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Eliminating Design Alternatives Based on Imprecise Information

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ABSTRACT

In this paper, the relationship between uncertainty and sets of alternatives in engineering design is investigated. In sequential decision making, each decision alternative actually consists of a set of design alternatives. Consequently, the decision-maker can express his or her preferences only imprecisely as a range of expected utilities for each decision alternative. In addition, the performance of each design alternative can be characterized only imprecisely due to uncertainty from limited data, modeling assumptions, and numerical methods. The approach presented in this paper recognizes the presence of both imprecision and sets in the design process by focusing on incrementally eliminating decision alternatives until a small set of solutions remains. This is a fundamental shift from the current paradigm where the focus is on selecting a single decision alternative in each design decision. To make this approach economically feasible, one needs efficient methods for eliminating alternatives—that is, methods that eliminate as many alternatives as possible given the available imprecise information. Efficient elimination requires that one account for dependencies between uncertain quantities, such as shared uncertain variables. In this paper, criteria for elimination with and without shared uncertainty are presented and compared. The set-based nature of design and the presence of imprecision are introduced, elimination criteria are discussed, and the overall set-based approach and elimination criteria are demonstrated with the design of a gearbox as an example problem.

INTRODUCTION

Design is a process of converting information about customer interests and requirements into a specification of a product. This process is complex because it involves searching through a very large, unstructured space of solutions (Tong and Sriram 1992) based on vague and uncertain knowledge about possible solution alternatives (Gupta and Xu 2002), their physical behavior (Aughenbaugh and Paredis 2004), their cost (Garvey 1999), and the decision-maker's preferences (Kirkwood and Sarin 1985; Otto and Antonsson 1992; Carnahan, Thurston et al. 1994; Seidenfeld, Schervish et al. 1995). The complexity of the design problem, including the presence of uncertainty, makes it impossible to arrive at a final design in one step.

Consider the following simple design problem with two design variables: vehicle type and engine type. There are two options for vehicle type: car or bike. There are three options for engine type: gasoline engine, diesel engine, or electric motor. If the designer, or decision-maker (hereafter abbreviated as DM), chooses the complete design in one step, he or she would choose from the set of six *design alternatives* shown in Figure 1. In the context of this example, each of these design alternatives is a fully detailed design of a final product. In order to choose the best design, the DM would need to evaluate and compare all six alternatives in detail. While easy in this simple example, the complexity of real engineering problems and the prohibitive cost of considering large numbers of alternatives require a sequential decision process.

SEQUENTIAL DECISION MAKING

It is impractical to enumerate and evaluate all design alternatives by considering all possible combinations of all solution principles for all the subsystems of a complex product. Consequently, the design process is broken down into a sequence of decisions to allow for efficient exploration of the design space. For example, in the previous vehicle design example, a DM can follow a sequential approach in which he or she first chooses the vehicle type, and then the engine type, as shown in Figure 2.

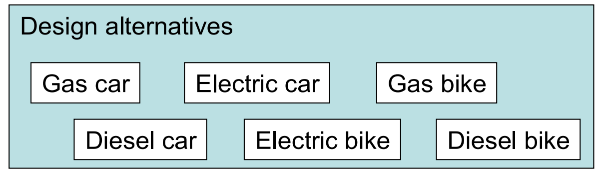

Figure 1: One stage decision

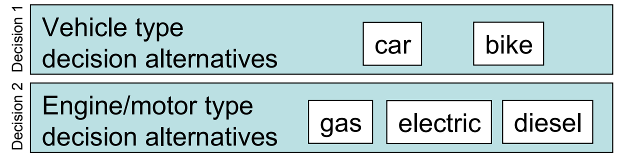

Figure 2: Sequential decisions

Note that it is important here to distinguish clearly between *decision* alternatives and *design* alternatives. A design alternative is one of the possible complete product design specifications, while each decision alternative corresponds to a set of design alternatives. For example, when choosing the vehicle type, the DM has two *decision alternatives*: car or bike. Each of these decision alternatives actually corresponds to a *set* of *design alternatives*, as shown in Figure 3. The decision alternative of vehicle type *car* includes the design alternatives gas car, diesel car, and electric car, because the vehicle type decision will be followed by the engine type decision. Once a decision is made to pursue, for example, a car design rather than a bike, the DM does not need to consider explicitly the design alternatives gas bike, electric bike, and diesel bike; these design alternatives are *eliminated* from consideration with the elimination of the decision alternative that contains them.

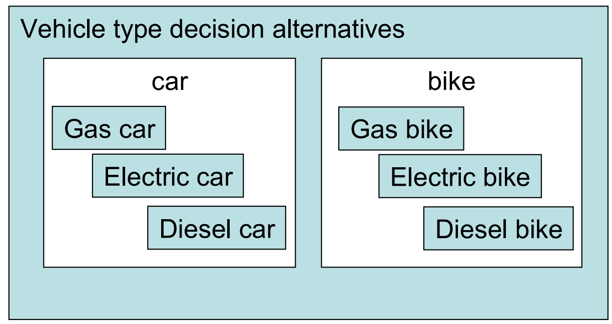

Figure 3: Sets of design alternatives

One limitation of a sequential decision process is that decisions are almost always coupled. In general, a DM really needs to know the selections made in future decisions in order to select the most preferred decision alternative for the current decision. For example, a fully designed gas car will have a certain maximum horsepower, but the set of car designs in Figure 3 has multiple horsepower maxima, each corresponding to a sub-design (gas car, electric car, and diesel car). The realized horsepower depends on the future design decision of engine type. Thus, when selecting type *car* rather than *bike*, a DM is not selecting a precisely characterized horsepower, but rather an imprecise horsepower, such as a set of values. Future decisions are one source of imprecision, a topic that is discussed in detail later in the paper. To lead eventually to the most preferred design, a design method must acknowledge the roles of both imprecision and sets of alternatives in design decisions.

