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THE UTILITY OF OPTIMIZATION TECHNIQUES IN THE DESIGN OF MAN-MACHINE SYSTEMS

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SUMMARY

The objective of this study was to examine a general procedure for the application of optimization techniques to the design of man-machine systems. Particular emphasis is placed on the development of appropriate criteria of design effectiveness. Criterion development is viewed as a stage of the overall design process.

The design procedure is examined by means of the specific example of instrument panel layout problems. Various layout criteria from the literature are examined in addition to the corresponding quantitative techniques. Emphasis is placed on computational feasibility of the optimization algorithms currently available.

Two new design criteria are examined. The Display Arrangement Hypothesis proposed by Clement, Jex and Graham (3) is translated into quantitative terms and appropriate computational techniques are suggested. In addition, a stochastic assignment problem is proposed in which frequency of fixation is treated as a random variable.

The current state of the art is examined and suggestions are made for areas of further research.

CHAPTER I

INTRODUCTION

A major characteristic of research in the area of human factors engineering and man-machine system design in recent years has been a trend towards the use of quantitative techniques. In particular, engineering techniques have been applied to behavioral problems. The results of such applications have been generally inconsistent but enough success has been realized to encourage further experimentation.

Perhaps the most widely known example of the application of an engineering model to a behavioral problem is the application by Hick in 1952 (12) of information theory as developed by Shannon (24) to the study of choice reaction times. Although the theory did not provide a completely acceptable explanation of the experimental findings, the results were sufficiently good to promote widespread interest. Indeed, the psychological and human factors literature of the ensuing years is filled with applications of information theory to diverse human performance problems.

A less well known but similar occurrence is the use of the theory of signal detectability (25) as a model for human performance. Originally developed by engineers as a theory of machine performance in a task of detecting signals in noise, considerable study has been devoted to the extent to which human observers approximate this model. While again the results are not conclusive, the studies have proven valuable in providing explanations of certain psychophysical phenomena.

Still another example is the use of the well developed theory of automatic control to the performance of the human controller in complex systems. This development can be attributed to the desire of the design engineer to describe the human operator in engineering terms. To this end, considerable effort has been spent in searching for the "human transfer function" (16).

The three examples presented above may be taken as indication of a general trend towards quantitative methods such as the development of mathematical models. Closely allied to this trend is the growing awareness of the importance of the "systems approach" in the study of man-machine systems. Evidence for this is seen in the increased emphasis given the area in the latest edition of McCormick's textbook (21) and in the publication of a new book by DeGreene (5).

Contributions of Operations Research

The field of operations research with its emphasis on mathematical models and quantitative methods is a likely area of interest for the new "systems psychology" (5). The operations research analyst's quest for optimality offers new opportunities for innovative research in the human factors area. A review of the literature reveals that operations researchers have contributed little to the area.

Several factors contribute to the lack of results. Behavioral scientists are not generally sophisticated in the techniques of systems analysis. For this reason they fail to incorporate the models into their work and hesitate to develop new ones.

Another aspect is an apparent lack of interest on the part of operations research analysts and systems engineers in problems of human behavior. As Ackoff (1) puts it, systems engineering is primarily concerned with the equipment in a man-machine problem, while operations research is primarily concerned with procedures. He says: "The operations research analyst looks at human beings, separately or collectively, as a black box whose input and output characteristics alone are of interest."

There appears to be a more fundamental reason that operations research and human factors engineering have followed divergent paths. As Toppmiller (26) says:

> Why then . . . does the Operations Researcher overlook human performance in his models? Without wanting to appear ridiculously parsimonious, I believe at least one reason is that human engineers have not developed appropriate measurement transforms of human engineering data so that these data can be readily assimilable into OR modeling frameworks.

This lack of data in the required form and the lack of appropriate criterion measures must be overcome if this powerful methodology is to be made available. Certainly future research efforts must be directed toward this end.

Contributions of the Behavioral Sciences

The behavioral sciences have made rather diverse contributions to the studies of human operator performance. These studies have, of course, addressed themselves to different aspects of performance. Psychologists have investigated the perceptual abilities of man as an information sensor and processor. Physiologists have been concerned with the physical abilities and limitations of man as well as the environmental conditions necessary for performance.

Some research areas have been more highly specialized. Anthropometric data has been widely used to design work areas for the human operator. The two primary sources of such data have been the physical anthropologists who provide studies of a general nature and other scientists who conduct quite specific experiments for design purposes.

Knowledge is not easily categorized and there are many examples of cooperation between separate disciplines. The relatively new field of biomechanics, for example, is based on the application of the engineering principles of mechanics to physiology (4). Similarly, bionics is the study of biological systems and the application of the principles to the design of mechanical devices (10). The application of basic research findings to the design and analysis of man-machine systems is generally referred to as human factors engineering or ergonomics.

The Criterion Problem

The development of operations research, primarily over the last quarter of a century, has been characterized by a readiness to apply quantitative techniques to real-world problems and a multi-disciplinary approach to such problems. In fact, these two characterizations are probably the terms which appear most often in definitions of operations research. Certainly they are the cornerstone of development of a powerful methodology which continues to expand.

Among the problems which have traditionally been attacked with this methodology, many are basically behavioral. The fundamental quantity to be observed in the study of an inventory system for example is the consumer demand for the particular product or products under consideration. Similarly, many applications of queueing theory are based on certain assumptions regarding the frequency with which customers arrive to be served and their behavior when confronted with a queue.

Measures of effectiveness often are readily apparent in applications of the methods of operations research. Since much work is concerned with industrial or economic problems, monetary value is a popular measure. The advantages of studying monetary value are that it is quantitative and is almost universally understood. Certainly any manager can easily understand when told that a particular inventory system will minimize his costs or, alternatively, maximize his profits. Similarly, the proprietor of a motion picture theater recognizes the value of minimizing the amount of time customers must wait in line. Time also has the virtues of being quantitative and universally known.

Even when intuitively appealing criteria are available, trouble often occurs. Often several such measures may be in conflict. It is well known, for example, that the minimum cost solution may not maximize profit. Similarly, the motion picture theater manager may have to weigh the desirability of minimizing waiting time against the added expense of additional ticket windows.

A more serious difficulty is that the traditional criteria may not be appropriate to the particular problem at hand. For example, utility theory was developed in order to examine certain economic situations in which monetary value alone did not seem sufficient to measure effectiveness. This theory retained the quantitative properties but made numerous compromises, not the least of which was the intuitive acceptance on the part of laymen.

Compromises of this nature are not uncommon since the techniques often applied in operations research such as simulation, game theory, mathematical programming and others are almost exclusively based on a quantitative approach. Clearly then in situations in which such a criterion is not readily available, either a less suitable but still quantitative criterion must be accepted or a new criterion must be synthesized which is measurable but which perhaps violates other desirable properties.

The Multi-Disciplinary Approach

That the criterion problem is of considerable importance in the application of operations research methodology to the analysis of behavioral systems can be illustrated by identification of two basic elements of the problem-solving procedure. The first of these elements considers a set of assumptions and the development of an appropriate criterion. Secondly, a technique or group of techniques suitable to the particular problem at hand is employed.

A fundamental characteristic of the operations research methodology concerns the allocation of responsibility for these decision-making elements. In particular, a multi-disciplinary approach has traditionally been employed with specialists from several fields participating. The group members contribute not only their expertise in a particular area but also allow a broader definition of the problem area. For example, a sociologist would not only answer specific questions relevant to his area, but would also suggest sociological implications of the problem which might not be obvious to other group members.

Of considerable importance is the manner in which the contributions of the several members are coordinated. Although certain aspects of the problem will be of particular interest to one or more of the members, it is essential that the entire team be involved at each level of analysis. Thus, rather complex feedback processes are necessary. It is the manner in which these feedback processes are controlled and the group is managed and directed that allows this approach to function successfully.

