

Final Report for Period: 07/2009 - 06/2010**Submitted on:** 08/30/2010**Principal Investigator:** Loss, Michael .**Award ID:** 0600037**Organization:** GA Tech Res Corp - GIT**Submitted By:**

Loss, Michael - Principal Investigator

Title:

Dynamics and Variational Problems

Project Participants**Senior Personnel****Name:** Loss, Michael**Worked for more than 160 Hours:** Yes**Contribution to Project:****Name:** Carlen, Eric**Worked for more than 160 Hours:** Yes**Contribution to Project:****Post-doc****Graduate Student****Undergraduate Student****Technician, Programmer****Other Participant****Research Experience for Undergraduates****Organizational Partners****Other Collaborators or Contacts****Activities and Findings****Research and Education Activities:**

The research program under this grant was completed with considerable success, resulting in more than 30 research publications, listed in the publications section, and moreover, several students were supported by this grant and two of them have completed their Ph.D's while supported by this grant.

Throughout the time they were supported by this grant, Carlen and Loss collaborated on problems in kinetic theory and functional analysis. In one of the highlights of this work Carlen and Loss, together with Jeff Geronimo, computed the exact spectral gap in the Kac model for physical collisions -- momentum and energy conserving -- in three dimensions [CL6]. The methodology builds on the previous work of Carlen, Carvalho and Loss, but introduced new techniques for using more detailed information about a kinematical 'correlation operator' to get bounds

on the spectrum of the generator of the Kac walk.

The research on the correlation operator arising in the Kac model has led to interesting unanticipated developments: Carlen, Loss and Geronimo have written a paper giving simple, completely elementary proofs of Gasper's and Bochner's theorem on the Markov sequence problems for the Jacobi polynomials [CL35], and this has led to further developments with more work in progress.

Findings:

Loss made contributions to a variety of fields. Jointly with Baker and Stolz he characterized the minimizing energy configurations in the random displacement model and showed that in two and higher dimensions these configurations are essentially unique [L11, L26]. This work is significant for two reasons. For one it introduces a new technique for analyzing the lowest Neumann eigenvalue as a function of the position of the potential (bubbles tend to the boundary) and, moreover, it is substantial progress towards proving localization in the random displacement model.

In [L2], which is joint work with Miyao and Spohn, the existence of ground states of moving atoms in non-relativistic quantum electrodynamics (QED) is proved. The significance of this paper is that the result holds under natural, non-technical conditions. All previous results assumed that the fine structure constant and the ultra-violet cutoff are sufficiently small. Likewise in [L27], again work with Miyao and Spohn, it is shown that various QED-systems have degenerate eigenvalues once the spin of the electron is included. This follows by a careful analysis of the Kramers degeneracy. It explains why, without and external magnetic field, there is no level splitting in the spin.

In [L18, L21] which is joint work with Dolbeault and Esteban, the Dirac hydrogen atom interacting with a strong magnetic field is analyzed in depth. In particular there is a critical size of the magnetic field at which the lowest eigenvalue penetrates the lower continuum. Curiously, the lowest Landau level approximation does not yield a very good result for the size of this critical field.

Some significant papers in pure analysis are [L8,L9] jointly with Esteban in which the old problem of self-adjointness for Coulomb-Dirac operators is put to rest. The results hold up and including the critical charge of the nucleus which is not the case with the previous results. In [L16], which is joint work with Dolbeault and Laptev, a simple proof of Lieb-Thirring inequalities is given with constants that, while not sharp, are the best known up to now. The paper [L10], with Benguria and Frank, deals with the Hardy-Sobolev-Maz'ya inequality in a half space for the special case of three dimensions. The surprising result is that the sharp constant in the inequality is the same as the Sobolev constant despite the fact that the derivative term is weakened by the Hardy term. In more than three dimensions this is no longer true. This development has piqued the interest of my graduate student Craig Sloane, who is investigating the analogous Hardy-Sobolev-Maz'ya inequalities for fractional derivatives. He has obtained a number of results which are published soon. Likewise, Amit Einav, another student of Loss, motivated by [CL31] has been investigating entropy production inequalities for the Kac model. He has already obtain some very nice results and is in the process of writing these up.

