# THE EFFECT OF DECK RIGIDITY
# IN AN OPEN-SPANDREL ARCH STRUCTURE

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James Thomas Haney

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THE EFFECT OF DECK RIGIDITY

IN AN OPEN-SPANDREL ARCH STRUCTURE

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# SUMMARY


Open-spandrel arches are highly complex structures consisting of a curved rib and a flat deck interconnected by vertical columns. The purpose of this study is to illustrate the interaction between the rib and deck of such a structure and to define a point at which this interaction can be safely ignored.

This study is limited to a structure with hinged columns and a structure in which the deck joins the rib at midspan. Only a vertical loading on the deck is investigated. The variables are: the number of columns, 2, 4, and 6; the rise-span ratio, 1/3, 1/6, 1/9, and 1/12; and the ratio of the moment of inertia of the deck, $I_d$, to the moment of inertia of the rib at midspan, $I_c$, 0.5, 1, 2, 4, and 8.

The goal of the analysis is to obtain influence ordinates over the entire span for all redundants. A structure is analyzed by the virtual work or dummy load method through numerical integration. First, the structure is made determinate and allowed to deform under load. The redundants are then replaced with sufficient magnitude to restore all deformations at points of redundancy. The deformations of the structure at a point of redundancy are evaluated by numerical integration. A computer program is developed to perform all calculations; the print-out is influence ordinates which are included in the appendix. This program is written in a compiler language, ALGOL, and used on a Burroughs 220 digital computer.

The interaction between the rib and deck is evaluated by comparing the influence areas, both positive and negative; the comparison being a redundant in an open-spandrel arch compared to an analogous redundant in a rib with no deck or a continuous beam on rigid supports.

Though all variables have an effect on the influence areas, the ratio $I_d/I_c$ has, by far, the greatest effect. And too, though stresses in the rib redundants are affected by interaction, the stresses induced in the deck redundants are far more critical. In general, all structures investigated in this study should be designed with an allowance for interaction between the rib and deck. However, if a maximum error of 10 per cent is acceptable, the effect of interaction may be ignored in the rib when $I_d/I_c$ is less than 0.3 and may be ignored in the deck when $I_d/I_c$ is less than approximately 0.05.

Because of the large stresses induced in the deck redundants from interaction between the rib and deck, further study should be made with $I_d/I_c$ ranging from 0 to 1.0. The computer program developed for this thesis is valid for such a study. A study should be conducted with more than six columns. When this computer program is used for a structure with more than six columns, all calculations must be made with 10 to 12 significant figures if valid results are to be obtained.

# CHAPTER I

# INTRODUCTION

## The Problem

Open-spandrel arches are highly complex structures consisting of a curved rib and a flat deck interconnected by vertical columns. The complexity of the structure stems from the curved bottom chord, as well as from the high degree of redundancy.

Simplifying assumptions have been used to bypass the complexity of analysis, the most common ones being that the columns were hinged and that the deck and rib were independent structures. The assumption that the columns are hinged is valid provided the construction permits the columns to act as hinged columns. However, assuming the deck and rib to act independently is, for many structures, invalid. Tests on models and prototypes have shown that this assumption yields large differences between analytical and actual stresses, particularly in regard to stresses in the deck.

The problem, then, is: when must deck and rib interaction be considered?

## History

"As has long been realized, the deck aids the arch rib. How harmful this assisting role may be to the deck is undetermined." This statement by A. H. Finlay[1] in 1932 epitomized quite well the lack of knowledge in regards to interaction between the rib and deck of an open-spandrel arch.

Prior to 1932, both full-sized and model tests had shed some light on the extent of interaction. An existing bridge over the Yadkin River in North Carolina indicated interaction to exist at some points[2]. Extensive model tests at the University of Illinois proved considerable interaction to exist between rib and deck on the structure tested[3]. Emphasis was, however, placed on the rib and columns with only a token investigation of the deck redundants.

Later bridges reflected this problem of interaction in their design. The Coos Bay and Yaquina Bay Bridges were constructed with six of the ten spandrel columns hinged[4]. The Colorado Freeway Arches spanning the Arroya Seco in California were constructed with all columns hinged at the deck and short columns hinged at the rib[5]. The deck of the arch bridge over the Grand Coulee Dam spillway consisted of a series of simple beams[6].

The difficulty encountered by earlier designers in designing an arch bridge with complete interaction between the rib and the deck was the tremendous amount of calculations involved. Simplifying assumptions circumvented this difficulty but the error in such assumptions was uncertain. Much research had been conducted on open-spandrel arches but only for a very limited range of shapes.

In 1953, A. F. Diwan[7] published an analytical solution for the model used in the University of Illinois test. This complete solution clearly revealed the error that may result when rib and deck interaction is ignored. The method of analysis represented an achievement in that the calculations required were less than in previous analytical methods. Each section was broken into a panel, allowed to deform, and then recombined with consistant deformations at the joints.

Because of the complexity of an open-spandrel arch, analysis to check a trial structure rather than a direct design is the method used in choosing a design. And, because of the complexity of an analysis, even by the method developed by Diwan, numerous trial sections for the best structure are prohibitive. With all of the research and even with the many structures built, one important fact is still missing: the effect of a change in geometry on the interaction between the rib and deck of an open-spandrel arch.

## Purpose

This study is an ivestigation of the interaction between the rib and deck of an open-spandrel arch. Since the extent of interaction will vary with the relative stiffness of the two members, the number of columns, the shape of the rib, etc., a large number of variables must be investigated. The variables will consist of rise-span ratio, number of columns, and the moment of inertia of the deck. The moment of inertia of the rib will be held constant. The study will be limited to structures in which the rib and deck intersect at midspan and to structures with hinged columns. Only one type of loading will be investigated, a vertical load applied to the deck.

The immediate goal is to obtain influence ordinates for all redundants; the final goal is to define a ratio of deck moment of inertia to rib moment of inertia, $I_d/I_c$, at which the interaction between the deck and rib can be ignored. If an analysis based on the rib and deck acting independently results in an error of less than 10 per cent for any force or moment, then interaction between the rib and deck is considered to be negligible. A Burroughs 220 digital computer will be used for performing the calculations.

## Review of Literature

An extensive search of professional literature has not revealed any research which would indicate when the interaction between the rib and deck of an open-spandrel arch may be ignored. Prior to 1953, many articles had been published which outlined a general relationship between the rib and deck. All of these articles pointed to the lack of a suitable method of incorporating interaction in the design. In 1934, an extensive model study was conducted at the University of Illinois on an open-spandrel arch. However, this study was for one structure and gave little information regarding the moments induced in the deck from interaction.

In 1953, the first published solution of an open-spandrel arch, considering complete interaction among all the components of the structure, clearly revealed the magnitude of forces and moments induced in the deck.

This analysis by Beaufoy[8] was an analytical solution for the structure tested at the University of Illinois. Analysis was accomplished by separating the structure into a series of panels, allowing these panels to deform under load, and then recombining the panels with compatible deformations at all panel joints.

In 1958, Diwan used the same structure and method of analysis as used by Beaufoy, but extended the solution to account for all forces, including temperature.

The analysis by Beaufoy illustrated a method of analyzing an open-spandrel arch with fixed columns. This illustration, like all previous work on arches, gave no indication as to how forces from rib and deck interaction varied when the geometry of the structure changed.

# CHAPTER II

# PROCEDURE

## Variables

### Modulus of Elasticity

The modulus of elasticity is assumed to be constant throughout the structure. If E of the rib does not equal E of the deck, the parameter $(EI)_d/(EI)_c$ should be substituted for the parameter $I_d/I_c$ used in this investigation.

### Rib

The rib centerline is assumed to be a parabola of the second degree. The moment of inertia varies along this centerline by the relationship

$$I_x = \frac{I_c}{\left[1 - m\left(\frac{x}{a}\right)^n\right]\cos\theta}$$

The value of the constants "m" and "n" was taken to be 0.7 and 2.0 as recommended for design by Chalos[9]. The expression for the rib moment of inertia is then

$$I_x = \frac{I_c}{\left[1 - 0.7\left(\frac{x}{a}\right)^2\right]\cos\theta}$$

The moment of inertia of the center portion of the structure, where the rib and deck join, is assumed to be the sum of the individual moments

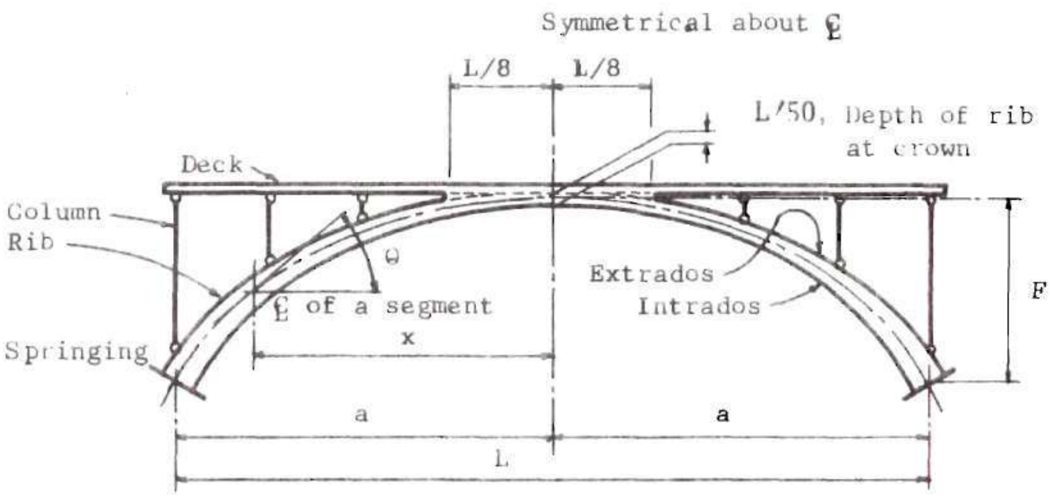


Figure 1.1 Arch dimensions,
6-Column Structure.

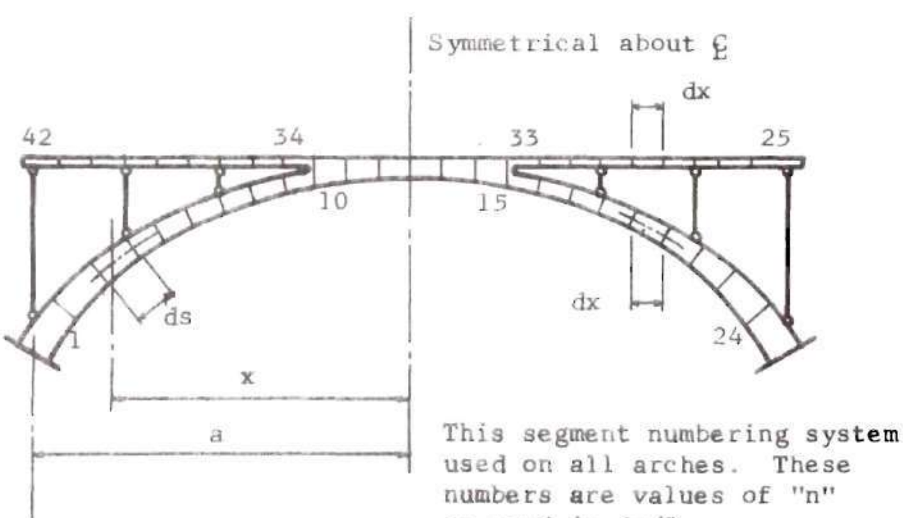


Figure 1.2 Arch Segments

of inertia of the rib and deck. The actual moment of inertia will, however, depend on the construction detail at this center section. The assumed moment of inertia should suffice as an adequate assumption.

The length of the composite center section, common to both rib and deck, was obtained as follows: with a rib thickness of L/50 at the crown, this length is the distance from the arch centerline to a point of intersection formed by the rib extrados and a line projected horizontally from the rib centerline at midspan. The average length of this distance is L/8, which gives a total length of L/4 for this section common to both rib and deck.

Columns

All columns are treated as hinged members, both top and bottom. Though fixed-end columns are a possibility, they are considered to be beyond the scope of this study.

Geometry

The geometry of the structure is varied by a change in the number of columns, the rise-span ratio, and $I_d/I_c$. The range of the variables considered is as follows: a structure with 2, 4, and 6 columns; a rise-span ratio of 1/3, 1/6, 1/9, and 1/12; and $I_d/I_c$ values of 0.5, 1.0, 2.0, 4.0, and 8.0.

Base Structure

The base structure with the least interaction among the redundants will, with a given number of significant figures, give the more accurate answer. This least interaction factor, then, is the main consideration in the choice of a base structure. The second factor is that the base structure should offer a method of solution which is readily programmed for a digital computer.

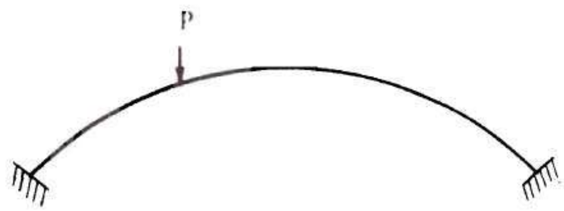


Figure 1.3 Fixed-End Arch with an External Load.

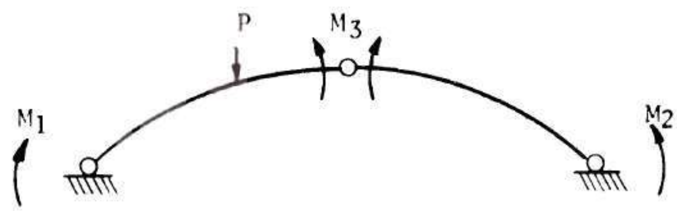


Figure 1.4 Base Structure A.

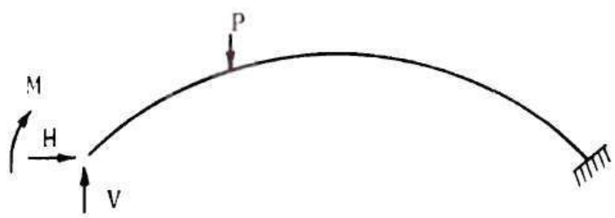


Figure 1.5 Base Structure B.

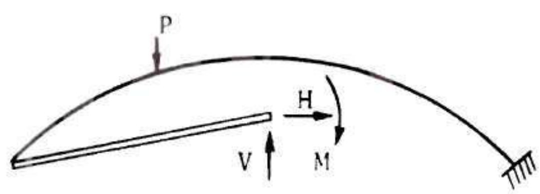


Figure 1.6 Base Structure C.

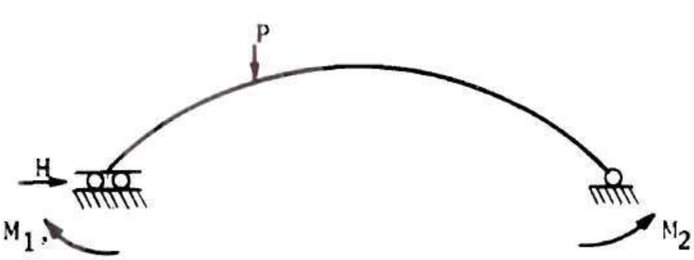


Figure 1.7 Base Structure D.

