FAULT STUDY AND GROUND FAULT PROTECTION OF A NETWORK SYSTEM,

TENNESSEE PUBLIC SERVICE COMPANY, KNOXVILLE 13.2 KV. DISTRIBUTION SYSTEM.

A Thesis

Submitted for the Degree of

ELECTRICAL ENGINEER

by

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Approved by:

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Chas H. Graham

Atlanta, Ga.,
May 1, 1932.

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Introduction

The Knoxville distribution system of the Tennessee Public Service Company receives 110 KV. power at the Washington primary substation. Transformation to 13.2 KV. is made here, and energy for domestic and industrial use is distributed throughout the network.

Fig. 1 is a representation of the lines and generating stations considered as supplying current to the Knoxville network under fault conditions. Other generating stations are interconnected, but they are sufficiently remote to be neglected. As a matter of fact, the group of Fig. 1 is a small section of a completely interconnected network of power systems, extending from Mississippi through Alabama, Tennessee, Georgia, North Carolina, and South Carolina, and up into Virginia. Some of the largest hydro and steam plants in the states are parts of this general network, and the capacity connected is enormous.

That the Knoxville 13.2 KV. network is in the nature of a shunt load connected by comparatively low capacity equipment to the general system considerably reduces fault KVA., and, as will be evident in the study, none of the stations supplying power are overloaded, unless operating at capacity at the time of the additional demand. The individual machines in the various plants therefore operate well within the range of their voltage regulators, and any demagnetizing action of fault current is readily compensated for by these regulators. This consideration obviates the necessity for a thorough investigation of machine characteristics, and warrants the conclusion that fault currents, as ascertained for the 13.2 KV network, may be regarded as values substantially sustained.

One source of power is the Waterville Hydroelectric plant of the Carolina Power and Light Company. Here are three generators, each rated 45,000 KVA., 80% power factor, 400 R.P.M., 13,800 volts, three phase, 60 cycles. Each generator connects directly to its bank of transformers, all paralleling being made on the high side. Transformers are single phase, 15,000 KVA., the bank connected delta-wye, ratio 13.2/110 KV., and the high side is solidly grounded. Generator voltage regulation is of the quick response type, and in addition, automatic frequency and load control, similar to that at the Norwood station in the eastern network of the same operating company, is provided. Of the several different types of control afforded by this "automatic operator", that usually in effect is constant station load with economic distribution between two or more machines in parallel.

Waterville power is transmitted to Arlington over two routes. The main line is direct, approximately 50 miles, and operates at 110 KV. This line is new, having been put up about the time Waterville was erected. The older line consists of a section operated at 66 KV., extending from Arlington to Newport, via Jefferson City. Auto-transformers at Newport transform to 110 KV., the line going thence to Bridgeport, where it taps onto the Waterville-Kingsport 110 KV. line. Comparatively, the older line is considerably "weaker" than the direct. One section is of #1 wire - that from Newport to

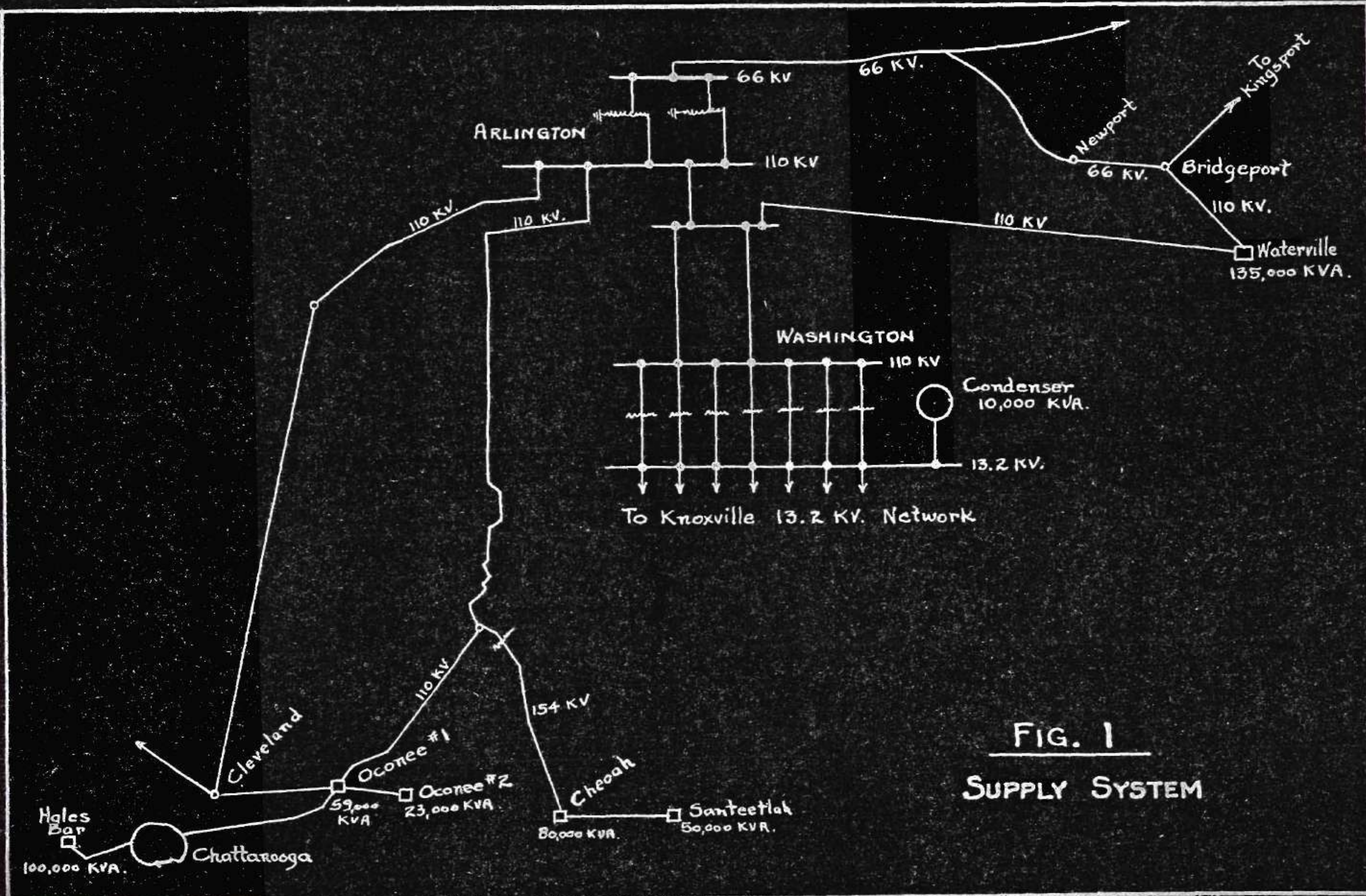


FIG. 1

SUPPLY SYSTEM

Jefferson City, while the section from Jefferson City to Arlington is of one size larger. The direct line is 250,000 cm.

Other than as shown in Fig. 1, the only information at hand regarding characteristics of the system supplying power to Arlington over the Maryville and Lenoir City lines is the short circuit KVA. for a three phase fault at Arlington 110 KV. bus, and ground current for single phase to ground fault at the same location. This is enough for the purpose of this study.

Two short parallel lines connect the 110 KV. busses of the Washington primary and Arlington substations.

Transformation to 13.2 KV. at Washington is made by six parallel banks of transformers, two 3750, two 5000 and two 10,000 KVA., a total of 37,500 KVA. High and low sides of all are connected wye. The two 5000 KVA. are three winding, tertiaries also connected wye, ungrounded. One of the 10,000 KVA. banks is three winding, its tertiary in delta, unloaded; low side of this bank is grounded through a resistance of 20 ohms, high side solidly grounded. This transformer provides the only path for ground current between the 110 and 13.2 KV. systems.

Eight feeders take off from the Washington 13.2 KV. bus, but two of these are dead end and are omitted from consideration in this study.

There are six substations within the 13.2 KV. network proper, the arrangement approximating a loop or ring system, with the addition of a few criss-crosses. Fig. 2 represents the general arrangement, shows the relative location

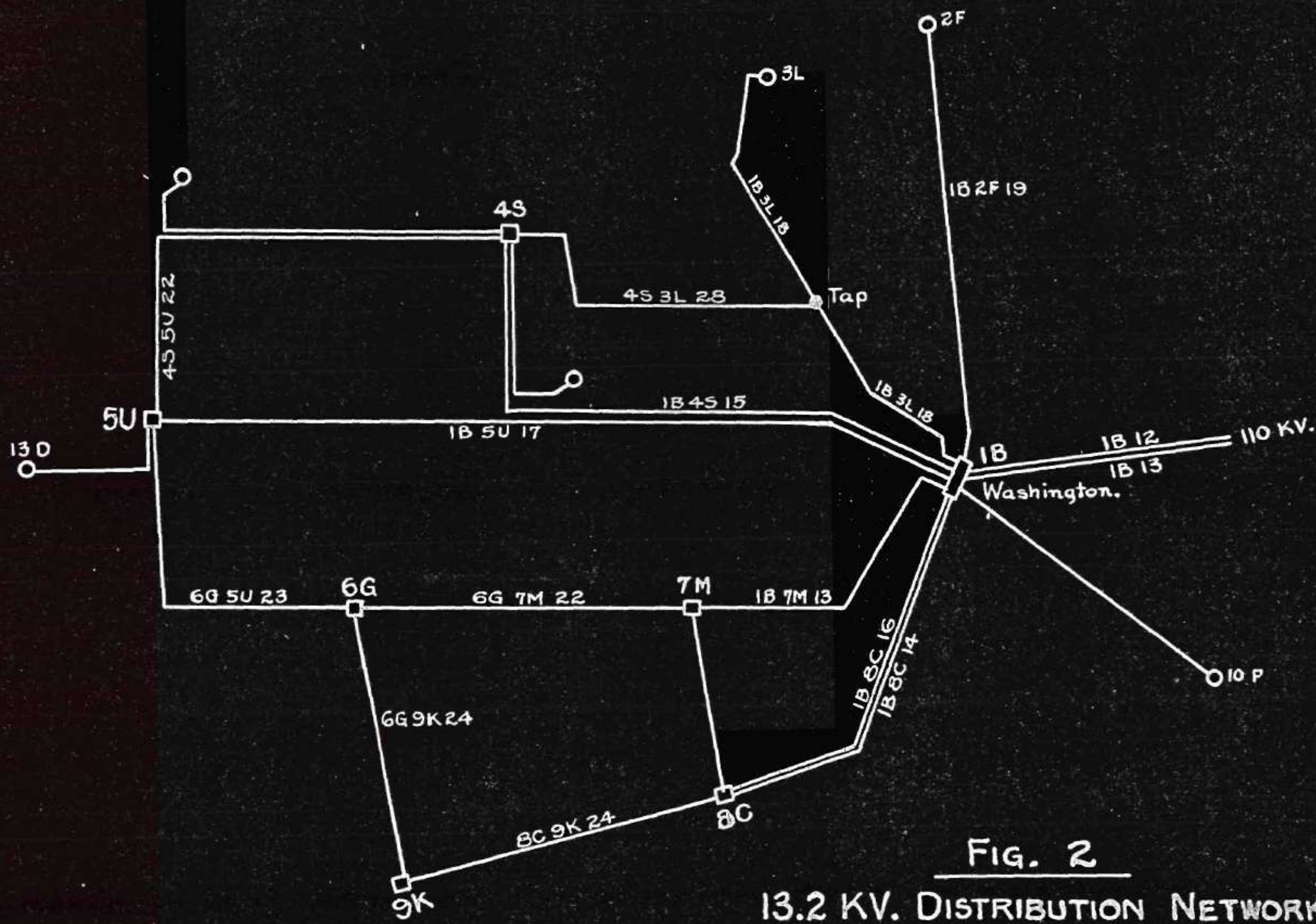


FIG. 2

13.2 KV. DISTRIBUTION NETWORK.

of the stations, and also indicates the few instances of parallel line construction.