ROBUST DESIGN

Researchers have recognized the limitations of sequential design processes and have proposed modifications in which the uncertainty about future design decisions is considered. For instance, Chen, Allen, and coauthors (1996) have introduced an approach based on robust design that seeks decision alternatives that are robust to future decisions. The idea is that since DMs lack knowledge about the outcomes of future design decisions, they should make their current decision in a way that yields a *satisficing* solution (Simon 1982)—a solution that is in some sense good enough regardless of future design decisions.

Robust design methods trade off optimal performance for consistent performance. This is a reasonable approach if the price one pays for robustness is relatively small—that is, if little performance is sacrificed for robustness. Unfortunately, robust design methods currently do not provide any indication of how large that price is. In this paper, we demonstrate an approach that helps the DM move toward the most preferred solution by actively managing the design space, rather than compromising high performance for robustness. This approach is inspired by set-based concurrent engineering.

SET-BASED CONCURRENT ENGINEERING

The perspective taken in this paper is that ideally the goal of the design process is to systematically eliminate inferior design alternatives from the set under consideration until only the most preferred alternative (or set of equally preferred alternatives) remains. This approach is derived from the paradigm of Set-Based

Concurrent Engineering (SBCE) (Sobek, Ward et al. 1999), a management approach used at Toyota. The guiding principle of SBCE is to begin the design process by selecting a broad set of solutions and gradually narrowing the set by eliminating weaker solutions as more information becomes available until converging on a final solution.

In traditional design practice, the emphasis is on selecting a *single* good design; engineers quickly converge on a single design and then iteratively modify that solution until it meets the design requirements. In SBCE, Toyota encourages its engineers to pursue multiple feasible design alternatives simultaneously. The consideration of multiple designs incurs more costs early in the design process than selecting a single robust design. However, these increased costs can be offset by two factors. First, the resulting design in SBCE is often much closer to optimal (has a much better performance) than the final designs in traditional methods. Second, SBCE avoids costly design tweaking and redesigns late in the development cycle. In some combination, these effects have enabled Toyota to use SBCE quite successfully (Parunak, Ward et al. 1997; Sobek 2004).

As implemented at Toyota, SBCE places a large responsibility on chief engineers to guide the process effectively, relying heavily on their implicit knowledge and expertise. Sobek, Ward, and Liker (1999) provide three broad principles for managing SBCE. One of these principles involves *narrowing sets gradually while increasing detail*, and a formal method has been introduced that uses predicate logic to eliminate infeasible designs (Finch 1997). However, little attention has been given to methods that guide elimination based on preferences—that is, methods that eliminate less-desirable, yet feasible, designs from the set under consideration. If the benefits seen at Toyota are to be generalized to other applications, a formal method of set-based design must be developed.

For a set-based approach to be efficient, the DM must be efficient at eliminating inferior solutions from the set under consideration; a DM should eliminate a solution as soon as he or she is confident that it cannot be the most preferred. If the DM does not eliminate such solutions, then he or she will continue to develop them in more detail, thereby incurring unnecessary costs. Since these elimination decisions must be made without complete knowledge about the solutions, traditional comparisons are inappropriate; different methods are needed. The remainder of this paper introduces the concept of imprecision, discusses elimination decision policies, and demonstrates an elimination-oriented, set-based design approach using the design of a gearbox as an example.

IMPRECISION IN DESIGN

Traditionally, the mathematical formalisms for design have been derived from the theory of decision analysis (Pratt, Raiffa et al. 1995), and decision-based design has recognized decisions as important milestones in the design process (Mistree, Smith et al. 1990; Hazelrigg 1998). In this paper, we focus on decision making while specifically considering the inherent uncertainty that exists in design, but unlike most previous research in decision-based design, we make a clear distinction between two different types of uncertainty: inherent variability and imprecision (Parry 1996; Nikolaidis 2005).

TYPES OF UNCERTAINTY

Variability, also called aleatory uncertainty (from the Latin aleator = dice thrower), is a description of naturally random behavior in a physical process or property (Oberkampf, DeLand et al. 2002). It is also known as objective uncertainty (Ferson and Ginzburg 1996) and irreducible uncertainty (Der Kiureghian 1989). Examples include manufacturing error, errors in communications systems, and radioactive decay. Inherent variability is best represented in stochastic terms, e.g., by a probability density function.

Imprecision, on the other hand, is due to a lack of knowledge or information (Parry 1996) and sometimes is called epistemic uncertainty (from the Greek episteme = knowledge), reducible uncertainty (Der Kiureghian 1989) or subjective uncertainty (Ferson and Ginzburg 1996). Imprecision often results from ignorance or from deliberate modeling decisions, such as including abstractions to reduce the computational complexity of a model. Imprecision is generally best represented in terms of intervals (Kreinovich, Ferson et al. 1999; Muhanna and Mullen 2004). While some authors doubt the philosophical distinction between variability and imprecision, such distinctions are useful in practice (Ferson and Ginzburg 1996; Hofer 1996; Winkler 1996; Aughenbaugh and Paredis 2005).

Almost every aspect of each design decision introduces imprecision. More specifically:

- The lack of knowledge about future design decisions introduce imprecision because each decision alternative defines a set of design alternatives.
- Behavioral simulations are imprecise abstractions of reality.
- Environmental factors are imprecise estimates based on limited measurements.
- Preferences are not fully elicited and therefore imprecise.
- Numerical solving of these models introduces additional imprecision.

Earlier, we introduced the importance of imprecision from future decisions with a simple horsepower characterization example. In a more complex problem, even once the horsepower is defined precisely, one is still referring to the entire set of engines with that specific horsepower. Other performance characteristics of that set of engines (e.g., mass or cost) will be inherently imprecise because they are different for each specific instance in the set.