In a continuous beam, moments at the supports as redundants give the least interaction. Thus, moments in the deck will be chosen as redundants. Unfortunately, the choice of redundants for an arch is not as obvious.

Four base structures for the arch of Fig. 1.3 are shown in Figs. 1.4 - 1.7. The structure of Fig. 1.6, with redundants acting at the elastic center, is a modification of the structure of Fig. 1.5. Basically, the choice lies between the structures of Figs. 1.4, 1.5, and 1.7. The structure of Fig. 1.7 is considered to have the least interaction among the redundants. Fortunately, this structure is the easiest to work with in that the moment diagrams for all redundants, except H, are linear.

The base structure of Fig. 1.7 is chosen because: the interaction among the redundants is considered to be a minimum, and the redundants can be readily obtained by the virtual work method with the aid of a computer.

## Analysis

### Method

The goal of the analysis is to obtain influence ordinates for all redundants.

The solution of the arch shown in Fig. 1.7 is obtained by the virtual work method using numerical integration. The equations for determining the redundants due to the given loading are:

$$M_1 d_{11} + M_2 d_{12} + H d_{13} = P d_{1p} \quad (1)$$

$$M_1 d_{21} + M_2 d_{22} + H d_{23} = P d_{2p} \quad (2)$$

$$M_1 d_{31} + M_2 d_{32} + H d_{33} = P d_{3p} \quad (3)$$

Equation 1 states that the movement at point "1" due to the redundants $M_1$, $M_2$, and H is equal to the movement of point "1" due to the load P. In other words, the base structure is allowed to deform under the applied load and values for the redundants are chosen which will restore this deformation. When a deck is added to the rib, the method or analysis is identical, but the additional redundants in the deck are added.

The general expression for a $d_{ij}$, including only flexural and axial strain energy is:

$$d_{ij} = \sum_{n=1}^{n=42} \left[ \frac{M\bar{M}ds}{EI} + \frac{N\bar{N}ds}{AE} \right] = d_{ij(f)} + d_{ij(a)}$$

The summation of n between 1 and 42 includes 24 segments of the arch and 18 segments of the deck.

Flexural Strain Energy

Since numerical integration is used, the incremental value of a $d_{ij(f)}$ must be obtained at the centerline of each segment; these incremental values must then be summed for the total value of a $d_{ij(f)}$.

The value of $d'_{21(f)}$ at n equal 9 is illustrated. From Fig. 1.8 and 1.9

$$M = 17L/48; \; \bar{M} = 31L/48; \; E = \text{unity}$$

and

$$\frac{ds}{I} = \left[ \frac{1}{24} - \frac{0.7(25 - 2n)^2}{13824} \right] L^1$$

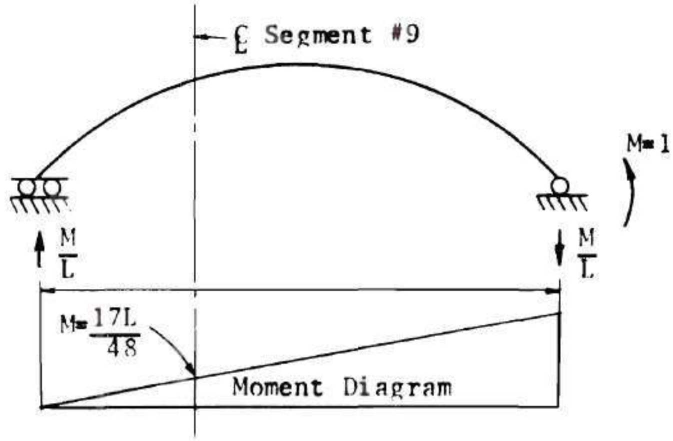


Figure 1.8 Moment Diagram for M=1.

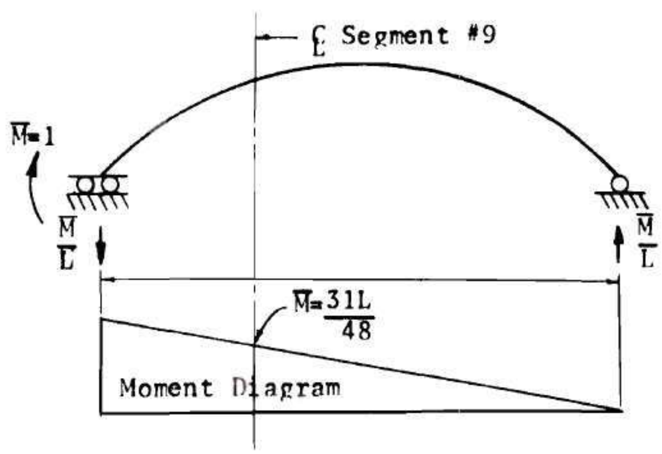


Figure 1.9 Moment Diagram for $\overline{M}$=1.

at n = 9:

$$d'_{21(f)} = \frac{(17)(31)L^3}{48^2}\left[\frac{1}{24} - \frac{0.7(25 - 18)^2}{13824}\right]$$

This procedure is repeated for n varying from 1 through 24 to obtain $d_{21(f)}$. A similar procedure is required for the evaluation of all other $d_{ij(f)}$'s.

Axial Strain Energy

From Fig. 1.11, axial force in the rib from a vertical load or from a moment in the rib is a function of sin θ; from Fig. 1.10, axial force in the rib from a horizontal thrust is a function of cos θ. Axial strain energy then is proportional to $\sin^2\theta$ from a moment redundant or external loading; $\cos^2\theta$ from a thrust redundant; and $(\sin\theta)(\cos\theta)$ from a thrust redundant in combination with a moment or external load. With the rise-span ratios covered in this study, $\sin^2\theta$ and $(\sin\theta)(\cos\theta)$ are negligible when compared to $\cos^2\theta$. For this reason, axial strain energy from thrust only is included in the analysis.

An incremental value of a $d_{ij(a)}$ is illustrated.

From Fig. 1.10, at n = 9:

$$d'_{33(a)} = \frac{(\cos^2\theta)ds}{AE};\ E = 1;\ \frac{ds}{A} \approx \frac{dx}{Ac} = \frac{50}{24b}^2$$

$$d_{33(a)} = \frac{50}{24b}\sum_{n=1}^{n=24} \cos^2\theta$$

1. See page 62 for derivation of this equation.

2. See page 64 for derivation of this equation.

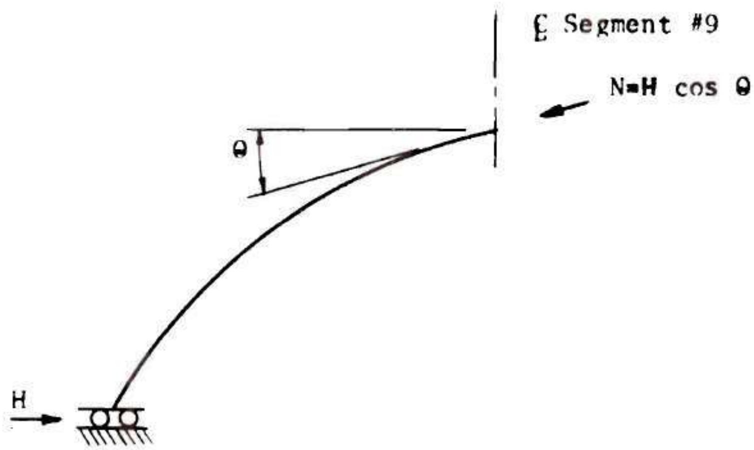


Figure 1.10 Normal Force Caused by a Horizontal Reaction.

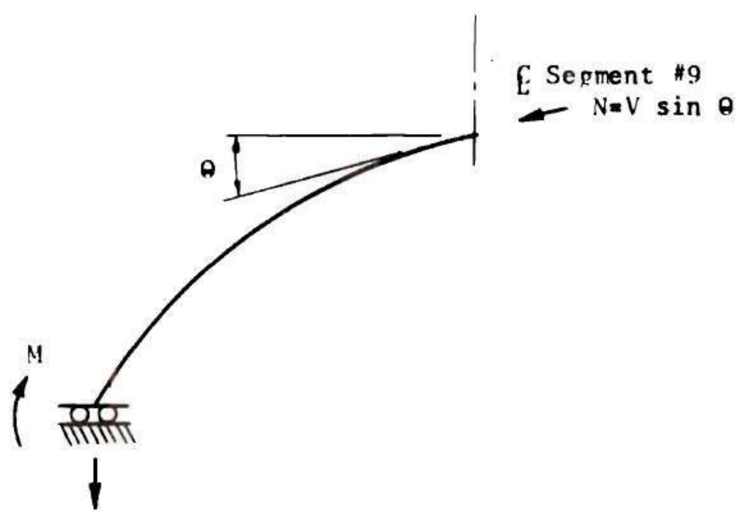


Figure 1.11 Normal Force Caused by a Moment Reaction.

Since a normal force does not exist in the deck, the summation in the above equation also applies from 1 to 24 when the deck is added.

Shear Strain Energy

All deformations associated with shearing forces are ignored in this study.

Influence Areas

Influence ordinates for each redundant are obtained for point loadings on the deck. A comparison of the areas under influence lines, without regard to the location of these areas, is the basis for illustrating the effect on redundants of a change in geometry. Both positive and negative moments are included.

Computer

The influence ordinates are obtained with the aid of a digital computer. The computer program is written in a compiler language, ALGOL, for use on a Burroughs 220 computer.

# CHAPTER III

# DISCUSSION

## Accuracy

In the chapter on Procedure, the statement was made that "The base structure with the least interaction among the redundants will, with a given number of significant figures, give the more accurate answer." The choice of redundants was based on this principle, and to increase the accuracy of the analysis, computer data to a power of $10^{-8}$ were used in all calculations. Influence ordinates were rounded off to a power of $10^{-5}$.

The accuracy of the analysis can be checked by comparing certain influence ordinates. Since the structure is symmetrical about the center-line, a load at the centerline should give identical influence ordinates for symmetrical moments. Agreement between the ordinates of symmetrical moments is excellent in the structures with two and four columns. However, the lack of agreement is noticeable in the structure with six columns.

The maximum difference between the ordinates of symmetrical moments, expressed as a per cent of the average ordinate of these moments, is 0.1, 0.3, and 3.0 per cent for the two-, four-, and six-column structure respectively. This error is commonly called round off error and exist in the solution of any system of simultaneous equations. With a given number of significant figures, the extent of the error is governed by the size of the system and by the relative size of the coefficients. The 3.0 per cent error in the six-column structure could be reduced by using data to a power higher than $10^{-8}$. The number of redundants and the interplay among redundants in the base structure of an eight-column structure would probably necessitate data to a power of $10^{-12}$ or higher for the influence ordinates to be valid. The only way to determine the number of significant figures is by an analysis.

## Influence Lines

Influence ordinates for all redundants are included in the appendix. The influence ordinates for all rib redundants are true influence ordinates in that the ordinate at any point is a function of the moment or force with a load at the point. The influence ordinates for deck moments in the two- and four-column structures are also true influence ordinates. However, the influence ordinates for deck moments in the six-column structure are the result of rib movement alone since ordinates were obtained only at the column centerlines.

Figs. 1.12 and 1.13 illustrate the difference between true influence lines and influence lines for the deck which are the result of rib movement alone. This difference exists only with an influence ordinate which is on the same side of the structure centerline as the redundant moment. When the ratio $I_d/I_c$ is small, an influence line should resemble the influence line of a continuous beam with one end hinged and the other end fixed. When the ratio $I_d/I_c$ is large, this resemblance will disappear. Fig. 1.12 illustrates this behavior. Fig. 1.13 illustrates an influence line resulting from rib movement alone and consequently does not have this resemblance. Figs. 17, 18, and 19 show the load points used in the analysis.

The greatest difference between these influence ordinates and true ordinates occurs at low values of $I_d/I_c$; the difference is negligible at $I_d/I_c$ of eight and less than ten per cent at $I_d/I_c$ of two.

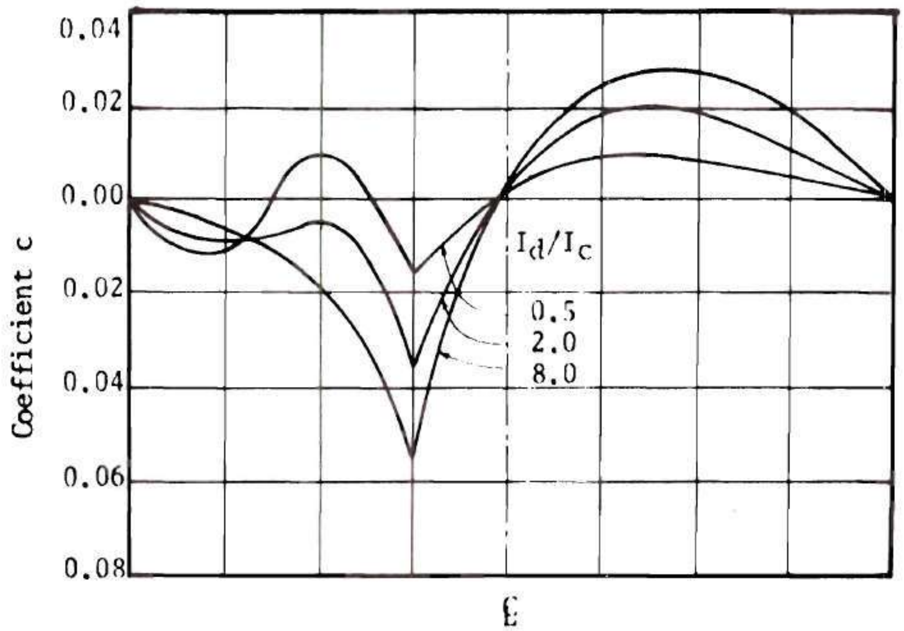


Figure 1.12 Influence Lines for Deck Moment, $X_1$, 4-Column Structure, L/F=6. M=cPL

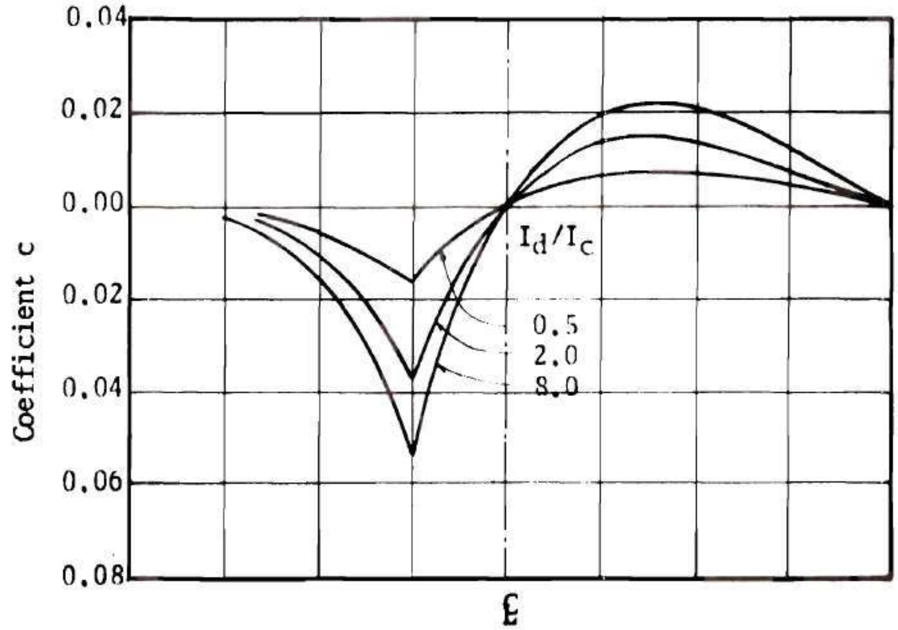


Figure 1.13 Influence Lines for Deck Moment, $X_4$, 6-Column Structure, L/F=6. M=cPL

Influence Areas

Influence areas for all redundants are shown in Figs. 2 to 16. These areas were obtained from influence lines plotted on 8 1/2" x 11" graph paper. All areas are scaled values and represent the total positive or negative area under an influence line.