Purpose of the Research

The purpose of this thesis is two-fold. An examination is made of the extent to which the methodology and analytical tools of operations research can be utilized in a specific man-machine system design problem. A second purpose is to analyze this specific problem and to use it as an example of the procedure involved in applying quantitative techniques to behavioral problems.

In actual practice, it is assumed that the development of a satisfactory design criterion is primarily the responsibility of the behavioral scientist. It is then necessary to express this criterion by means of an operational definition. The operations research analyst would then translate this operational definition into a mathematical objective function along with a set of constraints. The analyst would then employ suitable techniques in order to find the optimal solution to the mathematical problem. This solution is then interpreted in terms of the design requirements.

CHAPTER II

CURRENT LAYOUT CRITERIA

The specific problem to be considered in this thesis is that of the arrangement of displays on an instrument panel. The most common example is the panel in an airplane cockpit. From these dials and instruments, the pilot obtains the information about system output which enables him to take the appropriate control activities. The human factors implications of design of this particular man-machine system have been extensively studied. Many present and future studies may well be directed the area of space vehicle systems.

It should be noted that the problem is actually much more general. Not only do the results apply to a wide variety of instrument panel layouts, but to the more general problem of facilities design and space utilization. For example, the placement of controls in a manual control system or workplace would be approached in an analogous manner.

Design Criteria

In order to evaluate any design, it is necessary to have a criterion. This provides a metric against which the design effectiveness may be measured. If quantitative techniques are to be employed, a quantitative statement of the design is necessary.

The ideal criterion would consider all relevant factors both physical and psychological. In practice, this ideal is seldom if ever realized. Commonly, a set of general principles evolve which apply to all problems of a given class. Other more specific, criteria are developed for each individual situation.

In this chapter, criteria will be considered for the instrument panel layout problem which are general in nature. They would be of value in any problem of this type but would probably not be sufficient in themselves. Such quantitative criteria, used in conjunction with expert knowledge and experimental evaluation, can be of significant value.

Implicit is the concept of an optimum physical location. As McCormick (21) puts it:

> . . . there is a basic principle . . . namely, the promise of optimum locations of physical components. It is reasonable to hypothesize that any given type of activity using a physical component could be carried out best if the component is in a satisfactorily optimum general location as far as human sensory, anthropometric, and biomechanical characteristics are concerned. . . .

The determination of these optimum locations for the class of problems discussed here is the subject of this thesis.

McCormick's Principles

McCormick (21) presents four ideas which he calls "guiding principles or arrangement." The first of these states that components should be arranged with regard to their importance to the system objectives. Secondly, frequency-of-use data should be considered in placing components. Next, one should consider the function of each component in assigning its position. The final principle implies that the sequence-of-use of components should be examined when grouping them.

It is obvious that these principles are extremely general. Even after a thorough understanding of the principles has been realized, it is not clear how they should be incorporated into the design procedure. In other words, these principles do not have an unambiguous operational definition. Furthermore, it is possible for two or more of the principles to be in conflict. For example, it may be that the most important component is not the one most frequently used. In this case there is a question as to which takes precedence.

Freund-Sadosky Criteria

Freund and Sadosky (6) employed two "utility cost" concepts in approaching the problem. The first of these employs a distance measure which is the geometric distance from the center of the instrument panel to the center of the instrument location. The utility cost of assigning a given instrument to a given location is then defined to be the product of the distance measure for that location and the fixation frequency for that instrument. The sum of all such utility costs for a given set of assignments is the cost measure for that assignment set.

The second utility cost approach involves a different distance factor. In this case, the appropriate measure is the sum of the distances from the center of the instrument location to the center of each of the other locations in the panel.

Here transition frequencies are used rather than fixation frequencies. The cost of a single instrument-to-location assignment is the product of the distance factor and the probability of transition from that instrument to any other instrument on the panel. Clearly, the cost of a set of assignments is the summation of all such single assignments.

The objective in each of the above approaches is, of course, to minimize total utility cost. In each case this involves assigning one and only one component to each location in such a way as to minimize the cost function. The former case implicitly assumes that those instruments which are fixated most frequently should be placed near the panel center. The latter case assumes that instruments with high transition probabilities should be placed centrally with respect to other instruments.

Total Eye Movement Minimization Criterion

Hitchings (14) and Freund and Sadosky have considered the criterion of minimization of total eye movement. The data required in this case are the distances between each pair of locations and the frequency of transition between each pair of instruments. Total eye movement for a set of assignments is then defined to be the summation over all instrument-location pairs of the product of distance and transition frequency.

There seems to be little or no support for this specific criterion from the literature. No known study investigates the validity of the concept.

The concept of eye movement as used in this thesis requires some elaboration. There exist at this time rather sophisticated techniques for the recording of direction of gaze. Since visual acuity is greatest when the image falls upon the fovea, eye position can be used as an indication of the primary line of sight. This method, of course, discounts the use of peripheral vision.

Since measurement techniques are based on these considerations, eye movement will be taken to be a change in direction of primary line of sight. Clearly, the development of new measurement methods and instrumentation would allow a modification of this definition.

Senders-Clement, Jex and Graham Criteria

Senders (23) and Clement, Jex and Graham (3) have presented instrument layout criteria. Both schemes are based on investigations of operator performance in complex man-machine systems. Since they are quite similar in nature, they will be discussed jointly.

Senders, in discussing the overall problem of estimating operator workload, suggested two principles. He proposes that those instruments with high fixation frequencies should be located centrally and those with high transition probabilities should be located close together. It should be noted that this is actually a compound criterion.

Clement et al formulated a "Display Arrangement Hypothesis" based on a survey of pilot performance studies. The hypothesis is as follows:

1) locate centrally those displays having the highest probability of fixation;
2) locate peripherally adjacent to the center those displays having highest link values with the central display(s);
3) locate peripherally remote from the center those displays having lowest probability of fixation and/or lowest link values.

It is interesting to note that this scheme was used as the basis of the layout of instruments in an aircraft based on predictions from a model of visual monitoring behavior. The resulting layout was found to be in substantial agreement with the actual display arrangement adopted for the aircraft.

Summary

If optimization techniques are to be applied to the layout problem, the criterion must be expressed by an operational definition and translated into a mathematical expression. The ones described in this chapter meet these requirements to varying degrees. Furthermore, it will be shown that in some cases, two or more criteria may be in conflict.

By consideration of quantitative statements of the criteria, it is possible to examine in detail the methods by which they may be incorporated into the design process. In particular, suitable mathematical techniques may be identified in order to obtain optimal solutions with respect to the particular criterion. The remainder of this thesis is concerned with these problems.

CHAPTER III

COMPUTATIONAL TECHNIQUES

The mathematical programming techniques which are of greatest value in the present problem are generally referred to as assignment techniques. Within this class, a wide variety of algorithms are available which may well prove useful. In this chapter, some of these will be considered. A survey will be made of the techniques which have been applied to the criteria mentioned previously and some new methods will be explored.

Freund-Sadosky Computational Techniques

Freund and Sadosky (6) employed the standard assignment algorithm of linear programming to reach optimal solutions to the problem of minimization of total utility cost. The mathematical formulation of this problem is

$$\text{minimize} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to

$$\sum_{i=j}^{n} x_{ij} = 1 \qquad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, 2, \ldots, n$$

$$x_{ij} = 0, 1 \qquad i,j = 1, 2, \ldots, n$$

Consider the data matrix of Table 1. In this example, the first of the two utility cost concepts is used. The distance components represent distance from the center of the panel to the center of the given location. The frequency components are the frequency of fixation of the given instrument. Accordingly, each ij-cell represents the product of the i^{th} frequency component and the j^{th} distance component. This product will be designated c_{ij}.

