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Martin B. Roberts, Jr.

ANALYTICAL METHODS FOR THE DETERMINATION OF OPTIMUM MAINTENANCE IN TRANSIT OPERATIONS

A THESIS

Presented to

the Faculty of the Graduate Division

By

Martin Braddock Roberts, Jr.

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

June 1958

ANALYTICAL METHODS FOR THE DETERMINATION OF OPTIMUM MAINTENANCE IN TRANSIT OPERATIONS

Approved:

Date Approved by Chairman: May 30, 1958

ACKNOWLEDGMENTS

The author would like to express his thanks and appreciation to all those who have assisted him in the preparation of this thesis. He is grateful to the reading committee, Drs. P. T. Eaton and J. H. Wahab, and to the thesis advisor, Professor W. N. Cox, for their constructive criticism and friendly encouragement. He would like to thank Drs. J. Krol and J. H. MacKay for their assistance on the design of the mathematical models and general helpfulness, and Dr. B. M. Drucker for his generous aid with the operation of the I.B.M. 650 Data Processing Machine. Thanks are also due the management of the Atlanta Transit Systems, Inc., especially Messrs. W. P. Maynard, J. L. Johnson, W. D. Dean, H. L. Taylor and A. G. Mathis, for their kind cooperation and assistance in the gathering of the data and in the answering of numerous questions during the study. And finally, the author is particularly grateful to his wife for her patience and encouragement throughout this undertaking.

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SUMMARY

Maintenance problems have been the subject of many articles and books; however, the literature yields relatively little quantitative research on some of the basic problems. The purpose of this study was to investigate four of these basic problems as listed under the following subtitles. The objective was to show how analytical methods can be applied to maintenance problems to provide a more quantitative basis for maintenance policies and controls. The data used for these analyses were obtained from the records of the Atlanta Transit Systems, Inc. of Atlanta, Georgia.

A review of the transit situation in the United States over the last twenty years reveals that the transit companies are in a worse financial position today than they were during the depression years. The trend toward suburban living and rising prices have increased transit expenses per rider faster than the cost of fares have increased. On a national basis the transit income, after expenses and all taxes, dropped from 10.35% of operating revenue in 1940 to 3.91% in 1955. The transit executives are working hard to correct this situation, and they are looking forward to an expanding transit industry in the future.

(1) <u>Methods of Measuring Maintenance Effectiveness</u>.--Maintenance has long been a difficult function to evaluate. Cost comparison methods and the American Transit Association system using "standard pars"

both have limitations. There is probably no perfect system, but one that shows promise is production functions. The production function method, although still in the development stage, attempts to determine the magnitude of the contribution of the various factors contributing to productive output. It is a mathematical tool which was impractical before the advent of rapid data processing equipment because of the large volume of calculations which are necessary. This method is applied to cost data of Atlanta Transit and regression coefficients are obtained for operating expense, investment expense, maintenance expense, administrative expense, and power expense. Little significance is attached to the numerical answers obtained; rather, demonstration of the application of the method is the objective.

(2) Distribution of Unpreventable Breakdowns.--Preventive maintenance methods are used to reduce the number of breakdowns; yet, even with the best preventive programs, a certain number still occur. Planning for these "unpreventable" breakdowns is important for achieving minimum overall maintenance cost. A knowledge of the mathematical nature of these breakdowns is necessary when using certain mathematical techniques for calculating optimum conditions. In studying service calls for city trolleys and buses, it was found that the breakdowns, called pull-ins, appear to follow the Poisson distribution. The minor troubles, called roadcalls, apparently do not.

(3) Most Economical Number of Repair Facilities and Units of Spare Equipment.--The number of repair facilities and units of spare equipment

are factors which affect the overall maintenance cost. These two factors are interrelated and inseparable from an economic viewpoint. A mathematical model was devised for a hypothetical repair cycle to enable calculation of optimum conditions for various operating levels. Assumed cost values were used to develop comparative cost curves.

(4) Effect of Age of Equipment on Number of Breakdowns.--Age of equipment is an important factor in the number of breakdowns that occur. Chronological age and usage age are different. In studying five groups of vehicles, it was found that, when charting cumulative pull-ins per vehicle (Y axis) against cumulative mileage per vehicle (X axis), in each case the data could be closely approximated by a cubic parabola, $Y = aX^3 + bX^2 + cX$. Attempts to relate the parameters, a, b, and c, with three broad classifications of factors influencing breakdowns (equipment design, operating conditions, and type of maintenance) yielded inconclusive results.

CHAPTER I

INTRODUCTION

Much has been written on the subject of maintenance. Various periodicals are devoted primarily or exclusively to the multitude of problems that confront the plant engineer, maintenance superintendent, or master mechanic. Whatever the title of the man responsible for maintenance may be, the numerous aspects of his job have been discussed in more articles than a person could read in a lifetime. However, the large majority of these articles are of a "How We Solved This Problem" type, and very little research can be found on some of the basic problems of maintenance.

Such periodical material is helpful in that it helps to keep the maintenance executive informed of the rapid technological advances in his field. Also, it often provides helpful suggestions on how he might improve a feature of his operations, or stimulate his thinking about the possibilities of a new machine or technique. These things are necessary to an alert engineer. But he still must make many of his most important decisions without sufficient facts, guided largely by experience and judgment.

This same statement could be applied to a greater or lesser degree to any supervisor; yet in the case of the maintenance supervisor it is especially applicable. His decisions affect all other departments of the plant because he is usually responsible for keeping the machinery and facilities of the plant operating efficiently. Breakdowns cause production delays and result in pressure from production supervisors for more preventive maintenance. Yet, when the plant is running smoothly, management looks with a critical eye at the maintenance manpower and expense reports.

Much more needs to be done in the area of research on basic maintenance problems. Wherever men work with equipment the same problems arise: How much and what type of maintenance should be done?, What type of repair facilities should be set up and how many? How should the time of the mechanics and other personnel be scheduled so that the breakdowns will be repaired and the preventive maintenance accomplished with the minimum loss of production? These and similiar problems confront every maintenance supervisor. Some of them are broad, concerning the organizational design, and others are of a day-to-day nature. In many companies the data necessary to properly answer the question may not be recorded, or they might be on several forms in different files and require too much time to compile into usable form. The maintenance man then usually gathers whatever data are readily available, fills in the gaps with judgment and through conferences with his subordinates, and makes his decision. The approval of the other members of management is often needed where equipment purchases, manpower changes, or revised maintenance schedules are involved. This approval is sometimes hard to obtain where facts are lacking and the maintenance engineer cannot prove his need factually.

This brief summary of the position of the average maintenance supervisor illustrates the need for more research and understanding of the problems involved in maintaining the complex plant of today. The situation is not all gloomy, however, because in certain areas much effort has been devoted to improving the maintenance function. From experience certain procedures have been developed for organizing (1) and administering (2) maintenance operations. Accounting techniques (3) have been designed to help plan, budget, and control maintenance expenses. Books (4) and periodicals (5, 6, 7, 8, and 9) cover almost every phase of the maintenance activity, giving rules and axioms to follow to achieve an efficient maintenance operation.

This is not enough, because general rules sometimes conflict and are often hard to apply to a specific problem. A better understanding of the nature of some of the basic maintenance activities is needed. For instance, one of the larger problems of controlling maintenance costs lies in determining the most practical division of effort and expense between preventive and breakdown maintenance (10). This division is now a judgment factor, often hardly more than a guess, and at best, based on rule-of-thumb methods. Few companies even record the data necessary to determine this balance quantitatively. A knowledge of the nature of the recurrence of breakdowns and the effect of age and other factors on the maintenance requirements would greatly aid in the efficient scheduling of overhauls and other preventive measures, as well as the determination of the nature and extent of maintenance operations. The efficient scheduling of the time of the personnel so that they are readily available when breakdowns occur and occupied with preventive work in the meantime is closely associated with this knowledge. This knowledge is also necessary for the proper determination of the optimum inventory of spare parts and spare equipment so that the minimum investment and maximum availability of parts combination will be known.

The likelihood is that these problems will increase in quantity and importance in future years. The increasing trend toward more complete mechanization and the increasing complexity of the machines and equipment tend to increase the importance of maintenance and its problems. In the automatic plant of the future mistakes will be more costly, the maintenance function will have to be re-evaluated, and a higher efficiency will be demanded of the maintenance group (11). All of these considerations point to the need for a clearer understanding of the basic nature of the problem.

"When you cannot measure, your knowledge is of a meager and unsatisfactory kind." This famous statement by Lord Kelvin is appropriate to many maintenance problems. It was believed for many years that maintenance work was so varied and non-repetitive that the time study techniques which were being applied to productive operations could not be applied to maintenance. This fallacy has been exposed and many companies now not only have maintenance work standards, but also pay the maintenance personnel individually on an incentive plan. This same type of thinking has applied in many cases to the analysis of the other maintenance problems. The great variety of equipment in the average plant, the difficulties and expenses of obtaining, recording, and analyzing large volumes of data, and the lack of suitable techniques for quantitative analysis have hindered the development of a clear understanding of the basic problems. In recent years the widespread use of rapid data processing equipment and the advent of new mathematical techniques have increased the possibilities of putting the maintenance function on a more scientific basis.

Maintenance policies and procedures should be based on scientific studies of the problems rather than on experience or convenience. Maintenance supervision needs facts to guide it in its own policies and to convince management that certain expenditures are justified. A better method of determining maintenance effectiveness is needed to replace the traditional accounting system. The essential data that are needed on the equipment and working situations in order to determine optimum procedures must be investigated and resolved. These things are prerequisites for obtaining the most effective utilization and control of the money invested in equipment and maintenance expense.

The purpose of this study is to investigate four of these problem areas:

(1) Methods of measuring maintenance effectiveness

(2) The mathematical nature of service calls and breakdown data

(3) Method of determining the most economical number of repair facilities and spare units of equipment

(4) The effect of the age of the equipment on the number of equipment breakdowns

Mathematical techniques will be used wherever they are appropriate. The objective will be to show how analytical methods can be applied to maintenance problems to provide a quantitative basis for maintenance policies and controls.

The data used for these analyses were obtained from the records of the Atlanta Transit Systems, Inc. of Atlanta, Georgia. This source was selected because it had detailed maintenance records for a period of years on a large number of machines and the company personnel were cooperative in making the records available and in providing supplementary information concerning them.

CHAPTER II

A REVIEW OF TRANSIT OPERATIONS IN THE UNITED STATES

A review of the transit situation in the United States over the last twenty years reveals that the transit companies are in a worse financial position today than they were during the depression years. In recent years the American people have been moving into widely scattered suburban areas which are hard to service economically with buses and trolleys. They are driving their own automobiles to and from work and using the transit systems less and less. The table below from the Transit Fact Book, 1956 edition, shows that although revenues are increasing because of increasing cost of fares, the money left after expenses and taxes are paid is rapidly declining.

Table 1. Results of Transit Operation in the United States at Five Year Intervals, 1935-1955

Year	Operating Revenue (millions)	Operating Expenses Incl. Dpr. & Taxes (millions)	Operating Income (millions)	Rides per Capita of Pop.	Per Cent of Operating Revenue: Expenses & Taxes	Per Cent of Operating Revenue: Operating Income
1935	$ 681.4	$ 585.4	$ 96.0	171	85.91	14.09
1940	737.0	660.7	76.3	176	89.65	10.35
1945	1380.4	1231.7	148.7	312	89.23	10.77
1950	1452.1	1385.7	66.4	195	95.43	4.57
1955	1426.4	1370.7	55.7	124	96.10	3.90

This trend has forced the transit companies to re-valuate many of their problems and look for new ways to reduce expenses and increase the number of riders. There is unfortunately a limit to the reduction of expenses. As Mr. F. F. Cordone of the Syracuse Transit Corporation points out:

> With costs going up and revenues going down, management periodically calls for further reductions in maintenance expenditures. It has been truthfully said that you can't survive while spending more money than you take in. But there is a limit to reducing maintenance costs and still maintaining an economical operation over a long period of time. Experience has shown that the keystone of economical and effective maintenance is the servicing and replacement of units before they fail or before excessive wear cause damage to related parts (12).

With expenses drastically reduced, much of the effort has been directed toward increasing the number of paying passengers. Low family rate plans, express busses, more accurate schedules, and more courteous service are a few of the incentives the transit companies are offering (13). Most companies are operating at or near the breakeven point and anything that will reduce costs or increase the volume of riders is regarded favorably.

The Atlanta Transit Systems has had similar problems to those of other transit companies. Although the cost of fares increased by more than fifty per cent between 1950 and 1956, revenues were relatively constant. The income after expenses and before income taxes were paid, dropped from $574,000 to $190,000. Rising costs and wages cut profits from 6.45 per cent of revenue in 1950 to 2.15 per cent in 1956. A reorganization of the company in 1954 helped the situation somewhat by establishing a broader tax base and increasing the depreciation allowances, but income still dropped.

Figure 1 shows a comparison of Atlanta Transit data with national averages for several critical ratios. Chart A reveals that although both Atlanta Transit and the industry average figures on passengers per mile are steadily decreasing, Atlanta Transit is holding its riders better than the average for the industry. Chart B shows that the income per passenger in Atlanta is less than the national average. Chart C reveals that the revenue per vehicle mile is rising. For Atlanta Transit this increase in revenue per vehicle mile is not reflected in overall revenue, however, because of a reduction in miles driven from 17.6 million in 1950 to 15.2 million in 1956. This reduction was made necessary by insufficient fares on some scheduled runs, causing them to be shortened and combined with other runs or eliminated. The situation can be summarized by saying that costs are rising faster than revenues and unless the trend is reversed, the transit companies may soon be owned and operated by public agencies rather than private industry.

The transit industry is aware of its vulnerable position and is conducting a long range program to secure greater communter use. Besides the additional service items mentioned previously, the transit industry is investigating many possibilities of future improvement. The American Transit Association has appointed a special committee to design a more attractive bus for the future. Ideas are being accepted from transit men, professional designers, manufacturers, and the general public. The objective is to secure far greater comfort, convenience and styling. Softer lighting that will make passengers appear younger, improved ventilation, air conditioning, two-way telephones, and television

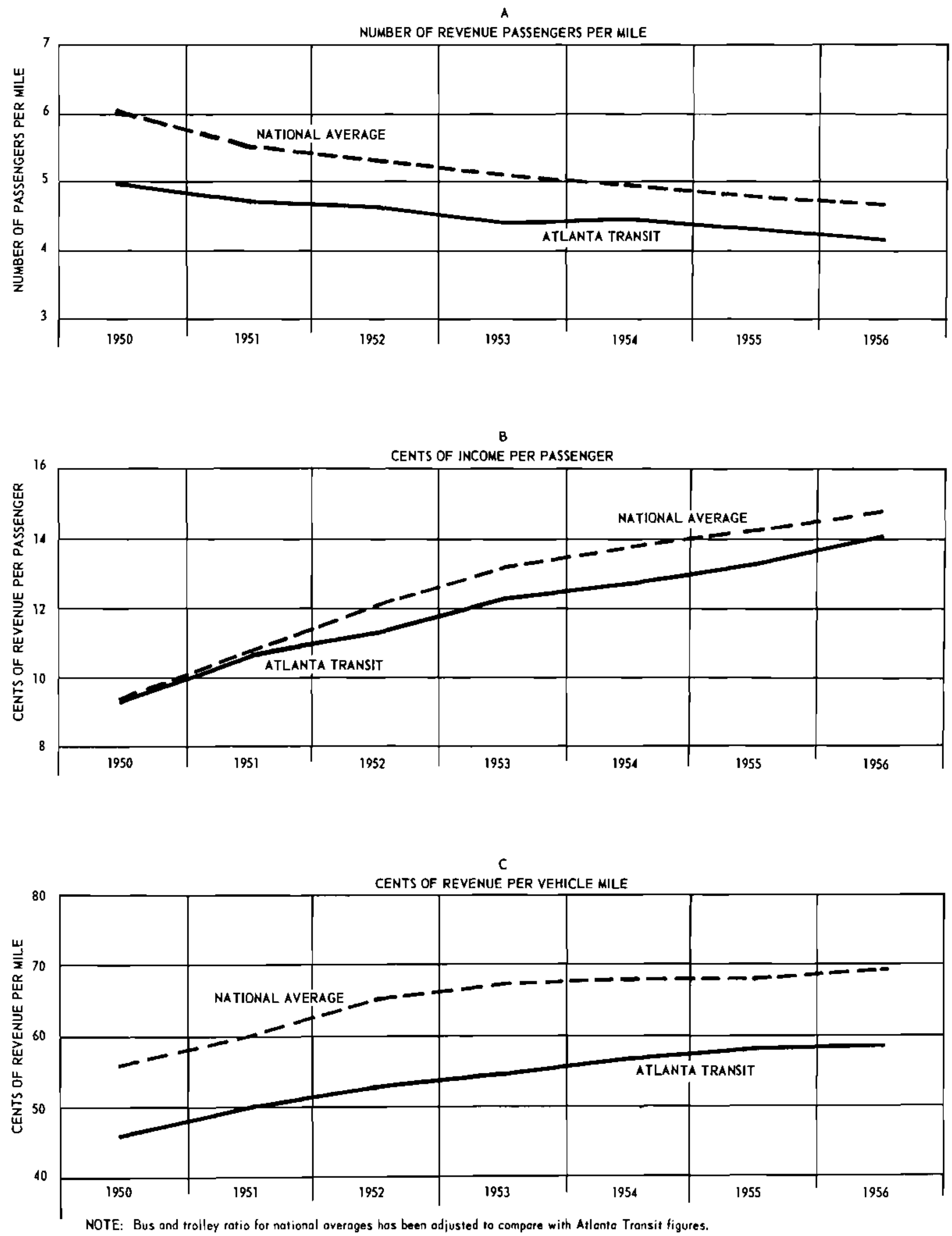


Figure 1. Comparison of Atlanta Transit Statistics with National Averages.

are some of the innovations being considered. Faster service and more economical operation will also be sought. The use of the gas turbine as motive power presents some intriguing possibilities if the present problems can be solved satisfactorily. These ideas and plans foster the belief among transit executives that the future holds great promise for them. They are looking toward an expanding industry that is now suffering temporary reverses (14).