Carlen has made several contributions in dissipative evolutions. In a paper with A. Blanchet and J. A. Carrillo [C34], the long time behavior of critical mass solutions of the Patlak-Keller-Segel system is studied, and basis of attraction are found for the steady state solutions. These steady state solutions are minimizers of the logarithmic Hardy-Littlewood-Sobolev inequality, and are also stationary solutions of a certain porous medium equation. The analysis in this work turns on a surprising relation between the Patlak-Keller-Segel system and the porous medium equation, and involves the development of a number of new functional inequalities. The paper makes use of optimal mass transportation techniques, and extends existing methods to work in what is a challenging 'critical' setting. Other contributions to the use of optimal mass transportation in variation problems are made in the paper [C11] by Carlen, Carvalho, Esposito, Lebowitz and Marra.

Together with Carvalho and Lu, Carlen has completed an investigation [C29] of the spatially homogeneous Boltzmann equation with soft potentials. This paper provides much improved bounds on both the growth of moments, and the rate of convergence to equilibrium for soft potentials, and introduced a range of new techniques. Soft potentials are physically relevant, but technically much more difficult to deal with than hard potentials.

While supported by this grant, Carlen has continued to work on optimal matrix inequalities. Results obtained in this are, in joint work with Lieb, are in [C19] and in [C22] which develops non-commutative analogs of Brascamp-Lieb inequalities. This paper uses an extension to the non-commutative setting of recent results of Carlen and Cordero [C25], also supported by this grant.

All citations in these sections are listed in the publications section.

Training and Development:

Outreach Activities:

Journal Publications

- [CL7] and [1] Carlen, E. A., Geronimo, J. and Loss, M., "Determination of the spectral gap in the Kac model for physical momentum and energy conserving collisions", SIAM Jour. Math. Analysis, p. 327-364, vol. 40, (2008). Published,
- [L8] and [2] Esteban, M. J. and Loss, M., "Selfadjointness of Dirac operators via Hardy-Dirac inequalities", J. Math. Phys., p. 112107, vol. 48, (2007). Published,
- [3] Benguria, R., Frank, R. and Loss, M., "The sharp constant in the Hardy-Sobolev-Maz'ya inequality in the three dimensional upper half-space", Math. Res. Letters, p. 613-622, vol. 15, (2008). Published,
- [4] Baker, J., Bellissard, J., Stolz, G. and Loss, M., "Bubbles tend to the boundary and an application to random Schrodinger operators", In preparation, p. , vol. , (2007). To be submitted in June,
- [5] Frank, R., Loss, M. and Weidl, T., "Eigenvalue estimates of the magnetic Laplacian in a domain", In preparation, p. , vol. , (2007). To be submitted in June,
- [6] Dolbeault, J., Felmer, P., Loss, M. and Patrel, E., "Lieb-Thirring type inequalities and Gagliardo-Nirenberg inequalities for systems", Jour. Funct. Anal., p. 193-220, vol. 238, (2006). Published,
- [L18] and [7] Dolbeault, J., Esteban, M. and Loss, M., "Relativistic hydrogenic atoms in strong magnetic fields", Annales Henri Poincare, p. 749-779, vol. 8, (2007). Published,
- [L3] and [8] Loss, M., Miyao, T. and Spohn, H., "Lowest energy in non-relativistic QED: atoms and ions in motion", Jour. Funct. Anal., p. 353-393, vol. 243, (2007). Published,
- [C12] and [9] Carlen, E. A., Gabetta, E. and Regazzini, E., "Probabilistic Investigation of the explosion of solutions of the Kac equation with infinite energy initial distribution", Advances in Applied Probability, p. 95-106, vol. 45, (2008). Published,
- [C4] and [10] Carlen, E. A., Gabetta, E. and Regazzini, E., "On the rate of explosion for infinite energy solutions of the spatially homogeneous Boltzmann equation", Jour. of Stat. Phys., p. 699-723, vol. 129, (2007). Published,
- [C13] and [11] Carlen, E. A., Carvalho, M.C., Esposito, R., Lebowitz, J.L. and Marra, R., "Displacement convexity and minimal fronts at phase boundaries", Arch Rat Mech Analysis (available in "online first" pages), p. 823-847, vol. 194, (2009). Published,
- [C14] and [12] Carlen, E. A., Carvalho, M.C., Esposito, R., Lebowitz, J.L. and Marra, R., "Droplet minimizers for a non-local free energy functional", Nonlinearity, p. 2919-2952, vol. 22, (2009). Published,
- [C25] and [13] Carlen, E. A. and Cordero-Erausquin, D., "Subadditivity of the Entropy and its Relation to Brascamp-Lieb Type Inequalities", GAFA, p. 373-405, vol. 19, (2009). Published,
- [C6] and [14] Ulusoy, S., "A new family of higher order nonlinear degenerate parabolic equations", Nonlinearity, p. 685-712, vol. 20, (2007). Published,
- [C5] and [15] Carlen, E.A. and Ulusoy, S., "Asymptotic equipartition and long time behavior of solutions of a thin-film equation", Journal of Differential Equations, p. 279-292, vol. 241, (2007). Published,
- [L11] Baker, J., Stolz, G. and Loss M., "Minimizing the ground state energy of an electron in a randomly deformed lattice", CMP, p. 397-415, vol. 283, (2008). Published,
- [L2] Loss, M., "Stability of Matter", Contemporary Mathematics, AMS, p. 171-199, vol. 437, (2007). Published,

