Modelling and Measurement of Process Errors in Micromilling

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Nomenclature

- T = Ideal transformation matrix
- F = Actual transformation matrix
- *D* = Compliance error transformation matrix
- W = Wear error transformation matrix
- V = Vibration error transformation matrix
- ε = Geometric error transformation matrix
- *E* = Elastic recovery transformation matrix
- δ = Beam deflection in μ m
- δ_{th} = Beam thermal elongation in μ m
- ϑ = Beam slope in °
- K =Stiffness in mN/ μ m
- F_c = Cutting force in N
- F_r = Radial force in N
- α = Coefficient of thermal expansion in μ m/m
- L = Beam length, tool length in mm
- d_{tool} = Tool nominal diameter in μ m
- I = Area moment of inertia in mm⁴
- M = Moment in Nm
- $u = unit power in J/mm^3$
- r = radial depth of cut in mm
- a = axial depth of cut in mm
- f = feed per tooth in mm
- n_t = number of teeth
- \overline{Nu} = Average Nusselt number
- Gr_d = Grashof number based on diameter
- Re ,= Rotating Reynolds number
- g = acceleration due to gravity
- θ = coefficient of thermal expansion for air = T_{∞}^{-1}
- T_{∞} = Air temperature in K

- T_d = Tool temperature in K
- v = Kinematic viscosity in m²/s
- $\overline{h_0}$ = Average convection coefficient in W/m²K
- k_a = Conductivity of air
- Ω = Angular velocity in rad/s
- κ = Buoyancy parameter

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1.Introduction

Error of a machined part is a combination of geometric and process errors. Error models that predict geometric, cutting force, fixture dependent and thermal errors in machine tools have been proposed for macro scale machining [1,2]. The effect of tool wear on cutting force in micro end-milling has been modelled [3]. Errors due to tool deflection for micro-end-milling tools have also been predicted [4-7]. Research on the effect of tool chatter on micro-milled parts is still in its nascent stages. A general framework to combine the effects of different error sources in machine tools based on HTMs and a first order approximation has been suggested by Soons *et al.* [8].

This study aims to meet the following objectives:

- Design rules for creation of 2D/3D artifacts using WEDM/EDM and/or milling processes
- Developing a method for separating machine tool error and process error with the function of variance evaluation
- Mathematical model and procedure for spatial error compensation based on the RSS method and sample implementation in MATLAB

In Chapter 1 an integrated model of machine tool errors is presented to combine the effect of different error sources. The model is based on Homogeneous Transformation Matrices (HTMs). This method has been widely used to model geometric and kinematic errors. In this chapter HTMs are used to model process errors such as thermal errors, vibration error (chatter or forced), tool wear, machine structure deflection, inertial effects and material elastic recovery. In comparison with the model developed in [8], the model developed in this chapter does not make a first order approximation. Furthermore, the model is applied to analyze error sources in micro milling applications. This model enables comparison of the contributions of various error sources in micro and macro machining to the total part error. This comparison provides insight into possible ways to increase the precision of the micro milling process.

Use of the model is demonstrated by an analysis of errors in the slot milling operation. Experiments are conducted to estimate the stiffness of micro tools. These stiffness values are used to calculate the error due to static deflection of the tool. Finite element analysis is used to estimate the thermal error in the tool during machining. These errors along with estimates of wear and geometric errors are used to analyze the relative significance of the different error sources on the final part feature accuracy of a groove. The form error of a groove is defined by errors in the surface of the side wall of the groove and errors in the depth of the groove. The effect of process parameters on surface location errors in micro end-milling has been studied in [9]. Hence, in this study the focus is on the effect of process and kinematic errors on the depth of the groove.

In Chapter 2, design rules are presented to isolate the effect various process errors. Artifacts are proposed to measure the errors caused due to elastic recovery of the workpiece material, deflection of the tool tip, thermal elongation of the tool tip due to heat generated in machining and tool wear during machining. The designed artifacts are then used to measure the significance of these process errors on the depth of micromilled grooves. The obtained values are used along with the RSS method to estimate the total variance in the groove depth.

2.Process Error Modeling and Comparison

For this study, process errors have been defined as errors that depend on process parameters such as spindle speed, feed, depth of cut, tool-workpiece combination, lubrication conditions, etc. Process errors can be further classified as follows:

Thermal errors: Errors in the part caused by thermal expansion or contraction of the machine structure including the tool and workpiece.

Vibration errors: These include tool chatter and errors caused by the dynamic excitation of the machine structure.

Static tool deflection: It has been suggested that micro-milling tools are susceptible to deflection due to cutting forces [5]. This deflection can result in errors in the machined part.

Tool wear: These are errors caused by the wear of the tool, which results in the reduction of the length and diameter of the tool. These in turn result in a reduction of the amount of machined material.

Elastic recovery: The machined surface exhibits an elastic spring back upon removal of the cutting tool. It has been suggested that this elastic recovery is a significant source of error in micro-milling [10].

Inertial effects: These are errors caused by the rigid body dynamics of accelerating machine tool components.