For the two-column structure, the curves which describe the variation of influence ordinates are plotted for a variation of $I_d/I_c$ ranging from 0.5 to 8.0. No plot for $I_d/I_c$ between 0.0 and 0.5 has been included. For the four- and six-column structures, the curves which describe the variation of influence areas are plotted for values of $I_d/I_c$ ranging from zero to eight. At $I_d/I_c$ of zero, the influence areas of rib moments and forces are those of a rib alone: the influence areas of deck moments are those of a continuous beam on non-yielding supports with one end hinged and the other end fixed. Since influence ordinates were obtained only at the column centerlines in the six-column structure, the influence lines for deck moments represent induced moments from rib movement alone. With $I_d/I_c$ of 4.0, or more, the influence lines are essentially true lines. However, at $I_d/I_c$ values of 0.5 and 1.0, the influence lines are not true lines. For this reason, the plots of $I_d/I_c$ of 0.5 and 1.0 are ignored in the curves which describe the variation of influence areas for deck moments in the six-column structure. The curves are formed by a smooth curve joining the points at $I_d/I_c$ of 0, 2, 4, and 8. The influence lines of rib redundants in the six-column structure are essentially true lines. All influence lines for the four-column structure are true lines since influence ordinates were obtained between columns.

## General Behavior

Regardless of the complexity of a structure, certain general relationships and behavior patterns can be described. The complexity shows up when an attempt is made to define a force or behavior as either critical or negligible; and if critical, how to evaluate the force or behavior.

The open-spandrel arches in this study are in this category—a general behavior can be defined, but because of the many variables, often with opposite influences on forces and moments in the structure, complexity abounds. For example, to say that ignoring interaction between the deck and rib will cause an error of 10 per cent in the springing moment of the rib, the number of columns, the sign of the moment being considered, the rise-span ratio and the value of $I_d/I_c$ must all be defined. If any one of these four variables is changed, the per cent of error also changes.

Figs. 2 through 4 illustrate the variation in influence areas of the springing moments for changes in variables. Influence areas decrease when the ratio of $I_d/I_c$ increases — the deck simply carries more of the load. The other variables have an effect on the extent of this decrease. The number of columns has only a slight effect on the reduction of negative area but a major effect on the reduction of positive area. For example, at $I_d/I_c$ of 8 and L/F of 12, the reduction of the positive coefficient, "c", in the two-column structure is seven times the reduction found in a six-column structure. An identical comparison of negative areas shows the reduction in the two-column structure to be 0.84 times the reduction found in the six-column structure.

The rise-span ratio influences the springing moment even with a deck stiffness of zero. The extent of this influence is illustrated at the origin of Figs. 3 and 4. Had axial strain energy been ignored, all curves would coincide at the origin. The curves would not, however, be coincident throughout the graph. A decrease in the rise-span ratio imposes beam action on the structure. This condition adds to the reduction in negative influence areas with increased $I_d/I_c$ ratios; it subtracts from the reduction in positive influence areas with increased $I_d/I_c$ ratios. This effect of F/L is vividly illustrated in Figs. 2 through 4.

Figs. 5 through 7 illustrate the variation in influence areas of the crown moments for changes in variables. Negative influence areas increase with an increase in $I_d/I_c$. This is logical since the crown moment is really a deck moment. However, positive influence areas increase or decrease, depending on the rise-span ratio. At low rise-span ratios, beam action predominates; at high rise-span ratios arch action predominates. The break point between these two behavior patterns is roughly at F/L of 1/9. The number of columns has no appreciable effect on the variation of negative influence areas and only a slight effect on the positive influence areas.

Figs. 8 through 10 illustrate the variation in thrust for changes in variables. Thrust is essentially directly proportional to the rise-span ratio. The only exception is the structure with a rise-span ratio of 1/12. At $I_d/I_c$ of 2.0, the ordinate of this curve is only 90 per cent of the ordinate at $I_d/I_c$ of 0.0.

Figs. 11 through 16 illustrate the variation in influence areas of the deck moments for changes in variables. The influence areas of deck moments in the two-column structure must be examined apart from the other structures. Fig. 11 illustrates the effect of rise-span ratio on the deck moment. The positive curve at F/L of 1/3 is the only curve in this entire study which neither increases nor decreases with increasing values of $I_d/I_c$. As with the influence areas describing rib moments in the two-column structure, the influence areas describing deck moment are started at $I_d/I_c$ of 0.5; the lowest value used in the analysis.

Figs. 12 through 16 are influence areas of four- and six-column structures. The most prominent feature of these curves is the steep slope of the negative curves near their origin. Except for the positive moments at the second interior column, X6 with four columns and X8 with six columns, the slope of the positive curves is also quite steep. The moments induced in the deck are, by far, the most critical condition imposed upon the structure from interaction between the rib and the deck.

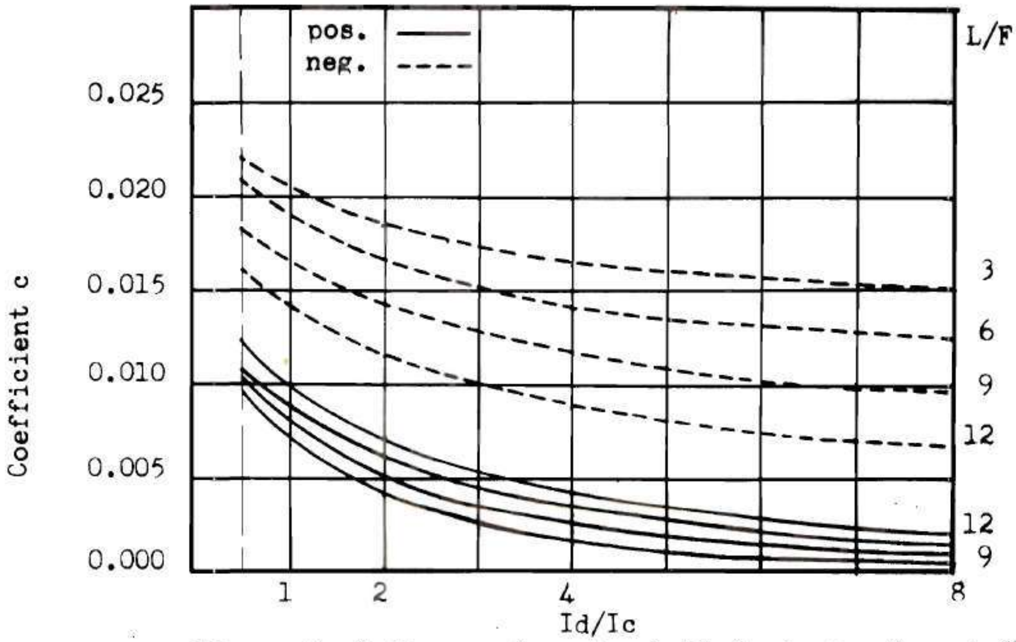


Figure 2 Influence Area for Left Springing Moment, $X_1$, 2-Column Structure. $M = cwL^2$

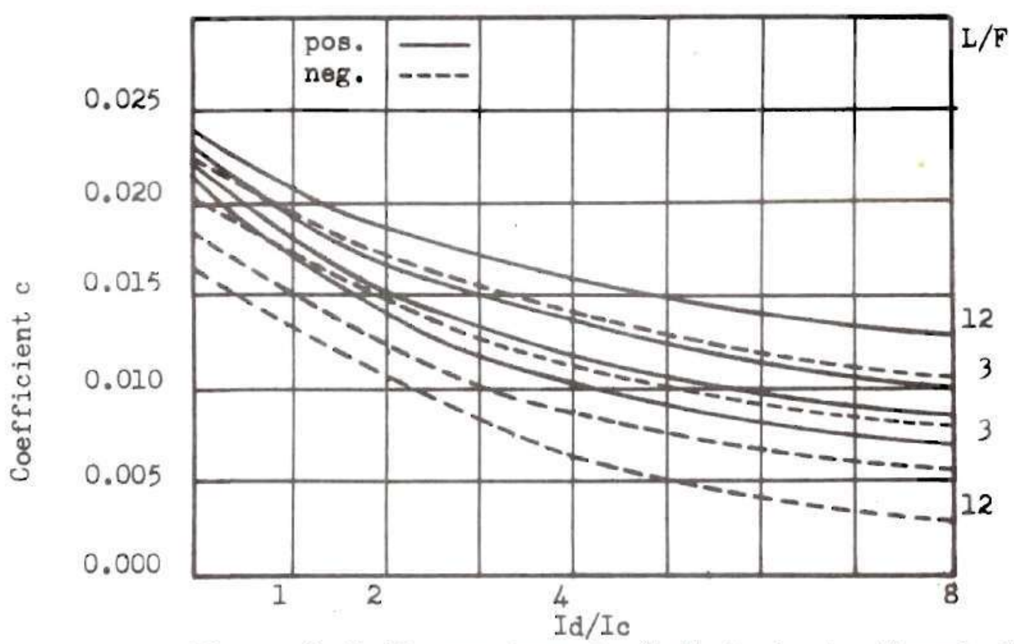


Figure 3 Influence Area for Left Springing Moment, $X_1$, 4-Column Structure. $M = cwL^2$

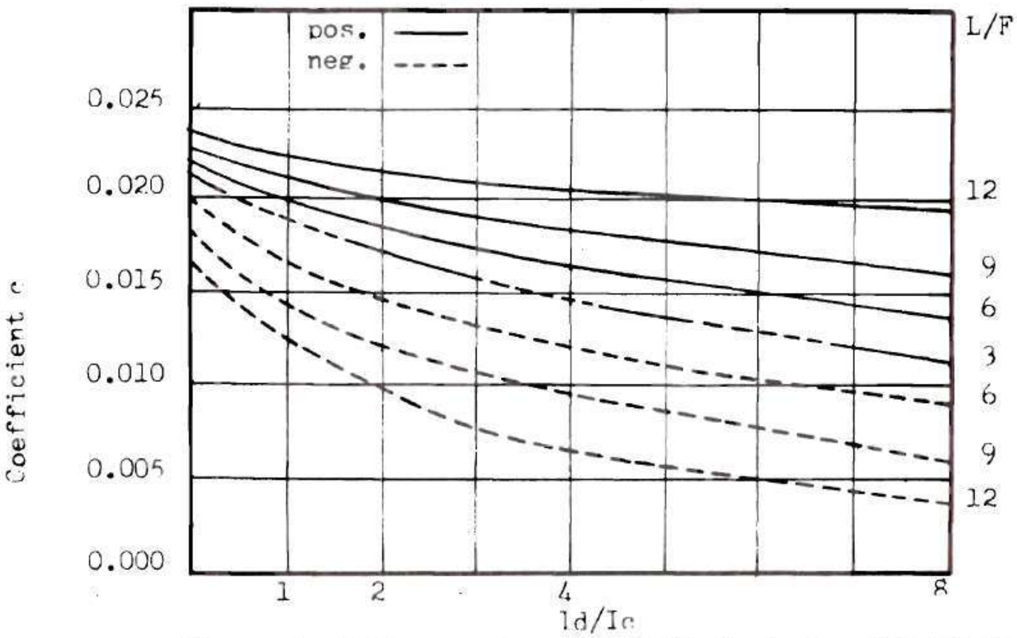


Figure 4 Influence Area for Left Springing Moment, $X_1$, 6-Column Structure. $M = cwL^2$