- [L24] Frank, R., Loss, M. and Weidl, T., "Polya's conjecture in the presence of a constant magnetic field", JEMS, p. 1365-1383, vol. 11, (2009). Accepted,
- Benguria, R.D., Depassier, M. C. and Loss, M., "Validity of the Brunet-Derrida formula for the speed of pulled fronts with a cutoff", Europhysics letters, p. , vol. , (2008). Accepted,
- [L16] Dolbeault, J., Laptev, A. and Loss, M., "Lieb-Thirring inequalities with improved constants", JEMS, p. 1121-1126, vol. 10, (2008). Published,
- [L17] Benguria, R.D., Loss, M. and Siedentop, H., "Stability of atoms and molecules in an ultrarelativistic Thomas-Fermi-Weizsaecker model", JMP, p. 012302, vol. 49, (2008). Published,
- [L21] Dolbeault, J., Esteban, M.J. and Loss, M., "Characterization of the critical magnetic field in the Dirac-Coulomb equation", J. Phys. A. Math. Theor., p. 13pp, vol. 41, (2008). Published,
- [C19] Carlen, E.A. and Lieb, E.H., "A Minkowski type trace inequality and strong subadditivity of the quantum entropy II", Lett. Math. Phys., p. 107-126, vol. 83, (2008). Published,
- [C23] Carlen, E.A., Carvalho M.C., "Strong convergence towards homogeneous cooling states for dissipative Maxwell models", Ann. Inst. H. Poincare, Nonlinear Analysis (available on line), p. 1675-1700, vol. 26, (2009). Published,
- [C20] Arnold, A., Carlen, E.A. and Ju, Q., "The large-time behavior of non-symmetric Fokker-Planck type equations", Communications on Stochastic Analysis, p. 153-175, vol. 2, (2008). Published,
- [C1] Carlen, E.A., "The rate of local equilibration in kinetic theory", Prospects in mathematical physics, Contemp. Math., p. 71-88, vol. 437, (2007). Published,
- [L26] J. Baker, M. Loss, and G. Stolz, "Low energy properties of the random displacement model", JFA, p. 2725-2740, vol. 256, (2009). Published,
- [L9] M. Esteban and M. Loss, "Self-adjointness via partial Hardy-like inequalities", Proceedings of q-math 10, World Scientific, p. , vol. , (2008). Published,
- [L27] M. Loss, T. Miyao and H. Spohn, "Kramers degeneracy theorem in nonrelativistic QED", LMP, p. 21-31, vol. 89, (2009). Published,
- [CL32] E. A. Carlen, J. Geronimo and M. Loss, "On the Markov sequence problem for Jacobi polynomials", Revised and extended version submitted to Advances in Math, p. , vol. , (2009). Submitted,
- [C28] E. A. Carlen and R. Vilela Mendes, "Signal reconstruction by random sampling in chirp space", Nonlinear Dynamics, p. 223-229, vol. 56, (2009). Published,
- [C22] E. A. Carlen and E. H. Lieb, "Bascamp-Lieb inequalities for non-commutative integration", Documenta Mathematica, p. 553-584, vol. 13, (2008). Published,
- [C29] E. A. Carlen, M. C. Carvalho and X. G. Lu, "On strong convergence to equilibrium for the Boltzmann equation with soft potentials", J. Stat. Phys., p. 681-736, vol. 135, (2009). Published,
- [CL31] E. A. Carlen, M. C. , Carvalho, J. Le Roux, M. Loss and C. Villani, "Entropy and chaos in the Kac model", Kinetic and Related Models, p. 65-122, vol. 3, (2009). Published,
- A. Blanchet, E. A. Carlen and J. A. Carrillo, "Long time behavior of the critical mass Patlak-Keller-Segal system", In preparation, p. , vol. , (2009). To be submitted before June 09,

