THE APPLICATION OF A RHEOELECTRIC ANALOGY TO
THE SOLUTION OF TWO DIMENSIONAL
ELASTICITY PROBLEMS

A THESIS

Presented to the Faculty
of the Graduate Division
by
Donald Walker O'Neal

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Aeronautical Engineering

Georgia Institute of Technology
September, 1959

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Date Approved by Chairman: Jan 7, 1960

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ACKNOWLEDGMENTS

The author wishes to express his appreciation to Doctor Frank M. White, Jr. for his assistance, guidance, and interest in the course of this study. To Professor James A. Strickland and Doctor Joseph P. Vidosic gratitude is extended for their helpful suggestions and critical review of the work.

The author also wishes to express his thanks to the Aeronautical Engineering Department for the donation of the conducting paper used in the experiments and to the Mechanical Engineering Research Department for the use of its Field Analog Plotter which was used to apply voltage and measure equipotential points.

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NOTATION

a	circular hole diameter, in.
A	cross sectional area, sq. in.
c	one half the width allowed for a concentrated load on the conducting sheet, in.
e	variable normal distance from a boundary to successive equipotential lines
E	variable normal distance from a boundary to points corresponding to a uniform equipotential line spacing
h	plate thickness, in.
k	stress concentration factor based on average stress
m	arbitrary equipotential line
n	variable which is normal to a boundary
p	maximum principal stress, psi
q	minimum principal stress, psi
R	specific resistance of a conductor, ohms per sq. in.
S	total applied load, lb.
V	electric potential function, volt
w	plate width, in.
x	rectangular coordinate axis, in.
y	rectangular coordinate axis, in.
F(x)	non-uniform loading function
w(x)	influence function
θ_{pq}	angles describing the direction of principal stresses, p and q, with respect to the direction of σ_x, degree

σ_{av} average stress at the minimum (net) cross section of an elastic plate, psi

$\sigma_{x,y}$ normal stress components parallel to the x and y axes, respectively, psi

$\sigma(x)$ normal stress at any point due to a non-uniform loading, psi

τ_{xy} shearing stress in rectangular coordinates, psi

ϕ angle describing the tangent to the E vs. e curve, degree

κ constant stress trajectory

∇^2 operator del-squared, where $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$

SUMMARY

Presented herein is a simplified analogy for the solution of plane stress problems involving thin elastic plates under various loads on two opposite edges. The type of loads discussed are uniform, concentrated, and non-uniform. The solutions presented are given in terms of stress concentrations relative to the average cross-sectional stress.

The basis for the analogy, the mathematical similarities between electric potential fields in plane conductors and plane stress fields in elastic plates, is quite well known and is established concisely in this study to reflect the identity between equipotential lines and stress trajectories. A graphic representation of a stress field is obtained by the use of an analog device to construct the lines of equal potential, and stress concentrations are discerned as proportional to contractions of these lines. A physical measurement of these contractions, and subsequent comparison with uniformly spaced stress trajectories, permits evaluation of stress concentration factors.

Several two-dimensional problems, previously solved by analytical and photoelastic methods, are investigated, using the method of this study, to show compatible results with previous work. The rheoelectric analogy is then shown to be adaptable to various other problems.

This simplified analogy is seen to provide a rapid and inexpensive method to solve certain plane stress problems and to give accurate results. In other cases of plane stress the stress trajectories are obtained accurately, but the method used to separate the stresses fails to give satisfactory results.

CHAPTER I

INTRODUCTION

Solutions to plane stress problems in the early studies of strength of materials neglected non-uniformities in the stress pattern near points of load application and at points of rapid change in cross section. The neglect of these stress disturbances could be tolerated in static elasticity problems, but when the dynamic problem is considered, these disturbances, or stress concentrations, become of major importance. Although analytical methods for solving finite plates and plates with discontinuities have improved considerably in recent times, there are still numerous problems of practical significance for which no rigorous solution has been obtained. Some of these analytical solutions are also very lengthy and, therefore, subject to error. Experimental methods, although approximate, have become the only practical means of solution. Although the principle of photoelasticity is widely used, the analogy methods have recently become of particular interest.

Frequently, two or more apparently different physical phenomena can be expressed in identical mathematical form. When this condition exists, the systems are said to be analogous, and the characteristics of one system can be determined through inspection of a mathematical model of the other. In this study, for example, the similarities between electric potential and stress fields are used to determine stress trajectories, and thereupon stress distributions, for problems where analytical means are not readily applicable. The ease with which electric potential fields can be measured gives this particular analogy a high degree of flexibility and simplicity.

The term "rheoelectric", a word coined by French scientists, is used to avoid confusion with other types of electric analogies. It implies the use of a continuous conducting medium for the creation of an electrical flow pattern.

The use of rheoelectric analogies has become classic in the last thirty years (1)[1]. Jacobsen (2) in 1925 determined the stress concentration factors about a circular fillet in a shaft subjected to torsion by utilizing the distribution of electric potential in a continuous conductor, steel plate with one surface flat and the other curved. In 1931 Peres (3) set up the first of several French laboratories to explore problems in aerodynamics, fluid mechanics, and mathematical physics using electrolytic tanks. A review of his work was presented at the International Congress of Mechanics in 1938, and since that time rheoelectric apparatus has become an accepted media for supplementing theoretical work in numerous fields. Recently the use of conducting sheets has come into prominence for the study of torsional stress concentration problems (4), and for supplementing data obtained with photoelastic equipment (5).

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[1]Numbers in parenthesis indicate references found in the Literature Cited section of the Bibliography.

Historically, the study of stress concentrations began with the investigation of railroad axle failures. A rapid change in diameter where the journals joined the body of the axle caused fatigue cracks to appear at the re-entrant corners, and failure resulted. In his early investigations of that problem Wohler (6) recognized the difficulty as stress concentration and the situation was corrected by fairing the journal joint. Since this formal recognition of the concentration problem, considerable effort has been expended toward its solution, but, unfortunately, not always with success. As an example, the Civil Aeronautics Administration has recently reported aircraft parts failures, attributable to stress fatigue, at the rate of 30 per month (7).

The purpose of this paper is to demonstrate an additional experimental method of stress study by examining the suitability of a simplified rheoelectric analogy to the solution of stress concentration problems for thin elastic plates with various edge conditions. The types of plates considered are those pierced by holes and subjected to uniform tension on two opposite edges, and those whose dimensions keep the average stress from being realized, subjected to concentrated and non-uniform loads. To this end, comparisons are made with existing solutions, and solutions are obtained for various problems without analytical solutions.

CHAPTER II

MATHEMATICAL ANALOGY

From the theory of a plane stress system the magnitude and direction of the principal stresses, p and q, in terms of normal and shearing stress components, σ_x, σ_y, and τ_{xy}, are given by

$$p = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad (1)$$

$$q = \tfrac{1}{2}(\sigma_x + \sigma_y) - \tfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad (2)$$

$$\tan \theta_{pq} = 2\tau_{xy} / (\sigma_x - \sigma_y) \qquad (3)$$

where θ_{pq} represents both θ_p and θ_q. Upon solution of equation (3), two roots are obtained, 90 degrees apart, which define the directions of the principal stress p and q. Curves whose tangents represent these same directions are defined in photoelasticity as principal stress trajectories (8). Since the principal stresses at each point in a stress system are mutually perpendicular, it follows that the stress trajectories form a system of orthogonal curves.