Creation of model

Developing an error model that considers all the errors in the complete machine tool structure would be time and memory intensive. Therefore, the first step in creating the model is to break down the machine tool structure into smaller sub-assemblies. The machine tool structure is broken into sub-assemblies such that they are connected by structural components that have a simple geometric shape and simple thermo-structural loading conditions.

The second step is to determine the location of the coordinate system for each component. Reference frames are placed at a point whose position is unaffected by the errors of *that* component. This point may move along with a component connected to the component under consideration. The error due to motion of parts connected to the component under consideration. The error due to motion of parts connected to the component under consideration. The error due to motion of parts connected to the second step is placed at the interface of the tool flutes and the tool shank. This point is not affected by the thermal, compliance, wear and geometric errors of the tool flutes.



Figure 2. 1: Locations of reference frames on a milling machine

The third step in creating the model is to determine the order of combination of the transformation matrices. For each component, the resultant transformation matrix from the reference frame of that component to the reference frame of the next component in the kinematic chain coordinate frame is affected by the errors in that component. Consider the case of the transformation matrix between the spindle and the spindle base in Fig. 1. Let ^{Sref}*T*₅ be the ideal transformation matrix relating the spindle base reference *S*_{ref} to the point *S* on the spindle where the error is to be calculated. Point *S* also acts as the reference frame for the tool holder, which is the next component in the kinematic chain. The final transformation *F* that relates the position of *S* with respect to the reference frame *S*_{ref} is given by Eq. 2.1. Transformation *F* is a function of the kinematic errors, denoted by ε , errors due to elastic deformation *D* of the machine tool components, thermal errors *H* and errors due to vibration *V*. Errors due to vibration can be superimposed on the deflection and is a periodic error of a frequency much higher than the tool feed. Hence, rather than including the instantaneous position of the vibrating tool, the model only considers the maximum amplitude of vibration of the component.

$$[^{Sref}F_S] = [V][D][\varepsilon][H][^{Sref}T_S]$$
(2.1)

Similar equations can be written for each of the components. For components other than the tool, the forces that cause elastic deformation are the weight of the component, the cutting forces and the forces induced by the assembly of the components. For micro milling, the cutting forces are small and hence, can be neglected for all components except the tool. On the other hand, the elastic deformation of the tool is affected more by the cutting forces than the weight of the tool. Furthermore, errors due to wear of the tool *W* are also present. Hence, the final transformation

between the tool reference frame A_{ref} and a point on the tool cutting surface *tool* is given by Eq. 2.2. The dimensions of the tool are altered by wear and thermal growth. Therefore, wear W and thermal errors H are included as scale factors in the model before deflection is considered. Scaling operations affect all matrices that they are pre-multiplied with. Hence, to avoid scaling the other error matrices, these scaling matrices are post-multiplied with the other error matrices.

$$[^{Aref}F_{tool}] = [V][D][H][W][\varepsilon][^{Aref}T_{tool}]$$
(2.2)

In Eq. 1 and 2, the order of combination of different error sources is not important for the case of small angular and linear errors. Typical values for geometric angular errors reported in literature for tool error are of the order of 0.01° and those for linear errors are of the order of a few microns [7, 11, 12]. Running the model for arbitrarily assumed error values indicate that the order of combination is important only if the translational error terms are greater than 50 µm and/or the angular errors are about 1°.

Consider the typical structure of a milling machine as shown in Fig. 2.1. Reference frames are placed at the spindle S_{ref} , spindle-holder interface *S*, holder T_{ref} , tool stem A_{ref} , base B_{ref} workpiece table L_{ref} , tool cutting surface *tool* and workpiece W_{ref} . Equation 2.1 can be used to determine the transformation *F* between any two consecutive reference frames except for the tool. Equation 2.2 is used for the transformation between A_{ref} and *tool*.

The relation between the workpiece reference frame W_{ref} and the tool tip reference frame *tool* can be expressed by Equations 2.3 to 2.5.

$$\begin{bmatrix} B^{ref}F_{tool} \end{bmatrix} = \begin{bmatrix} B^{ref}F_{Sref} \end{bmatrix} \begin{bmatrix} S^{ref}F_{S} \end{bmatrix} \begin{bmatrix} S^{F}F_{Tref} \end{bmatrix} \begin{bmatrix} T^{ref}F_{Aref} \end{bmatrix} \begin{bmatrix} A^{ref}F_{tool} \end{bmatrix}$$
(2.3)

$$[^{Bref}F_{Wref}] = [^{Bref}F_{Lref}][^{Lref}F_{Wref}]$$
(2.4)

$$[{}^{Wref}F_{tool}] = [{}^{Bref}F_{Wref}]^{-1}[{}^{Bref}F_{tool}]$$
(2.5)

Let ${}^{tool}P_t$ be the locus of points on the tool cutting surface with respect to the tool reference frame *tool*. The locus of all points on the tool cutting surface with respect to the workpiece reference frame W_{ref} will give an estimate of the machined surface using Eq. 2.6.

