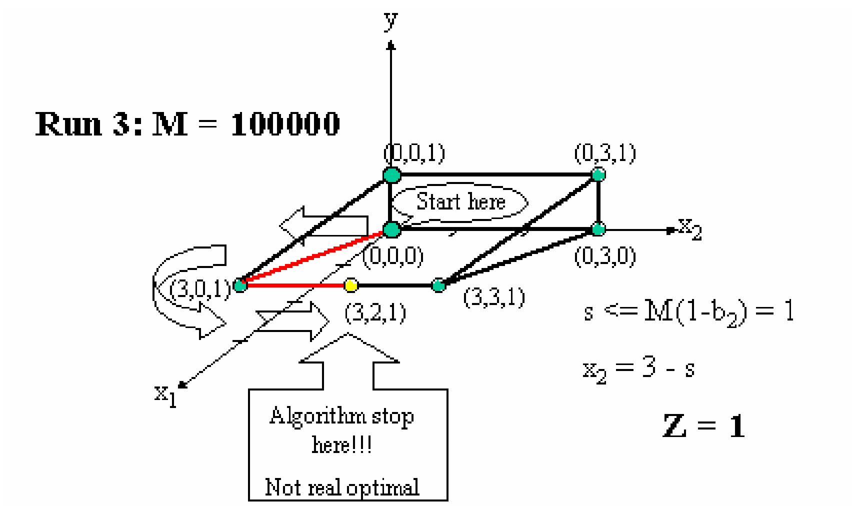


**Figure 6.12 Geometric Structure for the Numerical Error in Case (Run) 3**

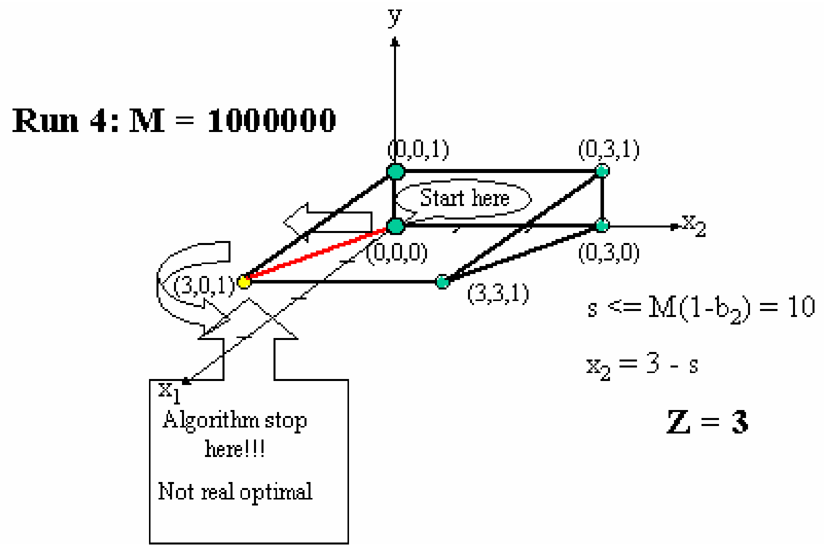


**Figure 6.13 Geometric Structure for the Numerical Error in Case (Run) 4**

These results demonstrate the ineffectiveness of big-M methodology for handling complementary slackness constraints, particularly for large instances where the big-M value is difficult to be bounded. An alternative methodology, named the Kuhn-Tucker Branch and Bound Approach, is presented in the next section.

This final version of the BLPP model can be solved without computational error by using the following branch and bound algorithm. This algorithm starts by solving the linear and complementary slackness relaxation problem. The branch and bound step will be performed if there is a violation in complementary slackness conditions. If all complementary slackness conditions are satisfied, the branch and bound step will also be performed if there is a violation in integrality constraints.

*Kuhn-Tucker Approach Algorithm for Bi-level Programming*

In the later study of Bard and Moore (1990), they developed an implicit approach to satisfying the complementary slackness constraints which is proved to be very effective. The methodology presented in this chapter is a modification of their original approach. The basic idea of this algorithm is to suppress the complementarity and integrality terms and solve the resulting linear sub-problem after adding KKT conditions. At all iterations, a check is made to see if complementary slackness conditions and integer restrictions are satisfied. If so, the corresponding point is in the inducible region (IR) and hence is a potential solution to the BLPP model. If not, a branch and bound scheme is used to implicitly examine all combinations of complementary slackness conditions and integer restrictions.

Before presenting the algorithm, we introduce some related notation. Let $W = \{1, 2, ..., q + m\}$ be the index set for the complementary slackness constraints $(u_i g_i = 0)$ in the BLPP model (see Section 2.4), and let $\overline{F}$ be the incumbent lower bound on the leader's objective function. At the $h^{\text{th}}$ node of the search tree on the complementary slackness conditions, we define a subset of indices $W_h \subset W$ and a path $P_h$ corresponding to an assignment of either $u_i = 0$ or $g_i = 0$ for $i \in W_h$. Let

$$\begin{aligned}
S_h^+ &= \{i \mid i \in W_h \text{ and } u_i = 0\} \\
S_h^- &= \{i \mid i \in W_h \text{ and } g_i = 0\} \\
S_h^0 &= \{i \mid i \notin W_h\}
\end{aligned}$$

For $i \in S_h^0$, the variable $u_i$ and $g_i$ are free to assume any nonnegative values in the solution of the BLPP model, so complementary slackness will not necessarily be satisfied.

*Kuhn-Tucker Branch and Bound Algorithm (Maximization Problem)*

Step 0: (Initialization) Set $k = 0,\ S_k^+ = \phi,\ S_k^- = \phi,\ S_k^0 = W$, $P_0 = 0$ and $\overline{F} = \Delta^L - 1$ where $\Delta^L$ is the lower bound on min-max regret from the first stage of the algorithm.

Step 1: (Iteration *k* on node *h*) Pick an active node from the current tree, which has parent node with a maximum objective function value for the LP and complementary slackness relaxation problem (in case *k* = 0, pick node 0 as the selected node) and let *h* be the index of this selected node. Set $u_i = 0$ for $i \in S_h^+$ and $g_i = 0$ for $i \in S_h^-$. Attempt to solve the linear and complementary slackness relaxation problem and store the objective value of node *h* into $F_h$. If the resultant problem is LP infeasible or $F_h \le \overline{F}$, go to Step 3; otherwise check if there exists $i \in S_h^0$ where $u_i g_i \ne 0$. If so, select the index, which attains the largest value, and label it as $i_1$ and perform branch and bound on this complementary slackness condition and identify two child nodes as node *k*+1 and node *k*+2. For node *k*+1, let $S_{k+1}^+ \leftarrow S_h^+ \cup \{i_1\}$, $S_{k+1}^- \leftarrow S_h^-$, $S_{k+1}^0 \leftarrow S_h^0 \setminus \{i_1\}$, and $P_{k+1} \leftarrow P_k \cup \{i_1\}$. For node *k*+2, let $S_{k+2}^+ \leftarrow S_h^+$, $S_{k+2}^- \leftarrow S_h^- \cup \{i_1\}$, $S_{k+1}^0 \leftarrow S_h^0 \setminus \{i_1\}$, and $P_{k+2} \leftarrow P_k \cup \{i_1\}$ and $k \leftarrow k+2$ and perform Step 1; otherwise, check if the resultant problem contains any integer variable, which violates the integer restrictions for leader problem. If so, perform the regular branch and bound on one of the violated variables and identify two child nodes as node *k*+1 and node *k*+2. Let $S_j^+ \leftarrow S_h^+, S_j^- \leftarrow S_h^-, S_j^0 \leftarrow S_h^0$ for $j = k+1$ and $k+2$, and let $k \leftarrow k+2$ and perform Step 1; otherwise go to Step 2.

Step 2: (Updating) $\overline{F} = F_h$

Step 3: (Cutting branch) Set node $h$ as non-active. If no active node exists, go to Step 4. Otherwise go to Step 1.

Step 4: (Termination) If $\overline{F} < \Delta^L$, there is no useful feasible solution to the BLPP model. Otherwise, declare the current feasible point associated with $\overline{F}$ the optimal solution to the BLPP model.

We also would like to point out that setting priorities on the branching variables is one of the important factors for improving the solution time of the BLPP model. For the BLPP model, we recommend branching priorities as follows: (1) complementary slackness conditions, (2) binary decisions on parameters' bounds, and (3) high effect to low effect infrastructure decision (site opening decisions to activate transportation arc decisions).

The following small example illustrates the use of the Kuhn-Tucker algorithm to solve the bi-level linear programming problem with discrete variables for the leader problem.

$$\max_{x_{11},x_{12},y_{11},y_{12}} (2x_{11} + 3x_{12} - 10y_{11} - 15y_{12} - \max_{x_{21},x_{22}}(2x_{21} + 3x_{22} - 10Y_{21\Omega} - 15Y_{22\Omega}))$$

*subject to*:

$$\begin{aligned}
x_{11} &\le 35y_{11} \\
x_{12} &\le 45y_{12} \\
2x_{11} + 3x_{12} &\le 100 \\
x_{21} &\le 35Y_{21\Omega} \\
x_{22} &\le 45Y_{22\Omega} \\
2x_{21} + 3x_{22} &\le 100 \\
x_{11}, x_{12}, x_{21}, x_{22} \ge 0 \quad &\text{and} \quad y_{11}, y_{12} \in \{0,1\} \\
\text{set } Y_{21\Omega} = 1 \quad &\text{and} \quad Y_{22\Omega} = 0
\end{aligned}$$

Now by adding KKT conditions to the problem, the linear and complementary slackness relaxation problem is demonstrated as follows.

$$
\begin{aligned}
&\max_{x_{11},x_{12},y_{11},y_{12},x_{21},x_{22}} \quad (2x_{11}+3x_{12}-10y_{11}-15y_{12}-2x_{21}-3x_{22}+10) \\
&\textit{subject to}: \\
&\quad x_{11} \le 35\mathrm{y}_{11} \\
&\quad x_{12} \le 45y_{12} \\
&\quad 2x_{11}+3x_{12} \le 100 \\
&\quad x_{21}+s_1=35 \\
&\quad x_{22}+s_2=0 \\
&\quad 2x_{21}+3x_{22}+s_3=100 \\
&\quad w_1+2w_3-a_1=2 \\
&\quad w_2+3w_3-a_2=3 \\
&\quad x_{11},x_{12},x_{21},x_{22},s_1,s_2,s_3,w_1,w_2,w_3,a_1,a_2 \ge 0 \quad \text{and} \quad y_{11},y_{12} \in [0,1]
\end{aligned}
$$

By performing the Kuhn-Tucker branch and bound algorithm, Figure 6.14 demonstrates the solution searching methodology of the algorithm.

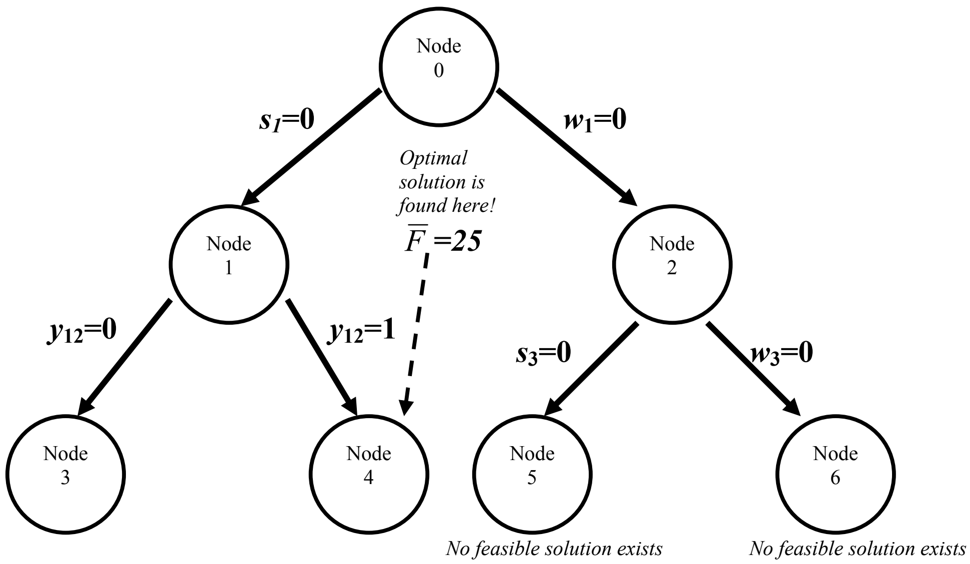


**Figure 6.14 Solution Searching Methodology of KKT Branch and Bound Algorithm**

The algorithm starts in step 0 by setting $k = 0$ and $\overline{F} = -\infty$. The algorithm performs step 1 next by picking node $h = 0$ as a selected node and start solving initial problem in node 0. The corresponding solution from node 0 is $F_0 = 98.88$, $x_{11} = 0$, $x_{12} = 33.33$, $x_{21} = 0$, $x_{22} = 0$, $y_{11} = 0$, $y_{12} = 0.7407$, $w_1 = 2$, $s_1 = 35$, $w_2 = 3$, $s_2 = 0$, $w_3 = 0$, $s_3 = 100$, $a_1 = 0$, and $a_2 = 0$. The algorithm performs the branching step on the most violated complementary slackness constraints ($w_1s_1 = 70 > 0$) and identifies two child nodes as node 1 ($s_1 = 0$) and node 2 ($w_1 = 0$).

The algorithm then performs step 1 next by picking node $h = 1$ as the selected node and solves the linear programming associated with node 1. The corresponding solution from node 1 is $F_1 = 28.88$, $x_{11} = 0$, $x_{12} = 33.33$, $x_{21} = 35$, $x_{22} = 0$, $y_{11} = 0$, $y_{12} = 0.7407$, $w_1 = 2$, $s_1 = 0$, $w_2 = 3$, $s_2 = 0$, $w_3 = 0$, $s_3 = 30$, $a_1 = 0$, and $a_2 = 0$. The algorithm performs the branching step on the violation on integrality restriction of $y_{12}$ and identifies two child nodes as node 3 ($y_{12} = 0$) and node 4 ($y_{12} = 1$).

The algorithm then continues to solve the corresponding linear programming problem of node 2. The corresponding solution from node 2 is $F_2 = 98.88$, $x_{11} = 0$, $x_{12} = 33.33$, $x_{21} = 0$, $x_{22} = 0$, $y_{11} = 0$, $y_{12} = 0.7407$, $w_1 = 0$, $s_1 = 35$, $w_2 = 0$, $s_2 = 0$, $w_3 = 1$, $s_3 = 100$, $a_1 = 0$, and $a_2 = 0$. The algorithm performs the branching step on the most violated complementary slackness constraints ($w_3s_3 = 100 > 0$) and identifies two child nodes as node 5 ($s_3 = 0$) and node 6 ($w_3 = 0$).

The algorithm then continues to solve the corresponding linear programming problems of node 5 and node 6. These linear programming problems are infeasible and are set as non-active nodes.

The algorithm then continues to solve the corresponding linear programming problem of node 3. The corresponding solution from node 3 is $F_3 = 0$, $x_{11} = 35$, $x_{12} = 0$, $x_{21} = 35$, $x_{22} = 0$, $y_{11} = 1$, $y_{12} = 0$, $w_1 = 2$, $s_1 = 0$, $w_2 = 3$, $s_2 = 0$, $w_3 = 0$, $s_3 = 30$, $a_1 = 0$, and $a_2 = 0$. Because the resultant solution satisfies all primal, dual and complementary slackness constraints and $-\infty = \overline{F} \leq F_3 = 0$, the algorithm then sets $\overline{F} = F_3 = 0$ and set node 3 as non-active node.

The algorithm then solves the corresponding linear programming problem of node 4. The corresponding solution from node 4 is $F_4 = 25$, $x_{11} = 0$, $x_{12} = 33.33$, $x_{21} = 35$, $x_{22} = 0$, $y_{11} = 0$, $y_{12} = 1$, $w_1 = 2$, $s_1 = 0$, $w_2 = 3$, $s_2 = 0$, $w_3 = 0$, $s_3 = 100$, $a_1 = 0$, and $a_2 = 0$. Because the resultant solution satisfies all primal, dual and complementary slackness constraints and $0 = \overline{F} \leq F_4 = 25$, the algorithm then sets $\overline{F} = F_4 = 25$ and set node 4 as non-active node.

Because no active node exists, the algorithm terminates with the optimal solution to the problem as $x_{11} = 0$, $x_{12} = 33.33$, $x_{21} = 35$, $x_{22} = 0$, $y_{11} = 0$, $y_{12} = 1$, $w_1 = 2$, $s_1 = 0$, $w_2 = 3$, $s_2 = 0$, $w_3 = 0$, $s_3 = 100$, $a_1 = 0$, and $a_2 = 0$ with the objective function value of 25.

When the Big-M method is used for solving this example with $M = 10,000,000$ by Xpress IVE, Xpress Optimizer, Xpress BCL and CPLEX 7.5 software, the misleading solution is generated as $x_{11} = 0$, $x_{12} = 33.33$, $x_{21} = 0$, $x_{22} = 0$, $y_{11} = 0$, $y_{12} = 1$, $w_1 = 2$, $s_1 = 35$, $w_2 = 3$, $s_2 = 0$, $w_3 = 0$, $s_3 = 100$, $a_1 = 0$, and $a_2 = 0$ with objective function value of 95. This example illustrates an unreliable solution from big-$M$ method compared with the optimal solution from Kuhn-Tucker branch and bound algorithm.

The following subsection gives the detail summary of all steps in the semi-continuous robust algorithm.

*Summary of the Semi-Continuous Robust Algorithm*

a) Determine which parameters define scenarios. From these parameters, determine which parameters are discrete and which parameters are continuous. For all discrete parameters, generate the initial set of scenarios based on the combination of finite numbers of all possible values of all discrete parameters (initial discrete scenarios). For all continuous parameters, determine their upper and lower bound values.

b) Choose a set of starting scenarios including the set of initial discrete scenarios and add them to the set $\Omega$.

c) Use the RPS model to solve each of the scenarios in $\Omega$ to optimality if an optimal solution has not already been obtained. If the RPS problem is infeasible for any scenario, the algorithm is terminated with the confirmation that no robust solution exists for the problem. Otherwise the optimal objective function value to the RPS problem for scenario $\omega$ is designated as $O^*_\omega$.

d) Solve the DRRPS model using all scenarios $\omega$ in $\Omega$. If the DRRPS model is infeasible, the algorithm is terminated with the confirmation that no robust solution exists for the problem. Otherwise obtain the robust solution, $Y_{\Omega,}$ and the corresponding DRRPS optimal objective function value as the lower bound, $\Delta^L$, from the set of scenarios $\Omega$ and proceed to step e.

e) From the $Y_\Omega$ information from step d, perform the pre-processing and elimination steps and solve the BLLP model for each scenario in the initial discrete scenarios.

f) If the optimal objective function values of all BLLP models are greater than or equal to zero, proceed to step g. Otherwise, add infeasible scenarios to set $\Omega$ and proceed to step c.

g) From the $Y_\Omega$ information forwarded from step f, perform the pre-processing algorithm and then solve BLPP models. Each model is associated with each scenario of initial discrete scenarios, to generate new scenarios, $\omega_i \ \forall i = 1,2,...,M$, and associated objective function value, which become upper bounds values $\Delta_i^U \ \forall i = 1,2,...,M$. Define new $\Delta^U$ as min ( $\max_{i=1,2,..,M} \{\Delta_i^U\}$, current $\Delta^U$). If $\{\Delta^U - \Delta^L\} \leq \varepsilon$ then stop and the robust solution that attains $\Delta^U$ in BLPP model is an ε-globally optimal robust solution. Otherwise add $\omega_i$ with $\Delta_i^U \geq \Delta^L \ \forall i = 1,2,...,M$ to set Ω and proceed to step c.

The following proposition provides the important result that this semi-continuous algorithm will always terminate at an ε-globally optimal robust solution in finite number of algorithm steps.

*Proposition* 8: The semi-continuous robust algorithm terminates at the robust optimal solution in finite number of steps by setting $\varepsilon = 0$.

*Proof*: One of the trivial but ineffective ways of obtaining the robust optimal solution for this problem is to enumerate all possible combinations of $Y_\Omega$ and then send these settings to the second stage and the third stage of the semi-continuous robust algorithm to check for feasibility and to solve for maximum regret associated with each setting. From among all these maximum regret values, pick the feasible setting of $Y_\Omega$ with minimum of maximum regret values as the robust optimal solution. If none exist, the problem has no robust solution.

Each time the semi-continuous robust algorithm executes the first stage, if the problem is RPS or DRRPS infeasible, the algorithm terminates with no robust solution. Otherwise, either a new or the same $Y_{\Omega}$ setting is generated. In the former case, this setting is sent to the second stage for feasibility check. The feasible setting is forwarded to calculate its maximum regret possible and the resultant scenarios are recorded and are always considered in the rest of the algorithm. In the later case, the semi-continuous robust algorithm is terminated with the robust optimal solution. Because there are finite numbers of possible combinations of $Y_{\Omega}$ settings, the claim is proven.

<u>*Corollary* 1:</u> By setting $\varepsilon > 0$, the semi-continuous robust algorithm terminates at the robust ε-optimal solution in finite number of steps.

<u>*Proof*:</u> the proof of this corollary uses the following facts:

1. $\Delta^L$ is a non-decreasing value because by adding constraints to the problem, the objective function value can only be worse or be the same. Because $\Delta^L$ is the objective function value of the relaxation problem, $\Delta^L$ is a lower bound on the minimum maximum regret.
2. $\Delta^U$ is a non-increasing value because the algorithm always keeps the minimum value of these upper bounds. Because $\Delta^U$ represents a maximum regret from one feasible setting of $Y_{\Omega}$, $\Delta^U$ is an upper bound on the minimum maximum regret.
3. The algorithm is terminated when $\Delta^U - \Delta^L \leq \varepsilon$.

From the results of Proposition 8, Fact 1, Fact 2, and Fact 3, the claim is proven.

The following section illustrates the use of the semi-continuous robust algorithm on some small problems for understanding purpose. The application of the semi-continuous robust algorithm on the large-scale case study is presented in Chapter VII.

### 6.6 Example Problems for the Semi-Continuous Robust Algorithm

*Tools Renting Problem*

Every morning, a carpenter has to make his decision on what type of tools he is going to rent for that specific day. There are two types of tools, $tool_1$ and $tool_2$, that he can rent. If he decides to rent $tool_1$, he can use it to produce $product_1$ up to $P_{21}$ units per one day which can be sold with the price of \$2 per unit. If he decides to rent $tool_2$, he can use it to produce $product_2$ up to $P_{22}$ units per day which can be sold with the price of \$$P_4$ per unit. The production of each product not only requires tools but also requires raw materials. By using $tool_1$, one units of $product_1$ requires 2 units of raw materials. By using $tool_2$, one units of $product_2$ requires $P_5$ units of raw materials ($tool_2$ is not very reliable). The numbers of raw material available are $P_3$ units per day. At the end of the day, this carpenter has to pay the rental fee for each rented tool. The rental fees of $tool_1$ and $tool_2$ are \$$P_1$ and \$15 per day respectively. Table 6.1 contains all distribution information of each model parameter. What tool should this carpenter rent at the beginning of each day?

This problem can be initially described by a stochastic mixed integer linear programming problem. Let $x_1$ and $x_2$ represents his decisions on daily production units of $product_1$ and $product_2$ respectively. Let $y_1$ and $y_2$ represents his decisions on renting $tool_1$ and $tool_2$ respectively where $y_i = 1$ if he rent $tool_i$ and 0 otherwise for $i = 1, 2$. Figure 6.15 illustrates this initial model.

**Table 6.1 Distribution Information of All Model Parameters**

| Random Parameters | Probability Distribution |
|---|---|
| $P_1$ | Uniform (8, 12) |
| $P_{21}$ | Unknown with UB = 38 and LB = 32 (Average ≈ 35) |
| $P_{22}$ | Unknown with UB = 50 and LB = 40 (Average ≈ 45) |
| $P_3$ | Triangular Distribution (90, 100, 110) |
| $P_4$ | Triangular Distribution (1, 2.5715, 4) |
| $P_5$ | $\Pr(P_5 = 2) = \Pr(P_5 = 4) = 0.5$ |

$$
\begin{aligned}
\max_{x} \quad & 2x_1 + P_4 x_2 - P_1 y_1 - 15 y_2 \\
s.t. \quad & x_1 \le P_{21} y_1 \\
& x_2 \le P_{22} y_2 \\
& 2x_1 + P_5 x_2 \le P_3 \\
& x_1, x_2 \ge 0 \quad y_1, y_2 \in \{0,1\}
\end{aligned}
$$

**Figure 6.15 Initial Stochastic Mixed Integer Linear Programming Model**

By applying semi-continuous robust algorithm, we start by considering four initial scenarios, which cover all possible values of the discrete random variable, $P_5$. Table 6.2 contains all parameter' values and $O^*_{\omega}$ for each scenario.

**Table 6.2 All Parameter' Values and $O^*_\omega$ for Four Initial Scenarios**

| Scenario | $P_1$ | $P_{21}$ | $P_{22}$ | $P_3$ | $P_4$ | $P_5$ | $x_1$ | $x_2$ | $y_1$ | $y_2$ | $O^*_\omega$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 32 | 40 | 90 | 1 | 2 | 32 | 0 | 1 | 0 | 56 |
| 2 | 12 | 38 | 50 | 110 | 4 | 2 | 0 | 50 | 0 | 1 | 185 |
| 3 | 8 | 32 | 40 | 90 | 1 | 4 | 32 | 0 | 1 | 0 | 56 |
| 4 | 12 | 38 | 50 | 110 | 4 | 4 | 0 | 27.5 | 0 | 1 | 95 |

By using this information in Table 6.2, the DRRPS model for these four scenarios can be optimally solved. Table 6.3 contains all solutions of this DRRPS model.

**Table 6.3 Solutions of the DRRPS Model for Four Initial Scenarios**

| Scenario | $x_{1\omega}$ | $x_{2\omega}$ | $y_{1\Omega}$ | $y_{2\Omega}$ | $O^*_\omega$ | $R_\omega$ | $O^*_\omega$- $R_\omega$ |
|---|---|---|---|---|---|---|---|
| 1 | 32 | 13 | 1 | 1 | 56 | 54 | 2 |
| 2 | 5 | 50 | 1 | 1 | 185 | 183 | 2 |
| 3 | 32 | 6.5 | 1 | 1 | 56 | 47.5 | 8.5 |
| 4 | 0 | 27.5 | 1 | 1 | 95 | 83 | 12 |

The candidate robust solution from the first stage is now $y_{1\Omega} = 1$ and $y_{2\Omega} = 1$ with the lower bound of 12. This information is then forwarded to the second stage of the algorithm for feasibility check. After performing the pre-processing step, all parameters can be fixed as follows: $P_{21} = 32$, $P_{22} = 40$, $P_3 = 90$, and $P_5 = 2$ or 4. Because these settings are already considered in scenario 1 and 3, the current candidate robust solution is already feasible for all possible scenarios. This current candidate robust solution and the lower bound are then forwarded to the third stage of the algorithm. At this stage, we are required to solve two BLPP models (case $P_5 = 2$ and $P_5 = 4$). By applying parameter pre-processing step, $P_1$ can be fixed to the value of 12. Figure 6.16 illustrates the initial form of the BLPP model and Figure 6.17 illustrates the final form of the BLPP model. Table 6.4 contains the optimal solution for these BLPP models. Because the upper bound resulting from this BLPP model is 12, the algorithm is then terminated with the robust optimal solution of $y_{1\Omega} = 1$ and $y_{2\Omega} = 1$ (the carpenter should rent both tools at the beginning of each day). Table 6.5 contains the comparison between the optimal robust solution and the optimal solution from the solution obtained from a standard mixed integer linear programming problem that uses average values for the uncertain parameters ($y_{1\Omega} = 0$ and $y_{2\Omega} = 1$).

$$
\begin{aligned}
\max_{\substack{x_{11},x_{12},y_1,y_2\\P_{21},P_{22},P_3,P_4}} \; & (2x_{11} + P_4 x_{12} - 12y_1 - 15y_2 - 2x_{21} - P_4 x_{22} + 27) \\
s.t. \quad & x_{11} \le P_{21} y_1 \\
& x_{12} \le P_{22} y_2 \\
& 2x_{11} + P_5 x_{12} \le P_3 \\
& x_{11}, x_{12} \ge 0 \quad y_1, y_2 \in \{0,1\} \\
& 32 \le P_{21} \le 38, \;\; 40 \le P_{22} \le 50 \\
& 90 \le P_3 \le 110, \; 1 \le P_4 \le 4 \\
& \max_{x_{21},x_{22}} (2x_{21} + P_4 x_{22} - 27) \\
& \quad s.t. \quad x_{21} \le P_{21} \\
& \quad\quad\quad\; x_{22} \le P_{22} \\
& \quad\quad\quad\; 2x_{21} + P_5 x_{22} \le P_3 \\
& \quad\quad\quad\; x_{21}, x_{22} \ge 0
\end{aligned}
$$

**Figure 6.16 The Initial Form of the BLPP Model ($P_5 = 2$ or $4$)**

$$\max_{x_{11},x_{12},y_1,y_2} (2x_{11} + PX_{412} - 12y_1 - 15y_2 - 2x_{21} - PX_{422} + 27)$$

$$\begin{array}{ll}
s.t.\quad x_{11} \le PY_{211} & x_{12} \le PY_{222} \\
2x_{11} + P_5 x_{12} \le P_3 & x_{21} + s_1 = P_{21} \\
x_{22} + s_2 = P_{22} & 2x_{21} + P_5 x_{22} + s_3 = P_3 \\
w_1 + 2w_3 - a_1 = 2 & w_2 + P_5 w_3 - a_2 = P_4 \\
PY_{211} - P_{21} \le 0 & -PY_{211} + P_{21} - 38(1 - y_1) \le 0 \\
PY_{211} \le 38y_1 & P_{21} \le 32 + 6y_1 \\
PY_{222} - P_{22} \le 0 & -PY_{222} + P_{22} - 50(1 - y_2) \le 0 \\
PY_{222} \le 50y_2 & P_{22} \le 40 + 10y_2 \\
PX_{412} - 4x_{12} \le 0 & -PX_{412} + 4x_{12} - 200(1 - bi_1) \le 0 \\
PX_{412} - x_{12} - 200bi_1 \le 0 & -PX_{412} + x_{12} \le 0 \\
PX_{422} - 4x_{22} \le 0 & -PX_{422} + 4x_{22} - 200(1 - bi_1) \le 0 \\
PX_{422} - x_{22} - 200bi_1 \le 0 & -PX_{422} + x_{22} \le 0 \\
-P_4 + 4 - 3(1 - bi_1) \le 0 & P_4 - 1 - 3bi_1 \le 0 \\
\multicolumn{2}{l}{x_{21}a_1 = 0,\ x_{22}a_2 = 0,\ w_1 s_1 = 0,\ w_2 s_2 = 0,\ w_3 s_3 = 0,} \\
\multicolumn{2}{l}{x_{11}, x_{12}, x_{21}, x_{22}, s_1, s_2, s_3, w_1, w_2, w_3, a_1, a_2 \ge 0, \quad y_1, y_2, bi_1 \in \{0,1\}} \\
\multicolumn{2}{l}{32 \le P_{21} \le 38,\ \ 40 \le P_{22} \le 50,\ 90 \le P_3 \le 110,\ 1 \le P_4 \le 4}
\end{array}$$

**Figure 6.17 The Final Form of the BLPP Model ($P_5$ = 2 or 4)**

**Table 6.4 The Optimal Solution for the BLPP Model**

| Decision Variable | $P_5 = 2$ | $P_5 = 4$ |
|---|---|---|
| $x_{11}$ | 0 | 0 |
| $x_{12}$ | 45 | 22.5 |
| $x_{21}$ | 0 | 32 |
| $x_{22}$ | 45 | 6.5 |
| $y_1$ | 0 | 0 |
| $y_2$ | 1 | 1 |
| $P_{21}$ | 32 | 32 |
| $P_{22}$ | 45 | 45 |
| $P_3$ | 90 | 90 |
| $P_4$ | 4 | 4 |
| $O^*_{\omega}$ - $R_{\omega}$ | 12 | 12 |

**Table 6.5 Comparison between the Semi-Continuous Robust Solution and the Optimal Solution from the Average Value Problem**

| | **Maximum Regret From Optimality** | **Objective Value under Average Value Scenario** |
|---|---|---|
| Solution for Problem using Average Values for Uncertain Parameters | 60.5 | 71.66 |
| Semi-Continuous Robust Solution | 12 | 71 |

For the average value problem, the decision makers ignore uncertainty in the model parameters and replace all random variables with their mean values. The results in Table 6.5 illustrate the superiority of the semi-continuous robust solution over the optimal solution for the average value problem.