The problem may be thought of as a decision process in which exactly one cell is chosen from each column and one from each row in such a manner that the sum of the costs in the cells selected is a minimum. The Hungarian method of linear programming provides an efficient algorithm for this problem although several other techniques are available. For this example, the cells marked with a circle constitute an optimal assignment and the corresponding value of the objective function is 580.

Two points should be noted regarding this formulation. First, it should be clear that the second utility cost concept leads to exactly the same problem formulation and is solved by the same techniques. Also, the nature of this particular problem allows much simplification of the algorithm and in fact allows the optimal assignment to be found by simple inspection. This feature of the problem will be examined in some detail.

Consider the data matrix of Table 2. Note that this matrix is the same as that of Table 1 except that the distance components have been arranged in non-decreasing order while the frequency components have been arranged in non-increasing order. The costs appearing in

Table 1. Data Matrix for Assignment Model

		Distance									
		I	II	III	IV	V	VI	VII	VIII	IX	X
		12	4	14	5	13	16	3	8	20	1
Frequency	A 7	84	28	98	35	(91)	112	21	56	140	7
	B 21	252	84	294	105	273	336	(63)	168	160	8
	C 14	168	(56)	196	70	182	224	42	112	280	14
	D 6	72	24	(84)	30	78	96	18	48	120	6
	E 21	252	84	294	105	273	336	63	168	420	(21)
	F 1	12	4	14	5	13	16	3	8	(20)	1
	G 2	24	8	28	10	26	(32)	6	16	40	2
	H 13	156	52	182	(65)	169	208	39	104	260	13
	I 7	(84)	28	98	35	91	112	21	56	140	7
	J 8	96	32	112	40	104	128	24	(64)	160	8

Optimal Assignment - E-X, B-VII, C-II, H-IV, J-VII, I-I, A-V,
D-III, G-VI, F-IX.

Cost = 580

Table 2. Revised Data Matrix for Assignment Model

		Distance									
		X	VII	II	IV	VIII	I	V	III	VI	IX
		1	3	4	5	8	12	13	14	16	20
Frequency	E 21	21	63	84	105	168	252	273	294	336	420
	B 21	21	63	84	105	168	252	273	294	336	420
	C 14	14	42	56	70	112	168	182	196	224	280
	H 13	13	39	52	65	104	156	169	182	208	260
	J 8	8	24	32	40	64	96	104	112	128	160
	I 7	7	21	28	35	56	84	91	98	112	140
	A 7	7	21	28	35	56	84	91	98	112	140
	D 6	6	18	24	30	48	72	78	84	96	120
	G 2	2	6	8	10	16	24	26	28	32	40
	F 1	1	3	4	5	8	12	13	14	16	20

Optimal Assignment - E-X, B-VII, C-II, H-IV, J-VIII, I-I, A-V,
D-III, G-VI, F-IX.

Cost = 580

the body of the matrix of Table 1 and that of Table 2 are identical but appear in different cells. It can be shown that in Table 2 the minimum cost assignment lies along the main diagonal and it is designated by circles. A proof of this property is found in Appendix A.

This solution method is known as the product method of linear programming. It should be clear that this method is applicable only when the cells contain products of two components and is not available for the general assignment problem.

Minimization of Total Eye Movement

The minimization of total eye movement criterion is computationally much more difficult than the utility cost problem. The standard assignment algorithms are not applicable to this formulation and the literature yields less than satisfactory methods.

Hitchings (14) considered a scheme that produces near-optimal results although a detailed description of his method was not presented. Freund and Sadosky (6) employed the simplex procedure of linear programming to solve a simple case. Their formulation, however, led to a complex system of constraints for even trivial cases. They were unable to devise constraint systems for the general problem.

In order to study the problem more closely, consider the objective of assigning n instruments to n fixed locations. Let

f_{ij} = frequency of transition from instrument i to instrument j

d_{kl} = Euclidean distance from center of location k to center of location l

$c_{ijkl} = f_{ij} \, d_{kl}$

The problem can then be formulated as

$$\text{minimize} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} c_{ijkl}\, x_{ij}\, x_{kl}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n$$

$$x_{ij} = 0, 1 \quad i, j = 1, 2, \ldots, n$$

This is a special case of the quadratic assignment problem originally formulated by Koopmans and Beckman (15). It is clear that the techniques applicable to the standard linear assignment problem considered previously are not in general applicable in the quadratic case. The linear assignment problem algorithms do not consider the possibility of activity between the facilities to be assigned.

Consider, for example, the data matrix of Table 3. Here the rows represent all possible pairwise instrument transition frequencies. Similarly, the columns represent pairwise location distances.

As before, the problem is one of selecting one cell from each row and one from each column so as to minimize the sum of the chosen costs. In this case, however, not every assignment made in this manner is acceptable since the method does not insure that each instrument is uniquely assigned to a location. For instance, if the cell A-E, II-IV is chosen and the cell A-B, III-V is also, no acceptable assignment is

Table 3. Data Matrix for Quadratic Assignment Problem

		Distance									
		II-V	II-IV	I-II	I-III	V-IV	I-V	III-IV	I-IV	II-III	V-III
		3	4	8	10	11	14	16	21	22	28
Frequency A-E	25	75	100	200	250	275	350	400	525	550	700
A-C	24	72	96	192	240	264	336	384	504	528	672
A-B	20	60	80	160	200	220	280	320	420	440	560
C-E	16	48	64	128	160	176	224	256	336	352	448
B-D	15	45	60	120	150	165	210	240	315	330	420
B-C	10	30	40	80	100	110	140	160	210	220	280
D-E	9	27	36	72	90	99	126	144	189	198	252
A-D	6	18	24	48	60	66	84	96	126	132	168
B-E	5	15	20	40	50	55	70	80	105	110	140
C-D	1	3	4	8	10	11	14	16	21	22	28

possible.

Gilmore (11), Lawler (18), Land (17), and Gavett and Plyter (8) have all developed branch-and-bound algorithms for the quadratic assignment problem. None of these can be considered computationally feasible for problems of significant size. For example, the Gavett and Plyter scheme, which is actually a modification of the travelling salesman algorithm developed by Little, Murty, Sweeney, and Karel (19), required 42 minutes of computer time with $n = 8$. Nugent, Vollmann and Ruml (22) suggest that computing time with $n = 15$ would approach fifty years. The other branch-and-bound algorithms require comparable computational effort.

Returning to the data matrix of Table 3 note that there are $N = \frac{n(n-1)}{2}$ rows and an equal number of columns. It is this matrix upon which the Land and the Gavett and Plyter procedures operate. Notice also that the rows are arranged in non-increasing order and the columns in non-decreasing order. It is therefore possible to use the product technique discussed earlier and make assignments along with the main diagonal. Of course, in general this will not result in an acceptable assignment. However, if the assignment is acceptable, then it is optimal and if it is not acceptable, it constitutes a lower bound on the minimum acceptable value of the objective function.

In the example problem, the value of the objective function when assignments are made along the main diagonal is 1207. Some study, however, leads to the finding that this is not an acceptable assignment. The optimal solution is indicated by circles and the objective function value is 1204.

The computational difficulties encountered indicate that near-optimal solutions should be considered. It is possible to guarantee that a solution is obtained which differs from the optimal by no more than a fixed percentage. For example, the branch-and-bound techniques might be employed to arrive at an answer which exceeds the minimum cost solution by no more than five percent. Until more powerful techniques are developed to deal with combinatorial problems of this magnitude, near-optimal solutions are probably the only realistic goal for problems of significant size.

Senders' Criterion

The Senders (23) compound criterion was to (i) place those instruments with high fixation frequencies near the center of the display and (ii) place those with high transition probabilities close together. Although this seems to be a rather explicit statement of the criterion, it does not translate directly into a mathematical statement.