$$\begin{bmatrix} W^{ref} P_t \end{bmatrix} = \begin{bmatrix} W^{ref} F_{tool} \end{bmatrix} \begin{bmatrix} tool P_t \end{bmatrix}$$
(2.6)

If the error due to elastic recovery is represented by *E*, the final machined surface ${}^{Wref}P_{work}$ can be expressed by Eq. 2.7.

$$\begin{bmatrix} W^{ref} P_{work} \end{bmatrix} = \begin{bmatrix} E \end{bmatrix} \begin{bmatrix} W^{ref} P_t \end{bmatrix}$$
(2.7)

Estimation of stiffness of micro milling tool

Analytical model

The analytical model for evaluation of stiffness of the end mill is based on Timoshenko beam theory. Euler beam theory is applicable for beams that have a length to diameter (I/d) ratio greater than ten. Most micro milling tools have a flute length to diameter ratio of about 3. In such cases shear forces are no longer negligible and have to be accounted for in the calculation of deflection of the beam. Based on Macaulay's method for a Timoshenko beam [13], the equations for deflection δ and slope ϑ for a beam with moment of inertia *I*, elastic modulus *E*, shear modulus *G* and length *L*, with a moment *M* and force *F*_r applied at its end are given by Eq. 2.8 and 2.9.

$$\delta = \frac{F_r L}{0.9AG} + \frac{1}{EI} \left[\frac{ML^2}{2} + \frac{F_r L^3}{3} \right]$$
(2.8)

$$\theta = \frac{F_r}{0.9AG} + \frac{1}{EI} \left[ML + \frac{F_r L^2}{2} \right]$$
(2.9)

As shown in Fig. 2.2, the micro end-mill is approximated as two cylindrical beams. The larger cylinder represents the shank while the smaller cylinder represents the flutes. The intermediate tapered portion seen in Fig. 2.1 has been neglected. For the fluted region, the radial load F_r is assumed to be equal to 30% of the cutting forces F_c experienced by the tool. The shank is subjected to the same force, but due to the length of the flutes L it also experiences a moment given by $F_r \times L$.

Experimental validation of model

The tool stiffness model developed in the preceding section is verified by experiments. As shown in Fig. 2.3, an experimental set-up was built to measure the stiffness of micro milling tools. A laser displacement sensor (Keyence, LK-G37) was used to measure the deflection of the tool when loaded with weights. This sensor has a resolution of 10 nm. The laser beam has a rectangular profile of size $30 \times 850 \mu$ m. Two sets of semicircular grooves of diameter 3.175 mm (0.125 in) are machined on either side of the locking screws. The tool is held in a clamp between the two semicircular grooves. A dummy bar of diameter equal to the tool shank diameter is inserted into the other groove. This ensures an even clamping force on the tool shank. The sensor is mounted on a translation stage to position the beam at the required location on the tool along the X-axis shown in Fig. 2.3. The sensor can be moved along the Z-axis using the slots machined in the support. The entire set-up is mounted on an active isolation table. Weights are hung from the tool using 50 µm nitinol wires attached to the tool using an adhesive.



Figure 2.2: Tool geometry considered for the Error and stiffness Model

Deflection readings are taken at two locations along the tool as shown in Fig. 2.4. The first reading δ_1 is taken at the tool tip. This gives the stiffness K_1 of the entire structure from the tool tip to the base of the support structure. The second measurement δ_2 is at the beginning of the fluted section and gives the stiffness value K_2 for the entire structure except the fluted region. Using these two readings, the stiffness of the fluted region can be calculated using Eq 2.10.

$$\frac{1}{K_f} = \frac{1}{K_1} - \frac{1}{K_2} \tag{2.10}$$

A 500 µm diameter, two fluted end mill was loaded with three different loads to evaluate the stiffness of the fluted region. The usable flute length of the tool as specified by the tool manufacturer is three times the tool diameter. However, on studying the tool geometry under a microscope, it was found that the portion of the tool that has a diameter equal to the nominal diameter of the tool is 2.65mm. Study of the geometry of tools with diameters ranging from 100 µm to 500 µm suggests that the actual length of the tool tip for deflection calculations is about five times the diameter of the tool. Hence, this value is used to calculate the stiffness of the tool using the model. Results of the experiment are shown in Table 1. The experimental stiffness values and the stiffness values obtained from the model are shown in Table 2.2. It can be seen that the Timoshenko beam model is in good agreement with the experimentally determined tool stiffness.



Figure 2.3: Experimental set up to measure tool stiffness



Figure 2.4: Schematic of loaded tool for experimental stiffness evaluation

Table 1: Experimental stiffness values for a 500 μm tool				
Load (N)	0.7848	0.5886	1.1772	
$K_1(mN/\mu m)$	184.73	323.41	265.63	
K₂ (mN/μm)	821.57	1375.98	1150.56	
K _f (mN/μm)	238.32	422.77	345.36	

Table 2.2: Comparison between experimental and predicted tool tip stiffness for a 500 µm tool

	Average experimental	From model	% error
<i>K_f</i> (mN/μm)	335.49	339.65	1.24

Estimation of Thermal Error in Milling Tools

Coupled thermo-structural finite element analysis was used to estimate the thermal error in the tool due to heating of the flutes during cutting. The geometry used to model the tool is shown in Fig. 2.5. Taking advantage of symmetry, only one fourth of the tool geometry was modeled.