*Example Comparing Semi-Continuous Robust Solutions and End-Point Robust Solutions*

In this section, we illustrate the comparison of the semi-continuous robust solution to the robust solution from the discrete robust algorithm that only considers each uncertain parameter at its boundaries. Figure 6.18 illustrates the initial form of the example problem.

$$
\begin{aligned}
&\max_{\substack{x_1,x_2,x_3\\ y_1,y_2,y_3}} (2x_1 + x_2 + x_3) \\
&s.t. \quad x_1 + x_2 \le 10y_1 \qquad x_1 \le 5y_2 \\
&\qquad\quad x_2 \le P_3 \qquad\qquad\quad x_1 \le x_2 \\
&\qquad\quad x_1 + x_3 \le 5y_3 \\
&\qquad\quad x_1, x_2, x_3 \ge 0 \quad y_1, y_2, y_3 \in \{0,1\} \quad P_3 \sim Uniform(0,10)
\end{aligned}
$$

**Figure 6.18 The Initial Form of the Problem**

Decision makers usually mislead themselves to the conclusion that setting the $P_3$ random variable value at its boundaries will generate the scenarios, which control the maximum regret from optimality. Figure 6.19 illustrates the discrete robust optimization model when considering $P_3$ value only at its boundaries.

$$
\begin{aligned}
&\min_{\substack{x_1,x_2,x_3\\ y_1,y_2,y_3}} \qquad \delta \\
&s.t. \quad \delta \ge 5 - 2x_{11} - x_{12} - x_{13} \quad \delta \ge 15 - 2x_{21} - x_{22} - x_{23} \\
&\qquad\quad x_{11} + x_{12} \le 10y_1 \quad x_{21} + x_{22} \le 10y_1 \\
&\qquad\quad x_{11} \le 5y_2 \qquad\qquad x_{21} \le 5y_2 \\
&\qquad\quad x_{12} \le 0 \qquad\qquad\;\; x_{22} \le 10 \\
&\qquad\quad x_{11} \le x_{12} \qquad\qquad x_{21} \le x_{22} \\
&\qquad\quad x_{11} + x_{13} \le 5y_3 \quad\;\; x_{21} + x_{23} \le 5y_3 \\
&\quad x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23} \ge 0 \quad y_1, y_2, y_3 \in \{0,1\}
\end{aligned}
$$

**Figure 6.19 The Discrete Robust Model (Setting $P_3$ at its Boundaries)**

The optimal solution to this robust model can be attained by setting $y_1 = 1$, $y_3 = 1$, $x_{12} = 10$, $x_{22} = 10$, $x_{23} = 5$ and all other variables at zero. The minimum maximum regret by using the discrete robust solution ($y_1 = 1$, $y_2 = 0$, $y_3 = 1$) is zero. Decision makers would be misled to the erroneous conclusion that this discrete robust solution is an optimal robust solution to the original problem. Misled decision makers would not be aware that the actual maximum regret for the original problem of this discrete robust solution might be really far away from zero (five in this case).

We will now apply the semi-continuous robust algorithm to this problem by using two initial scenarios ($P_3 = 0$ and $P_3 = 10$). The first stage of the algorithm will give the candidate robust solution by setting $y_1 = 1$, $y_2 = 0$, and $y_3 = 1$ with the lower bound of zero. This candidate robust solution is then forwarded to the second stage of the algorithm for feasibility check. After performing the pre-processing step, the $P_3$ parameter can be fixed at zero. Because this setting is already considered in scenario 1, the current candidate robust solution is already feasible for all possible scenarios. This current candidate robust solution and the lower bound are then forwarded to the third stage of the algorithm. At this stage, we are required to solve a BLPP model. After applying parameter pre-processing step and elimination step, the final form of the BLPP model is illustrated in Figure 6.20. Table 6.6 contains the optimal solution for this BLPP model.

$$
\begin{aligned}
& \max(2x_{11} + x_{12} + x_{13} - x_{22} - x_{23}) \\
s.t.\quad & x_{11} + x_{12} \le 10y_1 \qquad x_{11} \le 5y_2 \qquad x_{12} \le P_3 \\
& x_{11} - x_{12} \le 0 \qquad x_{11} + x_{13} \le 5y_3 \\
& x_{22} + s_1 = 10 \qquad x_{22} + s_2 = P_3 \qquad x_{23} + s_3 = 5 \\
& w_1 + w_2 - a_1 = 1 \qquad w_3 - a_2 = 1 \\
& x_{22}a_1 = 0,\ x_{23}a_2 = 0,\ w_1 s_1 = 0,\ w_2 s_2 = 0,\ w_3 s_3 = 0 \\
& x_{11}, x_{12}, x_{13}, x_{22}, x_{23}, w_1, w_2, w_3, s_1, s_2, s_3, a_1, a_2 \ge 0 \\
& y_1, y_2, y_3 \in \{0,1\}
\end{aligned}
$$

**Figure 6.20 The Final Form of the BLPP Model ($y_1 = 1, y_2 = 0, y_3 = 1$)**

**Table 6.6 The Optimal Solution for the BLPP Model ($y_1 = 1, y_2 = 0, y_3 = 1$)**

| $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $y_1$ | $y_2$ | $y_3$ | $P_3$ | $O^*_\omega - R_\omega$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5 | 0 | 0 | 5 | 5 | 1 | 1 | 1 | 5 | 5 |

Because the upper bound resulting from this BLPP model is 5, the algorithm forwards the setting of $P_3$ at 5 (scenario 3) and the upper bound of 5 to the first stage of the algorithm. The optimal solution under this new scenario is calculated next and the algorithm then solves the discrete robust optimization model under these three scenarios. The optimal solution to this discrete robust model is attained by setting $y_1 = 1$, $y_2 = 1$, $y_3 = 1$, $x_{11} = 5$, $x_{12} = 5$, $x_{21} = 5$, $x_{22} = 5$, $x_{31} = 5$, $x_{32} = 5$ and all other variables at zero. The first stage of the algorithm will give the candidate robust solution by setting $y_1 = 1$, $y_2 = 1$, and $y_3 = 1$ with the lower bound of zero. This candidate robust solution is then forwarded to the second stage of the algorithm for feasibility check. After performing the pre-processing step, the $P_3$ parameter can be fixed at zero. Because this setting is already considered in scenario 1, the current candidate robust solution is already feasible for all possible scenarios. This current candidate robust solution and the lower bound are then forwarded to the third stage of the algorithm. At this stage, we are required to solve a BLPP model. After applying parameter pre-processing step and elimination step, the final form of the BLPP model is illustrated in Figure 6.21.

$$
\begin{aligned}
& \max(2x_{11} + x_{12} + x_{13} - 2x_{21} - x_{22} - x_{23}) \\
s.t.\quad & x_{11} + x_{12} \le 10y_1 \quad x_{11} \le 5y_2 \quad x_{12} \le P_3 \\
& x_{11} - x_{12} \le 0 \quad x_{11} + x_{13} \le 5y_3 \\
& x_{21} + x_{22} + s_1 = 10 \quad x_{21} + s_2 = 5 \quad x_{22} + s_3 = P_3 \\
& x_{21} - x_{22} + s_4 = 0 \quad x_{21} + x_{23} + s_5 = 5 \\
& w_1 + w_2 + w_4 + w_5 - a_1 = 2 \\
& w_1 + w_3 - w_4 - a_2 = 1 \quad w_5 - a_3 = 1 \\
& x_{21}a_1 = 0,\ x_{22}a_2 = 0,\ x_{23}a_3 = 0,\ w_1 s_1 = 0 \\
& w_2 s_2 = 0,\ w_3 s_3 = 0,\ w_4 s_4 = 0,\ w_5 s_5 = 0 \\
& x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, w_1, w_2, w_3, w_4, w_5, s_1, s_2, s_3, s_4, s_5, a_1, a_2, a_3 \ge 0 \\
& y_1, y_2, y_3 \in \{0,1\}
\end{aligned}
$$

**Figure 6.21 The Final Form of the BLPP Model ($y_1 = 1, y_2 = 1, y_3 = 1$)**

Because the upper bound resulting from this BLPP model is zero, the algorithm is then terminated with the real robust optimal solution of $y_{1\Omega} = 1$, $y_{2\Omega} = 1$ and $y_{3\Omega} = 1$ to the original problem. These results illustrate the superior of the semi-continuous robust algorithm over the use of discrete robust optimization algorithm that only consider each parameter at its boundaries.

## 6.7 Summary

This chapter develops a new semi-continuous robust optimization algorithm for dealing with uncertainty in parameter values for reverse production system design problems and network infrastructure planning problems.

The semi-continuous robust algorithm is the first known approach to generate min-max regret robust solutions when the uncertain parameters in the mixed integer linear programming (MILP) network problem take their values from a real compact interval or a finite set of discrete values. The algorithm can be effectively used in designing robust network infrastructure for supply chain systems when the joint probability distributions of key parameters are unknown. The algorithm only requires the information on potential ranges and possible discrete values of uncertain parameters, which often are available in practice. The algorithm also involves many pre-processing steps, elimination steps and problem transformation procedures for improving its computational ability. The algorithm is proven to either terminate at an optimal robust solution or identify the inexistence of the robust solution in finite number of iterations.

The algorithm can easily be extended to generate the min-max regret robust solution to the problem when each uncertain continuous parameter takes its values from more than one compact interval (finite number of compact intervals). In this case, the initial discrete scenarios are generated based on the combination of all possible values of discrete parameters and all possible compact intervals of continuous parameters. In other words, each scenario in the initial discrete scenarios only contains one possible value of each discrete parameter and one possible compact interval of each continuous parameter. All remaining steps of the algorithm are the same.

In the next chapter, case studies are presented for illustrating the application of the algorithm on designing the robust supply chain network infrastructure for the realistically sized problem.

# CHAPTER VII

# CASE STUDIES OF SEMI-CONTINUOUS ROBUST ALGORITHM ON REVERSE SUPPLY CHAIN PROBLEMS

## 7.1 Introduction

The detail methodology of the semi-continuous robust algorithm was introduced in Chapter VI. In this chapter, two case studies are presented to illustrate the use of the semi-continuous robust algorithm on designing the robust infrastructure for reverse production systems. The first case study is an example of a moderate size traditional reverse production system problem with uncertainty in model parameters. The network represented is not meant to represent an existing system and is constructed only for illustrating the use of the semi-continuous robust algorithm on reverse supply chain problems. In this case study, the comparison of the solution quality between the semi-continuous robust solution and the average case solution is presented to illustrate the superiority of the semi-continuous robust solution over the average case solution. This case study also presents the statistical analysis of the relationship between the locations of uncertain parameters and the computational time required for solving the problem.

The second case study is a large Georgia television recycling network with uncertainty in supply of obsolete televisions, selling price of refurbished televisions and capacity of television refurbishing processes. The case study is solved using the semi-continuous algorithm. The comparisons of the solution time and problem size between the algorithm with and without pre-processing, elimination, and branching rules are also presented to illustrate the significant improvement in the BLPP model solution time by implementing these rules.

These case studies illustrate how a robust infrastructure can be generated for a strategic reverse production system under uncertainty in model parameters by implementing the semi-continuous robust algorithm. This chapter also illustrates the practical use of the algorithm for designing the robust infrastructure of a realistically sized reverse supply chain problem.

## 7.2 Case Study 1

In this case study, the government of city A is planning to construct a reverse supply chain infrastructure for the city. The resulting infrastructure is required to collect four types of obsolete materials from the city for recycling. These four types of obsolete materials can be collected from four different sections of the city. The supply information of these materials in each section of the city is provided in Table 7.1. Because of budget restrictions, the government only has three possible locations for collection centers and three possible locations for processing centers in the city. Table 7.3 contains all the information for each collection center. At each processing center, there are three possible alternative recycling processes. The information of each process is contained in Table 7.2 and the information of each processing center is contained in Table 7.4. The recycled materials can be resold to four possible different demand points inside and outside of the city. The demand information for these recycled materials at each demand point is provided in Table 7.5. Table 7.6 and Table 7.7 contain the distance information and the transportation cost information through the possible network respectively. Table 7.8 presents the fixed annual cost for opening and operating each collection center and processing center. Figure 7.1 illustrates the summary of the possible network infrastructure for the problem.

**Table 7.1 Supply Information at Each Section of the City**

| Section | Supply of Material 1 (lbs) | Supply of Material 2 (lbs) | Supply of Material 3 (lbs) | Supply of Material 4 (lbs) |
|---|---|---|---|---|
| Section 1 (So1) | Unknown distribution<br>UB = 15,000<br>LB = 10,000<br>Mean = 12,500 | 0 | 0 | 0 |
| Section 2 (So2) | 0 | 10,000 | 0 | 0 |
| Section 3 (So3) | 0 | 0 | 12,000 | 0 |
| Section 4 (So4) | 0 | 0 | 0 | Uniform (6000,8000) |

**Table 7.2 Process Information**

| Recycle Process | Process Inputs | Process Output |
|---|---|---|
| Process 1 | 50% Material 1<br>50% Material 2 | 100% Material 5 with prob = 0.5<br>or 80% Material 5 and<br>20% Material 8 with prob = 0.5 |
| Process 2 | 60% Material 3<br>40% Material 4 | 90% Material 6<br>10% Material 8 |
| Process 3 | 70% Material 5<br>30% Material 6 | 100% Material 7 |

**Table 7.3 Collection Center Information**

| Collection Center | Material | Collection Fee ($ per lbs) | ($) Fixed Collection Cost | Capacity Collection (lbs) |
|---|---|---|---|---|
| Collection Center 1 (Si1) | Material 1 | -10 | 6,000 | 20,000 |
| | Material 2 | -10 | 6,000 | 20,000 |
| | Material 3 | -12 | 8,000 | 20,000 |
| | Material 4 | -8 | 4,000 | 20,000 |
| Collection Center 2 (Si2) | Material 1 | -10 | 6,000 | 20,000 |
| | Material 2 | -10 | 6,000 | 20,000 |
| | Material 3 | -12 | 8,000 | 20,000 |
| | Material 4 | -8 | 4,000 | 20,000 |
| Collection Center 3 (Si3) | Material 1 | -10 | 6,000 | 20,000 |
| | Material 2 | -10 | 6,000 | 20,000 |
| | Material 3 | -12 | 8,000 | 20,000 |
| | Material 4 | -8 | 4,000 | 20,000 |

**Table 7.4 Processing Center Information**

| Processing Center | Recycle Process | Process Availability | Fixed Processing Cost ($) | Variable Processing Cost ($/lbs) | Capacity Process (lbs) |
|---|---|---|---|---|---|
| Processing Center 1 (Si4) | Process 1 | Yes | 12,000 | 5 | 30,000 |
| | Process 2 | Yes | 8,000 | 5 | Uniform(15000,30000) |
| | Process 3 | No | N/A | N/A | N/A |
| Processing Center 2 (Si5) | Process 1 | No | N/A | N/A | N/A |
| | Process 2 | Yes | 10,000 | 5 | 30,000 |
| | Process 3 | Yes | 12,000 | 6 | Uniform(10000,30000) |
| Processing Center 3 (Si6) | Process 1 | Yes | 12,000 | 5 | 30,000 |
| | Process 2 | No | N/A | N/A | N/A |
| | Process 3 | Yes | 15,000 | 6 | 30,000 |

**Table 7.5 Demand Information**

| Demand Point | Material | Price ($ per lbs) | Demand (lbs) |
|---|---|---|---|
| Demand Point 1 (C1) | Material 5 | Triangular(35,42.5,50) | Uniform(10000,12000) |
| Demand Point 2 (C2) | Material 6 | Triangular(40,42.5,45) | Uniform(15000,20000) |
| Demand Point 3 (C3) | Material 7 | Triangular(55,62.5,70) | Uniform(20000,23500) |
| Demand Point 4 (C4) | Material 1 | -5 | 30,000 |
| | Material 2 | -5 | 30,000 |
| | Material 3 | -5 | 30,000 |
| | Material 8 | -10 | 30,000 |

**Table 7.6 Distance Information**

| Distance (miles) | So1 | So2 | So3 | So4 | Si1 | Si2 | Si3 | Si4 | Si5 | Si6 | C1 | C2 | C3 | C4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| So1 | - | - | - | - | 200 | 150 | 100 | - | - | - | - | - | - | - |
| So2 | - | - | - | - | 150 | 150 | 150 | - | - | - | - | - | - | - |
| So3 | - | - | - | - | 100 | 150 | 200 | - | - | - | - | - | - | - |
| So4 | - | - | - | - | 50 | 150 | 250 | - | - | - | - | - | - | - |
| Si1 | 200 | 150 | 100 | 50 | - | 100 | 100 | 200 | 150 | 400 | - | - | - | - |
| Si2 | 150 | 150 | 150 | 150 | 100 | - | 100 | 200 | 200 | 350 | - | - | - | - |
| Si3 | 100 | 150 | 200 | 250 | 100 | 100 | - | 200 | 250 | 300 | - | - | - | - |
| Si4 | - | - | - | - | 200 | 200 | 200 | - | 100 | 200 | 100 | 150 | 80 | 70 |
| Si5 | - | - | - | - | 150 | 200 | 250 | 100 | - | 200 | 150 | 100 | 90 | 100 |
| Si6 | - | - | - | - | 400 | 350 | 300 | 200 | 200 | - | 70 | 120 | 100 | 150 |
| C1 | - | - | - | - | - | - | - | 100 | 150 | 70 | - | - | - | - |
| C2 | - | - | - | - | - | - | - | 150 | 100 | 120 | - | - | - | - |
| C3 | - | - | - | - | - | - | - | 80 | 90 | 100 | - | - | - | - |
| C4 | - | - | - | - | - | - | - | 70 | 100 | 150 | - | - | - | - |

**Table 7.7 Transportation Cost Information**

| Type of Transportation | Transportation Cost ($ per lbs mile) | Annual Capacity Transportation (lbs) |
|---|---|---|
| From So to Si | 0.05 | 30,000 |
| From Si to Si | 0.01 | 30,000 |
| From Si to C | 0.05 | 30,000 |

**Table 7.8 Site Opening and Operation Cost Information**

| Location | Fixed Annual Site Opening and Operating Cost |
|---|---|
| Collection Center 1 | Unknown Distribution<br>UB = 20,000<br>LB = 15,000<br>Mean = 17,500 |
| Collection Center 2 | Unknown Distribution<br>UB = 20,000<br>LB = 15,000<br>Mean = 17,500 |
| Collection Center 3 | Unknown Distribution<br>UB = 20,000<br>LB = 15,000<br>Mean = 17,500 |
| Processing Center 1 | 30,000 |
| Processing Center 2 | 30,000 |
| Processing Center 3 | 30,000 |

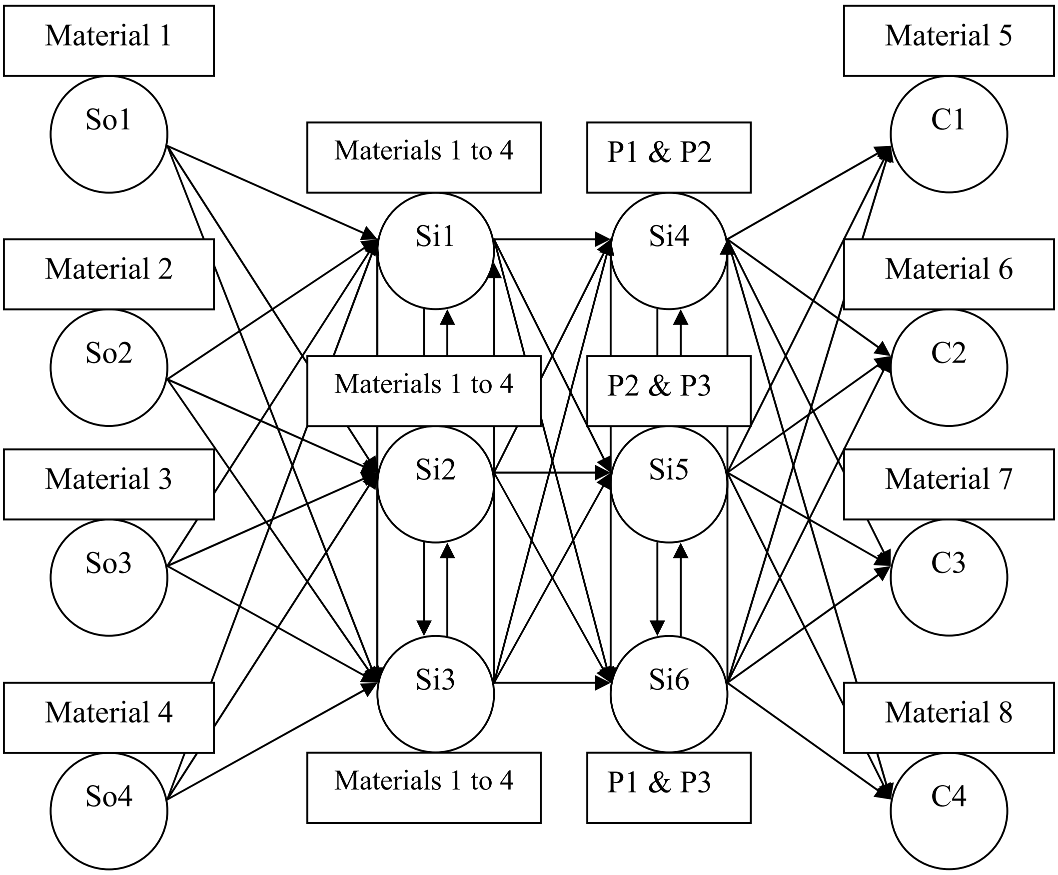


**Figure 7.1 Possible Network Infrastructure for the Problem**

*Comparison between Semi-Continuous Robust Solution and Average Case Solution*

The semi-continuous robust algorithm is first applied to this problem by using two initial scenarios that contain all possible discrete scenarios and capture all boundaries of continuous scenarios. Table 7.9 contains detail information for these two initial scenarios. The algorithm starts by solving two RPS models (one for each initial scenario) to obtain $O^*_1$ and $O^*_2$. The algorithm continues to solve the DRRPS model by considering only these two initial scenarios. Figure 7.2 illustrates the candidate robust solution to the problem (infrastructure solution from the current DRRPS model) with the lower bound on mini-max regret of 45,000. The optimal objective function value and the robust objective function value for each scenario are illustrated in Table 7.10.

**Table 7.9 Detail Information of Two Initial Scenarios**

| **Uncertain Parameter** | **Scenario 1** | **Scenario 2** |
|---|---|---|
| Fixed Opening Cost of Collection Center 1 | 20,000 | 15,000 |
| Fixed Opening Cost of Collection Center 2 | 20,000 | 15,000 |
| Fixed Opening Cost of Collection Center 3 | 20,000 | 15,000 |
| Price of Material 5 | 50 | 35 |
| Price of Material 6 | 45 | 40 |
| Price of Material 7 | 70 | 55 |
| Supply of Material 1 | 15,000 | 10,000 |
| Supply of Material 4 | 8,000 | 6,000 |
| Demand of Material 5 | 10,000 | 12,000 |
| Demand of Material 6 | 15,000 | 20,000 |
| Demand of Material 7 | 20,000 | 23,500 |
| Process 2 Capacity at Processing Center 1 | 15,000 | 30,000 |
| Process 3 Capacity at Processing Center 2 | 10,000 | 30,000 |
| Output of Process 1 | 100% Material 5 | 80% Material 5<br>20% Material 8 |

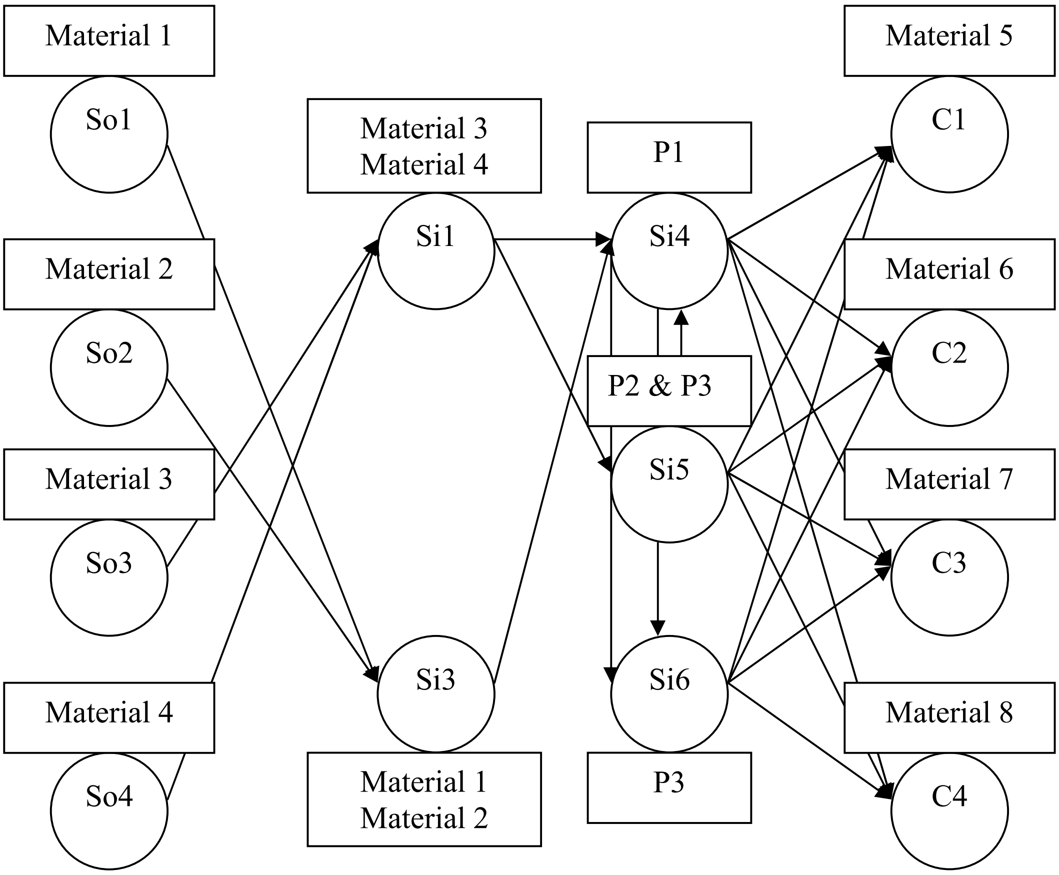


**Figure 7.2 First Candidate Robust Infrastructure to the Problem**

**Table 7.10 Optimal and the Robust Objective Function Values**

| **Scenario** | **Optimal** | **Robust** | **Regret** | **% of Optimal** |
|---|---|---|---|---|
| 1 | 695,500 | 665,500 | 30,000 | 95.69% |
| 2 | 148,892.857 | 103,892.857 | 45,000 | 69.78% |

The algorithm now forwards this candidate robust solution to the second stage of the algorithm to check for feasibility. Because scenario one includes the combination of high supply, low demand and low capacity and the candidate robust solution provides a feasible infrastructure for this scenario, this solution also provides feasible infrastructure for all possible scenarios. This candidate robust solution and the lower bound are then forwarded to the third stage of the algorithm. At this stage, we are required to solve two BLPP models (one for each discrete scenario). After applying parameter pre-processing step and elimination step, the BLPP models are solved by using the branch and bound steps presented in Chapter VI. The results from these BLPP models generate the upper bound of 45,000. Because there is no difference between upper bound and lower bound, the algorithm terminates with the candidate robust solution as the optimal robust solution. Table 7.12 contains two scenarios generated by this stage of the algorithm and Table 7.11 illustrates the performance of the optimal robust solution under all four scenarios. Table 7.13 illustrates the comparison of the BLPP model solution time between the algorithm with and without pre-processing, elimination, and branching rules. The results illustrate the significant improvement in the BLPP model solution time by using these rules.

**Table 7.11 Performance of the Optimal Robust Solution**

| Scenario | Objective under Robust Solution | Objective under Optimal Solution | Regret | % of Optimal | Time required by BLPP model (sec) | Number of Nodes Explored |
|---|---|---|---|---|---|---|
| 1 | 695,500 | 665,500 | 30,000 | 95.69% | N/A | N/A |
| 2 | 148,892.857 | 103,892.857 | 45,000 | 69.78% | N/A | N/A |
| 3 | 756,543.58 | 801,543.58 | 45,000 | 94.39% | 11,111 | 24,030,515 |
| 4 | 620,714.28 | 665,714.28 | 45,000 | 93.24% | 20,348 | 50,203,579 |

**Table 7.12 Two Scenarios Generated by the Third Stage of the Algorithm**

| Uncertain Parameter | Scenario 3 | Scenario 4 |
|---|---|---|
| Fixed Opening Cost of Collection Center 1 | 20,000 | 20,000 |
| Fixed Opening Cost of Collection Center 2 | 15,000 | 15,000 |
| Fixed Opening Cost of Collection Center 3 | 20,000 | 20,000 |
| Price of Material 5 | 35 | 50 |
| Price of Material 6 | 40 | 45 |
| Price of Material 7 | 70 | 70 |
| Supply of Material 1 | 10,000 | 10,000 |
| Supply of Material 4 | 8,000 | 8,000 |
| Demand of Material 5 | 12,000 | 12,000 |
| Demand of Material 6 | 15,000 | 20,000 |
| Demand of Material 7 | 23,499.765 | 22,857.143 |
| Process 2 Capacity at Processing Center 1 | 30,000 | 30,000 |
| Process 3 Capacity at Processing Center 2 | 23,499.765 | 22,857.143 |
| Output of Process 1 | 100% Material 5 | 80% Material 5<br>20% Material 8 |

**Table 7.13 Comparison of the BLPP model Solution Time with Different Rules**

| Average Solution Time (sec) of the BLPP Model | With Branching Rules | Without Branching Rules |
|---|---|---|
| With Pre-Processing and Elimination Rules | 15,729.5 | 100,800 |
| Without Pre-Processing and Elimination Rules | > 172,800 (with no solution) | N/A (CPU out of memory) |

Table 7.14 compares the semi-continuous robust solution and the optimal solution from the average value problem. The results illustrate the superiority of the semi-continuous robust solution over the optimal solution for the average value problem. The feasibility of the optimal solution from the average value problem is confirmed by feasibility of this solution under scenario one. Figure 7.3 illustrates the infrastructure solution from the average value problem.