CHAPTER III

METHODS OF MEASURING MAINTENANCE EFFECTIVENESS

Maintenance has long been a difficult function to evaluate. Because of the complex inter-relationships between production and maintenance, it is difficult to determine what the proper amount of maintenance should be. When equipment is over-maintained, maintenance costs go up, while insufficient maintenance results in breakdowns and production losses. To find the proper balance between the two is an elusive problem. Many techniques have been developed to help management with this decision. In addition to the traditional accounting procedures, many useful ratios and percentages have been determined by experience. Alford et al. note that:

> A primary requisite for adequate maintenance is sufficient men - but not an excessive number - of each craft to meet the demands under peak loads.
> .... Where maintenance work has not been setup on an organization basis, past history is an unreliable guide: frequently, unnecessarily large crews have been retained. On the other hand, an aging plant may require an increase in the crew to avoid the danger of undermaintenance.
> .... As equipment units become larger, heavier, and more intricate, the proportion of maintenance workers increases, but the claim is made that, in a given type of manufacturing, approximately equal ratios (maintenance forces to total plant forces) are found in well managed plants. ...
>
> A breakdown by crafts is useful in adjusting the maintenance force to the annual level of activity. Comparisons between plants, even when the product is similiar, should be made with caution. ...
>
> The Emerson Engineers state:
>
> Our experience over a period of years is that the annual cost of labor and material in maintenance of modern oil refineries,

> with well planned maintenance programs, tends to run between 5-1/2% and 6-1/2% of total plant cost, excluding land. Various parts of the plant, however, show widely different ratios (15).

These statements sum up the general thinking that only rough guide can be obtained from experience. Each situation has so many variables affecting the maintenance cost that it is difficult to know by which yardstick to measure maintenance effectiveness. D. E. Pierce, Chief Engineer of General Aniline and Film Corporation, in trying to find a quantitative method for comparing maintenance costs from year to year and from plant to plant, tried numerous ratios and indexes without success. He finally discovered that plant activity, as measured by kilowatt hours of electricity, was his best index to maintenance costs (16).

In the transit field members of the American Transit Association are participating in a plan to develop a reliable yardstick called "standard pars." As in the "par" on the golf course, the industry is attempting to set up man-hour measurement standards whereby management can evaluate effectively its maintenance performance. There have been about one hundred companies participating in the program since its beginning in 1952. The "par" is measured in man-hours of work required for the various types of activity.

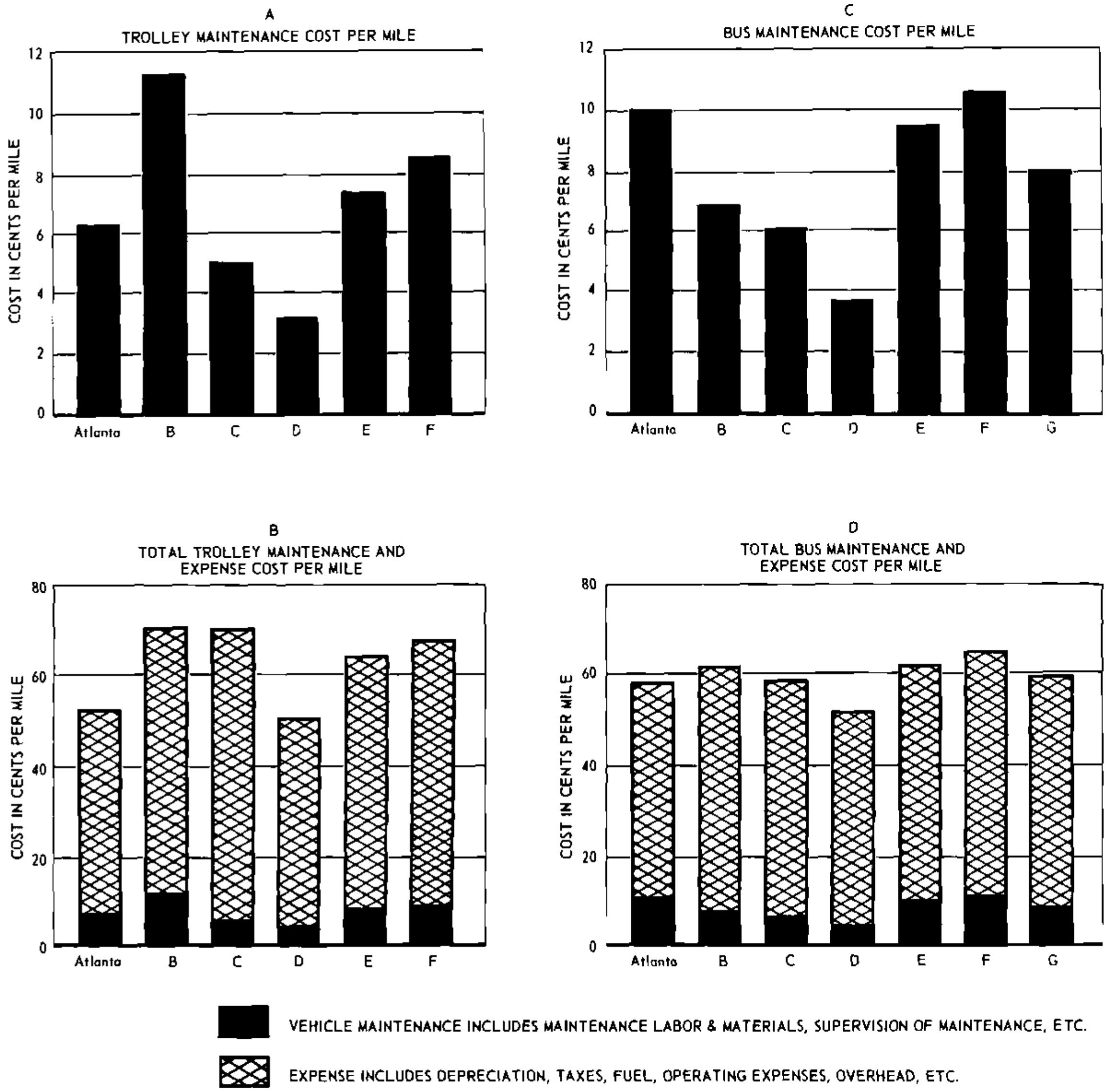
United Transit Company, Providence, Rhode Island, one of the participating companies, reports:

> Standard pars have worked out fairly well in measuring such items as platform costs, servicing, transportation supervision, office personnel and some others. These functions are about the same for companies of comparable size regardless of location.

> However, in applying 'pars' to maintenance, running repairs, and overhauls, complications arise from factors such as make, age, and size of vehicles, climate, terrain, type of operation, location and number of garages needed to properly serve the fleet (17).

Probably the most widely-used method of measuring maintenance effectiveness is by cost comparison, either between organizations or between different periods for the same organization. High maintenance costs are usually interpreted as poor maintenance effectiveness. This is not necessarily accurate, especially between different organizations, because of differences in equipment, operating conditions, and in accounting procedures. Figure 2, page 15, emphasizes this point. These cost data were obtained by sending questionnaires (see pages 76 and 77 in Appendix) to ten transit companies in cities approximately the size of Atlanta. Of the eight companies that replied, six furnished usable data. Notice the large variations in costs and then compare the costs with the operating conditions in Chart E.

It is difficult to determine a consistent relationship. For example, it might be expected that the company with the most vehicles would have the lowest cost per mile because of the economies possible with large scale operations. When the trolley maintenance cost per mile in Chart A is compared with the number of trolleys per company in Chart E, company B has the highest cost per mile and the fewest trolleys, also, Atlanta has the greatest number of trolleys but not the least cost per mile. This does not necessarily mean that Atlanta's maintenance is less effective, because many other factors, such as equipment design, accounting procedures, wage scales, and operating procedures, greatly affect the maintenance costs.



E

CITY	1950 POPULATION	SQ. MI. AREA	AVG. NO. VEH. PASS. PER YR. IN MILLIONS B & T	TROLLEY NUMBER OF VEHICLES	TROLLEY AVG. VEHICLE AGE IN YRS.	BUS NUMBER OF VEHICLES GAS	BUS NUMBER OF VEHICLES DIESEL	BUS AVG. VEHICLE AGE IN YRS. GAS	BUS AVG. VEHICLE AGE IN YRS. DIESEL
Atlanta	331,000	127.0	69.0	367	7.7	83	29	8.8	1.2
B	374,000	69.8	35.7	45	7.5	149	57	6.0	4.5
C	376,000	N.A.	44.9	220	11.0	65	37	7.0	1.6
D	396,000	121.1	53.1	218	9.3	41	83	13.2	4.5
E	326,000	66.9	35.0	137	6.9	137	5	8.1	.6
F	457,000	80.6	41.8	134	11.7	366		8.6	
G	334,000	99.4	32.3	--	---	6	236	8.5	7.3

NOTE: All vehicle figures are average for years 1954-6.

Figure 2. Comparison of Maintenance Cost Per Mile for Transit Systems in Cities of Approximately the Same Size.

From the data in Figure 2 it would be surmised that company D had the most efficient maintenance organization. This may or may not be accurate. A detailed study of the accounts and the maintenance activities of each company would be necessary to establish this point. This type of study is beyond the scope of this work. Even with the detailed study it would be hard to evaluate the effect of differences in equipment and operating conditions. Cost comparisons between maintenance groups must be used with caution. Within the same organization increased maintenance costs might be the results of aging equipment, price increases, personnel turnover, and other items that are sometimes beyond the control of the maintenance supervisor. The best cost system, with detailed costs on equipment and manpower, helpful though it is, still does not tell management if the best maintenance job is being done at the lowest overall cost.

What management needs in order to accurately evaluate maintenance is a system that not only considers maintenance costs but also takes into account the effect of maintenance on the productivity of the whole organization. Breakdowns of productive machinery create production losses, which are a productive cost just as the repairs are a maintenance cost. The idle productive labor and production delays might cost several times the maintenance cost for repairs. Again comes the question, "Where is the most effective cost balance between breakdowns and preventive maintenance?" This is a question of economics. It may be assessed by the analysis of maintenance costs and productive losses due to maintenance breakdowns, provided the detailed cost data are available. The author worked with a cost system of this type for about a year. Each breakdown was analyzed as to cause, and the department responsible for the breakdown was charged with the productive labor that was lost. The company finally discontinued the system because of the difficulties in determining the responsibility for each breakdown. It did not enable management to evaluate maintenance.

There is probably no system that will give all the information desired, for evaluating as complex a function as maintenance; however, one that shows promise is that of production functions.

The idea of production functions was originated by economist Paul H. Douglas, now Senator Douglas of Illinois. In studying the effect of increases in labor and capital on production, he determined a mathematical relationship which appeared to have validity for the total of all manufacturing industries of the United States. This relationship was

$$P' = 1.01\, L^{.75}\, C^{.25}$$

P' = computed production

L = labor expressed as number of wage earners

C = capital invested in machinery, tools, equipment, and buildings.

This relationship means that an increase of one per cent in the quantity of labor, with the quantity of capital constant, would lead to an increase of three-fourths of one per cent increase in production. Senator Douglas does not claim the discovery of a new economic law but states, "It is the purpose of this paper, then, not to state results but to illustrate a method of attack (18)."

Since the publication of this concept in 1928 there have been many studies of different industries in various countries which tend to substantiate it (19). Studies have also been made of individual firms which indicate that the concept of the production function is quite as applicable to the individual firm as it is to the industry (20). Not all economists agree with this mathematical approach (21 and 22), and it will probably require much more research to determine whether or not it is valid.

The production function concept has been extended by Tintner (23) to include variables other than labor and capital. In his derivation of production functions for various types of Iowa farms, he uses as variables acres of land, months of labor, farm improvements, liquid assets, working assets, and cash operating expenses. All of the variables except the first two are expressed in dollars. He says that "... the choice of our variables is to a certain degree arbitrary and could easily be replaced by another classification." The variables were expressed as a function, similar to the Douglas function, which was linear in logarithms. The regression coefficient for each of the variables was obtained. The larger the regression coefficient of a variable, the more important the variable. Tintner's results (24) are shown in Table 2.

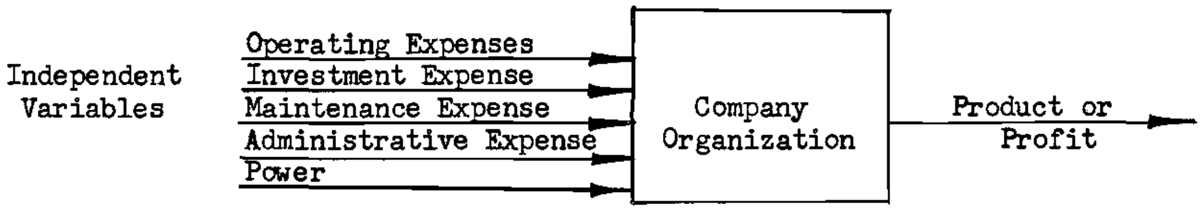
Table 2. Regression Coefficients for Four Types of Iowa Farms

Type of Farming	Land	Labor	Improve-ments	Liquid Assets	Working Assets	Cash Operating Expenses
Beef Feeders	.276	-.025	.097	.517	-.081	.004
Crops	.586	-.062	.045	.095	-.097	.203
Dairy	-.131	.469	.092	.197	.249	.254
Hogs	.278	.233	.041	.168	-.003	.183

The results were consistent with what might be deduced logically. Comparing the various types of farming, land was the most important in crop farming, whereas labor was the most important in dairy farming.

The validity of using the production function method to determine the magnitude of the contribution of the various factors contributing to production output has not yet been scientifically proven. On the other hand, favorable results have been obtained in many checks by different investigators (25, 26 and 27) and the majority of opinions indicate that it has considerable merit as an analytical tool.

This method will be used to demonstrate how the importance of the various functions of the organization might be determined, as measured by their contribution to the company's profit. The maintenance function will be of major interest. The organization will be visualized schematically as having certain independent input variables and one dependent output variable, as shown at the top of the next page.



The choice of the independent variables was more or less arbitrary. Each variable was selected so that the company's costs could be divided into logical areas of activity for which the costs are recorded. The details of this breakdown are shown on page 79 in the Appendix. A regression function which is linear in the logarithms will be used. This is the same type of function that Professor Douglas used in his studies except that no assumption of homogeneity will be made, i.e., the sum of the regression coefficients will not necessarily be equal to one. This type of regression function will immediately give the elasticities of the dependent variable with respect to each of the independent variables. That is, from the results it can be seen by how many per cent the dependent variable will change if one of the independent variables is increased one per cent. Elasticities are dimensionless numbers and independent of the units of measurement. The logarithmic transformation of the variables will also preserve their normality to a substantial degree if the errors in the data are small and normally distributed (28). If the errors are not normally distributed and not independent, the best linear estimate will still be obtained by the application of the method of least squares (29). The mathematical formula is:

$$R = b\,O^{k_1}\,I^{k_2}\,M^{k_3}\,A^{k_4}\,P^{k_5}$$

where, R = revenue derived from operations before income taxes are deducted

b = a constant

O = operating expenses K_1 = elasticity for O

I = investment expenses K_2 = elasticity for I

M = maintenance expenses K_3 = elasticity for M

A = administrative expenses K_4 = elasticity for A

P = power expenses K_5 = elasticity for P

The cost data for each of the variables for buses and trolleys were obtained from the records of the Atlanta Transit System for the years 1951-1956. The data were totaled at six-month intervals to decrease monthly fluctuations and still yield sufficient points for a regression analysis. The logarithms of the dollar figures were obtained. The data were punched into IBM cards and the regression analysis run on the IBM-650 Data Processing Machine at the Georgia Tech Engineering Experiment Station. The program for this analysis was obtained from the Engineering Experiment Station file. The results are tabulated below:

Table 3. Regression Coefficients for Types of Expense of Atlanta Transit Systems Vehicles

Item	Regression Coefficients	
	Trolleys	Buses
Operating Expenses	.7236	.7363
Investment Expense	.0504	-.0111

(continued)

Table 3. Regression Coefficients for Types of Expense of Atlanta Transit Systems Vehicles (continued)

Item	Regression Coefficients Trolleys	Buses
Maintenance Expense	.1316	.2753
Administrative Expense	.0992	-.0428
Power Expense	.2192	-.0904
Sum of Regression Coefficients	1.2240	.8673
b values	1/5.5	9.4

The formulas for the two types of vehicles would be,

for trolleys, Revenue $= \frac{1}{5.5} O^{.7236} I^{.0504} M^{.1316} A^{.0992} P^{.2192}$

for buses, Revenue $= 9.4 \quad \frac{O^{.7363} \quad M^{.2753}}{I^{.0111} \; A^{.0428} \; P^{.0904}}$

The multiple correlation coefficients for the analyses were .9573 for trolleys and .9321 for busses.

At this point it should be stressed that the purpose of this experiment was to show method. Little significance is attached to individual regression coefficients because no attempt was made to bring all dollar figures to a common index or to eliminate unusual costs. For instance, changes in fare rates, power rates, and fuel were not considered. A change in the policy of depreciation which greatly affected the amounts which could be charged to depreciation was not adjusted; unusual costs such as accident claims were not deducted. This type of analysis also assumes that if the independent variables are increased, the dependent variable, in this case the revenue, will vary as it has in the past. With transit operations this is probably not strictly correct. The scheduling of the vehicles to obtain the maximum revenue is limited by the number of riders on the various routes and increasing operations would bring a reduced income per mile operated.

A more thorough investigation of the company structure might also reveal some desirable changes in the variables involved. More or fewer variables might be better. Instead of using dollar figures for all the variables, it might be better to use man hours worked, number of vehicles in active use, or units of power. This type of investigation is beyond the scope of this study.

However limited the results may be, it is interesting to note the consistency of the coefficients for Operating Expense. This variable is largely made up of the vehicle operator's wages, and it is logical that the more the vehicles are operated, the more revenue will be obtained. This study indicates that one per cent increase in the logarithm of trolley Operating Expense, equivalent to $239,000, would yield about .73 per cent increase in the logarithm of Revenue, or $428,000, on the average. The results also indicate that one per cent increase in the log of Bus Maintenance, $22,000, will yield .275 per cent increase in the log of Bus Revenue, $23,100, whereas the same amount of money spent on Trolley Maintenance will increase Revenue only $18,000 (see sample calculation on page 84 in Appendix). If these figures were correct, this would mean that the trolleys were being over-maintained and that approximately a five per cent return could be obtained by increasing the Bus Maintenance by $22,000.

This type of information would obviously be very useful. A great-deal more investigation and research will be needed, however, before this method of analysis will be reliable. Prior to the advent of rapid data processing equipment the calculations which are necessary for this analysis made it impractical. As the use of computers increases and the trend toward more quantitative analytical methods continues, the production function analysis may prove to be a very useful tool.

CHAPTER IV

DISTRIBUTION OF UNPREVENTABLE BREAKDOWNS

Maintenance is often thought of in terms of repairs. Although the two words are associated, the meanings of the functions are quite different. Repairs are the remedial work needed to restore a piece of equipment or a building to a sound or satisfactory state after breakdown or deterioration. Maintenance is the noun form of "to maintain" or to keep in a desired condition, usually in a state of efficiency. Repairs are almost always needed in any maintenance program, and, probably for this reason, the usage of the two words has made them seem synonynous to most people.