The first part of the criterion might be approached by the linear assignment algorithms. The first utility cost concept of Freund and Sadosky might be considered as an interpretation of this criterion. In this case, however, it is implicitly assumed that the undesirability of a location increases linearly with distance from the center of the panel. This assumption may or may not be valid but other forms of cost function could be employed.

The second part of the objective function is perhaps most easily interpreted as the minimization of total eye movement. Again, however, this involves the assumption that some utility cost increases linearly with distance. It is certainly not clear that for a given transition frequency, the undesirability of placing instruments two units apart is twice that of placing them one unit apart.

A further difficulty is encountered in that the two stages of the criterion may be in conflict and, in fact, may be mutually exclusive. To see this, consider the layout of Figure 1. The link values indicate the transition frequencies between instruments and from the instruments to some point away from the panel. It is easy to see that in this simple case, total eye movement between instruments is minimized by placing the most frequently fixated instrument, C, in a non-central location.

If the above interpretations of the two parts of the compound criterion are accepted as valid, it is possible to combine them into a single expression by employing a special case of the quadratic assignment problem to include a linear term. In that case, the objective function would have the form

$$\text{minimize} \sum_{i=1}^{n} c_{il(i)} + \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}\, d_{l(i)l(j)}$$

$$\text{subject to } l(i) \in S_i \qquad i = 1, 2, \ldots, n$$

where $l(i), l(2), \ldots, l(n)$ is a permutation of $1, 2, \ldots, n$ and S_i is the set whose elements are the locations to which instrument i may be assigned. Hillier and Connors (13) demonstrate that this formulation can easily be obtained from the general statement of the quad-assignment problem.

Expressing the problem in this form does not, however, answer the questions regarding linearity of cost and distance. Furthermore, it is

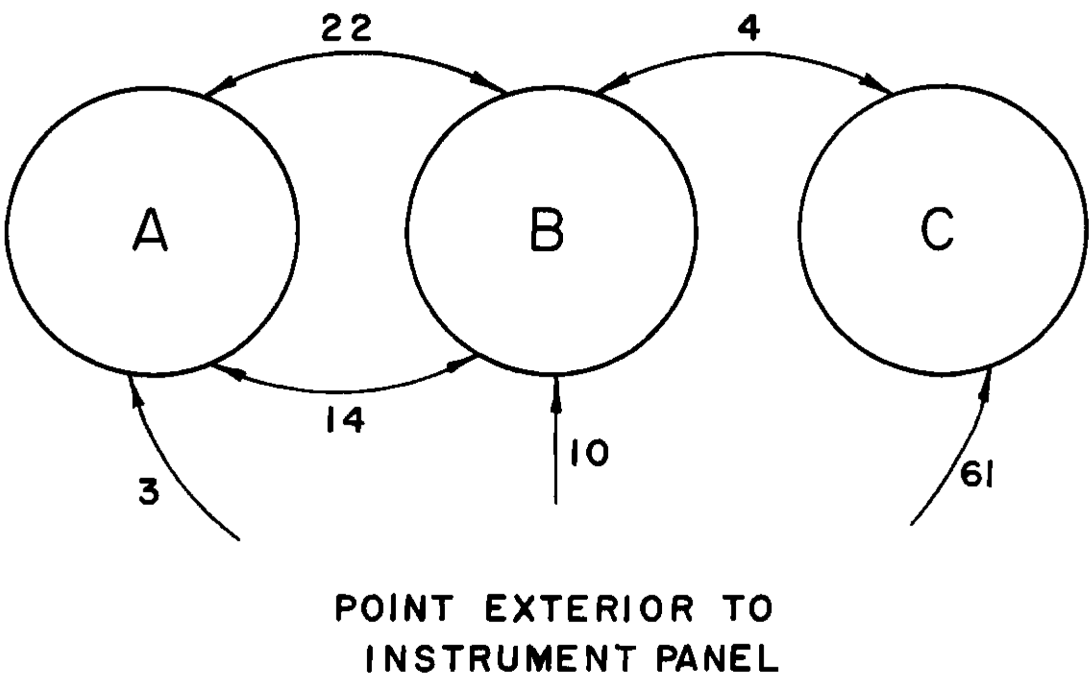

Figure 1. Hypothetical Link Valves.

possible to weight the two terms of the expression so that either can be made to predominate over the other. Until proper weighting factors are determined, it is not possible to solve real-world problems of this type.

McCormick's Principles

The remaining criteria from the previous chapter are those of McCormick (21) and Clement et al (3). The latter will be examined in some detail in the next chapter. At this point, some remarks about McCormick's principles are in order.

Of the four principles set forth by McCormick, the sequence-of-use and frequency-of-use principles have been accepted throughout the previous discussions. Perhaps the primary reason is that data can be collected on these aspects. On the other hand, the importance and function principles do not have an obvious metric against which they can be measured.

It has been stressed throughout that if optimization techniques are to be applied to instrument panel layout, it is not sufficient that a criterion be expounded. Only by means of an operational definition and a corresponding mathematical formulation of the problem can the methods be gainfully employed.

CHAPTER IV

TWO NEW DESIGN CRITERIA

In this chapter, two new operational criteria are developed for the instrument panel layout problem. For each criterion a general description is given followed by a mathematical formulation and an example problem. Computational techniques are discussed.

The first of the criteria discussed here is based on the Display Arrangement Hypothesis presented by Clement et al. A formulation is developed which is not a direct translation of the Clement hypothesis since this criterion is found to be somewhat ambiguous. The quantitative problem statement, however, is basically similar to the Clement statement.

A second criterion is presented which extends the Freund-Saodsky utility cost minimization concept to the stochastic case. The problem is found to be computationally more difficult but important in that it corresponds more closely to problems encountered in practice.

Clement's Display Arrangement Hypothesis

A characteristic common to many of the assignment procedures examined thus far is that they are often quite restrictive as to the number of variables which can be handled. One way of overcoming this constraint is to partition the problem into various sub-problems which can be solved independently. This approach seems particularly attractive in those cases where the layout criterion itself consists of several levels.

The Display Arrangement Hypothesis of Clement et al discussed previously is an example of such a multi-level criterion. In this section, this criterion is translated into quantitative terms as a decomposition of the overall procedure into several sub-stages. Thus, the proposed technique not only allows larger problems to be solved but allows a mathematical formulation which is structurally similar to the original problem definition.

Problem Formulation

Let f_i be the fixation frequency of instrument e_i where $i > j$ implies that $f_i \leq f_j$. That is, the sequence of instruments $e_1, e_2, \ldots, e_n$ is arranged in non-increasing order of frequency. Similarly, let d_i designate the Euclidean distance from the center of the panel to the center of location l_i where $i > j$ implies that $d_i \geq d_j$. Thus, the sequence $l_1, l_2, \ldots, l_n$ is non-decreasing order of distance.

Assume that of the n candidate locations, k_1 are designated as central locations where $1 \leq k_1 \leq n$. The identification of central locations is based, of course, on experimental studies. Clearly, the objective of the first stage of the procedure is to assign instruments e_1 through e_{k_1} collectively to locations l_1 through l_{k_1}. In particular, these instruments are to be assigned so as to minimize total eye movement.