Future design decisions can vary substantially, depending on which current decision alternative is selected. This impact is not just in the optimal solution of the problem, but also in the very nature of the design problems to be solved. For example, a choice between an electric motor and a diesel engine at one point in the process affects the future decisions, such as energy delivery. If an electric motor is chosen, the choice of volumetric fuel injection rate is meaningless.

Though important and often significant, future decisions are only one source of imprecision in engineering design. Behavioral models predict the performance of design alternatives imprecisely because these models, like all models, are only an abstraction of reality. For instance, an internal combustion engine is often modeled as an algebraic relationship between engine speed and torque, abstracting away the detailed physical phenomena, including airflow, gas-mixture combustion, friction and inertia. There is often also significant imprecision in the parameter values or inputs to these models. In addition, preference models, such as utility functions, may also be known only imprecise (Kirkwood and Sarin 1985; Otto and Antonsson 1992; Carnahan, Thurston et al. 1994; Seidenfeld, Schervish et al. 1995), and numerical methods have limited precision. All of these sources of imprecision should be recognized and incorporated into the design approach.

REPRESENTATIONS OF IMPRECISION

Although models and their parameters are usually only imprecisely known, there is often also variability due to inherent randomness. For example, an air resistance model may include as a parameter the air density, which fluctuates with the weather. To capture both inherent variability and imprecision, the theory of *imprecise probabilities,* as introduced by Peter Walley (1991), allows for intervals of probabilities and is a direct extension of traditional probability theory.

Imprecise probabilities can be interpreted as subjective probabilities—an expression of a DM's beliefs in terms of the DM's willingness to bet. The use of lower and upper probabilities (rather than just a single precise probability as in traditional probability theory) also reflects a DM's confidence in his or her beliefs—the larger the confidence, the smaller the difference between the lower and upper probabilities, i.e., the smaller the imprecision. Walley has axiomatically defined imprecise probabilities and has shown that they are rational in terms of avoiding a sure loss.

Unfortunately, imprecise probabilities as defined by Walley pose significant computational challenges that remain to be resolved. For engineering applications, it is crucial to adopt a mathematical formalism that is convenient and inexpensive for computation and decision making. Ferson and Donald (1998) have developed such a formalism, called probability bounds analysis (PBA) by imposing some additional restrictions on imprecise probabilities. Although PBA is not quite as expressive as imprecise probabilities, it can still represent both variability and imprecision and has been shown to be useful in engineering design (Aughenbaugh, Ling et al. 2005; Aughenbaugh and Paredis 2005). The major advantage is that probability bounds analysis is relatively computationally efficient, and commercial software exists to support it (Ferson 2002).

PBA expresses uncertainty in a structure called a probability-box, or p-box. Essentially, a p-box is an imprecise cumulative distribution function (CDF). Upper and lower CDF curves represent the bounds between which all possible probability distributions might lie. We believe that the p-box is an intuitive and convenient representation of imprecise probabilities. For example, a DM may have strong theoretical evidence that a random variable X is normally distributed but not know the parameters of the distribution precisely. If the DM believes that the mean is in the interval $\mu = [5,10]$ and the variance is in the interval $\sigma^2 = [1,4]$, one can extend the notation of probability $X \sim N(\mu, \sigma^2)$ and write:

$$X \sim N([5,\ 10],\ [1,\ 4]\ ) \tag{1}$$

INDETERMINACY IN DECISION MAKING

In this section, we focus on one important consequence of imprecision, namely, that it results in *indeterminacy*: based on the available information, one cannot determine which decision alternative is most preferred. Instead, one can develop rational arguments that support multiple alternatives as being the most preferred.

In general, there are three possible scenarios of preference between alternatives A and B. Either A is preferred to B, B is preferred to A, or the DM is indifferent between A and B. When utilities are used to reflect preference, these relationships can be determined by the inequality or equality of the expected utilities (von Neumann and Morgenstern 1944). However, when imprecision exists, the expected utilities become intervals (since the probabilities are not uniquely determined, neither is the mathematical expectation), and such comparisons become more complicated.

For example, consider the intervals of expected utility for two alternatives (A and B) shown in Figure 4(a). In this example, the intervals overlap. Since the true expected utility of B can lie anywhere in the given interval, the point labeled b_1 is possible. Similarly, both a_1 and a_2 are possible true values for the expected utility of A. Notice that a_1 is greater than b_1, but a_2 is less than b_1. Consequently, the available evidence is *indeterminate*; the DM cannot determine which alternative is the most preferred, nor can the DM determine that he or she is definitely indifferent. In order to make elimination decisions in the presence of imprecision, different methods are needed.

ELIMINATION DECISIONS WITH IMPRECISE INFORMATION

As demonstrated at the close of the previous section, standard numerical comparisons are insufficient for elimination under imprecision. Instead, a DM must turn to interval methods such as interval dominance, maximality, or E-admissibility.

INTERVAL DOMINANCE

An example of overlapping intervals was shown in Figure 4(a). Obviously, two intervals will not always overlap. In this case, shown in Figure 4(b), it does not matter where in the given interval the true expected utility of A falls—it will always be greater than any value in the interval for expected utility of B. This illustrates a situation referred to as *interval dominance*, (For a brief synopsis, see Zaffalon, Wesnes et al. 2003).