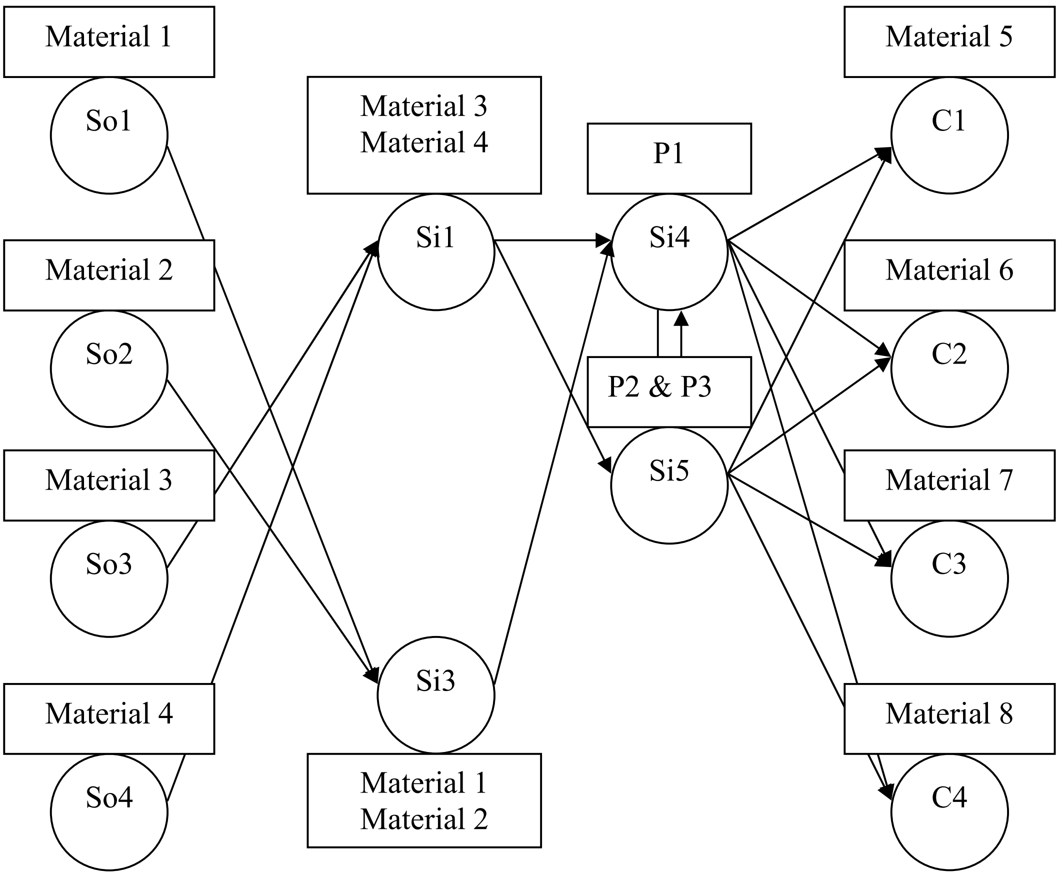


**Figure 7.3 Average Value Solution to the Problem**

**Table 7.14 Comparison between Robust Solution and Average Value Solution**

| | **Maximum Regret From Optimality** | **Objective Value under Average Value Scenario** |
|---|---|---|
| Robust Solution (RS) | 45,000 | 376,500 |
| Average Value Solution (AVS) | 440,975 | 400,500 |
| Difference | 395,375 | 24,000 |
| % (RS)/(AVS) | 10.2% | 94.01% |

In the next section, the statistical analysis of the relationship between parameter type and solution time of the problem is presented for this case study.

*Statistical Relationship between Parameter Type and Solution Time of the Problem*

In this section, the single replicate full factorial experimental design is implemented to find the statistical relationship between random parameter type (location in the model) and solution time required for the algorithm. The experiment starts by using the same problem presented in the last section with some parameter types being random and some parameters types being deterministic at the mean value. The five factors in this experiment are coefficients of discrete variables in the objective function ($P_1$), coefficients of discrete variables in the constraints ($P_2$), right hand side parameters ($P_3$), coefficients of continuous variables in the objective function ($P_4$), and coefficient of continuous variables in the constraints ($P_5$). Each factor is present at two levels (+ for random and – for deterministic). The design matrix and the response data obtained from a single replicate of the $2^5$ experiments are shown in Table 7.15.

**Table 7.15 Experimental Design Matrix and Response Data (Solution Time)**

| Run Number | Factor | | | | | Time (sec) | Min-Max Regret |
|---|---|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | | |
| 1 | – | – | – | – | – | 1 | 0 |
| 2 | – | – | – | – | + | 3 | 0 |
| 3 | – | – | – | + | – | 9.38 | 5,862.5 |
| 4 | – | – | – | + | + | 14.96 | 15,862.5 |
| 5 | – | – | + | – | – | 17.75 | 8,391.667 |
| 6 | – | – | + | – | + | 23.75 | 20,225 |
| 7 | – | – | + | + | – | 77,052.28 | 45,000 |
| 8 | – | – | + | + | + | 13,588.17 | 45,000 |
| 9 | – | + | – | – | – | 7.13 | 41912.5 |
| 10 | – | + | – | – | + | 1,641.11 | 45000 |
| 11 | – | + | – | + | – | 314.53 | 41,912.5 |
| 12 | – | + | – | + | + | 2,150.83 | 45,000 |
| 13 | – | + | + | – | – | 1,958.67 | 45,000 |
| 14 | – | + | + | – | + | 9,106.45 | 45,000 |
| 15 | – | + | + | + | – | 39,069.49 | 45,000 |
| 16 | – | + | + | + | + | 47,897.43 | 45,000 |
| 17 | + | – | – | – | – | 5.5 | 0 |
| 18 | + | – | – | – | + | 6 | 0 |
| 19 | + | – | – | + | – | 10.38 | 5,862.5 |
| 20 | + | – | – | + | + | 16.75 | 15,862.5 |
| 21 | + | – | + | – | – | 14.16 | 8,391.667 |
| 22 | + | – | + | – | + | 23.7 | 20,225 |
| 23 | + | – | + | + | – | 58,344.31 | 45,000 |
| 24 | + | – | + | + | + | 46,573.24 | 45,000 |
| 25 | + | + | – | – | – | 6 | 41,912.5 |
| 26 | + | + | – | – | + | 748.17 | 45,000 |
| 27 | + | + | – | + | – | 175.5 | 41,912.5 |
| 28 | + | + | – | + | + | 1,663.65 | 45,000 |
| 29 | + | + | + | – | – | 5,313.12 | 45,000 |
| 30 | + | + | + | – | + | 6,343.71 | 45,000 |
| 31 | + | + | + | + | – | 57,879.9 | 45,000 |
| 32 | + | + | + | + | + | 31,462 | 45,000 |

We begin the analysis of the experimental results be constructing a normal probability plot of the effect estimates. By using contrasts, we may estimate the 31 factorial effects as shown in Table 7.16.

**Table 7.16 Estimation of Effects Using Contrasts**

| Factor | Effect | Factor | Effect |
|---|---|---|---|
| $P_1$ | 983.135 | $P_1P_5$ | 568.305 |
| $P1P2$ | -802.334 | $P_2P_3$ | -202.909 |
| $P_1P_2P_3$ | -611.013 | $P_2P_3P_4$ | -2734.87 |
| $P_1P_2P_3P_4$ | -798.128 | $P_2P_3P_4P_5$ | 3,033.971 |
| $P_1P_2P_3P_4P_5$ | -5,084.97 | $P_2P_3P_5$ | 3,757.619 |
| $P_1P_2P_3P_5$ | -5,738.77 | $P_2P_4$ | -2,501.6 |
| $P_1P_2P_4$ | -764.056 | $P_2P_4P_5$ | 3,151.328 |
| $P_1P_2P_4P_5$ | -5,017.3 | $P_2P_5$ | 4,468.388 |
| $P_1P_2P_5$ | -5,893.68 | $P_3$ | 24,243.39 |
| $P_1P_3$ | 1,171.884 | $P_3P_4$ | 21,695.47 |
| $P_1P_3P_4$ | 987.5538 | $P_3P_4P_5$ | -6,373.55 |
| $P_1P_3P_4P_5$ | 1,375.931 | $P_3P_5$ | -5,646.64 |
| $P_1P_3P_5$ | 723.3888 | $P_4$ | 21,937.72 |
| $P_1P_4$ | 1,020.448 | $P_4P_5$ | -6,253.83 |
| $P_1P_4P_5$ | 1,444.175 | $P_5$ | -4,932.26 |
| $P_2$ | 627.085 | | |

The normal probability plot of these effects is shown in Figure 7.4. All of the effects that lie along the line are negligible, where as the large effects are far from the line.

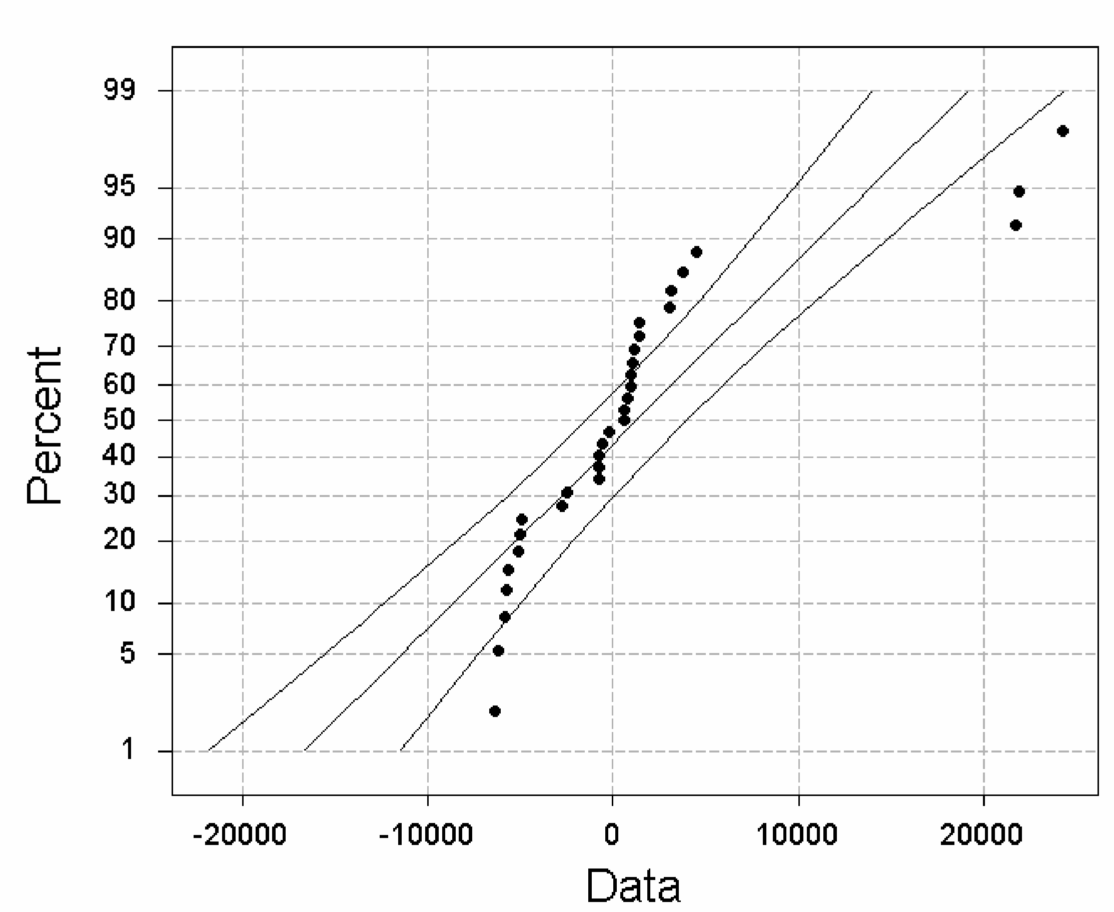


**Figure 7.4 Normal Probability Plot of the Effects**

There are several conclusions that can be drawn from this normal probability plot. The first conclusion is that the interaction effect of $P_3$ and $P_4$ has a significant effect on the solution time for solving this problem. When they both are introduced to this problem, the solution time required for solving this problem increases dramatically. The second conclusion is that the effect of the value of parameter of type $P_5$ has a strong influence on the solution time for this problem, which means that by changing some values for parameters of this type, the solution time for this problem varies widely. The third conclusion is that parameters of type $P_1$ and $P_2$ do not strongly influence the solution times for this problem. The conclusion for parameters of type $P_1$ is quite intuitive because all these parameters can be pre-processed to specific values before the algorithm starts solving the BLPP model. The reason that parameters of type $P_2$ have no strong effect to the solution time of this problem is caused by the low number of uncertain parameters of this type in this problem instance. Even though these conclusions are specific to this problem instance, they demonstrate that the solution time of the semi-continuous robust algorithm is influenced by the location of uncertain parameters in the model.

In conclusion, this case study illustrates the use of semi-continuous robust algorithm on the general moderate size reverse production system problem. The statistical analysis on the possible relationship between the solution time required and the location of uncertain parameters in the model is also presented. The next case study, Georgia television recycling, shows how the semi-continuous robust algorithm is applied to a realistically sized reverse production system problem.

### 7.3 Georgia Television Recycling Case Study

This case study concentrates on the robust design of reverse production system infrastructure for television recycling in the state of Georgia. We assume that no obsolete televisions may go deliberately uncollected, in other words the variables that represent the inflow of the material to the system must equal the amount available for collection. The outputs are in several categories of remanufactured units, component parts, and materials listed in Figure 7.5. The financial flows, depicting profits and costs in different shades are indicated.

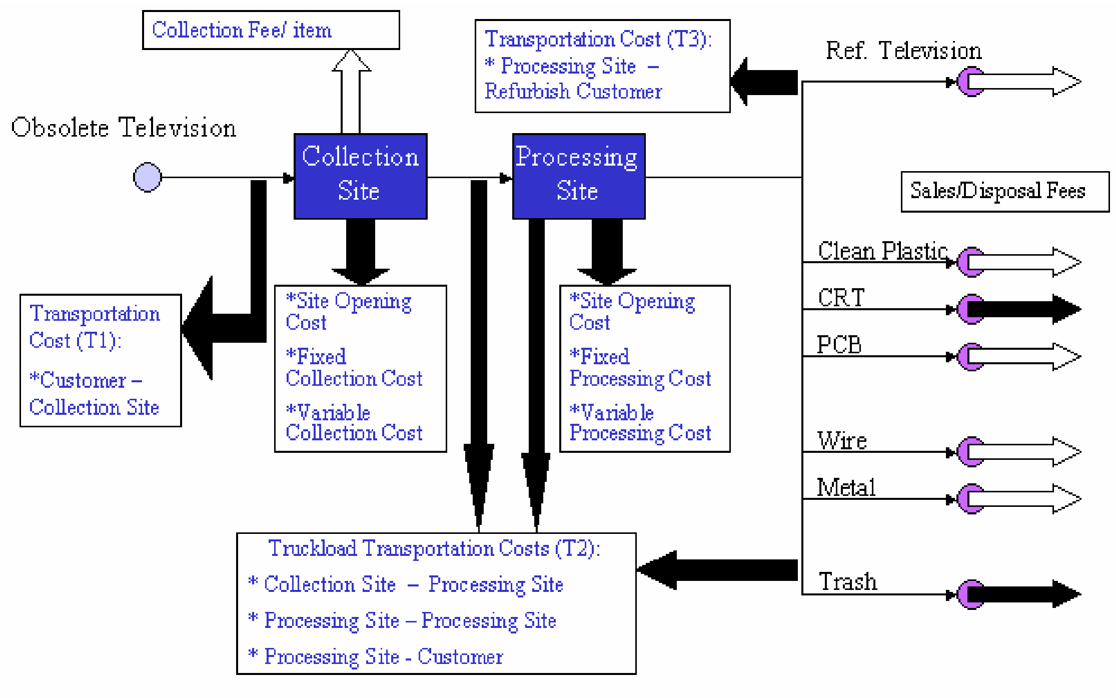


**Figure 7.5 Cash Flow Diagram with Costs (Black) and Profits (White)**

This case study divides the State of Georgia into 12 disjoint regions as shown in Figure 7.6 based on service delivery regions defined by Georgia's Department of Community Affairs (DCA). Each region represents a source of television waste streams, a centralized collection site and also a demand point for the units after refurbishing processes. The amount of obsolete televisions available for collection can be approximated from the population in each region.

For obsolete televisions originating in the state, the case study considers 12 potential state of Georgia government-collection centers located in the center county of each DCA region.

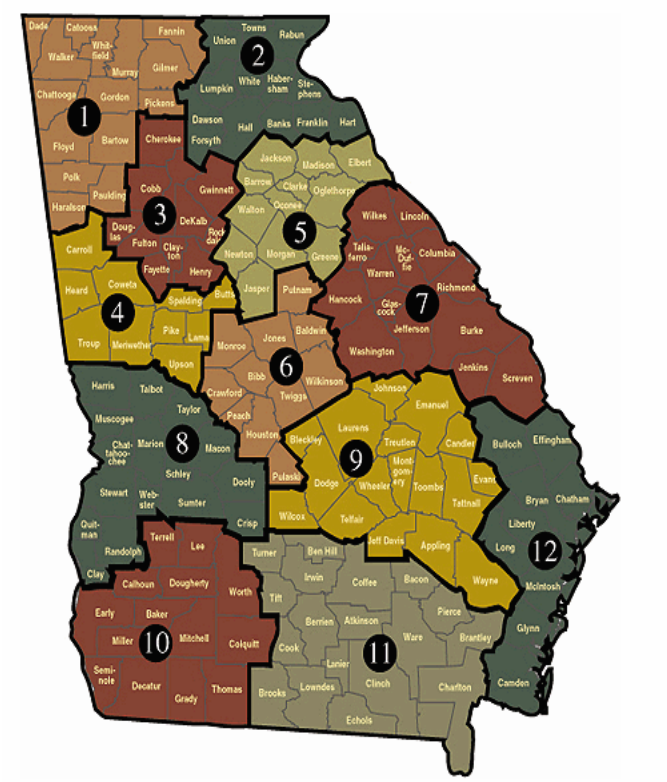


**Figure 7.6 Division of the State of Georgia into 12 DCA Regions**

Each collection center is assumed to collect obsolete televisions from the residential sources located within its 100 miles radius. Additionally, the case study includes nine potential processing centers throughout the state, which are able to perform the television refurbishing process and television demanufacturing process. Figure 7.7 shows all potential sites considered in the case study.


**Figure 7.7 All Potential Sites Considered in the Case Study**

*Supply Information*

The supply information is estimated by using the results from other studies that the supply of e-scrap by assuming that on average 6.2% of the households have an electronic item ready for recycling (Pasco County, Florida, Pilot Program, April 2000), and the assumption that exactly 25% of the total population in each region except region 1, 3, and

12 will participate in the collecting program. For these three specific regions, we assume that the participation rate of the collecting program is varied within 20% to 30%. The supply information of the obsolete televisions is then calculated by using the assumption that fifty percent of all products ready for recycle is comprised of obsolete television. The case study also assumes that the average weight for one television is 51.5 lbs (Alachua County Florida, Summary Report, October 1999). Table 7.17 shows the estimated supply information for obsolete televisions from each region under our assumptions.

**Table 7.17 Georgia Obsolete Television Supply Estimation**

| Region | Supply for TVs (lbs) |
|---|---|
| 1 | 84,000 – 126,000 |
| 2 | 68,600 |
| 3 | 413,200 – 619,800 |
| 4 | 60,900 |
| 5 | 66,000 |
| 6 | 66,300 |
| 7 | 65,500 |
| 8 | 53,300 |
| 9 | 41,000 |
| 10 | 53,100 |
| 11 | 54,900 |
| 12 | 65,400 – 98,100 |

** Televisions: Amount of supply = participation % × 6.2% × Number of households × Product proportion

*Collection Center Information*

Table 7.18 contains the collection center information used in this case study.

**Table 7.18 Collection Center Information**

| Description | Value |
|---|---|
| Fixed collection cost | $16,000 per year* |
| Collection cost | $0.01 per pound |
| Opening cost for government collection sites | $5,000 per year |
| Inspection cost per television | $0.5 per television |
| The collection fee charged for small business and residential sources | $5.28 per usable television<br>$15 per broken television |

* It is assumed that 1 worker per type of material collected with pay rate of $8 per hour working for 8 hours per day for 250 days per year.

*Processing Center Information*

The case study considers nine potential commercial processing centers (all sites located in Georgia). Each facility represents an actual refurbishing and/or demanufacturing facility located in Georgia. Table 7.19 contains the general information for all nine potential processing centers considered in the case study.

**Table 7.19 General Information for All Nine Potential Processing Centers**

| Processing Site Designation | State | County/City | Annualized Site Opening Cost | Number of facilities |
|---|---|---|---|---|
| 1A | Georgia | Catoosa | $28,800 | 1 |
| 2A | Georgia | Carroll | $28,800 | 1 |
| 3A | Georgia | Cobb | $57,600 | 2 |
| 4A | Georgia | Fulton | $144,000 | 5 |
| 5A | Georgia | DeKalb | $172,800 | 6 |
| 6A | Georgia | Gwinnett | $28,800 | 1 |
| 7A | Georgia | Washington | $28,800 | 1 |
| 8A | Georgia | Baldwin | $28,800 | 1 |
| 9A | Georgia | Richmond | $28,800 | 1 |

For each processing center, there are two main potential processes: television refurbishment and television demanufacturing. The information for these six processes is presented in Tables 7.20 and Table 7.21.

**Table 7.20 Variable Costs for Refurbishing and Demanufacturing Processes**

| Description | Value |
|---|---|
| Variable processing cost for refurbishing TVs | $0.23 per lbs* |
| Variable processing cost for demanufacturing TVs | $0.05 per lbs** |

* It is estimated by assuming the processing labor cost is $10 per hour and replacement costs are $8 for TVs. The testing process will take on average of 10 minutes and the refurbishing process will take on average of 20 minutes (DAAE30-98-C-1050, 2000)

** This information is the average of the information from Waters (1998), Pepi (1998), and Minnesota Office of Environmental Assistance (2001).

**Table 7.21 Fixed Processing Costs and Capacity for Each Processing Center**

| Sites | Description | Annualized Value |
|---|---|---|
| Commercial processing sites | Fixed processing cost for refurbishing all products | $8,820 per process (DAAE30-98-C-1050, 2000) |
| | Fixed processing cost for demanufacturing all products | $8,000 per process |
| | Refurbishing capacity per factory for processing center 3, 4, and 5 | 213,360 – 320,040 lbs per year |
| | Refurbishing capacity per factory for other processing centers | 266,700 lbs per year |
| | Demanufacturing capacity per factory | 800,000 lbs per year |

*Demand Information*

The processing centers provide an output of remanufactured equipment, parts, and recycled material to a set of demand locations. We consider three types of demand sources and estimate the quantities using the assumption that the demand for refurbished products in each region has a the positive correlation with the population in the region.

The first type of demand comes from people within Georgia who are interested in buying refurbished electronic equipment. For this type of demand, we use the same 12 DCA regions to designate the demand locations.

The second type of demand source is the group of recycling facilities interested in buying metal, plastic, CRT, and other demanufactured materials. We consider a total of five recyclers located in several states: Georgia (metal recycler), Florida (CRT products and electronics recycler), Texas (plastics recycler), and Ohio (CRT glass recycler).

The last type of demand describes landfills to which we can send the non-hazardous trash resulting from the demanufacturing. We consider eight landfills located in Georgia and group them into 5 demand points based on the DCA regions. (Landfill location information can be found at http://www.wastebyrail.com/network.html.). Table 7.22 illustrates the price information for each refurbished product and material.

**Table 7.22 Price Information for Refurbished Products and Materials**

| **Parameter** | **Value** |
|---|---|
| Selling price for plastic ($ per lb) | 0.175* |
| Selling price for PCB ($ per lb) | 0.9* |
| Selling price for CRT ($ per lb) | -0.1* |
| Selling price for metal ($ per lb) | 0.0175* |
| Selling price for wire ($ per lb) | 0.165* |
| Selling price for trash ($ per lb) (land fill tipping fee) | -0.028* |
| Selling price for used TV for region 1, 3, and 12 ($ per unit) | 48.00 – 72.00<br>(including shipping fee) |
| Selling price for used TV for other regions ($ per unit) | 60.00 |
| Selling price for broken television ($/lbs) | -0.25** |
| Selling price for usable television ($/lbs) | -0.25** |

* EPA-901-R-00-002, September 2000
** The data are from http://www.scrapcomputers.com
***The data are from http://www.boxq.net

*Transportation Information*

There are three types of transportation cost considered in this case study. The first type corresponds to the transportation cost for the people who travel to the collecting centers and drop off their used electronic equipment. This type of transportation cost is approximated by the gasoline cost ($0.15 per mile) and we assume that on average one trip can carry up to 50 lbs of electronic equipment. With this approximation, the transportation cost per lb per mile is $0.003.

The second type represents the transportation costs for moving material between collection centers and processing centers, the transportation costs for moving material between processing centers and recycler demand points, and the transportation costs for moving material between processing centers and landfill demand points. This type of transportation can be performed by a large truck with the cost of $2 per ton per mile or $0.0009 per lb per mile.

The last type corresponds to the transportation cost charged by United Parcel Service (UPS). This cost is about $0.26 per mile per item. This information can be found on the UPS website (www.ups.com).

The data for this Georgia television recycle case study represent a large-scale electronics recycling infrastructure design problem. The objective of the problem is to maximize net profit for the system while determining which collection and processing sites to utilize and then what quantities of each item type to process into what materials at each site.

The key uncertain parameters that we examine are described as follows.

1. *Participation rate.* The participation rate of the collection program for regions 1, 3, and 12 (the three regions with the highest number of population) are random variables that take values from 20% to 30% independently.

2. *Selling Price.* The selling price of the refurbished televisions in region 1, 3, and 12 (the three regions with the highest demand) are random variables that take values from $48 to $72 per television independently.

3. *Capacity of Refurbishing Process.* The capacities of the refurbishing processes in processing centers 3, 4, and 5 (the three processing centers with the highest number of facilities) are random variables that take values from 213,360 lbs to 320,040 lbs per year.

There are three observations for this case study problem which will help us determine the subset of scenarios that control feasibility of the robust solution over all possible scenarios without any additional calculation (BLLP model). First, the discrete solution, which can handle high supply scenarios, can also handle low supply scenarios in this case study. Second, the discrete solution, which is feasible under the extremely low capacity process scenario, is also feasible for the high capacity process scenario. Finally the discrete solution, which can handle the scenarios with extremely low demand, is also feasible for the high demand scenario because there is no restriction that all demands have to be met.

From these three observations, this case study can determine two initial scenarios which control feasibility of the solution and capture all possible bounds of all possible continuous scenarios for the semi-continuous robust algorithm. Table 7.23 contains the detail information for these two initial scenarios.

**Table 7.23 Two Initial Scenarios for the Semi-Continuous Robust Algorithm**

| Type of Random Variable | Location | Value (Scenario 1) | Value (Scenario 2) |
|---|---|---|---|
| Participation Rate | Region 1 | 30% | 20% |
| | Region 3 | 30% | 20% |
| | Region 12 | 30% | 20% |
| Selling Price of Refurbished Televisions | Region 1 | $48 per television | $72 per television |
| | Region 3 | $48 per television | $72 per television |
| | Region 12 | $48 per television | $72 per television |
| Annual Capacity of the Television Refurbishing Process per Facility | Processing Center 3 | 213,360 lbs | 320,040 lbs |
| | Processing Center 4 | 213,360 lbs | 320,040 lbs |
| | Processing Center 5 | 213,360 lbs | 320,040 lbs |

Our case study problems were solved by a Windows 2000-based Pentium 4 1.80GHz personal computer with 1GB RAM using C++ program and CPLEX 8.1 for the optimization process. MS-Access is used for the case study input and output database.

In the first iteration of the algorithm, the algorithm is required to solve two main mixed integer linear programming models (RPS and DRRPS for 2 initial scenarios) and one main linear programming model (RPSLP for 2 initial scenarios). The RPSLP model is used to re-optimize the problem under each scenario when all discrete parameters are fixed at the solution of the DRRPS model. The information on the size of each model and the solution time information are summarized in Table 7.24. Table 7.25 contains all solution information from the first stage of the algorithm in the iteration one.

**Table 7.24 Size of and Solution Time of Each Model (Iteration 1)**

| Model Type | Number of Discrete Variables | Number of Continuous Variables | Number of Constraints | Solution Time(sec) |
|---|---|---|---|---|
| RPS (Scenario1) | 1,174 | 11,849 | 14,182 | 2 |
| RPS (Scenario2) | 1,174 | 11,849 | 14,182 | 2 |
| DRRPS | 1,174 | 23,698 | 26,798 | 35 |
| RPSLP (Scenario1) | N/A | 11,849 | 12,608 | 1 |
| RPSLP (Scenario2) | N/A | 11,849 | 12,608 | 1 |

**Table 7.25 Solution Information for the First Stage (Iteration 1)**

| Scenario | Objective Value under Robust Solution | Optimal Objective Value | Regret | % From Optimal |
|---|---|---|---|---|
| 1 | 31,786.053 | 61,473.78 | 29,687.73 | 51.71% |
| 2 | 108,025.1 | 138,703.47 | 30,678.37 | 77.88% |

Table 7.26 contains the detail information on the candidate robust solution from the first iteration with the lower bound on min-max regret of 30,678.37.

**Table 7.26 Detail Information of the Candidate Robust Solution (Iteration 1)**

| Decision Type | Candidate Robust Solution |
|---|---|
| Site Opening Decision | Collection Centers 3, 5, 6, and 9<br>Processing Centers 6 and 8 |
| Collection Decision | Collection Centers 3, 5, 6, and 9 are required to collect all possible obsolete televisions from any source within 100 miles radius. |
| Process Decision | Collection Centers 3, 5, 6, and 9 are required to install the television inspection process. Processing Centers 6 and 8 are required to install both the television refurbishing process and the television demanufacturing process. |

Because scenario one is the scenario that controls the feasibility of the robust solution over all possible scenarios and this candidate robust solution is feasible under this scenario, this candidate robust solution can directly be forwarded to the third stage of the algorithm with no additional processing by the second stage.

At this stage, the algorithm is required to solve one BLPP model with the objective of finding the scenario with maximum regret of the candidate robust solution. Table 7.27 and Table 7.28 contain the detailed information for this stage of the algorithm.

**Table 7.27 Detail Information of the BLPP Model (Iteration 1)**

| Model Type | Number of Constraints | Number of Binary Variables + Complementary Slackness Conditions | Number of Continuous Variables |
|---|---|---|---|
| BLPP without rules | 38,691 | 1,177 + 24,457 | 60,763 |
| BLPP with rules | 2,359 | 328 + 486 | 593 |

**Table 7.28 Solution Information of the BLPP Model (Iteration 1)**

| Scenario | Robust Objective | Optimal Objective | Regret | % From Optimal | Solution Time (sec) |
|---|---|---|---|---|---|
| 3 | 158,156.1378 | 202,448.6378 | 44,292.5 | 78.12% | 13,306.69 |

The third stage of the algorithm generates one scenario and the upper bound on the min-max regret of 44,292.5 in the first iteration. This information is then forwarded back to the first stage to find another candidate robust solution or to confirm the optimality of the current best solution. At this stage, we introduce three scenarios to the problem: one scenario (Scenario 3) from the third stage and two scenarios (Scenario 4 and Scenario 5) derived on the basis of the author's expertise to accelerate the process. The algorithm solves the problem using 3 iterations and terminates at the solution with the maximum regret less than 3.33% from the maximum regret of the optimal robust solution. This solution is the same solution previously shown in Table 7.26. Table 7.29 contains the summary of all algorithm steps for the semi-continuous robust algorithm. Table 7.30 contains the summary of all scenarios used in the algorithm and Figure 7.8 illustrates the infrastructure of the 3.33% optimal robust solution generated by the semi-continuous robust algorithm. Table 7.31 summarizes the performance of the 3.33% optimal robust solution on all six scenarios considered by the algorithm.