At one time the main function of a plant maintenance and repair group was to get the machines back into operation after they had broken down. The emphasis was on speed of repairs rather than on effectiveness and permanence. Management tended to regard maintenance as the friction in the wheels of industry rather than the lubricant. More recently, however, management has recognized the production advantages of good maintenance and the trend has turned toward organized maintenance programs to prevent interruptions of operations and the subsequent losses of production (30).

These programs consist primarily of routine inspections combined with minor repairs and adjustments, and the periodic overhauling of the machinery according to a predetermined plan. The frequency of the inspections and overhauls is usually determined on a time or a usage basis, depending on which provides the best index as indicated by experience. The purpose of the planning, of course, is to replace worn or damaged parts before they cause breakdowns and also to schedule this maintenance work so that it interferes with production as little as possible.

This trend toward preventive maintenance has grown until many maintenance engineers feel that they are "over-maintaining" the equipment. They believe that, although preventive methods are necessary, preventive maintenance is subject to the law of diminishing returns, and that it is just as important to plan for the unexpected breakdowns as for the routine inspections and overhauls. It is these unanticipated breakdowns that cause emergencies and result in the majority of the productive losses. The productive cost for a breakdown might well be many times the cost of the repairs, and even with the best programs, some breakdowns still occur. The number of these breakdowns will depend on several factors such as the age of the equipment, the amount and quality of the preventive maintenance program, the operating conditions, and the type and design of the equipment. Mr. F. F. Cordone, Vice President in charge of equipment at Syracuse Transit Corporation, says:

> For the most part, the proper and economical periods between basic maintenance functions are practically fixed in the design of the vehicle, at the time of its manufacture. It does not appear that much can be done to stretch these periods. It is possible, of course, to reduce the quality of a given maintenance program and retain an outward appearance of a successful money-saving operation for a certain period of time. But sooner or later reductions mean increased costs and more failures (44).

An engineer in Great Britain supports the idea that the maintenance program should be adapted to the equipment to be serviced, and that it may vary widely among different plants, but for a given plant there should be a defined minimum cost associated with a particular degree of preventive maintenance. He says;

> From the viewpoint of the maintenance engineer, machinery and its parts fall into two of several categories. Two of them will deal with predictability:
> (i) Those with a predictable life.
> (ii) Those whose lifetime is random and unpredictable.
>
> Another pair of categories deals with inspectability:
> (a) Those, the observable conditions of which give warning of an approaching failure (wear, cracks, corrosion); these are inspectable.
> (b) Those that are 'uninspectable'.
>
> Thus there are four categories of parts:
> (ia) These fully qualify for preventive maintenance by inspection or replacement.
> (ib) These do not qualify for preventive maintenance.
> (iia) These qualify for inspection procedure.
> (iib) Should never inspected and should be replaced only on failure (31).

He also points out that overhauls and some types of inspections require the dismantling and reassembling of a complex machine, taking several hours, in order to inspect parts which may or may not need repairs or replacement. This can be a source of wasted labor. Another effect of the dismantling and reassembling is that it creates "disturbances" which sometimes cause recently overhauled machines to be more subject to breakdowns than ones which have not been touched. Some types of machines appear to be more susceptible to this phenomenon than others. It is believed that these disturbances are caused by inaccuracies or faults arising out of and during the reassembling of the machines. The skill of the mechanics will influence the number of the disturbances; however, many of the complex machines of today require a combination of skills which are not always easily obtainable.

Apparently Mr. Ferney has done some research on this problem and in his article he presents two charts, Figure 3, showing how these disturbances affect the number of breakdowns for delicate and for robust machinery. The chart for delicate machinery, such as complex radio and radar equipment, shows that immediately after an inspection the number of breakdowns, on the average, goes up and then gradually decreases. For robust and orthodox machinery (mechanical) a different pattern appears. Breakdowns tend to decrease after an inspection and then rise with the lapse of time. The significant point is that the rise in each case is not from zero, but from a fairly high level (60 to 80 per cent of the maximum reached). The implication is that a large portion of the breakdowns are caused by a multitude of factors which cannot be detected by inspections. This means that only a small proportion of the potential troubles can be prevented by inspections and overhauls. The preventable portion will vary, naturally, with the type and design of the machinery.

It is the purpose of this portion of the study to investigate the unpreventable breakdowns of certain groups of trolleys and buses of the Atlanta Transit System to determine their distributions. A knowledge of the distributions will aid in the planning and controlling of the work required to repair the breakdowns. The distributions will

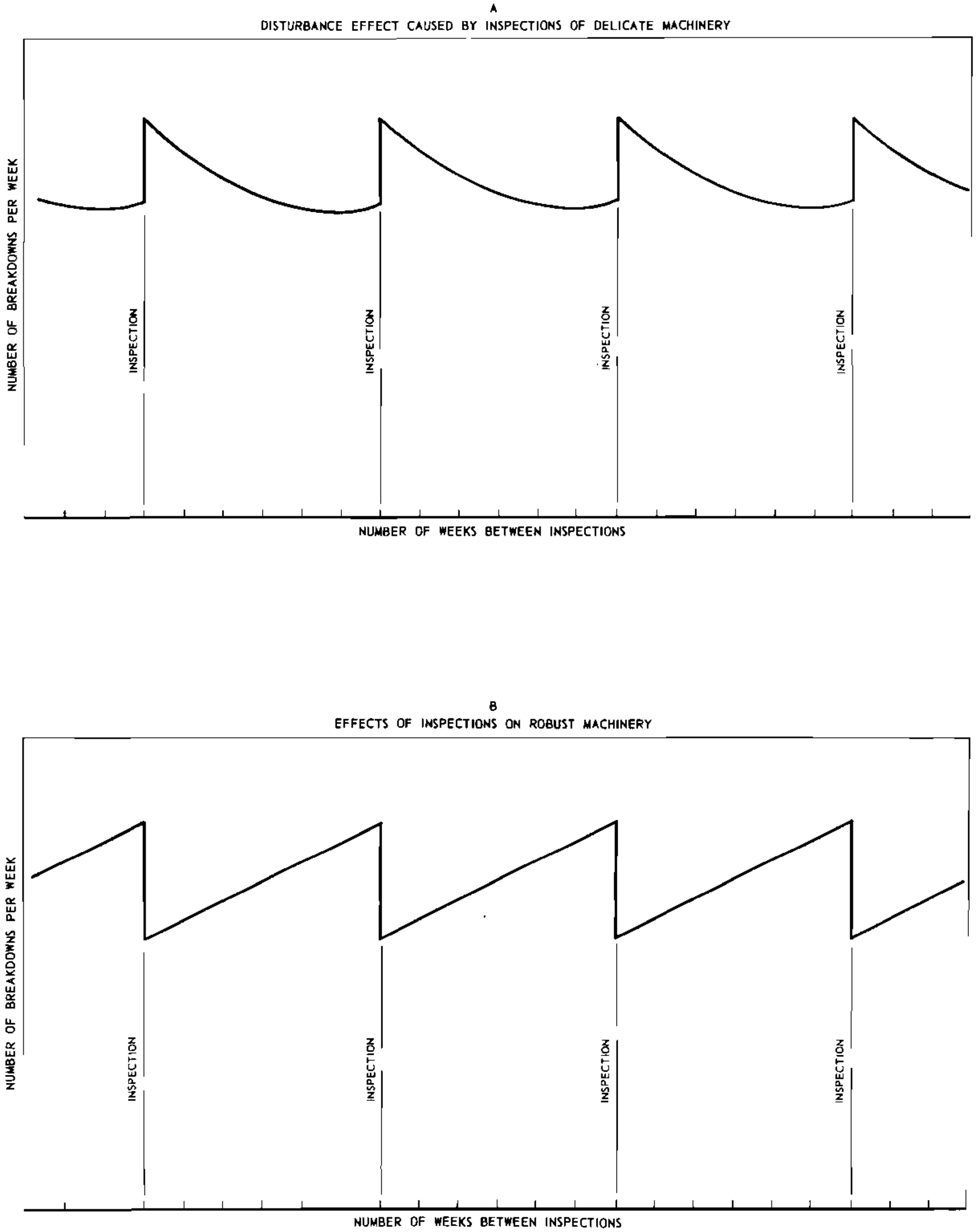


Figure 3. A Comparison of the Effect of Inspections for Delicate and Robust (Mechanical) Equipment.

also be used in determining optimum service facilities and optimum spare equipment needs in Chapter V.

The Atlanta Transit Systems has had an active maintenance program on all of their vehicular equipment for many years. The drivers of the vehicles report daily any malfunctions they notice. Minor repairs and adjustments are made daily at the service garages and major repairs are performed at the maintenance shop. Troubles encountered while the vehicle is in operation are serviced by two radio-equipped repair trucks each manned by two experienced mechanics.

A malfunction in the field is classified into one of two categories, roadcall or pull-in. Roadcalls consist of minor troubles that can be repaired on the spot and require not more than ten minutes delay of the vehicle. Pull-ins are the more serious troubles, which require more than ten minutes delay or necessitate the replacement of the vehicle in service.

A record is also kept of the type of unit which is defective. This information is recorded by type of vehicle and includes both roadcalls and pull-ins. The following table gives the number of service calls per vehicle that were experienced for a one-year period. The total number of roadcalls and pull-ins during this period was 12,597. At this rate, each repair truck would average slightly over seventeen service calls per day.

Table 4. Breakdown of All Field Calls by Type of Vehicle
March 1956 - February 1957

Unit Defective	Number of Field Calls per Vehicle			
	Trolleys		Buses	
	Type P	Type B	Gas	Diesel
Engine	--	--	5.19	1.63
Motor	.22	.23	--	--
Transmission	--	--	3.80	2.60
Controls	5.11	3.84	--	--
Overhead Trolley System	5.53	4.02	--	--
Body	2.98	3.29	2.88	2.67
Chassis	.47	.29	.87	.37
Brakes	2.80	2.29	3.84	3.17
Doors	1.65	2.22	1.99	1.99
Air System	.25	.24	.66	.94
Electrical System	1.71	2.12	1.89	1.37
Light System	.54	.47	.33	.66
Cooling & Heating System	.22	.22	2.60	1.59
Fuel System	--	--	1.11	.46
Tires	.32	.42	.65	.80
Others	2.36	2.18	2.90	2.89
Totals	24.16	21.83	28.71	21.14
Number of Vehicles in Group	114	245	83	30
Average Age of Vehicles in years	9.3	10.3	8.9	2.9

This table shows the items which cause the majority of breakdowns. Notice, also, the differences in frequency for the different designs; for instance, the controls for Type B trolleys are of a different make than those of Type P, and the Diesel engine is unlike the gas one. The design of the equipment is recognized by maintenance men as one of the most important factors which determine the amount of maintenance required. Age of the equipment and operating conditions also greatly affect the maintenance level, as will be shown in a later chapter of this study.

The breakdown distributions for roadcalls and pull-ins, shown on the following page, were obtained by summarizing the data on the daily Garage Foreman's Report of Trolley and Bus Trouble for a one year period, March 1, 1956 through February 28, 1957. Since only the data representing equal exposure rates were desired for plotting the distributions, the daily mileage figures for each group of vehicles were obtained from the Monthly Mileage Report and matched with the daily roadcall and pull-in data. The mileage driven on week-end days and some holidays was much lower than the week-day average because of light schedules on these days. Week-ends and holidays also generally experienced fewer breakdowns, as would be expected with a lower exposure rate. Since an equal daily exposure rate was desired, only the data within a selected mileage range was used to determine the distributions. This eliminated most of the data for week-ends and holidays.

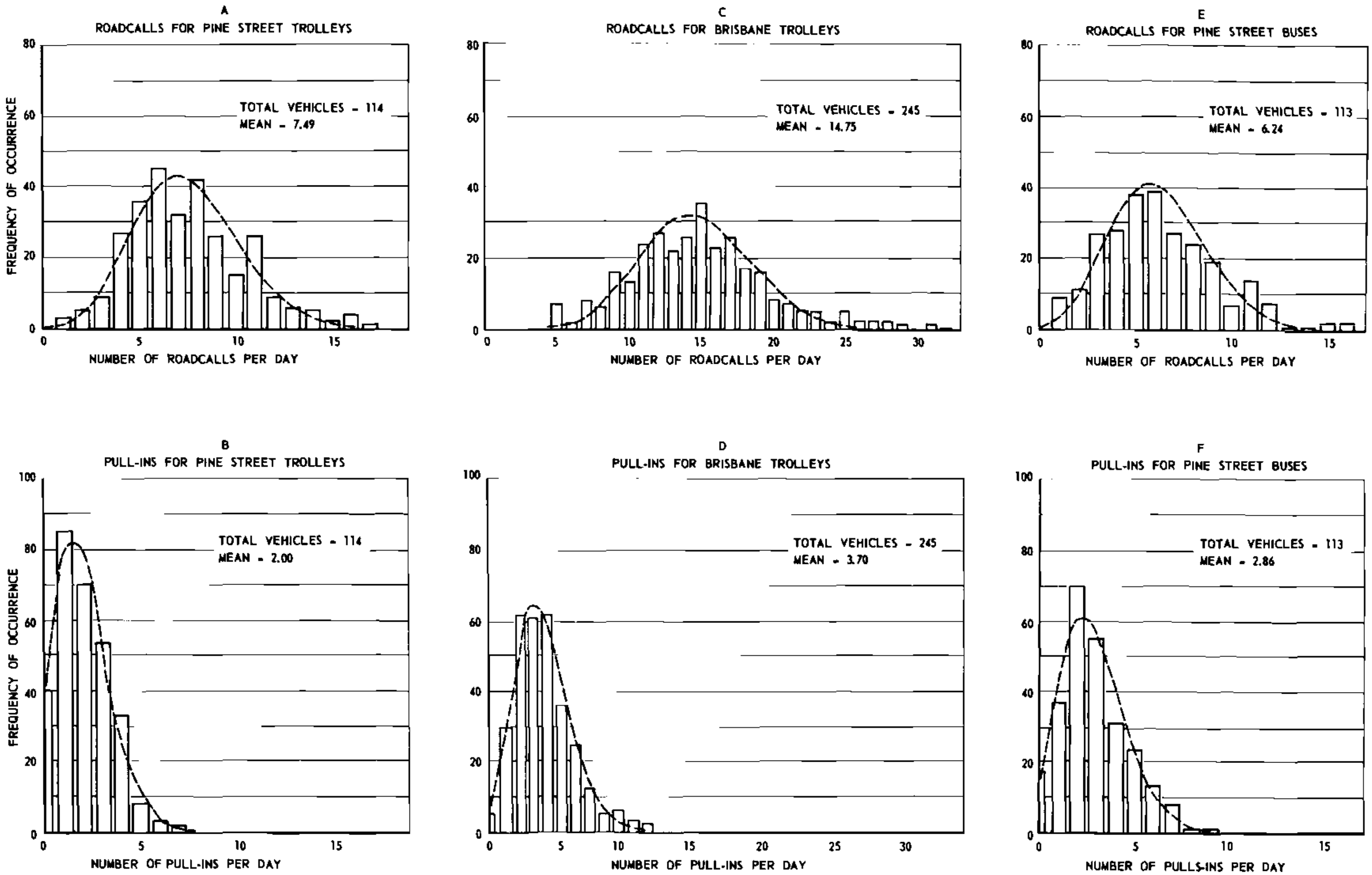


Figure 4. Distributions of Roadcalls and Pull-Ins for Two Groups of Trolleys and One Group of Buses Compared to the Poisson Curves Calculated from the Group Means. All Groups of Data Cover the Period March 1956 - February 1957.

The data were separated by type of service call, roadcall or pull-in, for each garage. Each garage services several types of vehicles. The data were put in the form of histograms for comparative purposes.

The expected distribution for these service calls, since we are dealing with the occurrence of isolated events in a continuum of time, would be the Poisson distribution. The values for the Poisson distribution were calculated for each group of data using the data mean. These calculated curves are shown on the histograms in dotted lines.

The distributions for the pull-ins compared very favorably with the Poisson distribution in each of the three cases. The roadcall distributions were peculiar in that they compared favorably for the middle portion of the distribution but differed considerably from the Poisson distribution at the extremes. A Chi-square test was made for each distribution and the results are given in Table 5 below:

Table 5. Results of Chi-square Tests for Roadcall and Pull-in Distributions and the Poisson Distribution

Garage	Distribution	Degrees of Freedom	Chi-square Value	Level of Significance
Pine Street Buses	Roadcall	9	24.87	.01
	Pull-in	6	6.86	NS*
Pine Street Trolleys	Roadcall	11	22.69	.02
	Pull-in	5	3.75	NS*
Brisbane Trolleys	Roadcall	15	40.14	.01
	Pull-in	7	8.29	NS*

*Not Significant even at .30 level.

These results show that the pull-in distributions are probably Poisson and the roadcall ones are not. The reason or reasons why the roadcall distributions do not compare favorably with the Poisson distribution is uncertain. The data is not sufficiently detailed to determine this. However, the maintenance men who work on the vehicle know from practical experience that the weather affects the number of service calls. Some operators, especially new ones, report troubles more frequently than others and some routes seem to give rise to more difficulties on the vehicles than others. It is probable that a combination of factors, weather conditions, human factors, and operating conditions, influences the minor service calls to a greater extent than the serious ones.

The distribution for the pull-ins in each grouping of vehicles compared very favorably with the Poisson distribution. This indicates that the number of pull-ins per day are about what we would expect due to chance and that the preventive maintenance program is effective at its present level. This does not mean, of course, that more preventive efforts would not reduce the average number of pull-ins per day; it may or may not depending on many factors. It does mean that with the present pull-in rate and the present preventive maintenance level, the distribution of the pull-ins is about what you would expect due to chance. It means, further, that it would be a waste of time to look for assignable causes on those days when the number of pull-ins is unusually high. These high-frequency days will occur occasionally due to chance variations, and this analysis shows that they do not occur more often than would be expected due to random chance causes.

Other advantages of knowing the nature of the distribution of breakdown data are:

(1) The maintenance supervisor may desire to set up statistical control charts to determine maintenance trends or when investigations are advisable to remedy out-of-control situations. To obtain the control limits for these charts it is necessary to know the type of distribution of the item being charted.

(2) In planning for optimum maintenance costs, mathematical techniques, such as Queuing and Monte Carlo, are often used. These techniques are based on probability. For the mathematical model to accurately represent the conditions under study the nature of the distribution must be known.

