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Decentralized Adaptive Control of Robot Manipulators with Robust Stabilization Design

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Abstract

Due to geometric nonlinearities and complex dynamics, a decentralized technique for adaptive control for multilink robot arms is attractive. Lyapunov-function theory for stability analysis provides an approach to robust stabilization. Each joint of the arm is treated as a component subsystem. The adaptive controller is made locally stable with servo signals including proportional and integral gains. This results in the bound on the dynamical interactions with other subsystems. A nonlinear controller which stabilizes the system with uniform boundedness is used to improve the robustness properties of the overall system. As a result, the robot tracks the reference trajectories with convergence. This strategy makes computation simple and therefore facilitates real-time implementation.

I. Introduction

Using conventional control methods, improvement in the quality of dynamic performance of robot manipulators is difficult to achieve due to the geometric nonlinearities and complex dynamics. Consequently, increased demand for manipulator performance results in the need for advanced control algorithm. One of such control algorithms which promise better tracking accuracy than the traditional control utilizes adaptive techniques that have been widely proposed [1-8, 10-12]. Model reference adaptive control (MRAC) is mostly adopted in robot motion. This results from an effective scheme given by the adaptive control to decouple and linearize the robot dynamic system.

It is usually assumed that the variation of robot dynamic parameters is slower than the speed of adaptation. Then, the robot dynamics can be characterized by a set of decoupled and linear time-varying equations of second order such that the gradient method can be used to adjust parameters of equations [1]. Alternatively, stability analysis based on hyperstability may be imposed to compensate nonlinear terms and decouple the dynamic interaction [2]. By implementing a computed torque method [9], the error dynamics system can thus match the reparametrization condition [6,7] in order to reduce computing time on the parameter. The Lyapunov technique is used for stability proof. However, the parameter adaptation still needs much real-time calculation. The signal-synthesis scheme may solve this problem with adaptive control. By separating the system into constant part and time-varying part, the feedback gains, which are adjustable, can be derived from the stability analysis [3,10], connection while the perfect model following conditions need to be satisfied [11]. Moreover, improvement of the system performance can be achieved by an auxiliary signal of integral errors [12].

Dynamically, the robot can be treated as the combination of individual link if the coupling of the system is neglected. Conventional PID control can thus be applied to each joint for trajectory tracking. The coupling terms in the dynamics degrade performance, however, to each joint for trajectory tracking [9]. The interconnection of the system dynamics deteriorates the performance of system responses, especially in high-speed motion or complex tasks with large nonlinear effects. An adaptive decentralized control algorithm which has a simple structure has been presented to adjust feedback gains corresponding to each joint [8,13]. However, the paper which follows provides several extensions of the existing work [8,10,13,14]. Each joint is considered as a subsystem so that the overall dynamic system can be expressed as a set of interconnected subsystems of interconnected terms. The decentralized constant gains are imposed to stabilize the subsystem locally, and the interconnected terms which are treated as uncertainty [14] thus have certain boundedness. Under satisfaction of the matching conditions [15] the adaptive control guarantees the state error between the reference model and the system to be uniformly bounded. However, the faster convergence rate of the state error can be achieved due to application of the integral gains. The simple structure of this controller significantly reduces computation time. Meanwhile, the variation of coupling terms of the robot dynamics need not be assumed slowly time-varying. Computer simulations will be used to compare results with and without the application of integral gains.

II. Dynamic Modeling and System Configuration

In the absence of actuator dynamics, friction and other disturbances, the system's dynamic behavior of an n-link rigid manipulator is formulated in terms of the kinetic and potential energies, expressed in terms of generalized coordinates, q_i, for each link-joint, i. Lagrange's equation is here imposed to derive the overall dynamic system, given by the following equation,

$$\sum_{j=1}^{n} m_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} c_{ijk}(q)\dot{q}_j\dot{q}_k + g_i(q) = \tau_i \qquad (2\text{-}1)$$

where

$i = 1,2,....,n$

$q^T = [q_1\ q_2\ ....\ q_n]$

m_{ij} = The inertia element

$c_{ijk}\dot{q}_j\dot{q}_k$ = Centrifugal and Coriolis forces

g_i = The gravity force

τ_i = The input torque

Thus, for the individual joint i, the dynamics equation (2-1) is of a second order nonlinear differential form. For the total system, the dynamic equations are written in the following matrix form

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = T \tag{2-2}$$

M(q), the nxn inertia matrix, is positive definite. Therefore, one can define a positive matrix β such that

$$\beta \geq ||M^{-1}(q) - \beta|| \ , \tag{2-3}$$

where $||\cdot||$ is an induced norm and β is diag(β_1,..,β_n).

