15:09:36 OCA PAD AMENDMENT - PROJECT HEADER INFORMATION 07/06/94

Active

Project #: E-24-X36 Cost share #: Rev #: 2
Center # : 10/24-6-R8019-0A0 Center shr #: OCA file #:
Work type : RES
Contract#: PO# P403247 Mod #: 1 Document : SUBCONT
Prime #: N00014-93-1-0234 Contract entity: GTRC

Subprojects ? : N CFDA: N/A
Main project #: PE #: N/A

Project unit: ISYE Unit code: 02.010.124
Project director(s):
MONTEIRO R DC ISYE (404)894-2300

Sponsor/division names: UNIVERSITY OF ARIZONA / TUCSON, AZ
Sponsor/division codes: 400 / 130

Award period: 930801 to 950930 (performance) 950930 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	36,756.00
Funded	0.00	36,756.00
Cost sharing amount		0.00

Does subcontracting plan apply ?: N

Title: POTENTIAL REDUCTION INTERIOR PT. METHODS FOR MIXED NONLINEAR COMPLEMENTARY...

PROJECT ADMINISTRATION DATA

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Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): N
Defense priority rating : N/A N/A supplemental sheet
Equipment title vests with: Sponsor GIT X
SEE SECTION (7), P. 3 OF SUBCONTRACT.
Administrative comments -
MOD 1 AUTHORIZES A 14-MOS EXTENSION TO 9/30/95, AS REQUESTED.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 11/01/95

Project No. E-24-X36 Center No. 10/24-6-R8019-0A0

Project Director MONTEIRO R DC School/Lab ISYE

Sponsor UNIVERSITY OF ARIZONA/TUCSON, AZ

Contract/Grant No. PO# P403247 Contract Entity GTRC

Prime Contract No. N00014-93-1-0234

Title POTENTIAL REDUCTION INTERIOR PT. METHODS FOR MIXED NONLINEAR COMPLEMENTAR

Effective Completion Date 950930 (Performance) 950930 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	Y	
Government Property Inventory & Related Certificate	N	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

Comments
NOTE***USE SPONSOR FORM FOR PATENT REPORT.

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

NOTE: ~~Final Patent Questionnaire sent to PDPI~~

E-24-X36
#1

Final Report for Grant No. N00014-93-1-0234
August 1, 1993 – September 30, 1995
(subcontract with the University of Arizona)

Potential Reduction Interior Point Methods for Mixed Nonlinear Complementarity Problems, Nonlinear Programs, and Extensions

by
Renato D.C. Monteiro

School of Industrial and Systems Engineering
Georgia Institute of Technology, Atlanta, GA 30332

I. Summary of Completed Project

A total of fourteen (14) papers and/or reports acknowledge the grant N00014-93-1-0234 among which nine (9) have already been accepted for publication. They are listed at the end of this report.

II. Main Research

The main purpose of this research project is to study interior-point algorithms for solving the nonlinear complementarity problem (NCP) and its extensions such as the mixed NCP and the constrained nonlinear equation problem, and to obtain more specialized convergence results for the particular case of the mixed NCP which arises from the Karush-Kuhn-Tucker (KKT) conditions of general nonlinear programming problems.

The papers [2, 3, 10, 11, 13] are directly related to the current project. The papers [2, 3, 13] have been done jointly with Professor Jong-Shi Pang and/or his graduate student Tao Wang. The other ones [10, 11] have been done jointly with my student Fangjun Zhou. I next give more specific comments on each of these works.

In the paper [3], which appeared on *SIAM Journal on Optimization*, we develop a new infeasible interior-point algorithm for solving the standard NCP. We have been able to derive convergence results for this algorithm when applied to NCP problems which are not necessarily monotone. This algorithm was tested on a number of equilibrium problems and randomly generated linear complementarity problems and we have found that the algorithm is computationally more efficient than a robust

algorithm, based on solving a sequence of successive quadratic programming problems, developed previously by Prof. Jong-Shi Pang and one of his collaborator.