As has been noted previously, the minimization of eye movement can be formulated as a quadratic assignment problem:

$$\text{minimize} \sum_{i=1}^{k_1} \sum_{j=1}^{k_1} \sum_{r=1}^{k_1} \sum_{s=1}^{k_1} c_{ijrs}\, x_{ij}\, x_{rs}$$

subject to

$$\sum_{i=1}^{k_1} x_{ij} = 1 \quad , \quad j = 1, 2, \ldots, k_1$$

$$\sum_{j=1}^{k_1} x_{ij} = 1 \quad , \quad i = 1, 2, \ldots, k_1$$

$$x_{ij}, x_{rs} = 0, 1, \quad i, j, r, s = 1, 2, \ldots, k_1$$

Here, c_{ijrs} is the cost of assigning instruments e_i and e_j to locations l_r and l_s. Thus, c_{ijrs} is calculated by multiplying t_{ij} by w_{rs} where t_{ij} is the transition frequency between instruments i and j and w_{rs} is the Euclidean distance between locations r and s. With the solution to this problem, the first stage is completed and instruments $e_1, e_2, \ldots, e_k$, have been uniquely assigned to locations $l_1, l_2, \ldots, l_{k_1}$.

The area which has been designated as peripherally adjacent to center contains k_2 locations where $1 \leq k_2 \leq k_1 + k_2 \leq n$. Now, consider the values of t_{ij} where $1 \leq i \leq k_1$ and $k_1 + 1 \leq j \leq k_1 + k_2$. These are the transition frequencies between instruments in the central locations and instruments in the peripherally adjacent locations.

It is convenient to define $t._j$ as $\sum_{i=1}^{k_1} t_{ij}$ for all $k_1 + 1 \leq j \leq k_1 + k_2$. The value of $t._j$ is interpreted as the transition frequency between instrument j and all instruments in the central area. Note that $t._j$ is the total frequency of transition regardless of direction. A concept similar to the second Freund-Sadosky utility cost function can now be applied to solve the following linear assignment problem:

$$\text{minimize} \sum_{r=k_1+1}^{k_1+k_2} \sum_{s=k_1+1}^{k_1+k_2} c_{rs}\, x_{rs}$$

$$\text{subject to} \sum_{r=k_1+1}^{k_1+k_2} x_{rs} = 1 \quad , \quad k_1 + 1 \leq s \leq k_1 + k_2$$

$$\sum_{s=k_1+1}^{k_1+k_2} x_{rs} = 1 \quad , \quad k_1 + 1 \leq r \leq k_1 + k_2$$

$$x_{rs} = 0,\ 1 \quad , \quad k_1 + 1 \leq r,\ s \leq k_1 + k_2$$

In the formulation above, c_{rs} is the product of $t._r$ and d_s. Hence, in the second stage, instrument-location pairings are made on a utility cost minimization basis where the utility cost is the summation over all instrument-location pairs of the product of a frequency component and a distance component. In particular, the frequency of transition between the given instrument and all instruments in the central area is multiplied by the distance from the location under consideration to the center of the panel.

At this point, instruments $e_{k_1+k_2+1}$, $e_{k_1+k_2+2}$, . . ., e_n remain to be assigned collectively to locations $l_{k_1+k_2+1}$, $l_{k_1+k_2+2}$, . . ., l_n. The final stage proceeds in a manner analogous to the previous one. The appropriate formulation of this problem is:

$$\text{minimize} \sum_{r=k_1+k_2+1}^{n} \sum_{s=k_1+k_2+1}^{n} c_{rs}\, x_{rs}$$

subject to $\sum_{r=k_1+k_2+1}^{n} x_{rs} = 1 \quad , \quad k_1 + k_2 + 1 \leq s \leq n$

$$\sum_{s=k_1+k_2+1}^{n} x_{rs} = 1 \quad , \quad k_1 + k_2 + 1 \leq r \leq n$$

$$x_{rs} = 0, 1 \quad , \quad k_1 + k_2 + 1 \leq r, s \leq n$$

The steps involved in the application of this procedure are summarized below. It should be noted that all steps involve techniques which have been discussed previously. Although some of these algorithms are not appropriate for large problems, the multi-stage nature of this approach allows problems of significant size to be handled.

Summary of Procedure

Stage 1(a) - Assign the k_1 instruments of highest fixation frequency collectively to the k_1 locations of the central area.

Stage 1(b) - Assign each of these k_1 instruments uniquely to one of the k_1 locations by solution of a quadratic assignment problem to minimize total eye movement within this group.

Stage 2(a) - Among all instruments not yet assigned, collectively assign the k_2 instruments of highest fixation frequency to the k_2 locations of the area peripherally adjacent to center.

Stage 2(b) - Employing the linear assignment problem formulation, uniquely assign these k_2 instruments to locations so as to minimize a utility cost measure which is defined as the summation over all instrument-location pairs of the product of the distance from the panel center to the location multiplied by the frequency of transition between the instrument under consideration and all instruments in the central area.

Stage 3(a) - Repeat stage 2(a) for the $n-k_1-k_2$ instruments remaining to be assigned to the peripherally remote area.

Stage 3(b) - Repeat stage 2(b) for the $n-k_1-k_2$ instruments assigned collectively to the locations of the peripherally remote area.

Example

Consider an example in which $k_1 = 5$ and $k_2 = 15$. There are 40 instruments so that $n - k_1 - k_2 = 20$. These instruments are to be uniquely assigned to 40 locations.

In Table 4, transition frequencies are given. The values given are those between each of the 40 instruments and the five instruments of highest fixation frequency. It should be noted that the actual frequencies are not given as they are not relevant to the procedure as long as the instruments are numbered in non-increasing order of fixation frequency. The right-hand column of Table 4 gives the values of $t_{\cdot j}$.

One additional point should be noted regarding the relationship between transition frequency and fixation frequency. It is not necessary that all transitions be from one instrument to another. For example, in an airplane the pilot may look at the fuel gauge and then look outside of the cockpit. For this reason, transition frequencies cannot be inferred from fixation frequencies.

The other information necessary to complete the assignment procedure is the matrix of distances between the five locations designated as central. This is given in Table 5.

Table 4. Transition Frequencies for Display Arrangement Problem

	TRANSITION FREQUENCY							TRANSITION FREQUENCY					
	e_1	e_2	e_3	e_4	e_5	$t._i$		e_1	e_2	e_3	e_4	e_5	$t._i$
e_1	0	10	3	1	15		e_{21}	2	4	4	3	1	14
e_2		0	6	8	4		e_{22}	6	2	0	2	4	14
e_3			0	0	10		e_{23}	3	4	1	1	4	13
e_4				0	3		e_{24}	5	2	1	1	5	14
e_5					0		e_{25}	4	1	3	2	1	11
e_6	0	4	2	0	1	7	e_{26}	4	1	1	4	2	12
e_7	3	2	7	0	0	12	e_{27}	2	0	3	3	1	9
e_8	9	2	0	4	0	15	e_{28}	0	1	1	4	2	8
e_9	2	1	0	1	1	5	e_{29}	1	1	3	4	1	10
e_{10}	0	2	0	6	2	10	e_{30}	0	2	2	3	0	7
e_{11}	2	4	4	1	1	12	e_{31}	2	1	1	2	0	6
e_{12}	0	1	0	0	4	5	e_{32}	0	1	1	4	1	7
e_{13}	2	1	1	2	0	6	e_{33}	2	1	1	3	1	8
e_{14}	0	2	4	0	0	6	e_{34}	0	1	4	0	0	5
e_{15}	2	1	0	0	1	4	e_{35}	2	1	0	0	1	4
e_{16}	0	1	1	0	0	2	e_{36}	1	0	2	2	1	6
e_{17}	0	1	1	0	1	3	e_{37}	2	0	0	1	0	3
e_{18}	0	1	1	0	0	2	e_{38}	4	1	1	2	0	8
e_{19}	2	0	0	1	2	5	e_{39}	2	0	0	1	0	3
e_{20}	1	0	1	4	2	8	e_{40}	0	0	1	1	0	2

Table 5. Distance Matrix for Display Arrangement Problem

	Distance				
	l_1	l_2	l_3	l_4	l_5
l_1	0	28	33	22	20
l_2		0	27	40	25
l_3			0	30	15
l_4				0	18
l_5					0

Clearly instruments e_1, e_2, e_3, e_4 and e_5 are to be assigned collectively to locations l_1, l_2, l_3, l_4 and l_5. The particular assignment which minimizes total eye movement is $e_1 - l_4$, $e_2 - l_1$, $e_3 - l_3$, $e_4 - l_2$, $e_5 - l_5$. Thus, five instruments have been uniquely assigned to locations and the first stage of the procedure is complete.