**Table 7.29 Summary of the Algorithm Steps**

| Iteration | Model | Solution Time (sec) | Number of Scenarios | Lower Bound | Upper Bound | % Difference | Max Regret by BLPP at each Iteration |
|---|---|---|---|---|---|---|---|
| 1 | RPS | 4 | 2 | 30,678.37 | 44,292.5 | 44% | 44,292.5 |
| | DRRPS | 35 | | | | | |
| | BLPP | 13,306.69 | | | | | |
| 2 | RPS | 10 | 5 | 42,074.95 | 44,292.5 | 5.27% | 44,303.8 |
| | DRRPS | 9,422.53 | | | | | |
| | BLPP | 61,661.86 | | | | | |
| 3 | RPS | 12 | 6 | 42,864.75 | N/A | < 3.33% | N/A |
| | DRRPS | 9,221.36 | | | | | |
| | BLPP | N/A | | | | | |

**Table 7.30 All Scenarios Considered by the Semi-Continuous Robust Algorithm**

| Type of Random Variable | Location | Scenario | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Participation Rate | Region 1 | 30% | 20% | 30% | 30% | 20% | 20% |
| | Region 3 | 30% | 20% | 30% | 30% | 20% | *22.452% |
| | Region 12 | 30% | 20% | 20% | 30% | 20% | 20% |
| Selling Price of Refurbished Television ($ per unit) | Region 1 | 48 | 72 | 48 | 72 | 48 | 48 |
| | Region 3 | 48 | 72 | 72 | 72 | 48 | 48 |
| | Region 12 | 48 | 72 | 48 | 72 | 48 | 48 |
| Annual Capacity of Television Refurbishing Process per Factory (lbs) | Processing Center 3 | 213,360 | 320,040 | 320,040 | 320,040 | 213,360 | 320,040 |
| | Processing Center 4 | 213,360 | 320,040 | 320,040 | 320,040 | 213,360 | 320,040 |
| | Processing Center 5 | 213,360 | 320,040 | 320,040 | 320,040 | 213,360 | 320,040 |

* The scenarios with maximum regret from the candidate solution need not be end-point scenarios. In this case, the end-point scenarios will result in less regret than this solution.

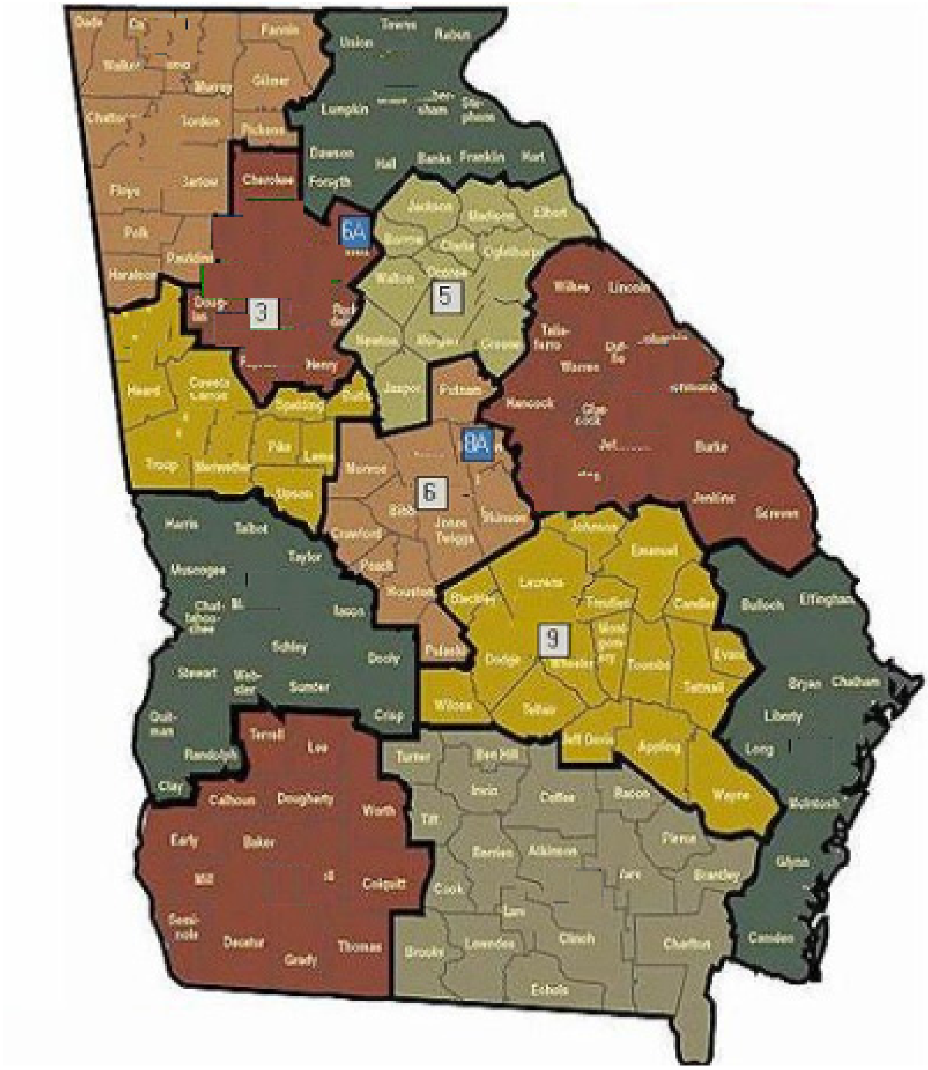


**Figure 7.8 3.33% Optimal Robust Infrastructure for Georgia Television Recycle**

**Table 7.31 Performance of the 3.33% Optimal Robust Solution on All Six Scenarios Considered by the Semi-Continuous Robust Algorithm**

| Scenario | Objective Value under Robust Solution | Optimal Objective Value | Regret | % From Optimal |
|---|---|---|---|---|
| 1 | 31,786.05 | 61,473.78 | 29,687.73 | 51.71% |
| 2 | 108,025.1 | 138,703.47 | 30,678.37 | 77.88% |
| 3 | 158,156.137 | 202,448.6378 | 44,292.5 | 78.12% |
| 4 | 163,608.67 | 207,474.4096 | 43,865.74 | 78.86% |
| 5 | 1,450.05 | 35,566.58656 | 34,116.54 | 4.08% |
| 6 | 9,162.088 | 44,720.82871 | 35,558.74 | 20.49% |

This case study illustrates the effectiveness of the semi-continuous robust algorithm for problems of practical size and structure. In the next chapter, we will introduce the combination of an algorithm called parameter space transformation algorithm and the semi-continuous robust algorithm for handling situations when correlations exist among parameters.

## 7.4 Summary

The case studies in this chapter are illustrative of the application of the semi-continuous robust algorithm to designing robust infrastructure for reverse production systems. The case studies shows that the algorithm can be applied to realistically sized problems.

The semi-continuous robust algorithm illustrates the innovation both in theory and in application to a real problem. The algorithm has the potential to be very useful and powerful for any area of supply chain strategic planning that can be modeled in the form of a mixed integer linear programming under uncertainty in model parameters where the joint probability distribution of the uncertain parameters is unknown.

# CHAPTER VIII

# PARAMETER SPACE TRANSFORMATION ALGORITHM

## 8.1 Introduction

One of the assumptions of the semi-continuous robust algorithm is that the algorithm requires all model parameters to be independent. In many practical problems, model parameters are correlated. For example in the reverse production system, the participation rate in the recycling program often has negative correlation with the collection fee collected per unit of an obsolete product. This chapter introduces a parameter space transformation algorithm in order to transform the parameter space from the original parameter space with high correlation to a new parameter space with low correlation (or approximately no correlation). After performing this algorithm, the semi-continuous robust algorithm can be applied to the problem under the new parameter space with less concern on the violation of the independency assumption. Section 8.2 presents the detailed methodology of the parameter space transformation algorithm. Section 8.3 demonstrates the implementation of the semi-continuous robust algorithm after applying the parameter space transformation algorithm. Section 8.4 illustrates the application of the algorithm to the sample problems.

## 8.2 Detailed Methodology for Parameter Space Transformation Algorithm

Let random variable $p_i \ \forall i \in \{1,2,...,n\}$ be the original model parameters that have correlation among one another and let $\bar{p}$ be the $n$ dimensional random vector such that its $i^{th}$ component is $p_i$. Let $p_i^U, p_i^L \in \Re^1 \forall i \in \{1,2,...,n\}$ be the upper bound and the lower bound of each model parameter respectively. By using the information from the sample data set for these model parameters, the algorithm steps are presented as follows:

Step 0: (Initialization step) let $\bar{a}_0$ be the vector in $\Re^n$ such that its $i^{th}$ component takes the value of $a_{i0} = \dfrac{p_i^U + p_i^L}{2}$ and let $\bar{x}$ be the $n$ dimensional random vector such that its $i^{th}$ component takes the value of $x_i = p_i - a_{i0}$. Let $e_i \in \Re^n \ \forall i \in \{1,2,...,n\}$ be the initial basis of the original parameter space.

Step 1: Use linear regression analysis to generate the approximated linear relationship among random variables $x_1$, $x_2$,…, $x_n$ with coefficients $a_i \ \forall i \in \{1,2,...,n-1\}$ where $x_n = a_1x_1 + a_2x_2 + ... + a_{n-1}x_{n-1} = \sum_{i=1}^{n-1} a_ix_i$. This function represents a linear subspace with dimension $n-1$.

Step 2: Identify sets $I = \{1,2,...,n-1\}$, $I_1 = \{i \in I \mid a_i = 0\}$ and $I_2 = \{i \in I \mid a_i \neq 0\}$. The alternative basis of this parameter space can also be represented by $e_i \ \forall i \in I_1$, $e_n$, and $g_i \ \forall i \in I_2$ where $g_i$ represents the $n$ dimensional vector such that its $i^{th}$ component takes the value of 1 and its $n^{th}$ component takes the value of $a_i$ and zero elsewhere. Perform the

Gram-Schmidt Orthogonalization algorithm starting with $g_i \;\; \forall i \in I_2$, $e_i \;\; \forall i \in I_1$, and $e_n$ to generate the new orthonormal basis for this parameter space. Let $u_i^{'} \;\; \forall i \in \{1,2,...,n\}$ represent this new orthonormal basis and let matrix $Q = [u_1^{'} \; u_2^{'} \; ..... \; u_n^{'}]$ represent an orthogonal matrix such that its $i^{th}$ column is the vector $u_i^{'} \;\; \forall i \in \{1,2,...,n\}$.

<u>Step 3:</u> Let random variable $p_i^{'} \;\; \forall i \in \{1,2,...,n\}$ be the new model parameters corresponding to the new basis that have low correlation (or approximately no correlation) among one another and let $\vec{p}^{\,'}$ be the $n$ dimensional random vector such that its $i^{th}$ component is $p_i^{'}$. The relationship between vectors $\vec{p}$ and $\vec{p}^{\,'}$ can be represented as $\vec{p} = Q\vec{p}^{\,'} + \vec{a}_0$.

<u>Step 4:</u> Transform all original random model parameters, $p_i \;\; \forall i \in \{1,2,...,n\}$, in the model to the new random model parameters, $p_i^{'} \;\; \forall i \in \{1,2,...,n\}$ by using the relationship in step 3. The upper bound and the lower bound of these new random parameters can be attained from the information containing in the sample data set.

This relationship between vectors $\vec{p}$ and $\vec{p}^{\,'}$ can be derived as follow. From the fact that $e_i = \sum_{j=1}^{n} (e_i^T \vec{u}_j^{\,'})\vec{u}_j^{\,'}$ and $[x_1, x_2, ..., x_n]^T = \sum_{i=1}^{n} x_i e_i = \sum_{i=1}^{n} x_i \left( \sum_{j=1}^{n} (e_i^T \vec{u}_j^{\,'})\vec{u}_j^{\,'} \right) = \sum_{j=1}^{n} \left( \sum_{i=1}^{n} (e_i^T \vec{u}_j^{\,'}) x_i \right) \vec{u}_j^{\,'}$, the relationship between $p_i^{'} \;\; \forall i \in \{1,2,...,n\}$ and $x_i \;\; \forall i \in \{1,2,...,n\}$ is obviously $p_j^{'} = \sum_{i=1}^{n} (e_i^T \vec{u}_j^{\,'}) x_i \;\; \forall j \in \{1,2,...,n\}$ or $\vec{p}^{\,'} = Q^T \vec{x}$. By using the fact that matrix $Q$ is an orthogonal matrix ($Q^{-1} = Q^T$) and the relationship between vector $\bar{p}$ and $\bar{x}$, the relationship between vectors $\bar{p}$ and $\bar{p}'$ can be attained. Figure 8.1 illustrates the algorithm steps of the parameter space transformation algorithm.

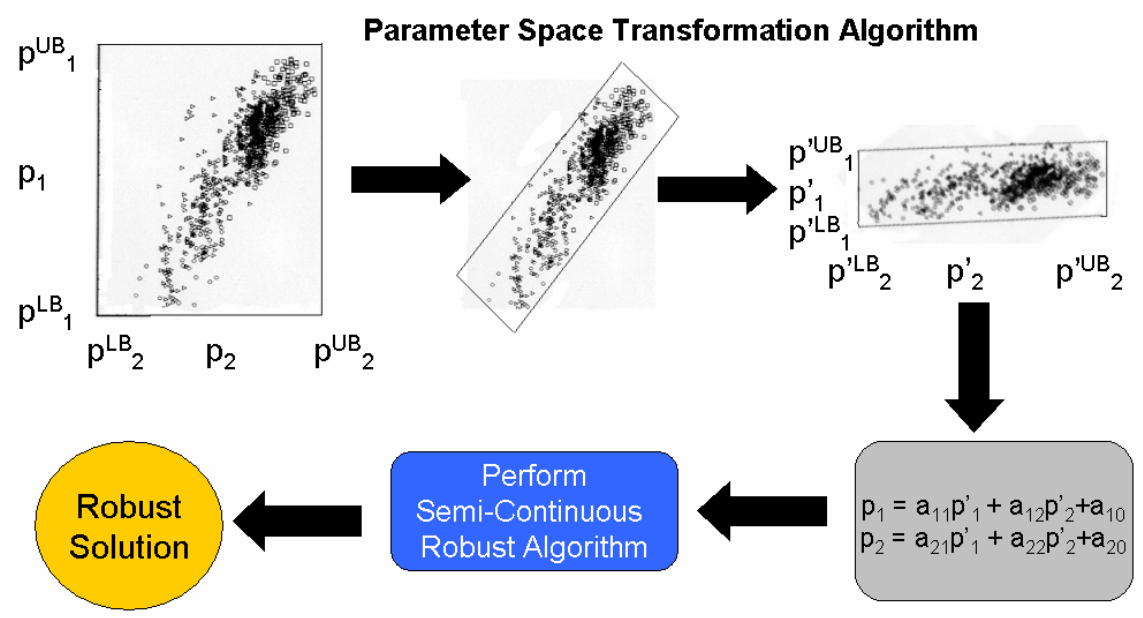


**Figure 8.1 Parameter Space Transformation Algorithm**

## 8.3 The Implementation of The Semi-Continuous Robust Algorithm After Applying The Parameter Space Transformation Algorithm

After applying the parameter space transformation algorithm to the problem with correlated parameters, the resulting model is now ready for the semi-continuous robust algorithm. This section contains the detail methodology for applying the semi-continuous robust algorithm to the transformed problem.

*Transformation in the First Stage of the Algorithm (RPS and DRRPS Models)*

Because this stage of the algorithm only solves the problem based on the predetermined finite set of discrete scenarios, there is no transformation necessary in this stage of the algorithm.

*Transformation in the Second Stage of the Algorithm (BLLP Model)*

After applying the parameter space transformation algorithm to the problem, each of the correlated original model parameters is transformed into the affine function of the uncorrelated new parameters. Figure 8.2 illustrates the model structure of the transformed BLLP model.

$$
\begin{aligned}
&\underset{\vec{p}}{\text{minimize}} && \delta \\
&\text{s.t.} && \vec{p}_L^{'} \le \vec{p}^{'} \le \vec{p}_U^{'} \\
& && \underset{\vec{x},\vec{s},\delta}{\text{maximize}} \quad \delta \\
& && \text{s.t.} \quad A\vec{x} \pm \vec{s} = Q\vec{p}^{'} + \vec{a}_0 \\
& && \qquad\; \delta\vec{1} \le \vec{s} \\
& && \qquad\; \vec{x} \ge \vec{0}
\end{aligned}
$$

**Figure 8.2 Transformed BLLP Model**

Perform pre-processing steps stated in Chapter VI on all uncorrelated original model parameters. If there still exist some random parameters in the transformed BLLP model that still cannot be fixed, the BLLP model can be further transformed into an easier problem by the results of the following lemma.

*Lemma* 2: The transformed BLLP model has at least one optimal solution $\bar{p}'^*$ in which each element of $\bar{p}'$ takes value at its bounds.

*Proof*: Let $\bar{p}'^*$ be an optimal solution of the transformed BLLP model such that an element $i$ does not take the value from its bounds or $p_i'^L < p_i'^* < p_i'^U$. There are only three possible cases to be considered.

Case 1: $\delta^* = s_j^*$ where $\left(A\bar{x}^*\right)_j \pm s_j^* = \sum_{l \neq i} a_{jl} p_l'^* + a_{ji} p_i'^* + a_{j0}$ and $a_{ji} \neq 0\ \exists j$.

In this case, we can easily show that $p_i'^*$ has already taken the value from its bounds. There are four sub-cases to be considered.

Sign is + and $a_{ji} > 0$:

If $p_i'^* > p_i'^L$, $\exists \varepsilon > 0$ such that $p_i'^* - \varepsilon \geq p_i'^L$ and $\left(A\bar{x}^*\right)_j + s_j^* > \sum_{l \neq i} a_{jl} p_l'^* + a_{ji}\left(p_i'^* - \varepsilon\right) + a_{j0}$.

The value of $\left(A\bar{x}^*\right)_j$ cannot be decreased because of the optimality of $\bar{p}'^*$ and $\bar{x}^*$. For this reason the value of $s_j^*$ can be decreased to $s_j^* - a_{ji}\varepsilon$. This contradicts the optimality of $\delta^*$.

Sign is + and $a_{ji} < 0$:

If $p_i'^* < p_i'^U$, $\exists \varepsilon > 0$ such that $p_i'^* + \varepsilon \leq p_i'^U$ and $\left(A\bar{x}^*\right)_j + s_j^* > \sum_{l \neq i} a_{jl} p_l'^* + a_{ji}\left(p_i'^* + \varepsilon\right) + a_{j0}$.

The value of $\left(A\bar{x}^*\right)_j$ cannot be decreased because of the optimality of $\bar{p}'^*$ and $\bar{x}^*$. For this reason the value of $s_j^*$ can be decreased to $s_j^* + a_{ji}\varepsilon$. This contradicts the optimality of $\delta^*$.

Sign is – and $a_{ji} > 0$:

If $p_i'^* < p_i'^U$, $\exists \varepsilon > 0$ such that $p_i'^* + \varepsilon \leq p_i'^U$ and $\left(A\bar{x}^*\right)_j - s_j^* < \sum_{l \neq i} a_{jl} p_l'^* + a_{ji}\left(p_i'^* + \varepsilon\right) + a_{j0}$.

The value of $\left(A\vec{x}^*\right)_j$ cannot be increased because of the optimality of $\vec{p}^{'*}$ and $\vec{x}^*$. For this reason the value of $s_i^*$ can be decreased to $s_j^* - a_{ji}\varepsilon$. This also contradicts the optimality of $\delta^*$.

Sign is – and $a_{ji} < 0$:

If $p_i^{'*} > p_i^{'L}$, $\exists \varepsilon > 0$ such that $p_i^{'*} - \varepsilon \geq p_i^{'L}$ and $\left(A\vec{x}^*\right)_j - s_j^* < \sum_{l \neq i} a_{jl} p_l^{'*} + a_{ji}\left(p_i^{'*} - \varepsilon\right) + a_{j0}$.

The value of $\left(A\vec{x}^*\right)_j$ cannot be increased because of the optimality of $\vec{p}^{'*}$ and $\vec{x}^*$. For this reason the value of $s_j^*$ can be decreased to $s_j^* + a_{ji}\varepsilon$. This contradicts the optimality of $\delta^*$.

Case 2: $\delta^* = s_j^*$ where $\left(A\vec{x}^*\right)_j \pm s_j^* = \sum_{l \neq i} a_{jl} p_l^{'*} + a_{ji} p_i^{'*} + a_{j0}$ and $a_{ji} = 0 \ \forall j$.

In this case, it is trivial to see that the value of $p_i^{'*}$ can be adjusted to either of its bound without any effect on the optimality and feasibility of the problem.

Case 3: $\delta^* < s_j^*$ where $\left(A\vec{x}^*\right)_j \pm s_j^* = \sum_{l \neq i} a_{jl} p_l^{'*} + a_{ji} p_i^{'*} + a_{j0}$.

In this case, the value of $p_i^{'*}$ can be adjusted to either of its bound without any effect on the optimality and feasibility of the problem. This statement is quite obvious from the optimality of $\vec{p}^{'*}$ and the structure of the BLLP model. □

The results from Lemma 2 greatly simplify the solution methodology of the transformed BLLP model. By adding dual constraints and a strong duality constraint for the follower problem into the BLLP model, the problem is transformed from a bi-level linear programming problem to a single level mixed integer linear programming problem as shown in Figure 8.3.

$$
\begin{aligned}
\text{minimize} \quad & \delta \\
\text{s.t.} \quad & \vec{p}_L^{'} \le \vec{p}^{'} \le \vec{p}_U^{'} \\
& A\vec{x} \pm \vec{s} = Q\vec{p}^{'} + \vec{a}_0 \\
& \delta \vec{1} \le \vec{s} \\
& A^T \vec{w}_1 \ge \vec{0} \\
& \vec{1}^T \vec{w}_2 = 1 \\
& \pm \vec{w}_1 - \vec{w}_2 = \vec{0} \\
& \delta = \left(Q\vec{p}^{'} + \vec{a}_0\right)^T \vec{w}_1 = \left(Q\vec{p}^{'} + \vec{a}_0\right)^T (\pm \vec{w}_2) \\
& \vec{w}_2,\ \vec{x} \ge \vec{0}
\end{aligned}
$$

**Figure 8.3 The New Transformed BLLP Model**

The nonlinear term in the constraint $\delta = \left(Q\vec{p}^{'} + \vec{a}_0\right)^T \vec{w}_1$ can be transformed into mixed integer linear constraints by using the results of Lemma 2 as shown in Figure 8.4 where $M$ is a significantly large number.

$$
\delta = \left(Q\vec{p}^{'} + \vec{a}_0\right)^T (\pm \vec{w}_2) \leftrightarrow \delta = \sum_i \left( \left( \sum_j a_{ij} p_j^{'} + a_{i0} \right) (\pm w_{2i}) \right) \leftrightarrow \delta = \sum_i \left( \sum_j a_{ij} p_j^{'} \left(\pm w_{2i}\right) + \left(\pm a_{i0} w_{2i}\right) \right)
$$

$$
\leftrightarrow \delta = \sum_i \left( \sum_j a_{ij} \left(\pm PW_{2ij}^{'}\right) + \left(\pm a_{i0} w_{2i}\right) \right)
$$

$$
\left.
\begin{array}{ll}
PW_{2ij}^{'} \le p_j^{'U} w_{2i} & -PW_{2ij}^{'} \le -p_j^{'U} w_{2i} + M(1 - bi_j) \\
PW_{2ij}^{'} \le p_j^{'L} w_{2i} + Mbi_j & PW_{2ij}^{'} \ge p_j^{'L} w_{2i} \\
bi_j \in \{0,1\} &
\end{array}
\right\} \forall j \text{ where } p_j^{'} \text{ cannot be preprocessed}
$$

If $p_j^{'}$ can be preprocessed at $\tilde{p}_j^{'}$ or have the constant value of $\tilde{p}_j^{'}, PW_{2ij}^{'} = \tilde{p}_j^{'} w_{2i}$

**Figure 8.4 Transformation of the Strong Duality Constraint in the BLLP Model**

*Transformation in the Third Stage of the Algorithm (BLPP Model)*

There are eight possible cases of correlations among model parameters that require different transformations in the BLPP model. The detail transformation steps of the BLPP model for these eight cases are presented as follow:

Case 1: The correlation exists among model parameters of type $p_1$ and these parameters are not correlated with other parameter types.

Let model parameters $p_{1i}\ \forall i \in A$ be correlated with one another and these parameters are not correlated with any other parameters in the model. These parameters appear in the original model objective function of the BLPP model as $\pm\left(\sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right)$.

After applying the parameter space transformation algorithm to the problem, this section of the transformed BLPP model can be rewritten as follows:

$$\pm\left(\sum_{i\in A}\left(\sum_{j=1}^{|A|} a_{ij}p'_{1j} + a_{i0}\right)y_{1i} - \sum_{i\in A}\left(\sum_{j=1}^{|A|} a_{ij}p'_{1j} + a_{i0}\right)y_{\Omega i}\right)$$

$$\equiv \pm\left(\sum_{j=1}^{|A|}\left(\sum_{i\in A} a_{ij}y_{1i}\right)p'_{1j} + \sum_{i\in A} a_{i0}y_{1i} - \sum_{j=1}^{|A|}\left(\sum_{i\in A} a_{ij}y_{\Omega i}\right)p'_{1j} - \sum_{i\in A} a_{i0}y_{\Omega i}\right)$$

$$\equiv \pm\left(\sum_{j=1}^{|A|}\left(\sum_{i\in A} a_{ij}y_{1i} - \sum_{i\in A} a_{ij}y_{\Omega i}\right)p'_{1j} + \sum_{i\in A} a_{i0}y_{1i} - \sum_{i\in A} a_{i0}y_{\Omega i}\right)$$

$$\equiv \pm\left(\sum_{j=1}^{|A|}\left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'_{1j} + \sum_{i\in A} a_{i0}(y_{1i} - y_{\Omega i})\right)$$

Because the value of $\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})$ is either negative or nonnegative, it is quite obvious that one of the optimal settings of $p'_{1j}$ is at its bound $\forall j = 1, 2, ..., |A|$. From these results, we can transform the BLPP model by using the following steps.

<u>Step 1:</u> Add variables $KP'_{1j}$ $\forall j = 1, 2, ..., |A|$ to the BLPP model and replace the term $\pm\left(\sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right)$ in the objective function of the original BLPP model by $\pm\left(\sum_{j=1}^{|A|} KP'_{1j} + \sum_{i\in A} a_{i0}(y_{1i} - y_{\Omega i})\right)$.

<u>Step 2:</u> Add binary variables $bi_{1j}$ $\forall j = 1, 2, ..., |A|$ and add the following constraints to the BLPP model $\forall j = 1, 2, ..., |A|$.

$$\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) \le \left(\sum_{i\in A} |a_{ij}|\right) bi_{1j}$$
$$\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) \ge \left(\sum_{i\in A} |a_{ij}|\right)(bi_{1j} - 1)$$

(If the sign is +)

$$p'_{1j} = (1 - bi_{1j})p'^{L}_{1j} + bi_{1j}p'^{U}_{1j}$$
$$KP'_{1j} - \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{U}_{1j} \le M(1 - bi_{1j}) \qquad -KP'_{1j} + \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{U}_{1j} \le M(1 - bi_{1j})$$
$$KP'_{1j} - \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{L}_{1j} \le Mbi_{1j} \qquad -KP'_{1j} + \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{L}_{1j} \le Mbi_{1j}$$

(If the sign is −)

$$p'_{1j} = bi_{1j}p'^{L}_{1j} + (1 - bi_{1j})p'^{U}_{1j}$$
$$KP'_{1j} - \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{U}_{1j} \le Mbi_{1j} \qquad -KP'_{1j} + \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{U}_{1j} \le Mbi_{1j}$$
$$KP'_{1j} - \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{L}_{1j} \le M(1 - bi_{1j}) \qquad -KP'_{1j} + \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i})\right)p'^{L}_{1j} \le M(1 - bi_{1j})$$

where $M = \left(\sum_{i\in A} |a_{ij}|\right)\left(|p'^{U}_{1j}| + |p'^{L}_{1j}|\right)$

Case 2: The correlation exists among model parameters of type $p_1$ and $p_4$ and these parameters are not correlated with other parameter types.

Let model parameters $p_{1i}$ $\forall i \in A$ and $p_{4i}$ $\forall i \in B$ (without lost of generality, we can modify the index such that $A \cap B = \varnothing$) be correlated with one another and these parameters are not correlated with any other parameters in the model. These parameters appear in the original model objective function of the BLPP model as $\pm\left(\sum_{i\in B} p_{4i}x_{1i} - \sum_{i\in B} p_{4i}x_{2i} + \sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right)$. After applying the parameter space transformation algorithm to the problem, this section of the transformed BLPP model can be rewritten as follow:

$$\pm\left(\sum_{i\in A}\left(\sum_{j=1}^{|A|+|B|} a_{ij}p'_{14j} + a_{i0}\right)y_{1i} - \sum_{i\in A}\left(\sum_{j=1}^{|A|+|B|} a_{ij}p'_{14j} + a_{i0}\right)y_{\Omega i} + \sum_{i\in B}\left(\sum_{j=1}^{|A|+|B|} a_{ij}p'_{14j} + a_{i0}\right)x_{1i} - \sum_{i\in B}\left(\sum_{j=1}^{|A|+|B|} a_{ij}p'_{14j} + a_{i0}\right)x_{2i}\right)$$

$$\equiv \pm\left(\sum_{j=1}^{|A|+|B|}\left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i})\right)p'_{14j} + \sum_{i\in A} a_{i0}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{i0}(x_{1i} - x_{2i})\right)$$

Because the value of $\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i})$ is either negative or nonnegative, it is quite obvious that one of the optimal settings of $p'_{14j}$ is at its bound $\forall j = 1, 2, ..., |A| + |B|$. From these results, we can transform the BLPP model by using the following steps.

Step 1: Add variables $KP'_{14j}$ $\forall j = 1, 2, ..., |A| + |B|$ to the BLPP model and replace the term $\pm\left(\sum_{i\in B} p_{4i}x_{1i} - \sum_{i\in B} p_{4i}x_{2i} + \sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right)$ in the objective function of the original BLPP model by $\pm\left(\sum_{j=1}^{|A|+|B|} KP'_{14j} + \sum_{i\in A} a_{i0}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{i0}(x_{1i} - x_{2i})\right)$.