If equations (1) and (2) are added, it is seen that

$$p + q = \sigma_x + \sigma_y \qquad (4)$$

The compatibility equation in terms of stress is

$$\nabla^2(\sigma_x + \sigma_y) = 0 \qquad (5)$$

where ∇^2 is $(\partial^2/\partial x^2 + \partial^2/\partial y^2)$. Substituting equation (4), equation (5) reduces to

$$\nabla^2(p + q) = 0 \tag{6}$$

At a point where p or q is zero, along a boundary or discontinuity, equation (6) becomes

$$\nabla^2 p = 0, \nabla^2 q = 0 \tag{7}$$

which is the well known Laplace equation.

If the continuous medium is a plate whose thickness h is small in comparison with its lateral dimensions, the potential V associated with the steady-state current flow satisfies the differential equation

$$\partial/\partial x(h/R \cdot \partial V/\partial x) + \partial/\partial y(h/R \cdot \partial V/\partial y) = 0 \tag{8}$$

See, for example, Hetenyi (9). If the thickness h is constant and R, the resistance of a cube of unit edge of the medium to a current entering one face of the cube and leaving by the opposite face, is held constant, equation (8) reduces to

$$\nabla^2(V) = 0 \tag{9}$$

By noting in equations (7) and (9) that the two systems can be made mathematically similar, it is concluded that equipotential and current lines, an orthogonal set, will represent the principal stress trajectories, another orthogonal set, provided suitable boundary conditions are established.

CHAPTER III

EQUIPMENT

A basic purpose of this study was to obtain quick and relatively inexpensive representations of a plane stress field in terms of stress trajectories. This purpose was accomplished by the use of a special conducting paper, called "Teledeltos" recording paper and manufactured by the Western Union Telegraph Company. Teledeltos paper is made of carbon or graphite impregnated stock and is formed to about 0.004 inch thickness. It is supplied in rolls of 100 feet, 200 feet, or greater. In most cases it is a good rule to make use of as large a plotting area as practicable, so that edge effects will be minimized.

There are three types of Teledeltos paper: L-39, L-48, and H. L-48 was used in this study. This type has a specific resistance of approximately 4000 ohms per square and has been found very well suited for use in the field plotter. It is a nearly homogeneous plane conductor relatively insensitive to atmospheric changes in temperature and humidity. In samples tested the uniformity of resistance of sample areas has been found to run within ±8 per cent maximum deviation from a mean value.[2] Greatest accuracy is obtained

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[2]All statements of properties of the equipment came from reference (10).

when the equipotential lines are oriented in the general direction of the roll or grain of the Teledeltos paper. Atmospheric changes can be eliminated by running all experiments in the same air-conditioned room.

In making a model, to insure contact between an electrode and the paper and to represent various holes and cut-outs, a silver paint of the type used for printing electronic circuits (General Cement, No. 21-1 or No. 21-2) was applied to the paper. This paint, which is almost pure silver, has a negligible resistance compared to that of the paper.

Applying the voltage and measuring the equipotential lines was accomplished by the use of the Analog Field Plotter. The plotter consists basically of an assembly of a d-c power supply unit and an instrument detecting unit as shown in Fig. 1. The only primary power source required for the plotter is a conventional 120-volt, 60 cycle outlet. A schematic diagram of the circuit is shown in Fig. 2.

Low voltage direct current is used for the excitation of the current sheet. Utilizing a regular 120-volt, 60 cycle supply circuit with an output of from 6 to 8 volts and a current rating-output up to 7 amperes, the plotter is considered a more than adequate power supply unit. Since the resistances of the plotting circuit are relatively high, the actual current supplied to the sheet is normally less than 50 milliamperes.

The instrument used as a zero voltage or "null" detector serves to indicate the locus of points of constant potential along the