Instruments e_6, e_7, . . ., e_{20} are now collectively assigned to locations l_6, l_7, . . ., l_{20}. Among these instruments and locations, individual assignments are made so as to minimize the product $t_{\cdot j}\ d_i$ summed over all instrument-location pairs. This is readily done by the product technique and the assignments are given in Table 6.

The final stage proceeds analogously with instruments e_{21}, e_{22}, . . ., e_{40} assigned to locations l_{21}, l_{22}, . . ., l_{40} so as to minimize $t_{\cdot j}\ d_i$. The results of this stage are also given in Table 6 which summarizes the assignment of all instruments.

A Stochastic Assignment Criterion

In their formulation of the utility cost minimization problem, Freund and Sadosky (6) employed the standard linear assignment algorithm. In doing so they implicitly assumed a completely deterministic system in that the frequency values were assumed to be precisely known. Clearly, this may not be the case in practice.

One might propose at least two reasons to hypothesize that the values are known only with respect to some frequency distribution. In the first place, any data collection of this type can only be considered as a sampling process from some universe. Therefore, statements can be made about these values only in a statistical context. Conversely, and

Table 6. Summary of Assignments for Remaining Instruments

ASSIGNMENTS for k_2 instruments	ASSIGNMENTS for k_3 instruments
$l_{6} - e_{8}$	$l_{21} - e_{21}$
$l_{7} - e_{11}$	$l_{22} - e_{22}$
$l_{8} - e_{7}$	$l_{23} - e_{24}$
$l_{9} - e_{10}$	$l_{24} - e_{23}$
$l_{10} - e_{20}$	$l_{25} - e_{26}$
$l_{11} - e_{6}$	$l_{26} - e_{25}$
$l_{12} - e_{13}$	$l_{27} - e_{29}$
$l_{13} - e_{14}$	$l_{28} - e_{27}$
$l_{14} - e_{9}$	$l_{29} - e_{33}$
$l_{15} - e_{12}$	$l_{30} - e_{38}$
$l_{16} - e_{19}$	$l_{31} - e_{28}$
$l_{17} - e_{15}$	$l_{32} - e_{30}$
$l_{18} - e_{17}$	$l_{33} - e_{32}$
$l_{19} - e_{16}$	$l_{34} - e_{31}$
$l_{20} - e_{18}$	$l_{35} - e_{36}$
	$l_{36} - e_{34}$
	$l_{37} - e_{35}$
	$l_{38} - e_{37}$
	$l_{39} - e_{39}$
	$l_{40} - e_{40}$

perhaps even more importantly, frequency-of-use data in a complex man-machine system typically varies as a function of changing task demands. Gainer and Obermayer (7) have shown, for example, that these frequencies vary significantly as an airplane pilot flies different maneuvers.

Thus the value of extending the utility cost minimization concept to the stochastic case is clearly indicated. Operationally this would amount to considering the frequencies to be random variables. It will be assumed that estimates of the means and variances of these random variables are available.

Before examining an example in detail it is informative to study a typical case which illustrates the concepts involved. Consider a group of instruments which are known to have fixation frequencies which are random variables with known distributions. These instruments are to be assigned to fixed locations in an instrument panel.

A possible objective is to choose that assignment which minimizes total expected utility cost. If this utility cost is defined to be the product of distance and the expected value of the frequency distribution, the problem is readily solved by the product technique. It is important to note that since the frequencies are random variables, the utility cost of any assignment will also be a random variable. In particular, the minimum expected cost assignment is a random variable with variance determined by the assignment.

In general, the assignment which minimizes mean utility cost does not minimize variance. Consider Figure 2 which shows typical utility cost distribution. Clearly, distribution A has the lower expected cost but the larger variance. If, for example, there is some cost level, k,

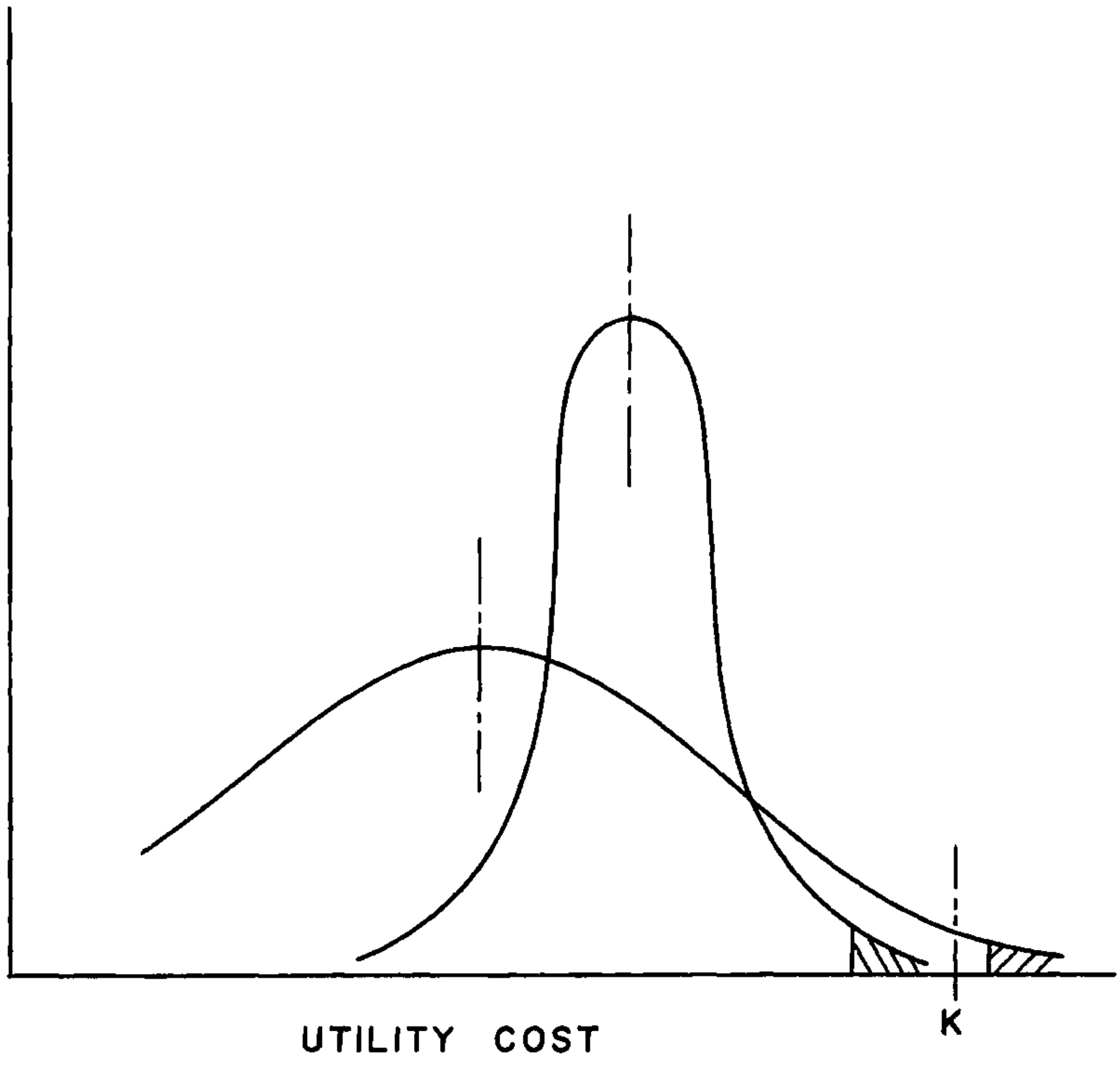

Figure 2. Typical Utility Cost Distributions.

which is the maximum level which is acceptable, distribution A exceeds this level with greater probability than does distribution B.