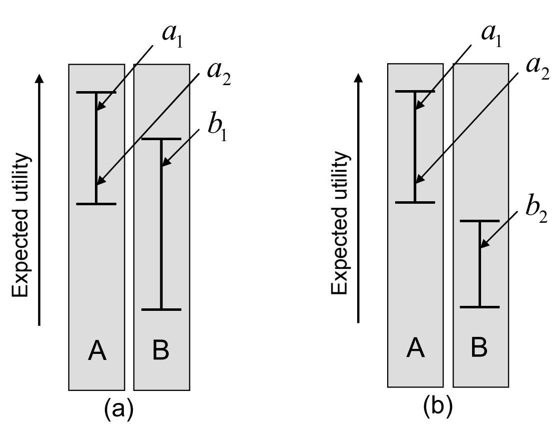

Figure 4: Intervals of expected utility

Interval dominance is obvious when there are only two alternatives, but it is more subtle when there are more alternatives, such as shown in Figure 5. At first glance, it may appear that no elimination is possible because there is significant overlap between intervals. However, comparing all pairs of alternatives, we discover that alternative D is dominated by alternative A, and hence can be eliminated. By using as a reference for comparison the alternative with the maximum lower-bound and then comparing this to the upper-bounds of all other alternatives, the complexity of the calculation is reduced from $O(n^2)$ to $O(n)$ for a problem with n alternatives. The result of applying this criterion is a set of alternatives whose intervals of expected utility all share some region of overlap.

Elimination using interval dominance supports the set-based approach, is consistent with imprecise information, and is relatively easy to compute; however, it may result in a large set of design alternatives. Although this is to be expected, especially during the early phases of design, it is important for the success of this approach that as many designs as possible and justifiable are eliminated as supported by the available knowledge and information; inefficiencies should be avoided. In the next section, we examine the criteria of maximality (Walley 1991) and E-Admissibility (Levi 1974), approaches that account for uncertainty shared across alternatives.

ACCOUNTING FOR SHARED UNCERTAINTY

In design, there are often uncertain conditions that influence the performance of all decision alternatives in a similar fashion, factors that we define as *shared uncertainty*. When uncertainty is shared among possible actions, it means that a particular future condition or event is independent of the current action taken by the DM. For example, ambient temperature is independent of the alternatives chosen—all potential final designs will have to operate over the same, but unknown, range of temperatures. Hence, the

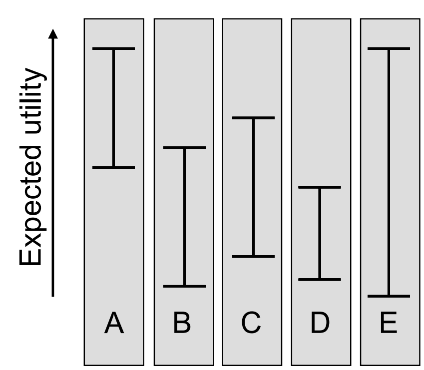

Figure 5: Many overlapping intervals

uncertainty is said to be *shared*. As an example of uncertainty that is not shared, consider the sequential decisions of designing first the engine and then the drive shaft. When designing the engine, the exact design of the drive shaft is unknown. However, this uncertainty is not shared by all engine alternatives, because the final design of the drive shaft will depend on the chosen engine design; the drive shaft must meet difference performances requirements depending on the power of the engine, for example.

Since temperature is a shared uncertainty, the performance of alternatives should be compared assuming they are operating at the same temperature. A similar argument favors paired statistical testing over pooled statistical testing to remove shared systematic errors (Devore 1995). The motivation is illustrated in the following example.

Consider two cars A and B, whose performance depends strongly on the uncertain ambient temperature T, such as shown in the top left of Figure 6. Note that for all values of the uncertain parameter, the utility of A is greater than the utility of B. Clearly then A is the superior alternative. However, if only the intervals of performance were compared without regard to shared uncertainty, such as shown in the top right of Figure 6, this superiority would not be detected.

The concept of shared uncertainty is similar to common random numbers (CRNs) in discrete-event simulations (Law and Kelton 2000). The goal of a simulation is usually to compare two scenarios or alternative designs by examining the difference in output for different combinations of control parameters.

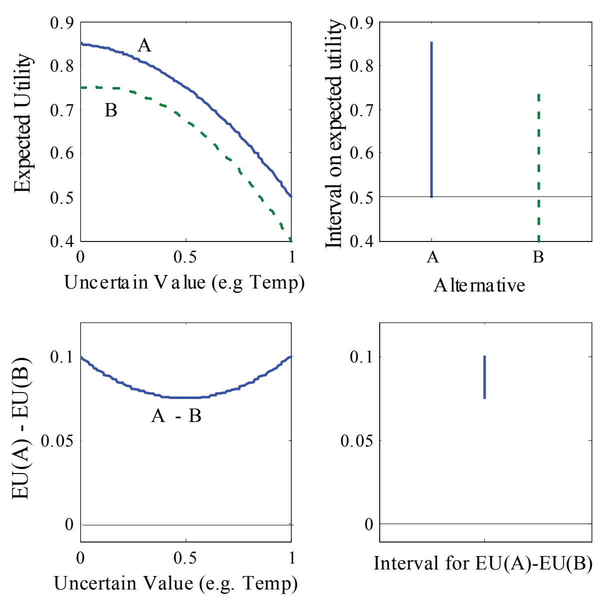

Figure 6: Comparing two alternatives with and without considering shared uncertainty.

If different random numbers are used in the simulations for the different alternatives, additional noise is introduced into the model. CRNs are used to induce correlation between scenarios, thereby reducing the variances of the results.

In engineering design, shared uncertainty is an inherent characteristic of the problem. Therefore, a DM does not have to add the commonality; he or she merely needs to recognize it and to take advantage of that additional property when it exists. One approach that considers shared uncertainty is maximality.

Maximality criterion

Since the intervals in the top right quadrant of Figure 6 overlap significantly, neither A nor B is eliminated according to interval dominance, even though it is clear from the curves in the top left quadrant that B should be eliminated. If the *difference* in performance across the uncertain parameter is considered, the elimination can be made, as shown in the lower left quadrant of Figure 6. Note that for any value of the shared uncertain variable, the difference between alternative A and alternative B is positive. In other words, A is always better than B, and B can be eliminated.