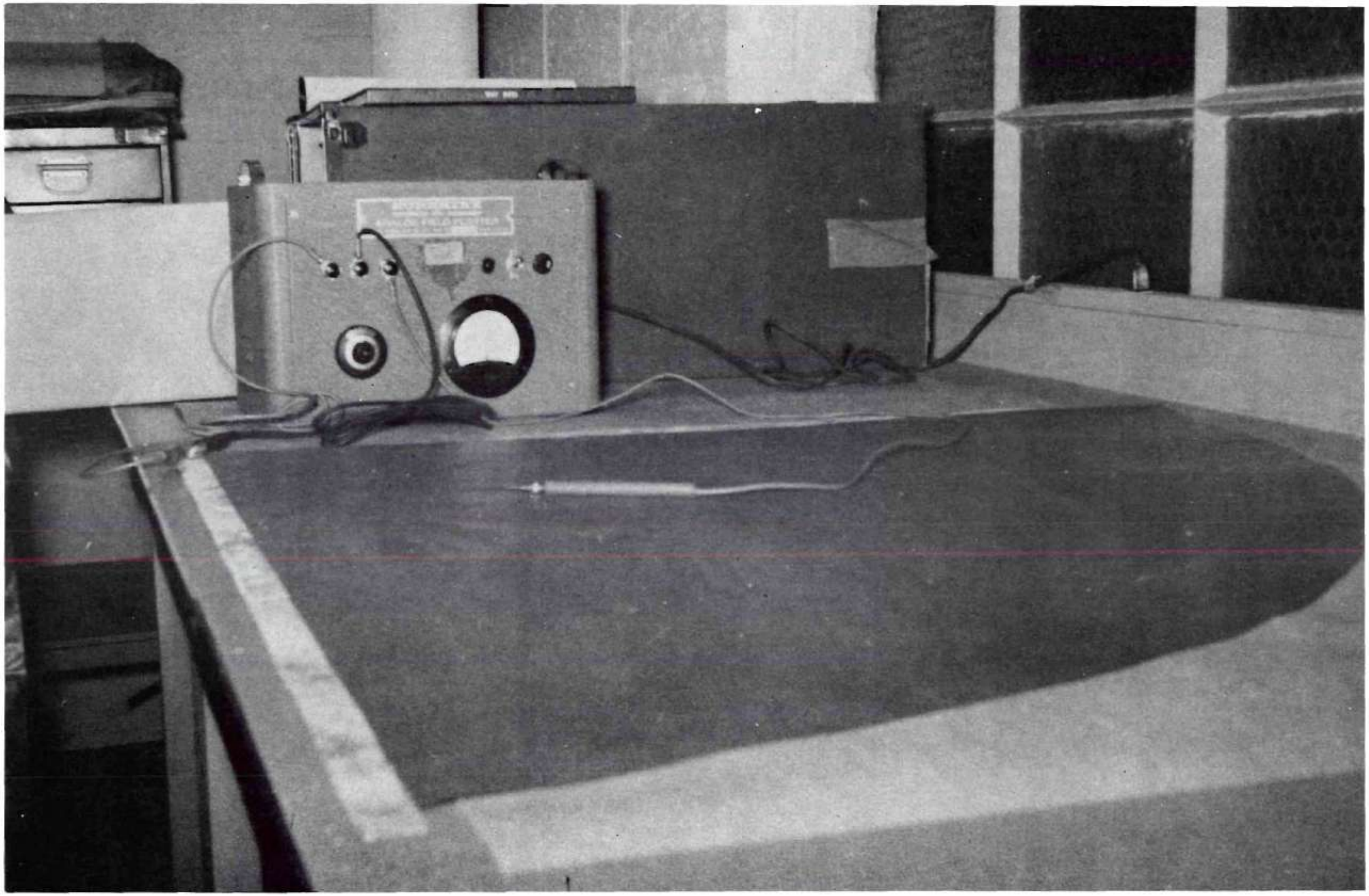

Figure 1. A General View of the Equipment with a Semi-Infinite Plate Model.

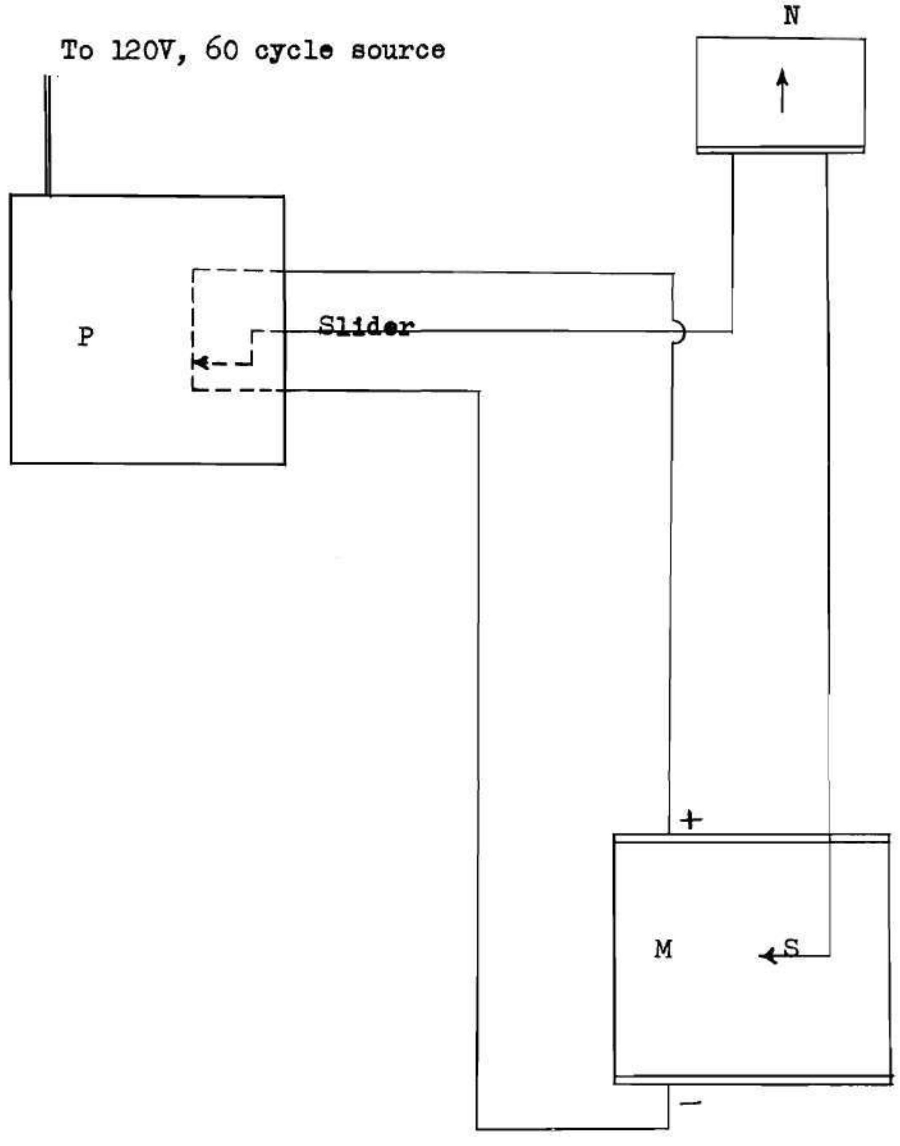

Figure 2. Schematic Diagram Showing Connection of Analog Field Plotter Components, Power Unit (P) and Null Detector (N) with Marking Stylus (S), to Conducting Sheet Model (M).

particular equipotential line being traced. This instrument consists of a small high-sensitivity, type D071, moving coil d-c microammeter having the zero reading point at center scale. It has a current rating of $\pm$10 microamperes full scale and a coil resistance of approximately 1700 ohms. A 5000-ohm resistor is used in series with the instrument and is adjusted to give nearly critical damping of the instrument. Adequate sensitivity is still retained to permit checking a null reading to within $\pm$0.01 inches under average conditions.

The voltage level of the particular equipotential being traced is determined by the particular slider position chosen on the voltage-dividing potentiometer. This level can be adjusted in small increments anywhere between 0 and 100 per cent of the voltage applied to the model. The "slide-wire" element of the voltage divider consists of 10 turns of a helical-wound resistance element, the slider traveling on a 10-turn lead screw and following around the complete length of the helix to give a long total effective slider travel. The potentiometer, having a total resistance of 1000 ohms, can be easily read to $\pm$0.2 per cent of the total resistance. Changes in the voltage applied to the model will not affect the null indicating system, since the potentiometer divides the voltage into percentages.

The searching probe serves both to explore and locate the zero potential balance points comprising the equipotential trajectories and also as a marking stylus, whose point is used to make small indentations in the plotting sheet surface in recording the equipotentials. These indentations, in general, are relatively small and have a negligible effect on the distribution pattern of the flow lines.

Lines thus marked can, if desired, be sketched in as a smooth curve with colored pencil or white ink for ease in studying or photographing. The stylus consists of a moderately sharp pointed brass rod with an insulating sleeve, about 5 inches long. It is connected by a flexible insulating lead to the slider of the voltage-dividing potentiometer.

All experiments were run with the Teledeltos paper lying on a table top insulated with a rubber covering to insure that no current flowed in a direction normal to the plane of the paper. A general view of the equipment is shown in Fig. 1.

CHAPTER IV

PROCEDURE

In considering a thin, finite, rectangular plate under various loads on two opposing edges, it will be noted that by Saint Venant's principle at a distant cross-section, usually twice the width or more, the average stress is realized. This study, however, deals with two cases in which the average stress is not realized--rectangular plates with discontinuities, such as circular holes, and square plates.

