GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF RESEARCH ADMINISTRATION

Date: 1 April 1970

RESEARCH PROJECT INITIATION

Project Title: Research Initiation - Mathematical Programming of Capital Budgeting Problems

Project No: B-1019 (E-24-605)

Project Director: Dr. Vernon E. Unger

Sponsor: National Science Foundation

Agreement Period: From 1 April 1970 until 30 September 1971

Type Agreement: Grant No. GK-5533

Amount: $15,000 NSF Funds (B-1019)
8,153 GIT Contribution (E-1006)
$23,153 Total Budget

Grant Administrator

Dr. Morris S. Ojalvo
Program Director
Special Programs
Division of Engineering
National Science Foundation
Washington, D.C. 20550

Reports Required

Annual - 1 April 1971; short, informal, letter technical report.

Final - Upon completion of project. See Appendix V to NSF 69-23 for items to be included.

Assigned to: School of Industrial and Systems Engineering

COPIES TO:

- [x] Project Director
- [x] School Director
- [x] Dean of the College
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Other: File B-1019

RA-3 (4/65)

A-344-2

GEORGIA INSTITUTE OF TECHNOLOGY

OFFICE OF RESEARCH ADMINISTRATION

RESEARCH PROJECT TERMINATION

Date: June 2, 1972

Project Title: Research Initiation - Mathematical Programming of Capital Budgeting Problems

Project No: E-24-605 (Old B-1019)

Principal Investigator: Dr. Vernon E. Unger

Sponsor: National Science Foundation

Effective Termination Date: March 31, 1972

Clearance of Accounting Charges: by March 31, 1972

Grant/Contract Closeout Actions Remaining:

Final Fiscal Report by June 30, 1972

Assigned to: School of Industrial and Systems Engineering

COPIES TO:

- Principal Investigator
- School Director
- Dean of the College
- Director, Research Administration
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- Patent and Inventions Coordinator
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- Rich Electronic Computer Center
- Photographic Laboratory
- Project File
- Other ______

RA-4 (6-71)

SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

FINAL REPORT

Research Grant No. GK-5533

Mathematical Programming of Capital Budgeting Problems

INSTITUTION: Georgia Institute of Technology
School of Industrial and Systems Engineering

PRINCIPAL INVESTIGATOR: V. E. Unger, Jr.

STARTING DATE: April 1, 1972

COMPLETION DATE: March 31, 1972

SUMMARY

Under this grant, research was conducted on the application of mathematical programming to investment decision problems. The research concentrated on the development of specialized solution techniques for various capital investment problems and on the economic interpretation of capital budgeting problems. The work in these two areas is summarized below.

Algorithm Development: An algorithm for a mixed zero-one integer programming capital budgeting problem was coded. This algorithm was extended to solve a quadratic mixed zero-one integer capital budgeting problem. Although designed for a particular capital budgeting problem, the solution technique could easily be adapted to solve general quadratic mixed zero-one integer programming problems. Based on this work with capital investment problems, an algorithm and computer code was developed for solving plant location problems.

Economic Interpretation: The duality concepts in integer programming developed by E. Balas were applied to a variety of capital budgeting problems with both linear and quadratic objective functions. Based on these conceptions, a dual solution procedure was developed for solving capital budgeting problems. This procedure permits a degree of decentralization of capital budgeting.procedures.

DISCUSSION OF RESULTS

A significant contribution to the formulation and analysis of capital budgeting problems was made by Weingartner (9) in his development of a mathematical programming formulation of the problem. The use of mathematical programming permits one to handle the complexities resulting from the financial interrelationships between the projects imposed by capital rationing as well as other complexities resulting from items such as indivisibility of the projects, various explicit dependencies between the projects, and risk.

Since Weingartner's important contribution to the formulation of constrained capital budgeting problems, linear and integer programming techniques have been used to develop systematic approaches for solving these problems and in the case of linear programming models, the concept of duality has provided interesting relationships for clarifying and interpreting many aspects of capital budgeting. These successful interpretations, however, have been based on the assumption of divisibility of projects and in the cases where indivisibilities occurs successful duality applications have been lacking.