Equation (2-2) can then be rewritten as

$$\ddot{q} = -M^{-1}(q)[C(q,\dot{q})+G(q)] + \beta T + (M^{-1}(q)-\beta)T \tag{2-4}$$

Now, we consider each joint i as a subsystem of the overall system (2-4). Defining state variables $x_i^T = [q_i \ \dot{q}_i]$ and a control input $\tau_i = u_i$, equation (2-4) can be divided into n equations for the n interconnected subsystems. Therefore, each subsystem is described by a first-order differential equation of the form

$$\dot{x}_i = A_i x_i + b_i u_i + F_i(X) + f_i(X)u_i \tag{2-5}$$

where

$i = 1,2,\ldots,n$

$X^T = [x_1^T \ x_2^T \ldots x_n^T]$

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{2-5a}$$

$$b_i = \begin{bmatrix} 0 \\ \beta_i \end{bmatrix} \tag{2-5b}$$

$F_i(X)$ = Coupling terms of $-M^{-1}(q)[C(q,\dot{q})$ $+G(q)]$ for subsystem i (2-5c)

$f_i(X)$ = Coupling terms of $(M^{-1}(q)-\beta)$ for subsystem i (2-5d)

$F_i(X)$ and $f_i(X)$ that are assumed to be bounded and are modelled as system uncertainties and are assumed to have the properties [15],

$$F_i(X) \triangleq F_i(X,\sigma) \tag{2-6}$$

$$f_i(X) \triangleq f_i(X,\sigma)$$

where $\sigma \in R^p$ represents the system uncertainty and is continuous on R^p as well as the uncertainty bounding set.

Moreover, there exist matrix functions D(•) and E(•) such that

$$F_i(X,\sigma) = b_i D_i(X,\sigma) \tag{2-7}$$

$$f_i(X,\sigma) = b_i E_i(X,\sigma)$$

$$\text{where } ||E_i|| < 1 \text{ from (2-3)} \tag{2-7a}$$

From the control point of view, the signal-synthesis adaptation implemented here assumes the satisfaction of the matching conditions (2-7). These conditions guarantee that the uncertainty vector does not influence the dynamics more than the control input does [16]. However, the uncertainties (2-6) which do not satisfy (2-7) can be tolerated if it does not exceed the mismatch threshold [17].

Therefore, the overall system takes the following matrix form

$$\dot{X} = AX + BU + BD(X,\sigma) + BE(X,\sigma) \tag{2-8}$$

where for i = 1,2,....,n

$A = \text{diagonal}(A_i)$,

$B = \text{diagonal}(B_i)$,

$D(X,\sigma) = \text{diagonal}(D_i(X,\sigma))$,

$E(X,\sigma) = \text{diagonal}(E_i(X,\sigma))$ and

$U^T = [u_1 \ u_2 \ldots u_n]$

III. Robust Adaptive Algorithm

The objective of model reference adaptive control is to eliminate the state error between the plant and the reference model so that the behavior of the plant and the model follow each other. Hence, consider the reference model first

$$\dot{x}_{mi} = A_{mi} x_{mi} + b_{mi} r_i \tag{3-1}$$

where for i = 1,2,....,n

$x_{mi}^T = [q_{mi} \ \dot{q}_{mi}]$

$$A_{mi} = \begin{bmatrix} 0 & 1 \\ a_{mi1} & a_{mi2} \end{bmatrix}$$

$$b_{mi} = \begin{bmatrix} 0 \\ \beta_{mi} \end{bmatrix}$$

and let

$$A_{mi} = A_i + b_i K_{xi} \tag{3-2}$$

$$b_{mi} = b_i K_{bi} \tag{3-3}$$

where K_{xi} and K_{bi} that are constant matrix have the corresponding dimension.

Also, A_{mi} which is a stable matrix satisfies the Lyapunov equation

$$A_{mi}^T P_i + P_i A_{mi} = -Q_i \tag{3-4}$$

where P_i and Q_i are positive definite and symmetric matrices.