The use of an electric analogy to represent a stress field is well established. No current can flow in or out of an insulated boundary. Such a boundary represents a current flow line and, due to orthogonal properties, is crossed at right angles by equipotential lines. Then

$$\partial V/\partial n = 0 \tag{10}$$

where n is taken normal to the boundary. The insulated edge of a model then corresponds to the loaded edge of an elastic plate. Since the maximum principal stress acts in the direction of the equipotential lines, when there is no shear on the boundary,

$$\partial p/\partial n = 0 \tag{11}$$

This boundary condition can also be useful when the problem has symmetry.

On the other hand, a conducting edge represents a potential flow line. So

$$V = \text{constant} \tag{12}$$

This then would correspond to an unloaded edge of an elastic plate and to a hole in the plate where all normal stress is considered of zero magnitude, that is

$$q = 0 \tag{13}$$

and the tangential stress describes a stress trajectory on which

$$p = \text{constant} \tag{14}$$

By imposing a constant potential difference on the edges of the Teledeltos model, the lines of equal potential will correspond to the maximum principal stress trajectories. The orthogonal set, current lines, will represent the minimum principal stress trajectories. The insulated edge, therefore, simulates the loaded edge of the plate under study.

In order to trace the equipotential lines, the free edges of the model are painted with silver paint. Care must be used to insure that the total length of the free edge is at the same potential. With the voltage-divider at a constant percentage of the applied voltage points of equal potential are established with the stylus and null reading dial. These points describe a smooth curve for a graphic picture of the principal stress trajectories. The principal stresses are then obtained from these stress trajectories.

The method used to obtain the stresses is explained in reference (11). They are in the form of stress concentration factors, the ratio of the stress at a point to the average stress, where

$$\sigma_{av} = S/A \tag{15}$$

S being the total load on the edge and A being the normal cross-sectional area. Briefly, the stress concentration factor may be described as

$$k = \lim_{e \to m} (E/e) = \tan \phi \tag{16}$$

where m is an arbitrary equipotential line used as a reference point, E is the variable normal distance from a boundary to points corresponding to a uniform equipotential line spacing, e is the variable normal distance from a boundary to successive equipotential lines, and ϕ is the angle describing the tangent to the E vs. e curve. This k is the slope of the tangent to the E vs. e curve and can be evaluated graphically as shown in Fig. 3. Sample problems utilizing this method are discussed in the Results and Appendix.

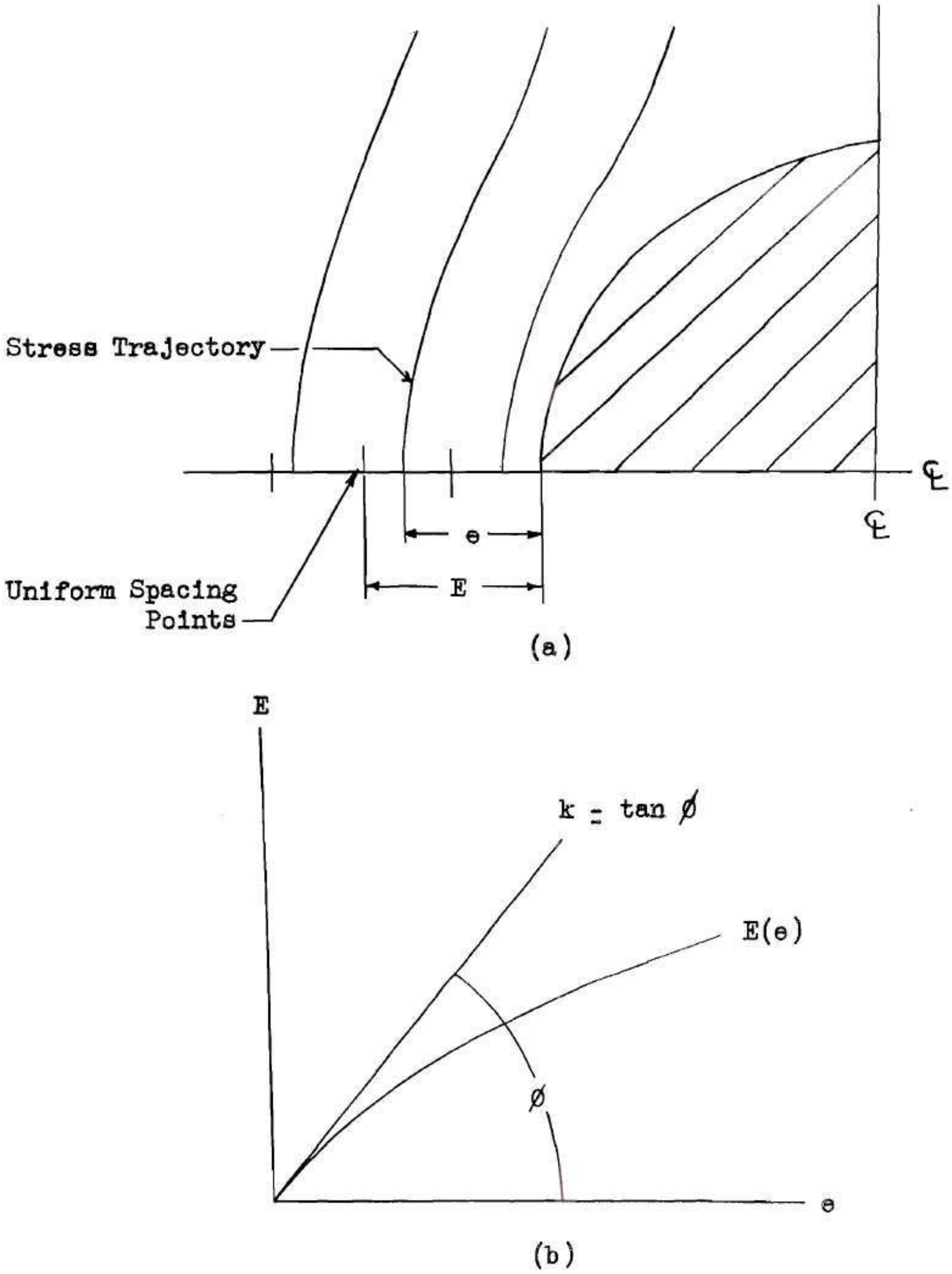

Figure 3. Sketches Showing: (a) The Measurement of Normal Distances E and e; (b) The Method of Determining the Stress Concentration Factor k as the Tangent to the E vs. e Curve at the Origin.

CHAPTER V

RESULTS

To compare values of k obtained with the conducting paper analogy with those values obtained by other means, the problem of a circular hole in a long finite width plate under uniform tension was used. A model was prepared with the ratio of the hole diameter to the plate width (a/w) of 0.2. Values of k across the entire minimum cross section were determined using equation (16) and were compared to those found by the analytical method of Howland (12) for models of identical geometry. A comparison of the results is presented in Fig. 4, and it will be noted that a maximum deviation from Howland's solution of 4.7 per cent, occurring at 0.8 of the width, was realized.