(3) A knowledge of the mathematical nature of a physical system gives the engineer a better understanding of the inter-relationships between its various components. In addition to giving him tools to calculate optimum conditions, it often helps him to interpret cause-and-effect relationships in day-to-day operations.

CHAPTER V

MOST ECONOMICAL NUMBER OF REPAIR FACILITIES AND UNITS OF SPARE EQUIPMENT

The number of repair facilities and units of spare equipment are factors which affect the overall maintenance cost. These two factors are interrelated and inseparable from an economic viewpoint. If the repair facilities are inadequate, equipment will have to wait in line for servicing, and a large number of spare units of equipment will be needed to maintain a desired level of operations. If there is no waiting for service, fewer spare units can maintain the same level of operations because the equipment being repaired will be back in service sooner, thus giving more effective utilization of the available equipment. In the later case, however, the repair facilities may not be used to their capacity.

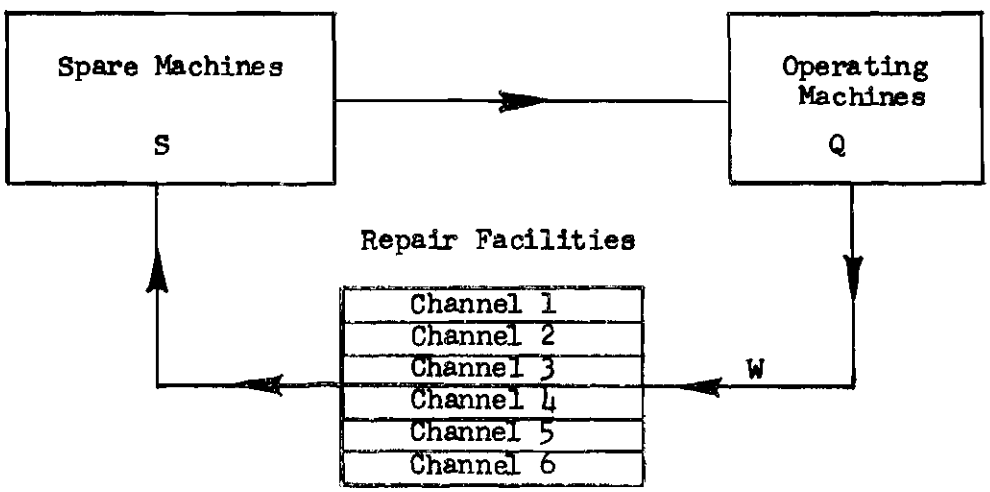
The problem is to balance these two factors so that the most economical operating point is obtained. Idle units of equipment and idle repair facilities are each unproductive and costly from an operating and maintenance standpoint respectively. Generally the cost of an hour of idle time for the repair facilities and for the equipment is different and the minimum cost solution is not obvious.

The usual method of resolving these factors is by trial and error or by using an industry average as a guide. For example, the experience factor for spare equipment for the transit industry is ten per cent of the operating vehicles. This would be used as a guide in the beginning and then varied as experience indicated. This process can be costly. Equally important is the fact that this rule-of-thumb type of decision often obscures the relationship between these factors, and a knowledge of this relationship is necessary for good control.

The purpose of this portion of the thesis is to show a method of determining the most economical balance between spare equipment costs and costs of repair facilities based on the operating conditions of a hypothetical company. Formulas for a general solution to the problem will be given and then applied to one phase of company's operation.

A simplified diagram of the basic maintenance repair cycle is shown below:

Diagram of Maintenance Repair Cycle



The operating machines in Q become unserviceable at a constant average rate, λ, which is distributed according to the Poisson distribution (as was indicated in the previous chapter). It is assumed that the unserviceable machines are repaired at a constant average rate, μ, which is distributed according to the normal distribution. The service time in each individual case would depend on the type of repairs needed.

Let: Q = number of machines in productive operation

W = number of machines waiting for repairs

C = number of repair channels

S = number of spare machines ready for replacement

R = number of machines that are in the process of being repaired at any one time

T = total number of machines in system

Then: total machines = T = Q + W + R + S, and

total number of spare machines = W + R + S

The probability of n machines being repaired or waiting for repairs (R + W) is designated by P_n and Churchman, et al (32) show this to be

$$P_n = \frac{P_o \left(\frac{\lambda}{\mu}\right)^n}{n!} \quad \text{for } n \leq C \qquad \text{(Eq. 1)}$$

$$P_n = \frac{P_o \left(\frac{\lambda}{\mu}\right)^n}{C! \quad C^{n-c}} \quad \text{for } n \geq C \qquad \text{(Eq. 2)}$$

P_o may be determined from the condition $\sum_{n=0}^{\infty} P_n = 1$

The expected number of machines at the various points in the system at any one time can be computed from, Expected number of machines being repaired or waiting for repairs $= E(R + W) = \sum_{n=o}^{T} nP_n$ (Eq. 3)

Expected number of machines being repaired $= E(R) = \sum_{n=o}^{T} nP_n - E(W)$ (Eq. 4)

Expected number of machines waiting for repairs $= E(W) = \sum_{n=C+1}^{T} (n-R)P_n$ (Eq. 5)

Let Q' = number of machines needed to perform the desired service, and

A = number of machines available for service.

Then, $A = T - W - R$, and the number of productive

$$\text{machines} = Q = \begin{cases} A \text{ if } Q \quad Q' \\ Q' \text{ otherwise} \end{cases}$$

The probability that any certain number, n, of machines would be available for service is

$$P\left\{T - W - R = n\right\} = P\left\{W + R = T-n\right\} = P_{T-n}$$

$$P\left\{Q = Q'\right\} = P\left\{A \geq Q'\right\} = \sum_{n=Q'}^{T} P_{T-n}$$

Expected number of machines in productive operation $= E(Q) = \sum_{n-o}^{Q'-1} nP_{T-n} + Q'\sum_{n=Q'}^{T} P_{T-n}$

To facilitate calculation of E(Q) from tables of P_n, let

$$j = T - n \begin{cases} T-Q' + 1 \text{ when } n = Q'-1 \\ T \text{ when } n = 0 \end{cases} \quad \text{and } n = T - j$$

$$E(Q) = \sum_{j=T-Q+1}^{T} (T-j)p_j + Q' \sum_{j=0}^{T-Q'} p_j$$

$$E(Q) = T\sum_{j=T-Q'+1}^{T} p_j + Q'\sum_{j=0}^{T-Q'} p_j - \sum_{j=T-Q'+1}^{T} jp_j \qquad \text{(Eq. 6)}$$

Expected number of spare machines ready for replacement $= E(S) = T - E(Q) - E(R-W)$

$$E(S) = T - \left[T\sum_{j=T-Q'+1}^{T} p_j + Q'\sum_{j=0}^{T-Q} p_j - \sum_{j=T-Q'+1}^{T} jp_j\right] - \sum_{n=0}^{T} nP_n \qquad \text{(Eq.7)}$$

The time factor is also important for the machines waiting for service and being serviced. The average time a vehicle is in the system will be denoted by $\bar{t}_w + \frac{1}{\mu}$, where $\bar{t}_w$ is the average time spent waiting in line and $\frac{1}{\mu}$ is the average servicing time. The formula for $\bar{t}_w$ for multichannel servicing facilities is given by Churchman, <u>et</u> <u>al</u> (33) to be

$$\bar{t}_w = \frac{P_o}{C(C!)\left[1 - \frac{\lambda}{\mu C}\right]^2} \left(\frac{\lambda}{\mu}\right)^C \qquad \text{(Eq. 8)}$$

The application will be made to the situation where

λ = 2.0 vehicles per day

μ = 1.5 vehicles per day per repair channel

$\frac{\lambda}{\mu} = \frac{4}{3}$

T = 106 machines

The expected number of machines at each point in the system for two to six channels for the above application is given in Table 6. It is necessary that Q', the number of machines needed to perform the desired service, be known before E(Q) and E(S) can be calculated. For this example Q' is assumed to be 100. See page 97 in the appendix for sample calculations.

Table 6. Expected Number of Machines at Various Points in Repair Cycle Where $\lambda/\mu = 4/3$ and Q' = 100

Symbol	Description	Number of Repair Channels 2	3	4	5	6
R	Number of machines being repaired	1.33	1.33	1.33	1.33	1.33
W	Number of machines waiting for repairs	1.05	.14	.03	.01	.001
Q	Number of productive operating machines	99.79	99.99	100.00	100.00	100.00
S	Number of spare machines ready for service	3.83	4.54	4.64	4.66	4.67
T	Total number of machines in system	106	106	106	106	106

It can be seen from the table that in this example where the total number of machines is fixed at 106, the required number of machines (100) would not be available, on the average, if less than four repair channels were used. A more realistic situation would be to determine operating requirements and then determine the number of spare machines which would be necessary in each case to maintain this level of operations. For instance, if it is assumed that 100 machines are required to perform a service and management desires that this level of service be maintained 98% of the time, then a certain number of spare machines would be needed. If this same service were required 99% of the time more spare units would be needed. Table 7 gives the number of spare units which would be required to maintain 100 machine service for various levels of operations.

Table 7. Number of Spare Units Which Would Be Required at Various Operating Levels to Maintain 100 Operating Machines, $\lambda/\mu = 4/3$

Per Cent of Time the Service of 100 Machines is Required	Number of Spare Units Required				
	Number of Repair Channels				
	2	3	4	5	6
90	5	3	3	3	3
93	6	4	3	3	3
95	7	4	3	3	3
98	9	5	4	4	4
99	11	6	5	5	5
99.99	18	11	9	8	7

Table 7 was obtained from the cumulative P_n column of the probability tables (see Appendix, page 92) for each channel. This column gives the cumulative probability for the occurrence of n machines waiting for service or being serviced (R + W). The first case in the table, for example, means that the probability is .8957 (rounded to .90), that for two channels five or less machines will be waiting for service or in process of being repaired. This result is limited, of course, to the arrival rate and the service rate for which the probability table was calculated.

The amount of time that the vehicles remain in the different stages of repair cycle will influence the costs. These times are given in Table 8 and were computed using Equation 8.

Table 8. Breakdown of Time Required for Repair Cycle

Symbol	Description	Number of Repair Channels				
		2	3	4	5	6
$\bar{t}_w$	Average number of days each vehicle spends waiting for repairs	.5340	.0711	.0130	.0023	.0004
$\frac{1}{\mu}$	Average number of days required to service each vehicle	.6667	.6667	.6667	.6667	.6667
$\bar{t}_w + \frac{1}{\mu}$	Total average number of days in system per breakdown	1.2007	.7378	.6797	.6690	.6671

The total cost of the maintenance operation, which we wish to minimize, consists of three major components; (1) labor costs, (2) vehicle downtime costs, and (3) costs associated with furnishing the space, tools, etc. for the repair facilities. The minimum cost point for any particular level of service can be obtained by determining the cost of each of these components and totaling for all of the alternatives which are being considered.

Figure 5 shows the comparative annual cost curves for equipment and facilities for two, three, four, five, and six channels at various operating levels. These curves are for the sum of items two and three above. The curves in Figure 6 are total comparative annual cost curves which include idle labor costs. The data that were calculated to plot these curves were based on the following cost assumptions:

Cost Constant	Description of Constant	Assumption
K_1 = \$10,220	Annual cost of labor per channel	It is assumed that two mechanics service each repair channel, that they work while a vehicle is in the channel they service, and that they are idle when there is no vehicle in their channel.*
K_2 = \$3650	Annual cost of depreciation, taxes, and other overhead per vehicle	Each vehicle is assumed to have an overhead cost of \$10 per day.
K_3 = \$800	Annual cost of depreciation, taxes, and other overhead costs per channel	Each channel is assumed to have an annual overhead cost of \$800.

The total cost of the maintenance operation is the sum of items one through five as given below.

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*This is not a realistic assumption in most cases because the idle mechanics would probably be assigned other work. This aspect will be discussed more fully later in this section.

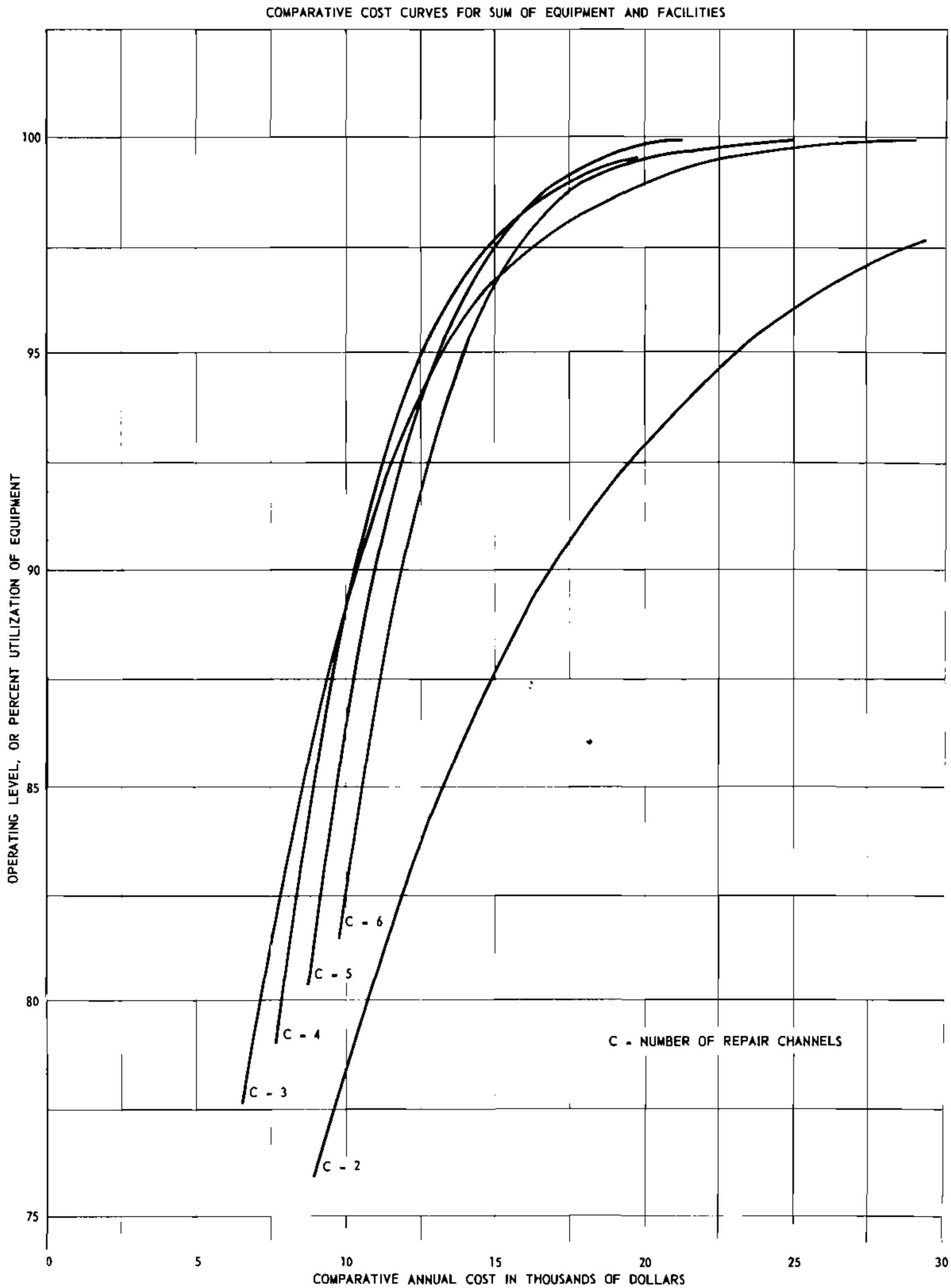

Figure 5. Comparative Annual Cost Curves for Sum of Equipment and Facilities for Two, Three, Four, Five, and Six Channel Repair Facilities at Various Operating Levels.

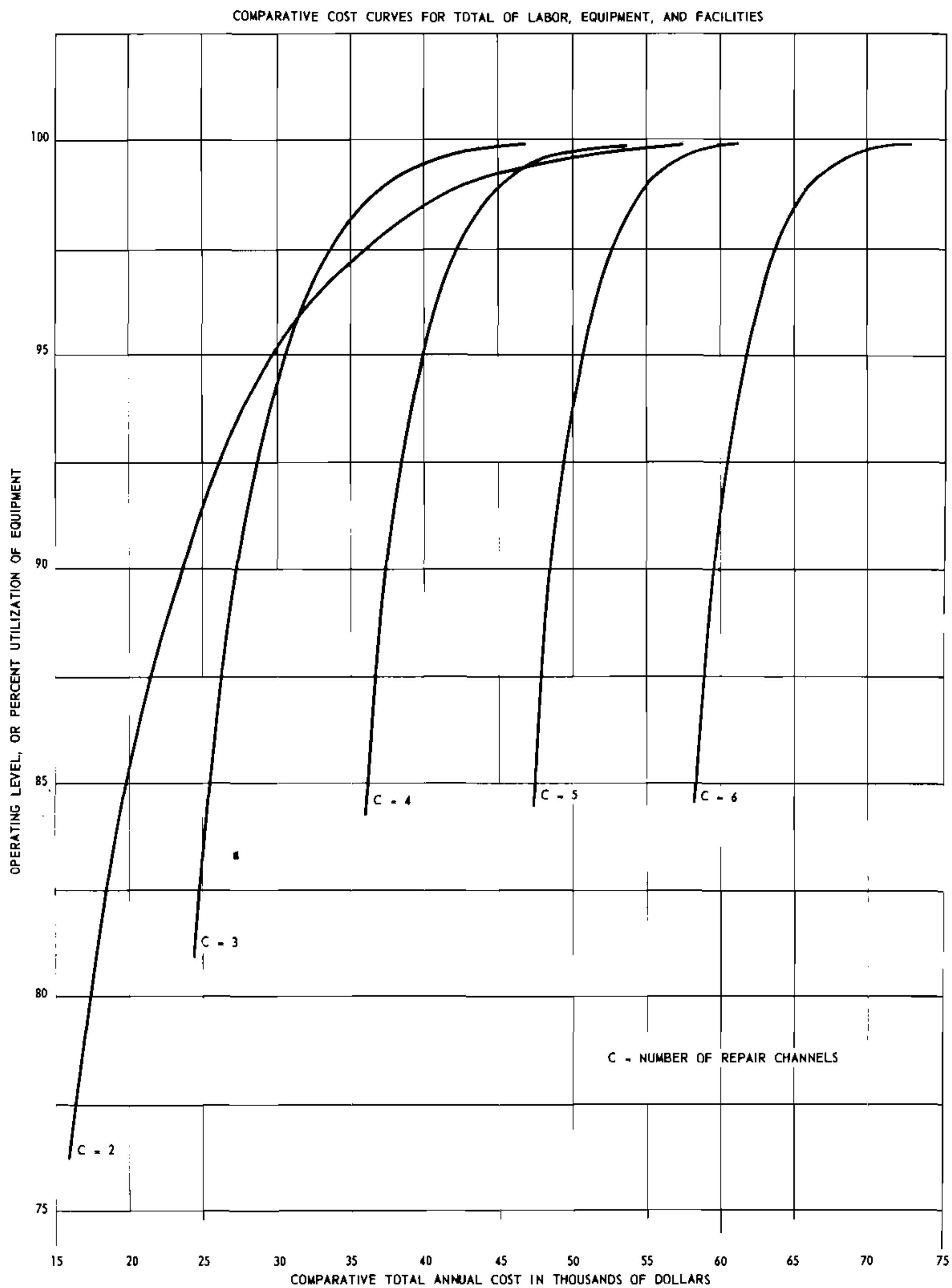

Figure 6. Comparative Total Annual Cost Curves for Two, Three, Four, Five, and Six Channel Repair Facilities at Various Operating Levels.