It is assumed that the portion of the flutes that is cutting heats up to 300°C. Hence, for the thermal analysis a temperature boundary condition of 300°C was applied to the bottom cutting region of the tool as shown in Fig. 6. The axial depth of cut is assumed to be equal to 10% of the diameter of the tool for microscale and equal to the tool diameter for macroscale analysis. A temperature boundary condition of 25°C was applied to the top surface of the tool. Heat flux on the planes of symmetry is zero.



Figure 2.5: Geometry and mesh used in coupled thermo-structural finite element analysis to estimate thermal error in tool

A convection boundary condition was applied to the outer cylindrical and conical surfaces except the portion of the flutes in contact with the workpiece. Convection coefficient for rotating microscale tools were estimated using Eq. 2.11 to 2.13 suggested by Shimada et al. [14]. The Reynolds number for the rotating microscale tools is between 3 and 535 under conditions mentioned in Table 2.3 and spindle speed 60,000 rpm. Equations for the convection coefficient at very low Reynolds numbers are not available. The closest to the range is Eq. 2.12 to 2.16 by Abu-Hijleh and Heilen [15]. However, this equation is valid for a buoyancy parameter κ greater than 0.1 as calculated by Eq. 2.15. For micro tools, the buoyancy parameter is of the order of 0.2×10^{-3} . Hence, this equation cannot be used. The limiting case of a Reynolds number of 5 and buoyancy parameter of 0.1 gives a convection coefficient of about 300 W/m²K. Hence, two analyses were run: one with the convection coefficient given by Eq. 2.11 and the other with a constant convection coefficient of 300 W/m^2 K. Both cases give similar results. Hence, the results corresponding to Eq. 2.11 are presented here. For macro scale the Reynolds number is between 2000 and 20000 for conditions mentioned in Table 2.3 and spindle speed 60,000 rpm. In this range, Eq. 2.17 by Becker [16] is valid and is used to calculate the convection coefficient. Properties of air at 300°C are taken from [17]. Reference temperature for thermal expansion and bulk air temperature is 25°C.



Figure 2.6: Boundary conditions in finite element analysis. (a) Thermal boundary conditions (b) Structural boundary conditions



Figure 2.7: Sample results of finite element analysis for 750 µm tool

$$\overline{Nu} = 0.046 \, Re_r^{0.7} \, \left(1 + \frac{8Gr}{Re_r^2}\right)^{0.95} \tag{2.11}$$

$$\overline{Nu} = \overline{h_0} \, \frac{d_{tool}}{k_a} \tag{2.12}$$

$$Re_r = \frac{\Omega d_{tool}^2}{2\nu} \tag{2.13}$$

 $\overline{Nu} = 1.586 + 0.05189 \, Re_r^{0.7072} [-0.4497 + 2.254 \, \kappa^{0.6729}]^{0.5978}$ (2.14)

$$\kappa = \frac{Gr_r}{Re_r^2} \tag{2.15}$$

$$Gr_d = \frac{g\beta(T_d - T_\infty)d_{tool}^3}{\nu^2}$$
(2.16)

$$\overline{Nu} = 0.119 R e_r^{2/3} \tag{2.17}$$



Figure 2.8: Geometric errors associated with the tool

	Micro scale	Macro scale
Tool diameter (mm)	0.25-3	6.35-19.05
Tool length (mm)	$5 \times d_{tool}$	$5 \times d_{tool}$
Shank diameter (mm)	3	d _{tool}
Shank length (mm)	15	15
Wear (µm)	10	100
CTE μm/m (°C)	5.2	5.2
Elastic modulus (GPa)	580	580
Shear modulus (GPa)	220	220
Shank tilt ϵ_s (°)	0.046	0.046
Number of teeth	4	4
Shank offset error (α_s) μm	7.2	7.2
Unit power for Aluminum (J/mm ³)	7.66	0.8737

Table 2.3: Tool properties and machining parameters

Thermal conductivity for the tungsten carbide tool is taken from [18]. The conductivity of tungsten carbide depends on the cobalt concentration and temperature. Also, thermal conductivity is expected to increase with increase in temperature. It is seen that higher values of conductivity results in higher thermal expansion values. Hence, to obtain a conservative estimate of tool thermal expansion, the lowest value of conductivity of tungsten carbide recorded in [18] is chosen for the analysis. This value is 84.02 W/mK.

The temperature distribution obtained from the thermal analysis is used in the structural finite element analysis to calculate the expansion of the tool. The top surface is constrained in all degrees of freedom fixed as shown in Fig 2.5. Symmetry boundary condition is applied on the planes of symmetry. The analysis was run for multiple diameters from 0.25 to 3 mm for the micro scale and 0.635 to 19.05 mm (0.25 to 0.75 inches) at the macro scale. The displacements at the tool tip and the beginning of the fluted portion of the tool are recorded after each run. These values are used in the error model. Sample results of a 750 µm tool are shown in Fig. 2.7.