To demonstrate the adaptability of the analogy to other than uniform loadings, a semi-infinite plate with a concentrated load was selected. This plate can be represented geometrically by a semicircle with a load applied at the origin of the radii (13). A model, shown in Fig. 3, 32.75 inches in radius was prepared with a uniform load over 0.5 inches representing the concentrated load. The solution of the stresses by equation (16) was highly inaccurate, varying up to 50 per cent in error. To check the accuracy of the trajectories, actually confocal hyperbolas (14), an equation for the exact solution in Lamb (15)

$$y^2/c^2\cos^2\chi \; - \; x^2/c^2\sin^2\chi = 1 \qquad (17)$$

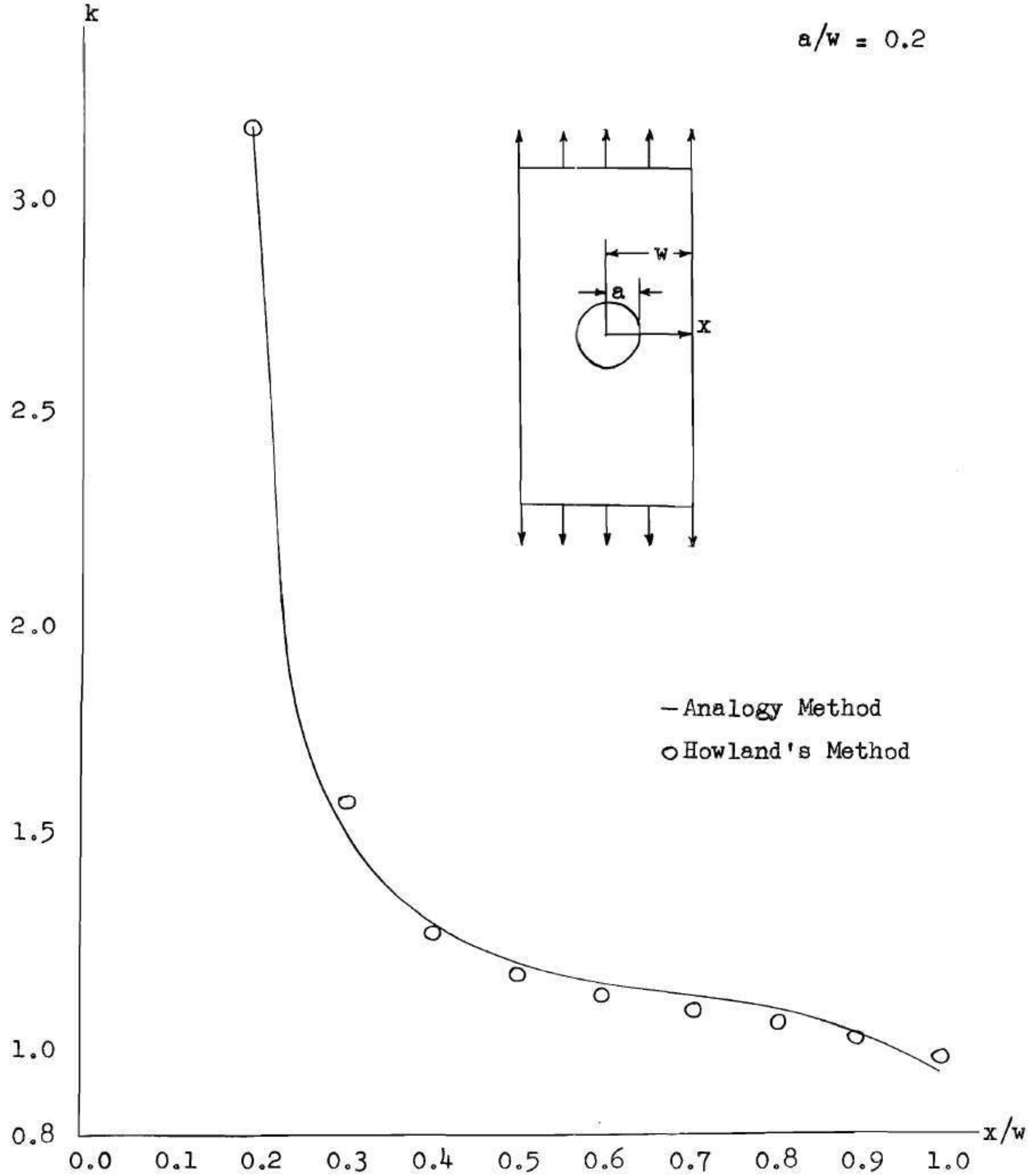

Figure 4. The Stress Concentration Factor k vs. Non-Dimensional Cross Section x/w for a Circular Hole. A Comparison of k, Found by the Analogy Method, to the Values Obtained by Howland.

was used, where y is along the diameter of the semicircle, x is in the direction of the applied load, c is one half the width allowed for the concentrated load, and ψ is defined by a constant stress trajectory. The comparison of some trajectories found using the model with the ones obtained by equation (17) is shown in Fig. 5, the maximum deviation being 8.0 per cent.

Examination of the suitability of this analogy to square plates under edge conditions, such as concentrated and non-uniform loads, is discussed in the Appendix under Sample Problems.