This type of comparison is formalized as the *maximality* (Walley 1991) criterion for elimination. First, we define a set of decision alternatives that are available for consideration, denoted D. We also distinguish between elements of shared uncertainty, $z_s \in Z_s$, and uncertainty that is specific to each alternative, $z_i \in Z_i$ for alternative $A_i \in D$. Recalling that the DM seeks to maximize expected utility, we can write the elimination rule of maximality as follows:

> A decision alternative $A_j \in D$ is dominated according to maximality, and hence the corresponding set of design alternatives can be eliminated, if for some $A_i \in D$, and $i \neq j$:
>
> $$\Big(\max_{\substack{z_s \in Z_s \\ z_i \in Z_i \\ z_j \in Z_j}} [EU(A_j, z_j, z_s) - EU(A_i, z_i, z_s)]\Big) < 0$$

Maximality is a stricter criterion than interval dominance, meaning that in general it leads to the elimination of more alternatives. Maximality eliminates alternatives that are dominated at all values of the uncertain parameter by any individual other alternative. In general, this requires the maximality condition to be checked for all pairs of decision alternatives. For example, consider the five decision alternatives whose expected utility is expressed as a function of a single shared imprecise parameter (fox example, ambient air

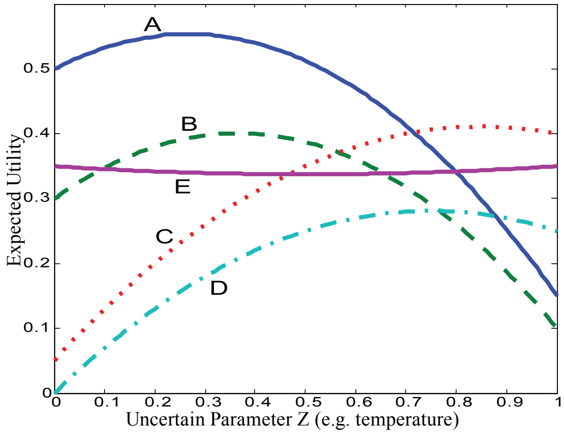

Figure 7: Performance of 5 alternatives influenced by a single uncertain parameter (e.g. temperature).

temperature: T) in Figure 7. If one were to use only A as a *reference design* (meaning comparing the other design to A), then only B could be eliminated because curves C, D, and E are higher than A for some values of z_s , but B is always lower. However, if C is used as the reference design, then D can be eliminated. Clearly to complete the elimination, both A and C must be used as reference designs in this case. In general, a DM must compare all combinations.

The difference in expected utility is often monotonic with respect to the uncertain variables. In this case, the maximum difference occurs at the boundary of the uncertainty region, making it easy to compute for a given pair of alternatives A_i and A_j . If the difference between the two alternatives is not monotonic then a complete optimization is necessary. However, commonly the uncertainty region is expressed in terms of intervals, in which case interval arithmetic can be used to determine the maximum efficiently (Moore 1979; Kearfott and Kreinovich 1996; Hansen and Walster 2004).

<u>E-admissibility criterion.</u>

A stricter criterion than maximality is E-admissibility (Levi 1974). According to E-admissibility, a solution is eliminated if at every value of the uncertain parameter there is at least one alternative with a higher expected utility. This is more easily understood by considering the converse—E-admissibility only accepts alternatives that for at least one value of the uncertain parameter have the greatest expected utility of all of the alternatives. Applying E-admissibility in general requires solving a mathematical programming problem, or at least proving that a feasible solution exists (Kyburg and Pittarelli 1996), making it at least as expensive as applying maximality.

For an example of applying E-admissibility, consider the alternatives in Figure 7 again. Alternatives eliminated using maximality are necessarily eliminated using E-admissibility since if at all temperatures there is a single alternative with higher expected utility, then the dominated alternative can never have the highest expected utility. Consequently, alternatives B and D are eliminated.

Notice that alternative A performs best in low temperatures, C performs best in high temperatures, while E performs consistently throughout the entire temperature range. Nevertheless, E will be eliminated based on the E-admissibility criterion because E is dominated by the set $\{A, C\}$, since either A or C (or both) is greater than E at every temperature; car A dominates E at low temperatures while C dominates E at high temperatures. The potential implications of eliminating the robust solution E are described in the following section.

RESOLVING REMAINING IMPRECISION

Although a DM can maintain a set of designs in the early stages of design, he or she must eventually select a particular alternative to finalize the design. Nevertheless, after applying elimination criteria, multiple alternatives usually will remain due to imprecision. In order to make a decision, a DM has two choices—the DM can collect additional information, thereby reducing imprecision, until only one alternative remains in the non-dominated set, or the DM can select one alternative arbitrarily. Traditional design approaches would require arbitrary elimination of non-dominated alternatives, while a set-based design approach allows a DM to delay elimination of alternatives until additional information is available.

When delaying decisions, the DM should carefully consider the tradeoff between the value of obtaining more information and the cost of doing so by applying information economics (Aughenbaugh, Ling et al. 2005). Although the cost of additional investigation is often worth the improved ability to make a more informed decision, the DM will reach a point at which the cost of gathering additional information outweighs the expected benefits. At that point, the DM should resort to the other option: arbitrary choice.

If a DM is unable to resolve the imprecision before needing to choose a single alternative from the set, he or she may need to make an arbitrary choice—a choice that is not uniquely determined by the DM's preferences, beliefs, and values (Walley 1991). Recall that the presence of indeterminacy implies that the available information does not uniquely identify a most preferred alternative. Consequently, any arbitrary choice from among indeterminate alternatives can be defended as rational.