Implementation of model

The model developed on page 2 is implemented in MATLAB^{*} to calculate the resultant error in the tool position with respect to the tool reference system T_{ref} for slot milling using a flat end mill. This involves calculating the error at the tool tip due to errors in the shank and the flute region as shown in Fig 2.2. The tool and shank are modeled as solid cylindrical beams. Geometric errors are shown in Fig. 8. Wear is assumed to be constant for each set of tool diameters examined. Values for wear in micro milling are of the order of those reported in [19]. Shank offset and tilt errors are assumed from results in [7].

Error due to tool deflection is calculated from Timoshenko beam theory [12]. Cutting force F_c (N) is calculated using Eq. 2.18 as a function of the number of teeth on the tool n_t , unit power of the material being cut u (J/mm³), radial depth of cut r (mm), axial depth of cut a (mm), feed per tooth f (mm), and tool diameter d_{tool} (µm). Force that causes deflection of the tool is the radial force assumed to be 0.3 times F_c as shown in Eq. 2.19. For a slotting operation, the radial depth of cut r is equal to d_{tool} . The axial depth of cut, a, is assumed to be equal to 10% of the tool diameter for micro milling. For macro scale milling the axial depth of cut is assumed to be equal to the tool diameter. This causes the feature size to scale with the tool diameter. The feed per tooth f is calculated as a linear function of d_{tool} using Eq. 2.20 for micro milling and Eq. 2.21 for macro milling. Eq. 2.20 is obtained by linearly interpolating between a feed value of 1 µm per tooth for a tool diameter of 250 µm and 5 µm per tooth for a tool diameter of 800 µm. Equation 2.21 is a linear interpolation between feed value of 0.05 mm (0.002 inches) per tooth for a tool diameter of 6.35 mm (0.25 inches) and 0.1 mm (0.004 inches) per tooth for a tool diameter of 19 mm (0.75 inches). These values are tabulated in Table 2.4.

Unit power for aluminum 6061-T6 was estimated experimentally by milling slots in an aluminum workpiece at different feeds and measuring the cutting force F_c . A 2 flute, 500 μ m tool was used in the experiment with an axial depth of cut of 50 μ m, radial depth of cut 500 μ m and spindle speed of 60,000 rpm. Forces were measured with a piezoelectric

force dynamometer (Kistler, 92656C2). Cutting force F_c is estimated by calculating the peak-to-valley variation of the force normal to the direction of cutting over a single revolution of the tool. The cutting force is measured for multiple instances during the cut. Substituting the mean of these values in Eq. 2.18, gives the unit power for the different feeds used as shown in Table 5. The average unit power value shown in Table 2.3 was used to calculate the cutting forces in the model for micro milling. For macro milling, the model uses a unit power value of 0.87 J/mm³ for aluminum, which is obtained from data handbooks. Parameters used in the model are shown in Table 2.3.

Tuble 2.4. Assumed recus for unreferred dumeters				
Tool diameter (mm)	0.25	0.8	6.35	19
Feed f (mm per tooth)	0.001	0.005	0.05	0.1

Table 2.4: Assumed feeds for different diameters

$$F_c = \frac{u \cdot n_t \cdot r \cdot a \cdot f}{(\pi \cdot d_{tool} \times 10^{-6})}$$
(2.18)

$$F_r = 0.3 F_c$$
 (2.19)

$$f = 0.45 \left(d_{tool} - 800 \right) + 0.005 \tag{2.20}$$

$$f = 4 \times 10^{-6} \left(d_{tool} - 19050 \right) + 0.1016 \tag{2.21}$$

Table 2.5: Force measurements for Different feeds					
Feed per tooth f (μ m)	0.8	1.7	2.8	4.2	5.8
$F_c(N)$	0.377	0.49	0.51	0.83	0.62
Unit power <i>u</i> (J/mm ³)	13.98	9.25	5.6	6.16	3.31

Discussion

The results of the simulation described in the previous section are presented in Fig. 2.9. The part feature dimension of interest in the simulation is the axial depth of cut. The simulation considers two ranges of tool diameters as indicated in Table 2.6. In all, six errors that contribute to the final position of the tool tip are studied, as shown in Table 2.6, which summarizes the maximum error at the tool tip caused by each of these errors and the maximum percentage contribution of each error to the sum of all six errors for each tool diameter. Figure 2.9 shows the error as a percentage of the axial depth of cut.

Figure 2.9 indicates that the error due to tool shank deflection as a percentage of the feature size increases with tool diameter for micro tools while it shows the opposite trend for macro tools. This is because micro tools have shanks of a

constant diameter of 3 mm, which deflects more with increasing cutting force. On the other hand, at the macro scale, the shank diameter increases with tool diameter and hence, the deflection reduces. The plots also show that the impact of tool geometric errors become more important at the micro scale. However, for the values assumed here, their magnitudes are still quite small.