Balas (1,2,3) has developed a duality theory for integer programming problems with the following desirable properties:

1) the dual of the dual is the primal
2) complimentary slackness conditions hold
3) if an optimal solution to the primal exists, then both a solution to the dual and a global saddle points exists for the saddle point function.

Balas has also shown how to formulate for any linear integer programming problem an equivalent linear problem. Through this equivalent linear programming problem, an economic interpretation of the integer programming problem can be penalties associated with the discrete variables. This duality theory was applied to various capital budgeting models including Weingartner's basic model and his terminal wealth model and both a linear and quadratic version of the Baumol and Quandt (4) model. For these models, it was possible to determine various properties that would exist between borrowing, lending, and dividend policies in any optimal solution. Futhermore, the dual variables could be related to the models internal interest or discount rates.

Based on Benders' partitioning procedure (5) a dual solution method was developed for Weingartner's terminal wealth model. This method permits a degree of decentralization in evaluating the individual investment projects of an organization.

The algorithm of (8) was extended to solve an integer programming version of Baumol and Quandt's model having a quadratic utility function. This procedure consists of an implicit search over the integer variables with the solution of quadratic programming problems in continuous variables at various points throughout the search.

To obtain some insight into the computational efficiency of these algorithms, and their extention to other similarly structured mixed zero-one integer programming problems, the algorithm was modified to solve the capacitated plant location problem. Computational results for plant location problems have been reported in the literature (6,7). The computer code developed was able to solve problems having up to sixteen plants and fifty destinations in under three minutes, the average time being less than one minute. These results are among the best reported and suggest that for "reasonably" sized problems, the basic algorithm is computationally efficient.

This research has demonstrated that computationally efficient algorithms can be developed by exploiting the structure of capital budgeting models and that by applying recent advances in duality theory for discrete programming, meaningful economic interpretations and insights can be obtained for these models. The research on plant location problems has shown that the results obtained for capital budgeting models may be readily extended to similarly structed mathematical programming problems. It is hoped that these results will permit broader and more effective utilization of mathematical programming in analyzing the capital budgeting decisions encountered by government agencies, corporations, and other organizations operating with limited sources of investment funds.

References

(1) Balas, E., "Duality in Discrete Programming" Technical Report No. 67-5, July 1967, Department of Operations Research Stanford University. Revised December 1967.

(2) _____, "Duality in Discrete Programming III: Nonlinear Objective Function and Constraints." Management Sciences Research Report No. 125, February 1968, Carnegie-Mellon University.

(3) _____, "Duality in Discrete Programming. IV: Applications." Management Sciences Research Report No. 145, October 1968, Carnegie-Mellon University.

(4) Baumol, W. J., and Quandt, R. E. , "Investment and Discount Rates Under Capital Rationing-A Programming Approach," The Economic Journal, Vol. 75, 317-329, 1965.

(5) Benders, J. F. , "Partitioning Procedures for Solving Mixed Variables Programming Problems," Numerische Mathematik, Vol. 4, 238-252, 1962.

(6) Ellwein, L. B. , "Fixed Charge Location-Allocation Problems with Capacity and Configuration Constraints," Stanford University Department of Industrial Engineering Technical Report 70-2 (1970).

(7) Spielberg, K., "Algorithms for the Simple Plant Location Problem with Some Side Conditions," Operations Research, Vol. 17, pp. 85-111 (1969).

(8) Unger, V. E. "Capital Budgeting and Mixed Zero-One Integer Programming," AIIE Transactions, Vol. II, pp. 28-36 (1970).

(9) Weingartner, H. Martin, Mathematical Programming and the Analysis of Capital Budgeting Problems (Englewood Cliffs, N. J. : Prentice-Hall, Inc., 1963). (Reprinted by Markham Publishing Company, Chicago, Illinois, 1967).

GRANT ACTIVITIES

Theses: As part of this research grant, the principal investigator directed the following theses (abstracts of these theses are presented in appendix A):

"Duality in Discrete Programming and Applications to Capital Budgeting", by Peter Georg Kastil.

"Capital Budgeting and Mixed Zero-One Integer Quadratic Programming", by S. R. Radhakrishnan.

"An Algorith for the Plant Location Problem", by Robert Lee Bulfin, Jr.