Arbitrary in this sense does not necessarily imply *without guidance* or *random*. Several policies are possible to guide arbitrary choice, including Γ-maximin (Berger 1985) and the Hurwicz-criterion (Arrow and Hurwicz 1972). A Γ-maximin policy says that given indeterminacy in a maximization problem, a DM should select the alternative with the highest lower-bound. This is a conservative policy in that it seeks to mitigate the worst-case. Robust design strategies that choose solutions that are insensitive to imprecision are also applicable at this stage. If the remaining uncertainty is extreme, it may be valuable to consider an alternative approach such as information gap theory (Ben-Haim 2001).

Returning to the vehicle design example in Figure 7, assume that the DM is unable to resolve the imprecise ambient temperature, yet has to choose between the alternatives (A and C) that remain after applying the E-admissibility criterion. The Γ-maximin policy would choose alternative A, because it has the highest lower-bound over the range of the uncertain parameter. However, notice that while A performs well in low temperatures, it performs quite badly at high temperatures. Notice also that alternative E performs moderately well at all temperatures—that is, alternative E is robust to temperature. However, using the E-admissibility criterion, this alternative was eliminated.

The elimination of alternative E is not a problem if the DM is able to resolve the imprecision before needing to choose a final design. Once the imprecision is eliminated, the DM knows at which temperature the cars are required to perform and can select the car that is most preferred at that temperature—which will always be either car A or C, the two alternatives that remain after applying the E-admissibility criterion. Therefore, if a DM knows that imprecision can be eliminated before making the final decision [an example of such imprecision is that from future decisions], then E-admissibility is an appropriate criterion.

However, information economic considerations will usually lead a DM to stop collecting information before removing all imprecision. In other cases, such elimination of imprecision is impossible. For example, there is in general no one temperature at which a care must operate, but rather a produced car could be subject to the entire range of temperatures during operation. The practical unlikelihood of removing all imprecision leads us to recommend the maximality elimination criterion for most design applications.

THE DESIGN PROBLEM

In this section, we study a gearbox design problem in order to demonstrate the different elimination criteria in

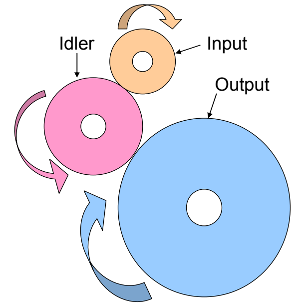

Figure 8: Gearbox configuration schematic

the context of a realistic design problem. The gearbox is intended for use in the drivetrain of an SAE Mini-Baja competition off-road vehicle. The basic configuration of the gearbox is shown in Figure 8. The objective of the design problem is to determine the geometries of the three gears such that the expected utility of the design is maximized.

A summary of our problem formulation is presented in Figure 9. Utility is formulated as the dollar earnings from constructing and using the gearbox in Georgia Tech's Mini-Baja vehicle for a long-distance race. There are five design variables and ten shared uncertain parameters, with uncertainty modeled as p-boxes, intervals, and precise probability distributions.

REPRESENTING AND COMPUTING WITH IMPRECISION

As noted earlier, probability bounds analysis (PBA) is an abstraction of imprecise probabilities in which uncertain information is represented as a p-box. While less expressive than imprecise probabilities, the p-box representation simplifies computation. In particular, PBA approaches allow for the computation of rigorous, "best-possible" bounds on functionally determined distributions (Williamson and Downs 1990; Berleant and Goodman-Strauss 1998). Unfortunately, these approaches are often inappropriate for realistic engineering problems. For example, they cannot be applied directly towards a black box analysis model.

An alternative to these methods is to use an entirely stochastic approach. For precise probabilistic problems, Monte Carlo sampling can be used to approximate the uncertainty in the output. For imprecise probabilistic problems, second-order (2D), also known as two-dimensional or double-loop, Monte Carlo sampling can be used to approximate the imprecise uncertainty in the output (Hoffman and Hammonds 1994). However, for the high dimensional problems typical in engineering design, computational expense could still be prohibitive.

For the gearbox example problem, we use an alternative approach in which we replace the outer loop of a 2D Monte Carlo simulation with an optimization algorithm. The inner loop remains a

Maximize

Expected Utility:

$$EU = E[U_t] * P\{\text{complete}\} - E[U_C]$$

where

- $U_t = (\text{Prize Money}) * \left(1 - \frac{1}{1+e^{16-4t}}\right)$, with the relationship determined by fitting a sigmoid function to past race finish times t.
- $P\{\text{complete}\}$ is the probability that the gearbox completes the race, i.e. the reliability
- U_C =the cost of constructing the gearbox

Select

Gear Ratio $N_g = [0.5, 5]$ (torque ratio)

Input Gear Diameter $d_{in} = [1.5, 15]\,cm$

Idler Gear Diameter $d_{id} = [1.5, 15]\,cm$

Gear Width $w = [1.00, 8.75]\,cm$

Gear Module $M = [1.27, 8.75]\,mm/tooth$

Where

Performance depends on 10 uncertain system parameters shared across all alternatives:

Total Mass (*kg*), M ~ Normal([200,215],[18,20])

External Drag Coefficient (*N/(m/s)2*),

$C_{D,e}$ = [0.27,0.28]

Internal Drag Coefficient (*N/rpm*), $C_{D,i}$ = [0,0.0075]

Course Roughness Coefficient, K_c ~ Normal(3,0.5)

Bending Strength Factor, J = [0.38,0.4]

Gear Quality, Q_v ~ Normal([8.25,8.75],1)

Cost Error (*$*), $Cost_{err}$ = [-5,5]

Uncorrected Bending Strength (*N/m2*),

S'_{fb} ~ Normal([197,203]x10^6, [30,35]x10^6)

Uncorrected Contact Strength (*N/m2*),

S'_{fc} ~ Normal([197,203]x10^6,[30,35]x10^6)

Application Factor, K_a = [1.68,1.70]

Figure 9: Formulation of Mini-Baja Gearbox Problem.