Step 2: Add binary variables $bi_{14j}$ $\forall j = 1, 2, ..., |A| + |B|$ and add the following constraints to the BLPP model $\forall j = 1, 2, ..., |A| + |B|$.

$$\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i}) \leq \left(\sum_{i\in A} |a_{ij}| + \sum_{i\in B} |a_{ij}| x_{1i}^U\right) bi_{14j}$$

$$\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i}) \geq \left(\sum_{i\in A} |a_{ij}| + \sum_{i\in B} |a_{ij}| x_{1i}^U\right)(bi_{14j} - 1)$$

(If the sign is +)

$$p'_{14j} = (1 - bi_{14j}) p'^L_{14j} + bi_{14j} p'^U_{14j}$$

$$KP'_{14j} - \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i})\right) p'^U_{14j} \leq M(1 - bi_{14j})$$

$$-KP'_{14j} + \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i})\right) p'^U_{14j} \leq M(1 - bi_{14j})$$

$$KP'_{14j} - \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i})\right) p'^L_{14j} \leq M bi_{14j}$$

$$-KP'_{14j} + \left(\sum_{i\in A} a_{ij}(y_{1i} - y_{\Omega i}) + \sum_{i\in B} a_{ij}(x_{1i} - x_{2i})\right) p'^L_{14j} \leq M bi_{14j}$$

(If the sign is −)

$$p'_{14j} = bi_{14j} p'^{L}_{14j} + (1 - bi_{14j}) p'^{U}_{14j}$$

$$KP'_{14j} - \left( \sum_{i \in A} a_{ij} (y_{1i} - y_{\Omega i}) + \sum_{i \in B} a_{ij} (x_{1i} - x_{2i}) \right) p'^{U}_{14j} \le M bi_{14j}$$

$$-KP'_{14j} + \left( \sum_{i \in A} a_{ij} (y_{1i} - y_{\Omega i}) + \sum_{i \in B} a_{ij} (x_{1i} - x_{2i}) \right) p'^{U}_{14j} \le M bi_{14j}$$

$$KP'_{14j} - \left( \sum_{i \in A} a_{ij} (y_{1i} - y_{\Omega i}) + \sum_{i \in B} a_{ij} (x_{1i} - x_{2i}) \right) p'^{L}_{14j} \le M (1 - bi_{14j})$$

$$-KP'_{14j} + \left( \sum_{i \in A} a_{ij} (y_{1i} - y_{\Omega i}) + \sum_{i \in B} a_{ij} (x_{1i} - x_{2i}) \right) p'^{L}_{14j} \le M (1 - bi_{14j})$$

where $M = \left( \sum_{i \in A} | a_{ij} | + \sum_{i \in B} | a_{ij} | x^{U}_{1i} \right) \left( | p'^{U}_{14j} | + | p'^{L}_{14j} | \right)$

Case 3: The correlation exists among model parameters of type $p_2$ and these parameters are not correlated with other parameter types.

Let model parameters $p_{2j} \ \forall j \in A$ be correlated with one another and these parameters are not correlated with any other parameters in the model. These parameters appear in the original model constraints of the BLPP model as $\sum_i x_{1i} \le p_{2j} y_{1j}$ and $\sum_i x_{2i} \le p_{2j} y_{\Omega j} \ \forall j \in A$. After applying the parameter space transformation algorithm to the problem, this section of the transformed BLPP model can be rewritten as follow:

$$\sum_i x_{1i} \le \left( \sum_{k=1}^{|A|} a_{jk} p'_{2k} + a_{j0} \right) y_{1j} \text{ and } \sum_i x_{2i} \le \left( \sum_{k=1}^{|A|} a_{jk} p'_{2k} + a_{j0} \right) y_{\Omega j}$$

$$\equiv \sum_i x_{1i} \le \left( \sum_{k=1}^{|A|} a_{jk} p'_{2k} \right) y_{1j} + a_{j0} y_{1j} \text{ and } \sum_i x_{2i} \le \left( \sum_{k=1}^{|A|} a_{jk} p'_{2k} \right) y_{\Omega j} + a_{j0} y_{\Omega j}$$

From these results, we can transform the BLPP model by using the following steps.

Step 1: Replace the constraints $\sum_i x_{2i} \le p_{2j} y_{\Omega j} \;\forall j \in A$ in the original BLPP model by $\sum_i x_{2i} \le \left( \sum_{k=1}^{|A|} a_{jk} p_{2k}^{'} + a_{j0} \right) y_{\Omega j} \;\forall j \in A$. Add variables $PY_{2j}^{'} \;\forall j \in A$ to the BLPP model and replace the constraints $\sum_i x_{1i} \le p_{2j} y_{1j} \;\forall j \in A$ in the original BLPP model by $\sum_i x_{1i} \le PY_{2j}^{'} + a_{j0} y_{1j} \;\forall j \in A$.

Step 2: Add the following constraints to the BLPP model $\forall j \in A$.

$$PY_{2j}^{'} \le M(1 - y_{1j}) + \left( \sum_{k=1}^{|A|} a_{jk} p_{2k}^{'} \right) \qquad -PY_{2j}^{'} \le M(1 - y_{1j}) - \left( \sum_{k=1}^{|A|} a_{jk} p_{2k}^{'} \right)$$

$$PY_{2j}^{'} \ge -My_{1j} \qquad PY_{2j}^{'} \le My_{1j} \quad \text{where } M = \left( \sum_{k=1}^{|A|} | a_{jk} | \max(| p_{2k}^{'U} |, | p_{2k}^{'L} |) \right)$$

Case 4: The correlation exists among model parameters of type $p_3$ and these parameters are not correlated with other parameter types.

Let model parameters $p_{3j} \;\forall j \in A$ be correlated with one another and these parameters are not correlated with any other parameters in the model. These parameters appear in the original model constraints of the BLPP model as $\sum_i x_{1i} \le p_{3j}$ and $\sum_i x_{2i} \le p_{3j} \;\forall j \in A$. After applying the parameter space transformation algorithm to the problem, this section of the transformed BLPP model can be rewritten as follow:

$$\sum_i x_{1i} \le \left( \sum_{k=1}^{|A|} a_{jk} p_{3k}^{'} + a_{j0} \right) \text{ and } \sum_i x_{2i} \le \left( \sum_{k=1}^{|A|} a_{jk} p_{3k}^{'} + a_{j0} \right)$$

From these results, we can transform the BLPP model by using the following steps.

Step 1: Replace the constraints $\sum_{i} x_{1i} \leq p_{3j} \ \forall j \in A$ and $\sum_{i} x_{2i} \leq p_{3j} \ \forall j \in A$ in the original BLPP model by $\sum_{i} x_{1i} \leq \left( \sum_{k=1}^{|A|} a_{jk} p_{3k}^{'} + a_{j0} \right)$ and $\sum_{i} x_{2i} \leq \left( \sum_{k=1}^{|A|} a_{jk} p_{3k}^{'} + a_{j0} \right) \ \forall j \in A$.

Case 5: The correlation exists among model parameters of type $p_4$ and these parameters are not correlated with other parameter types.

Let model parameters $p_{4i} \ \forall i \in A$ be correlated with one another and these parameters are not correlated with any other parameters in the model. These parameters appear in the original model objective function of the BLPP model as $\pm\left( \sum_{i \in A} p_{4i} x_{1i} - \sum_{i \in A} p_{4i} x_{2i} \right)$. After applying the parameter space transformation algorithm to the problem, this section of the transformed BLPP model can be rewritten as follow:

$$\pm\left( \sum_{i \in A} \left( \sum_{j=1}^{|A|} a_{ij} p_{4j}^{'} + a_{i0} \right) x_{1i} - \sum_{i \in A} \left( \sum_{j=1}^{|A|} a_{ij} p_{4j}^{'} + a_{i0} \right) x_{2i} \right)$$

$$\equiv \pm\left( \sum_{j=1}^{|A|} \left( \sum_{i \in A} a_{ij} x_{1i} \right) p_{4j}^{'} + \sum_{i \in A} a_{i0} x_{1i} - \sum_{j=1}^{|A|} \left( \sum_{i \in A} a_{ij} x_{2i} \right) p_{4j}^{'} - \sum_{i \in A} a_{i0} x_{2i} \right)$$

$$\equiv \pm\left( \sum_{j=1}^{|A|} \left( \sum_{i \in A} a_{ij} x_{1i} - \sum_{i \in A} a_{ij} x_{2i} \right) p_{4j}^{'} + \sum_{i \in A} a_{i0} x_{1i} - \sum_{i \in A} a_{i0} x_{2i} \right)$$

$$\equiv \pm\left( \sum_{j=1}^{|A|} \left( \sum_{i \in A} a_{ij} (x_{1i} - x_{2i}) \right) p_{4j}^{'} + \sum_{i \in A} a_{i0} (x_{1i} - x_{2i}) \right)$$

Because the value of $\sum_{i \in A} a_{ij} (x_{1i} - x_{2i})$ is either negative or nonnegative, it is quite obvious that one of the optimal settings of $p_{4j}^{'}$ is at its bound $\forall j = 1, 2, ..., |A|$. From these results, we can transform the BLPP model by using the following steps.

Step 1: Add variables $KP'_{4j}$ $\forall j = 1,2,...,|A|$ to the BLPP model and replace the term $\pm\left(\sum_{i\in A} p_{4i}x_{1i} - \sum_{i\in A} p_{4i}x_{2i}\right)$ in the objective function of the original BLPP model by $\pm\left(\sum_{j=1}^{|A|} KP'_{4j} + \sum_{i\in A} a_{i0}(x_{1i} - x_{2i})\right)$.

Step 2: Add binary variables $bi_{4j}$ $\forall j = 1,2,...,|A|$ and add the following constraints to the BLPP model $\forall j = 1,2,...,|A|$.

$$\sum_{i\in A} a_{ij}(x_{1i} - x_{2i}) \le \left(\sum_{i\in A} |a_{ij}|\, x_{1i}^U\right) bi_{4j}$$

$$\sum_{i\in A} a_{ij}(x_{1i} - x_{2i}) \ge \left(\sum_{i\in A} |a_{ij}|\, x_{1i}^U\right)(bi_{4j} - 1)$$

(If the sign is +)

$$p'_{4j} = (1 - bi_{4j})p_{4j}^{'L} + bi_{4j}p_{4j}^{'U}$$

$$KP'_{4j} - \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'U} \le M(1 - bi_{4j}) \qquad -KP'_{4j} + \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'U} \le M(1 - bi_{4j})$$

$$KP'_{4j} - \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'L} \le Mbi_{4j} \qquad -KP'_{4j} + \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'L} \le Mbi_{4j}$$

(If the sign is −)

$$p'_{4j} = bi_{4j}p_{4j}^{'L} + (1 - bi_{4j})p_{4j}^{'U}$$

$$KP'_{4j} - \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'U} \le Mbi_{4j} \qquad -KP'_{4j} + \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'U} \le Mbi_{4j}$$

$$KP'_{4j} - \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'L} \le M(1 - bi_{4j}) \qquad -KP'_{4j} + \left(\sum_{i\in A} a_{ij}(x_{1i} - x_{2i})\right)p_{4j}^{'L} \le M(1 - bi_{4j})$$

where $M = \left(\sum_{i\in A} |a_{ij}|\, x_{1i}^U\right)\left(|p_{4j}^{'U}| + |p_{4j}^{'L}|\right)$

Case 6: The correlation exists among model parameters of type $p_1$, $p_2$, $p_3$ and these parameters are not correlated with other parameter types.

Let model parameters $p_{1i}\ \forall i \in A$, $p_{2j}\ \forall j \in B$ and $p_{3k}\ \forall k \in C$ (without lost of generality, we can modify the index such that $A \cap B = A \cap C = B \cap C = \varnothing$) be correlated with one another and these parameters are not correlated with any other parameters in the model. These parameters appear in the original model objective function of the BLPP model as $\pm\left(\sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right)$ and appear in the model constraints of the BLPP model as $\sum_l x_{1l} \le p_{2j}y_{1j}$, $\sum_l x_{2l} \le p_{2j}y_{\Omega j}\ \forall j \in B$, $\sum_m x_{1m} \le p_{3k}$ and $\sum_m x_{2m} \le p_{3k}\ \forall k \in C$. After applying the parameter space transformation algorithm to the problem, this section of the transformed BLPP model can be rewritten as follow:

$$\pm\left(\sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right) \equiv \pm\left(\sum_{i\in A}\left(\sum_{n=1}^{|A|+|B|+|C|} a_{in}p'_{123n}\right)y_{1i} + \sum_{i\in A} a_{i0}y_{1i} - \sum_{i\in A}\left(\sum_{n=1}^{|A|+|B|+|C|} a_{in}p'_{123n}\right)y_{\Omega i} - \sum_{i\in A} a_{i0}y_{\Omega i}\right)$$

$$\sum_l x_{1l} \le p_{2j}y_{1j} \equiv \sum_l x_{1l} \le \left(\sum_{n=1}^{|A|+|B|+|C|} a_{jn}p'_{123n}\right)y_{1j} + a_{j0}y_{1j}\quad \forall j \in B$$

$$\sum_l x_{2l} \le p_{2j}y_{\Omega j} \equiv \sum_l x_{2l} \le \left(\sum_{n=1}^{|A|+|B|+|C|} a_{jn}p'_{123n}\right)y_{\Omega j} + a_{j0}y_{\Omega j}\quad \forall j \in B$$

$$\sum_m x_{1m} \le p_{3k} \equiv \sum_m x_{1m} \le \left(\sum_{n=1}^{|A|+|B|+|C|} a_{kn}p'_{123n} + a_{k0}\right)\quad \forall k \in C$$

$$\sum_m x_{2m} \le p_{3k} \equiv \sum_m x_{2m} \le \left(\sum_{n=1}^{|A|+|B|+|C|} a_{kn}p'_{123n} + a_{k0}\right)\quad \forall k \in C$$

From these results, we can transform the BLPP model by using the following steps.

Step 1: Add variables $PY'_{123i}$ $\forall i \in A$ to the BLPP model and replace the term $\pm\left(\sum_{i\in A} p_{1i}y_{1i} - \sum_{i\in A} p_{1i}y_{\Omega i}\right)$ in the objective function of the original BLPP model by $\pm\left(\sum_{i\in A} PY'_{123i} + \sum_{i\in A} a_{i0}y_{1i} - \sum_{i\in A}\left(\sum_{n=1}^{|A|+|B|+|C|} a_{in}p'_{123n}\right)y_{\Omega i} - \sum_{i\in A} a_{i0}y_{\Omega i}\right)$.

Step 2: Add the following constraints to the BLPP model $\forall i \in A$.

$$PY'_{123i} \le M(1-y_{1i}) + \left(\sum_{n=1}^{|A|+|B|+|C|} a_{in}p'_{123n}\right) \qquad -PY'_{123i} \le M(1-y_{1i}) - \left(\sum_{n=1}^{|A|+|B|+|C|} a_{in}p'_{123n}\right)$$

$$PY'_{123i} \ge -My_{1i} \qquad PY'_{123i} \le My_{1i} \quad \text{where } M = \left(\sum_{n=1}^{|A|+|B|+|C|} |a_{in}| \max(|p'^{U}_{123n}|, |p'^{L}_{123n}|)\right)$$

Step 3: Replace the constraints $\sum_{l} x_{2l} \le p_{2j}y_{\Omega j}$ $\forall j \in B$ in the original BLPP model by $\sum_{l} x_{2l} \le \left(\sum_{n=1}^{|A|+|B|+|C|} a_{jn}p'_{123n}\right)y_{\Omega j} + a_{j0}y_{\Omega j}$ $\forall j \in B$. Add variables $PY'_{123j}$ $\forall j \in B$ to the BLPP model and replace the constraints $\sum_{l} x_{1l} \le p_{2j}y_{1j}$ $\forall j \in B$ in the original BLPP model by $\sum_{l} x_{1l} \le PY'_{123j} + a_{j0}y_{1j}$ $\forall j \in B$.

Step 4: Add the following constraints to the BLPP model $\forall j \in B$.

$$PY'_{123j} \le M(1-y_{1j}) + \left(\sum_{n=1}^{|A|+|B|+|C|} a_{jn}p'_{123n}\right) \qquad -PY'_{123j} \le M(1-y_{1j}) - \left(\sum_{n=1}^{|A|+|B|+|C|} a_{jn}p'_{123n}\right)$$

$$PY'_{123j} \ge -My_{1j} \qquad PY'_{123j} \le My_{1j} \quad \text{where } M = \left(\sum_{n=1}^{|A|+|B|+|C|} |a_{jn}| \max(|p'^{U}_{123n}|, |p'^{L}_{123n}|)\right)$$

Step 5: Replace the constraints $\sum_m x_{1m} \le p_{3k} \ \forall k \in C$ and $\sum_m x_{2m} \le p_{3k} \ \forall k \in C$ in the original BLPP model by

$$\sum_m x_{1m} \le \left( \sum_{n=1}^{|A|+|B|+|C|} a_{kn} p'_{123n} + a_{k0} \right) \text{ and } \sum_m x_{2m} \le \left( \sum_{n=1}^{|A|+|B|+|C|} a_{kn} p'_{123n} + a_{k0} \right) \ \forall k \in C.$$

Case 7: The correlation exists among model parameters of type $p_4$ and ($p_2$ or $p_3$).

Unfortunately, in this case, the problem cannot be solved directly by using the parameter space transformation algorithm and semi-continuous robust algorithm because there will be the nonlinear terms appearing in the objective function of the transformed BLPP model. In this section, we present an approximation algorithm for solving the problem in this case.

The algorithm starts by approximately treating the possible values of parameters of type $p_4$ as initial discrete scenarios. Each scenario represents point values or small range values of parameters of type $p_4$ combining with the different possible point values or range values of other types of parameters based on the information from the sample data set. The scenarios are generated such that the parameters of type $p_4$ have approximately no correlation with the parameters of type $p_2$ and $p_3$ in each scenario. The BLPP model will then be solved individually using the previous algorithms for each scenario by assuming that no correlation exists among parameter of type $p_4$ and parameter of type $p_2$ and $p_3$. The following example illustrates the use of this approximation algorithm on a sample data set of model parameters. Figure 8.5 illustrates the scatter plots between a parameter of type $p_4$ ($y$ axis) and a parameter of type $p_2$ ($x$ axis). The information obviously shows that there exists positively high correlation between these two parameters and we already know that the parameter transformation algorithm will not be able to solve the problem in this case.

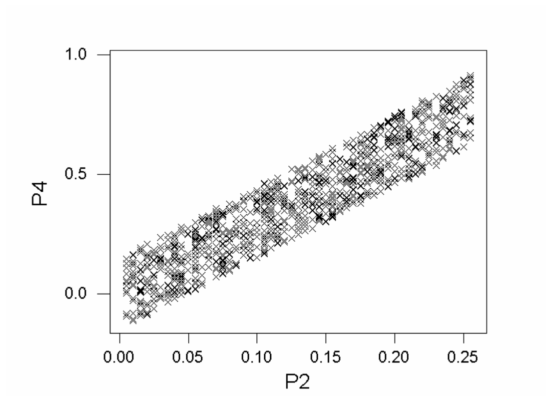


**Figure 8.5 Scatter Plots Between $p_4$ and $p_2$ in the Sample Data Set**

We will now apply the approximation algorithm previously presented in this section by treating the possible values of this $p_4$ parameter as discrete scenarios. We start by classifying the possible values of this $p_4$ parameter into eight scenarios. Figure 8.6 illustrates this classification of parameters values into scenarios (each box represents a scenario).

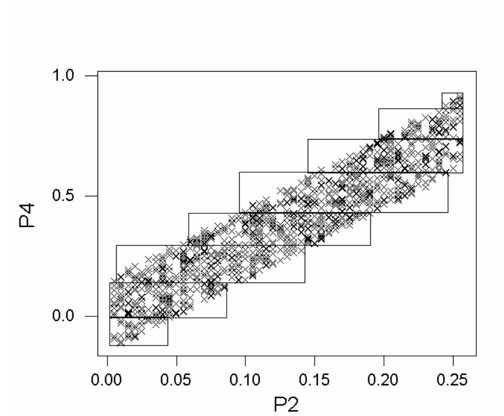


**Figure 8.6 Classification of Scenarios**

Figure 8.7 illustrates the scatter plots of these $p_4$ ($y$ axis) and $p_2$ ($x$ axis) parameters for all eight scenarios. The information from these scatter plots shows the significant reduction in correlation between these two parameters in each scenario.

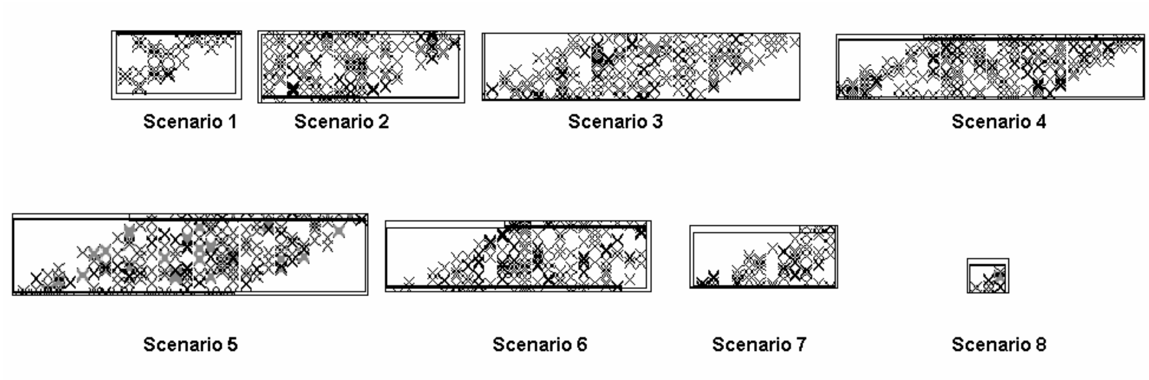


**Figure 8.7 Scatter Plots between $p_4$ and $p_2$ in Each Scenario**

The BLPP models will then be solve separately for each scenario by using the semi-continuous robust algorithm with the assumption that there exists no correlation between these $p_4$ and $p_2$ parameters in each scenario.

There is one important tradeoff for the use of this approximation algorithm. The higher the number of generated discrete scenarios, the more accurate the approximation will be. Unfortunately, the higher the number of scenarios generated, the greater number of BLPP models we are required to solve. With this tradeoff in mind, decision makers have to carefully select the number of scenarios generated for this approximation algorithm so that they will be able to solve for the high quality robust solutions in reasonable time.

Case 8: The correlation exists among model parameters of type $p_5$ and others.

Because the parameters of type $p_5$ are always discrete parameters in our consideration, the algorithm starts by treating the possible values of parameters of type $p_5$ as initial discrete scenarios. Each scenario will represent point values of parameters of type $p_5$ combining with the different possible point values or range values of other types of parameters based on the information from the sample data set. The scenarios are generated such that the parameters of type $p_5$ have approximately no correlation with the other types of parameters in each scenario. The BLPP model will then be solved individually using the previous algorithms for each scenario by assuming that no correlation exists among parameter of type $p_5$ and all other types of parameters.

## 8.4 Application of the Algorithms to the Sample Problems

*Tool Renting Problem with Correlation between Rental Fee and Capacity ($p_1$ and $p_2$)*

Every morning, a carpenter has to make a decision on what type of tools he is going to rent for that specific day. There are two types of tools, $tool_1$ and $tool_2$, that he can rent. If he decides to rent $tool_1$, he can use it to produce $product_1$ up to $P_{21}$ units per one day which can be sold with the price of \$2 per unit. If he decides to rent $tool_2$, he can use it to produce $product_2$ up to $P_{22}$ units per day which can be sold with the price of \$$P_4$ per unit. The production of each product not only requires tools but also requires raw materials. By using $tool_1$, one unit of $product_1$ requires 2 units of raw materials. By using $tool_2$, one units of $product_2$ requires $P_5$ units of raw materials ($tool_2$ is not very reliable). The amount of raw material available is $P_3$ units per day. At the end of the day, this carpenter has to pay the rental fee for each rented tool. The rental fees of $tool_1$ and $tool_2$ are \$$P_1$

and \$15 per day respectively. Table 8.1 contains the distribution information of uncorrelated model parameters. Figure 8.8 illustrates the scatter plot between parameter $P_1$ ($x$ axis) and $P_{21}$ ($y$ axis). The question the model seeks to answer is which tool(s) should this carpenter rent at the beginning of each day?

**Table 8.1 Distribution Information of Model Parameters**

| Random Parameters | Probability Distribution |
|---|---|
| $P_{22}$ | Unknown with UB = 50 and LB = 40 (Average ≈ 45) |
| $P_3$ | Triangular Distribution (90, 100, 110) |
| $P_4$ | Triangular Distribution (1, 2.5715, 4) |
| $P_5$ | $\Pr(P_5 = 2) = \Pr(P_5 = 4) = 0.5$ |

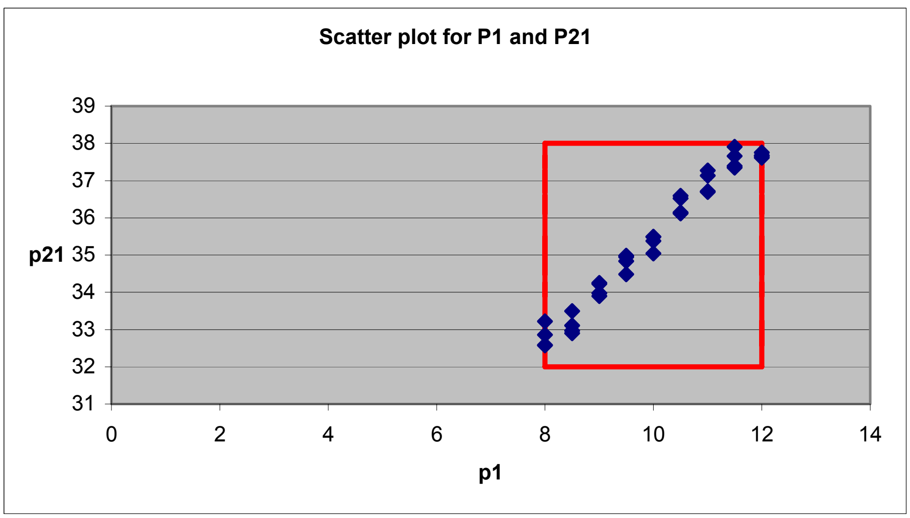


**Figure 8.8 Scatter Plots between $P_1$ ($x$ axis) and $P_{21}$ ($y$ axis)**

This problem can be initially described by a stochastic mixed integer linear programming problem. Let $x_1$ and $x_2$ represents his decisions on daily production units of product$_1$ and product$_2$ respectively. Let $y_1$ and $y_2$ represents his decisions on renting tool$_1$ and tool$_2$ respectively where $y_i = 1$ if he rents tool$_i$ and 0 otherwise for $i = 1, 2$. Figure 8.9 illustrates this initial model.

$$
\begin{aligned}
\max_{x} \quad & 2x_1 + P_4 x_2 - P_1 y_1 - 15 y_2 \\
s.t. \quad & x_1 \le P_{21} y_1 \\
& x_2 \le P_{22} y_2 \\
& 2x_1 + P_5 x_2 \le P_3 \\
& x_1, x_2 \ge 0 \quad y_1, y_2 \in \{0,1\}
\end{aligned}
$$

**Figure 8.9 Initial Stochastic Mixed Integer Linear Programming Model**

By applying semi-continuous robust algorithm, we start by considering four initial scenarios which cover all possible values of the discrete random variable, $P_5$. Table 8.2 contains all parameter values and $O^*_\omega$ for each scenario.

**Table 8.2 All Parameter' Values and $O^*_\omega$ for Four Initial Scenarios**

| Scenario | $P_1$ | $P_{21}$ | $P_{22}$ | $P_3$ | $P_4$ | $P_5$ | $x_1$ | $x_2$ | $y_1$ | $y_2$ | $O^*_\omega$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 32 | 40 | 90 | 1 | 2 | 32 | 0 | 1 | 0 | 56 |
| 2 | 12 | 38 | 50 | 110 | 4 | 2 | 0 | 50 | 0 | 1 | 185 |
| 3 | 8 | 32 | 40 | 90 | 1 | 4 | 32 | 0 | 1 | 0 | 56 |
| 4 | 12 | 38 | 50 | 110 | 4 | 4 | 0 | 27.5 | 0 | 1 | 95 |

By using this information in Table 8.2, the DRRPS model for these four scenarios can be optimally solved. Table 8.3 contains all solutions of this DRRPS model.

**Table 8.3 Solutions of the DRRPS Model for Four Initial Scenarios**

| Scenario | $x_{1\omega}$ | $x_{2\omega}$ | $y_{1\Omega}$ | $y_{2\Omega}$ | $O^*_{\omega}$ | $R_{\omega}$ | $O^*_{\omega}-R_{\omega}$ |
|---|---|---|---|---|---|---|---|
| 1 | 32 | 13 | 1 | 1 | 56 | 54 | 2 |
| 2 | 5 | 50 | 1 | 1 | 185 | 183 | 2 |
| 3 | 32 | 6.5 | 1 | 1 | 56 | 47.5 | 8.5 |
| 4 | 0 | 27.5 | 1 | 1 | 95 | 83 | 12 |

The candidate robust solution from the first stage is now $y_{1\Omega} = 1$ and $y_{2\Omega} = 1$ with the lower bound of 12. This information is then forwarded to the second stage of the algorithm for a feasibility check. Now we apply the parameter space transformation algorithm to the problem. The algorithm starts by calculating the vector $\vec{a}_0 = \begin{bmatrix} \frac{12+8}{2} = 10 \\ \frac{38+32}{2} = 35 \end{bmatrix}$ and vector $\vec{P}'_{12} = \begin{bmatrix} P_1 \\ P_{21} \end{bmatrix} - \vec{a}_0$. In the next step, the linear regression relationship between $P'_{12,1}$ and $P'_{12,2}$ is established. The resulting linear relationship is $P'_{12,2} = (1.34)P'_{12,1}$. The Gram-Schmidt Orthogonalization algorithm is applied on vectors $\vec{g}_1 = \begin{bmatrix} 1 \\ 1.34 \end{bmatrix}$ and $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The resulting vectors are $\vec{u}'_1 = \begin{bmatrix} 0.59809 \\ 0.80144 \end{bmatrix}$ and $\vec{u}'_2 = \begin{bmatrix} -0.80144 \\ 0.59809 \end{bmatrix}$.

The algorithm then produces two affine functions as follows:

$$P_1 = (0.59809)P'_{12,1} + (-0.80144)P'_{12,2} + 10$$

$$P_{21} = (0.80144)P'_{12,1} + (0.59809)P'_{12,2} + 35$$

Figure 8.10 illustrates the scatter plots of parameter $P'_{12,1}$ and $P'_{12,2}$, which support the validity of independency assumption for the transformed problem.