Item	Description and Formula
1	D_L = Total annual labor cost = cost of productive maintenance labor + cost of idle labor $D_L = K_1 C = K_1 \frac{\lambda}{\mu} + K_1 \sum_{n=0}^{C} (C-n)P_n$
Note:	Since the cost of productive labor is constant for all alternatives, only the cost of idle time will be included in the total comparative cost formula.
2	D_W = Annual cost of idle time of vehicles that are waiting for repairs $D_W = K_2 W \bar{t}_w$
3	D_R = Annual cost of idle time of vehicles that are in process of being repaired $D_R = K_2 \frac{R}{\mu}$
4	D_S = Annual cost of idle time of spare vehicles that are ready for service $D_S = K_2 S$
5	D_F = Annual overhead cost for repair facilities $D_F = K_3 C$

Comparative Total Annual Cost = $D_L + D_W + D_R + D_S + D_F$

$$\text{C.T.A.C.} = K_1 \sum_{n=0}^{C} (C-n)P_n + K_2\left(W\bar{t}_w + \frac{R}{\mu} + S\right) + K_3 C$$

Inspection of the curves for the comparative total annual costs and for the equipment costs reveals some interesting points. The only difference in data between the two sets of curves is that the total cost curves include costs of idle labor, whereas the equipment cost curves do not. The wider range of the total cost curves shows the importance of labor in this type of study. The horizontal distance between the curves for a certain operating level indicates the relative differences in dollars between the alternatives. Since the operating level of most companies will vary from time to time, this type of analysis would appear to be very helpful in determining optimum equipment requirements and labor scheduling policies.

Once this type of mathematical model for a particular repair system is devised, of course, it must be verified as accurately representing the physical system. After this is ascertained valid calculations can be made for various operating conditions to determine:

(1) The most economical number of repair channels

(2) The most economical number of units of spare equipment

(3) The most economical crew size and labor scheduling policies

(4) The average number of machines at the various points in the system

(5) The anticipated cost for alternative methods of operations

Since the operating level of most companies will vary from time to time due to economic cycles, this method of analysis would provide a quantitative basis for decisions involving maintenance costs and policies. Technological changes and changes in operating conditions would naturally necessitate periodical restudying to preserve the validity of the model.

As was previously noted, the assumption regarding idle labor was not realistic for a transit company. Normally, when the mechanics were not working on breakdowns they would be performing routine preventive maintenance work or other jobs. This would bring in problems of scheduling the preventive work and balancing the preventive and breakdown portions of the overall maintenance job. These factors could be analyzed be using Monte Carlo techniques instead of Queuing. The method of calculations for the various costs would be similar to the one described.

CHAPTER VI

EFFECT OF AGE OF EQUIPMENT ON NUMBER OF BREAKDOWNS

Someone has said that machines are always marching down the road to the junk heap, and that some machines reach the end of the road every year. The problems of deterioration and depreciation of equipment with the passage of time are familiar to both the accountant and the business-man. They set aside sums of money each year for the purpose of replacing worn out machinery. The production man copes with the problem every day as he assigns the less exacting tasks to the older machines, and has to reschedule work when they break down. The maintenance man, particularly, is aware of the increasing trouble and expense of aging machinery as he tries to keep it running and producing satisfactorily.

The need for information on equipment failures and their costs, both in repair and lost production, is recognized by most companies. Replacement analysis, production planning, preventive maintenance programs, and the preparation of maintenance budgets are some of the functions which utilize equipment records.

The records kept on productive equipment are generally of two types. One type is for recording the history of the machine. Items are recorded such as name of machine, number, date of purchase, make and model, location in plant, cost installed, alterations, and other pertinent machine data. This record is used for certain production planning functions and accounting purposes. The other type of equipment record is used for entering all the details of the preventive maintenance and repair work that are performed on the machine. Data such as date of repairs, description of repairs, inventory numbers and cost of spare parts, hours of labor required, and repair order number are commonly found on the second type of equipment record. This information is used to provide detailed data of the machine repairs for cost control and preventive maintenance uses (34).

The point of this discussion of equipment records is to emphasize that often the age, or usage, which is responsible for the wear of the productive equipment is not recorded. The chronological age of the equipment is usually a minor factor in breakdowns except in cases of deterioration of standby equipment. The usage of productive machinery in most manufacturing plants will vary with production schedules, machine assignments, and other plant conditions. A certain machine may be operated ten hours one week and forty-eight the next. In plants operating three shifts, the machines would receive much more wear per month than in a plant operating one shift. Records of machine usage are usually kept only on certain types of machinery; such as the mileage of rolling equipment, flying hours of airplane engines, and pounds of product produced by continuous or semi-continuous machines.

The problems and costs involved in obtaining data on machine usage for many types of equipment are probably a major reason why there has been little done on the analysis of breakdowns versus age of equipment. The literature yields relatively few articles on the subject.

Probably the most complete analysis of equipment replacement costs in recent years is the M.A.P.I. (Machinery & Allied Products Institute) study by George Terborgh (35). Mr. Terborgh shows the relation between age or accumulated usage and repair costs for eight types of machinery including a group of city buses. With regard to the plotting of chronological age and repair costs he says:

> ...The curves are all of the type $y = k - a.b^x$ (modified exponential trend), which besides giving a good fit over the whole range of observations in each of the eight cases shown has certain properties which commend it on theoretical grounds.[2] As read from this curve type, repair costs per unit of use increase continuously with rising service age but the annual increment declines by a constant percentage ratio. This means that the curve if extrapolated to the right (for higher and higher service ages) asymptotically approaches a constant figure. For average repair costs, this is a plausible assumption.
>
> [2]In this equation x is the year of service. The other symbols are constants, k being the asymptote which y, the cost, approaches as a limit with the increase of x (36).

He also points out that the relation of repairs to accumulated use is a better measure than their relation to age. One of the prime reasons for this contention is the decreasing utility and consequently the decreasing intensity of use of the machinery as it ages. As Terborgh indicates, ".... when we relate repair costs per unit of service to <u>equal</u> increments of attained age we are relating it to <u>diminishing</u> increments of accumulated use" (italics are Terborgh's)(37).

When plotting the repair cost per mile (only normal repairs included - not servicing, tire and tube expense, or repairs necessitated by accidents) that have been adjusted for changes in maintenance wage levels but not in materials prices, against accumulated usage, Terborgh obtained approximately a straight line for city buses. A constant average increase of approximately $0.0267 per vehicle mile for each additional thousand miles of usage was recorded for ten groups of buses, 763 units in all (29). This type of information is very useful; however it soon becomes obsolete because of changing material prices and wage levels.

A Rand Corporation study by Davis is more concerned with the statistical nature of breakdown data (38). He investigated the probability distribution of time-to-failure data of several types of equipment. This study included data on trolley motors for one group of 191 vehicles operated by a large city transit company. His definition of failure was when "... some part broke and the motor would not run; or, ... when the maximum power produced, as measured by a dynamometer fell below a fixed percentage of the normal rated value." (39)

His rationale includes two failure theories which are applicable to the systems he considers. These are: (1) the normal theory of failure - so named because the familiar probability density function is assumed to describe the failure distribution. Systems following this theory have a conditional density function which has zero value in the early phase but increases at an accelerated rate throughout its life. (2) The exponential theory of failure. Since preventive maintenance techniques are usually instituted at a time when the systems tend to become prone to failure, Mr. Davis proposes an exponential theory of failure under which it is assumed that the conditional density function is constant throughout the life of the system. He points out that "... systems which are governed by the exponential theory of failure, produce a Poisson distribution of number of failures during equal intervals of time if systems which have failed are instantly replaced." (40)

Davis' study is of particular interest because of the similiarity of findings in his study and this one. For instance, in a previous portion of this thesis it was determined that the distributions of pull-ins for both trolleys and buses were probably Poisson. The data for these distributions were of vehicles that had been in operation for a number of years and, therefore, the second theory would be applicable. His explanations as to why the exponential theory applies is also interesting and pertinent.

He visualizes failures as the result of three variables (1) the environment under which the system is operated, (2) the failure resistance of the system's parts, and (3) the complexity of the system which might be thought of as the number of ways in which the system can fail.

His reasons for selecting these two particular theories of failure are best presented in his own words.

> Preventive maintenance procedures, by which parts are replaced after a fixed lifetime or at a given state of wear are justified by a normal theory of failure. Literature on mechanical wear indicates that the time rate of wear increases with the cumulative amount of wear. If the conditional density function of failure of a system is assumed to increase with increased cumulative wear of some of the system components, then the replacement of worn components at a specified degree of wear will decrease the conditional density failure function for such systems (41).

Mr. Davis' study included types of data in which human error or failure predominates, such as student typing errors, bank statement and ledger errors, etc., as well as mechanical failures of linotype machines, vacuum failures in radar sets, bus motor failures, etc. His general conclusions are: (1) The exponential theory of failure appears to describe systems where failure is caused predominately by human errors or where a careful and well developed operating technique to minimize failure is in use. Also the systems which are subject to a wide range of environmental severity appear to follow this pattern. (2) The normal theory of failure appears to apply to systems which exhibit small variations in failure resistance among the individuals within the group and which are subject to small variations in environmental severity. Further, the failure resistance of the mechanism deteriorates with time and operational procedure requires that each item be used until failure.

He states that the oversimplified exponential hypothesis of failure does not precisely describe the observed phenomena; however, the discrepancies between the theory and the data are small enough so that the exponential theory may be regarded as a useful approximation of certain classes of actual failure distributions (42).

Considering only the bus motor failures, he found that the normal curve approximated the distribution of the first bus motor failure, while the distributions of the third, fourth, and fifth appear to be approximated by the exponential distribution. The distribution of the second failures is not described by either theoretical curve but looks like a combination of both.

He reasons that this peculiarity is plausible.

> ... A normal distribution of mileage to first failure can be expected, for the moving parts of the motor slowly abrade each other until, when a sufficient amount of metal is worn away, they either break or no longer perform their function with satisfactory efficiency. This expectation is confirmed by first failure being caused singly or in combination by worn cylinders, pistons, piston rings, valves, camshafts, connecting rod or crankshaft bearings, etc. There were, however, a considerable number of failures in which moving parts fractured at low mileages, indicating manufacturing errors or inadequate or improper maintenance or repair. These later conditions were exponential type occurrences which should follow a different distribution. Unfortunately these data on the cause of failure were so fragmentary and incomplete that segregation of the two types of failures could not be accomplished. ......
>
> An exponential distribution of the third and subsequent failures appears logical in that by the time the motor has been overhauled twice it consists of components in a scattered state of wear. The individual parts may each exhibit a normal distribution of failure with operating time, but with components in random stages of wear a motor has an equally likely chance of failing during any period of operation. This does not mean that the chance of failure during equal operating periods is the same after each overhaul. The mean mileage to failure after the first overhaul (71,000) is less than that of first failure (97,000), and after each subsequent overhaul, a shorter mean mileage to failure is exhibited (54,000 miles after second overhaul, 42,000 after third, and 36,000 after fourth).
>
> The second failures are not predominantly due to either exponential or normal theory causes and can therefore be expected to exhibit a distribution characteristic of neither cause (43).

This method of attack provides the maintenance man with information from which he can calculate the average probability of motor failure (after he has determined the mean time or mileage to failure) for a machine or vehicle during any portion of its life. Then, knowing his cost factors, he can determine the most economical time for motor overhaul or replacement.

A less detailed method of analysis of breakdowns is the plotting of the cumulative breakdowns against the cumulative age or mileage. From an extrapolation of the resulting curve the average number of breakdowns over any period of time or usage can be obtained. The shape, or equation, of the curve is also useful for cost analysis work and for preventive maintenance planning. This method will be used here.

Annual mileage data and number of pull-ins (for definition of pull-in see page 30) were obtained for five groups of vehicles, three groups of buses and two groups of trolleys. The bus data covers a ten-year period and the trolley data a nine-year period. The average annual mileage per vehicle and the average annual number of pull-ins per vehicle were calculated by dividing the annual data by the number of vehicles in the group. All vehicles in each group remained in operation for the entire period except for the usual intervals of maintenance and repairs.

When the cumulative pull-ins per vehicle were plotted against the cumulative mileage per vehicle, a series of points was obtained for each group of vehicles. Each of these groups of points appeared to follow a similiar pattern but each had a different slope. A smooth curve was drawn by eye through each series of points.

Data from these smooth curves was analyzed to determine the nature of the curve. From knowledge of the data, the curve was expected to pass through point (0,0) and the analysis indicated that it was probably parabolic in form.

In attempting to find the formula that would give the best fitting curve, the power function $Y = aX^b$ was investigated first. The method of least squares was used with the logarithms of X and Y to determine the variables a and b. The curve obtained with this function approximated the data fairly well; however, the variations between the computed curve and the transit data were of the same nature for each curve. This led to the investigation of another form of parabola.

The failure theory of Davis' in which he visualizes failures as the results of three variables (operating conditions, failure resistance of system's parts, and complexity of system) together with more mathematical analysis suggested the possibilities of a cubic parabola. This form of the parabola, $Y = aX^3 + bX^2 + cX$, possesses a three-constant flexibility in combination with the independent variable. The method of selected points was used to determine the constants a, b, and c for each of the five groupings of vehicle data. Using these constants, the curve for each group of vehicles was calculated and plotted. In each case the curve followed the original data very closely. Figure 7 shows the comparison of the fit of the original data with the cubic parabola. The constants for the vehicle groups are given in Table 9.

Table 9. Values of Parameters for Cubic Parabolas

$$Y = aX^3 + bX^2 + cX$$

Vehicle Group	$a \times 10^{-6}$	$b \times 10^{-6}$	c
Buses 34-S	.1167	416.7	.04216
38-S	-1.2900	1239.5	.02595
41-S	- .9580	1187.8	.03084
Trolleys WH	.350	57.5	.08325
GE	.378	-122.3	.08095

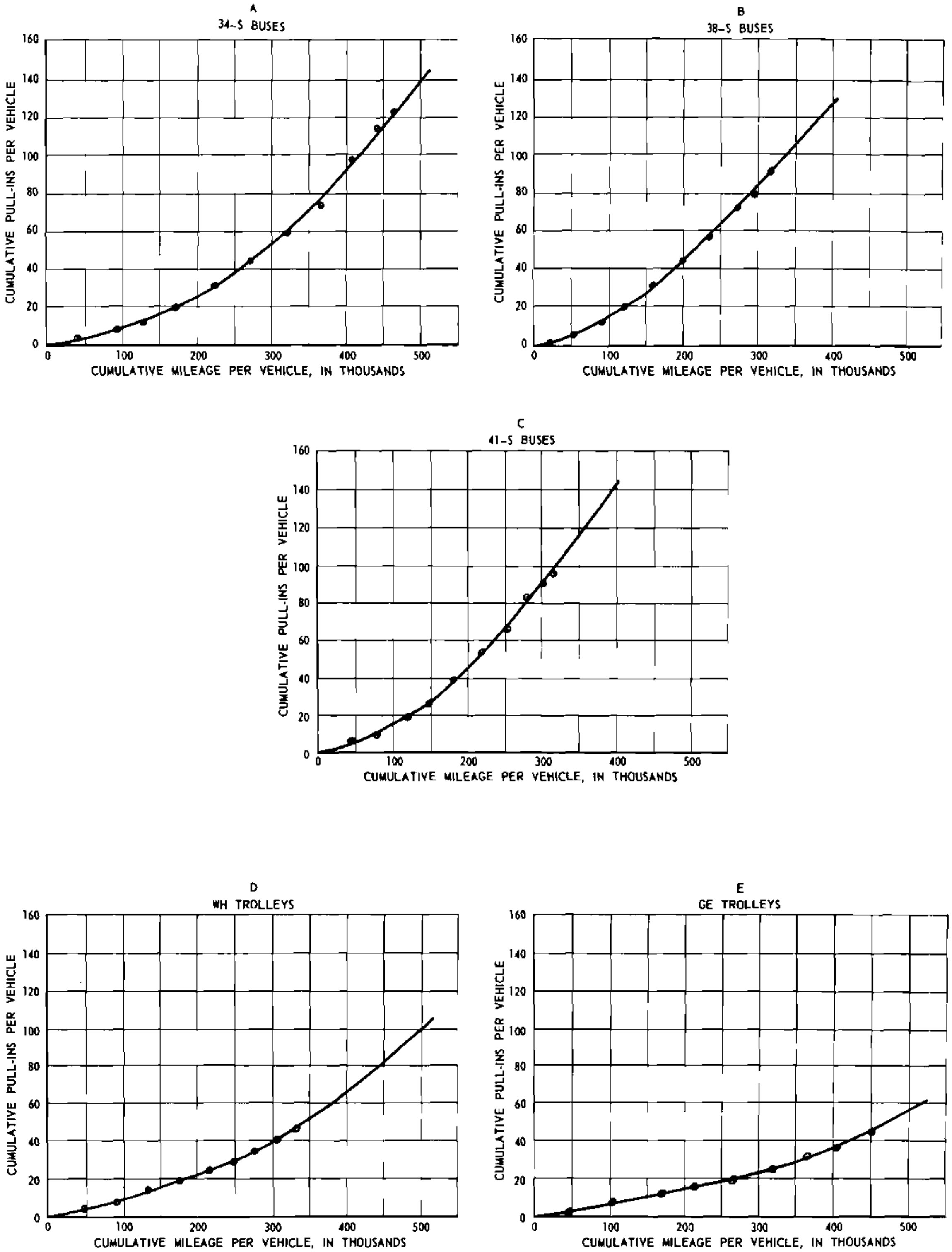

Figure 7. Comparison of Points of Data With Cubic Parabala for Three Groups of Buses and Two Groups of Trolleys. Note That the Curves for all Figures are of the Cubic Parabala Type,

$y = ax^3 + bx^2 + cx.$

The similiarity between some of the constants within the bus or trolley groupings would be expected because the shape of the curves are similiar. It is obvious that variable c is of the largest magnitude, however, it does not always have the greatest effect on the shape of the curve. For the buses, the term containing parameter b has the greatest influence on the rise in breakdowns throughout the entire period. For the trolleys, the term containing parameter c has the greatest effect on the increase in breakdowns, but the term with parameter a increases more rapidly as mileage increases and at the upper end of the curve it approaches the effect of the term with parameter c. The negative terms, of course, retard the increase of pull-ins. The detailed tables for the terms are shown on page 106 in the Appendix.