All line construction within the network is wood pole, standard N.E.L.A. medium voltage arms. Single circuit uses a pole top pin with 7 ft. arm on second gain; double circuit utilizes a 10 ft. arm on second gain, 7 ft. arm on first, each circuit completely on its side of the pole.

All transformer banks within the 13.2 KV. network, and also at the termination of dead end 13.2 KV. feeders, for transforming to various subsidiary lower voltage distribution systems, are connected delta-delta. This is important in that a path for zero sequence current is not provided by them.

Present protection is straight overcurrent line and ground relays on all feeders taking off from the Washington bus. Directional overcurrent line and ground relays are provided for all network lines at the other substations. Dead end feeders from other substations are protected by overcurrent and/or overcurrent ground relays, but these are not considered in this investigation.

The distribution system originally operated ungrounded, with consequent impossibility to automatically detect and sectionalize any portion in which a single phase might be faulted to ground. Shifting of the neutral caused incorrect operation of the directional overcurrent relays.

The system neutral was then solidly grounded, but difficulty was experienced in getting sufficient potential to operate ground relays. Recently a 20 ohm resistor was installed in the neutral of the grounded transformer. What currents and voltages exist within the network under fault conditions, now that the neutral is grounded through this resistor? This is the question considered in the study.

Study

The primary purpose of this study is the determination of the magnitude and direction of residual current and voltage in the network branches upon occurrence of a ground fault within the 13.2 KV. network of the Tennessee Public Service Company's Knoxville Distribution System.

The network is solved for the four types of faults, and factors are given for the ready determination of all currents in all branches, but the results are discussed only in connection with the ground relay scheme.

The method of symmetrical components is now in general use for system short circuit studies, and is employed whether the actual calculations are made on the network calculating board or by mathematical analysis. The latter is more adaptable to small systems, and is followed in this study.

Complex notation is used for all vector quantities, except wherein one of the components is less than 1/5 the other, in which case the actual modulus is considered all real or all imaginary, according to the predominant component of the original vector. This applies particularly to the zero sequence network impedances, computations being carried out with values as scalar quantities, but the result is converted to the complex notation by average multipliers, in order to more closely approximate the actual result.

A summary of the procedure may be of assistance in following the study:

(1) Collection and tabulation of all system data, as received.

(2) Conversion of data as received into form required for making computations.

(3) Tabulation of system constants.

(4) Single line diagrams of supply positive, negative and zero sequence networks.

(5) Reduction of supply system component networks into "equivalent sources".

(6) Single line diagram of distribution system positive, negative and zero sequence networks.

(7) Reduction of distribution system component networks to "equivalent impedances" to point of fault, and determination of "distribution factors" of all branches, for various locations of faults.

(8) Determination of the sequence component currents for the various types of faults.

(9) Tabulation of distribution factors, with component sequence currents, for the various faults at the different locations.

(1) System data, as received.

Table 1 gives transmission and distribution line characteristics as received. Single and double circuit wood pole construction within the 13.2 KV network is indicated in Fig. 3. One column of Table 1 indicates, by the notation S or D, which of Fig. 3 applies. This same column for the high tension lines carries the equivalent spacing, as there is some question about the actual arrangement of conductors for these lines.

Table 2 collects data on the various transformers within the supply network. There are no transformers within the 13.2 KV. network having bearing on this study.

Complete and definite information is at hand only on the Waterville generators. This is:

Rating: 45,000 KVA., 80% power factor, 3 phase,
60 cycle, 13,800 volts, 400 R.P.M.

Characteristics, on rating base.

Positive sequence -

Synchronous reactance,	X_d	-	107%
Transient reactance,	X'_d	-	28%
Subtransient reactance,	X''_d	-	14%
Negative Sequence reactance	X_2	-	16%
Zero sequence reactance,	X_0	-	18%

Time constants -

Open circuit transient	T_{do}	-	8.7 sec.
Short circuit transient	T'_d	-	2.3 sec.
Short circuit subtransient	T''_d	-	.035 sec.

The 10,000 KVA. condenser on the Washington 13.2 KV. bus. Reactance given as 80% on 20,000 KVA. base.

The only information regarding the system supplying power to the Arlington 110 KV. bus over the Maryville and Lenoir City lines is as follows:

For 3 phase short circuit on Arlington 110 KV. bus.

	Maximum	Minimum
Maryville line	300,000 KVA.	110,000 KVA.
Lenoir City line	135,000 KVA.	125,000 KVA.

Ground current for single phase to ground fault at Arlington 110 KV bus.

	Maximum	Minimum
Maryville line	816 amps.	234 amps.
Lenoir City line	507 amps.	440 amps.

"The difference between maximum and minimum values shown above is due chiefly to the fact that Alcoa is considered in parallel for the maximum conditions."

"As a result of tests run jointly by the Tennessee Public Service Company and the Southern Bell Telephone and Telegraph Company, it was found that earth conductivity could be taken as 10^{-14} " abmhos per cm^3.

(2) Conversion of data.

System studies may be made upon an arbitrarily chosen KVA. or voltage base. Whichever is used, it is necessary to convert all constants into figures referred to the chosen base before computations may be made. The KVA. base is more adaptable to solution by means of the calculating board, whereas the voltage base is better adapted to the mathematical solution.

Since the distribution network under consideration operates at 13.2 KV., this voltage is used as the reference base, and constants of all equipment are corrected accordingly.

Calculated values of percent reactance and zero sequence impedance check the constants as received in all but a few instances, in which cases, calculated values are accepted. Furthermore, received figures cover the reactance component only (in the positive and negative sequence networks) whereas resistance component is comparable with reactance, particularly in the 13.2 KV network.

Line or Section	Operating KV.	Length 10^3 ft.	Size Cond.	Const. Fig. 3	Percent Reactance 20000 KVA. base	Zero Seq. Impedance, Ohms.
Waterville - Bridgeport.	110	66	250	Eq. Spa. 18.3 ft	1.7	$10.5+j\,35.5$
Bridgeport - Newport.	110	23.75	2/0 A.C.S.R.	–	–	–
Newport - Jefferson City.	66	110.3	#1	Eq. Spa. 7.6 ft.	7.9	$61.4+j\,104$
Jefferson City - Arlington	66	141.5	1/0	Eq. Spa. 7.6 ft.	9.82	$39.2+j\,106$
Waterville - Arlington direct	110	280	250	Eq. Spa. 18.3 ft.	7.3	$26.8+j\,162.5$
1B 12	110	10.56	2/0	Eq. Spa. 8.82 ft.	0.3	$1.5+j\,6.0$
1B 13	110	10.56	2/0	Eq. Spa. 8.82 ft.	0.3	$1.5+j\,6.0$
1B 3L 18 (to tap)	13.2	5.0	2/0	S	–	–
4S 3L 28	13.2	6.3	2/0	S	14	–
1B 4S 15	13.2	9.9	4/0	D	17	–
1B 5U 17	13.2	15.0	4/0	D-1/2 S-1/2	22	–
1B 7M 13	13.2	5.4	2/0	S	8	–
1B 8C 14	13.2	7.2	4/0	D	10	–
1B 8C 16	13.2	7.2	4/0	D	10	–
6G 7M 22	13.2	6.0	2/0	S	9	–
7M 8C 21	13.2	3.2	2/0	S	8	–
8C 9K 24	13.2	5.2	2/0	S	6	–
9K 6G 24	13.2	6.0	2/0	S	9	–
6G 5U 23	13.2	7.2	2/0	D	10	–
4S 5U 22	13.2	12.0	4/0	D	17	–

TABLE 1

TRANSMISSION AND DISTRIBUTION LINES
DATA AS RECEIVED - SUPPLY & NETWORK.