It can be seen from Table 2.6 that wear and thermal errors of the tool are a significant source of error at both the macro and micro scales relative to the other error sources considered here. Tool thermal error is the sum of thermal errors due to the shank and the fluted portion of the tool. Hence, the effective thermal error at the tool tip due to the expansion of both the shank and the flutes is about 61.93% of the feature size for micro milling and 36.92% of the feature size for macro milling. At the micro scale, increased rubbing at the tool tip can lead to high temperatures and rapid tool wear.



Figure 2.9: Variation of errors with tool diameter. Error plotted as a percentage of feature size (depth of cut)

It is interesting to note that the errors due to tool deflection have a negligible effect on the slot depth. This can be explained by using Fig. 2.10. For a 500 μ m tool, F_r of 0.21 N causes a deflection δ of 0.6 μ m and a slope ϑ of 0.02°. Change in slot depth however, is $\delta \tan(\vartheta/2)$ which is only $0.1 \times 10^{-3} \mu$ m. It is to be noted that the tools used in [5] were about 1.5 times as long as the tools used for this study. Furthermore, the workpiece material in [5] was hardened steel while this study considers aluminum.



Figure 2.10: Relationship between tool deflection and change in axial depth of cut

Chapter summary

In this chapter a model was developed to predict the error in the complete machine tool structure based on homogeneous transformation matrices. This model integrates both kinematic and process induced errors. The model was illustrated for a range of milling tool diameters ranging from the micro to the macro scale. This method can be extended to the whole machine tool structure. Experiments were conducted to verify the assumption that Timoshenko beam theory can be used to evaluate the stiffness of the micro tool. From the results of the simulation, the following conclusions can be drawn:

- 1. Tool wear and thermal growth errors account for the majority of the change in slot depth.
- 2. Errors due to tool deflection have a negligible effect on tool depth.

Measurements of the tool length suggest that when calculating the deflection for a micro tool of flute length three times the nominal diameter of the tool using beam theory, the length of the tool should be taken as five times the diameter.

In the next chapter artifacts are developed to isolate the effect of each process error and measure the contribution of each error to the total error in micro-milled grooves.

	Table 2.6: Comparison of errors at tool tip at macro and micro scales				
Error	Maximum effec	t at tool tip (μm)	Max. percentage	of sum of all errors	
	Micro-scale	Macro-scale	Micro-scale	Macro-scale	
Shank thermal	6.2591	5.077	23.84	4	
Shank geometric	0.0096	0.035	0.04	0.02	
Shank deflection	0.012	0.009	0.036	0.007	
Tool thermal	10.05	58.805	38.16	35.18	
Tool wear	10	100	82.71	78.83	
Tool deflection	0.001	0.0048	0.004	0.003	

3.Design rules for 2D/3D artifacts

In this chapter rules are presented to design artifacts that separate the effect of each process error from the total error. Three artifacts are designed to isolate the effect of elastic recovery, tool static deflection, thermal error and wear. Results from experiments where these artifacts were used to measure the error sources are then presented. Thus, the significance of each error source to the total error in the depth of micro-milled grooves is evaluated.

Elastic recovery

Machining forces during micromachining elastically deform the material being machined. When the tool advances, this elastically deformed material returns to its undeformed position. This introduces an error in the machined surface. In the case of micromilled grooves, it causes a reduction in the depth of the groove.

The artifact designed to isolate the effect of elastic recovery is based on the principle that material will deform significantly only if the machining forces are sufficiently high. Consider two overlapping holes milled in a workpiece with the same axial depth of cut as shown in Fig. 3.1. The material at the base of the first hole deforms elastically during machining. When the tool is withdrawn from the hole, the material returns to its undeformed state. In the case of the second, overlapping hole, material deforms in the first case. However, in the region of overlap, the machining forces are small. Hence, the elastically deformed material from the first hole gets machined. The remaining material from the second hole springs back when the tool is withdrawn. This results in a step in the machined surface as shown in Fig. 3.1, which can be used to measure the error due to elastic recovery. The effect of thermal elongation of the tool tip and wear of the tool is small due to the short machining cycle. Effect of tool deflection is reduced by using a stub length tool.



Figure 3.1: Proposed artifact for elastic recovery

The artifact to measure elastic recovery was machined in Aluminum 6061-T6 using a 2 flute, 500 μ m tool. A 30 μ m hole was milled at 10 mm/minute axial depth of cut, and spindle speed of 60krpm. Then the tool was lifted and moved

by half the diameter of the tool. Another hole of the same depth was milled. The bottom surface of the holes were analyzed using a white light interferometer (ZYGO). As seen in the surface profile in Fig. 3.2. identifying the step was difficult , conical shape of the bottom of the hole. Hence, the artifact was modified to a set of overlapping grooves as shown in Fig. 3.3. The workpiece material and machining conditions were the same except that the axial depth of cut was 100 µm and the radial feed was 100 mm/minute. The machined artifact is shown in Fig.3.4. The bottom surface of the grooves was measured using the ZYGO and then analyzed using MATLAB. The expected surface profile is shown in Fig. 3.5.