Monte Carlo sampling from parameterized distributions, but instead of determining the parameters of these distributions by an outer loop of Monte Carlo sampling, we use optimization to find the set of distribution parameters that will give us the largest (and the smallest) expected values. The results of these calculations are then used to make the comparisons in the elimination criteria.

Our approach, while computationally more efficient, assumes independence between uncertain variables. While not ideal, we believe that this assumption is reasonable for large classes of engineering models. Another limitation of this approach is the presence of local minima in typical engineering problems. For the gearbox example problem, we were able to overcome this difficulty by using several starting points for the optimization. A more detailed explanation of the approach is available in (Bruns 2006).

DEMONSTRATION OF ELIMINATION CRITERIA

The first part of the example demonstrates the reduction of the design space for the first design variable—the gear ratio. This design problem is slightly different from the examples mentioned earlier because it deals with a continuous variable. For a continuous design variable, it is ranges of values that are eliminated, rather than discrete alternatives. The initial problem statement specifies the design space of gear ratios in the interval [0.5, 5.0]. In the first step of the sequential decision process, the DM seeks to reduce this interval as much as possible while retaining in the range the most preferred—though currently unidentifiable—solution.

We first consider the application of interval dominance by the DM. Figure 10 contains a plot of expected utility versus gear ratio. The two curves represent the upper and lower bounds on expected utility at a given gear ratio. In the plot, the highest point on the lower-bound, or the Γ-maximin solution, occurs at a ratio of about 1.5. The DM draws a horizontal line at the lower expected utility at this gear ratio. By the condition of interval dominance, any gear ratio with an upper-bound on expected utility that is below this line should be eliminated. For example, two expected utility intervals are indicated in Figure 10. The leftmost interval is located at the Γ-maximin solution. The DM compares all other decision alternatives to this interval. The Γ-maximin solution clearly dominates any of the expected utility intervals in the shaded regions. Therefore, the DM can eliminate gear ratios in both shaded regions from the design space.

By taking into account that the uncertain parameters described in Figure 9 are shared between different design possibilities, we can make further eliminations in the design space using the maximality criterion. In theory, the DM would need to make pairwise comparisons between all alternatives to eliminate all that are dominated under maximality. Of course, this is impossible for design problems with continuous design variables. In practice, a DM should therefore perform comparisons between a well-chosen discrete set of design alternatives across the entire design space.

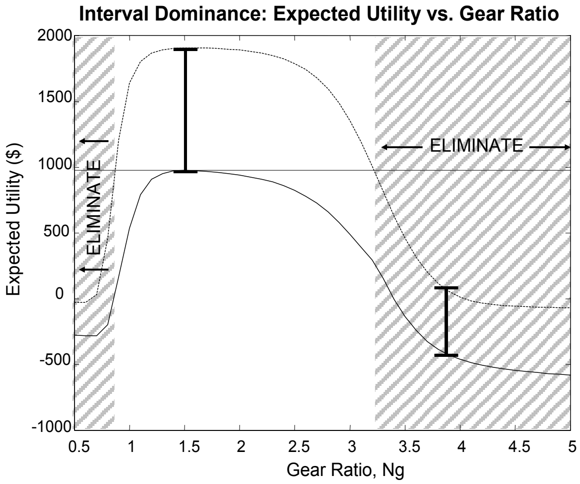

Figure 10: Elimination using interval dominance

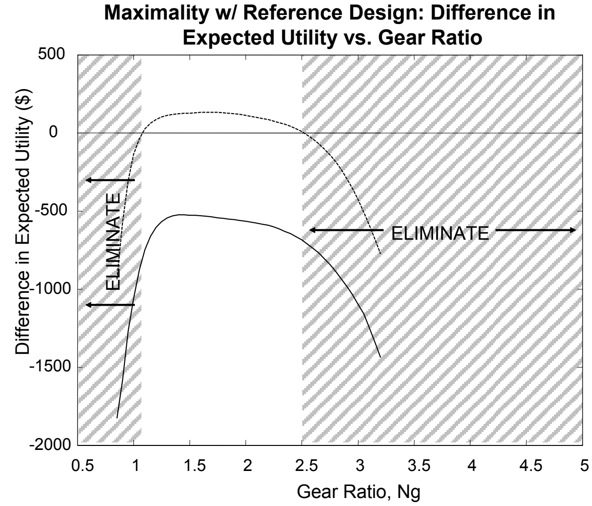

Figure 11: Eliminating using maximality

In this example, the DM computes the bounds on the expected difference in utility between each gear ratio and the Γ-maximin gear ratio of 1.5. Recall that the maximality elimination criterion specifies that the DM should eliminate any alternative (in this case, gear ratio) with an upper bound on expected difference less than zero. Figure 11 contains a demonstration of maximality elimination over a continuous variable. The DM draws a horizontal line at an expected difference in utility of zero. The shaded regions correspond to gear ratios that are always dominated by designs with the reference gear ratio of 1.5. Therefore, the DM can eliminate all decision alternatives that fall in the shaded regions in Figure 11. The two curves represent upper and lower expected *differences in utility*.

The calculation of a difference in expected utility requires two alternatives—the one being tested, and a reference. In order to increase the efficiency of elimination, we choose a detailed reference design (Rekuc 2005). The idea is to develop one promising alternative to a greater level of detail than the others, thereby reducing the imprecision from future decisions for that alternative. The narrower intervals of utility for this design will often enable more elimination. In this paper, the Γ-maximin solution is used as the reference design. Specifically, comparisons are made to the reference design of $N_g = 2.1$, $d_{in} = 1.5cm$, $d_{id} = 1.5cm$, $w = 1.25cm$, and $M = 6.35mm$. The mechanics of the resulting elimination decisions are the same as those described earlier in this paper, with all comparisons made to this reference design.