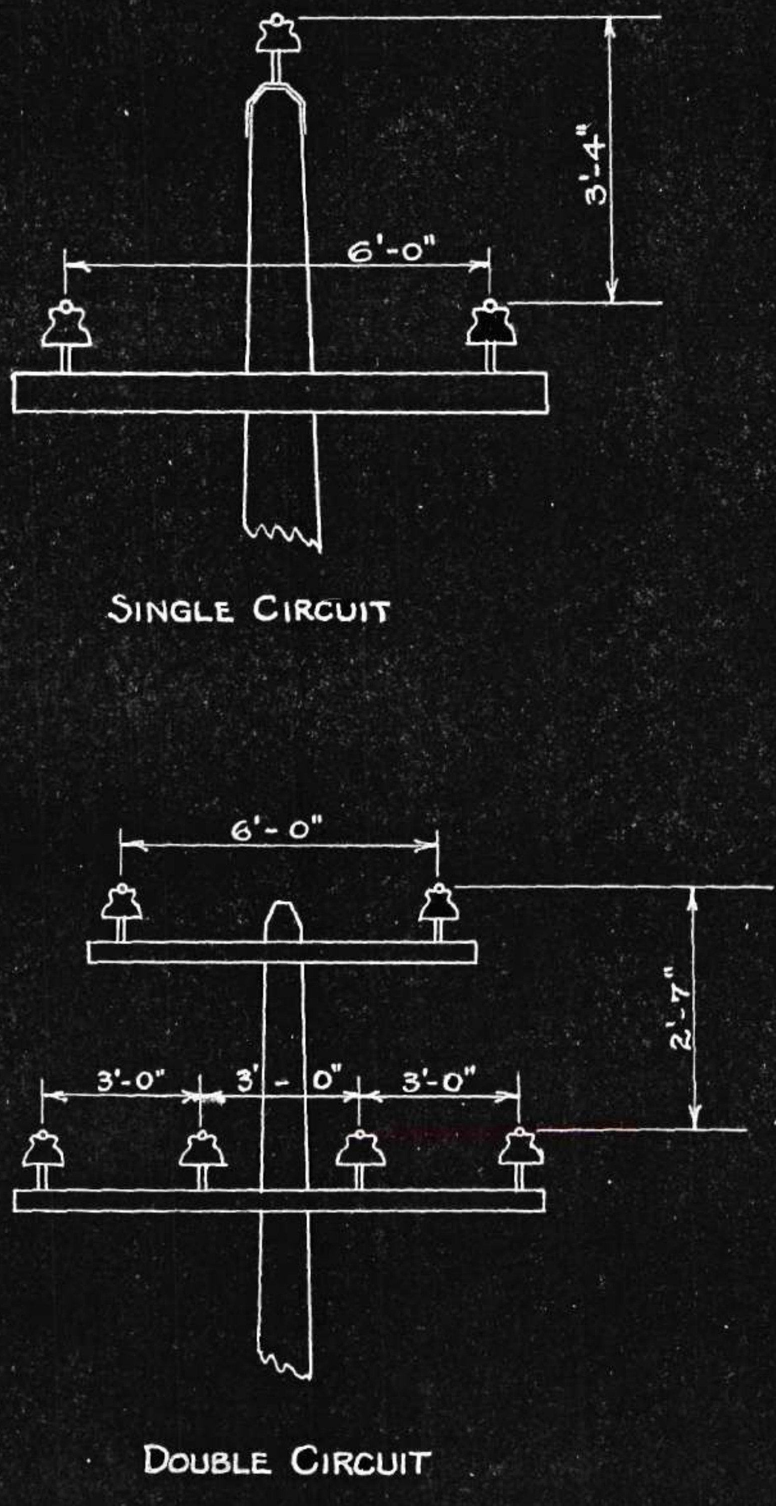


Fig. 3
13.2 KV. Line Construction

Location of Transformer	3 Phase KVA. of Main Windings	Winding	Connection			Line KV.		Percent Reactance 20,000 KVA. Base.		
			High	Low	Ter.	High	Low	High to Low	High to Ter.	Low to Ter
Waterville	45,000	2	Y⏚	Δ		110	13.2	3.61	–	–
Waterville	45,000	2	Y⏚	Δ		110	13.2	3.61	–	–
Waterville	45,000	2	Y⏚	Δ		110	13.2	3.61	–	–
Newport	9,000	Auto-3	Y⏚	Y⏚	Δ	110	66	12.4	30.7	21.3
Arlington	7,500	Auto-3	Y⏚	Y⏚	Δ	110	66	8	45.0	35.4
Arlington	7,500	Auto-3	Y⏚	Y⏚	Δ	110	66	8	45.0	35.4
Washington	3,750	2	Y	Y		110	13.2	60	–	–
Washington	3,750	2	Y	Y		110	13.2	59	–	–
Washington	5,000	3	Y	Y	Y	110	13.2	48	?	?
Washington	5,000	3	Y	Y	Y	110	13.2	47	?	?
Washington	10,000	2	Y	Y		110	13.2	22	–	–
Washington	10,000	3	Y⏚	Y⏚	Δ	110	13.2	22	23	6.3

TABLE 2

TRANSFORMERS.

DATA AS RECEIVED – SUPPLY AND NETWORK

The positive and negative sequence impedance of all lines (the two are equal) are determined from standard 60 cycle reactance tables and are recorded in one column of Table 3. The method employed is that usually followed for the customary impedance of transmission lines, and no particular comment is necessary.

Zero sequence impedance is calculated by the formula:

$$Z_0 = R + .0543 + j0.1587 \log_{10} \frac{De}{G.M.R.}, \quad \text{ohms per 1000 ft. per phase, 60 cycles.}$$

Earth conductivity for the supply system is given as 10^{-14} abmhos per cm^3, corresponding to an "equivalent depth" of 8800 ft. Conductivity of the earth under the distribution system is assumed 10^{-13}, the value for damp soil, because of the proximity of rivers. This corresponds to a De of 2800 ft.

R in the above formula is determined from standard wire tables in the same manner as for the positive and negative sequence impedances. About all now required is determination of G.M.R., the group geometric mean radius of the three line conductors. To to this -

First, find conductor G.M.R. (example: line 1B4S15)

Conductor G.M.R. = conductor radius x factor

$$= \frac{.528}{2} \times .759, \text{ for 4/0, 19 strand.}$$

$$= .202 \text{ inches}$$

$$= .0167 \text{ ft.}$$

Then,

$$\text{Group G.M.R.} = [\text{Conductor G.M.R.} \times (\text{Equivalent Spacing})^2]^{1/3}$$

The equivalent spacing for 1B4S15 is:

$$[3 \times 3 \times 2.585]^{1/3}$$

or 2.985 ft.

Therefore:

$$\text{Group G.M.R.} = [.0167 \times (2.985)^2]^{1/3}$$
$$= 0.530 \text{ ft.}$$

The zero sequence impedance:

$$Z_0 = .053 + .0543 + j\ 0.1587 \log_{10} \frac{2800}{.53}$$

$$= .1073 + j\ .572 \text{ ohms per 1000 ft. one conductor.}$$

$$= 1.06 + j\ 5.85 \text{ for the total length of 1B4S15.}$$

Since the line operates on 13.2 KV., this value is already upon the proper voltage base, and no reduction is required.

1B4S15 and 1B5U17 are in parallel for 7.5×10^3 ft. To determine the mutual zero sequence impedance, it is first necessary to find the geometric mean distance between the two circuits.

$$\text{G.M.D.} = [d_{1a} \times d_{1b} \times d_{1c} \times d_{2a} \times d_{2b} \times d_{2c} \times d_{3a} \times d_{3b} \times d_{3c}]^{1/9}$$

which, for the double circuit wood pole construction, is

$$= [3 \times 6 \times 5.17 \times 6 \times 9 \times 7.92 \times 5.17 \times 7.92 \times 6]^{1/9}$$
$$= 5.98 \text{ ft.}$$

the mutual impedance, zero sequence -

$$Z_{0m} = .0543 + j\ 0.1587 \log_{10} \frac{De}{\text{G.M.D.}}, \text{ ohms per 1000 ft. per phase, 60 cycles.}$$

which, for the lines in question, becomes:

$$Z_{0m} = 0.0543 + j\ 0.1587 \log_{10} \frac{2800}{5.98}$$

$$= .0543 + j\ 0.424 \text{ ohms per 1000 ft., one conductor.}$$

or

$Z_{0m} = 0.406 + j\ 3.18$ for the 7.5×10^3 ft. the two lines are together. These lines are operated at the reference voltage, and no reduction is necessary.

1B8C16 and 1B8C14 are in parallel for their entire length. The zero sequence (self) impedance of the two groups is:

$$Z_0 = \frac{R}{2} + .0543 + j\ 0.1587 \log_{10} \frac{De}{[(G.M.R.)\ (G.M.D.)]^{1/2}}\text{, ohms}$$

per 1000 ft., per phase, 60 cycles.

G.M.R. is group G.M.R., = 0.53 ft. for #4/0 wire double circuit construction. G.M.D. is 5.98 ft.

Hence

$$Z_0 = 0.0264 + 0.0542 + j\ 0.1587 \log_{10} \frac{2800}{[0.53 \times 5.98]^{1/2}}\text{, ohms}$$

per 1000 ft.

$= 0.58 + j\ 3.65$ ohms, total length of the lines.

These lines are within the 13.2 KV network and the impedance as determined is upon the reference base.

Table 3 is a compilation of the transmission and distribution line constants.

For transformers, it is customary to neglect the resistance component of the impedance, as it is an inappreciable part of the whole.

The five ungrounded transformers at the Washington substation are most conveniently treated in parallel. The impedances of these devices, on 20,000 KVA. base, are, respectively, 22, 60, 48, 47 and 59 percent. Hence the impedance of the parallel group is

$$Z = \frac{1}{\frac{1}{22}\quad\frac{1}{60}\quad\frac{1}{48}\quad\frac{1}{47}\quad\frac{1}{59}}$$

$= 8.25\%.$

Converting into ohms, on 13.2 KV. base

$$\text{ohms} = \frac{8.25 \times 13.2^2 \times 10}{20{,}000}$$

$$= j\ 0.718 \text{ ohms.}$$

And this value is recorded in Table 4 for the group.

There is no path for zero sequence current through any of the above transformers, and the zero sequence impedance is therefore infinity, represented on the network diagram by an open circuit, as shown in Table 4.

The 10,000 KVA. three winding transformer at Washington:

Windings Considered	Given Percent Reactance 20,000 KVA. base	Corresponding ohms on 13.2 KV. base
110 to 13.2	22	j 1.91
110 to Ter.	23	j 2.0
13.2 to Ter.	63	j 5.47

Therefore, reactance of the individual windings

$$X_{110} = j\ \frac{1.91 + 2.0 - 5.47}{2} = -j\ .78 \text{ ohms}$$

$$X_{13.2} = j\ \frac{1.91 + 5.47 - 2.0}{2} = j\ 2.69 \text{ ohms}$$

$$X_{Ter.} = j\ \frac{5.47 + 2.0 - 1.91}{2} = j\ 2.78 \text{ ohms}$$

The 20 ohm resistor in neutral of the 13.2 KV. side is applied in the zero sequence network as 60 ohms, connected to the transformer 13.2 KV. lead.

The portion of appendix A devoted to constants of transformers explains in greater detail the treatment.

Table 4 gives representation on the component network diagrams for the various transformers or transformer groupings.

It is necessary to replace the Maryville and Lenoir City lines by an "equivalent source". Using values for the maximum condition, these sources are determined as follows:

For the Maryville line - maximum for a three phase short circuit on the Arlington 110 KV. bus is 300,000 KVA.

The theory of symmetrical components demonstrates that only positive sequence impedances limit current supplied to a three phase short. Therefore, referred to the 13.2 KV. base, the positive sequence impedance of the Maryville line is

$$Z_1 = \frac{13,300^2}{300,000,000} = 0.58 \text{ ohms.}$$

Experience indicates that negative sequence system impedance may be assumed as 90% the positive sequence impedance. Hence

$$Z_2 = 0.58 \times .9 = 0.52 \text{ ohms.}$$

Also from symmetrical component theory, the zero sequence current for a single phase to ground fault is limited by the sum of the three sequence impedances. Since the ground current, given, is three times the zero sequence current, and since line voltage is 110 KV.:

$$\Sigma \text{ component sequence impedances} = \frac{110,000 \times 3}{816 \times [3]^{1/2}} \text{ ohms.}$$

$$= 234 \text{ ohms on 110 KV. base}$$

$$= 234 \times \left(\frac{13.2}{110}\right)^2 \text{ ohms on 13.2 KV. base}$$

$$= 3.37 \text{ ohms.}$$

$$Z_1 + Z_2 = \underline{1.1} \text{ ohms.}$$

$$\therefore \quad Z_0 = 2.27 \text{ ohms.}$$

These impedances are nearly all reactive, and are used as j values.

Line or Section	Impedance on 13.2 KV. Base: Positive & Negative R + j X	Zero R + j X
Waterville-Bridgeport.	.042 + j .154	.093 + j .556
Bridgeport-Newport.	.044 + j .054	.063 + j .208
Newport-Jefferson City.	.570 + j .675	.810 + j 2.96
Jefferson City-Arlington.	.595 + j .856	.901 + j 3.02
Total-Waterville-Jefferson City-Arlington line.	1.251 + j 1.739	1.867 + j 6.744
Waterville–Arlington direct.	.179 + j .651	.398 + j 2.35
1B 12	.006 + j .012	.015 + j .081
1B 13	.006 + j .012	
1B 3L 18 (to tap)	.421 + j .68	.693 + j 2.86
4S 3L 28	.531 + j .856	.872 + j 3.605
1B 4S 15	.524 + j 1.18	1.062 + j 5.85
1B 5U 17	.793 + j 1.875	1.61 + j 8.68
1B 7M 13	.454 + j .735	.747 + j 3.09
1B 8C 14	.381 + j .857	.58 + j 3.65
1B 8C 16	.381 + j .857	
6G 7M 22	.505 + j .816	.830 + j 3.43
7M 8C 21	.269 + j .435	.443 + j 1.83
8C 9K 24	.437 + j .707	.720 + j 2.96
9K 6G 24	.505 + j .816	.830 + j 3.43
6G 5U 23	.606 + j .900	.997 + j 4.29
4S 5U 22	.635 + j 1.43	1.29 + j 7.08

Zero Sequence Mutual Impedence between 1B4S15 and 1B5U17, parallel for 7.5×10^3 ft. = .406 + j 3.18

TABLE 3

TRANSMISSION AND DISTRIBUTION LINE CONSTANTS.