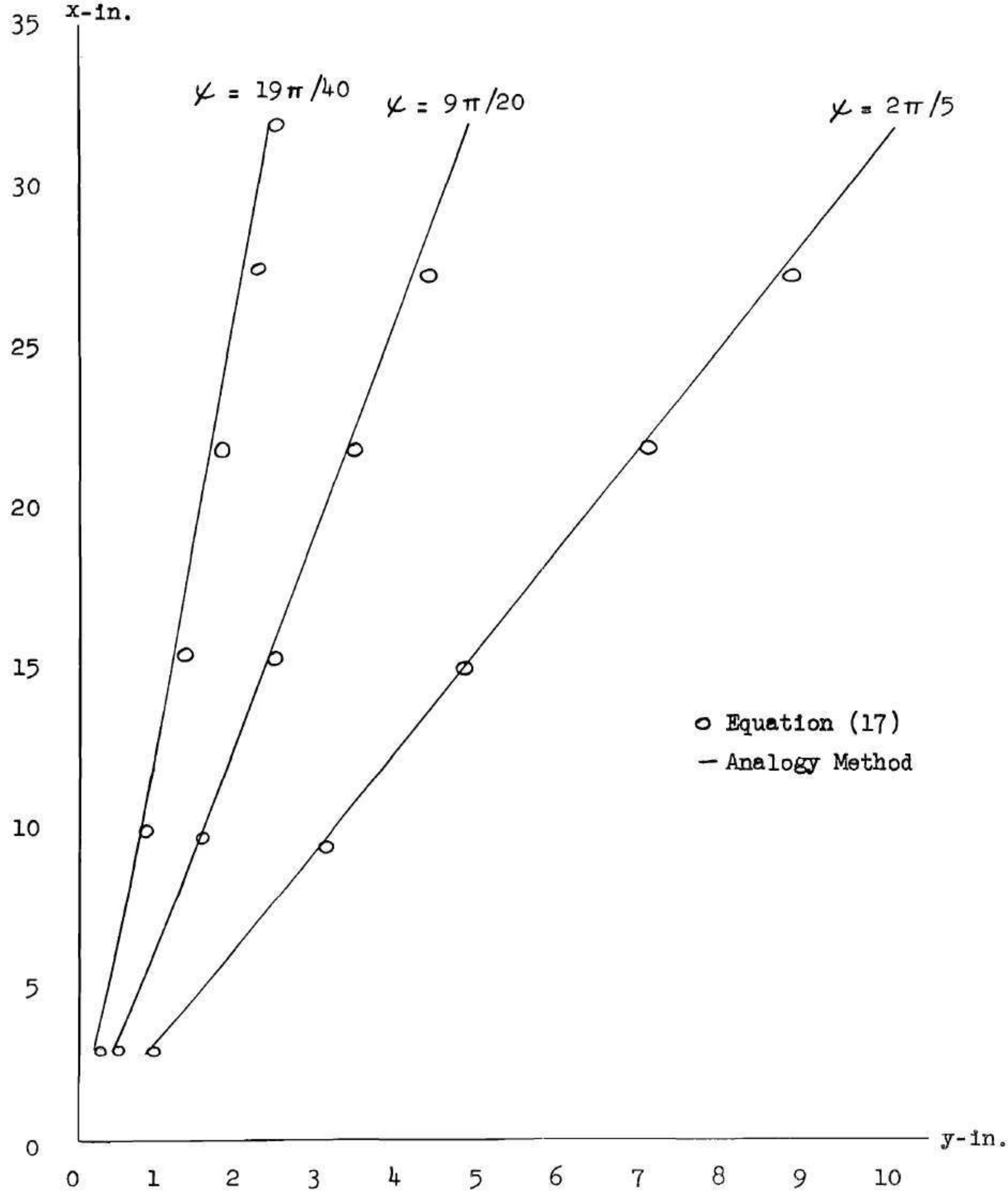

Figure 5. Semi-Infinite Plate Stress Trajectories at $\psi = 19\pi/40$, $9\pi/20$, $2\pi/5$. A Comparison of Stress Trajectories Found By the Analogy to those Found by Flow Theory.

CHAPTER VI

CONCLUSIONS

On the basis of tests run using this rheoelectric analogy, the following conclusions are reached.

1. A graphic representation of a two dimensional stress field, in terms of principal stress trajectories, may be readily obtained by using the conducting paper analogy.

2. The stress concentration factor k can be evaluated by measuring the contraction of the principal stress trajectories in a uniform tension field.

3. Stress concentrations across an entire cross section are easily determined for a finite plate with a circular hole under uniform tension.

4. Values of k obtained through the use of this analogy for other than uniform loadings are inaccurate when compared with those derived by theoretical and photoelastic means.

CHAPTER VII

RECOMMENDATIONS

To determine stresses accurately, the following recommendations are made.

1. For a concentrated load the method of Theocaris (18) should be investigated as a means for separating principal stresses instead of using equation (16).

2. If this method gives satisfactory stresses, equation (18) in the Appendix should be investigated further as a solution for non-uniform loads.

3. The mathematical solution for $\nabla^2(p + q) = 0$ should be investigated for the case when p or q does not vanish at the boundary.

APPENDIX

SAMPLE PROBLEMS

For the purpose of a sample problem, square plates with concentrated loads at various points and with a parabolic load were chosen.

Because of symmetry, only one half of the plate is prepared in model form; six models representing twenty inch squares were prepared. Each model was loaded with one concentrated load applied at 0, 2, 4, 6, 8, and 10 inches from the left edge. A comparison of k values across the middle cross section with those obtained in an approximate analytical solution by Goodier (16) for a square plate with a center load, the only available solution, is illustrated in Fig. 6. It may be noted that there are very large deviations, although the average ordinate of each curve was the average stress, 1.0.

The stress distribution due to a non-uniform load was attempted, using the experimental values of stress due to the six concentrated loads, Fig. 7. By cross plotting these values of stress, an influence function, $w(x)$, is obtained, where x varies from the left edge of the plate. Letting $F(x)$ be the function for a non-uniform load, a parabola in this case, the stress at any point x is given by