SEQUENTIAL REDUCTION OF THE DESIGN SPACE

We conclude our examination of the example problem with a sequential decision making process, sketched in Figure 12, to reduce the set of feasible designs. A single step in this process was described in the last section, in which we reduced the design interval for the gear ratio. Now we proceed to reduce the set of non-dominated design alternatives sequentially through each of the remaining four design variables.

The advantage of sequential elimination is that with each reduction in the uncertainty associated with a single design variable, the uncertainty in expected utility is reduced. This, in turn, allows the DM to identify more dominated decision alternatives in the subsequent decisions, and so on.

In step 1, the DM reduces the interval for the gear ratio based upon the initial design uncertainty for the other four design variables. In step 2, the DM reduces the uncertainty for input gear diameter based upon the reduced uncertainty for the gear ratio and the initial uncertainty for the other three design variables. The DM repeats this process sequentially until the design spaces for all design variables have been reduced via elimination. The process could then be repeated for further reductions. The right column contains the

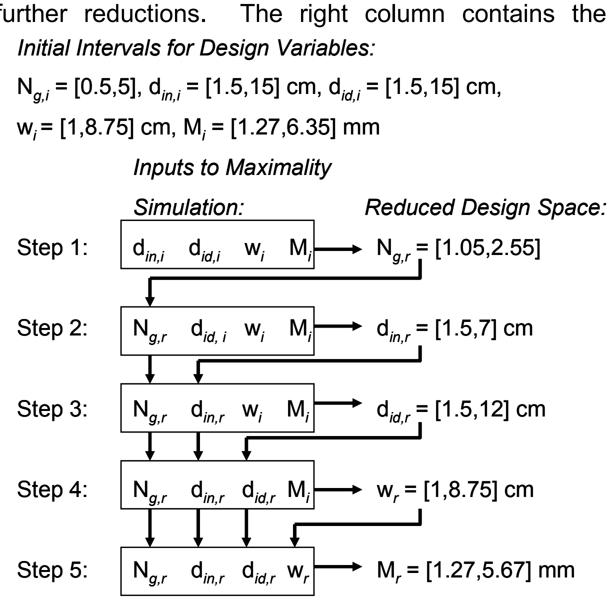

Figure 12: Sequential reduction process.

intervals representative of the reduced design space for one iteration. We address additional aspects of this process in the discussion and future work section.

SUMMARY OF EXAMPLE

This section has demonstrated the process of making eliminating design alternatives in the context of gearbox design for an SAE Mini-Baja competition off-road vehicle. The problem is relatively rich in that it contains five decision variables and ten uncertain parameters that illustrate a range of possible uncertainty characterizations. The goal of the example was to introduce the rationale of the methods and illustrate how such decisions can be made in a practical example. In the next section, we discuss the results and present directions for future work.

DISCUSSION AND FUTURE WORK

This paper has motivated and demonstrated a sequential, set-based design approach in which the decision maker (DM) explicitly considers imprecision. The DM incurs costs in additional computation time as well as the expenditure of additional resources for developing and evaluating sets of design alternative in exchange for the benefit of converging on the most preferred design alternative and avoiding costly redesign. Before adopting these methods, a DM should answer whether these benefits outweigh the costs. This question requires further research, and the answer to the question will likely depend on the development of efficient means for managing and organizing the sequence of decisions.

Specifically, it would be valuable to develop a formal set-based design model that goes beyond the general management principles of SBCE (Sobek, Ward et al. 1999), feasibility-based elimination (Finch 1997), and the elimination methods presented in this paper. Such a model would define the partitioning of the design problem into sets, the sequence of decisions, and definitive rules for elimination and arbitrary selection. For example, the order in which the design variables are explored in Figure 12 can significantly affect the results.

It was noted earlier that one factor in choosing between the maximality and E-admissibility criteria was whether or not imprecision can be eliminated before a final decision is made. In practice, it costs resources to eliminate imprecision, and resources are always limited. At some point, the cost of additional information collection will exceed the expected benefit in increased performance of the design solution. A DM must choose when to stop expending resources to eliminate imprecision and to select a final design solution arbitrarily, taking guidance from information economics in engineering design (Aughenbaugh, Ling et al. 2005). However, the issue remains as to how to make this final arbitrary decision in a manner that works effectively in different classes of engineering problems.

There is also substantial room for improvement in the computational method for propagating imprecise probabilities through our model. The current method models all uncertain variables as independent. This may be sufficient for certain problems, but how would the computations change for dependent uncertain variables? In addition, with the current method, it is uncertain how close our computed upper and lower expected utilities are to the actual expected utilities or to the so-called rigorous, “best-possible” bounds for unknown dependence between the uncertain inputs. Finally, more efficient computational methods would be necessary for uncertainty propagation in complex, computationally expensive models. In order to solve all of these problems, it seems likely that a fundamentally different computational approach will need to be developed.

SUMMARY

In this paper, we discussed how the presence of sequential decision making necessitates a set-based approach to engineering design. Due to imprecision introduced from behavioral models, uncertain parameters, preferences, and future design decisions, a decision maker’s emphasis should be on eliminating undesirable design alternatives, rather than directly selecting the most preferred alternative. We introduced and discussed several decision criteria, including recommending one for use in engineering design. As an example, we have presented a gearbox design problem involving multiple design variables and sources of uncertainty. Using this method, we have demonstrated how interval methods can guide the elimination of alternatives in set-based design. Finally, we have identified the priorities for future work.

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