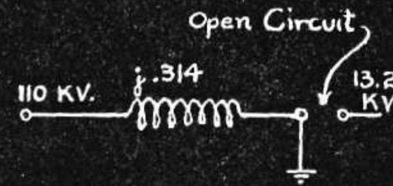
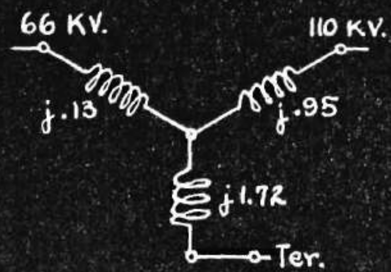
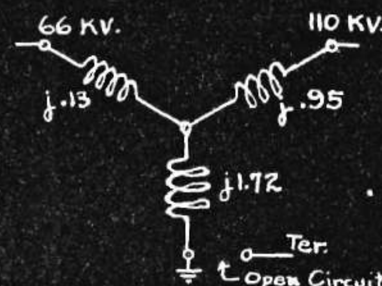
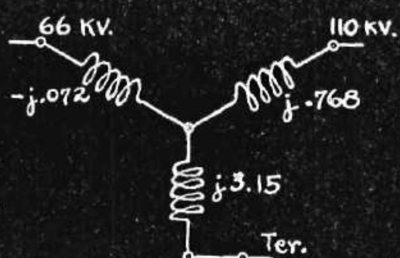
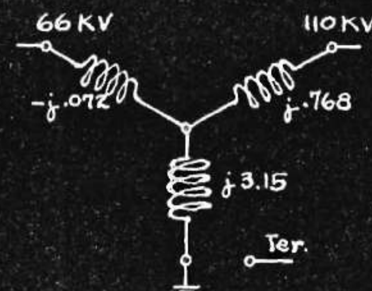
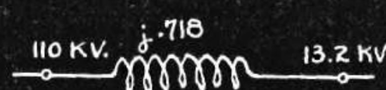
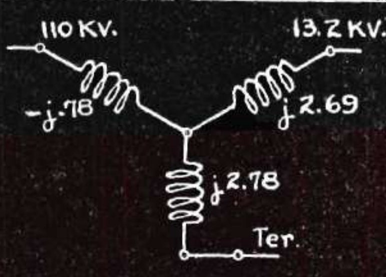
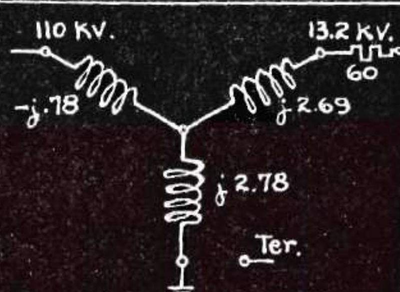
Transformer or Transformer Group	Representation on Positive & Negative Sequence Networks	Representation on Zero Sequence Networks
Waterville 45,000 KVA. 13.2/110 KV.  Representation for each Transformer.		
Newport Auto- 9,000 KVA. 110/66/(ter.) KV. 		
Arlington Auto- 7,500 KVA. 110/66/(ter.) KV.  Representation-each.		
Washington Five Transformers Total Capacity 27,000 KVA. 110/13.2 KV. 		
Washington 10,000 KVA. 110/13.2/(Ter.) KV. 		

TABLE 4.
TRANSFORMER CONSTANTS

Constants of the system behind the Lenoir City line are determined in the same manner, hence, for maximum conditions:

	Z_1	Z_2	Z_0
Maryville line	j 0.58	j 0.52	j 2.27
Lenoir City line	j 1.29	j 1.16	j 2.96

Generator constants are available only for the Waterville machines. The short circuit subtransient time constant corresponds to about 2 cycles, and for the purposes of this study, the current during this interval may be disregarded.

Transient reactance is 28% on machine KVA. base, or 1.08 ohms on 13.2 KV. base.

The negative and zero sequence reactances are likewise found to be, respectively, 0.62 and 0.7 ohms. Because the Waterville transformers are connected delta on the generator side, no path is provided for zero sequence currents between transformer and generator, so the zero sequence impedance of the circuit is infinity. This is covered in the transformer connection by an open circuit on the delta side, in the zero sequence network.

A preliminary calculation indicates Waterville plant will not supply more than approximately 50,000 KVA. to a three phase fault within the Knoxville 13.2 KV. network. This corresponds to a system impedance of 270% on the Waterville plant KVA. rating.

Reference to standard decrement curves indicates that, for system impedances above 100%, initial fault KVA. may be considered as sustained. This, in conjunction with the fact that Waterville generators are provided with quick response excitation and corresponding voltage regulators, indicates no correction is necessary in order to determine KVA. at any particular time after the occurrence of a fault within the Knoxville 13.2 KV. network.

Since machine reactances are considered as external to the source of E.M.F., it is necessary to determine the internal voltage. This may be approximated by the following:

$$E_i = [(E_T + IX_d' \sin \theta)^2 + (IX_d' \cos \theta)^2]^{1/2}$$

in which

E_i = Internal voltage.

I = Load current, before fault.

E_T = Terminal voltage, before fault.

$\cos \theta$ = Power factor

X_d' = Transient reactance.

It has been stated that the fault currents to be determined are to be considered as sustained values. On this basis, the correct internal voltage would be that determined by using the synchronous reactance, X_d, in the above equation, instead of the transient reactance. If this were done, the synchronous reactance would also be used in the network. However, only transient values may be determined from the data given for the Maryville and Lenoir City lines, and as a consequence it is necessary to use corresponding values for the Waterville machines. The difference in the current calculated by the two sets of constants is not as much as seems at first, because both the internal voltage and the system impedance are increased when using synchronous constants. However, current determined by these higher values is somewhat less than as found by the method to be followed, but does not take into consideration the effect of voltage regulators. Not that using transient values makes such correction, but using transient values and considering the resultant current sustained does.

At best the answer is indefinite. With all machine, regulator, prime mover and accessory constants at hand, it would be possible to determine the exact value of fault current at any instant. It is unnecessary to be so thorough. It is customary to make approximations at this point in all studies, and at best the standard decrement curves are average values to be used when system conditions indicate a decrement will occur.

Returning to the initial voltage, assuming machines approximately 80% loaded, 80% power factor, $E_T = 13,200/1.732 = 7620$ volts, $X_d' = 1.08$ ohms, the internal voltage is found to be 8700 volts. This is the "reference" voltage to neutral, and consequently is all real.

Machine constants as discussed above determine R.M.S. values of a current wave symmetrical about the axis of zero current. As is well known, fault currents during the first few cycles may or may not be symmetrical, depending upon the point of the voltage wave at which the fault occurs. It is necessary to take the D.C. component of current into consideration for a number of purposes, but, since current transformers do not transform D.C., the relay current will be same as though the line current were of equal amplitude above and below the zero line, except for the effect caused by saturation of the current transformer iron.

The condenser at Washington primary supplies a little current under fault conditions. The given reactance is 80% on a 20,000 KVA. base, or 40% on the machine rating. This is one of the positive sequence reactances, and probably is the transient, since the figure is an average value for condensers. 40% corresponds to j 7.0 ohms on the 13.2 KV. base. The negative sequence reactance may be assumed as 60% of the positive sequence transient reactance, or j 4.2 ohms. For the zero sequence reactance, notation at hand indicates the machine is un - grounded, hence the zero sequence impedance is infinity, and representation on the network diagram is by an open circuit.

(5) Reduction of Supply System network.

Figures 4 and 5 pertain, respectively, to the supply system positive and negative sequence networks. Group (a) of each figure is a single line diagram of the network with impedances shown in accordance with the arrangement of equipment in the original system, and marked with values appropriate to the individual parts.

The difference between the two is first that there is no source of E.M.F. in the negative sequence network and second impedances of machines or parts involving machines are different for the negative network than for the positive.

Generating equipment produces only positive sequence voltages; the only means by which negative (and zero) sequence voltages get into the systems is by transformation of part of the positive sequence voltage at the point of fault.

Groups (b) and (c) show two of the steps in reducing the original network to "equivalent" values, shown in (d). The method is by successively substituting equivalent single line impedances for two or more in parallel, and no details are required.

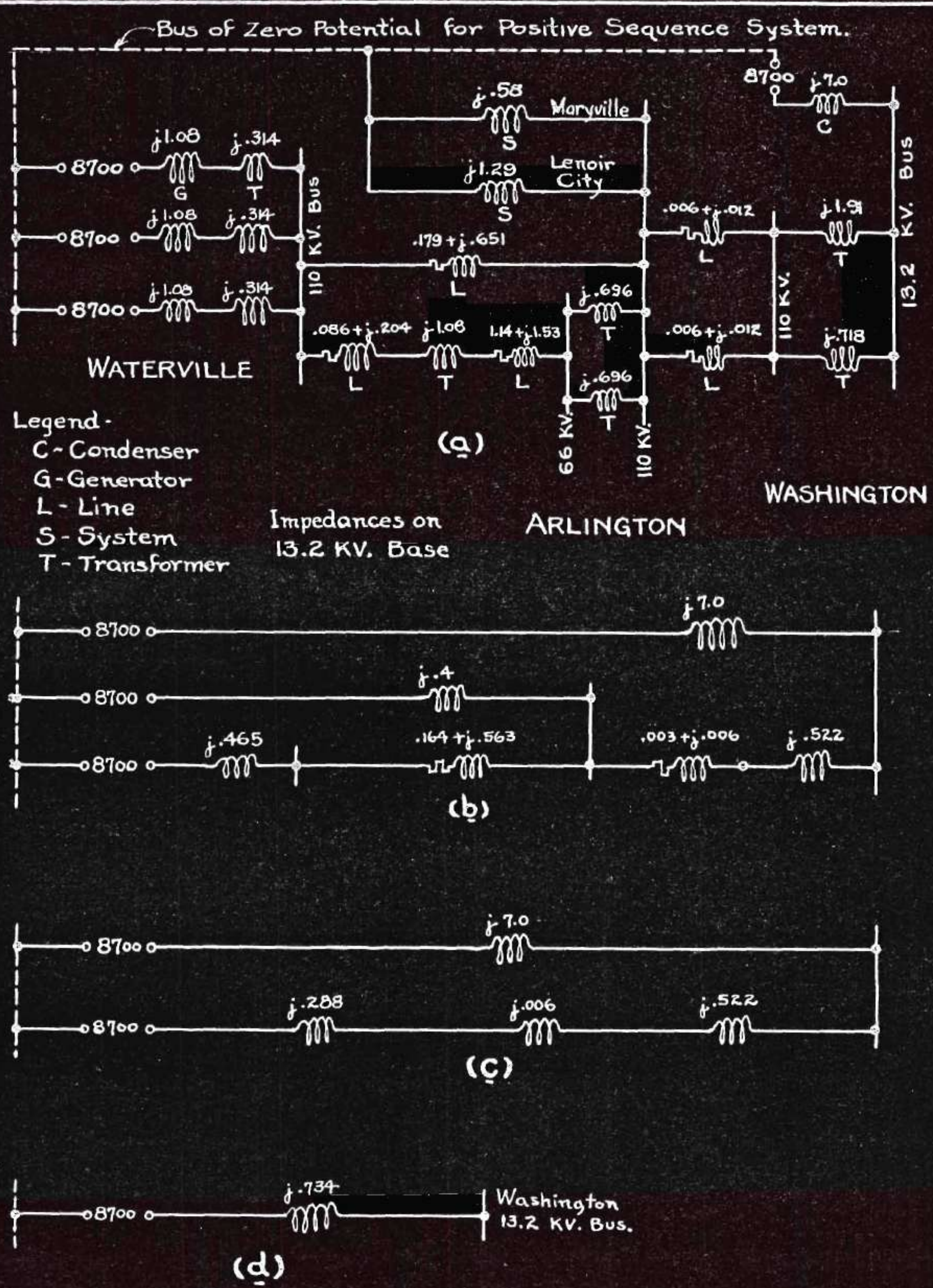


FIG. 4

POSITIVE SEQUENCE SUPPLY SYSTEM REDUCTION TO "EQUIVALENT SOURCE."