$$\sigma(x) = \int_{0}^{1.0} F(x)w(x)dx \tag{18}$$

where 1.0 is the non-dimensional width. Values of $w(x)$ obtained

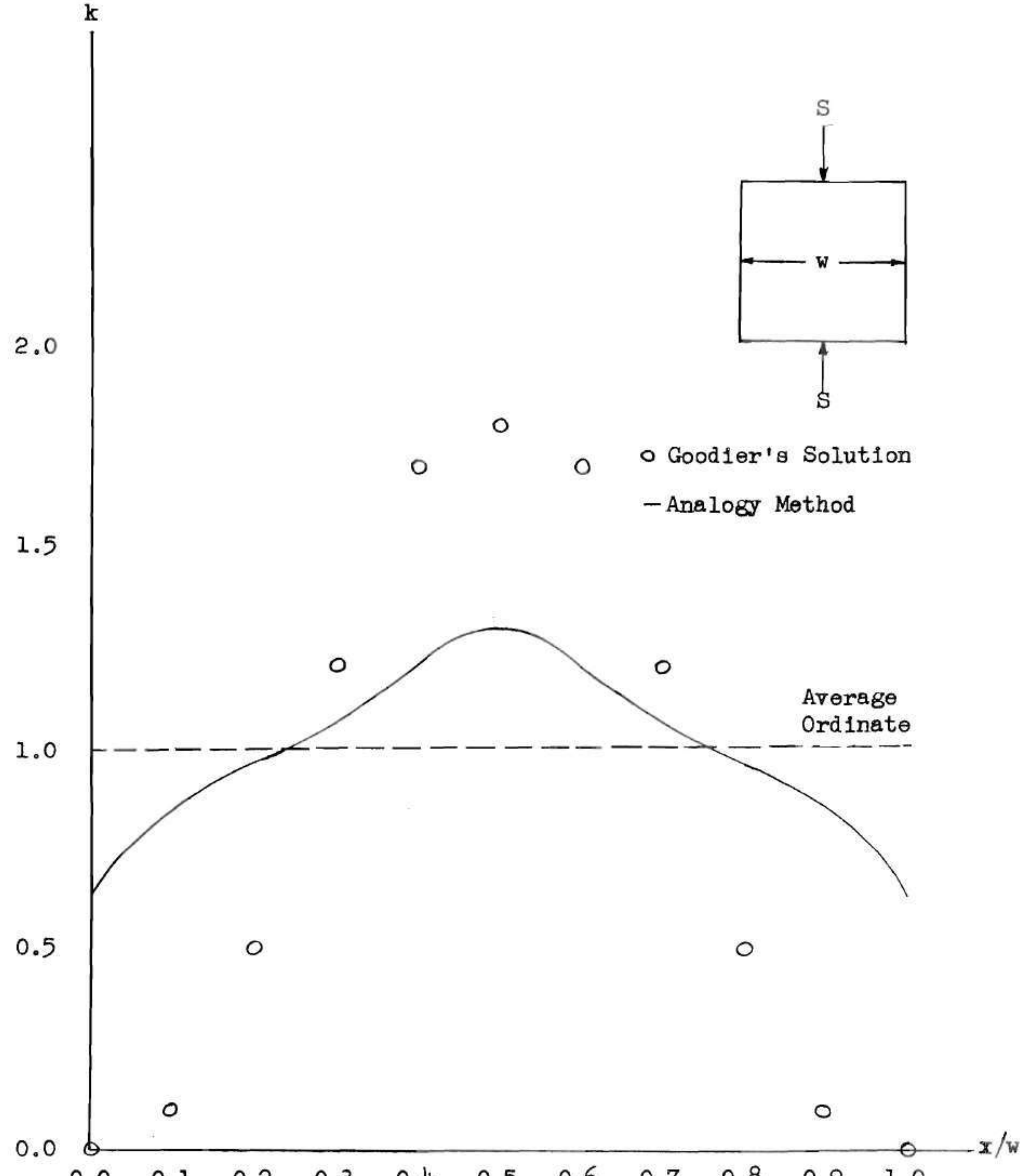

Figure 6. The Stress Concentration Factor k vs. Non-Dimensional Cross Section x/w for a Square Plate Under a Center Concentrated Load. A Comparison of k, Found by the Analogy Method, to the Values Obtained by Goodier.

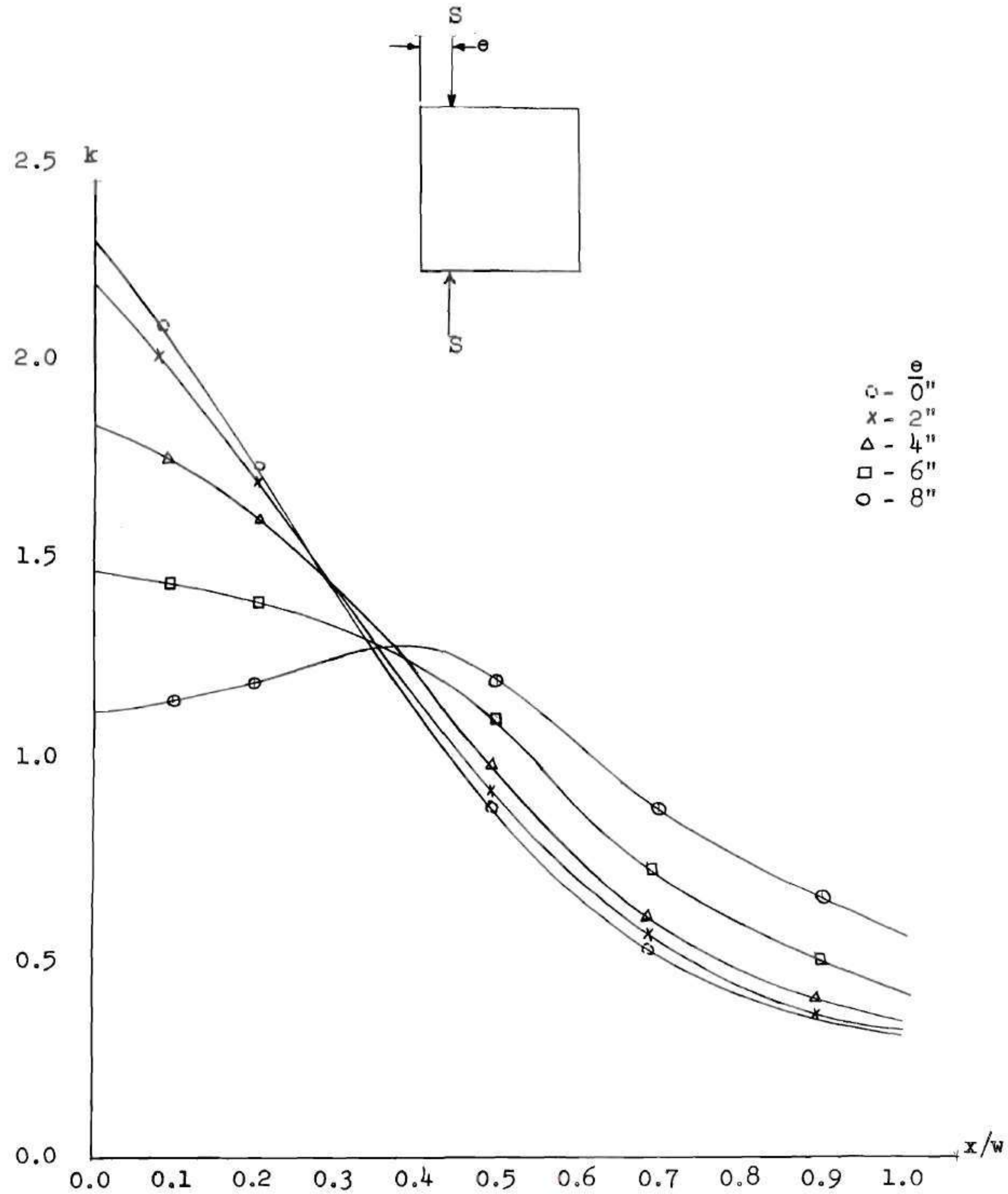

Figure 7. The *Stress Concentration Factor k vs. Non-Dimensional Cross* Section x/w for a Square Plate Under Various Concentrated Loads.

are shown in Fig. 8, while a comparison of $\sigma(x)$, middle cross sectional stress, with that obtained in an approximate analytical solution by Timoshenko (17) is illustrated in Fig. 9. The maximum error in $\sigma(x)$ is ± 15 per cent.