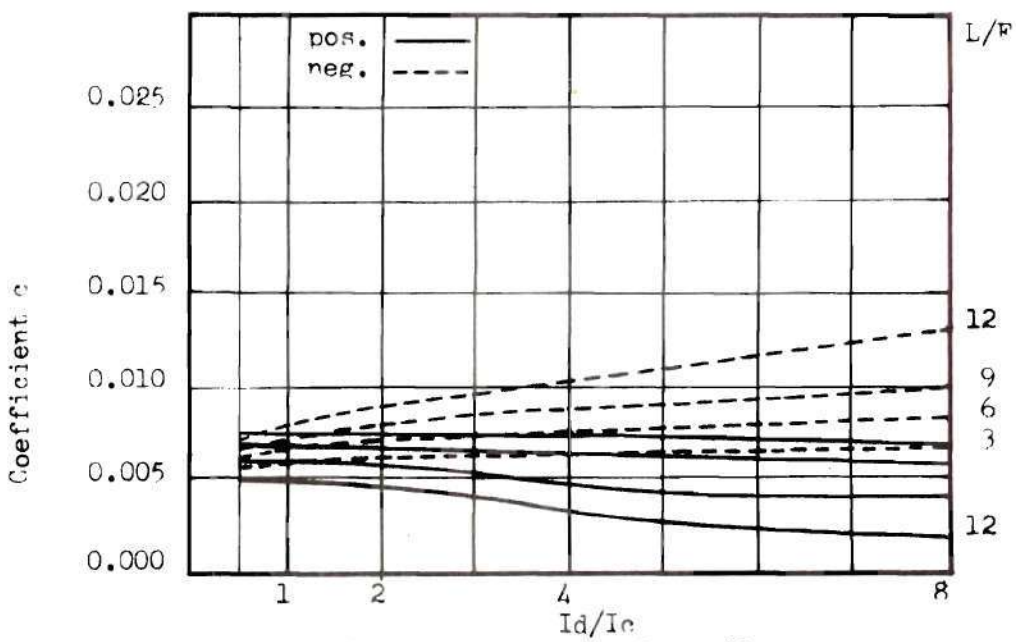


Figure 5 Influence Area for Crown Moment, 2-Column Structure. $M = cwL^2$

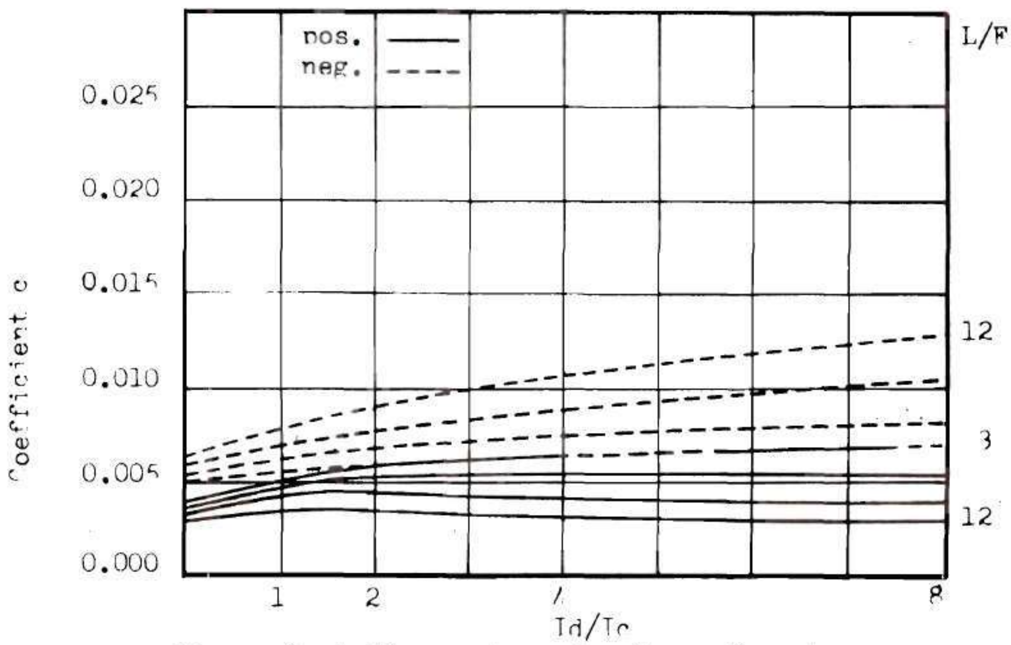


Figure 6 Influence Area for Crown Moment,
4-Column Structure. $M = cwL^2$

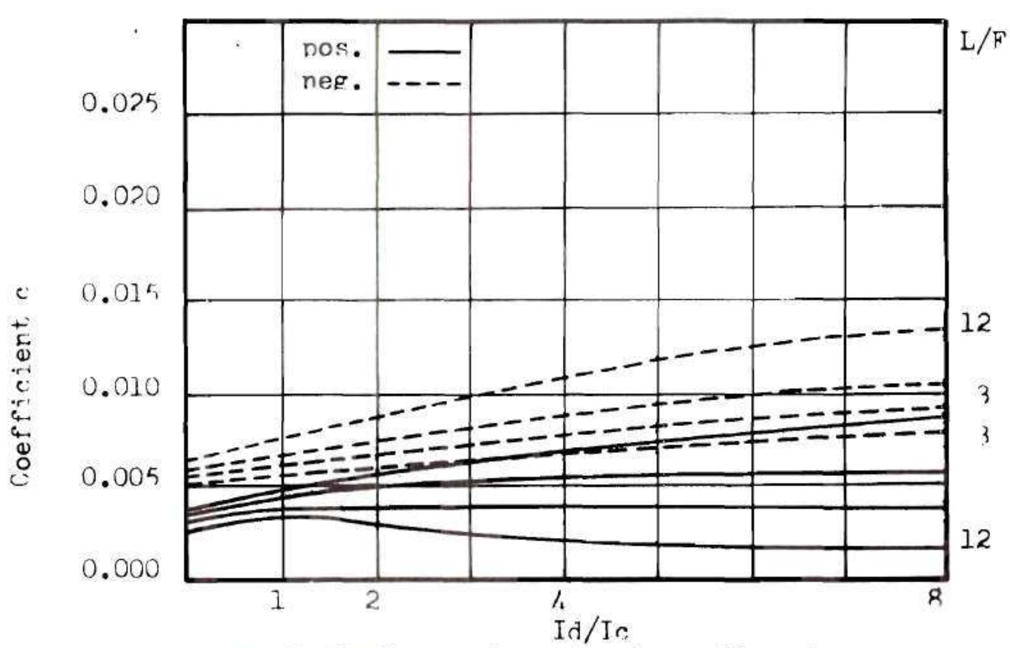


Figure 7 Influence Area for Crown Moment,
6-Column Structure. $M = cwL^2$


Figure 8 Influence Area for Thrust, $X_3$, 2-Column Structure. $H = kwL^2$

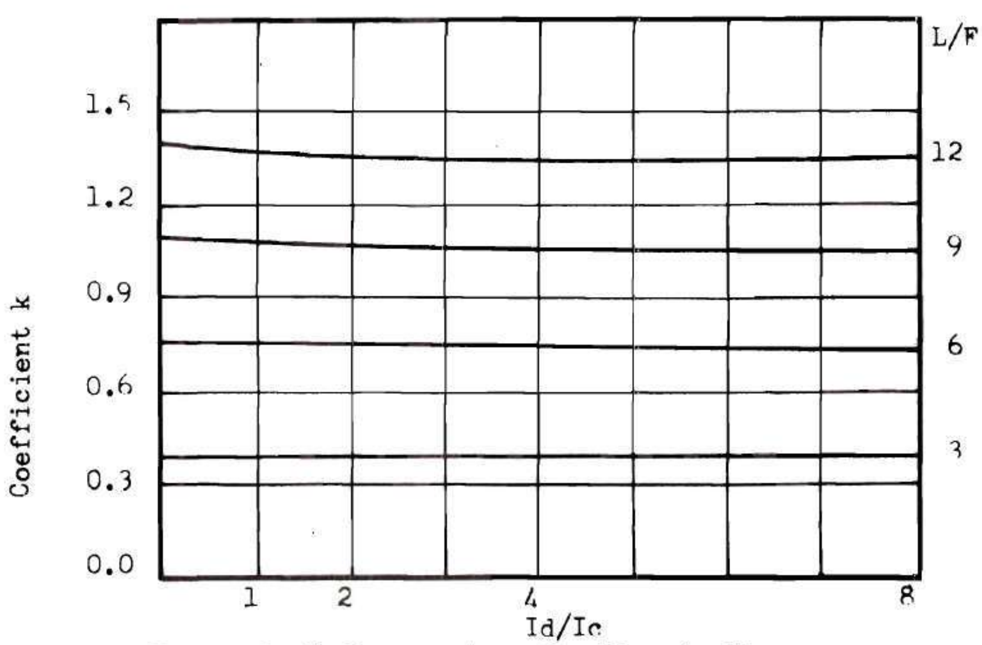


Figure 9 Influence Area for Thrust, $X_3$, 4-Column Structure. $H = kwL^2$

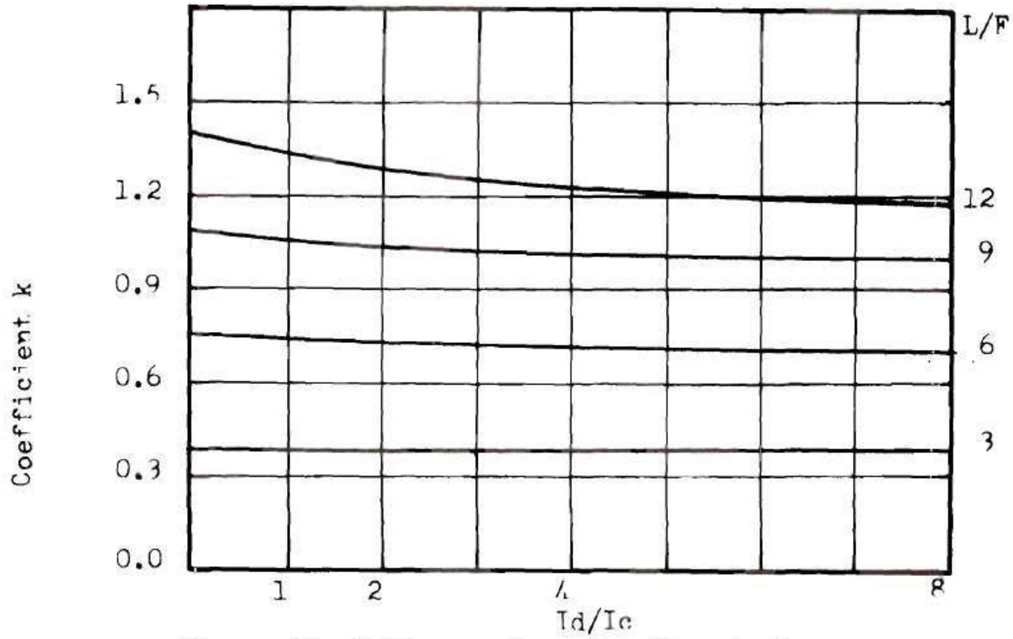


Figure 10 Influence Area for Thrust, $X_3$, 6-Column Structure. $H = kwL^2$

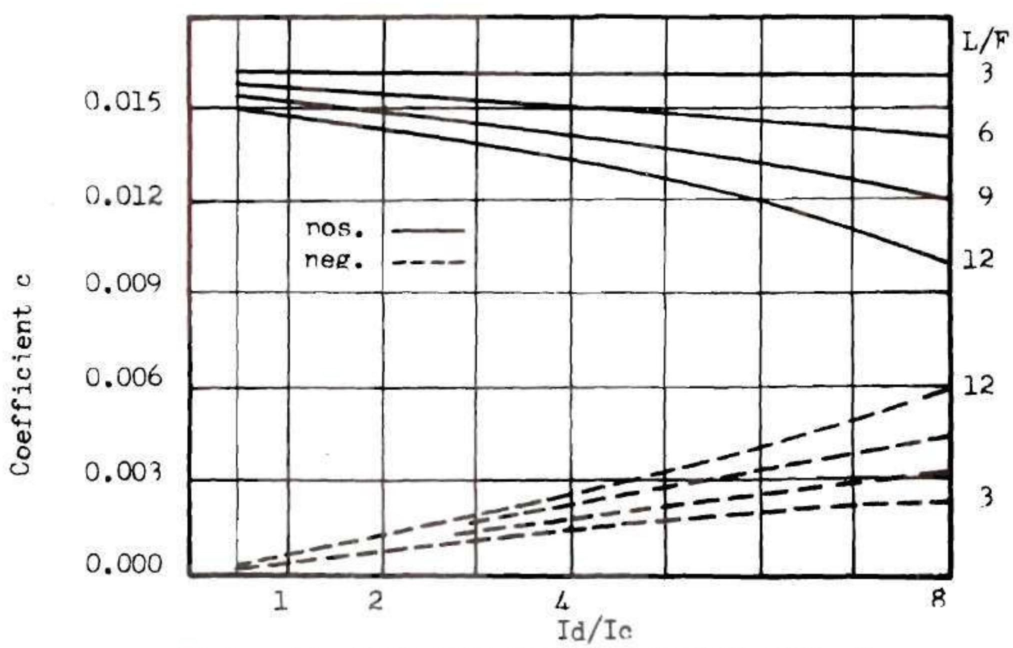


Figure 11 Influence Area for Deck Moment, $X_4$, 2-Column Structure. $M = cwL^2$

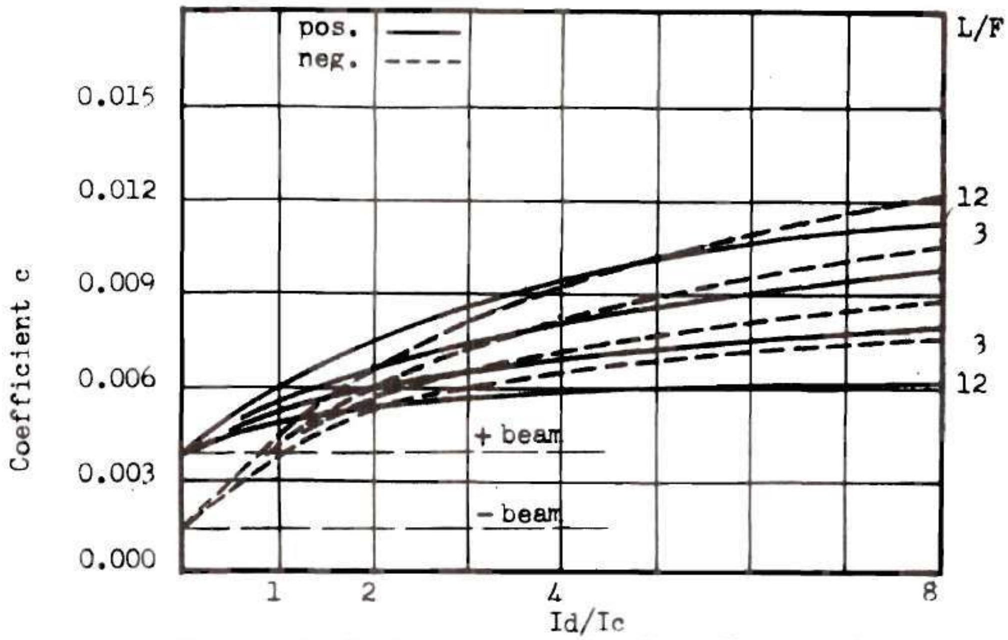


Figure 12 Influence Area For Deck Moment, $X_4$, 4-Column Structure. $M = cwL^2$

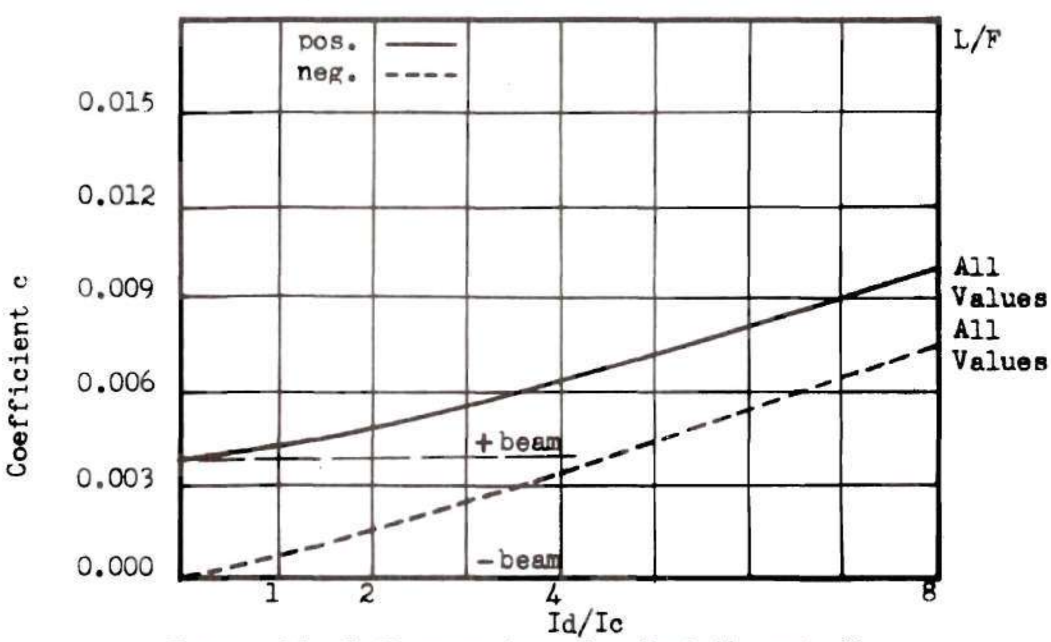


Figure 13 Influence Area for Deck Moment, $X_6$, 4-Column Structure. $M = cwL^2$

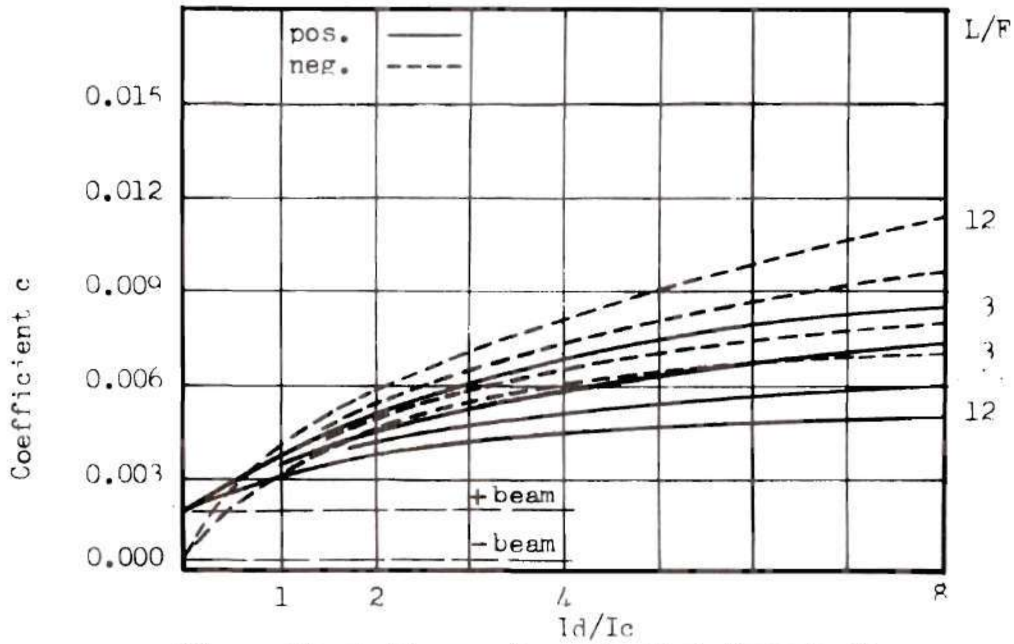


Figure 14 Influence Area for Deck Moment, $X_4$, 6-Column Structure. $M = cwL^2$

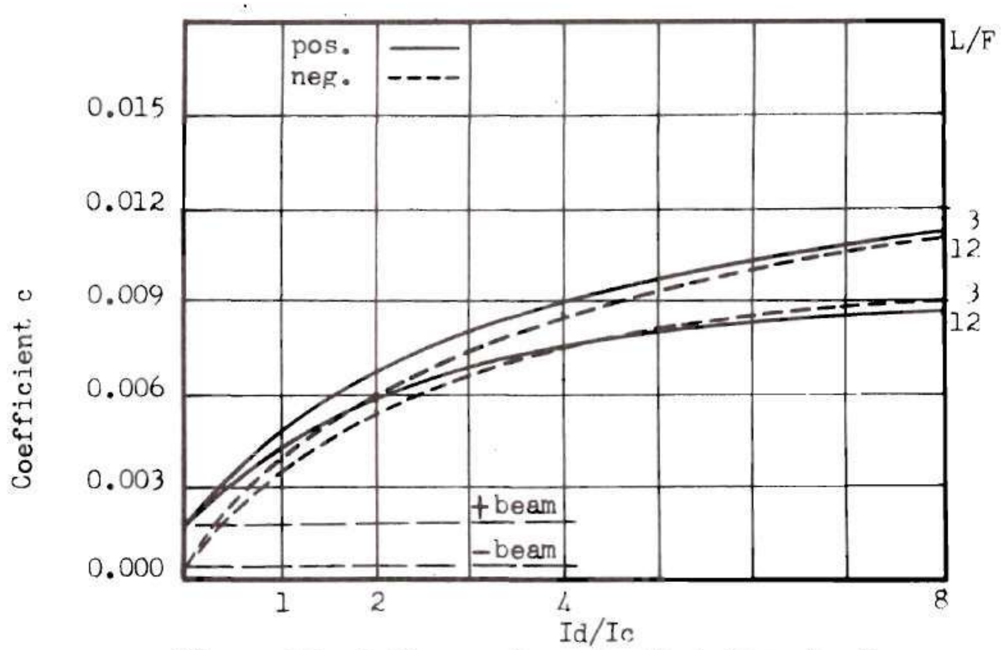


Figure 15 Influence Area for Deck Moment, $X_6$, 6-Column Structure. $M = cwL^2$

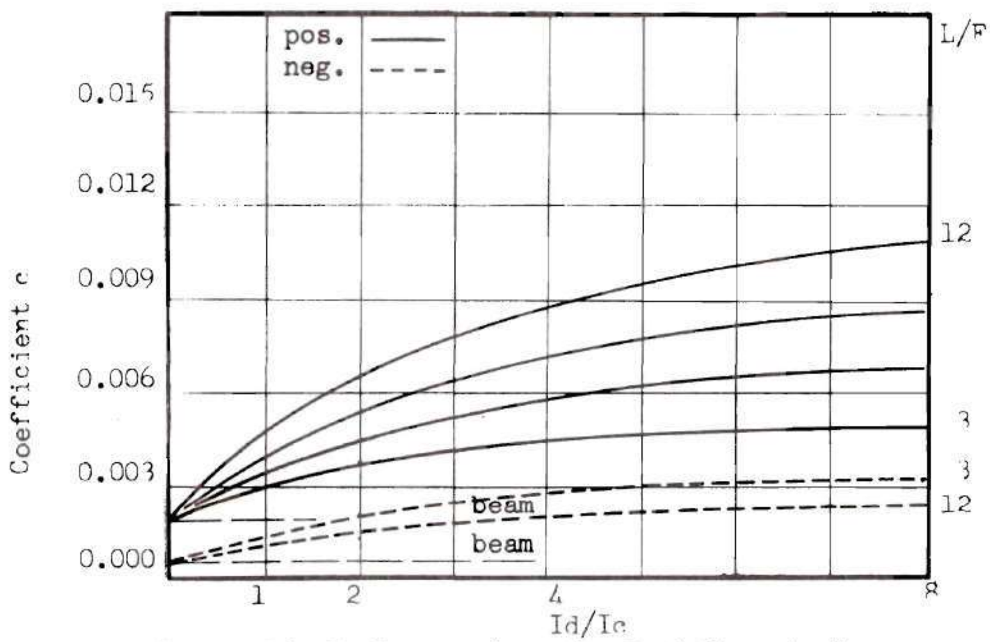


Figure 16 Influence Area for Deck Moment, $X_8$, 6-Column Structure. $M = cwL^2$

# CHAPTER IV

## CONCLUSIONS

The results of this study verify the opening statement in the introduction; "Open-spandrel arches are highly complex structures..." For this reason, only three conclusions can be safely made and they are limited to the range of variables covered in this study.

1. Any open-spandrel arch should be analyzed with a consideration for interaction between the deck and rib. The interrelationship of all variables, which creates a separate structure for each combination of variables, necessitates such an analysis. If a maximum error of 10 per cent can be accepted, than conclusions two and three are true.
2. The rib may be analyzed with the deck excluded when the moment of inertia of the deck is less than 30 per cent of the moment of inertia of the rib.
3. The deck may be analyzed as a beam when the moment of inertia of the deck is less than approximately 5 per cent of the moment of inertia of the rib. This figure of 5 per cent is approximate because the analysis was not concentrated in this low range of $I_d/I_c$.

# CHAPTER V

## RECOMMENDATIONS

This study, reveals certain points which require further evaluation. Because of the tremendous moments induced in the deck from rib-deck interaction, further study should be concentrated on structures with a deck moment of inertia varying from 0 to 100 per cent of the moment of inertia of the rib. The computer program used in this study is valid for such an evaluation.

The maximum number of columns investigated was only six. This variable should be increased. However, the data used in the computer program will have to be in terms of $10^{-12}$ and higher. The computer program included in the appendix is valid provided the round-off error is not excessive.

The extent of this round-off is indicated by any lack of agreement between the influence ordinates of symmetrical moments for a load at the centerline of the structure.

# APPENDIX A

# COMPUTER PROGRAM

This program is written in a compiler language, ALGOL, for the Burroughs 220 digital computer. The program is written in general terms so that the variation of the rib moment of inertia, the number of columns, the rib centerline, and deck moment of inertia may be changed with little or no program changes.

A program for the solution of a rib is readily available. The conditions on the variable "c" arise from the discontinuity of the elastic weight near the center of the structure where the rib and deck join. With no deck, the elastic weight is described by the function "ELWT (E)" throughout the problem. Therefore, all the conditions on "c" are omitted for a rib.

Any rib centerline may be programmed by appropriate changes in the vertical ordinates to a segment centerline, y(P), and the cosine $\theta$, F(L,P). Any variation in moment of inertia may be programmed by an appropriate change in the elastic weight function ELWT (E).

Program Variables

| | |
|---|---|
| I | Degree of indeterminancy |
| J | I plus the number of influence points |
| K | Number of segments |
| M(A,C) | The value of the moment at the centerline of segment C due to a unit load or moment applied at point A |
| M(B,C) | The value of the moment at the centerline of segment C due to a unit load or moment applied at point B |
| SUM | $\frac{M\bar{M}ds}{I}$ for a given value of C (I as used here denotes moment of inertia) |
| NOR | $\frac{N\bar{N}ds}{A}$ for a given value of C |

| | |
|---|---|
| T(A,B) | SUM + NOR, i.e. the movement at point A due to a unit load or moment at point B |
| Y(P) | The vertical ordinate for the centerline of segment P (P = C) for F/L equal one |
| R(L) | Rise-span ratio, (L) denotes various rise-span ratios |
| G(D) | The moment of inertia of the deck with respect to the rib crown, $I_d/I_c$ in the analysis |
| F(L,P) | Cosine $\theta$ at a segment centerline for a given value of P and L, P=C which defines a segment and L defines a rise-span ratio |
| H(L,D,Z) | Elastic weight of a segment common to both rib and deck, L defines a rise-span ratio, D defines a deck moment of inertia, Z defines a segment number where rib and deck join |
| DW | Elastic weight |
| DA | ds/A |
| W | Iteration constant with maximum value equal to the number of influence points |
| SIMEQ | Rich Electronic Computer Center procedure for the solution of simultaneous equations |
| EXTRA | Storage identifier for a W by N array |

Input Identifiers

| | |
|---|---|
| DEGRE | Identifier for variables I, J, and K |
| MOM | Value of the moment at a segment centerline in the base structure caused by: 1) a moment or force of unity acting at a point of redundancy; 2) a vertical load acting at an influence point. |
| GBA | Elastic weight at the saddle, all other elastic weights are computed in the program |

RISE Ordinate to a segment centerline for F/L equal one