Graduate Students Supported by Grant:

R. L. Bulfin, Jr.

R. A. Dunn

Publications:

"An Algorithm for the Plant Location Problem", R. L. Bulfin and V.E. Unger, presented at 39th national meeting of ARSA.

"Duality Results for Zero-One Capital Budgeting Models", V. E. Unger, in preparation.

A publication on capital budgeting and mixed zero-one integer quadratic programming is also planned.

Copies of all reports and publications resulting from this research will be forwarded to the Foundation when they become available.

Related Activities: Some experimental work on the application of group theortic methods for solving integer programming problems was conducted on this grant and this work is currently being continued.

APPENDIX A

DUALITY IN DISCRETE PROGRAMMING AND APPLICATIONS
TO CAPITAL BUDGETING

Peter Georg Kastil

ABSTRACT

A duality concept for discrete programming due to Balas is introduced for the case of two "dual" problems with a Lagrangian-type objective function, where the min-max/max-min of a linear function is to be found over a domain defined by linear inequalities. The variables are constrained to belong to arbitrary sets of real numbers, i.e. some or all of the variables may be discrete. After a discussion of the consequences of including indivisibilities in linear economic models for the application of shadow-price systems and decentralized decision making procedures, economic interpretations are presented for the case of a mixed-integer programming problem, whose dual has been formulated following Balas's duality concepts. A generalized shadow-price system, involving nonnegative dual variables and unrestricted subsidies and penalities, is introduced.

Balas's duality concept and its economic implications then are applied to four specific capital budgeting models. The first three models are based on formulations by Weingartner and Baumol and Quandt; they include, however, the additional assumption of indivisible projects and lead to new results in the interpretation of their optimal solutions. A finite decentralized decision making procedure is one of these results. By combining and extending the mentioned capital budgeting models, a formulation is obtained, the solution of which yeilds the optimal investment, dividend, and financing policy of a firm interacting with an imperfect capital market.

CAPITAL BUDGETING AND MIXED ZERO-ONE INTEGER QUADRATIC PROGRAMMING

S. R. Radhakrishnan

ABSTRACT

The objective of investment analysis is to select, from the set of available proposals, a feasible package to be undertaken, which is preferred above all others. Most of the work done in the area of "Capital Budgeting" assumes linearity, independency of projects, and deterministic cashflows. A quadratic objective function is justified in the following cases to build realism into the models developed:

1. Where the objective is to maximize the utility and the utility function is quadratic instead of linear.
2. Where the interrelationship between projects are considered explicitly in the models.
3. Where risk factor is taken into consideration in project selection.

The objective of this research is to analyze capital budgeting problem as a mixed zero-one integer quadratic programming problem by relaxing the independency assumption and linear utility assumption in the models developed by Weingartner, Bernhard, and Baumol and Quandt. The duality concepts developed by Balas for the quadratic case are used to analyze the properties of optimal solutions to the models. Solution techniques for a class of capital budgeting problems are also discussed. Specifically, the problem involves the allocation of limited amounts of capital among a specified set of investment opportunities in such a way as to maximize the utility of a discounted sum of cash distributions made to the firm's shareholders.

AN ALGORITHM FOR THE PLANT LOCATION PROBLEM

R. L. Bulfin, Jr.

ABSTRACT

This dissertation considers the plant location problem, which involves the selection of a subset of plants to be opened from a set of possible plant locations. Each plant has a fixed cost and an upper bound on its production capability. Fixed costs are incurred when a plant is opened (i.e. allowed to produce) and transportation costs are incurred in shipping the product from plants to points of demand. The objective is to select the subset of plants to open which minimizes total cost, while satisfying demand.

A solution technique for this problem is presented which gives optimal solutions. The problem is partitioned into an integer problem and a linear problem by using Benders' procedure. An implicit enumeration scheme is then applied to the integer problem. The linear problem is only solved for those subsets of plants which pass certain feasibility and optimality tests, and then it is used to generate constraints which bound the objective functions of the integer problem. Methodology is also developed and implemented for combining these constraints to form a surrogate constraint which is "better" than the individual constraints.

Computational results for the algorithm are presented.