An attempt was made to relate the parameters of the cubic parabolas with maintenance theory and experience to see if they might be used as measures of the factors causing breakdowns. Although there are a multitude of independent factors influencing equipment breakdowns, they can be generally grouped under three broad classifications; (1) equipment design, (2) operating conditions, and (3) amount and quality of maintenance that the equipment receives. Table 10 summarizes the equipment under study for each of these classifications.

Comparison of the factors for the groups of buses reveals that the basic design for all the vehicles is identical except for body size, that the 34-S group has different operating conditions, and that all the vehicles are similiarily maintained. For the trolley groups, all factors are similiar except design.

Table 10. Summary of the Factors Affecting Pull-ins for Equipment Being Studied

Vehicle Group		Design	Operating Conditions	Type of Maintenance
Buses	34-S	34 passenger body	feeder routes*	routine preventive and break-down maintenance
	38-S	38 passenger body 6 cyl. model F gas engine Spicer 916 hyd. trans.	city schedules	routine preventive and break-down maintenance
	41-S	41 passenger body 6 cyl. model F gas engine Spicer 916 hyd.trans.	city schedules	routine preventive and break-down maintenance
Trolleys	WH	45 passenger body pneumatic acceleration controls series motor type A electric braking	city schedules	routine preventive and break-down maintenance
	GE	45 passenger body electric acceleration controls compound motor type B electric braking	city schedules	routine preventive and break-down maintenance

*Feeder routes are suburban routes having less strenuous operating conditions than city routes; less starting, less stopping, less traffic, etc.

Attempts to relate the parameters in Table 9 with the factors in Table 10 yielded inconclusive results. The comparison indicates that if a relationship exists between them it is either a complex interrelationship or the relationship will change with a change in type of equipment. For example, for the groups of buses it appears that the term containing parameter a may indicate a measure of the effectiveness of the maintenance program, the term with b may indicate the operating conditions, and the one containing c the design. However, for the trolleys, the term with a appears to be more a measure of the operating conditions, the term containing b seems to indicate the effect of maintenance, and the one with c the design. These are mere interpretations based on the author's judgment and experience, and any definite conclusions regarding the possible relationships between the parameters and the three broad classifications of factors are not possible from this study.

Regardless of what the parameters of the cubic parabolas measure, it appears that this type of curve best fits the cumulative pull-in data in each of the five cases tested. The reasons why this particular type of equation describes the data is not known. Further studies on different data might help to clarify the reasons and also might determine if this relationship can be applied generally to cumulative breakdown data.

Since the cumulative pull-ins per vehicle data appears to follow a predictable pattern, the maintenance supervisor can use this information to help him plan for future needs. After the vehicles have been in operation long enough to furnish data for the calculation of the parameters the expected number of pull-ins per period for each group of vehicles could be approximately predicted. These estimates of future pull-ins would aid in the following ways:

(1) Prediction of future maintenance workloads and expenses for pull-ins

(2) Determination of the most economical replacement age (because maintenance cost is an important factor in replacement analysis)

(3) Provide a quantitative basis for the comparison of different groups of vehicles or different designs.

Atlanta Transit estimates that each pull-in costs the company approximately one hundred dollars. Remember that each pull-in causes at least a ten-minute delay or requires the replacement of the vehicle by another to complete the schedule. A large portion of this estimated cost is made up of intangible costs such as public goodwill which is reduced by interrupted service, and the costs of interruptions to planned maintenance work which the replacements require. The out-of-pocket maintenance costs per pull-in are estimated at about twenty dollars. During 1955* Atlanta Transit had 2177 pull-ins for trolleys and 1298 for buses. The estimated out-of-pocket maintenance costs for these pull-ins are $43,540 and $25,960 respectively. Total estimated cost to the company, based on one hundred dollars per pull-in, would have been $347,500, or an average of about $713 per vehicle per year. More accurate predictions of the changes in these costs from year to year would increase the accuracy of forecasting maintenance needs and expenses.

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*A year picked at random.

CHAPTER VII

CONCLUSIONS

The purpose of this study was to investigate four problem areas of maintenance with the objective of applying analytical methods to show how these methods could be used to obtain a more quantitative basis for maintenance policies and controls. The general conclusions are that mathematical models and techniques, properly used and verified as accurately representing the physical conditions, are valuable tools which the maintenance engineer can use to help obtain optimum results from a maintenance system. The specific conclusions for each of the four areas, which are given at the ends of the respective chapters, are summarized.

(1) Methods of Measuring Maintenance Effectiveness.--The application of the production function technique for the evaluation of productive output factors appears to have considerable merit. Although this tool is still in the development stage and there are recognized limitations in the cost data that was used for this study, the results that were obtained agree to a limited extent with what one would expect by inductive reasoning. The regression coefficients for operating expenses for both trolleys and buses were larger than those for any of the other productive factors, indicating that operating expense is the most important factor contributing to revenue. This is logical because

operating expense is composed largely of vehicle drivers' salaries. Also, the regression coefficient for bus maintenance was larger than the one for trolley maintenance, indicating that bus maintenance is a more critical factor than trolley maintenance. This again appears logical, because the cost data for this study covers a period during which the average cost per mile for bus maintenance was greater than the average cost per mile for trolley maintenance. These results indicate that, with more research, the production function technique may prove to be a very useful tool for analyzing productive output factors.

(2) Distribution of Unpreventable Breakdowns.--A breakdown of one vehicle in a group of operating vehicles is an isolated event in a continuum of time, and therefore, the expected distribution for vehicular breakdowns would be the Poisson distribution. This study of two groupings of trolleys and one of buses indicates that in each case the distributions of minor troubles, labeled roadcalls, does not compare favorably with the Poisson distribution; however, in each case the distributions of the breakdowns, labeled pull-ins, does compare favorably with the Poisson distribution. Chi-square tests indicate a level of significance of 0.02, or lower, for the pull-in distributions, and the roadcall distributions are not significant at the 0.30 level. The data is not sufficiently detailed to determine the reasons for the differences between these two types of maintenance troubles.

(3) Most Economical Number of Repair Facilities and Units of Spare Equipment.--The mathematical model and cost curves developed in this

study indicate that this method of analysis is a valuable tool for the evaluation of maintenance systems. The study also indicates that, as would be expected, labor is an important factor in determining optimum conditions from an economic viewpoint. It appears that the mathematical model including the proper cost data can be used to determine:

1. The most economical number of repair channels
2. The most economical number of units of spare equipment
3. The most economical maintenance crew size
4. The average number of machines at the various points in the repair system
5. The anticipated cost for alternative methods of operations.

(4) Effect of Age of Equipment on Number of Breakdowns.--When cumulative pull-ins per vehicle (Y axis) were plotted against cumulative mileage per vehicle (X axis), the resulting points of data appear to follow the cubic parabola type of curve, $Y = aX^3 + bX^2 + cX$, in each of the five groupings of vehicles studied. Attempts to determine a relationship between the parameters of the cubic parabolas, a, b, and c, and three classifications of factors which influence equipment breakdowns (equipment design, operating conditions, and type of maintenance) yielded inconclusive results. Since the cumulative pull-ins per vehicle data appears to follow a predictable pattern, the knowledge of this pattern can be used to forecast the expected number of future pull-ins for a certain period of time. This information would aid in maintenance planning in the following ways:

1. Prediction of future maintenance workloads and expenses for pull-ins
2. Determination of most economical replacement age (because maintenance cost is an important factor in replacement analysis)
3. Providing of a quantitative basis for the comparison of different groups of vehicles of different designs

CHAPTER VIII

RECOMMENDATIONS

Although these recommendations are limited to the four problem areas within this study, it is acknowledged that there are many other maintenance problem areas in which additional research would seem to offer great possibilities.

The general use of "production functions" as a tool for the evaluation of productive output factors will require investigations beyond the scope of this thesis. It is believed that one of the major problems is the determination of the proper factors for use in such evaluation. If dollar units are used, measures must be taken to adjust for non-representative or unusual data. The possibility of expressing the factors in non-monetary units, such as number of riders instead of revenue, or man-hours of maintenance labor instead of maintenance expense, should be investigated. If desirable, the non-monetary units for a series of years could be converted to equivalent present dollar values. In the author's opinion, this approach, cautiously and properly applied, has considerable possibilities for the evaluation of productive output factors.

Several studies, including this one, indicate that equipment-breakdown data can be approximated with acceptable accuracy by one of the standard statistical distributions, such as the normal or Poisson distribution. As plants become increasingly automatic and optimum maintenance becomes more critical, knowledge of the nature of the unpreventable breakdowns will become more imperative. This knowledge will help in analyzing the factors contributing to the breakdowns so that corrective steps can be taken. Studies to determine the mathematical nature of various types of equipment-breakdown data will furnish additional information on this subject. The investigation may also include a study of the reasons why the breakdown data is of a certain type. With further research, it is believed that general principles can be developed to evaluate not only existing maintenance systems but also to predict the results of systems in the design stage.

In determining optimum conditions with the queuing method, the mathematics becomes rather involved and sometimes situations arise which are difficult of solution with the formal approach. The Monte Carlo technique is easier from a mathematical viewpoint and can be used for queuing type problems. One disadvantage of the Monte Carlo technique is that, although the calculations are mostly arithmetical, the volume of calculations makes this technique laborious for some problems. In such problems, the use of electronic computer equipment is recommended.

For future studies to determine optimum maintenance conditions, it is recommended that the Monte Carlo technique be applied. With this method labor scheduling problems and the preventive maintenance portion of the repair cycle can be included readily. This approach should yield more complete information on the practical problems of day-to-day operations, as well as provide a good estimate of the results to be expected from alternative systems.

The analysis of equipment-breakdown data using the graphical presentation of cumulative breakdowns versus cumulative units of age appears to have merit. Studies of other data for vehicles of similiar types will be necessary to determine if the cubic parabola can be applied generally to this kind of equipment-breakdown data for transit vehicles. Statistical correlation of the parameters of the cubic parabolas and the factors affecting equipment breakdowns is recommended for such studies.

A P P E N D I X

DATA FOR COMPARISON OF ATLANTA TRANSIT STATISTICS WITH NATIONAL AVERAGES

Data for Atlanta Transit was obtained from the Engineering Department and accounting files of the Atlanta Transit Systems, Inc. of Atlanta, Georgia.

Data for National Average was compiled from the February issues of Bus Transportation for the respective years and adjusted, as shown on the following page, to compare with Atlanta Transit figures.

Table 11. Data for Comparison of Atlanta Transit Statistics with National Averages

CHART DATA

A
Number of Revenue Passengers Per Mile

	1950	1951	1952	1953	1954	1955	1956
N. A.	6.04	5.56	5.37	5.11	4.97	4.77	4.69
A. T.	4.98	4.70	4.62	4.42	4.45	4.36	4.19

B
Cents of Income Per Passenger

	1950	1951	1952	1953	1954	1955	1956
N. A.	9.35	10.83	12.13	13.17	13.81	14.28	14.85
A. T.	9.32	10.65	11.38	12.30	12.70	13.32	14.07

C
Cents of Revenue Per Vehicle Mile

	1950	1951	1952	1953	1954	1955	1956
N. A.	56.41	60.07	65.04	67.18	68.45	68.02	69.48
A. T.	46.46	50.09	52.60	54.32	56.57	58.01	58.92

(continued)

Table 11. Data for Comparison of Atlanta Transit Statistics with National Averages (continued)

BASIC DATA

Number of Annual Revenue Passengers in Millions

	1950	1951	1952	1953	1954	1955	1956
National Totals							
Bus	8,434	7,470	7,138	6,658	6,125	5,758	5,595
Trolley	1,169	1,226	1,205	1,135	1,034	937	892
Atlanta Transit							
Bus		16.81	13.67	11.59	10.99	10.25	10.88
Trolley		69.33	70.78	67.05	62.19	58.25	54.29

Number of Annual Vehicle Miles in Millions

	1950	1951	1952	1953	1954	1955	1956
National Totals							
Bus	1,882	1,897	1,877	1,831	1,743	1,711	1,727
Trolley	182	206	211	210	196	184	176
Atlanta Transit							
Bus		3.442	3,368	3.009	2.965	2.866	3.116
Trolley		14.118	14.987	14.234	13.698	12.542	12.035

Dollars of Annual Revenue in Millions

	1950	1951	1952	1953	1954	1955	1956
National Totals							
Bus	798.9	833.5	888.6	912.6	893.4	881.0	915.3
Trolley	109.0	131.9	145.5	148.4	141.3	132.1	129.8
Atlanta Transit							
Bus		1.553	1.481	1.386	1.272	1.331	1.483
Trolley		7.341	7.803	7.779	7.006	7.300	7.328

(continued)

Table 11. Data for Comparison of Atlanta Transit Statistics with National Averages (continued)

SAMPLE CALCULATIONS FOR ADJUSTED NATIONAL AVERAGES

1. Number of Revenue Passengers Per Mile (1956)

No. of National Revenue Pass./Mi. for Bus = 5595/1727 = 3.24

No. of National Revenue Pass./Mi. for Trolley = 892/176 = 5.07

Per Cent of Atlanta Transit Mileage for Bus = 3.116/15.151 = 20.57%

Per Cent of Atlanta Transit Mileage for Trolley = 12.035/15.151 = 79.43%

Adjusted National Avg. for Bus = 20.57% x 3.24 = .666
Adjusted National Avg. for Trolley = 79.43% x 5.07 = 4.027

Total Adjusted National Avg. Revenue Passenger Per Mile = 4.693

2. Cents of Income Per Passenger (1956)

Dollars of National Revenue/Pass. for Bus = $915.3/5595 = $.1636

Dollars of National Revenue/Pass. for Trolley = $129.8/892 = $.1455

Per Cent of Atlanta Transit Passengers Riding Bus = 10.88/65.17 = 16.69%

Per Cent of Atlanta Transit Passengers Riding Trolley = 54.29/65.17 = 83.31%

Adjusted National Avg. for Bus = 16.69% x $.1636 = $.0273
Adjusted National Avg. for Trolley = 83.31% x $.1455 = .1212

Total Adjusted National Avg. Income Per Passenger = $.1485

3. Cents of Revenue Per Mile (1956)

Dollars of National Revenue/Mile for Bus = $915.3/1727 = $.5300

Dollars of National Revenue/Mile for Trolley = $129.8/176 = $.7375

Adjusted National Avg. for Bus = 20.57% x $.5300 = $.1090
Adjusted National Avg. for Trolley = 79.43% x $.7375 = .5858

Total Adjusted National Avg. Revenue Per Mile = $.6948

QUESTIONNAIRE FORM NO. 1

CITY TROLLEYS

Maintenance Data

This data is requested so that the following questions may be answered:

1. What per cent of total cost of operations is the maintenance cost of equipment?
2. What is average annual maintenance cost per vehicle?
3. What is average annual maintenance cost per vehicle mile?

Item	1954	1955	1956
Total Annual Collars - Cost of Maintenance of Vehicles (Labor, Materials, Supervision of Maintenance, etc.)	________	________	________
Total Annual Dollars Expense Allocated to These Vehicles (Including Depreciation, Taxes, Fuel, Operating Expenses, Overhead, etc.)	________	________	________
Number of Vehicles in Active Service	________	________	________
Number of Miles Per Year for All Vehicles in Active Service	________	________	________
Approximate Average Age of Vehicles in Years	________	________	________
Total Annual Number of Revenue Passengers	________	________	________

* *

Total Number of Maintenance Personnel			
1. Supervision	________	________	________
2. Mechanics	________	________	________
3. Inspectors	________	________	________
4. Others	________	________	________

QUESTIONNAIRE FORM NO. 2

CITY BUSES

Maintenance Data

This data is requested so that the following questions may be answered:

1. What per cent of total cost of operations is the maintenance cost of equipment?
2. What is average annual maintenance cost per vehicle?
3. What is average annual maintenance cost per vehicle mile?

Item		1954	1955	1956
Total Annual Dollars - Cost of Maintenance of Vehicles (Labor, Materials, Supervision of Maintenance, etc.)		______	______	______
Total Annual Dollars Expense Allocated to These Vehicles (Including Depreciation, Taxes, Fuel, Operating Expense, Overhead, etc.)		______	______	______
Number of Vehicles in Active Service	Gas	______	______	______
	Diesel	______	______	______
Number of Miles Per Year for All Vehicles in Active Service	Gas	______	______	______
	Diesel	______	______	______
Approximate Average Age of Vehicles in Years	Gas	______	______	______
	Diesel	______	______	______
Total Annual Number of Revenue Passengers		______	______	______

COMPARISON OF MAINTENANCE DATA FOR TRANSIT SYSTEMS IN CITIES OF APPROXIMATELY THE SAME SIZE

Data for Atlanta Transit was obtained from the Accounting Department records of the Atlanta Transit Systems, Inc. of Atlanta, Georgia.

Data for the other companies (B, C, D, E, F and G) was obtained by sending questionnaires to transit companies in ten cities whose population was approximately equal to that of Atlanta. Seven replies were received but only six had usable data. Sample questionnaires are included in this Appendix.