LOF Rise-span (F/L) ratio

GBM Moment of inertia of deck in reference to moment of inertia of rib at structure centerline

NORM Cosine of $\theta$ at a segment centerline for a given value of F/L

All integers appearing as subscripts in a subscripted variable are used for iteration.

ALGOL Program for Burroughs 220 Digital Computer

```
COMMENT SOLUTION OF SIMULTANEOUS EQUATIONS BY GAUSS-JORDAN RREDUCTION  $
PROCEDURE SIMEQ (N,A(,) $ X())                                         $
BEGIN INTEGER I,J,K,N,                                                 $
       FOR K = (N,-1,1)                                                $
BEGIN FOR J = (1,1,K)                                                  $
   X(J) = A(1,J+1)/A(1,1)                                              $
       FOR I = (1,1,N-1)                                               $
       FOR J = (1,1,K)                                                 $
   A(I,J) = A(I+1,J+1) - A(I+1,1).X(J)                                 $
       FOR J = (1,1,K)                                                 $
A(N,J) = X(J)                  END                                     $
       FOR I = (1,1,N)                                                 $
X(I) = A(I,1)                                                          $
       RETURN   END   SIMEQ()
INTEGER A,B,C,D,I,J,K,L,P,Z,W,N,O,Q,S,U                                $
ARRAY M(13,42),R(4),Y(24),H(4,5,6),T(9,13),G(5),F(4,24),X(9),EXTRA(9,9)$
FUNCTION ELWT(E)=0.04166667-((0.7)((25.0-2.0(E))(25.0-2.0(E))))/13824.0$
READ($$DEGRE) $     READ($$MOM)  $     READ($$GBA)  $     READ($$RISE) $
READ($$LOF)  $     READ($$GBM)  $     READ($$NORM)                     $
       FOR L=(1,1,4)                                                   $
       FOR D=(1,1,5)                                                   $
BEGIN FOR A=(1,1,I)                                                    $
       FOR B=(1,1,J)                                                   $
BEGIN T(A,B)=0.0                                                       $
       FOR C=(1,1,K)                                                   $
```

```
BEGIN  NOR=0.0                                    $
   SUM=0.0                                        $
   V=FLOAT(C)                                     $
   IF V LSS 10.0                                  $
   DW=ELWT(V)                                     $
   IF V GTR 15.0                                  $
   (IF V LSS 25.0                                 $
   DW=ELWT(V))                                    $
   C=FIX(V)                                       $
   IF C LSS 16                                    $
   (IF C GTR 9                                    $
   (Z=C-9                                         $
   DW=H(L,D,Z))$)                                 $
   IF C GTR 24                                    $
   DW=1/(24.0)G(D)                                $
   IF C LSS 25                                    $
BEGIN P=C                                         $
   IF A EQL 3                                     $
   M(A,C)=(Y(P)).R(L)                             $
   IF B EQL 3                                     $
   M(B,C)=(Y(P)),R(L)                             $
   IF A EQL 3                                     $
   (IF B EQL 3                                    $
   NOR=((F(L,P))(F(L,P)))/(720000.0))             $
END                                               $
   SUM=(M(A,C))(M(B,C))DW                         $
```

```
   T(A,B)=T(A,B)+SUM+NOR            END                                $
END                                                                    $
       FOR  O=(1,1,I)                                                  $
   (FOR  Q=(1,1,I)                                                     $
   EXTRA(O,Q)=T(O,Q))                                                  $
       FOR  W=(1,1,4)                                                  $
BEGINFOR N=(1,1,I)                                                     $
   T(N,I+1)=T(N,I+W)                                                   $
   SIMEQ(5,T(,)$ X())                                                  $
   HEADING ($$TITLE)                                                   $
   WRITE($$SET,FMT)                                                    $
       FOR  S=(1,1,I)                                                  $
       (FOR U=(1,1,I                                                   $
   T(S,U)=EXTRA(S,U))                                                  $
END                                                                    $
END                                                                    $
STOP 999                                                               $
OUTPUT SET ( X(1), X(2), X(3), X(4), X(5) )                            $
FORMAT FMT(5X10.5,WO)                                                  $
FORMAT TITLE (B4,*X1*,B8,*X2*,B8,*X3*,B8,*X4*,B8,*X5*,WO)              $
INPUT DEGRE(I,J,K)                                                     $
INPUT MOM (FORB=(1,1,J)                                                $
       FOR  C=(1,1,K)                                                  $
M(B,C))                                                                $
INPUT GBA (FOR L=(1,1,4)                                               $
```

```
        FOR  D=(1,1,5)                                    $
        FOR  Z=(1,1,6)                                    $
H(L,D,Z))                                                 $
INPUT RISE (FOR P=(1,1,24)                                $
Y(P))                                                     $
INPUT LOF (FOR L=(1,1,4)                                  $
R(L))                                                     $
INPUT GBM (FOR D=(1,1,5)                                  $
G(D))                                                     $
INPUT NORM (FOR L=(1,1,4)                                 $
FOR P=(1,1,24)                                            $
F(L,P))                                                   $
FINISH                                                    $
```

APPENDIX B

INFLUENCE ORDINATES

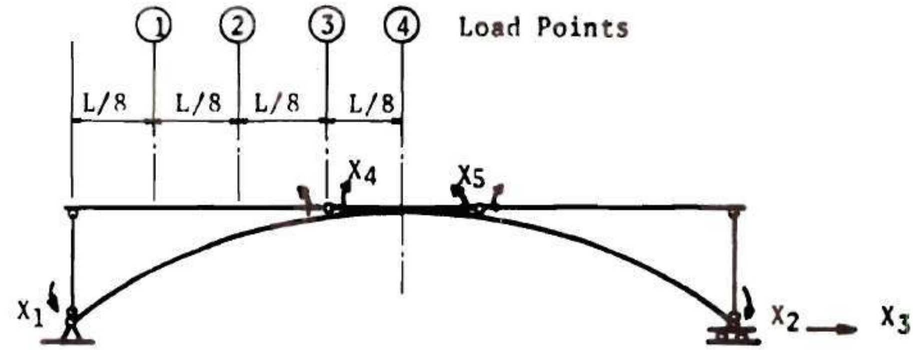


Figure 17 Base Structure, 2-Column Structure.