[L15] R. D. Benguria, M. C. Depassier and M. Loss, "Upper and lower bounds for the speed of pulled fronts with a cut-off", European Physical Journal B - Condensed Matter and Complex Systems, p. 331-334, vol. 61, (2008). Published,

[C30] A. Ayyer, E. A. Carlen, J. L. Lebowitz, P. K. Mohanty, D. Mukamel, and E..R. Speer, "Phase Diagram of the ABC Model on an Interval", J. Stat. Phys., p. 1166-1204, vol. 137, (2009). Published,

[C33] E. A. Carlen, S. N. Chow, and A. Grigo, "The dynamics and the hydrodynamic limit of the dissipative Boltzmann equation", Nonlinearity, p. , vol. , (2010). Submitted,

[C34] A. Blanchet, E. A. Carlen and J. A. Carrillo, "Functional inequalities, thick tails and asymptotics for the critical mass Patlak-Keller-Segal system", Invent. Math., p. , vol. , (2010). Submitted,

[C35] J. Bandyopadhyay, "Concentration for $SU(1, 1)$ Coherent State Transforms and An Analogue of the Lieb-Wehrl Conjecture for $SU(1, 1)$ ", Comm. Math. Phys., p. 1065-1086, vol. 28, (2009). Published,

Books or Other One-time Publications

[16] Carlen, E.A. and Carvalho, M.C., "Linear Algebra, From the Beginning", (2006). Book, Published
Bibliography: W.H. Freeman and Co., New York, New York

Web/Internet Site

URL(s):

www.math.gatech.edu/~loss and www.math.rutgers.edu/~carlen/

Description:

Besides publication in refereed journals, our work is disseminated through the archive <http://arxiv.org/> and on the PIs URLs listed above. All of these papers acknowledge NSF support.

Other Specific Products

Contributions

Contributions within Discipline:

Contributions to Other Disciplines:

Contributions to Human Resource Development:

Loss's student Craig Sloane is progressing towards his thesis, with papers in progress as noted above, as is Amit Einav. Carlen's student Suleyman Ulusoy graduated with his Ph.D. in August 2007, spent two years a postdoctoral fellow at the Centre of Mathematics for Application at the University of Oslo, and is now at the University of Maryland in a second postdoctoral position. Two papers of his, one joint with Carlen, were written while Ulusoy was supported by this grant. A second student of Carlen, Jogia Bandyopadhyay, defended her Ph.D. thesis in April 2008, and is now a postdoctoral fellow at the University of Helsinki. Her supported thesis work is published in Comm. Math. Phys., 28 no. 3 (2009) 1065-1086.

Contributions to Resources for Research and Education:

Contributions Beyond Science and Engineering:

Conference Proceedings

Categories for which nothing is reported:

Organizational Partners

Activities and Findings: Any Training and Development

Activities and Findings: Any Outreach Activities

Any Product

Contributions: To Any within Discipline

Contributions: To Any Other Disciplines

Contributions: To Any Resources for Research and Education

Contributions: To Any Beyond Science and Engineering

Any Conference