Figure 3.2: Surface profile of bottom of artifact

The results of the analysis of the elastic recovery artifact are shown in Fig. 3.5. In all, twelve overlapping sections were analyzed. As indicated in Fig. 3.5, no clear steps were noted. In addition, the groove depth variation in the region of overlap is not significantly different from the variation is other regions of the groove. These results suggest that error due to elastic recovery is not significant as predicted by the model in Chapter 2.



Figure 3.3: Revised artifact for elastic recovery



Figure 3.4: Machined artifact for elastic recovery



Figure 3.5: Measured surface profile at base of elastic recovery artifact

Tool static deflection error

Error due to tool deflection is caused due to bending of the tool tip due to machining forces. This deflection causes a reduction in the depth of the micromilled groove. The artifact to measure deflection is based on the principle that the amount of deflection is a function of the transverse forces on the tool. Consider the case when the tool enters the workpiece from the side. Before contact with the workpiece, there are no machining forces on the tool. As the tool is feed radially in to the workpiece, the force increases until the entire tool diameter has entered the workpiece. Similarly, when the tool exits the workpiece, the radial forces reduce back to zero. This variation in radial forces suggests a variation in the groove depth as shown in Fig. 3.6. This variation can be used as a measure of error due to deflection.



Figure 3.6: Expected shape of groove base

The artifact used to measure the error due to tool tip deflection is shown in Fig. 3.7. It consists of short walls machined in Aluminum 6061-T6. These walls allow the tool to enter and exit the workpiece in the radial direction. The short length of the wall reduces the effect of wear and thermal elongation. As shown in the previous section, effects of elastic recovery are negligible. Grooves, 30 μ m in depth are micromilled in the walls with a standard length tool as shown in Fig. 3.8. The groove bottom surface is analyzed using the ZYGO.



Fig. 3.7: Artifact for deflection

Results from the analysis are shown in Fig. 3.8. In all, four grooves were analyzed. No consistent results are seen in the artifacts. This suggests that the effects due to deflection are smaller than the stage errors.



Figure 3.8: Groove bottom profile of deflection artifact

Thermal and wear errors

Error due to thermal expansion is due to the thermal elongation of the tool tip as it gets heated during machining. This causes an increase in the depth of the groove. Similarly, as machining progresses, the tool wears off, leading to a reduction in its cutting length. This leads to a reduction in the depth of micromilled grooves. Of the errors considered in this study, thermal error is the only error that leads to an increase in depth of the groove. This fact is used to isolate the effect of thermal error.

The artifact used to measure the effect of thermal and wear errors is shown in Fig. 3.9. The first feature to be milled is the reference surface. The purpose of the reference surface is to account for repeatable stage errors. The principle is shown in Fig. 3.10. A plane surface is first milled on the surface of the workpiece. This surface would be flat in the absence of geometric and process errors. However, due to the presence of these errors, it will not be perfectly flat. All process errors become negligible when the axial depth of cut is made very small. Hence, if the previously machined surface is milled again with a very small increment of depth of cut, the surface produced will be free from the effects of process errors. Hence, error in the reference surface is due to stage errors. Consider the case that process errors are absent and that the stage errors are repeatable. In this case, a groove machined on the reference surface will be parallel to the reference surface as shown in Fig.3.10. The error in groove depth at any section of the groove will be zero. In reality some error will be present due to process errors and stage repeatability errors. If a stage is used such that it has a repeatability value much smaller than the errors being measured, then stage repeatability errors can be considered negligible. Thus, process errors can be separated from repeatable errors.



Material: Al6061T6 Axial depth of cut: 50 μm Axial feed: 10 mm/min Radial feed: 100 mm/min Spindle speed: 60 krpm Tool: 500 μm, 2 flute, standard length

Figure 3.9: : Artifact for measuring thermal and wear errors

The reference surface in Fig.3.9 is machined first with a 30 μ m depth of cut. The depth of cut is then increased by 4 μ m and the surface is machined again to remove any process errors. The generated surface is then machined again with no increase in depth of cut to ensure a surface that is free from process errors. The first short groove is then milled with an axial depth of cut of 50 μ m and a length of 3 mm. The same path is traced three more times to ensure that this first groove is free from errors due to deflection and wear. The tool tip is cooled with water at 15°C to reduce the effect of thermal error on the depth of the groove. The long grooves are now machined without cooling in a continuous path from the first to last groove. Each groove is 14 mm long and there are 20 grooves. Finally, the second short groove is

machined with cooling. As in the case of the first short groove, the tool path of the second groove is also retraced three times to ensure that the surface is free from defection and wear errors.



Figure 3.10: Reference surface to account for repeatable stage errors



Figure 3.11: Combined effect of wear and thermal errors.