The signal-synthesis method is implemented here to control the system by adjusting the input u_i which is described in the following equation

$$u_i = K_{xi} x_i + K_{bi} r_i + \psi_i(e_i) \tag{3-5}$$

where $e_i = x_{mi} - x_i$ is referred to as state error and the function ψ_i is the control input to compensate the system uncertainty. Thus, let ψ_i be

$$\psi_i(e_i) = \begin{cases} \dfrac{b_i^T P_i e_i}{||b_i^T P_i e_i||} \rho_i(X,e_i), & \text{when } ||b_i^T P_i e_i|| > \delta_i \\ \dfrac{b_i^T P_i e_i}{\delta_i} \rho_i(X,e_i), & \text{when } ||b_i^T P_i e_i|| \leq \delta_i \end{cases} \tag{3-6}$$

where δ_i is a prescribed positive constant and ρ_i is a positive constant which will be specified subsequently.

As a result, the error dynamics of subsystem is derived from the difference between equations (3-1) and (2-5) along with (2-7) as well as (3-5)

$$\dot{e}_i = \dot{x}_{mi} - \dot{x}_i \tag{3-7}$$

$$= [A_{mi}x_{mi} + b_{mi}r_i] - [(A_i + b_iK_{xi})x_i$$

$$+ b_iK_{bi}r_i + b_iD_i(X)$$

$$+ b_iE_i(X)(K_{xi}x_i + K_{bi}r_i + \psi_i(e_i))]$$

$$= A_{mi}e_i - b_i(\psi_i + \upsilon_i)$$

$$\text{where } \upsilon_i = D_i(X) + E_i(X)[K_{xi}x_i + K_{bi}r_i + \psi_i(e_i)] \tag{3-8}$$

Due to boundedness of the state variable x_i and the control input u_i, equations (3-8) and (3-6) give the following inequality

$$||\upsilon_i|| \leq ||D_i(X)|| + ||E_i(X)||(||K_{xi}x_i||$$

$$+ ||K_{bi}r_i|| + ||\psi_i(e_i)||) \tag{3-9}$$

$$\triangleq \rho_i(X, e_i)$$

The definition of ρ_i in (3-9) is valid, i.e. (3-9) can be solved since (2-7a) is satisfied. Therefore, we have

$$\rho_i = (1 - ||E_i||)^{-1}[||D_i|| + ||E_i||(||K_{xi}x_i||$$

$$+ ||K_{bi}r_i||)] \tag{3-10}$$

Consequently, the stabilization of the error dynamics for the subsystem is as shown below.

Lemma 1 : The error dynamics (3-7) of subsystem is locally uniformly bounded, if the control input u_i is given by (3-5) with bounded reference states.

Proof : The system is said to be uniformly bounded, iff the candidate Lyapunov function V is positive definite and V is negative semi-definite [18].

So, choose

$$V = \sum_{j=1}^{n} V_i = \sum_{j=1}^{n} e_i^T P_i e_i \tag{3-11}$$

and then

$$\dot{V}_i = \dot{e}_i^T P_i e_i + e_i^T P_i \dot{e}_i \tag{3-12}$$

$$= e_i^T(A_{mi}^T P_i + P_i A_{mi})e_i - 2e_i^T P_i b_i(\psi_i + \upsilon_i)$$

$$= -e_i^T Q_i e_i - 2e_i^T P_i b_i(\psi_i + \upsilon_i)$$

From (3-6), we have

$$e_i^T P_i b_i(\psi_i + \upsilon_i) \geq ||e_i^T P_i b_i||\rho_i -$$

$$||e_i^T P_i b_i||\ ||\upsilon_i|| \geq 0$$

Therefore, (3-12) becomes

$$\dot{V}_i \leq -e_i^T Q_i e_i \leq 0$$

$$\dot{V} = \sum_{i=1}^{n} \dot{V}_i \leq 0 \qquad \text{Q.E.D.}$$

Concerning the choice of a Lyapunov function candidate V, it is sufficient that V is continuously differentiable and positive definite. If the conditions imposed on $\dot{V}$ are also satisfied, then V is referred to as a Lyapunov function [19]. Henceforth, an auxiliary signal $w_i(t)$ which is time-varying and differentiable is added to the control input u_i (3-5) for the improvement of convergence rate of equation (3-7) and the candidate V is chosen as a function of e_i and w_i. The equation (3-5) is now expressed as