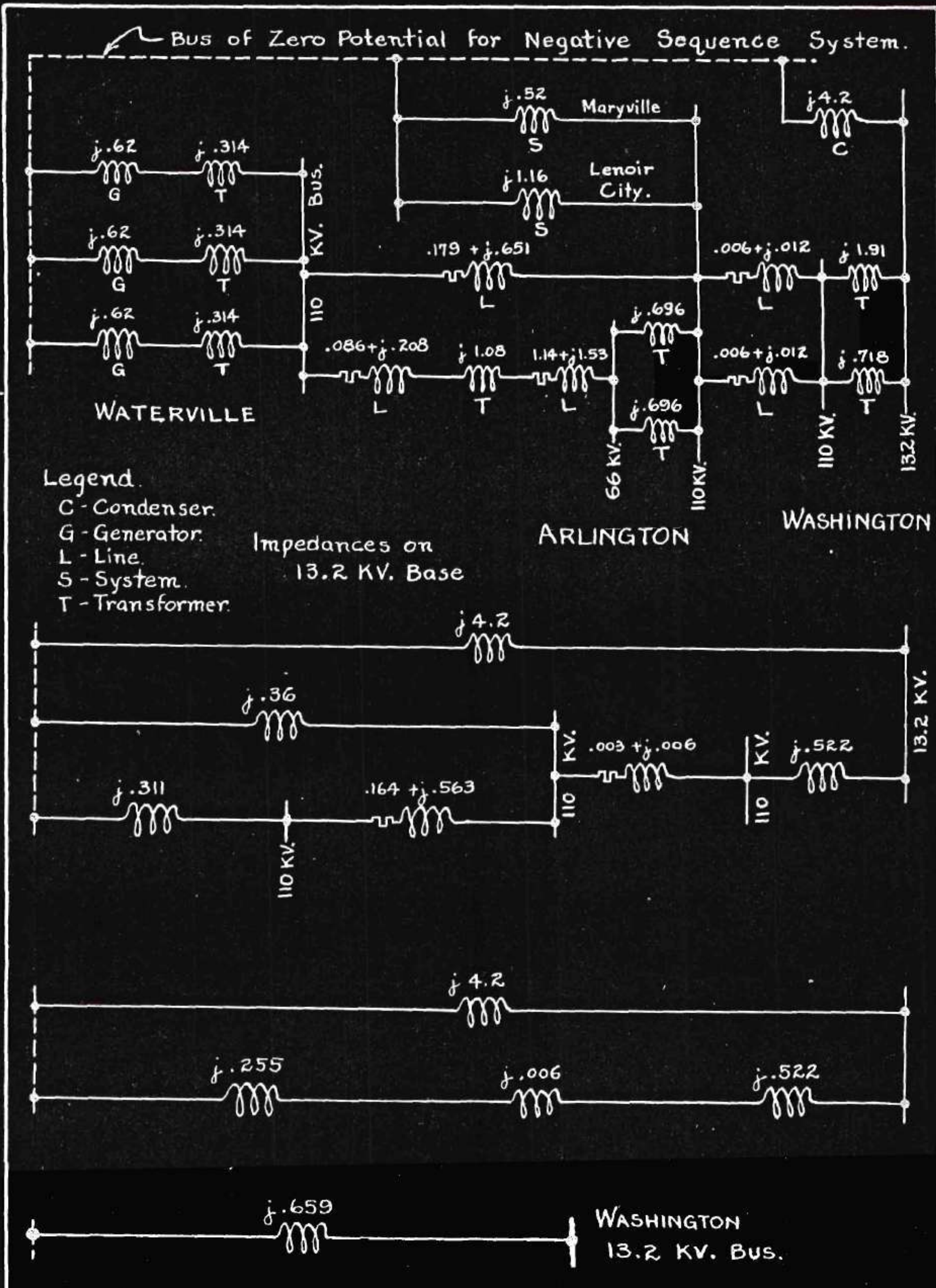


FIG. 5

NEGATIVE SEQUENCE SUPPLY SYSTEM.
REDUCTION TO "EQUIVALENT IMPEDANCE."

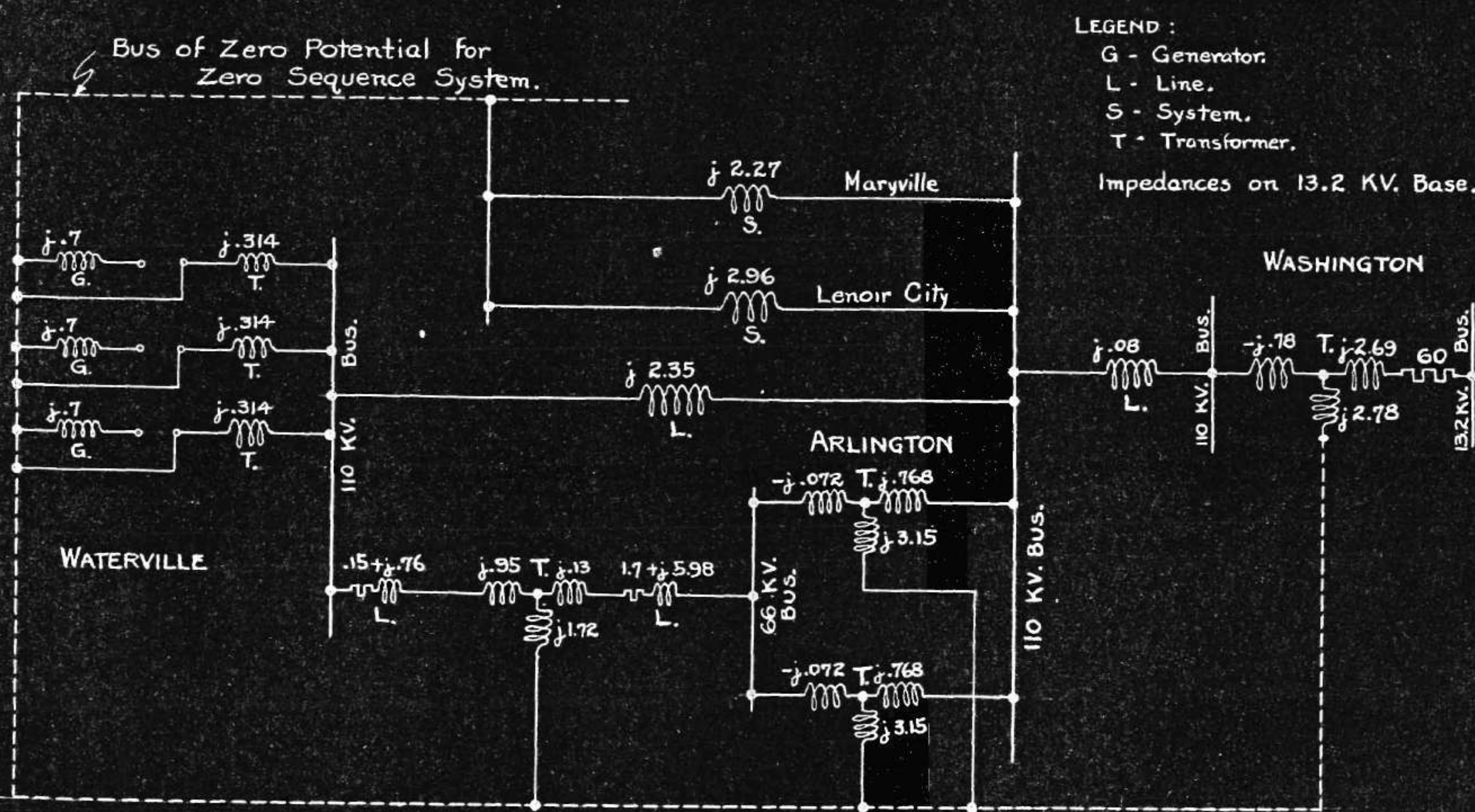


FIG. 6

ZERO SEQUENCE SUPPLY SYSTEM.

It should be noted three winding transformers symbols are not incorporated in the positive and negative sequence networks in the manner indicated by Table 4. This is because the tertiaries do not connect to supply or load, and therefore have no effect in the positive and negative sequence systems.

The zero sequence supply system is shown in Fig. 6. It is not necessary to make computations to reduce this network to an equivalent single line impedance, as by inspection it is apparent a value sufficiently close may be had by disregarding all the j components, and considering only the resistance of the 60 ohms plus resistance of the network. These latter resistances were neglected in making up the diagram because, adding at right angles to the predominant inductive component, they were inappreciable. However, now that the real is the major, the j component is omitted, and real component of the network impedance adds directly to the 60 ohms. The network resistances are small, however, and may just as well be approximated as 1 ohm, making the value considered as supply system zero sequence impedance 61.0 ohms.

(7) Reduction of Distribution System Networks.

Fig. 7 shows the positive and negative, and zero sequence single line diagrams for the 13.2 KV. distribution network. The stations and lines are designated in accordance with Fig. 2 and in addition, sequence impedance of the lines is noted. Arrows indicate an arbitrarily selected direction of current for which values are to be computed. Necessarily the current direction in all branches will not be as shown for all the different fault locations, but suitable correction is made in the distribution factors by putting a minus sign before the factor pertaining to a line in which fault current for the particular location is the reverse of the direction noted by the arrow in Fig. 7. Therefore, multiplying a fault current by any distribution factor results in a value indicating current in the direction shown by the arrows.

Figures 8 through 17 show steps taken in the reduction of the component sequence networks for faults at the different stations. On some of these diagrams arrows are shown, but these refer to direction of current supplied to the fault, and are not to be confused with those of Fig. 7.

The main use to which these reduction diagrams may be put is in computing voltages at the different stations for line relay studies. The problem at hand concerns only ground relays, and it is not required to carry computations of the zero sequence voltage through the distribution network, because practically all the zero sequence impedance is considered in the supply system, so this component of voltage is about the same at all stations within the network.

It may be of some value to give a sample of network reduction. Consider a fault at 4S bus. The positive (and negative) sequence network will be reduced to an "equivalent" impedance, and the distribution factors determined.

From Fig. 7, positive and negative sequence network, it is seen lines 1B3L18 plus 4S3L28 and 1B4S15 are in parallel. Replace these by a single impedance whose value is

$$Z = \frac{1}{\dfrac{1}{.951 + j\,1.54} + \dfrac{1}{.542 + j\,1.18}}$$

$$= .345 + j\,.672 \text{ ohms.}$$

Also, it is necessary to substitute a wye network for the delta formed by lines connecting stations 1B, 7M and 8C. The delta is shown in Fig. 8-(a), the equivalent star in Fig. 8-(b).