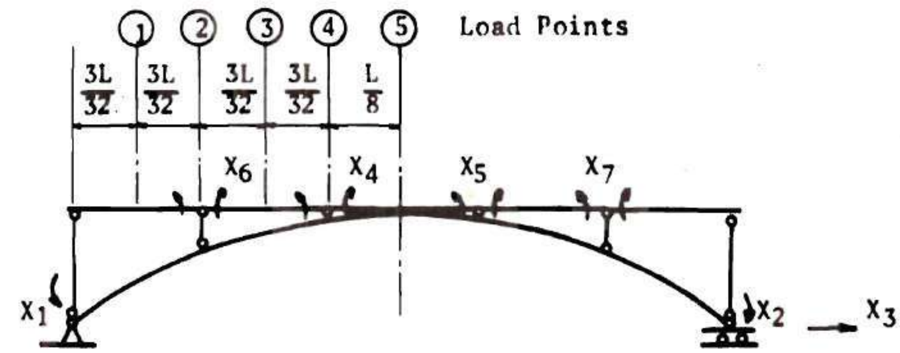


Figure 18 Base Structure, 4-Column Structure.

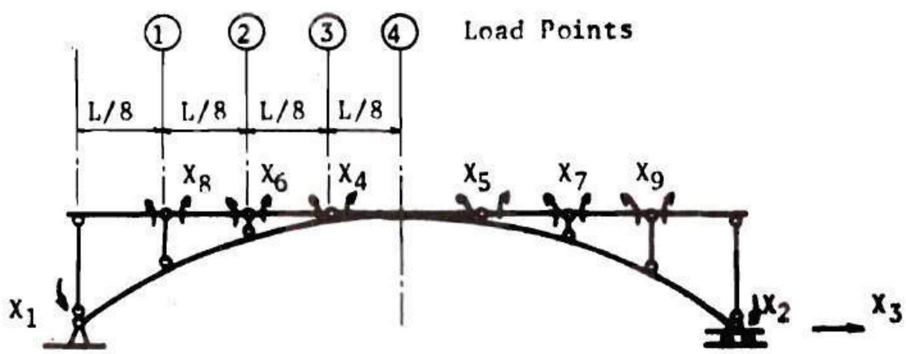


Figure 19 Base Structure, 6-Column Structure.

Table 1 Influence Ordinates
Rib Alone

| Ordinates | | | Point | L/r |
|---|---|---|---|---|
| X1 | X2 | X3 | | |
| .07881 | -.01587 | -.10769 | 1 | |
| X1 | X2 | X3 | | |
| .07037 | -.04849 | -.37495 | 2 | |
| X1 | X2 | X3 | | 3 |
| .01310 | -.06525 | -.64147 | 3 | |
| X1 | X2 | X3 | | |
| -.04331 | -.04339 | -.75119 | 4 | |
| X1 | X2 | X3 | | |
| .07935 | -.01533 | -.21103 | 1 | |
| X1 | X2 | X3 | | |
| .07226 | -.04660 | -.73475 | 2 | |
| X1 | X2 | X3 | | 6 |
| .01633 | -.06201 | -1.25701 | 3 | |
| X1 | X2 | X3 | | |
| -.03953 | -.03960 | -1.47201 | 4 | |
| X1 | X2 | X3 | | |
| .08007 | -.01460 | -.30783 | 1 | |
| X1 | X2 | X3 | | |
| .07478 | -.04408 | -1.07178 | 2 | |
| X1 | X2 | X3 | | 9 |
| .02064 | -.05769 | -1.83360 | 3 | |
| X1 | X2 | X3 | | |
| -.03448 | -.03454 | -2.14722 | 4 | |
| X1 | X2 | X3 | | |
| .08103 | -.01364 | -.39508 | 1 | |
| X1 | X2 | X3 | | |
| .07811 | -.04074 | -1.37557 | 2 | |
| X1 | X2 | X3 | | 12 |
| .02633 | -.05199 | -2.35334 | 3 | |
| X1 | X2 | X3 | | |
| -.02781 | -.02786 | -2.75585 | 4 | |

Table 2 Influence Ordinates
Two Columns L/F=3

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .02644 | -.02905 | -.22702 | -.04486 | -.00398 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .03488 | -.05067 | -.44305 | -.05445 | -.00711 | 2 | 0.5 |
| X1 | X2 | X3 | X4 | X5 | | |
| .00733 | -.05744 | -.63713 | .00649 | -.00855 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03793 | -.03796 | -.73033 | -.00546 | -.00549 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02067 | -.02671 | -.23321 | -.03763 | -.00728 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02662 | -.04641 | -.45012 | -.04416 | -.01274 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00314 | -.05208 | -.63442 | .01152 | -.01458 | 3 | 1.0 |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03492 | -.03492 | -.71840 | -.00844 | -.00849 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01345 | -.02321 | -.23971 | -.02847 | -.01204 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01619 | -.04030 | -.45736 | -.03092 | -.02066 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00249 | -.04514 | -.63091 | .01864 | -.02244 | 3 | 2.0 |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03165 | -.03161 | -.70490 | -.01147 | -.01155 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00615 | -.01908 | -.24476 | -.01897 | -.01739 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00555 | -.03332 | -.46257 | -.01699 | -.02936 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00855 | -.03785 | -.62651 | .02686 | -.03046 | 3 | 4.0 |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02879 | -.02871 | -.69181 | -.01363 | -.01374 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00022 | -.01531 | -.24707 | -.01081 | -.02185 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00314 | -.02705 | -.46392 | -.00480 | -.03642 | 2 | 8.0 |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01366 | -.03166 | -.62032 | .03485 | -.03643 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02665 | -.02654 | -.67923 | -.01413 | -.01427 | 4 | |

Table 3 Influence Ordinates
Two Columns L/F = 6

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .02763 | -.02785 | -.44408 | -.04484 | -.00396 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .03721 | -.04831 | -.86649 | -.05440 | -.00706 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 0.5 |
| .01070 | -.05401 | -1.24558 | .00657 | -.00847 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03404 | -.03406 | -1.42777 | -.00537 | -.00540 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02193 | -.02544 | -.45545 | -.03756 | -.00721 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02906 | -.04394 | -.87881 | -.04400 | -.01259 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 1.0 |
| .00661 | -.04855 | -1.23796 | .01177 | -.01434 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03096 | -.03095 | -1.40182 | -.00816 | -.00821 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01480 | -.02185 | -.46674 | -.02825 | -.01183 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01877 | -.03768 | -.89023 | -.03050 | -.02024 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 2.0 |
| .00110 | -.04149 | -1.22722 | .01929 | -.02181 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02761 | -.02756 | -1.37112 | -.01076 | -.01083 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00760 | -.01762 | -.47396 | -.01842 | -.01685 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00829 | -.03054 | -.89542 | -.01593 | -.02830 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 4.0 |
| -.00481 | -.03406 | -1.21187 | .02839 | -.02895 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02462 | -.02454 | -1.33812 | -.01195 | -.01205 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00178 | -.01374 | -.47355 | -.00955 | -.02060 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00020 | -.02409 | -.88883 | -.00239 | -.03402 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 8.0 |
| -.00972 | -.02767 | -1.18758 | .03817 | -.03312 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02230 | -.02219 | -1.30030 | -.01050 | -.01062 | 4 | |

Table 4 Influence Ordinates
Two Columns L/F = 9

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .02923 | -.02624 | -.64591 | -.04480 | -.00392 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04034 | -.04518 | -1.26026 | -.05433 | -.00699 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 0.5 |
| .01520 | -.04950 | -1.81152 | .00668 | -.00836 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02888 | -.02889 | -2.07649 | -.00525 | -.00528 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02362 | -.02374 | -.66089 | -.03745 | -.00709 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .03232 | -.04066 | -1.27514 | -.04379 | -.01238 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 1.0 |
| .0'121 | -.04393 | -1.79612 | .01208 | -.01403 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02574 | -.02573 | -2.03385 | -.00782 | -.00786 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01659 | -.02005 | -.67451 | -.02795 | -.01152 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02220 | -.03425 | -1.28644 | -.02991 | -.01965 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 2.0 |
| .00583 | -.03674 | -1.77320 | .02010 | -.02099 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02231 | -.02226 | -1.98110 | -.00985 | -.00991 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00950 | -.01572 | -.68001 | -.01767 | -.01610 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01188 | -.02694 | -1.28461 | -.01451 | -.02688 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 4.0 |
| .00005 | -.02918 | -1.73839 | .03033 | -.02701 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01925 | -.01916 | -1.91950 | -.00981 | -.00990 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00378 | -.01174 | -.67053 | -.00791 | -.01895 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00354 | -.02034 | -1.25848 | .00070 | -.03092 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 8.0 |
| -.00471 | -.02266 | -1.68126 | .04232 | -.02896 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01682 | -.01671 | -1.84085 | -.00596 | -.00607 | 4 | |

Table 5 Influence Ordinates
Two Columns L/F = 12

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .03133 | -.02414 | -.82580 | -.04475 | -.00387 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04444 | -.04107 | -1.61123 | -.05423 | -.00689 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 0.5 |
| .02110 | -.04359 | -2.31591 | .00682 | -.00822 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02211 | -.02212 | -2.65466 | -.00509 | -.00511 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02583 | -.02153 | -.84232 | -.03730 | -.00695 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .03659 | -.03639 | -1.62516 | -.04351 | -.01209 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 1.0 |
| .01722 | -.03791 | -2.28903 | .01248 | -.01363 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01893 | -.01891 | -2.59201 | -.00736 | -.00740 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01892 | -.01772 | -.85510 | -.02755 | -.01112 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .02664 | -.02980 | -1.63082 | -.02915 | -.01889 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 2.0 |
| .01195 | -.03061 | -2.24778 | .02115 | -.01993 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01548 | -.01542 | -2.51133 | -.00867 | -.00873 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01191 | -.01330 | -.85412 | -.01671 | -.01514 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .01645 | -.02237 | -1.61346 | -.01270 | -.02506 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 4.0 |
| .00624 | -.02299 | -2.18326 | .03279 | -.02454 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01242 | -.01233 | -2.41070 | -.00710 | -.00717 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00624 | -.00928 | -.82843 | -.00587 | -.01691 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00817 | -.01571 | -1.55477 | .00452 | -.02709 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 8.0 |
| .00147 | -.01647 | -2.07693 | .04744 | -.02382 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01005 | -.00994 | -2.27406 | -.00035 | -.00045 | 4 | |

Table 6 Influence Ordinates
Four Columns L/F = 3

| Ordinates | | | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .05088 | -.01490 | -.10367 | .00936 | -.00282 | -.01432 | .00038 | 1 | |
| .07425 | -.03193 | -.24786 | .00433 | -.00621 | .00729 | .00046 | 2 | |
| .05476 | -.05028 | -.44694 | -.01370 | -.00998 | -.00946 | -.00004 | 3 | 0.5 |
| .01251 | -.05784 | -.62853 | .01494 | -.01116 | -.00500 | -.00137 | 4 | |
| -.03678 | -.03684 | -.72622 | -.00408 | -.00414 | -.00401 | -.00399 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .04576 | -.01520 | -.11392 | .00836 | -.00492 | -.00924 | .00023 | 1 | |
| .06687 | -.03129 | -.26060 | .00628 | -.01031 | .01359 | -.00011 | 2 | |
| .04953 | -.04702 | -.45066 | -.00458 | -.01570 | -.00736 | -.00141 | 3 | 1.0 |
| .01197 | -.05235 | -.61912 | .02348 | -.01685 | -.00726 | -.00371 | 4 | |
| -.03222 | -.03227 | -.70883 | -.00543 | -.00550 | -.00718 | -.00715 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .03823 | -.01496 | -.12771 | .00700 | -.00783 | -.00162 | -.00072 | 1 | |
| .05599 | -.02943 | -.27766 | .00797 | -.01557 | .02356 | -.00223 | 2 | |
| .04172 | -.04174 | -.45539 | .00507 | -.02220 | -.00256 | -.00498 | 3 | 2.0 |
| .01081 | -.04450 | -.60638 | .03312 | -.02253 | -.00880 | -.00838 | 4 | |
| -.02621 | -.02624 | -.68589 | -.00594 | -.00602 | -.01166 | -.01164 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .02885 | -.01353 | -.14273 | .00557 | -.01118 | .00804 | -.00307 | 1 | |
| .04234 | -.02559 | -.29614 | .00914 | -.02120 | .03683 | -.00680 | 2 | |
| .03163 | -.03429 | -.46028 | .01339 | -.02829 | .00546 | -.01142 | 3 | 3.0 |
| .00858 | -.03495 | -.59240 | .04204 | -.02701 | -.00846 | -.01547 | 4 | |
| -.01977 | -.01977 | -.66126 | -.00536 | -.00544 | -.01673 | -.01671 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .01928 | -.01086 | -.15573 | .00448 | -.01428 | .01807 | -.00672 | 1 | |
| .02825 | -.01998 | -.31207 | .00994 | -.02619 | .05110 | -.01343 | 2 | |
| .02079 | -.02569 | -.46430 | .01953 | -.03315 | .01537 | -.01978 | 3 | 5.0 |
| .00534 | -.02535 | -.58013 | .04898 | -.02998 | -.00612 | -.02360 | 4 | |
| -.01423 | -.01422 | -.64001 | -.00417 | -.00424 | -.02121 | -.02121 | 5 | |