$$u_i = K_{xi}x_i + K_{bi}r_i + \psi_i(e_i) + w_i \tag{3-13}$$

and the error dynamics (3-7) also becomes

$$\dot{e}_i = A_{mi}e_i - b_i(\psi_i + \upsilon_i + w_i) \tag{3-14}$$

where υ_i still has the same form as (3-9) and is bounded in ρ_i. The auxiliary signal $w_i(t)$ is a function of the state error given by

$$\dot{w}_i = \alpha_i b_i^T P_i e_i \qquad ,\alpha_i > 0 \tag{3-15}$$

Thus, the stability of (3-14) and (3-15) has to be analyzed. The augmented dynamic system becomes

$$\begin{bmatrix} \dot{e}_i(t) \\ \dot{w}_i(t) \end{bmatrix} = \begin{bmatrix} A_{mi} & -b_i \\ \alpha_i b_i^T P_i & 0 \end{bmatrix} \begin{bmatrix} e_i \\ w_i \end{bmatrix} + \begin{bmatrix} b_i \\ 0 \end{bmatrix} (\psi_i + \upsilon_i) \tag{3-16}$$

Theorem 1 : The system (3-16) is uniformly bounded.

Proof : Choose

$$V_i = e_i^T P_i e_i + w_i^T \alpha_i^{-1} w_i \tag{3-17}$$

$$\dot{V}_i = e_i^T(A_{mi}^T P_i + P_i A_{mi})e_i + 2w_i^T \alpha_i^{-1}\dot{w}_i \tag{3-18}$$

$$= -e_i^T Q_i e_i - 2e_i^T P_i b_i(\psi_i + \upsilon_i + w_i) + 2w_i^T \alpha_i^{-1}\dot{w}_i$$

$$= -e_i^T Q_i e_i - 2e_i^T P_i b_i(\psi_i + \upsilon_i) + 2w_i^T[\alpha_i^T \dot{w}_i$$

$$- b_i^T P_i e_i]$$

From (3-15), the equation (3-18) is

$$\dot{V}_i = -e_i^T Q_i e_i - 2e_i^T P_i b_i(\psi_i + \upsilon_i)$$

From lemma 1,

$$\dot{V} \leq 0 \qquad \text{Q.E.D.}$$

The convergence rate in the error dynamics (3-17), (3-14) can be compared by a positive value $-\dot{V}/V$. Obviously, the Lyapunov function candidate V_i in (3-16) is larger than that in (3-11), while they have the same $\dot{V}_i$; i.e. the control input with an auxiliary signal, (3-13) has a faster convergence rate. However, the nonlinear control ψ_i in (3-6) is to improve robustness of the overall system.

To summarize the procedure for robust stabilization design : The inertia matrix can be acquired from the dynamic equation (2-1) so that β is determined from (2-3) to form b_i in equation (2-5) and A_i is (2-5a). According to equations (3-2) and (3-3), the constant feedback gains K_{xi} and K_{bi} are calculated respectively and the reference model A_{mi} and b_{mi} can be chosen to be stable. P_i and Q_i are from the matrix Lyapunov equation (3-4). The reference input r_i can directly be derived from the inverse dynamics of the reference model; i.e. $r_i = \beta_{mi}^{-1}(\ddot{q}_{mi} - a_{mi2}\dot{q}_{mi} - a_{mi1}q_{mi})$ at each time.

Finally, the control input u_i is given by equation (3-5) and with an auxiliary signal given by equation (3-13). Note that ρ_i satisfies inequality (3-10) and those bounds can be determined from the workspace domain of robot manipulators.

IV. Case study

In the following, a case study is performed for the control of a light-weight arm with parallel mechanism existing in the Flexible Automation Laboratory at Georgia Tech. In exchange for light weight in the design of the links the system flexibility needs to be considered. However, if the payload on the end is light, the link flexibility can be treated as a bounded uncertainty to the rigid-body system [20]. The computer simulation which is based on the rigid-body dynamic model is thus used to illustrate the results of the control algorithm derived in this paper for robot manipulators, neglecting the system flexibility.

The robot structure [Fig.1] consists of two 3m long links made of aluminum tubing and a parallel actuating link made of rectangular aluminum tubing [21]. To simplify the analysis, the cylindrical sleeves at the connection of the lower link and the upper link are modelled as concentrated masses. The lower and the upper links are 12 kg and 13 kg respectively, while the point masses at each end of links are 20 kg and 30 kg. This system is assumed to have motion in the vertical plane from 0 rad. to 1.5 rad. in 5 seconds, following a standard trapezoidal velocity profile with maximum velocity 0.4 rad./sec.