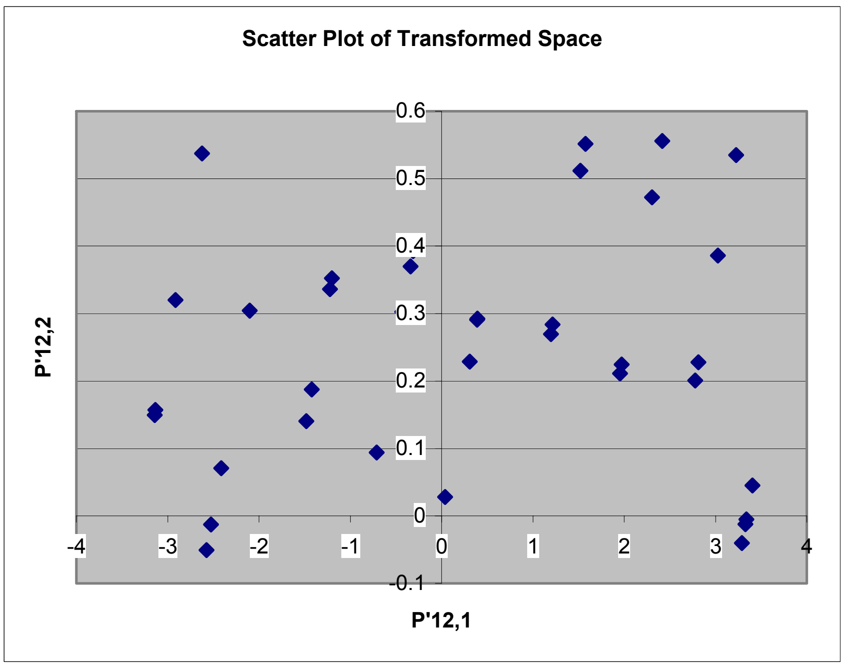


**Figure 8.10 Scatter Plots between $P'_{12,1}$ (*x* axis) and $P'_{12,2}$ (*y* axis)**

By using the information from the sample data set, the upper and the lower bounds of $P'_{12,1}$ and $P'_{12,2}$ are identified as follow: $P'^{U}_{12,1} = 3.41$, $P'^{L}_{12,1} = -3.15$, $P'^{U}_{12,2} = 0.56$, $P'^{L}_{12,2} = -0.051$. Because these settings ($P_{21}$ = 32, $P_{22}$ = 40, $P_3$ = 90, and $P_5$ = 2 or 4) are already considered in Scenarios 1 and 3, the current candidate robust solution is already feasible for all possible scenarios. This current candidate robust solution and the lower bound are then forwarded to the third stage of the algorithm. At this stage, we are required to solve two transformed BLPP models (Cases $P_5$ = 2 and $P_5$ = 4). Figure 8.11 illustrates the initial form of the transformed BLPP model and Figure 8.12 illustrates the final form of the transformed BLPP model.

$$
\begin{aligned}
&\max_{\substack{x_{11},x_{12},y_1,y_2\\P_{21},P_{22},P_3,P_4}} \begin{pmatrix} 2x_{11}+P_4x_{12}-\left(\left(0.59809\right)P_{12,1}^{'}+\left(-0.80144\right)P_{12,2}^{'}+10\right)y_1-15y_2-2x_{21}-P_4x_{22} \\ +\left(0.59809P_{12,1}^{'}-0.80144P_{12,2}^{'}\right)+25 \end{pmatrix}\\
&s.t.\quad x_{11}\le\left(\left(0.80144\right)P_{12,1}^{'}+\left(0.59809\right)P_{12,2}^{'}+35\right)y_1\\
&\qquad x_{12}\le P_{22}y_2\\
&\qquad 2x_{11}+\ P_5x_{12}\le P_3\\
&\qquad x_{11},x_{12}\ge 0\qquad y_1,y_2\in\{0,1\}\\
&\qquad \text{-}3.15\le P_{12,1}^{'}\le 3.41,\quad \text{-}0.051\le P_{12,2}^{'}\le 0.56,\quad 40\le P_{22}\le 50\\
&\qquad 90\le P_3\le 110,\ \ 1\le P_4\le 4\\
&\qquad \max_{x_{21},x_{22}}(2x_{21}+P_4x_{22}-27)\\
&\qquad s.t.\quad x_{21}\le\left(0.80144\right)P_{12,1}^{'}+\left(0.59809\right)P_{12,2}^{'}+35\\
&\qquad\qquad x_{22}\le P_{22}\\
&\qquad\qquad 2x_{21}+P_5x_{22}\le P_3\\
&\qquad\qquad x_{21},x_{22}\ge 0
\end{aligned}
$$

**Figure 8.11 The Initial Form of the Transformed BLPP Model ($P_5$ = 2 or 4)**

$$\max_{x_{11},x_{12},y_1,y_2} (2x_{11} + PX_{412} - PY'_{12,1} - 10y_1 - 15y_2 - 2x_{21} - PX_{422} + \left(0.59809P'_{12,1} - 0.80144P'_{12,2}\right) + 25)$$

$$\begin{aligned}
s.t.\quad & x_{11} \le PY'_{12,2} + 35y_1 \qquad && x_{12} \le PY_{222} \\
& 2x_{11} + P_5 x_{12} \le P_3 && x_{21} + s_1 = \left(0.80144P'_{12,1} + 0.59809P'_{12,2} + 35\right) \\
& x_{22} + s_2 = P_{22} && 2x_{21} + P_5 x_{22} + s_3 = P_3 \\
& w_1 + 2w_3 - a_1 = 2 && w_2 + P_5 w_3 - a_2 = P_4 \\
& PY_{222} - P_{22} \le 0 && -PY_{222} + P_{22} - 50(1-y_2) \le 0 \\
& PY_{222} \le 50y_2 && P_{22} \le 40 + 10y_2 \\
& PX_{412} - 4x_{12} \le 0 && -PX_{412} + 4x_{12} - 200(1-bi_1) \le 0 \\
& PX_{412} - x_{12} - 200bi_1 \le 0 && -PX_{412} + x_{12} \le 0 \\
& PX_{422} - 4x_{22} \le 0 && -PX_{422} + 4x_{22} - 200(1-bi_1) \le 0 \\
& PX_{422} - x_{22} - 200bi_1 \le 0 && -PX_{422} + x_{22} \le 0 \\
& -P_4 + 4 - 3(1-bi_1) \le 0 && P_4 - 1 - 3bi_1 \le 0 \\
& PY'_{12,1} \le \left(2.48829\right)(1-y_1) + \left(0.59809P'_{12,1} - 0.80144P'_{12,2}\right) && \\
& -PY'_{12,1} \le \left(2.48829\right)(1-y_1) - \left(0.59809P'_{12,1} - 0.80144P'_{12,2}\right) && \\
& PY'_{12,1} \ge -\left(2.48829\right)y_1 && PY'_{12,1} \le \left(2.48829\right)y_1 \\
& PY'_{12,2} \le \left(3.06784\right)(1-y_1) + \left(0.80144P'_{12,1} + 0.59809P'_{12,2}\right) && \\
& -PY'_{12,2} \le \left(3.06784\right)(1-y_1) - \left(0.80144P'_{12,1} + 0.59809P'_{12,2}\right) && \\
& PY'_{12,2} \ge -\left(3.06784\right)y_1 && PY'_{12,2} \le \left(3.06784\right)y_1 \\
& x_{21}a_1 = 0,\ \ x_{22}a_2 = 0,\ \ w_1 s_1 = 0,\ \ w_2 s_2 = 0,\ \ w_3 s_3 = 0, && \\
& x_{11}, x_{12}, x_{21}, x_{22}, s_1, s_2, s_3, w_1, w_2, w_3, a_1, a_2 \ge 0, \quad y_1, y_2, bi_1 \in \{0,1\} && \\
& \text{-}3.15 \le P'_{12,1} \le 3.41,\ \ \text{-}0.051 \le P'_{12,2} \le 0.56,\ \ 40 \le P_{22} \le 50,\ \ 90 \le P_3 \le 110,\ \ 1 \le P_4 \le 4 &&
\end{aligned}$$

**Figure 8.12 The Final Form of the Transformed BLPP Model ($P_5$ = 2 or 4)**

Table 8.4 contains the optimal solution for these BLPP models. Because the upper bound resulting from this BLPP model is 12, the algorithm is then terminated with the robust optimal solution of $y_{1\Omega} = 1$ and $y_{2\Omega} = 1$ (the carpenter should rent both tools at the beginning of each day). Table 8.5 contains the comparison between the optimal robust solution and the optimal solution from the average value problem ($y_{1\Omega} = 0$ and $y_{2\Omega} = 1$).

**Table 8.4 The Optimal Solution for the BLPP Model**

| Decision Variable | $P_5 = 2$ | $P_5 = 4$ |
|---|---|---|
| $x_{11}$ | 0 | 0 |
| $x_{12}$ | 45 | 22.5 |
| $x_{21}$ | 0 | 0 |
| $x_{22}$ | 45 | 22.5 |
| $y_1$ | 0 | 0 |
| $y_2$ | 1 | 1 |
| $P'_{12,1}$ | 3.41 | 3.41 |
| $P'_{12,2}$ | -0.051 | -0.051 |
| $P_1$ | 12 | 12 |
| $P_{21}$ | 37.7 | 37.7 |
| $P_{22}$ | 50 | 40 |
| $P_3$ | 90 | 90 |
| $P_4$ | 4 | 4 |
| $O^*_\omega - R_\omega$ | 12 | 12 |

**Table 8.5 The Comparison between the Semi-Continuous Robust Solution and the Optimal Solution from the Average Value Problem**

| | **Maximum Regret From Optimality** | **Objective Value under Average Value Scenario** |
|---|---|---|
| Solution From Average Value Problem | 57.045 | 71.66 |
| Semi-Continuous Robust Solution | 12 | 71 |

The results in Table 8.5 illustrate the superior of the semi-continuous robust solution over the optimal solution from the average value problem.

*Tools Renting Problem with Correlation between Selling Price and Supply ($p_4$ and $p_3$)*

For this example, we consider the same tools renting example presented in Chapter VI with the additional correlation between selling price of product per unit and the amount of the raw material available at the beginning of each day. This example assumes that if the amount of the raw material available at the beginning of each day is high, it is highly likely that there will be a lot of competition on that day which will cause the selling price of product to drop down and vice versa. Because there is no exact method of solving this problem, this carpenter creates four discrete scenarios to capture this correlation. Table 8.6 contains the information on these scenarios.

**Table 8.6 Information on Four Discrete Scenarios to Represent Correlation**

| | $P_3$ | $P_4$ |
|---|---|---|
| Scenario1 | [105, 110] | 1 |
| Scenario 2 | [100, 105] | 2 |
| Scenario 3 | [95, 100] | 3 |
| Scenario 4 | [90, 95] | 4 |

By applying the semi-continuous robust algorithm, we start by considering 32 initial scenarios, which cover all possible values of the discrete random variable, $P_5$ and the correlated parameters. Table 8.7 contains all the parameters' values and $O^*_\omega$ for each scenario.

**Table 8.7  All Parameter' Values and $O^*_\omega$ for Thirty Two Initial Scenarios**

| Scenario | $P_1$ | $P_{21}$ | $P_{22}$ | $P_3$ | $P_4$ | $P_5$ | $O^*_\omega$ |
|---|---|---|---|---|---|---|---|
| 1 | 8 | 32 | 40 | 105 | 1 | 2 | 61.5 |
| 2 | 8 | 32 | 40 | 100 | 2 | 2 | 77 |
| 3 | 8 | 32 | 40 | 95 | 3 | 2 | 112 |
| 4 | 8 | 32 | 40 | 90 | 4 | 2 | 147 |
| 5 | 8 | 32 | 40 | 105 | 1 | 4 | 56 |
| 6 | 8 | 32 | 40 | 100 | 2 | 4 | 59 |
| 7 | 8 | 32 | 40 | 95 | 3 | 4 | 64.25 |
| 8 | 8 | 32 | 40 | 90 | 4 | 4 | 75 |
| 9 | 12 | 38 | 50 | 105 | 1 | 2 | 64 |
| 10 | 12 | 38 | 50 | 100 | 2 | 2 | 85 |
| 11 | 12 | 38 | 50 | 95 | 3 | 2 | 127.5 |
| 12 | 12 | 38 | 50 | 90 | 4 | 2 | 165 |
| 13 | 12 | 38 | 50 | 105 | 1 | 4 | 64 |
| 14 | 12 | 38 | 50 | 100 | 2 | 4 | 64 |
| 15 | 12 | 38 | 50 | 95 | 3 | 4 | 64 |
| 16 | 12 | 38 | 50 | 90 | 4 | 4 | 75 |
| 17 | 8 | 32 | 40 | 110 | 1 | 2 | 64 |
| 18 | 8 | 32 | 40 | 105 | 2 | 2 | 82 |
| 19 | 8 | 32 | 40 | 100 | 3 | 2 | 117 |
| 20 | 8 | 32 | 40 | 95 | 4 | 2 | 152 |
| 21 | 8 | 32 | 40 | 110 | 1 | 4 | 56 |
| 22 | 8 | 32 | 40 | 105 | 2 | 4 | 61.5 |
| 23 | 8 | 32 | 40 | 100 | 3 | 4 | 68 |
| 24 | 8 | 32 | 40 | 95 | 4 | 4 | 80 |
| 25 | 12 | 38 | 50 | 110 | 1 | 2 | 66 |
| 26 | 12 | 38 | 50 | 105 | 2 | 2 | 85 |
| 27 | 12 | 38 | 50 | 100 | 3 | 2 | 135 |
| 28 | 12 | 38 | 50 | 95 | 4 | 2 | 175 |
| 29 | 12 | 38 | 50 | 110 | 1 | 4 | 64 |
| 30 | 12 | 38 | 50 | 105 | 2 | 4 | 64 |
| 31 | 12 | 38 | 50 | 100 | 3 | 4 | 67 |
| 32 | 12 | 38 | 50 | 95 | 4 | 4 | 80 |

By using this information in Table 8.7, the DRRPS model for these four scenarios can be optimally solved. The candidate robust solution from the first stage is now $y_{1\Omega} = 1$ and $y_{2\Omega} = 1$ with the lower bound of 12. This information is then forwarded to the second stage of the algorithm for a feasibility check. Because these settings ($P_{21} = 32$, $P_{22} = 40$, $P_3 = 90$, and $P_5 = 2$ or 4) are already considered in these thirty two scenarios, the current candidate robust solution is already feasible for all possible scenarios. This current candidate robust solution and the lower bound are then forwarded to the third stage of the algorithm. At this stage, we are required to solve eight BLPP models (cases $P_5 = 2$ and $P_5 = 4$ combined with four possible scenarios generated by correlation). Table 8.8 contains the optimal objective function value of these eight BLPP models.

**Table 8.8 Optimal Objective Function Value of these Eight BLPP Models**

| **Maximum Regret** | $\boldsymbol{P_4 = 1}$ $P_3 \in [105,110]$ | $\boldsymbol{P_4 = 2}$ $P_3 \in [100,105]$ | $\boldsymbol{P_4 = 3}$ $P_3 \in [95,100]$ | $\boldsymbol{P_4 = 4}$ $P_3 \in [90,95]$ |
|---|---|---|---|---|
| $P_5 = 2$ | 0.5 | 12 | 12 | 12 |
| $P_5 = 4$ | 7.75 | 3 | 0.75 | 12 |

Because the upper bound resulting from these BLPP models is 12, the algorithm is then terminated with the robust optimal solution of $y_{1\Omega} = 1$ and $y_{2\Omega} = 1$ (the carpenter should rent both tools at the beginning of each day).

## 8.5 Summary

This chapter introduces the new parameter space transformation algorithm which can be used together with the semi-continuous robust algorithm for solving the mini-max robust optimization problem with correlated uncertain parameters.

The algorithm is constructed based on the idea of transforming the original parameter space with high correlation to the new parameter space with low or no correlation. The methodology for handling each possible case of correlations among uncertain parameters is presented in the chapter. Small examples are also presented with the purpose of giving the readers a clear understanding of the algorithm.

The algorithm can easily be extended to generate the min-max regret robust solution to the problem when each uncertain continuous parameter takes its values from more than one compact interval (finite number of compact intervals). In this case, the initial discrete scenarios are generated based on the combination of all possible values of discrete parameters and all possible compact intervals of continuous parameters. The parameter space transformation algorithm and approximation algorithm are then applied to each scenario in the initial discrete scenarios separately. All remaining steps of the algorithm are the same.

In conclusion, the parameter space transformation algorithm and the semi-continuous robust algorithm give a good theoretical value to the methodology of solving the mini-max robust optimization problem with correlated uncertain parameters. Further studies are required for improving the computational ability of the algorithm to handle the realistically sized problem.

# CHAPTER IX

# KEY CONTRIBUTIONS AND FUTURE RESEARCH

## 9.1 Key Contributions

Current existing robust optimization methodologies with the deviation robustness definition assume that the model uncertainty either can be discretized into the finite set of discrete scenarios or is represented by the variation of model parameters which take their value within bounded ranges (Newton, 2000). The problem of applying the algorithm in the former case is that the discrete robust optimization model size grows exponentially with the number of uncertain parameters. In the later case, the existing algorithms can handle only limited types of parameters and cannot represent the uncertainty represented by the combination of discrete and continuous scenarios at the same time. The existing algorithms are not comprehensive and do not address a significant class of practical problems.

In Chapter IV, we developed a scenario relaxation heuristic algorithm and explored the use of accelerated Benders' decomposition algorithm for this discrete robust optimization approach when dealing with a large number of scenarios. The results from the case studies illustrate the significant improvement in computational time for the large discrete robust optimization problems.

In Chapter VI, we develop a semi-continuous robust algorithm that is capable of solving the robust optimization problems with the deviation robustness definition for dealing with the problem with continuous ranges of random parameters and discrete valued random parameters. This new algorithm can handle all variations (discrete scenarios, continuous scenarios and their combinations) in uncertain parameters for mixed integer linear programming network problems. This dissertation explicitly includes mathematical models and detailed solution methodologies required for the problem. These mathematical models and solution methodologies provide a great tool for network infrastructure planning that explicitly deals with uncertainty through the use of parameter ranges and fixed discrete parameter values without knowing the information on the parameters' joint probability distributions. This type of approach can be useful in network infrastructure planning where the joint probability distributions of key parameters are unknown and the only information available are the parameters' ranges and fixed discrete values of parameters. The algorithm is a significant advance beyond the current state of the art in robust mathematical programming with a mini-max regret objective.

The semi-continuous robust algorithm can also be used to provide the bounds (both upper and lower bound) on the value of minimum maximum regret between optimal setting and the robust configuration setting. Terminating the algorithm anytime after the third stage has been completed at least once will provide these bounds. If the decision makers do not intentionally terminate the algorithm, the algorithm is proven to terminate either at an optimal robust solution, or by confirming that no existing robust solution exists, in a finite number of iterations.

In Chapter VII, the semi-continuous robust algorithm has been applied for solving many case study problems of designing the robust reverse logistic infrastructure. The results illustrate the computational efficiency of the algorithm to the problems.

In Chapter VIII, the parameter space transformation algorithm has been introduced with the capability of transforming the original parameter space with correlation into the new parameter space with approximately no correlation. The methodologies of combining this algorithm and the semi-continuous robust algorithm are also explicitly presented with the capability of solving the robust optimization problem when correlations exist among parameters.

Overall this research facilitates the robust design of network supply chain systems (with reverse production systems as one of their subsystems) under a min-max regret objective. The resultant system has the potential to be more financially and operational viable because for each realization of the parameters, the system still tries to be close to the optimal settings.

## 9.2 Future Research

In Chapter VI we presented a solution methodology for the third stage of the semi-continuous robust algorithm. Within the algorithm, we are required to solve a number of BLPP models (one for each possible discrete scenario). By solving each of these BLPP models in parallel, one can make the significant improvement in computational time required by the algorithm. Another interesting idea of the parallel computing is to assign different processors to work on different parts of the solution tree when solving the BLPP model. These steps can be achieved by developing the computer codes that assign required tasks that can be processed in parallel to different computer processors and combine these solutions for the further use in the algorithm.

Although our research introduces many effective parameter pre-processing steps, variables and constraints elimination steps, lower bound setting techniques, and priority branch and bound steps, which have been shown in Chapter VII to be quite effective in computational time reduction of the BLPP model, the computational time of the BLPP model is still considered to be one of the bottlenecks of the semi-continuous robust algorithm. A future research opportunity is the search for improved pre-processing and branching rules for further improvement in computational ability of the BLPP models.

In the current semi-continuous robust algorithm, we require that the variation of parameters of type $p_5$ (coefficient of continuous variables in model functional constraints) to be represented by their possible discrete values. This requirement suits the nature of our RPS model perfectly, for the same reasons given in Chapter VI. For general mixed integer linear programming problems, this requirement may be too restrictive. The search for new or modified methodology that is able to solve the problem for semi-continuous robust solution without this restriction for general stochastic mixed integer linear programming problem should be further explored.

In Chapter VIII, we present the combination of the parameter space transformation algorithm and the semi-continuous robust algorithm for solving the problems when correlations exist among parameters. Although these algorithms have high theoretical value and introduce the innovative solution methodology to the problem, an interesting future research topic is the development of an improved/modified methodology to handle realistically sized problems.

There are still many different directions this research could lead to in the future. Incorporating game theory to look at the interactions of the company's actions and government's actions within the reverse supply chain system and to find the robust supply chain infrastructure of the system are one of the areas to which this research could be expanded. Considering different definitions of robustness and developing new robust optimization approach for the problem is another way of expanding this research problem.

# APPENDIX A

## The BLPP Model for Semi-Continuous Robust Algorithm of the RPS Model

**Table A1  Model Indices**

| | |
|---|---|
| s | Supplier |
| i | Sites |
| c | Customer |
| j | Material type |
| m | Transportation mode |
| p | Process type |
| t | Time period |

**Table A2  Model Superscripts**

| | |
|---|---|
| Co | Collection |
| Sa | Selling |
| St | Storage |
| Tr | Transportation |
| Pr | Process |
| Su | Supplier |
| Si | Site |
| Cu | Customer |
| 1 | Leader problem |
| 2 | Follower problem |
| UB | Upper bound value |
| LB | Lower bound value |
| Ind | Indicator if this value cannot be pre-processed |
| * | Pre-processed value |

**Table A3 Model Parameters**

$S_{sjt}^{(Su)\,UB}$ = Upper bound on amount of material j that is supplied at supplier s at time period t

$S_{sjt}^{(Su)\,LB}$ = Lower bound on amount of material j that is supplied at supplier s at time period t

$D_{cjt}^{(Cu)\,UB}$ = Upper bound on amount of material j that is demanded at customer j at time period t

$D_{cjt}^{(Cu)\,LB}$ = Lower bound on amount of material j that is demanded at customer j at time period t

$P_{cjt}^{(Cu)\,UB}$ = Upper bound on selling Price offered per standard unit of material j from customer c at time period t

$P_{cjt}^{(Cu)\,LB}$ = Lower bound on selling Price offered per standard unit of material j from customer c at time period t

$P_{cjt}^{(Cu)\,*}$ = Pre-processing value on selling Price offered per standard unit of material j from customer c at time period t, 0 if this value cannot be pre-determined

$P_{cjt}^{(Cu)\,Ind}$ = 1 if the selling Price offered per standard unit of material j from customer c at time period t cannot be predetermined, 0 otherwise

$V_{ijt}^{(St)\,UB}$ = Upper bound on storage cost per standard unit of material j per time period at site i at time period t

$V_{ijt}^{(St)\,LB}$ = Lower bound on storage cost per standard unit of material j per time period at site i at time period t

$V_{ijt}^{(St)\,*}$ = Pre-processing value on storage cost per standard unit of material j per time period at site i at time period t, 0 if this value cannot be pre-determined

$V_{ijt}^{(St)\,Ind}$ = 1 if the storage cost per standard unit of material j per time period at site i at time period t cannot be predetermined, 0 otherwise

$V_{ijt}^{(Co)\,UB}$ = Upper bound on collection cost per standard unit of material j at site i at time period t

$V_{ijt}^{(Co)\,LB}$ = Lower bound on collection cost per standard unit of material j at site i at time period t

$V_{ijt}^{(Co)*}$ = Pre-processing value on collection cost per standard unit of material j at site i at time period t, 0 if this value cannot be pre-determined

$V_{ijt}^{(Co)\,Ind}$ = 1 if the collection cost per standard unit of material j at site i at time period t cannot be predetermined, 0 otherwise

$V'^{(Co)\,UB}_{ijt}$ = Upper bound on collection fee per standard unit of material j at site i at time period t

$V'^{(Co)\,LB}_{ijt}$ = Lower bound on collection fee per standard unit of material j at site i at time period t

$V'^{(Co)*}_{ijt}$ = Pre-processing value collection fee per standard unit of material j at site i at time period t, 0 if this value cannot be pre-determined

$V'^{(Co)\,Ind}_{ijt}$ = 1 if the collection fee per standard unit of material j at site i at time period t cannot be predetermined, 0 otherwise

$V_{ipt}^{(Pr)\,UB}$ = Upper bound on processing cost per standard unit for process p at site i at time period t

$V_{ipt}^{(Pr)\,LB}$ = Lower bound on processing cost per standard unit for process p at site i at time period t

$V_{ipt}^{(Pr)*}$ = Preprocessing value on processing cost per standard unit for process p at site i at time period t, 0 if this value cannot be pre-determined

$V_{ipt}^{(Pr)\,Ind}$ = 1 if the processing cost per standard unit for process p at site i at time period t cannot be predetermined, 0 otherwise

$V_{simt}^{(Tr)\,UB}$ = Upper bound on transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t

$V_{simt}^{(Tr)\,LB}$ = Lower bound on transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t

$V_{simt}^{(Tr)*}$ = Preprocessing value on transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t, 0 if this value cannot be pre-determined

$V_{simt}^{(Tr)\,Ind}$ = 1 if the transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t cannot be predetermined, 0 otherwise

$V_{ii'mt}^{(Tr)\,UB}$ = Upper bound on transportation cost per standard unit per distance from site i to i' using transportation mode m at time period t

$V_{ii'mt}^{(Tr)\,LB}$ = Lower bound on transportation cost per standard unit per distance from site i to i' using transportation mode m at time period t

$V_{ii'mt}^{(Tr)*}$ = Preprocessed value on transportation cost per standard unit per distance from site i to i' using transportation mode m at time period t, 0 if this value cannot be pre-determined

$V_{ii'mt}^{(Tr)\,Ind}$ = Lower bound on transportation cost per standard unit per distance from site i to i' using transportation mode m at time period t cannot be predetermined, 0 otherwise

$V_{icmt}^{(Tr)\,UB}$ = Upper bound on transportation cost per standard unit per distance from site i to customer j using transportation mode m at time period t

$V_{icmt}^{(Tr)\,LB}$ = Lower bound on transportation cost per standard unit per distance from site i to customer j using transportation mode m at time period t

$V_{icmt}^{(Tr)*}$ = Preprocessing value on transportation cost per standard unit per distance from site i to customer j using transportation mode m at time period t, 0 if this value cannot be pre-determined

$V_{icmt}^{(Tr)\,Ind}$ = 1 if the transportation cost per standard unit per distance from site i to customer j using transportation mode m at time period t cannot be predetermined, 0 otherwise

$d_{sim}$ = Distance from supplier s to site i by transportation mode m

$d_{ii'm}$ = Distance from site i to i' by transportation mode m

$d_{icm}$ = Distance from site i to customer c by transportation mode m

$F_{it}^{(Si)*}$ = Pre-processed value on fixed site operating cost if site i is opened at time period t

$F'^{(Si)*}_{it}$ = Pre-processed value on fixed site opening cost of site i at time period t

$F''^{(Si)*}_{it}$ = Pre-processed value on fixed site closing cost of site i at time period t

$F_{ijt}^{(St)*}$ = Pre-processed value on fixed storage cost of material j at site i at time period t

$F_{ijt}^{(Co)*}$ = Pre-processed value on fixed collecting cost of material j at site i at time period t

$F_{ipt}^{(Pr)*}$ = Pre-processed value on fixed processing cost for process p at site i at time period t

$F_{simt}^{(Tr)*}$ = Pre-processed value on fixed cost for transportation from supplier s to site i using transportation mode m at time period t

$F_{ii'mt}^{(Tr)*}$ = Pre-processed value on fixed cost for transportation from site i to site i' using transportation mode m at time period t

$F_{icmt}^{(Tr)\,*}$ = Pre-processed value on fixed cost for transportation from site i to customer c using transportation mode m at time period t

$C_{ijt}^{(Co)\,UB}$ = Upper bound on maximum collection capacity to collect material type j at site i at time period t

$C_{ijt}^{(Co)\,LB}$ = Lower bound on maximum collection capacity to collect material type j at site i at time period t

$C_{ijt}^{(Co)*}$ = Preprocessing value on maximum collection capacity to collect material type j at site i at time period t, 0 if this value cannot be pre-determined

$C_{ijt}^{(Co)\,Ind}$ = 1 if the maximum collection capacity to collect material type j at site i at time period t cannot be predetermined, 0 otherwise

$C_{ijt}^{(St)\,UB}$ = Upper bound on maximum amount of material type j that can be stored at site i in at time period t

$C_{ijt}^{(St)\,LB}$ = Lower bound on maximum amount of material type j that can be stored at site i in at time period t

$C_{ijt}^{(St)*}$ = Preprocessing value on maximum amount of material type j that can be stored at site i in at time period t, 0 if this value cannot be pre-determined

$C_{ijt}^{(St)\,Ind}$ = 1 if the maximum amount of material type j that can be stored at site i in at time period t cannot be predetermined, 0 otherwise

$C_{simt}^{(Tr)\,UB}$ = Upper bound on maximum amount of material that can be shipped for supplier s to site i using transportation mode m at time period t

$C_{simt}^{(Tr)\,LB}$ = Lower bound on maximum amount of material that can be shipped for supplier s to site i using transportation mode m at time period t

$C^{(Tr)*}_{simt}$ = Preprocessing value on maximum amount of material that can be shipped for supplier s to site i using transportation mode m at time period t, 0 if this value cannot be pre-determined

$C^{(Tr)\,Ind}_{simt}$ = 1 if the maximum amount of material that can be shipped for supplier s to site i using transportation mode m at time period t cannot be predetermined, 0 otherwise

$C^{(Tr)\,UB}_{ii'mt}$ = Upper bound on maximum amount of material that can be shipped for site i to i' using transportation mode m at time period t

$C^{(Tr)\,LB}_{ii'mt}$ = Lower bound on maximum amount of material that can be shipped for site i to i' using transportation mode m at time period t

$C^{(Tr)*}_{ii'mt}$ = Preprocessing value on maximum amount of material that can be shipped for site i to i' using transportation mode m at time period t, 0 if this value cannot be pre-determined

$C^{(Tr)\,Ind}_{ii'mt}$ = 1 if the maximum amount of material that can be shipped for site i to i' using transportation mode m at time period t cannot be predetermined, 0 otherwise

$C^{(Tr)\,UB}_{icmt}$ = Upper bound on maximum amount of material that can be shipped for site i to customer c using transportation mode m at time period t

$C^{(Tr)\,LB}_{icmt}$ = Lower bound on maximum amount of material that can be shipped for site i to customer c using transportation mode m at time period t

$C^{(Tr)*}_{icmt}$ = Preprocessing value on maximum amount of material that can be shipped for site i to customer c using transportation mode m at time period t, 0 if this value cannot be pre-determined

$C_{icmt}^{(Tr)\,Ind}$ = 1 if the maximum amount of material that can be shipped for site i to customer c using transportation mode m at time period t cannot be predetermined, 0 otherwise

$C_{ipt}^{(Pr)\,UB}$ = Upper bound on maximum amount of material that process p can produce at site i at time period t

$C_{ipt}^{(Pr)\,LB}$ = Lower bound on maximum amount of material that process p can produce at site i at time period t

$C_{ipt}^{(Pr)*}$ = Preprocessing value on maximum amount of material that process p can produce at site i at time period t, 0 if this value cannot be pre-determined

$C_{ipt}^{(Pr)\,Ind}$ = 1 if the maximum amount of material that process p can produce at site i at time period t cannot be predetermined, 0 otherwise

$\rho_{jp}$ = proportion of material type j consumed by process p

$\rho'_{jp}$ = proportion of material type j produced by process p

$y_{ijt}^{(Co)\Omega}$ = 1 if collection of material type j is performed at site i at time period t in robust solution from scenario set Ω and 0 otherwise

$y_{simt}^{(Tr)\Omega}$ = 1 if shipment is used between supplier s and site i using transportation mode m at time period t in robust solution from scenario set Ω and 0 otherwise

$y_{ii'mt}^{(Tr)\Omega}$ = 1 if shipment is used between sites i and i' using transportation mode m at time period t in robust solution from scenario set Ω and 0 otherwise

$y^{(Tr)\Omega}_{icmt}$ = 1 if shipment is used between sites i and customer c using transportation mode m at time period t in robust solution from scenario set Ω and 0 otherwise

$y^{(Pr)\Omega}_{ipt}$ = 1 if process p is used at site i at time period t in robust solution from scenario set Ω and 0 otherwise

$y^{(St)\Omega}_{ijt}$ = 1 if storage is used for material type j at site i at time period t in robust solution from scenario set Ω and 0 otherwise