The machined surface is now measured using the ZYGO and analyzed using MATLAB to find the depth of the grooves. Four measurements are made for the short grooves and seven measurements are made at the first and last long grooves. Let d_{w1} and d_{w2} be the mean groove depths of the first and second short grooves respectively. Similarly, let d_1 and d_2 be the mean groove depths of the first and second long grooves respectively. Then, thermal error δ_{th} and wear δ_w are given by Eq. 3.1 and 3.2.

$$\delta_w = d_{w1} - d_{w2} \tag{3.1}$$

$$d_2 = d_1 + \delta_{th} - \delta_w$$

$$\delta_{th} = d_2 - d_1 + \delta_w$$
(3.2)

A total of six artifacts were machined and analyzed. The calculated thermal and wear values are shown in Table 2.1. Mean and standard deviation values for various groove depths in artifact 6 and shown in Table 3.2 to illustrate the typical total error seen in the grooves machined in the artifact. These values indicate that the total error in the groove depth is less than 3 μ m or 6% of axial depth of cut. Thermal error is about 2 μ m and wear is about 1 μ m for the length of cut and machining parameters used in this study. Significant variation is seen in the values which could be because of the stage repeatability which is about 0.2 μ m.

Table 3.1: Thermal and wear error measurements

Artifact No.	Thermal error	Wear error
1	1.9923	0.6308
2	1.3084	-0.1886
3	1.1843	0.6875
4	0.2073	-0.1514
5	-0.5056	1.0263
6	1.7785	1.0549

Table 3.2: Mean groove depths and standard deviations for artifact 6

Dimension	Average (µm)	Standard dev (µm)
<i>d</i> _{w1}	52.637	0.024
<i>d</i> _{w2}	51.582	0.139
<i>d</i> ₁	49.975	0.282
<i>d</i> ₂	50.698	0.151









Tool before artifact 5 Tool after artifact 5 Tool after artifact 6 Figure 3.12: Tool condition after machining artifacts

Error variance and RSS compensation

The RSS method [20] suggests that the cumulative standard deviation in the error in the dimensions of a batch of parts is the square root of the sum of the squares of the standard deviation of each of the contributing errors. Hence, if

 σ_e is the variance in the part, σ_{th} is the variance induced by thermal errors, σ_w is the variance induced by wear, then the cumulative error due to process errors is given by Eq. 3.3.

$$\sigma_e^2 = \sigma_{th}^2 + \sigma_w^2 \tag{3.3}$$

For the obtained measurements of thermal and wear errors, σ_{th} is 0.961 µm and σ_w is 0.554 µm. This gives the variance in groove depth due to process errors to be 1.109 µm. To compensate for these errors, the depth of cut should be reduced by 1.109 µm.

Chapter summary

In this chapter design rules were presented for measuring the contribution of process errors to the total error in the depth of micromilled grooves. Artifacts were designed for elastic recovery, tool static deflection, thermal elongation and wear. The artifacts were machined and estimates of the error values were obtained. From these measurements the following conclusions can be drawn:

- Total error in the groove depth is less than 3 μ m or 6% of axial depth of cut.
- Deflection effects are negligible in comparison with stage errors.
- Elastic recovery error is negligible.
- No significant wear visible on tool.
- Thermal error is about 2 μm.
- Wear error is about 1 μm.
- The stage repeatability is about 0.2 μm which is 0.25 times the error being measured. Hence, for better statistical significance, larger lengths need to be cut so that the errors are much greater than the stage repeatability error.
- Based on the RSS method, the variance in part dimensions will be 1.109 μm for the given machining conditions and material.

4.Conclusions

This report summarizes the study conducted on process errors in micromilling. The envisioned aims of the study were as follows:

- Design rules for creation of 2D/3D artifacts using WEDM/EDM and/or milling processes
- Developing a method for separating machine tool error and process error with the function of variance evaluation
- Mathematical model and procedure for spatial error compensation based on the RSS method and sample implementation in MATLAB

Chapter 1 presented a model to predict the error in the complete machine tool structure based on homogeneous transformation matrices. This model was used to study the relative importance of process errors on the depth of micromilled grooves. The model used experimentally measured stiffness values of micromilling tools to verify the stiffness values predicted by Timoshenko beam theory. Finite element analysis was used to estimate the thermal error in micro and macro machining. These values along with estimates of other errors were then used in the model to evaluate the significance of process errors on the total error in the depth of cut of micromilled grooves. From the results of the simulation, the following conclusions can be drawn:

- 1. Tool wear and thermal growth errors account for the majority of the change in slot depth.
- 2. Errors due to tool deflection have a negligible effect on tool depth.

Chapter 2 presented the design rules for separating machine tool error and process errors. Artifacts were developed for elastic recovery, tool deflection, thermal elongation of tool tip and tool wear. These artifacts were machined and the errors were estimated. The error measurements verify the predictions of the model that wear and thermal errors are the most significant of the process errors. Deflection and elastic recovery do not have a significant impact on the depth of the machined grooves. Finally, the measured values of tool thermal expansion and wear are used to obtain the variance of the part based on the RSS method.

As stated in Chapter 2, the repeatability of the micromilling stage is about 25% of the errors being measured. This could account for the high variance in the estimates of thermal and wear errors. The effect of the stage repeatability can be reduced by machining for a longer time, to increase the amount of tool wear and probably also thermal error. Experiments will be conducted to verify this hypothesis and based on these revised values, the machining code will be modified to compensate for the errors. The results of these experiments will be sent as an update by January 31, 2010.

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