The inertia matrix M(q) in (2-3) has eigenvalues between 37.6 and 1805.4. β_1 and β_2 are chosen as 0.001 satifying the inequality (2-3). The reference model in (3-1) for each joint is $a_{mi1} = -1$, $a_{mi2} = -2$ and $\beta_{mi} = 1$ so that the constant gains K_{xi} and K_{bi} are $[-1000\ \ -2000]^T$ and 1000 respectively. With Q = 2I in (3-4), the matrix Lyapunov equation is solved to yield

$$P_i = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \qquad (4\text{-}1)$$

The upper bound for ρ_i which is determined from (3-9) and (3-10) is here chosen as 1000, while the prescribed constant δ_i in (3-6) is 5. Note that the exact value for ρ_i is computed from the functional property of the bound of uncertainty and the assumption of the existence of the bound. The appropriate value of P_i will be investigated in the future. Finally, the adaptive integral gains α_1 and α_2 are 2000 and 1000 respectively such that the state errors converge faster than without the auxiliary signal. However, this algorithm which can accommodate the system uncertainties and the time-varying inertia matrix leads to an adaptive approach with high gains, but demonstrates a simple control structure without increasing on-line computation. Therefore, it is believed that this technique is more suitable for the control of the light-weight arm than the conventional manipulator.

Different sets of simulations have been carried out, one with (3-5) and the other with (3-13). First, consider the case (Fig. 2 - Fig. 3) of the decentralized controller (3-5) without the integral gain which is similar to other work [14]. The joint position for this two-link manipulator is bounded and converge to the reference states (Fig. 2), while the joint velocity is bounded but oscillatory (Fig. 3). This agrees with our analysis for the adaptive controller with robust stabilization design. Due to the auxiliary integral signal, the state errors are shown to have faster convergence. For comparative purposes, Fig. 4 - Fig. 7 are the joint position and velocity errors with and without the signal.

V. Conclusions

A decentralized adaptive control with robust stabilization has been presented for motion control of large-scale robot manipulators. Under consideration of the uncertainty for interconnected terms of each subsystem, the dynamic system of robot motion is illustrated to be bounded, while an auxiliary input with the integral gain has faster convergence rate and smaller steady-state error. The possible magnitude of the uncertainty is presumed known, making the statistical information for a stochastic approach unnecessary.

Without adaptation of parameters, this decentralized adaptive control has a simple control structure for reducing real-time calculation. Simulations show satisfying responses. Conceptually, this control algorithm is entirely applicable to light-weight manipulators, provided that modifications to the system dynamics which involves flexibility are made.

VI. Acknowledgement

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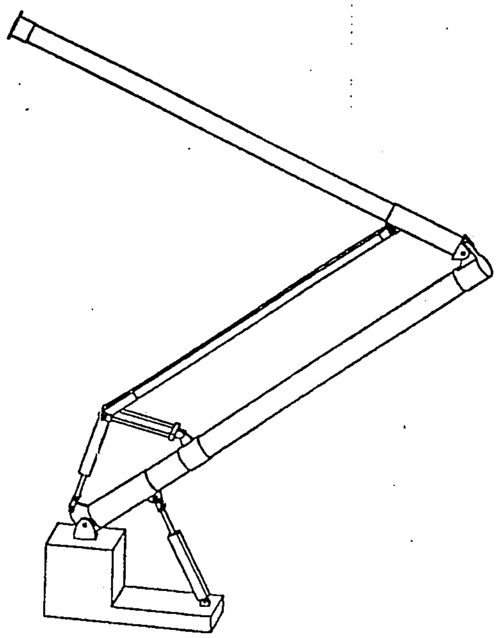

Figure 1. Two (very large) Robotic Arms.

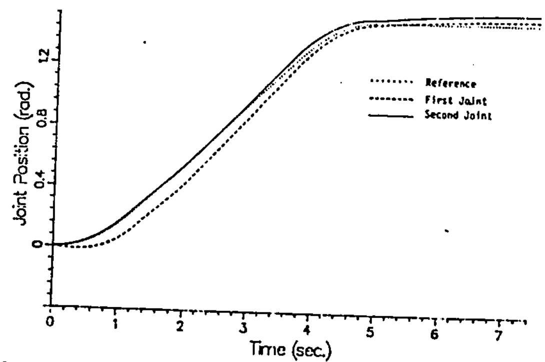

Figure 2. The Time Responses of the Joint Position (without the integral gain).