Transformation is made as follows:

Sum of the delta impedances $= .913 + j\ 1.6$ ohms.

Then, with T as neutral of the equivalent star-

$$Z_{1B-T} = \frac{(.454 + j\ .735)\ (.19 + j\ .428)}{.913 + j\ 1.6} = .096 + j\ .197 \text{ ohms}$$

$$Z_{8C-T} = \frac{(.19 + j\ .428)\ (.269 + j\ .435)}{.913 + j\ 1.6} = .057 + j\ .117 \text{ ohms}$$

$$Z_{7M-T} = \frac{(.454 + j\ .735)(.269 + j\ .435)}{.913 + j\ 1.6} = .132 + j\ .2 \text{ ohms}$$

The substitutions noted above are now incorporated in the complete network diagram, shown in group (c) of Fig. 8. An equivalent impedance is substituted for the two parallel branches between T and 6G, and the 1B-T-5U branch is replaced by the series of impedances shown in group (d). This group is now in parallel with line 1B5U17, so the two may be replaced by a single line whose impedance is determined as was that replacing lines 1B3L18 plus 4S3L28 in parallel with 1B4S15. Substituting in the network, group (e) results, which is reduced immediately to (f), the equivalent impedance.

It is now required to determine distribution factors. Assume $1.0 + j\ 0.0$ ampere to go into the network at the 1B bus and leave at the 4S bus. The division of current between the two branches of group (e) is inversely as the impedances, or, current through upper branch is

$$\frac{(1.0 + j\ 0.0)(.266 + j\ .527)}{.345 + j\ .672}$$

$$= .778 \text{ amperes.}$$

Current through the lower branch should be calculated in order to check,

$$\frac{(1.0 + j\ 0.0)(.266 + j\ .527)}{1.11 + j\ 2.34}$$

$= .222$ amperes.

The .778 amperes is through 1B3L18 plus 4S3L28 in parallel with 1B4S15. Dividing the .778 amperes in a manner indicated above, current through 1B3L18 plus 4S3L28 is 0.325 amperes. Comparing the direction of current in this line for fault at 4S bus with the arrow indicating positive direction of current, as shown in Fig. 7, it is seen the two accord, hence 0.325 is the proper sign.

From above, we have the distribution factor of line 4S5U22 is -0.222, since direction of current to a fault at 4S bus is opposite to that indicated in Fig. 7 and the distribution table carries the figure with minus sign in front.

The 0.222 amperes divides on the 5U bus between line 1B5U17 and the 1B-T-6G-5U branch of group (c). The division is determined as before, and found to be half and half, or 0.111 amperes through the 1B-T-6G-5U branch. From this it is seen distribution factor for 6G5U23 is 0.111.

Again, currents through T-7M-6G and T-8C-9K-6G branches are found, respectively, to be .0685 and .0425 amperes, permitting the distribution factors for lines 6G7M22, 8C9K24 and 6G9K24 to be written.

It is now necessary to determine distribution of current in the delta for which the wye was substituted.

Voltage drop from:

$$1B \text{ to } T = 0.111\ (.096 + j\ .197) = .01065 + j\ .02185 \text{ volts}$$

$$T \text{ to } 7M = .0685\ (.132 + j\ .2) = .00904 + j\ .01370 \text{ volts}$$

$$T \text{ to } 8C = .0425\ (.057 + j\ .117) = .00242 + j\ .00498 \text{ volts}$$

Therefore, voltage drop-

$$\text{From } 1B \text{ to } 7M = .01065 + j\ .02185 + .00904 + j\ .01370$$

$$= .0197 + j\ .0356 \text{ volts.}$$

Therefore, current in 1B7M13

$$= \frac{.0197 + j\ .0356}{.454 + j\ .735}$$

$$= .0474 \text{ amperes,}$$

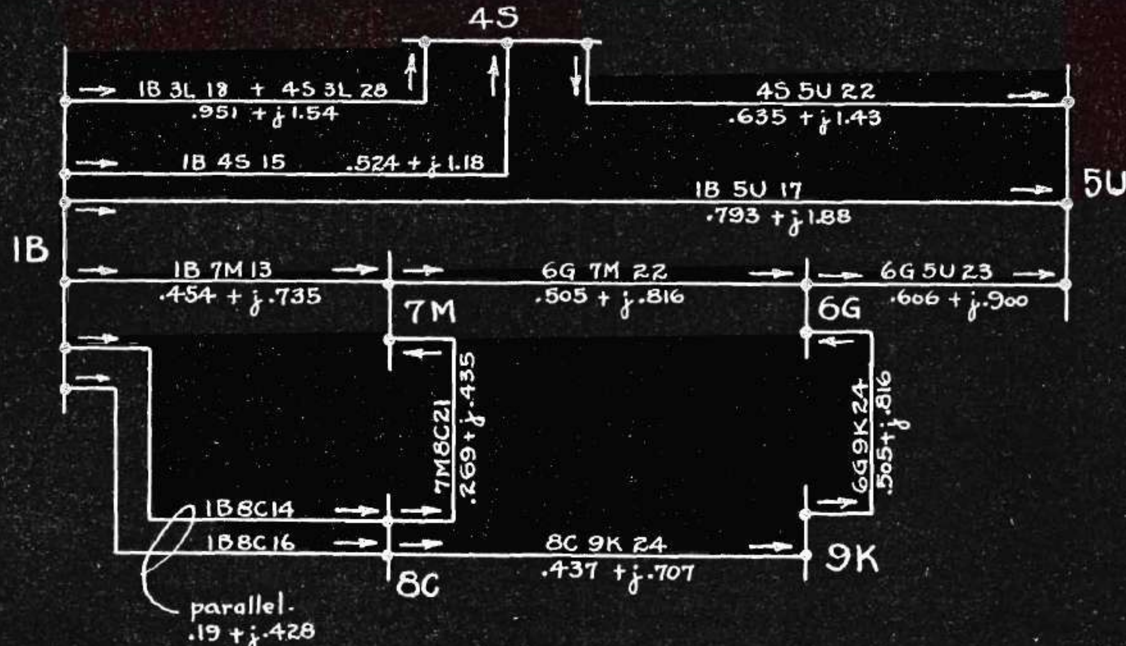
and distribution factor for 1B7M13 is .0474. The same procedure is followed in determining currents in the other branches.

The zero sequence system is carried out with impedances considered as wholly reactive, because they are nearly so, and the error is well within limits reasonable for a study of this nature. Networks for faults at the different locations are reduced, and distribution factors determined in the same manner as for the positive networks.

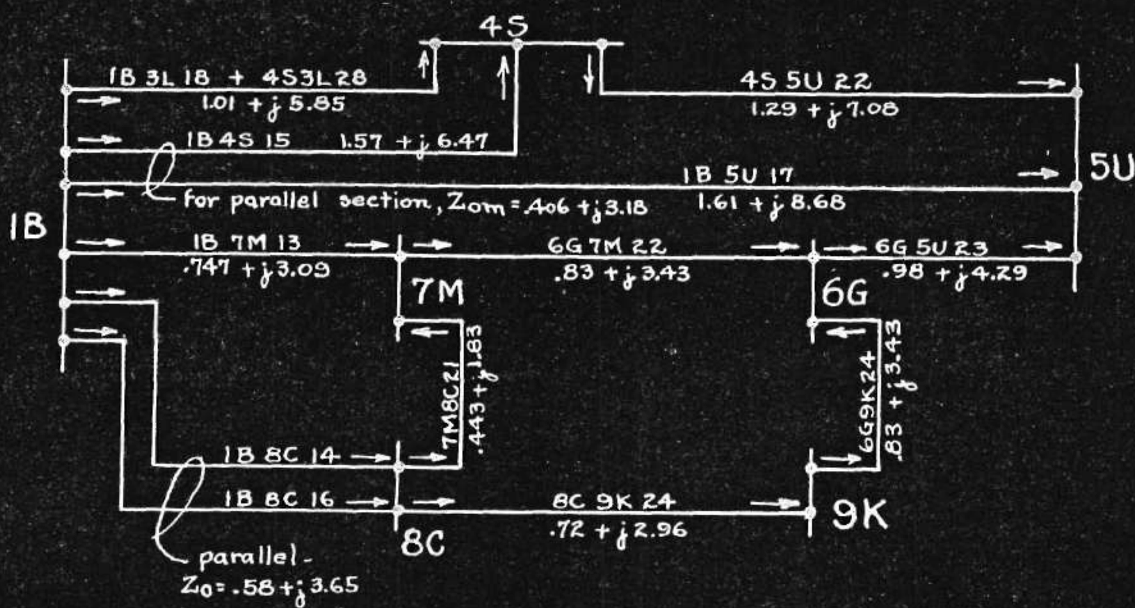
The 1B3L18 plus 4S3L28 line cannot be paralleled with 1B4S15 in the zero sequence network, however. It will be noticed that 1B4S15 and 1B5U17 are carried on the same pole for considerable distance, and it is therefore necessary to take into account the mutual zero sequence impedance. This is done by replacing the actual arrangement of the two, as shown in Fig. 13-(a), by an equivalent network, Fig. 13-(b).

(8) Determination of fault component currents.

The sequence impedances for the supply system are added to corresponding impedances pertaining to the distribution



Positive & Negative Sequence



Zero Sequence

Fig. 7

Distribution Component Sequence Networks.