In paper [2], which has been accepted for publication on *Mathematics of Operations Research*, Pang and I study the properties of an interior-point mapping associated with a new complementarity problem. This mapping has been systematically studied by Kojima and his collaborators in the context of the standard NCP. The introduction of this new complementarity problem allows us to generalize their work to several other complementarity problems including the mixed NCP (and hence, the Karush-Kuhn-Tucker (KKT) conditions of a nonlinear program) and the horizontal LCP. The understanding of this interior-point mapping is crucial in the development of path-following algorithms for solving these more general complementarity problems.

In [13], Pang, Wang and I develop a potential reduction interior-point algorithm for the constrained nonlinear equation problem, which provides an excellent framework for understanding primal-dual interior-point algorithms for solving NCPs and nonlinear programs. We then specialize the algorithm and its convergence results to the context of the mixed NCP and the KKT condition associated with a nonlinear program.

The work [10] studies existence conditions and the convergence behavior for the central path of convex programming problems in which both the objective and constraint functions are analytic.

Finally, the paper [11] establishes the superlinear convergence of primal-dual infeasible path following algorithms for solving linearly constrained convex programs satisfying the following conditions: 1) an optimal solution satisfying strict complementarity exists; 2) the Hessian of the objective function satisfies a certain invariance property.

III. Related Research.

Dr. Steve Wright from the Argonne National Laboratory and I have studied superlinear convergence of interior-point algorithms for monotone LCP. We have written three papers [7, 8, 9] on the subject and two of them [8, 9] have already appeared in press. In the work [8], we show that a large class of feasible and/or infeasible interior-point methods can not converge superlinearly when the LCP is degenerate. We have also studied in connection with degenerate LCP problems the issue of finite termination for interior-point algorithms and the rate of convergence of a feasible predictor-corrector method. In the other two papers [7, 9], we have developed primal-

dual feasible and infeasible affine scaling superlinearly convergent algorithms for the monotone LCP.

I have also been involved in a joint research with Dr. T. Tsuchiya from the Institute of Statistical Mathematics in Japan. We have written three joint papers [4, 5, 12]. The paper [4] studies the limiting behavior of certain continuous trajectories associated with the monotone LCP. The work [5] proves global convergence of the second-order affine scaling algorithm for convex quadratic programming. The paper [12] analyzes the effect of the step size length on the convergence of the affine scaling method and presents a two-step superlinearly convergent variant of the primal affine scaling algorithm for linear programs.

Jointly with my student Yanhui Wang, I have written two papers [6, 14]. In the work [6], we study a trust region affine scaling method for linearly constrained convex or concave problems. The paper [14] studies several types of degeneracy for linear programs.

Finally, in the paper [1], we present a novel approach for proving polynomial convergence of primal-dual path following algorithms for semidefinite programming. In particular, we extend for the first time the long-step primal-dual path following algorithms for linear programs to the context of semidefinite programming.

IV. Miscellaneous

This grant has partially supported one graduate student, Fangjun Wang, a Ph.D. student in the Industrial and Systems Engineering Dept. at Georgia Tech. I have been advising two Ph.D. students at Georgia Tech.

I have collaborated with several investigators including: Steve Wright (Argonne National Laboratory), Takashi Tsuchiya (Institute of Statistical Mathematics – Japan) and Jong-Shi Pang (John Hopkins University).

I was elected vice-chair of the area of linear programming and complementarity for the new Optimization section of INFORMS. I was also elected cluster chair for the ORSA/TIMS meeting of Phoenix (November 1993) and for the INFORMS meeting of Atlanta (November 1996).

I have been offered summer positions for the summer of 1992 and 1993 at Argonne National Laboratory.

I have moved from University of Arizona to a new faculty position at Georgia Tech.

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[2] R. D. C. Monteiro and J.-S. Pang. Properties of an interior-point mapping for mixed complementarity problems. manuscript, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0205, USA, December 1993. To appear on *Mathematics of Operations Research.*

[3] R. D. C. Monteiro, J.-S. Pang, and T. Wang. A positive algorithm for the nonlinear complementarity problem. *SIAM Journal on Optimization*, 5:129–148, 1995.

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[14] Y. Wang and R. D. C. Monteiro. Nondegeneracy of polyhedra and linear programs. Technical Report, School of ISyE, Georgia Institute of Technology, Atlanta, GA 30332, USA, August 1994. Accepted for publication on *Computational Optimization and Applications.*