Table 12. Comparison of Maintenance Data for Transit Systems in Cities of Approximately the Same Size

CHART DATA

A
Trolley Maintenance Cost Per Mile in Cents

Company	Atl. Tr.	B	C	D	E	F	G
Maint. Cost.	6.25	11.23	4.92	3.11	7.38	8.51	No Trolleys

B
Total Trolley Maintenance and Expense Cost Per Mile in Cents

Company	Atl. Tr.	B	C	D	E	F	G
Maint. Cost	52.39	71.77	71.36	50.47	63.88	68.74	No Trolleys

C
Bus Maintenance Cost Per Mile in Cents

Company	Atl. Tr.	B	C	D	E	F	G
Maint. Cost	10.01	6.95	6.13	3.76	9.57	10.67	8.05

D
Total Bus Maintenance and Expense Cost Per Mile in Cents

Company	Atl. Tr.	B	C	D	E	F	G
Maint. Cost	57.98	61.47	58.11	51.38	62.42	66.24	59.86

DETAILS OF COST CLASSIFICATION FOR PRODUCTION FUNCTION ANALYSIS

For the production function analysis the Atlanta Transit cost records were divided into five classifications as follows:

Table 13. Details of Cost Classification for Production Function Analysis

Classification	Atl. Transit Accounts Included	Details of Atlanta Transit Accounts
Operating Expense	Transportation Expense	Vehicle operators, their supervision, their supplies, and miscellaneous other expenses.
	Operating Taxes	State and Federal Gasoline tax, federal excise and state sales tax, property taxes, vehicle licenses, miscellaneous vehicle taxes, Social Security, state and federal unemployment tax.
Investment	Depreciation	Depreciation of all vehicles, buildings and equipment as allowed by state and federal laws.
	Rent of Revenue Equipment	Expenses for leased vehicles
Maintenance	Maintenance of Equipment	Supervision, labor and materials for maintenance and repair of all vehicles, shops and garages.
	Maintenance of Power Facilities	Supervision, labor and materials for maintenance and repairs to poles, trolley overhead systems, signals, etc. (trolleys only)

(continued)

Table 13. Details of Cost Classification for Production Function Analysis (continued)

Classification	Atl. Transit Accounts Included	Details of Atlanta Transit Accounts
Administration	Administrative and General Expenses	Salaries and expenses of general officers and employees, office rents, general legal services, general supplies, assoc. dues, expenses for company bulletin and miscellaneous general expenses.
	Traffic Promotion	Supervision salaries and expenses, all advertising and miscellaneous promotion expenses.
	Claims, Insurance and Safety Expense	Claim department salaries and expenses, legal fees, insurance, workman's compensation, etc.
Power	Operating Garage Expense	Electric power, gasoline and diesel fuel, lubricants labor for servicing of vehicles and miscellaneous expenses.

Table 14. Cost Data for Production Function Analysis

Trolley (Actual Cost Figures)

Dollar figures are accumulated for six-months and year-end intervals.

	REVENUE Dollars	REVENUE Logs	OPERATING EXPENSE Dollars	OPERATING EXPENSE Logs
6 mos.	3,579,171	6.55378	1,625,628	6.21101
1951 ye	3,761,954	6.57529	1,657,803	6.21953
6 mos.	3,909,071	6.59208	1,637,948	6.21429
1952 ye	3,893,781	6.59037	1,660,779	6.22031
6 mos.	3,881,887	6.58904	1,603,550	6.20508
1953 ye	3,897,607	6.59080	1,647,381	6.21679
6 mos.	3,916,024	6.59284	1,706,736	6.23216
1954 ye	3,089,562	6.48990	1,347,394	6.12949
6 mos.	3,709,573	6.56932	1,551,803	6.19083
1955 ye	3,590,577	6.55516	1,516,555	6.18086
6 mos.	3,745,523	6.57351	1,586,724	6.20050
1956 ye	3,582,159	6.55415	1,519,933	6.18182

	INVESTMENT Dollars	INVESTMENT Logs	MAINTENANCE Dollars	MAINTENANCE Logs
6 mos.	185,244	5.26774	520,482	5.71649
1951 ye	191,067	5.28118	546,349	5.73747
6 mos.	185,535	5.26842	673,265	5.82818
1952 ye	188,176	5.27456	516,804	5.71332
6 mos.	255,631	5.40761	547,887	5.73869
1953 ye	431,524	5.63500	557,181	5.74599
6 mos.	416,737	5.61986	580,634	5.76391
1954 ye	322,470	5.50849	460,633	5.66335
6 mos.	526,087	5.72106	512,120	5.70937
1955 ye	517,966	5.71430	486,506	5.68708
6 mos.	516,815	5.71334	514,998	5.71181
1956 ye	503,219	5.70184	478,915	5.68026

	POWER Dollars	POWER Logs	ADMINISTRATION Dollars	ADMINISTRATION Logs
6 mos.	429,768	5.63325	375,211	5.57427
1951 ye	452,868	5.65597	439,781	5.64324
6 mos.	435,325	5.63882	442,665	5.64608
1952 ye	454,997	5.65800	454,347	5.65739
6 mos.	420,539	5.62381	489,039	5.68935
1953 ye	434,037	5.63753	466,579	5.66892

(continued)

Table 14. Cost Data for Production Function Analysis
(continued)

Trolley (Actual Cost Figures)

Dollar figures are accumulated for six-months and year-end intervals.

	POWER		ADMINISTRATION	
	Dollars	Logs	Dollars	Logs
6 mos.	419,311	5.62253	451,093	5.65426
1954 ye	346,321	5.53948	497,050	5.69640
6 mos.	382,183	5.58227	449,074	5.65231
1955 ye	400,066	5.60213	395,206	5.59683
6 mos.	373,458	5.57224	429,245	5.63271
1956 ye	382,408	5.58253	390,981	5.59216

Table 15. Cost Data for Production Function Analysis

Bus (Actual Cost Figures)

Dollar figures are accumulated for six-months and year-end intervals.

	REVENUE		OPERATING EXPENSE	
	Dollars	Logs	Dollars	Logs
6 mos.	755,858	5.87844	494,743	5.69438
1951 ye	796,902	5.90140	513,769	5.71077
6 mos.	767,394	5.88502	482,672	5.68365
1952 ye	713,652	5.85348	448,672	5.65193
6 mos.	678,255	5.83139	418,259	5.62145
1953 ye	708,043	5.85005	437,085	5.64056
6 mos.	696,456	5.84289	437,935	5.64140
1954 ye	575,684	5.76018	369,409	5.56751
6 mos.	647,277	5.81109	412,889	5.61583
1955 ye	683,784	5.83492	440,016	5.64347
6 mos.	722,084	5.85860	477,132	5.67864
1956 ye	760,830	5.88129	498,240	5.69744

(continued)

Table 15. Cost Data for Production Function Analysis (continued)

Bus (Actual Cost Figures)

Dollar figures are accumulated for six-months and year-end intervals.

	INVESTMENT		MAINTENANCE	
	Dollars	Logs	Dollars	Logs
6 mos.	41,727	4.62042	172,114	5.23581
1951 ye	43,644	4.63992	186,038	5.26960
6 mos.	38,645	4.58709	181,018	5.25770
1952 ye	38,713	4.58785	175,561	5.24442
6 mos.	35,909	4.55521	164,134	5.21519
1953 ye	35,973	4.55598	174,316	5.24134
6 mos.	34,488	4.53767	173,040	5.23815
1954 ye	29,384	4.46811	128,828	5.11001
6 mos.	94,969	4.97754	141,643	5.15119
1955 ye	90,384	4.95609	135,314	5.13135
6 mos.	113,064	5.05369	151,106	5.17928
1956 ye	129,840	5.11341	166,530	5.22149

	POWER		ADMINISTRATION	
	Dollars	Logs	Dollars	Logs
6 mos.	124,629	5.09562	86,693	4.93799
1951 ye	129,600	5.11261	90,810	4.95813
6 mos.	116,764	5.06731	104,960	5.02103
1952 ye	115,698	5.06330	102,386	5.01023
6 mos.	103,276	5.01400	112,586	5.05189
1953 ye	105,715	5.02413	112,047	5.04940
6 mos.	102,327	5.01000	110,969	5.04520
1954 ye	87,285	4.94094	111,879	5.04875
6 mos.	80,658	4.90665	105,117	5.02167
1955 ye	79,615	4.90100	101,206	5.00521
6 mos.	76,585	4.88415	112,917	5.05275
1956 ye	84,419	4.92644	114,500	5.05881

Sample Calculations for Production Function Example

The production function is linear in logarithms.

Average Values for Six-Month Periods

(Averages for items from Tables 14 and 15)

Item	Averages for Trolleys Dollars (Millions)	Trolleys Logs	Buses Dollars (Millions)	Buses Logs
Revenue	$3.71	6.56885	$.71	5.84906
Operating Expense	1.59	6.20022	.45	5.65392
Maintenance	.53	5.72466	.16	5.20796

1% of log of trolley O.E. = (.01)(6.20022) = 0.06200

(0.06200 + 6.20022) = 6.26222, antilog = $1,829,000

Additional O.E. = $1,829,000 - $1,590,000 = $239,000

.73% of log of trolley revenue = (0.0073)(6.56885) = 0.04795

(0.04795 + 6.56885) = 6.61680, antilog = $4,138,000

Additional revenue = $4,138,000 - $3,710,000 = $428,000

1% of log of Bus Maintenance = (.01)(5.20796) = 0.05208

(0.05208 + 5.20796) = 5.26004, antilog = $182,000

$182,000 - $160,000 = $22,000 = increase in bus maintenance

.275% of log of Bus Revenue = (0.00275)(5.84906) = 0.01608

(0.01608 + 5.84906) = 5.86514, antilog = $733,100

$733,100 - $710,000 = $23,100 = increase in bus revenue

$22,000 + $530,000 = $552,000, log $552,000) = 5.74225

5.74225 - 5.72466 = 0.01759 = difference in trolley maintenance log equivalent to $22,000

$$\frac{0.01759}{5.72466} \times 100 = 0.307\% = \text{per cent increase in trolley maintenance log}$$

$$\frac{0.307\%}{X} = \frac{1\%}{0.1316}$$

X = 0.0404 = per cent increase in trolley revenue log

(0.000404)(6.56885) = 0.00265

(0.00265 + 6.56885) = 6.57150, antilog = $3,728,000

$3,728,000 - $3,710,000 = $18,000

Table 16. Distribution Data for Pull-Ins of Pine Street Buses

Number of Pull-Ins Per Day (n)	Relative Frequency Recorded (O)		Total Pull-Ins Recorded	Poisson Frequency Calculated (E)		$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	17		0	14.67		5.429	.37007
1	37		37	41.94		24.404	.58188
2	70		140	59.97		100.601	1.67752
3	55		165	57.16		4.666	.08163
4	31		124	40.86		97.220	2.37934
5	23		115	23.37		.137	.00586
6	13		78	11.14		3.460	.31059
7	8		56	4.55			
8	1		8	1.63		9.923	1.44861
9	1	10	9	.52	6.85		
10	0		0	.15			
	256		732	255.96			6.85550

$$\text{Mean} = \frac{732}{256} = 2.85938 = m$$

$$e^m = 17.451$$

$$e^{-m} = .05730$$

Degrees of Freedom = 6

$$\text{Poisson Probability} = P_n = \frac{m^n e^{-m}}{n!}$$

$$\text{Poisson Frequency Calculated} = nP_n$$

Table 17. Distribution Data of Roadcalls for Pine Street Buses

Number of Roadcalls Per Day (n)	Relative Frequency Recorded (O)		Total Roadcalls Recorded	Poisson Frequency Calculated (E)		$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	0	20	0	.50	13.35	44.223	3.31258
1	9		9	3.12			
2	11		22	9.73			
3	27		81	20.23		45.833	2.26560
4	28		112	31.55		12.603	.39946
5	38		190	39.36		1.850	.04700
6	39		234	40.93		3.725	.09101
7	27		189	36.47		89.681	2.45903
8	24		192	28.44		19.714	.69318
9	19		171	19.71		.504	.02557
10	7		70	12.30		28.090	2.28374
11	14		154	6.97		49.421	7.09053
12	7	13	84	3.63	6.60	40.960	6.20606
13	1		13	1.74			
14	1		14	.78			
15	2		30	.32			
16	2		32	.13			
	256		1597	255.91			24.87376

$$\text{Mean} = \frac{1597}{256} = 6.2383 = m$$

$$e^m = 511.975$$

$$e^{-m} = .001953$$

Degrees of Freedom = 9

Table 18. Distribution Data of Pull-Ins for Pine Street Trolleys

Number of Pull-Ins Per Day (n)	Relative Frequency Recorded (O)		Total Pull-Ins Recorded	Poisson Frequency Calculated (E)		$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	40		0	39.79		.0441	.00111
1	85		85	79.58		29.3764	.36914
2	70		140	79.58		91.7764	1.15326
3	53		159	53.05		.0003	.00001
4	33		132	26.53		41.8609	1.57787
5	8		40	10.61		6.8121	.64205
6	3		18	3.54			
7	2	5	14	1.01	4.80	.0400	.00833
8	0		0	.25			
TOTALS	294		588	293.94			3.75177

$$\text{Mean} = \frac{588}{294} = 2.00000 = m$$

$$e^{m} = 7.3891$$

$$e^{-m} = .13534$$

Degrees of Freedom = 5

Table 19. Distribution Data of Roadcalls for Pine Street Trolleys

Number of Roadcalls Per Day (n)	Relative Frequency Recorded (O)		Total Roadcalls Recorded	Poisson Frequency Calculated		$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	0		0	.16			
1	3	8	3	1.22	5.97	4.121	.69028
2	5		10	4.59			
3	9		27	11.45		6.003	.52428
4	27		108	21.45		30.803	1.43604
5	36		180	32.14		14.900	.46360
6	45		270	40.13		23.717	.59100
7	32		224	42.94		119.684	2.78724
8	42		336	40.21		3.204	.07968
9	26		234	33.47		55.801	1.66719
10	15		150	25.08		101.606	4.05128
11	26		286	17.08		79.566	4.65843
12	9		108	10.66		2.756	.25854
13	6		78	6.14		.020	.00326
14	5		70	3.29			
15	2		30	1.64			
16	4	12	64	.77	6.18	33.872	5.48091
17	1		17	.34			
18	0		0	.14			
	293		2195	292.90			22.69173

Mean $= \frac{2195}{293} = 7.491468 = m$

$e^{m} = 1792.68$

$e^{-m} = .0005578$

Degrees of Freedom = 11

Table 20. Distribution Data of Pull-Ins for Brisbane Trolleys

Number of Pull-Ins Per Day (n)	Relative Frequency Recorded (O)		Total Pull-Ins Recorded	Poisson Frequency Calculated (E)		$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	5		0	7.63		6.917	.90655
1	30		30	28.22		3.168	.11226
2	62		124	52.18		96.432	1.84806
3	61		183	64.32		11.022	.17136
4	62		248	59.46		6.452	.10851
5	36		180	43.98		63.680	1.44793
6	24		144	27.11		9.672	.35677
7	12		84	14.32		5.382	.37584
8	5		40	6.62			
9	6		54	2.50			
10	3	16	30	.93	10.44	30.914	2.96111
11	2		22	.30			
12	0		0	.09			
	308		1139	307.66			8.28839

Mean $= \frac{1139}{308} = 3.698 = m$

$e^{m} = 40.367$

$e^{-m} = .024773$

Degrees of Freedom = 7

Table 21. Distribution Data of Roadcalls for Brisbane Trolleys

Number of Roadcalls Per Day (n)	Relative Frequency Recorded (O)		Total Roadcalls Recorded	Poisson Frequency Calculated (E)		$(O-E)^2$	$\frac{(O-E)^2}{E}$
0	0		0	0.00			
1	0		0	0.00			
2	0		0	0.01			
3	0	16	0	0.06	6.36	92.930	14.61164
4	0		0	0.24			
5	7		35	0.70			
6	1		6	1.72			
7	8		56	3.63			
8	6		48	6.69		.476	.07115
9	16		144	10.97		25.301	2.30638
10	13		130	16.18		10.112	.62497
11	24		264	21.71		5.244	.21455
12	27		324	26.68		.102	.00382
13	22		286	30.28		68.558	2.26413
14	26		364	31.91		34.928	1.09458
15	35		525	31.38		13.104	.41759
16	23		368	28.94		35.284	1.21921
17	26		442	25.11		.792	.03154
18	17		306	20.58		12.816	.62274
19	16		304	15.98		.0004	.00003
20	8		160	11.79		14.364	1.21832
21	7		147	8.28		1.638	.19783
22	5		110	5.55		.303	.00054
23	5		115	3.56			
24	2		48	2.19			
25	5		125	1.29			
26	2		52	.73		130.645	15.24446
27	2	20	54	.40	8.57		
28	2		56	.21			
29	1		29	.11			
30	0		0	.05			
31	1		31	.03			
	307		4529	306.96			40.14348

$$\text{Mean} = \frac{4529}{307} = 14.75244 = m$$

$$e^m = 2,552,140$$

$$e^{-m} = .000000391828$$

Degrees of Freedom = 15

Table 22. Various Combinations of Machines Requiring Service at a Given Time for $\lambda/\mu = 4/3$

Two Repair Channels

No. Machines Being Serviced and Waiting (n = (R + W))	No. Machines Being Serviced (R)	n - R (W)	$\frac{P_n}{P_o}$	P_n	Cum. P_n
0	0	0	1.0000	.20023	.20023
1	1	0	1.333333	.26697	.46720
2	2	0	.888889	.17798	.64518
3	2	1	.592600	.11866	.76384
4	2	2	.395100	.07911	.84295
5	2	3	.263400	.05274	.89569
6	2	4	.175600	.03516	.93085
7	2	5	.117100	.02345	.95430
8	2	6	.078040	.01562	.96992
9	2	7	.052030	.01042	.98034
10	2	8	.034680	.00694	.98728
11	2	9	.023120	.00463	.99191
12	2	10	.015420	.00309	.99500
13	2	11	.010280	.00206	.99706
14	2	12	.006851	.00137	.99843
15	2	13	.004567	.00091	.99934
16	2	14	.001522	.00030	.99964
17	2	15	.001015	.00020	.99984
18	2	16	.000677	.00014	.99998
TOTALS			4.994162	.999980	

$$\sum \frac{P_n}{P_o} = \frac{1}{P_o} = 4.994162$$

$$P_o = \frac{1}{4.994162} = .20023$$

Table 23. Various Combinations of Machines Requiring Service at a Given Time for $\lambda/\mu = 4/3$

Three Repair Channels

No. Machines Being Serviced and Waiting (n = (R + W))	No. Machines Being Serviced (R)	n - R (W)	$\frac{P_n}{P_o}$	P_n	Cum. P_n
0	0	0	1.00000	.25425	.25425
1	1	0	1.333333	.33900	.59325
2	2	0	.888889	.22600	.81925
3	3	0	.395060	.10045	.91970
4	3	1	.175580	.04464	.96434
5	3	2	.078040	.01984	.98418
6	3	3	.034680	.00882	.99300
7	3	4	.015410	.00392	.99692
8	3	5	.006851	.00174	.99866
9	3	6	.003045	.00077	.99943
10	3	7	.001353	.00034	.99977
11	3	8	.000602	.00015	.99992
12	3	9	.000267	.00007	.99999
TOTALS			3.933108	.99999	

$$\sum \frac{P_n}{P_o} = \frac{1}{P_o} = 3.933108$$

$$P_o = \frac{1}{3.933108} = .25425$$

Table 24. Various Combinations of Machines Requiring Service at a Given Time for $\lambda/\mu = 4/3$