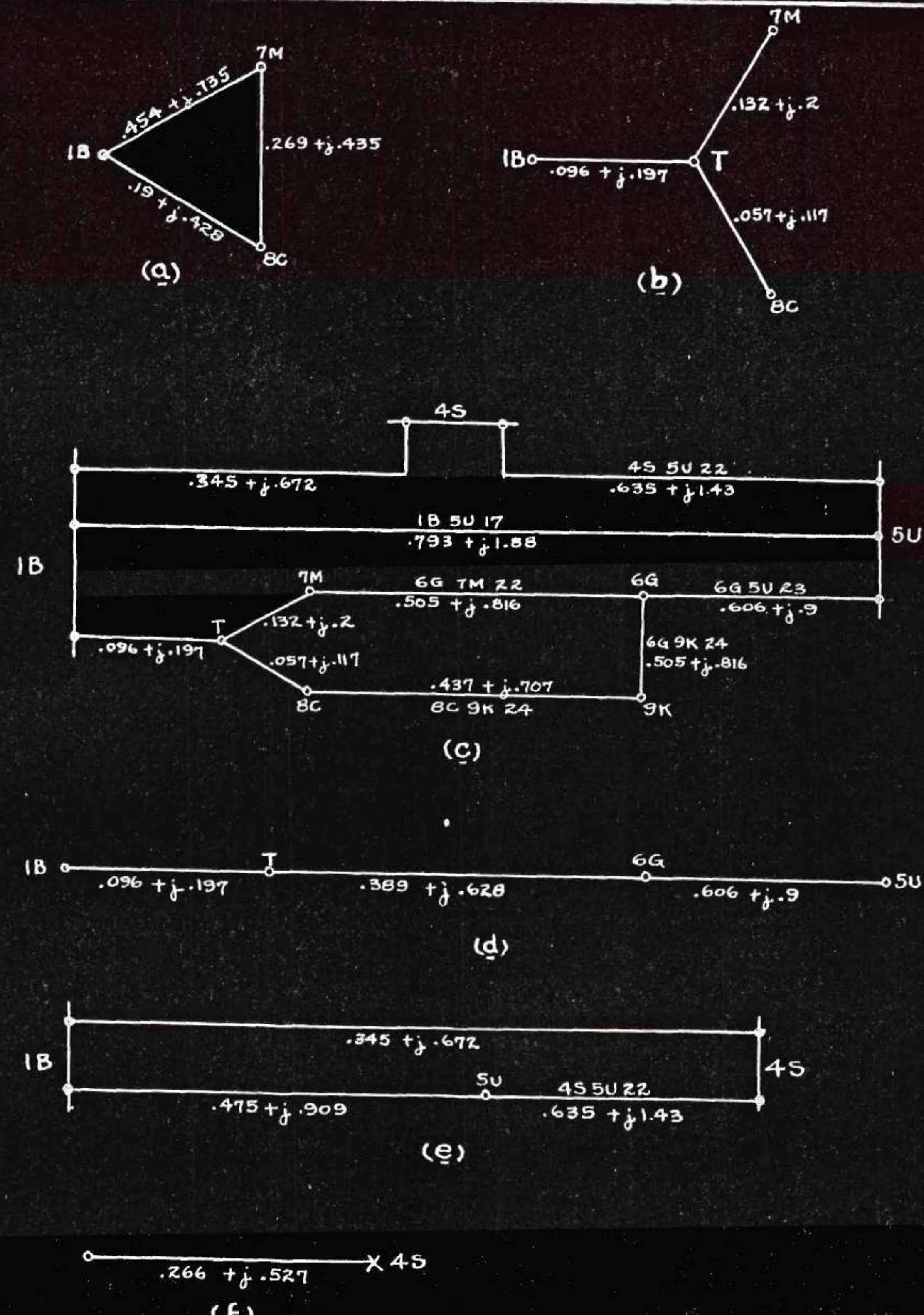
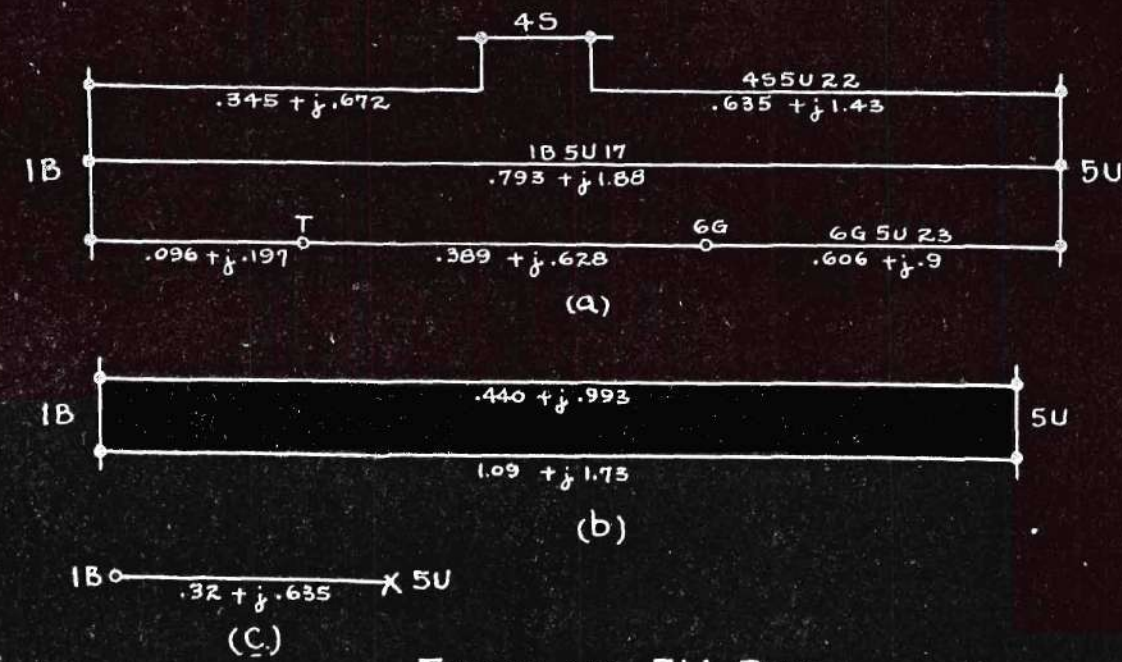
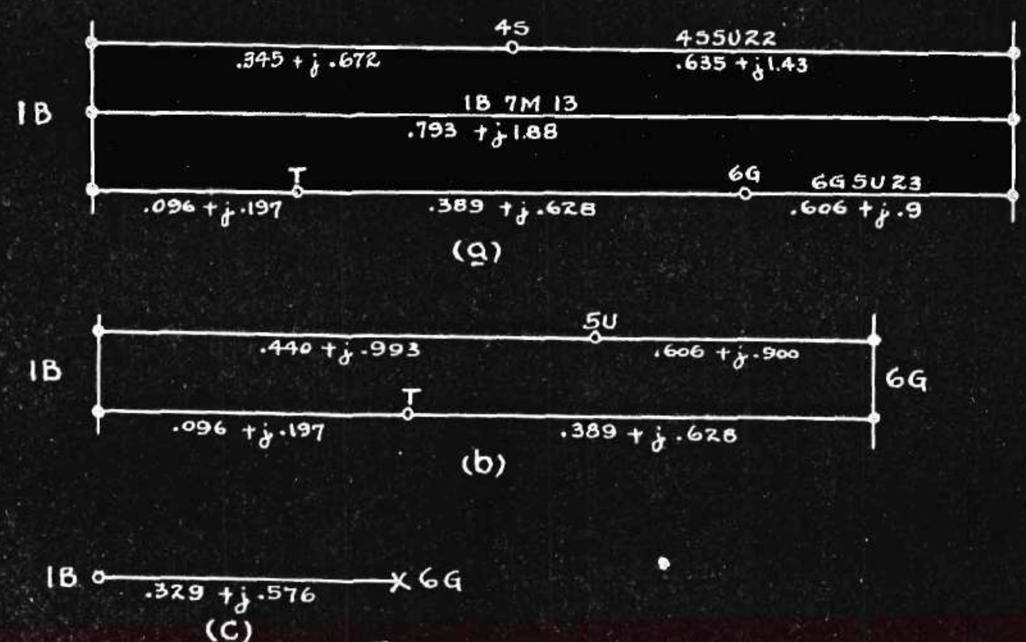


FIG. 8

DISTRIBUTION NETWORK REDUCTION.
POSITIVE & NEGATIVE SEQUENCE.
FAULT AT 4S BUS.



FAULT AT 5U BUS.



FAULT AT 6G BUS.

FIG. 9

DISTRIBUTION NETWORK REDUCTION
POSITIVE & NEGATIVE SEQUENCE.
FAULT AT 5U BUS; AT 6G BUS.

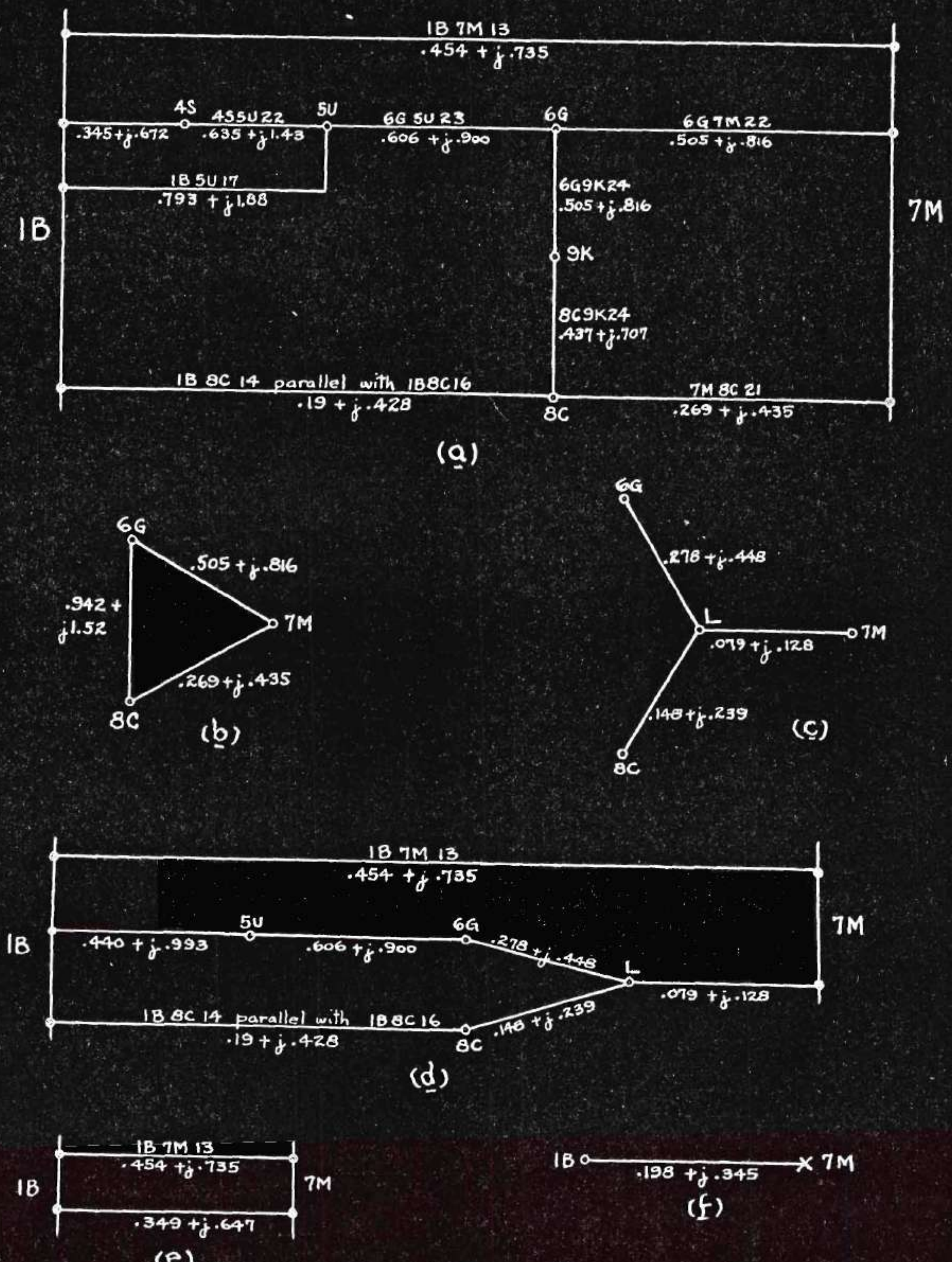


FIG. 10

DISTRIBUTION NETWORK REDUCTION
POSITIVE & NEGATIVE SEQUENCE.
FAULT AT 7M BUS.