This result suggests the criterion to be considered here. In particular, expected utility cost may be minimized subject to the condition that the probability of exceeding a specified level is held below a certain probability. In effect, the original minimization problem has an additional constraint added.

Problem Formulation

Consider the following problem formulation:

$$\text{minimize} \quad E \sum_i \sum_j c_{ij} x_{ij} = \sum_i \sum_j \bar{c}_{ij} x_{ij}$$

$$\text{subject to} \quad \sum_i x_{ij} = 1 \qquad \text{for all } j$$

$$\sum_j x_{ij} = 1 \qquad \text{for all } i$$

$$x_{ij} = 0,\ 1 \qquad \text{for all } i,\ j$$

$$\text{Prob}\left(\sum_i \sum_j c_{ij} x_{ij} > k\right) \leq p$$

where c_{ij} is a random variable, $\bar{c}_{ij}$ is the mean value of c_{ij}, k is the upper limit of the allowable cost and p is a stated probability.

The chance constraint in this problem is not in a form that can be incorporated into a solution scheme. Thus a new constraint is needed which is equivalent to the present one but is in a usable form. In order to construct such a constraint, first note that the total utility cost is a random variable with mean $\sum_i \sum_j \bar{c}_{ij} x_{ij}$. If the individual random variables are mutually independent, then the variance of the sum is equal to the sum of the individual variances. Letting s_{ij}^2 be the variance of c_{ij}, it follows that

$$\mathrm{Var}(c_{ij}) = \sum_i \sum_j s_{ij}^2 \, x_{ij}$$

If each of the c_{ij} are normally distributed, the total utility cost is normally distributed. It is now clear that

$$\mathrm{Prob}\ (\sum_i \sum_j c_{ij}\, x_{ij} > k) \leq p$$

is equivalent to the non-stochastic constraint

$$\sum_i \sum_j \bar{c}_{ij}\, x_{ij} + t\ (\sum_i \sum_j s_{ij}^2\, x_{ij})^{\frac{1}{2}} \leq k.$$

where t is the standard normal deviate corresponding to an upper-tail probability of p. See Vajda (27) for a discussion of the construction of constraints of this type.

Computational Methods

The problem statement above is of the form of a 0-1 integer programming problem. The primary solution techniques available are based on linear objective functions and a set of linear constraints. In this formulation, however, the chance constraint contains a non-linearity. It is possible that modification of one of the well-known 0-1 integer programming algorithms such as that of Geoffrion (9) might lead to a suitable computational technique. This approach, however, is not pursued in this thesis.

Murty (20) has presented an algorithm for ranking all possible solutions to the linear assignment problem in order of non-decreasing cost. This algorithm, which is based on a branch and bound process, is discussed briefly in Appendix B. In the context of the present formulation, Murty's algorithm may be employed to generate solutions to the linear assignment problem sequentially. As each assignment is generated, it is tested against the chance constraint. Since assignments are ranked by cost in non-decreasing order, the first assignment which satisfies the chance constraint is the optimal solution to the chance-constrained problem.

The efficiency of this approach varies widely with the particular problem under consideration. If the optimal solution to the stochastic problem is of relatively low expected cost, the algorithm may be quite efficient. If, however, the optimal stochastic solution is of relatively high expected cost, many assignments may be generated before the optimal solution is identified. The technique is used in this thesis because it is illustrative of the nature of the problem and it guarantees an optimal solution if one exists.

<u>An Example</u>

Table 7 presents hypothetical frequency-of-use data for a set of five instruments and distance measures for five locations. In this case, the distance parameter represents geometric distance from instrument panel center to the individual instrument center. Note that frequencies are expressed in terms of probability density functions where $N(\mu, \delta^2)$ signifies a normal distribution with mean μ and variance δ^2.

The ij^{th} cell in Table 8 gives the product of the i^{th} frequency

Table 7. Frequency-of-Use Distributions and Distance Measures

Component	Frequency-of-Use Distribution
A	N(10.00, 0.49)
B	N(8.00, 0.36)
C	N(7.00, 5.76)
D	N(3.00, 0.25)
E	N(2.00, 1.00)

Location	Distance from Panel Center
I	1
II	3
III	4
IV	7
V	8

Table 8. Matrix of Expected Utility Costs

			Distance				
			1	3	4	7	8
			I	II	III	IV	V
Frequency	10	A	10	30	40	70	80
	8	B	8	24	32	56	64
	7	C	7	21	28	49	56
	3	D	3	9	12	21	24
	2	E	2	6	8	14	16

and the j^{th} distance. Since the frequency values in this table are expected values while the distances are constants, the cell values are expected values of utility cost components. The frequencies have been arranged in non-increasing order and the distances have been arranged in non-decreasing order so that, as noted previously, the minimum cost assignment lies along the main diagonal. Letting a(k) denote the k^{th} best assignment, a(1) = (A,I), (B,II), (C,III), (D,IV), (E,V). Similarly, letting c(k) denote the expected cost of the k^{th} best assignment, c(1) = 99.

On the basis of experimental studies, it was decided that a desirable design criterion is the Freund-Sadosky utility cost minimization procedure. Additionally, studies indicated that it was highly undesirable to operate at any time at a utility cost level greater than 120. Thus an additional constraint was imposed which states that the probability of a utility cost level greater than 120 should not exceed 0.05.

Consider Table 9 in which the ij^{th} cell is the component of utility cost variance which results from the ij^{th} assignment. Derivation of this variance matrix is given in Appendix C. Based on the independence assumption, the variance of the total utility cost distribution is the sum of the variances of the appropriate cells for a given assignment. Letting v(k) denote the variance of the cost distribution for assignment a(k), it follows that v(1) = 0.49, + 3.24 + 92.16 + 12.25 + 64.00 = 172.14.

The standard normal deviate corresponding to an upper-tail probability of 0.05 is equal to 1.65. Thus, in order to satisfy the chance

Table 9. Matrix of Utility Cost Variances

Variance of Frequency Distribution		(Distance)^2 1 I	9 II	16 III	49 IV	64 V
0.49	A	0.49	4.41	7.84	24.01	31.36
0.36	B	0.36	3.24	5.76	17.64	23.04
5.76	C	5.76	51.84	92.16	282.24	368.64
0.25	D	0.25	2.25	4.00	12.25	16.00
1.00	E	1.00	9.00	16.00	49.00	64.00

constraint, the point 1.65 standard deviations above the mean of the distribution must be below 120 utility cost units. In this case, the standard deviation is 13.12 and thus, letting b(k) = t(v(k)) + c(k), b(1) = 120.65 $>$ 120.00.

Table 10 gives a(k), $\overline{c}$(k), v(k), $\sqrt{v(k)}$, and b(k) for k = 1, 2, 3. Note that the assignments a(2) and a(3) are of equal cost. Thus they would be of equal desirability from an expected cost viewpoint. However, b(2) = 120.92 $>$ 120.00 while b(3) = 119.12 $<$ 120.00. Clearly, a(3) is the optimal assignment to the chance constrained problem.