Four Repair Channels

No. Machines Being Serviced and Waiting (n = (R + W))	No. Machines Being Serviced (R)	n - R (W)	$\frac{P_n}{P_o}$	P_n	Cum. P_n
0	0	0	1.000000	.26214	.26214
1	1	0	1.333333	.34951	.61165
2	2	0	0.888889	.23302	.84469
3	3	0	0.395100	.10357	.94824
4	4	0	0.131700	.03452	.98276
5	4	1	0.043900	.01151	.99427
6	4	2	0.014630	.00384	.99811
7	4	3	0.004877	.00128	.99939
8	4	4	0.001626	.00043	.99982
9	4	5	0.000542	.00014	.99996
10	4	6	0.000181	.00004	1.00000
TOTALS			3.814756	1.00000	

$$\sum \frac{P_n}{P_o} = \frac{1}{P_o} = 3.814756$$

$$P_o = \frac{1}{3.814756} = .26214$$

Table 25. Various Combinations of Machines Requiring Service at a Given Time for $\lambda/\mu = 4/3$

Five Repair Channels

No. Machines Being Serviced and Waiting (n = (R + W))	No. Machines Being Serviced (R)	n - R (W)	$\frac{P_n}{P_o}$	P_n	Cum. P_n
0	0	0	1.000000	.26338	.26338
1	1	0	1.333333	.35117	.61455
2	2	0	.888889	.23412	.84867
3	3	0	.395060	.10405	.95272
4	4	0	.131690	.03468	.98740
5	5	0	.035120	.00925	.99665
6	5	1	.009364	.00247	.99912
7	5	2	.002497	.00066	.99978
8	5	3	.000666	.00017	.99995
9	5	4	.000178	.00005	1.00000
TOTALS			3.796797	1.00000	

$$\sum \frac{P_n}{P_o} = \frac{1}{P_o} = 3.796797$$

$$P_o = \frac{1}{3.796797} = .26338$$

Table 26. Various Combinations of Machines Requiring Service at a Given Time for $\lambda/\mu = 4/3$

Six Repair Channels

No. Machines Being Serviced and Waiting (n = (R + W))	No. Machines Being Serviced (R)	n - R (W)	$\frac{P_n}{P_o}$	P_n	Cum. P_n
0	0	0	1.000000	.26357	.26357
1	1	0	1.333333	.35143	.61500
2	2	0	.888889	.23429	.84929
3	3	0	.395060	.10413	.95342
4	4	0	.131690	.03471	.98813
5	5	0	.035120	.00926	.99739
6	6	0	.007804	.00206	.99945
7	6	1	.001734	.00045	.99990
8	6	2	.000385	.00010	1.00000
TOTALS			3.794015	1.00000	

$$\sum \frac{P_n}{P_o} = \frac{1}{P_o} = 3.794015$$

$$P_o = \frac{1}{3.794015} = .26357$$

SAMPLE CALCULATIONS FOR EXPECTED NUMBER OF MACHINES AT VARIOUS POINTS IN REPAIR CYCLE WHERE $\lambda/\mu = 4/3$ and Q' = 100

All sample calculations will be made for two repair channels.

Expected number of machines being repaired = E(R).

$$E(R) = \sum_{n=0}^{T} n\, P_n - \sum_{n=C+1}^{T} (n - R)P_n$$

For two repair channels, or C = 2, and T = 106

$$\sum_{n=0}^{106} n\, P_n = 2.38 \text{ and} \sum_{n=3}^{106} (n - R)P_n = 1.05$$

$$E(W) = 2.38 - 1.05 = 1.33$$

Expected number of machines being repaired = E(W)

$$E(W) = \sum_{n=C+1}^{T} (n - R)P_n$$

$$E(W) = \sum_{n=3}^{106} (n - R)P_n = 1.05$$

Expected number of machines in productive operation = E(Q)

$$E(Q) = T \sum_{j=T-Q'+1}^{T} p_j + Q' \sum_{j=0}^{T-Q'} p_j - \sum_{j=T-Q'+1}^{T} j\, p_j$$

$$E(Q) = 106 \sum_{j=7}^{106} p_j + 100 \sum_{j=0}^{6} p_j - \sum_{j=7}^{106} j\, p_j$$

$$E(Q) = 106(.06913) + 100\,(.93085) - (.61063)$$

$$E(Q) = 7.30078 + 93.085 - .61063 = 99.78$$

Expected number of spare machines ready for service = E(S)

$$E(S) = T - E(R) - E(w) - E(Q)$$

$$E(S) = 106 - 1.33 - 1.05 - 99.78 = 3.84$$

SAMPLE CALCULATIONS OF COST FACTORS FOR COMPARATIVE ANNUAL COST OF REPAIR CYCLE

Cost of Idle Labor

D_L = Dollar cost of labor waiting for vehicles to be repaired.

$$D_L = K_1 \sum_{n=0}^{C} (C - n)P_n$$

D_L = (Daily wage rate/mechanic)(Number of Mechanics)(365 days/year)

(Number of idle repair channels)(Per Cent of time channel or channels are idle)

The following table of data was obtained from the probability tables for the various number of repair channels.

Table 27. Number of Idle Repair Channels and Per Cent of Time They are Idle

X = Number of Idle Channels	Per Cent of Time Repair Channels are Idle				
	Number of Repair Channels				
	2	3	4	5	6
1	.26697	.22600	.10357	.03468	.00926
2	.20023	.33900	.23302	.10405	.03471
3	--	.25425	.34951	.23412	.10413
4	--	--	.26214	.35117	.23429
5	--	--	--	.26338	.35143
6	--	--	--	--	.26357
$\sum$(X)(Per Cent)	.66743	1.66675	2.66670	3.66672	4.66680
D_L =	$6821	$17,034	$27,254	$37,474	$47,695

Cost of Idle Time of Vehicles Waiting for Repairs

D_W = Annual dollar cost of idle equipment waiting in line for repairs.

$D_W = W \bar{t}_w K_2$

$D_W = (W)(\bar{t}_w)$(Overhead cost/vehicle/day)(365 days/year).

Overhead cost/vehicle/day = $10

Sample calculation for two repair channels:

$$\bar{t}_w = \frac{P_o}{C(C!)\left[1 - \frac{\lambda}{\mu C}\right]^2} \left(\frac{\lambda}{\mu}\right)^C$$

$$\bar{t}_w = \frac{.20023}{(1.5)(2)(2!)\left[1 - \frac{2}{3}\right]^2} \left(\frac{4}{3}\right)^2 = .5340$$

$$D_W = (1.05)(.5340)(\$10)(365)$$

$$D_W = \$2047$$

Table 28. Average Number of Idle Vehicles, Average Waiting Time and Average Annual Cost of Idle Vehicle Time

Number of Channels	W	$\bar{t}_w$	D_W
2	1.05	.5340	$2047
3	.14	.0711	36
4	.03	.0130	1
5	.01	.0023	0
6	.001	.0004	0

Cost of Idle Time of Vehicles in Repairs

D_R = Annual dollar cost of idle equipment in the process of being repaired.

$D_R = K_2 \frac{R}{\mu}$

D_R = (Overhead cost/vehicle/day)(365 days/year) $\frac{R}{\mu}$

Overhead cost/vehicle/day = $10

$\frac{1}{\mu}$ = Average servicing time/vehicle = .6667 days

D_R = ($10)(365)(.6667 days/vehicle)(1.33 vehicles)

D_R = $3236 (constant for all numbers of channels)

Cost of Idle Time of Spare Equipment

D_S = Annual cost of idle spare equipment, S, that is waiting to be put into service

$D_S = K_2 S$

D_S = (Overhead cost/vehicle/day)(365 days/year)(S machs.)

Assumed overhead cost/vehicle/day = $10

S = Total number of spare machines − R − W

R = 1.33 machines for all numbers of channels

W varies as follows:

	Number of Channels				
	2	3	4	5	6
W =	1.05	.14	.03	.01	.001

Sample calculation for two channel repair facilities with three spare vehicles:

S = 3 − 1.33 − 1.05 = .62

D_S = (.62)($10)(365) = $2263

Cost of Repair Facilities

D_F = Annual overhead cost for repair facilities

$D_F = K_3 C$

D_F = (Annual overhead cost per channel)(Number of channels)

Sample calculation for two repair channels:

D_F = (\$800)(2) = \$1600

	Number of Channels				
	2	3	4	5	6
D_F =	\$1600	\$2400	\$3200	\$4000	\$4800

Table 29. Comparative Annual Costs for Various Numbers of Repair Channels at Different Operating Levels

Two Repair Channels

Opr. Level in Per Cent	Total No. of Spare Vehicles Required	D_W	D_R	D_S	D_F	D_L	C.A.Cost of Equip. & Facilities	C.A.Cost of Equip., Facilities & Labor
76.38	3	$2047	$3236	$ 2,263	$1600	$6821	$ 9,146	$15,967
84.30	4			5,913			12,796	19,617
89.57	5			9,563			16,446	23,267
93.09	6			13,213			20,096	26,917
95.43	7			16,863			23,746	30,567
96.99	8			20,513			27,396	34,217
98.03	9			24,163			31,046	37,867
98.73	10			27,813			34,696	41,517
99.19	11			31,463			38,346	45,167
99.50	12			35,113			41,996	48,817
99.71	13			38,763			45,646	52,467
99.84	14			42,413			49,296	56,117
99.93	15			46,063			52,946	59,767

Table 30. Comparative Annual Costs for Various Numbers of Repair Channels at Different Operating Levels

Three Repair Channels

Opr. Level in Per Cent	Total No. of Spare Vehicles Required	D_W	D_R	D_S	D_F	D_L	C.A.Cost of Equip. & Facilities	C.A.Cost of Equip., Facilities & Labor
81.93	2	$36	$3236	$ 1,935	$2400	$17,034	$ 7,607	$24,641
91.97	3			5,585			11,257	28,291
96.43	4			9,235			14,907	31,941
98.42	5			12,885			18,557	35,591
99.30	6			16,535			22,207	39,241
99.69	7			20,185			25,857	42,891
99.89	8			23,835			29,507	46,541
99.94	9			27,485			33,157	50,191
99.98	10			31,135			36,807	53,841
99.99	11			34,785			40,457	57,491

Table 31. Comparative Annual Costs for Various Numbers of Repair Channels at Different Operating Levels

Four Repair Channels

Opr. Level in Per Cent	Total No. of Spare Vehicles Required	D_W	D_R	D_S	D_F	D_L	C.A.Cost of Equip. & Facilities	C.A.Cost of Equip., Facilities & Labor
84.47	2	$1	$3236	$ 2,336	$3200	$27,254	$ 8,773	$36,027
94.82	3	↓	↓	5,986	↓	↓	12,423	39,677
98.28	4	↓	↓	9,636	↓	↓	16,073	43,327
99.43	5	↓	↓	13,286	↓	↓	19,723	46,977
99.81	6	↓	↓	16,936	↓	↓	26,073	53,327
99.94	7	↓	↓	20,586	↓	↓	27,023	54,277
99.98	8	↓	↓	24,236	↓	↓	30,673	57,927

Table 32. Comparative Annual Costs for Various Numbers of Repair Channels at Different Operating Levels

Five Repair Channels

Opr. Level in Per Cent	Total No. of Spare Vehicles Required	D_W	D_R	D_S	D_F	D_L	C.A.Cost of Equip. & Facilities	C.A.Cost of Equip., Facilities & Labor
84.87	2	$0	$3236	$ 2,446	$4000	$37,474	$ 9,682	$47,156
95.27	3	↓	↓	6,096	↓	↓	13,332	50,806
98.74	4	↓	↓	9,746	↓	↓	16,982	54,456
99.67	5	↓	↓	13,396	↓	↓	20,632	58,106
99.91	6	↓	↓	17,046	↓	↓	24,282	61,756
99.98	7	↓	↓	20,696	↓	↓	27,932	65,406
99.99	8	↓	↓	24,346	↓	↓	31,582	69,056

Table 33. Comparative Annual Costs for Various Numbers of Repair Channels at Different Operating Levels

Six Repair Channels

Opr. Level in Per Cent	Total No. of Spare Vehicles Required	D_W	D_R	D_S	D_F	D_L	C.A. Cost of Equip. & Facilities	C.A. Cost of Equip., Facilities & Labor
84.93	2	$0	$3236	$ 2,446	$4800	$47,695	$10,482	$58,177
95.34	3	↓	↓	6,096	↓	↓	14,132	61,827
98.81	4	↓	↓	9,746	↓	↓	17,782	65,477
99.74	5	↓	↓	13,396	↓	↓	21,432	69,127
99.95	6	↓	↓	17,046	↓	↓	25,082	72,777
99.99	7	↓	↓	20,696	↓	↓	28,732	76,427

Table 34. Comparison of Points of Data with Cubic Parabola

Original Data					Calculated Data for Cubic Parabola $y = ax^3 + bx^2 + cx$				
Year	Mileage Per Vehicle	Pull-Ins Per Vehicle	Cum. Miles Per Vehicle	Cum. Pull-Ins Per Vehicle	x	ax^3	bx^2	cx	Y
34-S Buses (Sample of 4 Buses)									
1947	42,418	2.25	42,418	2.25	50	.02	1.04	2.11	3.15
1948	51,157	6.25	93,575	8.50	100	.12	4.16	4.22	8.50
1949	34,553	2.75	128,128	11.25	150	.39	9.38	6.32	16.09
1950	43,193	8.75	171,321	20.00	200	.93	16.67	8.43	26.03
1951	53,418	12.75	224,739	32.75	250	1.82	26.04	10.54	38.40
1952	47,157	12.25	271,896	45.00	300	3.15	37.50	12.65	53.30
1953	46,868	14.25	318,764	59.25	350	5.00	51.05	14.76	70.81
1954	45,821	13.25	364,585	72.50	400	7.47	66.67	16.86	91.00
1955	43,208	25.00	407,793	97.50	450	10.63	84.38	18.97	113.98
1956	31,392	18.25	439,185	115.75	500	14.59	104.18	21.08	139.85
1957	24,705	8.00	463,890	123.75					
38-S Buses (Sample of 21 Buses)									
1948	20,237	2.00	20,237	2.00	50	-.16	3.09	1.30	4.23
1949	35,389	3.00	55,626	5.00	100	-1.29	12.40	2.60	13.71
1950	34,293	6.29	89,919	11.29	150	-4.35	27.89	3.89	27.43
1951	33,730	8.24	123,649	19.53	200	-10.32	49.58	5.19	44.45
1952	35,841	11.57	159,490	31.10	250	-20.16	77.47	6.49	63.80
1953	39,598	13.33	199,088	44.43	300	-34.83	111.56	7.79	84.52
1954	38,410	12.48	237,498	56.91	350	-55.31	151.84	9.08	105.61
1955	32,704	14.88	270,202	71.79	400	-82.56	198.32	10.38	126.14
1956	21,430	7.86	291,632	79.65	450	-117.55	251.10	11.68	145.23
1957	29,748	11.29	321,380	90.94	500	-161.20	309.80	12.97	161.57

(continued)

Table 34. Comparison of Points of Data with Cubic Parabola

	Original Data				Calculated Data for Cubic Parabola $y = ax^3 + bx^2 + cx$				
Year	Mileage Per Vehicle	Pull-Ins Per Vehicle	Cum. Miles Per Vehicle	Cum. Pull-Ins Per Vehicle	x	ax^3	bx^2	cx	Y
41-S Buses (Sample of 12 Buses)									
1948	43,506	6.00	43,506	6.00	50	-.12	2.97	1.54	4.39
1949	39,397	4.08	82,903	10.08	100	-.96	11.88	3.08	14.00
1950	35,706	9.00	118,609	19.08	150	-3.23	26.73	4.63	28.13
1951	29,951	8.75	148,560	27.83	200	-7.66	47.51	6.17	46.02
1952	33,488	10.92	182,048	38.75	250	-14.97	74.24	7.71	66.98
1953	36,042	15.67	218,090	54.42	300	-25.87	106.90	9.25	90.28
1954	34,663	13.17	252,753	67.59	350	-41.07	145.51	10.79	115.23
1955	29,343	16.58	282,096	84.17	400	-61.31	191.85	12.33	142.87
1956	19,382	6.33	301,478	90.50	450	-87.30	240.53	13.88	167.11
1957	16,677	6.67	318,155	97.17	500	-119.80	296.43	15.43	192.06
WH Trolleys (Sample of 120 Trolleys)									
1948	1,790	.21	1,790	.21	50	.04	.01	4.16	4.22
1949	46,879	3.29	48,669	3.50	100	.35	.58	8.33	9.25
1950	43,024	4.70	91,693	8.20	150	1.18	1.29	12.49	14.96
1951	43,277	5.83	134,970	14.03	200	2.80	2.30	16.65	21.75
1952	40,557	5.06	175,527	19.09	250	5.47	3.59	20.81	29.88
1953	38,509	4.52	214,036	23.61	300	9.45	5.18	24.98	39.60
1954	34,089	5.38	248,125	28.99	350	15.01	7.04	29.14	51.19
1955	30,224	6.03	278,349	35.02	400	22.40	9.20	33.30	64.90
1956	28,507	5.94	306,856	40.96	450	31.89	11.64	37.46	81.00
1957	29,370	6.18	335,226	47.14	500	43.80	14.39	41.60	99.79

(continued)

Table 34. Comparison of Points of Data with Cubic Parabola

Original Data					Calculated Data for Cubic Parabola $y = ax^3 + bx^2 + cx$				
Year	Mileage Per Vehicle	Pull-Ins Per Vehicle	Cum. Miles Per Vehicle	Cum. Pull-Ins Per Vehicle	x	ax^3	bx^2	cx	Y
GE Trolleys (Sample of 20 Trolleys)									
1949	43,560	3.00	43,560	3.00	50	.05	-.03	4.05	4.07
1950	62,561	4.50	106,121	7.50	100	.38	-1.22	8.10	7.26
1951	58,844	4.85	164,965	12.35	150	1.28	-2.75	12.14	10.67
1952	52,938	3.60	217,903	15.95	200	3.02	-4.89	16.19	14.32
1953	48,430	3.80	266,333	19.75	250	5.91	-7.64	20.24	18.51
1954	52,592	5.90	318,925	25.65	300	10.21	-11.01	24.29	23.49
1955	44,242	6.80	363,167	32.45	350	16.21	-14.98	28.33	29.56
1956	42,606	5.15	405,773	37.60	400	24.19	-19.56	32.38	37.01
1957	46,502	7.10	452,275	44.70	450	34.45	-24.77	36.43	46.11
					500	47.25	-30.60	40.49	57.14

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