$y'^{(Si)\Omega}_{it}$ = 1 if site i is opened at period t in robust solution from scenario set Ω and 0 otherwise

$y''^{(Si)\Omega}_{it}$ = 1 if site i is closed down at period t in robust solution from scenario set Ω and 0 otherwise

$y^{(Si)\Omega}_{it}$ = 1 if site i is operated at time period t in robust solution from scenario set Ω and 0 otherwise

$x^{(Co)\,1UB}_{ijt}$ = Upper bound on amount of material collected of type j at site i at time period t for the leader problem

$x^{(Co)\,2UB}_{ijt}$ = Upper bound on amount of material collected of type j at site i at time period t for the follower problem

$x^{(St)\,1UB}_{ijt}$ = Upper bound on amount of material stored of type j at site i at time period t for the leader problem

$x^{(St)\,2UB}_{ijt}$ = Upper bound on amount of material stored of type j at site i at time period t for the follower problem

$x^{(Sa)\,1UB}_{cjt}$ = Upper bound on amount of material sold of type j to customer c at time period t for the leader problem

$x_{cjt}^{(Sa)\,2UB}$ = Upper bound on amount of material sold of type j to customer c at time period t for the follower problem

$x_{sjimt}^{(Tr)\,1UB}$ = Upper bound on amount of material shipped from supplier s to site i of type j using transportation mode m at time period t for the leader problem

$x_{sjimt}^{(Tr)\,2UB}$ = Upper bound on amount of material shipped from supplier s to site i of type j using transportation mode m at time period t for the follower problem

$x_{iji'mt}^{(Tr)\,1UB}$ = Upper bound on amount of material shipped from site i to site i’ of type j using transportation mode m at time period t for the leader problem

$x_{iji'mt}^{(Tr)\,2UB}$ = Upper bound on amount of material shipped from site i to site i’ of type j using transportation mode m at time period t for the follower problem

$x_{ijcmt}^{(Tr)\,1UB}$ = Upper bound on amount of material shipped from site i to customer c of type j using transportation mode m at time period t for the leader problem

$x_{ijcmt}^{(Tr)\,2UB}$ = Upper bound on amount of material shipped from site i to customer c of type j using transportation mode m at time period t for the follower problem

$x_{ipt}^{(Pr)\,1UB}$ = Upper bound on amount of material processed by process p at site i at time period t for the leader problem

$x_{ipt}^{(Pr)\,2UB}$ = Upper bound on amount of material processed by process p at site i at time period t for the follower problem

**Table A4 Model Variables**

$x_{ijt}^{(Co)1}$ = Amount of material collected of type j at site i at time period t for the leader problem

$x_{ijt}^{(Co)2}$ = Amount of material collected of type j at site i at time period t for the follower problem

$x_{ijt}^{(St)1}$ = Amount of material stored of type j at site i at time period t for the leader problem

$x_{ijt}^{(St)2}$ = Amount of material stored of type j at site i at time period t for the follower problem

$x_{cjt}^{(Sa)1}$ = Amount of material sold of type j to customer c at time period t for the leader problem

$x_{cjt}^{(Sa)2}$ = Amount of material sold of type j to customer c at time period t for the follower problem

$x_{sjimt}^{(Tr)1}$ = Amount of material shipped from supplier s to site i of type j using transportation mode m at time period t for the leader problem

$x_{sjimt}^{(Tr)2}$ = Amount of material shipped from supplier s to site i of type j using transportation mode m at time period t for the follower problem

$x_{iji'mt}^{(Tr)1}$ = Amount of material shipped from site i to site i' of type j using transportation mode m at time period t for the leader problem

$x_{iji'mt}^{(Tr)2}$ = Amount of material shipped from site i to site i' of type j using transportation mode m at time period t for the follower problem

$x_{ijcmt}^{(Tr)1}$ = Amount of material shipped from site i to customer c of type j using transportation mode m at time period t for the leader problem

$x^{(Tr)2}_{ijcmt}$ = Amount of material shipped from site i to customer c of type j using transportation mode m at time period t for the follower problem

$x^{(Pr)1}_{ipt}$ = Amount of material processed by process p at site i at time period t for the leader problem

$x^{(Pr)2}_{ipt}$ = Amount of material processed by process p at site i at time period t for the follower problem

$y^{(Co)1}_{ijt}$ = 1 if collection of material type j is to be performed at site i at time period t 0 otherwise for the leader problem

$y^{(Tr)1}_{simt}$ = 1 if shipment is to be used between supplier s and site i using transportation mode m at time period t, 0 otherwise for the leader problem

$y^{(Tr)1}_{ii'mt}$ = 1 if shipment is to be used between sites i and i' using transportation mode m at time period t, 0 otherwise for the leader problem

$y^{(Tr)1}_{icmt}$ = 1 if shipment is to be used between sites i and customer c using transportation mode m at time period t, 0 otherwise for the leader problem

$y^{(Pr)1}_{ipt}$ = 1 if process p is to be used at site i at time period t, 0 otherwise for the leader problem

$y^{(St)1}_{ijt}$ = 1 if storage is to be used for material type j at site i at time period t 0 otherwise for the leader problem

$y'^{(Si)1}_{it}$ = 1 if site i is decided to be opened at period t, 0 otherwise for the leader problem

$y''^{(Si)1}_{it}$ = 1 if site i is decided to be closed down at period t, 0 otherwise for the leader problem

$y^{(Si)1}_{it}$ = 1 if site i is operated at time period t, 0 otherwise for the leader problem

$S_{sjt}^{(Su)}$ = Amount of material j that is supplied at supplier s at time period t that make the maximum regret of robust solution from stage 1

$D_{cjt}^{(Cu)}$ = Amount of material j that is demanded at customer j at time period t that make the maximum regret of robust solution from stage 1

$P_{cjt}^{(Cu)}$ = Selling Price offered per standard unit of material j from customer c at time period t that make the maximum regret of robust solution from stage 1

$V_{ijt}^{(St)}$ = Storage cost per standard unit of material j per time period at site i at time period t that make the maximum regret of robust solution from stage 1

$V_{ijt}^{(Co)}$ = Collection cost per standard unit of material j at site i at time period t that make the maximum regret of robust solution from stage 1

$V'^{(Co)}_{ijt}$ = Collection fee per standard unit of material j at site i at time period t that make the maximum regret of robust solution from stage 1

$V_{ipt}^{(Pr)}$ = Processing cost per standard unit for process p at site i at time period t that make the maximum regret of robust solution from stage 1

$V_{simt}^{(Tr)}$ = Transportation cost per standard unit per distance from supplier s to site i using transportation mode m at time period t that make the maximum regret of robust solution from stage 1

$V_{ii'mt}^{(Tr)}$ = Transportation cost per standard unit per distance from site i to i' using transportation mode m at time period t that make the maximum regret of robust solution from stage 1

$V_{icmt}^{(Tr)}$ = Transportation cost per standard unit per distance from site i to customer j using transportation mode m at time period t that make the maximum regret of robust solution from stage 1

$C_{ijt}^{(Co)}$ = Maximum collection capacity to collect material type j at site i at time period t that make the maximum regret of robust solution from stage 1

$C_{ijt}^{(St)}$ = Maximum amount of material type j that can be stored at site i in at time period t that make the maximum regret of robust solution from stage 1

$C_{simt}^{(Tr)}$ = Maximum amount of material that can be shipped for supplier s to site i using transportation mode m at time period t that make the maximum regret of robust solution from stage 1

$C_{ii'mt}^{(Tr)}$ = Maximum amount of material that can be shipped for site i to i' using transportation mode m at time period t that make the maximum regret of robust solution from stage 1

$C_{icmt}^{(Tr)}$ = Maximum amount of material that can be shipped for site i to customer c using transportation mode m at time period t that make the maximum regret of robust solution from stage 1

$C_{ipt}^{(Pr)}$ = Maximum amount of material that process p can produce at site i at time period t that make the maximum regret of robust solution from stage 1

$w_{ijt}^{(Balance)}$ = Dual variable for material j balance equality constraint for site i in time period t for the follower problem

$w_{sjt}^{(Supply)}$ = Dual variable for material j Supply equality constraint for source s in time period t for the follower problem

$w_{cjt}^{(Demand)}$ = Dual variable for material j Demand inequality constraint for customer c in time period t for the follower problem

$w_{ijt}^{(Collect)}$ = Dual variable for material j Capacity collection inequality constraint for site i in time period t for the follower problem

$w_{ipt}^{(Process)}$ = Dual variable for process p Capacity processing inequality constraint for site i in time period t for the follower problem

$w_{ijt}^{(Storage)}$ = Dual variable for material j Capacity storage inequality constraint for site i in time period t for the follower problem

$w_{simt}^{(Tr1)}$ = Dual variable for Capacity transportation inequality constraint from source s to site i by mode m in time period t for the follower problem

$w_{ii'mt}^{(Tr2)}$ = Dual variable for Capacity transportation inequality constraint from site i to site i' by mode m in time period t for the follower problem

$w_{icmt}^{(Tr3)}$ = Dual variable for Capacity transportation inequality constraint from site i to customer c by mode m in time period t for the follower problem

$sl_{cjt}^{(Cu)2}$ = Slack variable for material j Demand inequality constraint for customer c in time period t for the follower problem

$sl_{ijt}^{(Co)2}$ = Slack variable for material j Capacity collection inequality constraint for site i in time period t for the follower problem

$sl_{ipt}^{(Pr)2}$ = Slack variable for process p Capacity processing inequality constraint for site i in time period t for the follower problem

$sl_{ijt}^{(St)2}$ = Slack variable for material j Capacity storage inequality constraint for site i in time period t for the follower problem

$sl^{(\mathrm{Tr1})2}_{\mathrm{simt}}$ = Slack variable for Capacity transportation inequality constraint from source s to site i by mode m in time period t for the follower problem

$sl^{(\mathrm{Tr2})2}_{\mathrm{ii'mt}}$ = Slack variable for Capacity transportation inequality constraint from site i to site i' by mode m in time period t for the follower problem

$sl^{(\mathrm{Tr3})2}_{\mathrm{icmt}}$ = Slack variable for Capacity transportation inequality constraint from site i to customer c by mode m in time period t for the follower problem

$sd^{(\mathrm{St})}_{\mathrm{ijt}}$ = Slack variable for dual problem of the follower corresponding to variable $x^{(\mathrm{St})2}_{\mathrm{ijt}}$

$sd^{(\mathrm{Pr})}_{\mathrm{ipt}}$ = Slack variable for dual problem of the follower corresponding to variable $x^{(\mathrm{Pr})2}_{\mathrm{ipt}}$

$sd^{(\mathrm{Tr1})}_{\mathrm{sjimt}}$ = Slack variable for dual problem of the follower corresponding to variable $x^{(\mathrm{Tr})2}_{\mathrm{sjimt}}$

$sd^{(\mathrm{Tr2})}_{\mathrm{iji'mt}}$ = Slack variable for dual problem of the follower corresponding to variable $x^{(\mathrm{Tr})2}_{\mathrm{iji'mt}}$

$sd^{(\mathrm{Tr3})}_{\mathrm{ijcmt}}$ = Slack variable for dual problem of the follower corresponding to variable $x^{(\mathrm{Tr})2}_{\mathrm{ijcmt}}$

$CY^{(\mathrm{Co})1}_{\mathrm{ijt}} \equiv C^{(\mathrm{Co})}_{\mathrm{ijt}} y^{(\mathrm{Co})1}_{\mathrm{ijt}}$

$CY^{(\mathrm{Pr})1}_{\mathrm{ipt}} \equiv C^{(\mathrm{Pr})}_{ipt} y^{(\mathrm{Pr})1}_{\mathrm{ipt}}$

$CY^{(\mathrm{St})1}_{\mathrm{ijt}} \equiv C^{(St)}_{ijt} y^{(\mathrm{St})1}_{\mathrm{ijt}}$

$CY^{(\mathrm{Tr})1}_{\mathrm{simt}} \equiv C^{(Tr)}_{simt} y^{(\mathrm{Tr})1}_{\mathrm{simt}}$

$CY^{(\mathrm{Tr})1}_{\mathrm{ii'mt}} \equiv C^{(Tr)}_{ii'mt} y^{(\mathrm{Tr})1}_{\mathrm{ii'mt}}$

$CY^{(\mathrm{Tr})1}_{\mathrm{icmt}} \equiv C^{(Tr)}_{icmt} y^{(\mathrm{Tr})1}_{\mathrm{icmt}}$

$PX^{(\mathrm{Sa})\mathrm{k}}_{\mathrm{cjt}} \equiv P^{(Cu)}_{cjt} x^{(\mathrm{Sa})\mathrm{k}}_{\mathrm{cjt}} \quad \forall k = 1,2$

$VX^{(\mathrm{St})\mathrm{k}}_{\mathrm{ijt}} \equiv V^{(St)}_{ijt} x^{(\mathrm{St})\mathrm{k}}_{\mathrm{ijt}} \quad \forall k = 1,2$

$VX^{(\mathrm{Co})\mathrm{k}}_{\mathrm{ijt}} \equiv V^{(Co)}_{ijt} x^{(\mathrm{Co})\mathrm{k}}_{\mathrm{ijt}} \quad \forall k = 1,2$

$VX'^{(\mathrm{Co})\mathrm{k}}_{\mathrm{ijt}} \equiv V'^{(Co)}_{ijt} x^{(\mathrm{Co})\mathrm{k}}_{\mathrm{ijt}} \quad \forall k = 1,2$

$VX^{(\mathrm{Pr})\mathrm{k}}_{\mathrm{ipt}} \equiv V^{(\mathrm{Pr})}_{ipt} x^{(\mathrm{Pr})\mathrm{k}}_{\mathrm{ipt}} \quad \forall k = 1,2$

$VX^{\text{(Tr)k}}_{\text{sjimt}} \quad \equiv V^{(Tr)}_{sjimt} x^{\text{(Tr)k}}_{\text{sjimt}} \quad \forall k = 1,2$

$VX^{\text{(Tr)k}}_{\text{iji'mt}} \quad \equiv V^{(Tr)}_{iji'mt} x^{\text{(Tr)k}}_{\text{iji'mt}} \quad \forall k = 1,2$

$VX^{\text{(Tr)k}}_{\text{ijcmt}} \quad \equiv V^{(Tr)}_{ijcmt} x^{\text{(Tr)k}}_{\text{ijcmt}} \quad \forall k = 1,2$

$bi^{\text{(Cu)}}_{\text{cjt}} \quad = 1$ when $P^{(Cu)}_{cjt} = P^{(Cu)UB}_{cjt}$, 0 when $P^{(Cu)}_{cjt} = P^{(Cu)LB}_{cjt}$

$bi^{\text{(St)}}_{\text{ijt}} \quad = 1$ when $V^{(St)}_{ijt} = V^{(St)UB}_{ijt}$, 0 when $V^{(St)}_{ijt} = V^{(St)LB}_{ijt}$

$bi^{\text{(Co)}}_{\text{ijt}} \quad = 1$ when $V^{(Co)}_{ijt} = V^{(Co)UB}_{ijt}$, 0 when $V^{(Co)}_{ijt} = V^{(Co)LB}_{ijt}$

$bi^{\text{'(Co)}}_{\text{ijt}} \quad = 1$ when $V^{'(Co)}_{ijt} = V^{'(Co)UB}_{ijt}$, 0 when $V^{'(Co)}_{ijt} = V^{'(Co)LB}_{ijt}$

$bi^{\text{(Pr)}}_{\text{ipt}} \quad = 1$ when $V^{(\text{Pr})}_{ipt} = V^{(\text{Pr})UB}_{ipt}$, 0 when $V^{(\text{Pr})}_{ipt} = V^{(\text{Pr})LB}_{ipt}$

$bi^{\text{(Tr)}}_{\text{sjimt}} \quad = 1$ when $V^{(Tr)}_{sjimt} = V^{(Tr)UB}_{sjimt}$, 0 when $V^{(Tr)}_{sjimt} = V^{(Tr)LB}_{sjimt}$

$bi^{\text{(Tr)}}_{\text{iji'mt}} \quad = 1$ when $V^{(Tr)}_{iji'mt} = V^{(Tr)UB}_{iji'mt}$, 0 when $V^{(Tr)}_{iji'mt} = V^{(Tr)LB}_{iji'mt}$

$bi^{\text{(Tr)}}_{\text{ijcmt}} \quad = 1$ when $V^{(Tr)}_{ijcmt} = V^{(Tr)UB}_{ijcmt}$, 0 when $V^{(Tr)}_{ijcmt} = V^{(Tr)LB}_{ijcmt}$

## Table A5 Mathematical Model

*Maximize (Objective)* *Maximize regret*

$$\sum_t \sum_c \sum_j P^{(Cu)*}_{cjt} x^{(Sa)1}_{cjt} + \sum_t \sum_c \sum_j P^{(Cu)Ind}_{cjt} PX^{(Sa)1}_{cjt} - \sum_t \sum_c \sum_j P^{(Cu)*}_{cjt} x^{(Sa)2}_{cjt} - \sum_t \sum_c \sum_j P^{(Cu)Ind}_{cjt} PX^{(Sa)2}_{cjt}$$

**- (**Sales Revenue for leader and follower)

$$-\sum_{\text{t}} \sum_{\text{j}} \sum_{i} (F^{(Co)*}_{ijt} y^{(Co)1}_{ijt} + F^{(St)*}_{ijt} y^{(St)1}_{ijt}) + \sum_{\text{t}} \sum_{\text{j}} \sum_{i} (F^{(Co)*}_{ijt} y^{(Co)\Omega}_{ijt} + F^{(St)*}_{ijt} y^{(St)\Omega}_{ijt})$$

$$-\sum_t \sum_i (F^{(Si)*}_{it} y^{(Si)1}_{it} + F'^{(Si)*}_{it} y'^{(Si)1}_{it} + F''^{(Si)*}_{it} y''^{(Si)1}_{it}) + \sum_t \sum_i (F^{(Si)*}_{it} y^{(Si)\Omega}_{it} + F'^{(Si)*}_{it} y'^{(Si)\Omega}_{it} + F''^{(Si)*}_{it} y''^{(Si)\Omega}_{it})$$

$$-\sum_{t}\sum_{p}\sum_{i} F_{ipt}^{(Pr)\,*}\, y_{ipt}^{(Pr)\,1}$$

$$+\sum_{t}\sum_{p}\sum_{i} F_{ipt}^{(Pr)\,*}\, y_{ipt}^{(Pr)\,\Omega}$$

$$-\sum_{t}\sum_{m}\sum_{s}\sum_{i} F_{simt}^{(Tr)*} y_{simt}^{(Tr)1} - \sum_{t}\sum_{m}\sum_{i}\sum_{i'\neq i} F_{ii'mt}^{(Tr)*} y_{ii'mt}^{(Tr)1}$$

$$+\sum_{t}\sum_{m}\sum_{s}\sum_{i} F_{simt}^{(Tr)*} y_{simt}^{(Tr)\Omega} + \sum_{t}\sum_{m}\sum_{i}\sum_{i'\neq i} F_{ii'mt}^{(Tr)*} y_{ii'mt}^{(Tr)\Omega}$$

$$-\sum_{t}\sum_{m}\sum_{i}\sum_{c} F_{icmt}^{(Tr)*} y_{icmt}^{(Tr)1} + \sum_{t}\sum_{m}\sum_{i}\sum_{c} F_{icmt}^{(Tr)*} y_{icmt}^{(Tr)\Omega}$$ - (Fixed Costs for leader and follower)

$$-\sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(St)*} x_{ijt}^{(St)1} - \sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(St)Ind} VX_{ijt}^{(St)1}$$ - (Storage Costs for leader and follower)

$$+\sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(St)*} x_{ijt}^{(St)2} + \sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(St)Ind} VX_{ijt}^{(St)2}$$

$$-\sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(Co)*} x_{ijt}^{(Co)1} + \sum_{t}\sum_{j}\sum_{i} V'^{(Co)*}_{ijt}\, x_{ijt}^{(Co)1}$$ - (Collection Costs and Fees

$$-\sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(Co)Ind} VX_{ijt}^{(Co)1} + \sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(Co)Ind} VX'^{(Co)1}_{ijt}$$ for leader and follower)

$$+\sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(Co)*} x_{ijt}^{(Co)2} - \sum_{t}\sum_{j}\sum_{i} V'^{(Co)*}_{ijt}\, x_{ijt}^{(Co)2}$$

$$+\sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(Co)Ind} VX_{ijt}^{(Co)2} - \sum_{t}\sum_{j}\sum_{i} V_{ijt}^{(Co)Ind} VX'^{(Co)2}_{ijt}$$

$$-\sum_{t}\sum_{p}\sum_{i} V_{ipt}^{(\mathrm{Pr})*} x_{ipt}^{(\mathrm{Pr})1} - \sum_{t}\sum_{p}\sum_{i} V_{ipt}^{(\mathrm{Pr})\ Ind} VX_{ipt}^{(\mathrm{Pr})1}$$ - (Processing Costs for leader

$$+\sum_{t}\sum_{p}\sum_{i} V_{ipt}^{(\mathrm{Pr})*} x_{ipt}^{(\mathrm{Pr})\,2} + \sum_{t}\sum_{p}\sum_{i} V_{ipt}^{(\mathrm{Pr})\ Ind} VX_{ipt}^{(\mathrm{Pr})\,2}$$ and follower)

$$-\sum_{t}\sum_{m}\sum_{i}\sum_{j}\sum_{s} V_{simt}^{(Tr)*} x_{sjimt}^{(Tr)1} d_{sim}$$

$$-\sum_{t}\sum_{m}\sum_{i}\sum_{j}\sum_{s} V_{simt}^{(Tr)Ind} VX_{sjimt}^{(Tr)1} d_{sim}$$

$$+\sum_{t}\sum_{m}\sum_{i}\sum_{j}\sum_{s} V_{simt}^{(Tr)*} x_{sjimt}^{(Tr)2} d_{sim}$$

$$+\sum_{t}\sum_{m}\sum_{i}\sum_{j}\sum_{s} V_{simt}^{(Tr)Ind} VX_{sjimt}^{(Tr)2} d_{sim}$$

$$-\sum_t\sum_m\sum_i\sum_j\sum_{i'\neq i}V^{(Tr)*}_{ii'mt}x^{(Tr)1}_{iji'mt}d_{ii'm}$$
$$-\sum_t\sum_m\sum_i\sum_j\sum_{i'\neq i}V^{(Tr)Ind}_{ii'mt}VX^{(Tr)1}_{iji'mt}d_{ii'm}$$
$$+\sum_t\sum_m\sum_i\sum_j\sum_{i'\neq i}V^{(Tr)*}_{ii'mt}x^{(Tr)2}_{iji'mt}d_{ii'm}$$
$$+\sum_t\sum_m\sum_i\sum_j\sum_{i'\neq i}V^{(Tr)Ind}_{ii'mt}VX^{(Tr)2}_{iji'mt}d_{ii'm}$$

$$-\sum_t\sum_m\sum_i\sum_j\sum_c V^{(Tr)*}_{icmt}x^{(Tr)1}_{ijcmt}d_{icm}$$
- (Shipping Costs for leader and follower)
$$-\sum_t\sum_m\sum_i\sum_j\sum_c V^{(Tr)Ind}_{icmt}VX^{(Tr)1}_{ijcmt}d_{icm}$$
$$+\sum_t\sum_m\sum_i\sum_j\sum_c V^{(Tr)*}_{icmt}x^{(Tr)2}_{ijcmt}d_{icm}$$
$$+\sum_t\sum_m\sum_i\sum_j\sum_c V^{(Tr)Ind}_{icmt}VX^{(Tr)2}_{ijcmt}d_{icm}$$

**Subject to:**

$$x^{(St)1}_{ijt} = x^{(St)1}_{ij(t-1)} + \sum_s\sum_m x^{(Tr)1}_{sjimt} + \sum_{i'\neq i}\sum_m x^{(Tr)1}_{i'jimt}$$
$$-\sum_{i'\neq i}\sum_m x^{(Tr)1}_{iji'mt} - \sum_c\sum_m x^{(Tr)1}_{ijcmt} + \sum_p \rho'_{jp}\, x^{(\mathrm{Pr})1}_{ipt}$$
$$-\sum_p \rho_{jp} x^{(\mathrm{Pr})1}_{ipt} \qquad \forall i,j,t \quad \text{(Leader)}$$

$$S^{(Su)}_{sjt} = \sum_i\sum_m x^{(Tr)1}_{sjimt} \qquad \forall s,j,t \quad \text{(Leader)}$$

$$D^{(Cu)}_{cjt} \geq \sum_i\sum_m x^{(Tr)1}_{ijcmt} \qquad \forall c,j,t \quad \text{(Leader)}$$

$$x^{(Sa)1}_{cjt} = \sum_i\sum_m x^{(Tr)1}_{ijcmt} \qquad \forall c,j,t \quad \text{(Leader)}$$

$$x^{(Co)1}_{ijt} = \sum_s\sum_m x^{(Tr)1}_{sjimt} \qquad \forall i,j,t \quad \text{(Leader)}$$

$$\mathrm{y}_{\mathrm{ijt}}^{(\mathrm{Co})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, i,j,t$$
$$\mathrm{y}_{\mathrm{ipt}}^{(\mathrm{Pr})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, i,p,t$$
$$\mathrm{y}_{\mathrm{ijt}}^{(\mathrm{St})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, i,j,t$$
$$\mathrm{y}_{\mathrm{sjimt}}^{(\mathrm{Tr})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, s,i,j,m,t$$
$$\mathrm{y}_{\mathrm{iji'mt}}^{(\mathrm{Tr})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, i,i',j,m,t \qquad \text{(Leader)}$$
$$\mathrm{y}_{\mathrm{i'jimt}}^{(\mathrm{Tr})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, i,i',j,m,t$$
$$\mathrm{y}_{\mathrm{ijcmt}}^{(\mathrm{Tr})1} \leq y_{\mathrm{it}}^{(\mathrm{Si})1} \qquad \forall\, i,c,j,m,t$$
$$\mathrm{y}_{\mathrm{it}}^{(\mathrm{Si})1} - y_{i(t-1)}^{(Si)1} \leq y'^{(Si)1}_{it} \qquad \forall\, i,t$$
$$\mathrm{y}_{\mathrm{i(t-1)}}^{(\mathrm{Si})1} - y_{it}^{(Si)1} \leq y''^{(Si)1}_{it} \qquad \forall\, i,t$$

$$x_{ijt}^{(Co)1} \leq C_{ijt}^{(Co)*} y_{ijt}^{(Co)1} + C_{ijt}^{(Co)Ind} CY_{ijt}^{(Co)1} \qquad \forall\, i,j,t$$
$$x_{ipt}^{(\mathrm{Pr})1} \leq C_{ipt}^{(\mathrm{Pr})*} y_{ipt}^{(\mathrm{Pr})1} + C_{ipt}^{(\mathrm{Pr})Ind} CY_{ipt}^{(\mathrm{Pr})1} \qquad \forall\, i,p,t$$
$$x_{ijt}^{(St)1} \leq C_{ijt}^{(St)*} y_{ijt}^{(St)1} + C_{ijt}^{(St)Ind} CY_{ijt}^{(St)1} \qquad \forall\, i,j,t$$
$$\sum_j x_{sjimt}^{(Tr)1} \leq C_{simt}^{(Tr)*} y_{simt}^{(Tr)1} + C_{simt}^{(Tr)Ind} CY_{simt}^{(Tr)1} \qquad \forall\, s,i,j,m,t$$
$$\sum_j x_{iji'mt}^{(Tr)1} \leq C_{ii'mt}^{(Tr)*} y_{ii'mt}^{(Tr)1} + C_{ii'mt}^{(Tr)Ind} CY_{ii'mt}^{(Tr)1} \qquad \forall\, i,i',j,m,t \qquad \text{(Leader)}$$
$$\sum_j x_{ijcmt}^{(Tr)1} \leq C_{icmt}^{(Tr)*} y_{icmt}^{(Tr)1} + C_{icmt}^{(Tr)Ind} CY_{icmt}^{(Tr)1} \qquad \forall\, i,c,j,m,t$$

$$\begin{aligned} x_{ijt}^{(St)2} &= x_{ij(t-1)}^{(St)2} + \sum_s \sum_m x_{sjimt}^{(Tr)2} + \sum_{i' \neq i} \sum_m x_{i'jimt}^{(Tr)2} \\ &- \sum_{i' \neq i} \sum_m x_{iji'mt}^{(Tr)2} - \sum_c \sum_m x_{ijcmt}^{(Tr)2} + \sum_p \rho'_{jp}\, x_{ipt}^{(\mathrm{Pr})2} \\ &- \sum_p \rho_{jp} x_{ipt}^{(\mathrm{Pr})2} \qquad \forall i,j,t \qquad \text{(Leader)} \end{aligned}$$

$$\sum_i \sum_m x_{sjimt}^{(Tr)2} = S_{sjt}^{(Su)} \qquad \forall s,j,t \qquad \text{(Leader)}$$

$$\sum_i \sum_m x_{ijcmt}^{(Tr)2} + sl_{cjt}^{(Cu)2} = D_{cjt}^{(Cu)} \qquad \forall c,j,t \qquad \text{(Leader)}$$

$$x_{cjt}^{(Sa)2} = \sum_i \sum_m x_{ijcmt}^{(Tr)2} \qquad \forall c,j,t \qquad \text{(Leader)}$$

$$x_{ijt}^{(Co)2} = \sum_s \sum_m x_{sjimt}^{(Tr)2} \qquad \forall i,j,t \qquad \text{(Leader)}$$

$$x_{ijt}^{(Co)2} + sl_{ijt}^{(Co)2} = C_{ijt}^{(Co)*} y_{ijt}^{(Co)\Omega} + C_{ijt}^{(Co)Ind} C_{ijt}^{(Co)} y_{ijt}^{(Co)\Omega} \quad \forall i,j,t$$

$$x_{ipt}^{(\mathrm{Pr})2} + sl_{ipt}^{(\mathrm{Pr})2} = C_{ipt}^{(\mathrm{Pr})*} y_{ipt}^{(\mathrm{Pr})\Omega} + C_{ipt}^{(\mathrm{Pr})Ind} C_{ipt}^{(\mathrm{Pr})} y_{ipt}^{(\mathrm{Pr})\Omega} \quad \forall i,p,t$$

$$x_{ijt}^{(St)2} + sl_{ijt}^{(St)2} = C_{ijt}^{(St)*} y_{ijt}^{(St)\Omega} + C_{ijt}^{(St)Ind} C_{ijt}^{(St)} y_{ijt}^{(St)\Omega} \quad \forall i,j,t$$

$$\sum_j x_{sjimt}^{(Tr)2} + sl_{simt}^{(Tr1)2} = C_{simt}^{(Tr)*} y_{simt}^{(Tr)\Omega} + C_{simt}^{(Tr)Ind} C_{simt}^{(Tr)} y_{simt}^{(Tr)\Omega} \quad \forall s,i,j,m,t \quad \text{(Follower)}$$

$$\sum_j x_{iji'mt}^{(Tr)2} + sl_{ii'mt}^{(Tr2)2} = C_{ii'mt}^{(Tr)*} y_{ii'mt}^{(Tr)\Omega} + C_{ii'mt}^{(Tr)Ind} C_{ii'mt}^{(Tr)} y_{ii'mt}^{(Tr)\Omega} \quad \forall i,i',j,m,t$$

$$\sum_j x_{ijcmt}^{(Tr)2} + sl_{icmt}^{(Tr3)2} = C_{icmt}^{(Tr)*} y_{icmt}^{(Tr)\Omega} + C_{icmt}^{(Tr)Ind} C_{icmt}^{(Tr)} y_{icmt}^{(Tr)\Omega} \quad \forall i,c,j,m,t$$