Table 10. Results for Three Best Assignments

k	a(k)	c(k)	v(k)	$\sqrt{v(k)}$	p(k)
1	(A,I), (B,II), (C,III), (D,IV), (E,V)	99.00	172.14	13.12	120.65
2	(A,I), (B,II), (C,III), (D,V), E,IV)	100.00	160.89	12.68	120.92
3	(A,I), (B,III), (C,II), (D,IV), (E,V)	100.00	134.34	11.59	119.12

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this thesis has been to develop a broad conceptual framework for the application of the methodology of operations research to problems in the design of man-machine systems. Particular emphasis has been placed on the development of measures of design effectiveness and the selection of appropriate quantitative techniques. The approach has been illustrated by consideration of the specific example of the layout of instrument panels.

Any attempt to apply this methodology to human activity systems must be based on the fundamental assumption that behavior can be quantified. The French mathematician Henry Bergson has expressed this premise eloquently. Bergson (2) has said:

> I have sometimes wondered what would have happened if modern science, instead of turning from mathematics in the direction of mechanics, astronomy, physics, and chemistry, and focusing the whole of its effort on the study of matter, had concentrated instead on the study of the human mind. Our knowledge of psychology would probably bear much the same relation to our existing psychology as modern physics bears of Aristotle.
>
> The place of mathematics in the human sciences is already an important one and before long it will become predominant. Into such diverse fields as psychology, economics, semantics, and philology, mathematics brings clarity and precision of method. Operations research will be the future science of action.

Assumptions of Existing Criteria

It has been repeatedly pointed out that any measure of design effectiveness includes certain assumptions. In this section certain of the assumptions common to all the criteria mentioned in this thesis will be examined.

Gainer and Obermayer (7) have presented two problems which are not considered in the criteria examined in this thesis. They term these "looking without seeing" and "seeing without looking." Basically, this means that the operator may focus on an instrument without actually obtaining any information from it or he may obtain information by peripheral vision without directly focusing on the instrument. The corresponding assumption has been that such factors are negligible.

Another assumption has been that all focuses are at the center of the instrument and that all eye movements are in straight lines. Although there have been numerous studies of eye movement patterns, most of these are laboratory investigations. There is a shortage of studies of such eye movements patterns in actual system operation.

It is tacitly assumed that all eye movements of a given distance are equivalent regardless of direction. Thus, movements in the horizontal and vertical planes are treated exactly the same.

All of the points mentioned above are subject to assumption at the present time because adequate experimental data is not available. It is only by way of such studies that the validity or lack of validity of the assumptions can be determined.

Areas for Further Research

Many areas of potentially valuable research are indicated. These areas fall primarily into two distinct categories. The first category concerns experimental studies to develop new design criteria and to evaluate the ones which now exist. Secondly, new computational procedures should be examined.

The assumptions introduced above indicate several possible research areas. Other areas may concentrate more specifically on the information processing aspects of human operator performance. For example, studies of the temporal factors in instrument monitoring might examine the distribution of eye fixation times. Certainly, some instruments require more time to obtain information than do others. The various utility cost concepts must be studied to determine which are most highly correlated with operator performance. Similarly, new utility cost models may be considered.

A far more basic problem is to establish the extent to which eye movements are indicative of performance. This area has not been fully explored and can only be verified on a rigorous empirical basis.

A primary problem in the area of computational procedures is the inability of existing algorithms to handle combinatorial problems of this magnitude efficiently. This is certainly the most potentially promising area of research. Since the general area of assignment algorithms continues to be one of vigorous research, improved techniques may well be available in the future.

One approach to the improvement of such techniques is the development of a more efficient ranking algorithm to replace the Murty procedure. Murty's algorithm is general in that it can be applied to the general assignment problem. A procedure developed particularly for the product-type assignment problem could take advantage of the characteristics of that matrix. An efficient technique of this nature could be used not only for the stochastic assignment problem but also for the quadratic assignment problem.

The two general areas mentioned above provide many opportunities for research. With further investigation of design criteria and improved computational algorithms, the application of operations research methodology to the design of man-machine systems will become increasingly prevalent.

APPENDIX A

PROOF OF THE PRODUCT TECHNIQUE

Theorem: Let $\{a_i\} = a_1, a_2, \ldots, a_n$ be a sequence of n elements arranged in non-decreasing order. Similarly, let $b_1, b_2, \ldots, b_n$ be a non-increasing sequence of n elements. It follows that $\sum_{i=1}^{n} a_i b_i$ is at least as small as the corresponding sum of pair-wise products of any permutations of the two sequences.

Proof: Clearly, $i < j$ implies that $a_i \leq a_j$ and that $b_i \geq b_j$. Consider $\{a_i'\}$, a particular permutation of $\{a_i\}$, in which $a_i = a_i'$ for all $i \neq j$, $i \neq k$, and $a_j = a_k'$, $a'_j = a_k$.

$$\text{Now, } \sum_{i=1}^{n} a_i b_i - \sum_{i=1}^{n} a_i' b_i = a_j b_j + a_k b_k - a_j' b_k - a_k' b_k$$

$$= a_j b_j + a_k b_k - a_k b_j - a_j b_k$$

$$= (a_j - a_k)(b_j - b_k)$$

If $j < k$, then $(a_j - a_k) \leq 0$ and $(b_j - b_k) \geq 0$.

If $j > k$, then $(a_j - a_k) \geq 0$ and $(b_j - b_k) \leq 0$.

In either case, $(a_j - a_k)(b_j - b_k) \leq 0$.

Thus, $\sum_{i=1}^{n} a_i b_i - \sum_{i=1}^{n} a_i' b_i \leq 0$.

$$\sum_{i=1}^{n} a_i b_i \leq \sum_{i=1}^{n} a_i' b_i$$

Any permutation $\{a_i^*\}$ of $\{a_i\}$ can be obtained by a succession of such transpositions. Hence, $\sum_{i=1}^{n} a_i b_i \leq \sum_{i=1}^{n} a_i^* b_i$ for any permutation $\{a_i^*\}$.

A similar result holds for any permutation $\{b_i^*\}$.

Q.E.D.

APPENDIX B

DESCRIPTION OF MURTY'S ALGORITHM

Murty (20) has presented an algorithm for the ranking of all assignments to an assignment problem in order of increasing cost. It is important to note that this algorithm operates only on the linear problem and is not available for the quadratic assignment problem.

The technique is based on a branch-and-bound procedure. If $X = (x_{ij})$ is the assignment matrix, it is known that an acceptable assignment requires that X be made up of exactly one unit entry in each row and each column. A particular assignment is called the minimal assignment if the sum of the costs associated with the cells with unit entries is a minimum.

A partial assignment consists of a matrix X as above with one or more rows and an equal number of columns deleted. It is clear that the minimum cost completion for any partial assignment is obtained by solving the assignment problem consisting of the deleted rows and columns. In this manner, a lower bound can be found for the completion to any partial solution.

It is in this manner that the branch-and-bound procedure is utilized. The partial solutions are nodes for which lower bounds are computed by obtaining the minimum cost solution from the minimal completion. By a partitioning scheme, Murty is able to systematically generate assignments in order of increasing cost.

APPENDIX C

DERIVATION OF VARIANCE MATRIX

The purpose of this appendix is to briefly outline the procedure for obtaining the variance matrix in the stochastic assignment procedure. In particular, Table 9 of the text is such a matrix.

The fundamental property which is used in this development concerns the variance of a random variable. If X is a random variable, and a is a constant, then variance (aX) = a^2 variance (X). This property is developed in standard elementary statistics texts and the proof is omitted here.

Within the context of the stochastic assignment problem, utility cost is defined as the product of frequency and distance. Since frequency is a random variable and distance is a constant, it follows that utility cost is a random variable with variance equal to the variance of the frequency multiplied by the square of the distance.

Since total utility cost is the sum of such random variables, it is useful to employ the fact that the variance of the sum of independent random variables is equal to the sum of the variances. Thus, under the independence assumption, the appropriate entries in the variance matrix may be added to obtain the variance of the total utility cost for a particular assignment.

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