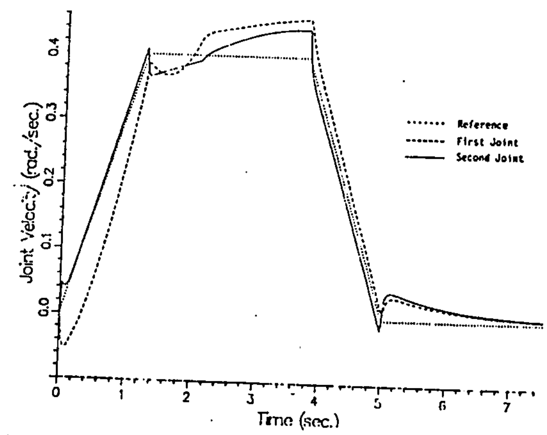

Figure 3. The Time Responses of the Joint Velocity (without the integral gain).

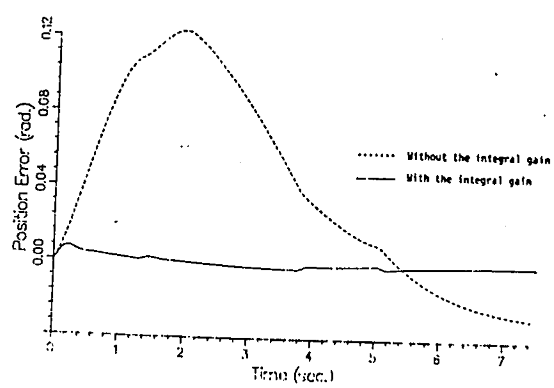

Figure 4. The Position Error Responses of the First Joint.

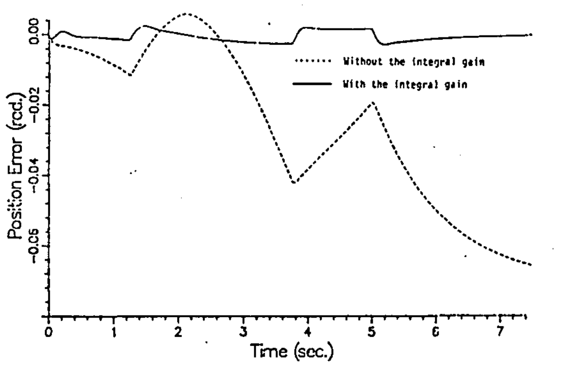

Figure 5. The Position Error Responses of the Second Joint.

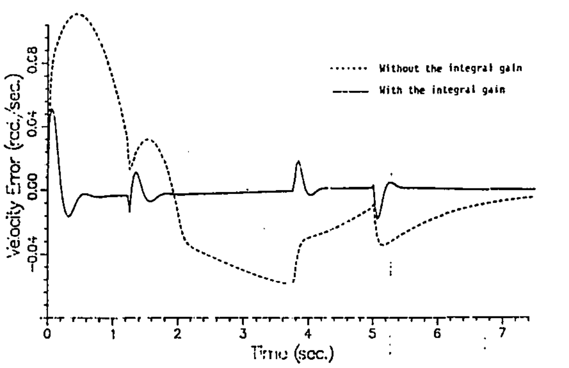

Figure 7. The Velocity Error Responses of the Second Joint.

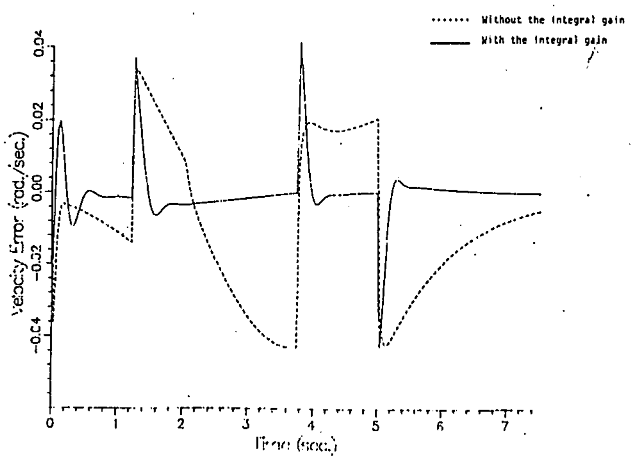

Figure 6. The Velocity Error Responses of the First Joint.