$$-w_{ijt}^{(Balance)} + w_{cjt}^{(Demand)} + w_{icmt}^{(Tr3)} - sd_{ijcmt}^{(Tr3)} = -V_{icmt}^{(Tr)} d_{icm} + P_{cjt}^{(Cu)} \quad \forall i,j,c,m,t \quad \text{(Follower)}$$

$$w_{ijt}^{(Balance)} + w_{sjt}^{(Supply)} + w_{ijt}^{(Collect)} + w_{simt}^{(Tr1)} - sd_{sjimt}^{(Tr1)} = V_{ijt}^{'(Co)} - V_{ijt}^{(Co)} - V_{simt}^{(Tr)} d_{sim} \quad \forall s,j,i,m,t \quad \text{(Follower)}$$

$$-w_{ijt}^{(Balance)} + w_{i'jt}^{(Balance)} + w_{ii'mt}^{(Tr2)} - sd_{iji'mt}^{(Tr2)} = -V_{ii'mt}^{(Tr)} d_{ii'm} \quad \forall i,j,i',m,t \quad \text{(Follower)}$$

$$\sum_p (\rho_{jp}^{'} - \rho_{jp}) w_{ijt}^{(Balance)} + w_{ipt}^{(\mathrm{Pr}ocess)} - sd_{ipt}^{(\mathrm{Pr})} = -V_{ipt}^{(\mathrm{Pr})} \quad \forall i,p,t \quad \text{(Follower)}$$

$$w_{ijt+1}^{(Balance)} - w_{ijt}^{(Balance)} + w_{ijt}^{(Storage)} - sd_{ijt}^{(St)} = -V_{ijt}^{(St)} \quad \forall i,j,t \quad \text{(Follower)}$$

$$x_{ijt}^{(Co)l}, x_{ijt}^{(St)l}, x_{cjt}^{(Sa)l}, x_{sjimt}^{(Tr)l}, x_{iji'mt}^{(Tr)l}, x_{ijcmt}^{(Tr)l}, x_{ipt}^{(\mathrm{Pr})l} \geq 0 \quad \forall s,i,c,j,m,p,t,i' \neq i$$

$$sd_{ijt}^{(St)}, sd_{sjimt}^{(Tr1)}, sd_{iji'mt}^{(Tr2)}, sd_{ijcmt}^{(Tr3)}, sd_{ipt}^{(\mathrm{Pr})} \geq 0 \quad l \in \{1,2\}$$

$$w_{cjt}^{(Demand)}, w_{ijt}^{(Collect)}, w_{ipt}^{(\mathrm{Pr}ocess)}, w_{ijt}^{(Storage)}, w_{simt}^{(Trl)}, w_{ii'mt}^{(Tr2)}, w_{icmt}^{(Tr3)} \geq 0$$

$$sl_{cjt}^{(Cu)2}, sl_{ijt}^{(Co)2}, sl_{ipt}^{(\mathrm{Pr})2}, sl_{ijt}^{(St)2}, sl_{simt}^{(Trl)2}, sl_{ii'mt}^{(Tr2)2}, sl_{icmt}^{(Tr3)2} \geq 0$$

$$y_{ijt}^{(Co)1}, y_{ijt}^{(St)1}, y_{simt}^{(Tr)1}, y_{ii'mt}^{(Tr)1}, y_{icmt}^{(Tr)1}, y_{ipt}^{(Pr)1}, \quad \forall s,i,c,j,m,p,t,i' \neq i$$

$$y_{it}^{(Si)1}, y'^{(Si)1}_{it}, y''^{(\mathrm{Si})1}_{\mathrm{it}} \in \{0,1\}$$

$$w_{cjt}^{(Demand)} sl_{cjt}^{(Cu)2} = 0 \quad \forall c,j,t$$

$$w_{icmt}^{(Tr3)} sl_{icmt}^{(Tr3)2} = 0 \qquad \forall i, c, m, t$$

$$sd_{ijcmt}^{(Tr3)} x_{ijcmt}^{(Tr3)2} = 0 \qquad \forall i, j, c, m, t$$

$$w_{ijt}^{(Collect)} sl_{ijt}^{(Co)2} = 0 \qquad \forall i, j, t$$

$$w_{simt}^{(Tr1)} sl_{simt}^{(Tr1)2} = 0 \qquad \forall s, i, m, t$$

$$sd_{sjimt}^{(Tr1)} x_{sjimt}^{(Tr1)2} = 0 \qquad \forall s, j, i, m, t$$

$$w_{ii'mt}^{(Tr2)} sl_{ii'mt}^{(Tr2)2} = 0 \qquad \forall i, i', c, m, t$$

$$sd_{iji'mt}^{(Tr2)} x_{iji'mt}^{(Tr2)2} = 0 \qquad \forall i, j, i', m, t$$

$$w_{ipt}^{(\Pr ocess)} sl_{ipt}^{(\Pr)2} = 0 \qquad \forall i, p, t$$

$$sd_{ipt}^{(\Pr)} x_{ipt}^{(\Pr)2} = 0 \qquad \forall i, p, t$$

$$w_{ijt}^{(Storage)} sl_{ijt}^{(St)2} = 0 \qquad \forall i, j, t$$

$$sd_{ijt}^{(St)} x_{ijt}^{(St)2} = 0 \qquad \forall i, j, t$$

$$\mathrm{S}_{\mathrm{sjt}}^{(\mathrm{Su})\,\mathrm{LB}} \le \mathrm{S}_{\mathrm{sjt}}^{(\mathrm{Su})} \le \mathrm{S}_{\mathrm{sjt}}^{(\mathrm{Su})\,\mathrm{UB}} \qquad \forall s, j, t$$

$$\mathrm{D}_{\mathrm{cjt}}^{(\mathrm{Cu})\,\mathrm{LB}} \le \mathrm{D}_{\mathrm{cjt}}^{(\mathrm{Cu})} \le \mathrm{D}_{\mathrm{cjt}}^{(\mathrm{Cu})\,\mathrm{UB}} \qquad \forall c, j, t$$

$$\mathrm{P}_{\mathrm{cjt}}^{(\mathrm{Cu})\,\mathrm{LB}} \le \mathrm{P}_{\mathrm{cjt}}^{(\mathrm{Cu})} \le \mathrm{P}_{\mathrm{cjt}}^{(\mathrm{Cu})\,\mathrm{UB}} \qquad \forall c, j, t$$

$$\mathrm{V}_{\mathrm{ijt}}^{(\mathrm{St})\,\mathrm{LB}} \le \mathrm{V}_{\mathrm{ijt}}^{(\mathrm{St})} \le \mathrm{V}_{\mathrm{ijt}}^{(\mathrm{St})\,\mathrm{UB}} \qquad \forall i, j, t$$

$$\mathrm{V}_{\mathrm{ijt}}^{(\mathrm{Co})\,\mathrm{LB}} \le \mathrm{V}_{\mathrm{ijt}}^{(\mathrm{Co})} \le \mathrm{V}_{\mathrm{ijt}}^{(\mathrm{Co})\,\mathrm{UB}} \qquad \forall i, j, t$$

$$\mathrm{V'}_{\mathrm{ijt}}^{(\mathrm{Co})\,\mathrm{LB}} \le \mathrm{V'}_{\mathrm{ijt}}^{(\mathrm{Co})} \le \mathrm{V'}_{\mathrm{ijt}}^{(\mathrm{Co})\,\mathrm{UB}} \qquad \forall i, j, t$$

$$\mathrm{V}_{\mathrm{ipt}}^{(\mathrm{Pr})\,\mathrm{LB}} \le \mathrm{V}_{\mathrm{ipt}}^{(\mathrm{Pr})} \le \mathrm{V}_{\mathrm{ipt}}^{(\mathrm{Pr})\,\mathrm{UB}} \qquad \forall i, p, t$$

$$\mathrm{V}_{\mathrm{simt}}^{(\mathrm{Tr})\,\mathrm{LB}} \le \mathrm{V}_{\mathrm{simt}}^{(\mathrm{Tr})} \le \mathrm{V}_{\mathrm{simt}}^{(\mathrm{Tr})\,\mathrm{UB}} \qquad \forall s, i, m, t$$

$$\mathrm{V}_{\mathrm{ii'mt}}^{(\mathrm{Tr})\,\mathrm{LB}} \le \mathrm{V}_{\mathrm{ii'mt}}^{(\mathrm{Tr})} \le \mathrm{V}_{\mathrm{ii'mt}}^{(\mathrm{Tr})\,\mathrm{UB}} \qquad \forall i, i', m, t$$

$$\mathrm{V}_{\mathrm{icmt}}^{(\mathrm{Tr})\,\mathrm{LB}} \le \mathrm{V}_{\mathrm{icmt}}^{(\mathrm{Tr})} \le \mathrm{V}_{\mathrm{icmt}}^{(\mathrm{Tr})\,\mathrm{UB}} \qquad \forall i,c,m,t$$

$$\mathrm{C}_{\mathrm{ijt}}^{(\mathrm{Co})\,\mathrm{LB}} \le \mathrm{C}_{\mathrm{ijt}}^{(\mathrm{Co})} \le \mathrm{C}_{\mathrm{ijt}}^{(\mathrm{Co})\,\mathrm{UB}} \qquad \forall i,j,t$$

$$\mathrm{C}_{\mathrm{ijt}}^{(\mathrm{St})\,\mathrm{LB}} \le \mathrm{C}_{\mathrm{ijt}}^{(\mathrm{St})} \le \mathrm{C}_{\mathrm{ijt}}^{(\mathrm{St})\,\mathrm{UB}} \qquad \forall i,j,t$$

$$\mathrm{C}_{\mathrm{simt}}^{(\mathrm{Tr})\,\mathrm{LB}} \le \mathrm{C}_{\mathrm{simt}}^{(\mathrm{Tr})} \le \mathrm{C}_{\mathrm{simt}}^{(\mathrm{Tr})\,\mathrm{UB}} \qquad \forall s,i,m,t$$

$$\mathrm{C}_{\mathrm{ii'mt}}^{(\mathrm{Tr})\,\mathrm{LB}} \le \mathrm{C}_{\mathrm{ii'mt}}^{(\mathrm{Tr})} \le \mathrm{C}_{\mathrm{ii'mt}}^{(\mathrm{Tr})\,\mathrm{UB}} \qquad \forall i,i',m,t$$

$$\mathrm{C}_{\mathrm{icmt}}^{(\mathrm{Tr})\,\mathrm{LB}} \le \mathrm{C}_{\mathrm{icmt}}^{(\mathrm{Tr})} \le \mathrm{C}_{\mathrm{icmt}}^{(\mathrm{Tr})\,\mathrm{UB}} \qquad \forall i,c,m,t$$

$$\mathrm{C}_{\mathrm{ipt}}^{(\mathrm{Pr})\,\mathrm{LB}} \le \mathrm{C}_{\mathrm{ipt}}^{(\mathrm{Pr})} \le \mathrm{C}_{\mathrm{ipt}}^{(\mathrm{Pr})\,\mathrm{UB}} \qquad \forall i,p,t$$

$$\left.\begin{pmatrix} CY_{ijt}^{(Co)1} - C_{ijt}^{(Co)} - |\min(0, C_{ijt}^{(Co)\,\mathrm{LB}})|\,(1 - y_{ijt}^{(Co)1}) \le 0 \\ -CY_{ijt}^{(Co)1} + C_{ijt}^{(Co)} - C_{ijt}^{(Co)\,\mathrm{UB}}(1 - y_{ijt}^{(Co)1}) \le 0 \\ CY_{ijt}^{(Co)1} \le C_{ijt}^{(Co)\,\mathrm{UB}} y_{ijt}^{(Co)1} \\ C_{ijt}^{(Co)} \le C_{ijt}^{(Co)\,\mathrm{LB}} + y_{ijt}^{(Co)1}(C_{ijt}^{(Co)\,\mathrm{UB}} - C_{ijt}^{(Co)\,\mathrm{LB}}) \end{pmatrix} C_{ijt}^{(Co)Ind} \right\} \forall i,j,t$$

$$\left.\begin{pmatrix} CY_{ipt}^{(\Pr)1} - C_{ipt}^{(\Pr)} - |\min(0, C_{ipt}^{(\Pr)\,\mathrm{LB}})|\,(1 - y_{ipt}^{(\Pr)1}) \le 0 \\ -CY_{ipt}^{(\Pr)1} + C_{ipt}^{(\Pr)} - C_{ipt}^{(\Pr)\,\mathrm{UB}}(1 - y_{ipt}^{(\Pr)1}) \le 0 \\ CY_{ipt}^{(\Pr)1} \le C_{ipt}^{(\Pr)\,\mathrm{UB}} y_{ipt}^{(\Pr)1} \\ C_{ipt}^{(\Pr)} \le C_{ipt}^{(\Pr)\,\mathrm{LB}} + y_{ipt}^{(\Pr)1}(C_{ipt}^{(\Pr)\,\mathrm{UB}} - C_{ipt}^{(\Pr)\,\mathrm{LB}}) \end{pmatrix} C_{ipt}^{(\Pr)Ind} \right\} \forall i,p,t$$

$$\left.\begin{pmatrix} CY_{ijt}^{(St)1} - C_{ijt}^{(St)} - |\min(0, C_{ijt}^{(St)\,\mathrm{LB}})|\,(1 - y_{ijt}^{(St)1}) \le 0 \\ -CY_{ijt}^{(St)1} + C_{ijt}^{(St)} - C_{ijt}^{(St)\,\mathrm{UB}}(1 - y_{ijt}^{(St)1}) \le 0 \\ CY_{ijt}^{(St)1} \le C_{ijt}^{(St)\,\mathrm{UB}} y_{ijt}^{(St)1} \\ C_{ijt}^{(St)} \le C_{ijt}^{(St)\,\mathrm{LB}} + y_{ijt}^{(St)1}(C_{ijt}^{(St)\,\mathrm{UB}} - C_{ijt}^{(St)\,\mathrm{LB}}) \end{pmatrix} C_{ijt}^{(St)Ind} \right\} \forall i,j,t$$

$$\left.\begin{pmatrix} CY_{simt}^{(Tr)1} - C_{simt}^{(Tr)} - |\min(0, C_{simt}^{(Tr)\,\mathrm{LB}})|\,(1 - y_{simt}^{(Tr)1}) \le 0 \\ -CY_{simt}^{(Tr)1} + C_{simt}^{(Tr)} - C_{simt}^{(Tr)\,\mathrm{UB}}(1 - y_{simt}^{(Tr)1}) \le 0 \\ CY_{simt}^{(Tr)1} \le C_{simt}^{(Tr)\,\mathrm{UB}} y_{simt}^{(Tr)1} \\ C_{simt}^{(Tr)} \le C_{simt}^{(Tr)\,\mathrm{LB}} + y_{simt}^{(Tr)1}(C_{simt}^{(Tr)\,\mathrm{UB}} - C_{simt}^{(Tr)\,\mathrm{LB}}) \end{pmatrix} C_{simt}^{(Tr)Ind} \right\} \forall s,i,m,t$$

$$\left.\begin{pmatrix} CY_{ii'mt}^{(Tr)1} - C_{ii'mt}^{(Tr)} - |\min(0, C_{ii'mt}^{(Tr)\,\mathrm{LB}})|\,(1 - y_{ii'mt}^{(Tr)1}) \le 0 \\ -CY_{ii'mt}^{(Tr)1} + C_{ii'mt}^{(Tr)} - C_{ii'mt}^{(Tr)\,\mathrm{UB}}(1 - y_{ii'mt}^{(Tr)1}) \le 0 \\ CY_{ii'mt}^{(Tr)1} \le C_{ii'mt}^{(Tr)\,\mathrm{UB}} y_{ii'mt}^{(Tr)1} \\ C_{ii'mt}^{(Tr)} \le C_{ii'mt}^{(Tr)\,\mathrm{LB}} + y_{ii'mt}^{(Tr)1}(C_{ii'mt}^{(Tr)\,\mathrm{UB}} - C_{ii'mt}^{(Tr)\,\mathrm{LB}}) \end{pmatrix} C_{ii'mt}^{(Tr)Ind} \right\} \forall i,i',m,t$$

$$\left.\left(\begin{array}{l} CY_{icmt}^{(Tr)1} - C_{icmt}^{(Tr)} - |\min(0, C_{icmt}^{(Tr)\,\mathrm{LB}})|(1 - y_{icmt}^{(Tr)1}) \le 0 \\ -CY_{icmt}^{(Tr)1} + C_{icmt}^{(Tr)} - C_{icmt}^{(Tr)\,\mathrm{UB}}(1 - y_{icmt}^{(Tr)1}) \le 0 \\ CY_{icmt}^{(Tr)1} \le C_{icmt}^{(Tr)\,\mathrm{UB}} y_{icmt}^{(Tr)1} \\ C_{icmt}^{(Tr)} \le C_{icmt}^{(Tr)\,\mathrm{LB}} + y_{icmt}^{(Tr)1}(C_{icmt}^{(Tr)\,\mathrm{UB}} - C_{icmt}^{(Tr)\,\mathrm{LB}}) \end{array}\right) C_{icmt}^{(Tr)Ind} \quad \right\} \forall i, c, m, t$$

$$\left.\left(\begin{array}{l} PX_{cjt}^{(Sa)k} - P_{cjt}^{(Cu)UB} x_{cjt}^{(Sa)k} \le 0 \\ -PX_{cjt}^{(Sa)k} + P_{cjt}^{(Cu)UB} x_{cjt}^{(Sa)k} - (P_{cjt}^{(Cu)UB} x_{cjt}^{(Sa)kUB} + |\min(0, P_{cjt}^{(Cu)LB})|\, x_{cjt}^{(Sa)kUB})(1 - bi_{cjt}^{(Cu)}) \le 0 \\ PX_{cjt}^{(Sa)k} - P_{cjt}^{(Cu)LB} x_{cjt}^{(Sa)k} - (P_{cjt}^{(Cu)UB} x_{cjt}^{(Sa)kUB} + |\min(0, P_{cjt}^{(Cu)LB})|\, x_{cjt}^{(Sa)kUB})(bi_{cjt}^{(Cu)}) \le 0 \\ -PX_{cjt}^{(Sa)k} + P_{cjt}^{(Cu)LB} x_{cjt}^{(Sa)k} \le 0 \end{array}\right) P_{cjt}^{(Cu)Ind} \quad \right\} \forall c, j, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -P_{cjt}^{(Cu)} + P_{cjt}^{(Cu)UB} - (P_{cjt}^{(Cu)UB} - P_{cjt}^{(Cu)LB})(1 - bi_{cjt}^{(Cu)}) \le 0 \\ P_{cjt}^{(Cu)} - P_{cjt}^{(Cu)LB} - (P_{cjt}^{(Cu)UB} - P_{cjt}^{(Cu)LB})(bi_{cjt}^{(Cu)}) \le 0 \end{array}\right) P_{cjt}^{(Cu)In} \quad \right\} \forall c, j, t$$

$$\left.\left(\begin{array}{l} VX_{ijt}^{(St)k} - V_{ijt}^{(St)UB} x_{ijt}^{(St)k} \le 0 \\ -VX_{ijt}^{(St)k} + V_{ijt}^{(St)UB} x_{ijt}^{(St)k} - (V_{ijt}^{(St)UB} x_{ijt}^{(St)kUB} + |\min(0, V_{ijt}^{(St)LB})|\, x_{ijt}^{(St)kUB})(1 - bi_{ijt}^{(St)}) \le 0 \\ VX_{ijt}^{(St)k} - V_{ijt}^{(St)LB} x_{ijt}^{(St)k} - (V_{ijt}^{(St)UB} x_{ijt}^{(St)kUB} + |\min(0, V_{ijt}^{(St)LB})|\, x_{ijt}^{(St)kUB})(bi_{ijt}^{(St)}) \le 0 \\ -VX_{ijt}^{(St)k} + V_{ijt}^{(St)LB} x_{ijt}^{(St)k} \le 0 \end{array}\right) V_{ijt}^{(St)Ind} \quad \right\} \forall i, j, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V_{ijt}^{(St)} + V_{ijt}^{(St)UB} - (V_{ijt}^{(St)UB} - V_{ijt}^{(St)LB})(1 - bi_{ijt}^{(St)}) \le 0 \\ V_{ijt}^{(St)} - V_{ijt}^{(St)LB} - (V_{ijt}^{(St)UB} - V_{ijt}^{(St)LB})(bi_{ijt}^{(St)}) \le 0 \end{array}\right) V_{ijt}^{(St)Ind} \quad \right\} \forall i, j, t$$

$$\left.\left(\begin{array}{l} VX_{ijt}^{(Co)k} - V_{ijt}^{(Co)UB} x_{ijt}^{(Co)k} \le 0 \\ -VX_{ijt}^{(Co)k} + V_{ijt}^{(Co)UB} x_{ijt}^{(Co)k} - (V_{ijt}^{(Co)UB} x_{ijt}^{(Co)kUB} + |\min(0, V_{ijt}^{(Co)LB})|\, x_{ijt}^{(Co)kUB})(1 - bi_{ijt}^{(Co)}) \le 0 \\ VX_{ijt}^{(Co)k} - V_{ijt}^{(Co)LB} x_{ijt}^{(Co)k} - (V_{ijt}^{(Co)UB} x_{ijt}^{(Co)kUB} + |\min(0, V_{ijt}^{(Co)LB})|\, x_{ijt}^{(Co)kUB})(bi_{ijt}^{(Co)}) \le 0 \\ -VX_{ijt}^{(Co)k} + V_{ijt}^{(Co)LB} x_{ijt}^{(Co)k} \le 0 \end{array}\right) V_{ijt}^{(Co)Ind} \quad \right\} \forall i, j, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V_{ijt}^{(Co)} + V_{ijt}^{(Co)UB} - (V_{ijt}^{(Co)UB} - V_{ijt}^{(Co)LB})(1 - bi_{ijt}^{(Co)}) \le 0 \\ V_{ijt}^{(Co)} - V_{ijt}^{(Co)LB} - (V_{ijt}^{(Co)UB} - V_{ijt}^{(Co)LB})(bi_{ijt}^{(Co)}) \le 0 \end{array}\right) V_{ijt}^{(Co)In} \quad \right\} \forall i, j, t$$

$$\left.\left(\begin{array}{l} VX'^{(Co)k}_{ijt} - V'^{(Co)UB}_{ijt} x_{ijt}^{(Co)k} \le 0 \\ -VX'^{(Co)k}_{ijt} + V'^{(Co)UB}_{ijt} x_{ijt}^{(Co)k} - (V'^{(Co)UB}_{ijt} x_{ijt}^{(Co)kUB} + |\min(0, V'^{(Co)LB}_{ijt})|\, x_{ijt}^{(Co)kUB})(1 - bi_{ijt}^{(Co)}) \le 0 \\ VX'^{(Co)k}_{ijt} - V'^{(Co)LB}_{ijt} x_{ijt}^{(Co)k} - (V'^{(Co)UB}_{ijt} x_{ijt}^{(Co)kUB} + |\min(0, V'^{(Co)LB}_{ijt})|\, x_{ijt}^{(Co)kUB})(bi_{ijt}^{(Co)}) \le 0 \\ -VX'^{(Co)k}_{ijt} + V'^{(Co)LB}_{ijt} x_{ijt}^{(Co)k} \le 0 \end{array}\right) V'^{(Co)Ind}_{ijt} \quad \right\} \forall i, j, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V'^{(Co)}_{ijt} + V'^{(Co)UB}_{ijt} - (V'^{(Co)UB}_{ijt} - V'^{(Co)LB}_{ijt})(1 - bi_{ijt}^{(Co)}) \le 0 \\ V'^{(Co)}_{ijt} - V'^{(Co)LB}_{ijt} - (V'^{(Co)UB}_{ijt} - V'^{(Co)LB}_{ijt})(bi_{ijt}^{(Co)}) \le 0 \end{array}\right) V'^{(Co}_{ijt} \quad \right\} \forall i, j, t$$

$$\left.\begin{array}{l}\left(\begin{array}{l} VX^{(\Pr)k}_{ipt} - V^{(\Pr)UB}_{ipt} x^{(\Pr)k}_{ipt} \le 0 \\ -VX^{(\Pr)k}_{ipt} + V^{(\Pr)UB}_{ipt} x^{(\Pr)k}_{ipt} - (V^{(\Pr)UB}_{ipt} x^{(\Pr)kUB}_{ipt} + |\min(0, V^{(\Pr)LB}_{ipt})|\, x^{(\Pr)kUB}_{ipt})(1 - bi^{(\Pr)}_{ipt}) \le 0 \\ VX^{(\Pr)k}_{ipt} - V^{(\Pr)LB}_{ipt} x^{(\Pr)k}_{ipt} - (V^{(\Pr)UB}_{ipt} x^{(\Pr)kUB}_{ipt} + |\min(0, V^{(\Pr)LB}_{ipt})|\, x^{(\Pr)kUB}_{ipt})(bi^{(\Pr)}_{ipt}) \le 0 \\ -VX^{(\Pr)k}_{ipt} + V^{(\Pr)LB}_{ipt} x^{(\Pr)k}_{ipt} \le 0 \end{array}\right) V^{(\Pr)Ind}_{ipt} \end{array}\right\} \forall i, p, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V^{(\Pr)}_{ipt} + V^{(\Pr)UB}_{ipt} - (V^{(\Pr)UB}_{ipt} - V^{(\Pr)LB}_{ipt})(1 - bi^{(\Pr)}_{ipt}) \le 0 \\ V^{(\Pr)}_{ipt} - V^{(\Pr)LB}_{ipt} - (V^{(\Pr)UB}_{ipt} - V^{(\Pr)LB}_{ipt})(bi^{(\Pr)}_{ipt}) \le 0 \end{array}\right) V^{(\Pr)}_{ipt} \right\} \forall i, p, t$$

$$\left.\begin{array}{l}\left(\begin{array}{l} VX^{(Tr)k}_{sjimt} - V^{(Tr)UB}_{sjimt} x^{(Tr)k}_{sjimt} \le 0 \\ -VX^{(Tr)k}_{sjimt} + V^{(Tr)UB}_{sjimt} x^{(Tr)k}_{sjimt} - (V^{(Tr)UB}_{sjimt} x^{(Tr)kUB}_{sjimt} + |\min(0, V^{(Tr)LB}_{sjimt})|\, x^{(Tr)kUB}_{sjimt})(1 - bi^{(Tr)}_{sjimt}) \le 0 \\ VX^{(Tr)k}_{sjimt} - V^{(Tr)LB}_{sjimt} x^{(Tr)k}_{sjimt} - (V^{(Tr)UB}_{sjimt} x^{(Tr)kUB}_{sjimt} + |\min(0, V^{(Tr)LB}_{sjimt})|\, x^{(Tr)kUB}_{sjimt})(bi^{(Tr)}_{sjimt}) \le 0 \\ -VX^{(Tr)k}_{sjimt} + V^{(Tr)LB}_{sjimt} x^{(Tr)k}_{sjimt} \le 0 \end{array}\right) V^{(Tr)Ind}_{sjimt} \end{array}\right\} \forall s, j, i, m, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V^{(Tr)}_{sjimt} + V^{(Tr)UB}_{sjimt} - (V^{(Tr)UB}_{sjimt} - V^{(Tr)LB}_{sjimt})(1 - bi^{(Tr)}_{sjimt}) \le 0 \\ V^{(Tr)}_{sjimt} - V^{(Tr)LB}_{sjimt} - (V^{(Tr)UB}_{sjimt} - V^{(Tr)LB}_{sjimt})(bi^{(Tr)}_{sjimt}) \le 0 \end{array}\right) V^{(Tr)In}_{sjimt} \right\} \forall s, j, i, m, t$$

$$\left.\begin{array}{l}\left(\begin{array}{l} VX^{(Tr)k}_{iji'mt} - V^{(Tr)UB}_{iji'mt} x^{(Tr)k}_{iji'mt} \le 0 \\ -VX^{(Tr)k}_{iji'mt} + V^{(Tr)UB}_{iji'mt} x^{(Tr)k}_{iji'mt} - (V^{(Tr)UB}_{iji'mt} x^{(Tr)kUB}_{iji'mt} + |\min(0, V^{(Tr)LB}_{iji'mt})|\, x^{(Tr)kUB}_{iji'mt})(1 - bi^{(Tr)}_{iji'mt}) \le 0 \\ VX^{(Tr)k}_{iji'mt} - V^{(Tr)LB}_{iji'mt} x^{(Tr)k}_{iji'mt} - (V^{(Tr)UB}_{iji'mt} x^{(Tr)kUB}_{iji'mt} + |\min(0, V^{(Tr)LB}_{iji'mt})|\, x^{(Tr)kUB}_{iji'mt})(bi^{(Tr)}_{iji'mt}) \le 0 \\ -VX^{(Tr)k}_{iji'mt} + V^{(Tr)LB}_{iji'mt} x^{(Tr)k}_{iji'mt} \le 0 \end{array}\right) V^{(Tr)Ind}_{iji'mt} \end{array}\right\} \forall i, j, i', m, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V^{(Tr)}_{iji'mt} + V^{(Tr)UB}_{iji'mt} - (V^{(Tr)UB}_{iji'mt} - V^{(Tr)LB}_{iji'mt})(1 - bi^{(Tr)}_{iji'mt}) \le 0 \\ V^{(Tr)}_{iji'mt} - V^{(Tr)LB}_{iji'mt} - (V^{(Tr)UB}_{iji'mt} - V^{(Tr)LB}_{iji'mt})(bi^{(Tr)}_{iji'mt}) \le 0 \end{array}\right) V^{(Tr)In}_{iji'mt} \right\} \forall i, j, i', m, t$$

$$\left.\begin{array}{l}\left(\begin{array}{l} VX^{(Tr)k}_{ijcmt} - V^{(Tr)UB}_{ijcmt} x^{(Tr)k}_{ijcmt} \le 0 \\ -VX^{(Tr)k}_{ijcmt} + V^{(Tr)UB}_{ijcmt} x^{(Tr)k}_{ijcmt} - (V^{(Tr)UB}_{ijcmt} x^{(Tr)kUB}_{ijcmt} + |\min(0, V^{(Tr)LB}_{ijcmt})|\, x^{(Tr)kUB}_{ijcmt})(1 - bi^{(Tr)}_{ijcmt}) \le 0 \\ VX^{(Tr)k}_{ijcmt} - V^{(Tr)LB}_{ijcmt} x^{(Tr)k}_{ijcmt} - (V^{(Tr)UB}_{ijcmt} x^{(Tr)kUB}_{ijcmt} + |\min(0, V^{(Tr)LB}_{ijcmt})|\, x^{(Tr)kUB}_{ijcmt})(bi^{(Tr)}_{ijcmt}) \le 0 \\ -VX^{(Tr)k}_{ijcmt} + V^{(Tr)LB}_{ijcmt} x^{(Tr)k}_{ijcmt} \le 0 \end{array}\right) V^{(Tr)Ind}_{ijcmt} \end{array}\right\} \forall i, j, c, m, t, k = \{1,2\}$$

$$\left.\left(\begin{array}{l} -V^{(Tr)}_{ijcmt} + V^{(Tr)UB}_{ijcmt} - (V^{(Tr)UB}_{ijcmt} - V^{(Tr)LB}_{ijcmt})(1 - bi^{(Tr)}_{ijcmt}) \le 0 \\ V^{(Tr)}_{ijcmt} - V^{(Tr)LB}_{ijcmt} - (V^{(Tr)UB}_{ijcmt} - V^{(Tr)LB}_{ijcmt})(bi^{(Tr)}_{ijcmt}) \le 0 \end{array}\right) V^{(Tr)In}_{ijcmt} \right\} \forall i, j, c, m, t$$

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# VITA

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