Table 7 Influence Ordinates
**Four** Columns L/F = 6

| Ordinates | | | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .05148 | -.01430 | -.20241 | .00943 | -.00275 | -.01438 | .00033 | 1 | |
| .07568 | -.03049 | -.48387 | .00452 | -.00602 | .00716 | .00032 | 2 | |
| .05735 | -.04766 | -.87232 | -.01337 | -.00964 | -.00969 | -.00028 | 3 | 0.5 |
| .01617 | -.05413 | -1.22637 | .01542 | -.01067 | -.00532 | -.00171 | 4 | |
| -.03252 | -.03257 | -1.41701 | -.00353 | -.00357 | -.00439 | -.00437 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .04648 | -.01447 | -.22174 | .00852 | -.00476 | -.00934 | .00013 | 1 | |
| .06852 | -.02963 | -.50717 | .00663 | -.00995 | .01337 | -.00034 | 2 | |
| .05239 | -.04412 | -.87677 | -.00396 | -.01507 | -.00773 | -.00180 | 3 | 1.0 |
| .01592 | -.04832 | -1.20399 | .02437 | -.01596 | -.00777 | -.00425 | 4 | |
| -.02766 | -.02769 | -1.37846 | -.00442 | -.00448 | -.00777 | -.00775 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .03913 | -.01405 | -.24739 | .00730 | -.00753 | -.00177 | -.00087 | 1 | |
| .05795 | -.02744 | -.53771 | .00864 | -.01490 | .02325 | -.00256 | 2 | |
| .04496 | -.03845 | -.88157 | .00619 | -.02107 | -.00307 | -.00551 | 3 | 2.0 |
| .01515 | -.04008 | -1.17320 | .03466 | -.02097 | -.00946 | -.00909 | 4 | |
| -.02126 | -.02128 | -1.32702 | -.00420 | -.00426 | -.01243 | -.01241 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .02998 | -.01239 | -.27450 | .00615 | -.01059 | .00788 | -.00324 | 1 | |
| .04468 | -.02321 | -.56938 | .01036 | -.01998 | .03650 | -.00715 | 2 | |
| .03529 | -.03059 | -.88459 | .01533 | -.02635 | .00497 | -.01194 | 3 | 4.0 |
| .01331 | -.03015 | -1.13775 | .04461 | -.02444 | -.00906 | -.01612 | 4 | |
| -.01445 | -.01445 | -1.26997 | -.00250 | -.00256 | -.01742 | -.01742 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .02062 | -.00951 | -.29628 | .00556 | -.01320 | .01804 | -.00676 | 1 | |
| .03095 | -.01726 | -.59355 | .01212 | -.02400 | .05105 | -.01350 | 2 | |
| .02481 | -.02162 | -.88269 | .02282 | -.02984 | .01532 | -.01986 | 3 | 8.0 |
| .01038 | -.02024 | -1.10215 | .05319 | -.02576 | -.00614 | -.02367 | 4 | |
| -.00863 | -.00862 | -1.21585 | .00047 | .00042 | -.02125 | -.02125 | 5 | |

Table 8 Influence Ordinates
Four Columns L/F = 9

| Ordinates | | | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .05227 | -.01349 | -.29366 | .00954 | -.00264 | -.01445 | .00025 | 1 | |
| .07759 | -.02857 | -.70199 | .00477 | -.00577 | .00699 | .00015 | 2 | |
| .06080 | -.04420 | -1.26550 | -.01292 | -.00918 | -.00010 | -.00059 | 3 | 0.5 |
| .02102 | -.04927 | -1.77905 | .01605 | -.01003 | -.00576 | -.00215 | 4 | |
| -.02692 | -.02696 | -2.05561 | -.00280 | -.00283 | -.00489 | -.00488 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .04744 | -.01351 | -.32035 | .00873 | -.00455 | -.00946 | .00000 | 1 | |
| .07071 | -.02743 | -.73266 | .00711 | -.00946 | .01308 | -.00063 | 2 | |
| .05618 | -.04032 | -1.26652 | -.00312 | -.01423 | -.00823 | -.00230 | 3 | 1.0 |
| .02113 | -.04310 | -1.73908 | .02552 | -.01480 | -.00845 | -.00494 | 4 | |
| -.02169 | -.02172 | -1.99108 | -.00311 | -.00315 | -.00856 | -.00854 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .04032 | -.01285 | -.35506 | .00771 | -.00711 | -.00196 | -.00106 | 1 | |
| .06054 | -.02485 | -.77172 | .00954 | -.01400 | .02284 | -.00298 | 2 | |
| .04920 | -.03420 | -1.26513 | .00766 | -.01958 | -.00374 | -.00619 | 3 | 2.0 |
| .02079 | -.03442 | -1.68346 | .03664 | -.01899 | -.01035 | -.00999 | 4 | |
| -.01487 | -.01488 | -1.90418 | -.00197 | -.00201 | -.01344 | -.01343 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .03144 | -.01093 | -.39029 | .00692 | -.00982 | .00768 | -.00344 | 1 | |
| .04771 | -.02018 | -.80951 | .01196 | -.01836 | .03609 | -.00756 | 2 | |
| .03999 | -.02587 | -1.25758 | .01782 | -.02384 | .00433 | -.01259 | 3 | 4.0 |
| .01935 | -.02408 | -1.61732 | .04783 | -.02119 | -.00987 | -.01694 | 4 | |
| -.00769 | -.00768 | -1.80526 | .00110 | .00107 | -.01833 | -.01833 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .02231 | -.00781 | -.41553 | .00694 | -.01181 | .01802 | -.00678 | 1 | |
| .03433 | -.01387 | -.83242 | .01490 | -.02121 | .05100 | -.01355 | 2 | |
| .02985 | -.01657 | -1.23784 | .02695 | -.02569 | .01525 | -.01993 | 3 | 8.0 |
| .01667 | -.01394 | -1.54543 | .05836 | -.02056 | -.00621 | -.02374 | 4 | |
| -.00169 | -.00167 | -1.70486 | .00617 | .00616 | -.02133 | -.02134 | 5 | |

## Table 9 Influence Ordinates
### Four Columns L/F = 12

| Ordinates | | | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .05332 | -.01245 | -.37420 | .00767 | -.00250 | -.01454 | .00016 | 1 | |
| .08008 | -.02607 | -.89450 | .00509 | -.00544 | .00677 | -.00007 | 2 | |
| .06530 | -.03969 | -1.61253 | -.01234 | -.00859 | -.01040 | -.00099 | 3 | 0.5 |
| .02734 | -.04293 | -2.26685 | .01688 | -.00920 | -.00632 | -.00272 | 4 | |
| -.01961 | -.01963 | -2.63926 | -.00184 | -.00186 | -.00555 | -.00554 | 5 | |
| X1 | X2 | X3 | X6 | X5 | X6 | X7 | | |
| .04868 | -.01227 | -.40596 | .00900 | -.00427 | -.00963 | -.00016 | 1 | |
| .07354 | -.02459 | -.92840 | .00774 | -.00884 | .01271 | -.00100 | 2 | |
| .06107 | -.03541 | -1.60486 | -.00205 | -.01314 | -.00888 | -.00295 | 3 | 1.0 |
| .02785 | -.03636 | -2.20357 | .02700 | -.01330 | -.00934 | -.00583 | 4 | |
| -.01399 | -.01400 | -2.52289 | -.00141 | -.00143 | -.00957 | -.00956 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .04183 | -.01134 | -.44616 | .00824 | -.00658 | -.00220 | -.00130 | 1 | |
| .06383 | -.02155 | -.96969 | .01068 | -.01284 | .02232 | -.00350 | 2 | |
| .05459 | -.02879 | -1.58964 | .00954 | -.01768 | -.00459 | -.00706 | 3 | 2.0 |
| .02797 | -.02721 | -2.11519 | .03914 | -.01646 | -.01148 | -.01113 | 4 | |
| -.00674 | -.00674 | -2.39253 | .00086 | .00085 | -.01472 | -.01472 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .03325 | -.00911 | -.48466 | .00788 | -.00885 | .00743 | -.00368 | 1 | |
| .05148 | -.01640 | -1.00523 | .01397 | -.01635 | .03558 | -.00808 | 2 | |
| .04584 | -.02000 | -1.56155 | .02094 | -.02070 | .00354 | -.01338 | 3 | 4.0 |
| .02689 | -.01652 | -2.00812 | .05184 | -.01715 | -.01088 | -.01796 | 4 | |
| .00073 | .00075 | -2.24148 | .00557 | .00558 | -.01946 | -.01947 | 5 | |
| X1 | X2 | X3 | X4 | X5 | X6 | X7 | | |
| .02436 | -.00577 | -.50742 | .00862 | -.01012 | .01799 | -.00681 | 1 | |
| .03843 | -.00976 | -1.01647 | .01826 | -.01783 | .05095 | -.01361 | 2 | |
| .03594 | -.01047 | -1.51146 | .03196 | -.02066 | .01518 | -.02001 | 3 | 8.0 |
| .02428 | -.00632 | -1.88691 | .06461 | -.01428 | -.00630 | -.02384 | 4 | |
| .00670 | .00673 | -2.08156 | .01307 | .01309 | -.02143 | -.02144 | 5 | |

Table 10 Influence Ordinates
Six Columns L/F = 3

| Ordinates | | | | | | |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | Point | Id/Ic |
| .07391 | -.01671 | -.11923 | .00000 | -.00230 | 1 | |
| .06660 | -.04588 | -.37845 | .00412 | -.00670 | 2 | |
| .01242 | -.05927 | -.63110 | .01521 | -.00820 | 3 | 0.5 |
| -.03932 | -.03841 | -.73323 | -.00193 | -.00176 | 4 | |
| .06966 | -.01730 | -.12891 | -.00000 | -.00401 | 1 | |
| .06329 | -.04393 | -.38187 | .00621 | -.01063 | 2 | |
| .01215 | -.05487 | -.62312 | .02372 | -.01242 | 3 | 1.0 |
| -.03614 | -.03469 | -.71959 | -.00237 | -.00221 | 4 | |
| .06261 | -.01790 | -.14419 | -.00028 | -.00657 | 1 | |
| .05768 | -.04087 | -.38788 | .00848 | -.01540 | 2 | |
| .01184 | -.04840 | -.61137 | .03316 | -.01681 | 3 | 2.0 |
| -.03128 | -.02930 | -.69943 | -.00221 | -.00215 | 4 | |
| .05232 | -.01783 | -.16470 | -.00037 | -.01008 | 1 | |
| .04910 | -.03618 | -.39670 | .01073 | -.02065 | 2 | |
| .01113 | -.03993 | -.59672 | .04188 | -.02069 | 3 | 4.0 |
| -.02490 | -.02263 | -.67392 | -.00131 | -.00141 | 4 | |
| .03961 | -.01617 | -.18704 | -.00010 | -.01431 | 1 | |
| .03789 | -.02957 | -.40704 | .01302 | -.02606 | 2 | |
| .00946 | -.03029 | -.58191 | .04888 | -.02390 | 3 | 8.0 |
| -.01805 | -.01589 | -.64746 | .00000 | -.00024 | 4 | |

Table 10 cont.

| Ordinates | | | | Point | Id/Ic |
|---|---|---|---|---|---|
| X6 | X7 | X8 | X9 | | |
| .00409 | -.00168 | .00221 | .00162 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .01597 | -.00519 | -.00999 | .00393 | 2 | |
| X6 | X7 | X8 | X9 | | 0.5 |
| .00131 | -.00807 | -.00572 | .00381 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00635 | -.00712 | .00113 | .00007 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .00669 | -.00308 | .00504 | .00254 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .02521 | -.00877 | -.01412 | .00553 | 2 | |
| X6 | X7 | X8 | X9 | | 1.0 |
| .00230 | -.01307 | -.00883 | .00480 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00982 | -.01104 | .00071 | -.00086 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01011 | -.00535 | .01067 | .00328 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .03586 | -.01363 | -.01626 | .00608 | 2 | |
| X6 | X7 | X8 | X9 | | 2.0 |
| .00382 | -.01907 | -.01194 | .00401 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01352 | -.01516 | -.00123 | -.00326 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01421 | -.00870 | .01999 | .00292 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .04638 | -.01950 | -.01393 | .00400 | 2 | |
| X6 | X7 | X8 | X9 | | 4.0 |
| .00595 | -.02516 | -.01386 | .00013 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01669 | -.01852 | -.00509 | -.00737 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01882 | -.01303 | .03236 | .00043 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .05583 | -.02595 | -.00661 | -.00153 | 2 | |
| X6 | X7 | X8 | X9 | | 8.0 |
| .00865 | -.03070 | -.01366 | -.00673 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01898 | -.02064 | -.01012 | -.01231 | 4 | |

Table 11 Influence Ordinates
**Six Columns** L/F = 6

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .07462 | -.01599 | -.23255 | .00006 | -.00224 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06888 | -.04356 | -.73794 | .00430 | -.00649 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 0.5 |
| .01626 | -.05537 | -1.23012 | .01552 | -.00783 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03483 | -.03390 | -1.42921 | -.00156 | -.00133 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07054 | -.01641 | -.25037 | .00002 | -.00387 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06592 | -.04126 | -.74143 | .00658 | -.01022 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 1.0 |
| .01648 | -.05046 | -1.20912 | .02434 | -.01172 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.03111 | -.02964 | -1.39632 | -.00166 | -.00141 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06379 | -.01671 | -.27810 | -.00002 | -.00630 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06086 | -.03764 | -.74774 | .00918 | -.01464 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 2.0 |
| .01690 | -.04325 | -1.17767 | .03434 | -.01555 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02544 | -.02346 | -1.34729 | -.00087 | -.00071 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .05395 | -.01619 | -.31433 | .00017 | -.00952 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .05305 | -.03221 | -.75666 | .01207 | -.01925 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 4.0 |
| .01713 | -.03390 | -1.13713 | .04399 | -.01850 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01808 | -.01586 | -1.28420 | .00106 | .00108 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04181 | -.01399 | -.35175 | .00101 | -.01316 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04270 | -.02480 | -.76506 | .01549 | -.02353 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 8.0 |
| .01637 | -.02343 | -1.09267 | .05252 | -.02017 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01033 | -.00830 | -1.21568 | .00405 | .00394 | 4 | |

Table 11 cont.

| Ordinates | | | | Point | Id/Ic |
|---|---|---|---|---|---|
| X6 | X7 | X8 | X9 | | |
| .00412 | -.00166 | .00210 | .00152 | 1 | 0.5 |
| X6 | X7 | X8 | X9 | | |
| .01608 | -.00514 | -.01034 | .00361 | 2 | |
| X6 | X7 | X8 | X9 | | |
| .00150 | -.00798 | -.00631 | .00326 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00613 | -.00701 | .00044 | -.00056 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .00676 | -.00303 | .00483 | .00234 | 1 | 1.0 |
| X6 | X7 | X8 | X9 | | |
| .02542 | -.00864 | -.01475 | .00494 | 2 | |
| X6 | X7 | X8 | X9 | | |
| .00266 | -.01284 | -.00986 | .00382 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00940 | -.01077 | -.00049 | -.00197 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01025 | -.00524 | .01028 | .00291 | 1 | 2.0 |
| X6 | X7 | X8 | X9 | | |
| .03626 | -.01332 | -.01731 | .00507 | 2 | |
| X6 | X7 | X8 | X9 | | |
| .00449 | -.01855 | -.01359 | .00242 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01276 | -.01456 | -.00314 | -.00505 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01452 | -.00842 | .01934 | .00230 | 1 | 4.0 |
| X6 | X7 | X8 | X9 | | |
| .04715 | -.01881 | -.01548 | .00251 | 2 | |
| X6 | X7 | X8 | X9 | | |
| .00717 | -.02406 | -.01618 | -.00211 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01532 | -.01725 | -.00774 | -.00987 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01950 | -.01236 | .03148 | -.00040 | 1 | 8.0 |
| X6 | X7 | X8 | X9 | | |
| .05734 | -.02447 | -.00851 | -.00333 | 2 | |
| X6 | X7 | X8 | X9 | | |
| .01089 | -.02849 | -.01635 | -.00928 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01649 | -.01816 | -.01315 | -.01512 | 4 | |