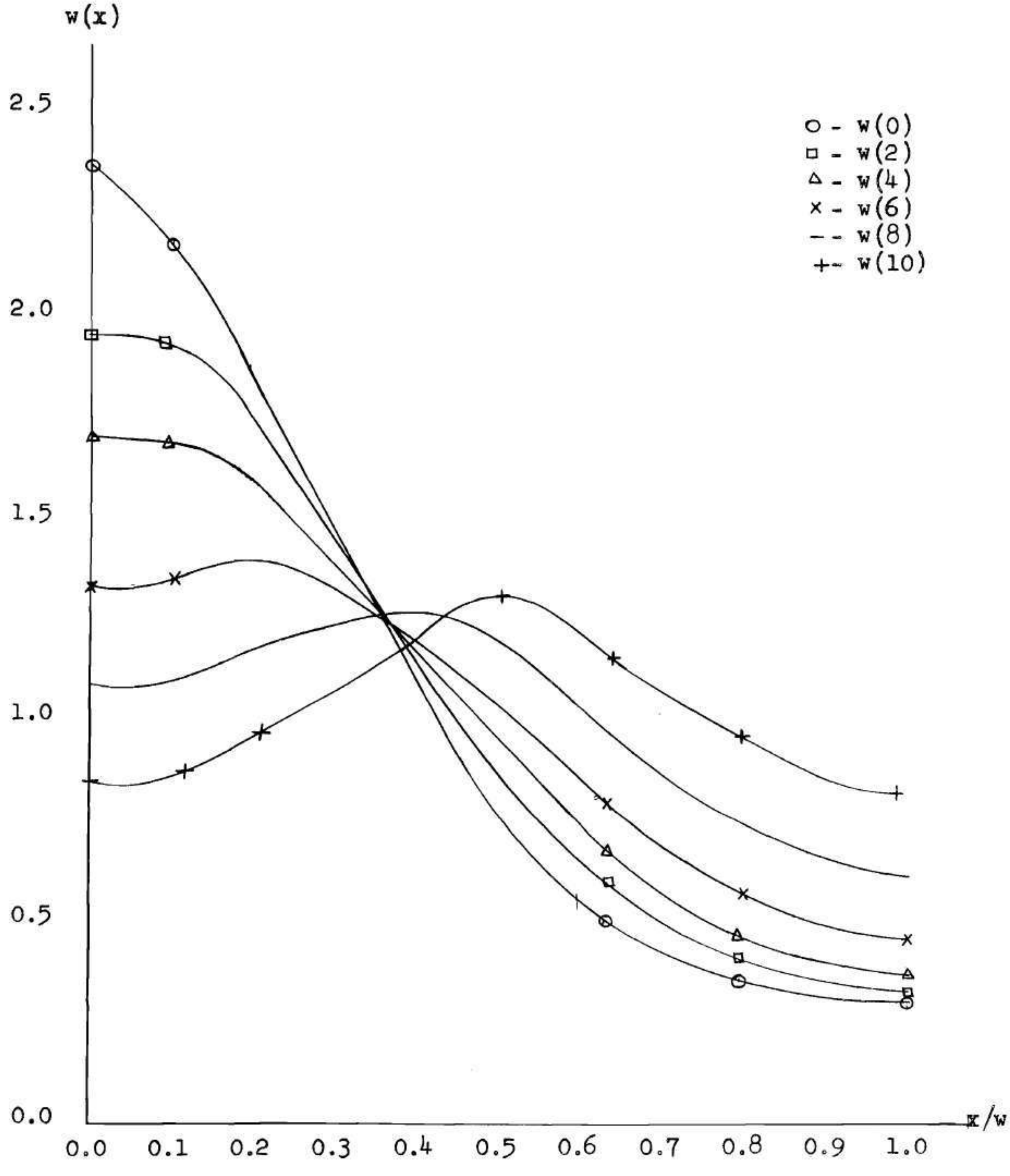

Figure 8. Loading or Influence Function vs. Non-Dimensional Cross Section x/w for a Square Plate Under Various Concentrated Loads.

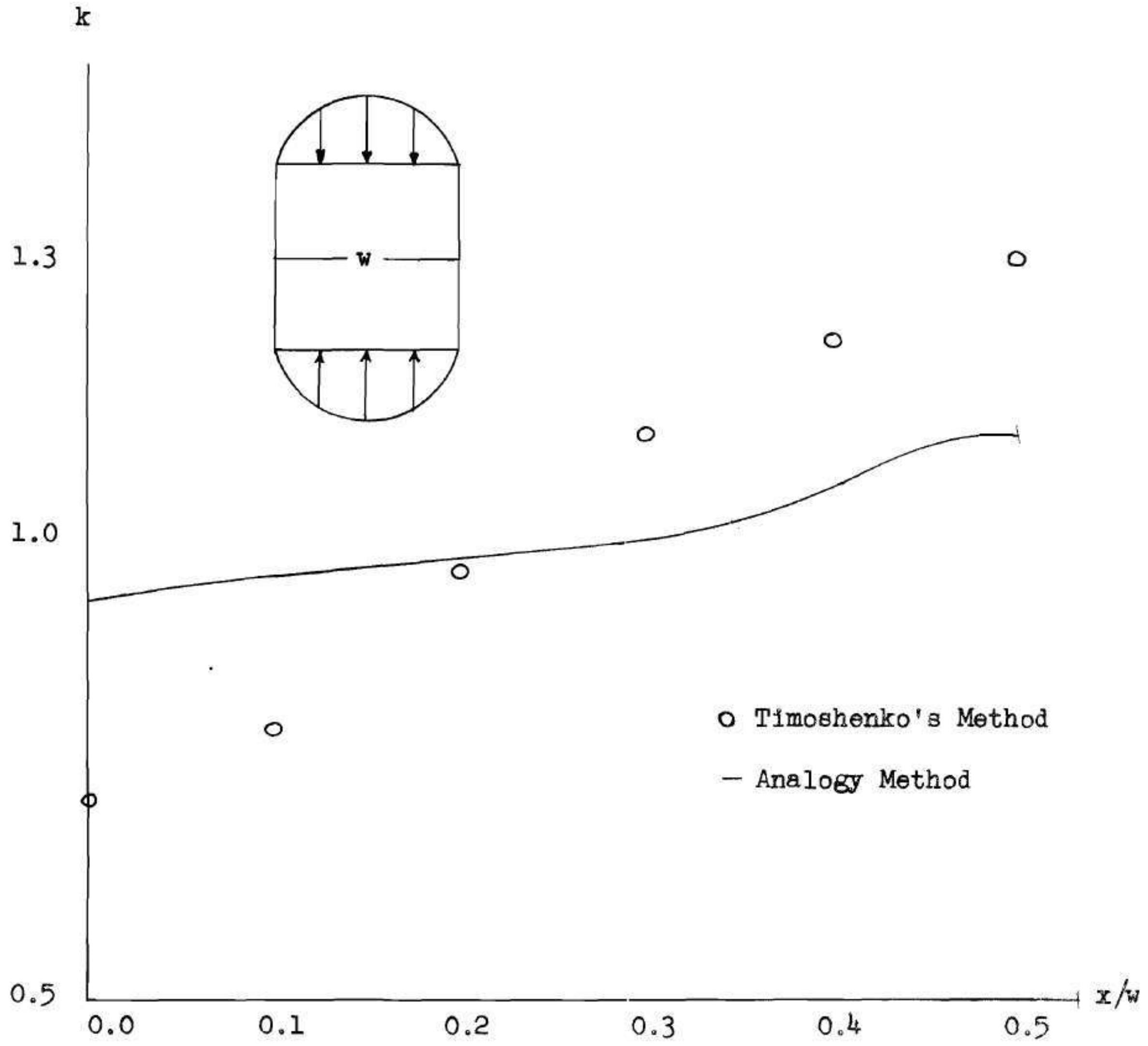

Figure 9. The Stress Concentration Factor k vs. Non-Dimensional Cross Section x/w for a Square Plate Under a Parabolic Load. A Comparison of k, Found by the Analogy Method, to the Values Obtained by Timoshenko.

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