Table 12 Influence Ordinates
Six Columns L/F = 9

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .07558 | -.01503 | -.33692 | .00014 | -.00214 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07192 | -.04051 | -1.06911 | .00454 | -.00620 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 0.5 |
| .02133 | -.05028 | -1.78204 | .01593 | -.00734 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02894 | -.02799 | -2.07047 | -.00109 | -.00077 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07171 | -.01523 | -.36067 | .00018 | -.00369 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06939 | -.03778 | -1.06799 | .00706 | -.00968 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 1.0 |
| .02215 | -.04477 | -1.74149 | .02513 | -.01083 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02456 | -.02307 | -2.01111 | -.00074 | -.00037 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06533 | -.01517 | -.39690 | .00032 | -.00592 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06500 | -.03350 | -1.06706 | .01011 | -.01363 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 2.0 |
| .02343 | -.03672 | -1.68033 | .03581 | -.01395 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01797 | -.01599 | -1.92235 | .00081 | .00112 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .05603 | -.01413 | -.44254 | .00088 | -.00877 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .05805 | -.02724 | -1.06520 | .01379 | -.01745 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 4.0 |
| .02465 | -.02641 | -1.60057 | .04658 | -.01578 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00958 | -.00742 | -1.80758 | .00399 | .00415 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04451 | -.01134 | -.48632 | .00241 | -.01172 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04857 | -.01903 | -1.05765 | .01853 | -.02040 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 8.0 |
| .02476 | -.01518 | -1.51032 | .05689 | -.01568 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00099 | .00088 | -1.68034 | .00890 | .00894 | 4 | |

Table 12 cont.

| Ordinates | | | | Point | Id/Ic |
|---|---|---|---|---|---|
| X6 | X7 | X8 | X9 | | |
| .00417 | -.00164 | .00195 | .00139 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .01623 | -.00506 | -.01081 | .00318 | 2 | |
| X6 | X7 | X8 | X9 | | 0.5 |
| .00175 | -.00786 | -.00709 | .00255 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00584 | -.00687 | -.00046 | -.00138 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .00686 | -.00298 | .00455 | .00208 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .02570 | -.00847 | -.01559 | .00417 | 2 | |
| X6 | X7 | X8 | X9 | | 1.0 |
| .00312 | -.01255 | -.01122 | .00256 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00887 | -.01043 | -.00206 | -.00342 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01044 | -.00509 | .00978 | .00243 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .03678 | -.01291 | -.01867 | .00379 | 2 | |
| X6 | X7 | X8 | X9 | | 2.0 |
| .00532 | -.01789 | -.01574 | .00040 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01181 | -.01380 | -.00560 | -.00736 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01493 | -.00805 | .01853 | .00153 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .04813 | -.01790 | -.01744 | .00067 | 2 | |
| X6 | X7 | X8 | X9 | | 4.0 |
| .00867 | -.02269 | -.01912 | -.00488 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01364 | -.01570 | -.01106 | -.01300 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .02035 | -.01151 | .03042 | -.00139 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .05920 | -.02261 | -.01083 | -.00548 | 2 | 8.0 |
| X6 | X7 | X8 | X9 | | |
| .01356 | -.02583 | -.01966 | -.01235 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01352 | -.01520 | -.01683 | -.01853 | 4 | |

Table 13 Influence Ordinates
Six Columns L/F = 12

| Ordinates | | | | | Point | Id/Ic |
|---|---|---|---|---|---|---|
| X1 | X2 | X3 | X4 | X5 | | |
| .07683 | -.01377 | -.42856 | .00024 | -.00203 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07588 | -.03653 | -1.35986 | .00486 | -.00582 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 0.5 |
| .02793 | -.04365 | -2.26661 | .01646 | -.00671 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.02127 | -.02029 | -2.63346 | -.00047 | -.00004 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07322 | -.01372 | -.45535 | .00039 | -.00345 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07385 | -.03330 | -1.34832 | .00768 | -.00897 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 1.0 |
| .02942 | -.03747 | -2.19850 | .02615 | -.00967 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.01616 | -.01465 | -2.53890 | .00043 | .00096 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06726 | -.01323 | -.49517 | .00076 | -.00545 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .07020 | -.02830 | -1.33122 | .01129 | -.01236 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 2.0 |
| .03162 | -.02852 | -2.09619 | .03766 | -.01195 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| -.00859 | -.00661 | -2.39812 | .00293 | .00342 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .05856 | -.01161 | -.54291 | .00175 | -.00786 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .06415 | -.02118 | -1.30672 | .01588 | -.01525 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 4.0 |
| .03382 | -.01730 | -1.96331 | .04974 | -.01246 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00078 | .00287 | -2.21724 | .00755 | .00789 | 4 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .04767 | -.00823 | -.58381 | .00405 | -.01004 | 1 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .05545 | -.01227 | -1.26961 | .02211 | -.01672 | 2 | |
| X1 | X2 | X3 | X4 | X5 | | 8.0 |
| .03458 | -.00552 | -1.81283 | .06200 | -.01042 | 3 | |
| X1 | X2 | X3 | X4 | X5 | | |
| .00995 | .01162 | -2.01689 | .01459 | .01479 | 4 | |

Table 13 cont.

| Ordinates | | | | Point | Id/Ic |
|---|---|---|---|---|---|
| X6 | X7 | X8 | X9 | | |
| .00423 | -.00161 | .00176 | .00122 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .01642 | -.00497 | -.01142 | .00263 | 2 | |
| X6 | X7 | X8 | X9 | | 0.5 |
| .00207 | -.00769 | -.00810 | .00163 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00547 | -.00668 | -.00164 | -.00245 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .00698 | -.00290 | .00418 | .00174 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .02606 | -.00824 | -.01665 | .00318 | 2 | |
| X6 | X7 | X8 | X9 | | 1.0 |
| .00371 | -.01217 | -.01296 | .00095 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.00819 | -.00010 | -.00408 | -.00528 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01069 | -.00489 | .00914 | .00183 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .03744 | -.01239 | -.02038 | .00218 | 2 | |
| X6 | X7 | X8 | X9 | | 2.0 |
| .00637 | -.01706 | -.01844 | -.00213 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01061 | -.01285 | -.00869 | -.01025 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .01543 | -.00759 | .01754 | .00060 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .04934 | -.01679 | -.01982 | -.00158 | 2 | |
| X6 | X7 | X8 | X9 | | 4.0 |
| .01048 | -.02102 | -.02270 | -.00826 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01159 | -.01381 | -.01511 | -.01681 | 4 | |
| X6 | X7 | X8 | X9 | | |
| .02136 | -.01051 | .02917 | -.00255 | 1 | |
| X6 | X7 | X8 | X9 | | |
| .06139 | -.02044 | -.01354 | -.00800 | 2 | |
| X6 | X7 | X8 | X9 | | 8.0 |
| .01670 | -.02272 | -.02353 | -.01595 | 3 | |
| X6 | X7 | X8 | X9 | | |
| -.01003 | -.01173 | -.02114 | -.02253 | 4 | |

# APPENDIX C

# DERIVATION OF EQUATIONS

## Derivation of Elastic Weight Term

The variation of the moment of inertia of the rib, as recommended by Chalos[9], is:

$$I_x = \frac{I_c}{\left(1-0.7\left(\frac{x}{a}\right)^2\right)} \cos\theta \qquad (4)$$

$$\frac{ds}{I_x} = \frac{ds \cos\theta\left(1-0.7\left(\frac{x}{a}\right)^2\right)}{I_c}$$

$$\frac{ds}{I_x} = \frac{dx\left(1-0.7\left(\frac{x}{a}\right)^2\right)}{I_c} \qquad (5)$$

$$a = \frac{L}{2} \qquad \text{therefore} \qquad \frac{x}{a} = \frac{2x}{L}$$

The value of x can be expressed as:

$$x = \left(\frac{25-2n}{2}\right)dx = \left(\frac{25-2n}{2}\right)\frac{L}{24} \qquad (6)$$

where dx is a segment length, n is a segment number, and x is the distance from the arch centerline to the segment center.

$$\frac{x}{a} = \frac{2x}{L} = \left(\frac{24-2n}{24}\right) \qquad (7)$$

Since all moments of inertia, both rib and deck, are relative to $I_c$, this term may be omitted.

Substitute equation (7) into equation (5), then

$$\frac{ds}{I_x} = dx\left(1-0.7\left(\frac{25-2n}{24}\right)^2\right) \qquad (8)$$

$$dx = \frac{L}{24}$$

$$\frac{ds}{I_x} = \left(\frac{1}{24} - \frac{0.7(25-2n)^2}{13824}\right)L \qquad (9)$$

This expression is valid for all values of n between 1 and 24 inclusive since the term $(25-2n)^2$ is always positive.

## Derivation of the Term $\frac{ds}{A_x}$

The exact expression for $\frac{ds}{A_x}$ is:

$$\frac{ds}{A_x} = \frac{dx\ A_x}{\cos\theta}$$

where

$$ds = \frac{dx}{\cos\theta}$$

For convenience, the value of $A_x$ should be in terms of $I_x$.

$$I_x = \frac{bh_x^3}{12} \qquad h_x = \left(\frac{12I_x}{b}\right)^{1/3}$$

$$A_x = bh_x = b\left(\frac{12I_x}{b}\right)^{1/3}$$

$$\frac{ds}{A_x} = \frac{dx}{b\cos\theta}\left(\frac{b}{12I_a}\right)^{1/3} \qquad (10)$$

Substitute the expression for $I_x$ as given in equation (4) into equation (10) and obtain:

$$\frac{ds}{A_x} = \frac{dx}{b \cos\theta}\left(\frac{b\left(1 - 0.7\left(\frac{x}{a}\right)^2\right)\cos\theta}{12\ I_c}\right)^{1/3} \tag{11}$$

The rib height at the crown is assumed to be L/50, thus:

$$I_c = \frac{b}{12}\left(\frac{L}{50}\right)^3 = \frac{b\ L^3}{12(50)^3}$$

Replace $I_c$ in (11) by its value as expressed above, thus:

$$\frac{ds}{A_x} = \frac{50\ dx}{L\ b \cos\theta}\left(\left(1 - 0.7\left(\frac{x}{a}\right)^2\right)\cos\theta\right)^{1/3} \tag{12}$$

An exact evaluation of equation (12) for $\frac{ds}{A_x}$ is not warranted in view of the influence which rib shortening has on the determination of the redundants. The maximum influence occurs at F/L equal to 1/3 and is no more than 25 per cent. If an approximate expression is used in place of equation (12) which deviates from this equation by 20 per cent, an error of only 5 per cent results.

An approximate expression is used in lieu of equation (12). The derivation of this expression is:

$$A_c = \frac{12\ I_c}{h_c^{\ 2}} = \frac{12\ I_c}{\left(\frac{L}{50}\right)^2} = \frac{30{,}000 I_c}{L^2}$$

$$ds \approx dx \qquad\qquad A_x \approx A_c$$

$$\frac{ds}{A_x} \approx \frac{dx}{A_c} = \frac{L^2\ dx}{30{,}000\ I_c} = \frac{L^3}{720{,}000\ I_c} \qquad (13)$$

$$\frac{dx}{A_c} = \frac{(50)^3\ L^3(12)}{720{,}000\ b\ L^3} = \frac{50}{24b}$$

The exact expression for $\frac{ds}{A_x}$ can be expressed as:

$$\frac{ds}{A_x} = \frac{24\ dx}{L\ \cos\ \theta}\left[\left(1-0.7\left(\frac{x}{a}\right)^2\right)\cos\ \theta\right]^{1/3} \quad \frac{dx}{A_c} = (e)\frac{dx}{A_c}$$

Bu appropriate substitution, the following values of "e" are obtained.

| segment number | "e" L/F = 6 | "e" L/F = 12 |
|---|---|---|
| 2 | 0.839 | 0.875 |
| 5 | 0.918 | 0.934 |
| 8 | 0.971 | 0.977 |
| ave. | 0.909 | 9.925 |

Thus, the approximate value, $\frac{dx}{A_c}$, as used in the analysis differs from the exact value, $\frac{ds}{A_x}$, by less than 10 per cent. If the normal force represents 25 per cent of the influence on redundants (flexural forces would then represent 75 per cent), then the error caused by using the approximate value of $\frac{ds}{A_x}$ is 2.5 per cent.

# LIST OF SYMBOLS

| | |
|---|---|
| $a$ | One-half the length |
| $A_c$ | Area of rib at crown |
| $A_x$ | Area of rib at distance x from structure centerline |
| $b$ | Width of rib, assumed equal to one |
| $ds$ | Straight line segment length along the rib centerline |
| $dx$ | Horizontal projection of ds |
| $d_{ij}$ | Movement in base structure at point i from applied redundant or external force at point j |
| $d_{ij(a)}$ | $d_{ij}$ movement caused by axial strain energy |
| $d_{ij(f)}$ | $d_{ij}$ movement caused by flexural strain energy |
| $d'$ | Incremental value of a d at a segment centerline, i.e. $d_{ij} = \sum_a^b d'_{ij}$ |
| $E$ | Modulus of elasticity (E used as iteration constant in computer program) |
| $F$ | Rise of arch, distance from springing to the rib centerline at the crown |
| $H_c$ | Rib depth at crown |
| $I$ | Moment of inertia |
| $I_d$ | I of deck |
| $I_c$ | I of rib at crown |
| $I_x$ | I of rib at distance x from centerline of structure |
| $M$ | Moment at a segment centerline in the base structure due to the application of a redundant |
| $\bar{M}$ | Moment at a segment centerline in the base structure due to the application of a unit force |
| $N$ | Normal force analogous to M |
| $\bar{N}$ | Normal force analogous to $\bar{M}$ |

| | |
|---|---|
| n | Segment number |
| $\tan\theta$ | Slope of the rib centerline |
| x | Distance from arch centerline to centerline of a segment |
| $X_i$ | Redundant at point i |
| $\theta$ | Angle between a horizontal line and a line tangent to the rib centerline |

SIGN CONVENTION

| | |
|---|---|
| Positive moment | Tension on the bottom face |
| Negative moment | Tension on the top face |
| Negative sign on an influence ordinate | Indicates redundant force acts in opposite direction from assumed direction on base structure figure |

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