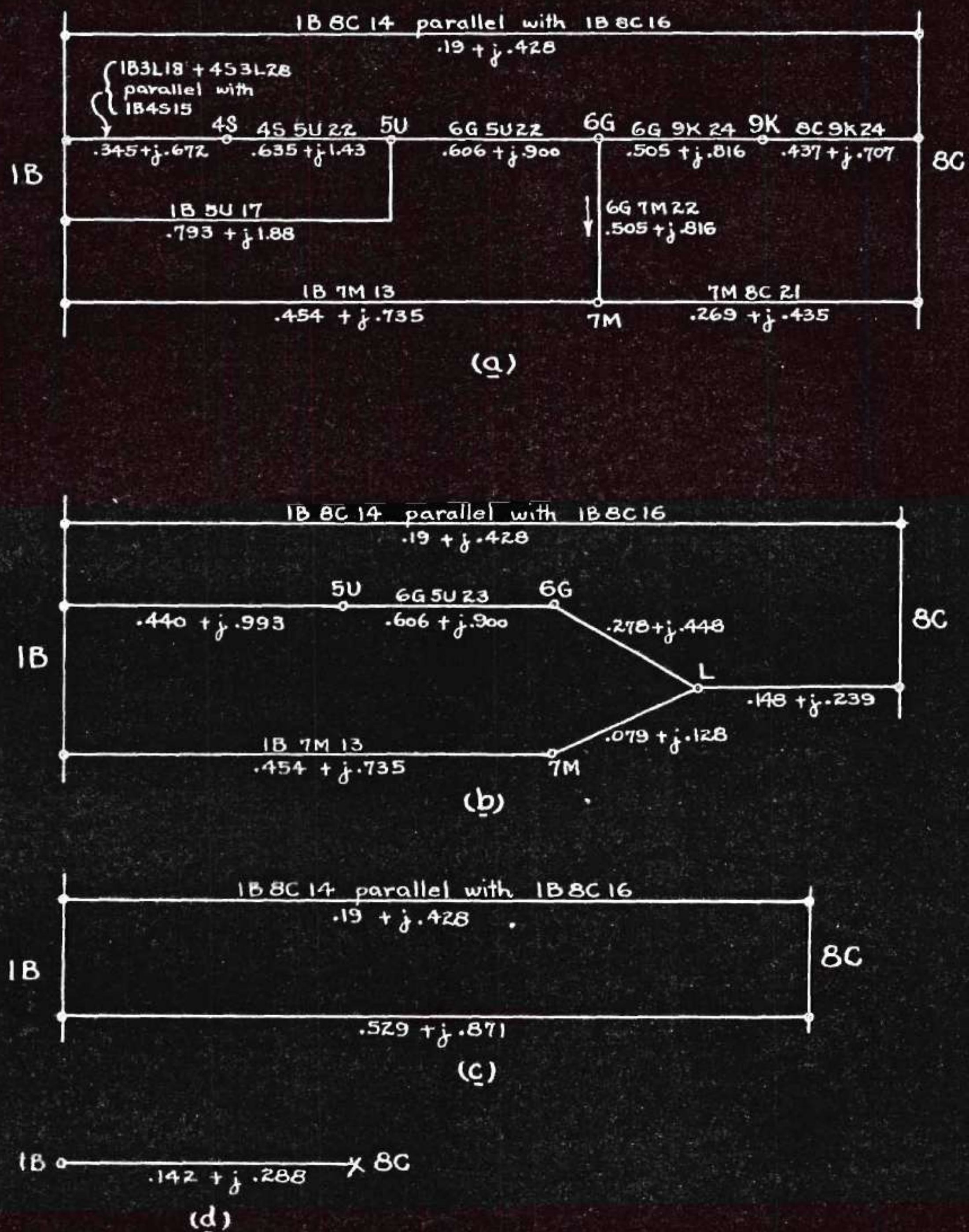


FIG. 11

DISTRIBUTION NETWORK REDUCTION
POSITIVE & NEGATIVE SEQUENCE.
FAULT AT 8C BUS.

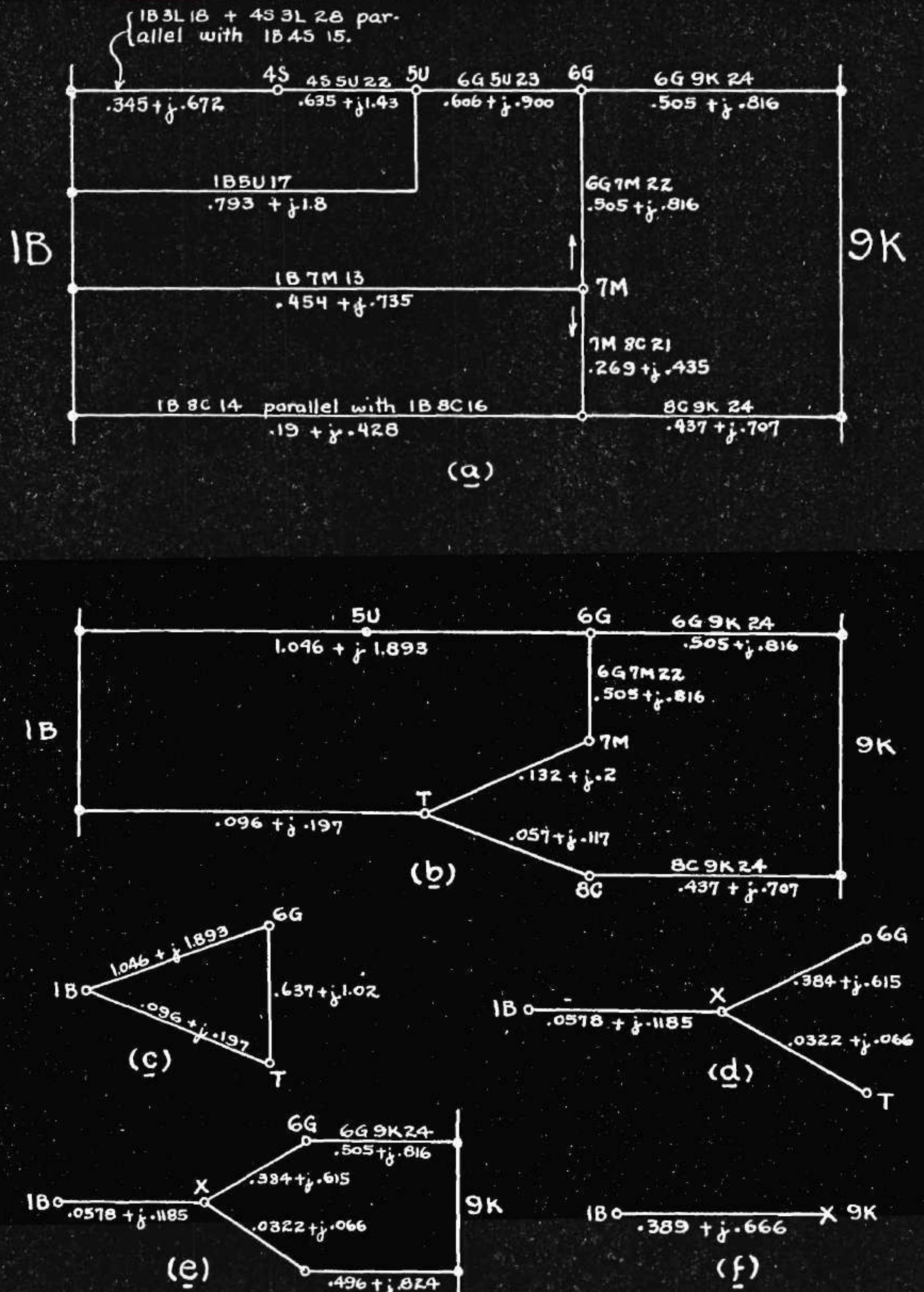


FIG. 12

DISTRIBUTION NETWORK REDUCTION.
POSITIVE & NEGATIVE SEQUENCE
FAULT AT 9K BUS

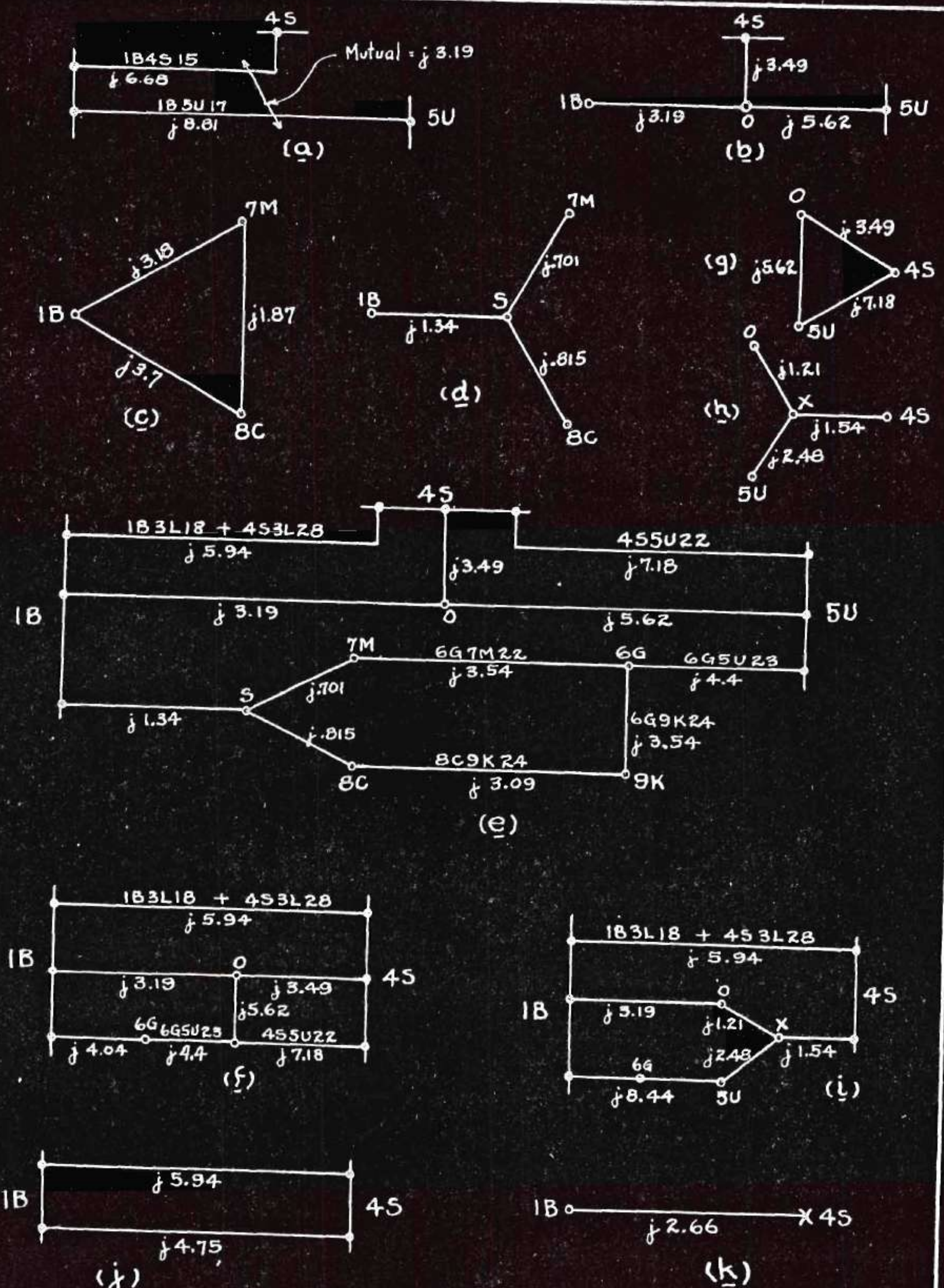


FIG. 13

DISTRIBUTION NETWORK REDUCTION.
ZERO SEQUENCE
FAULT AT 4S BUS.

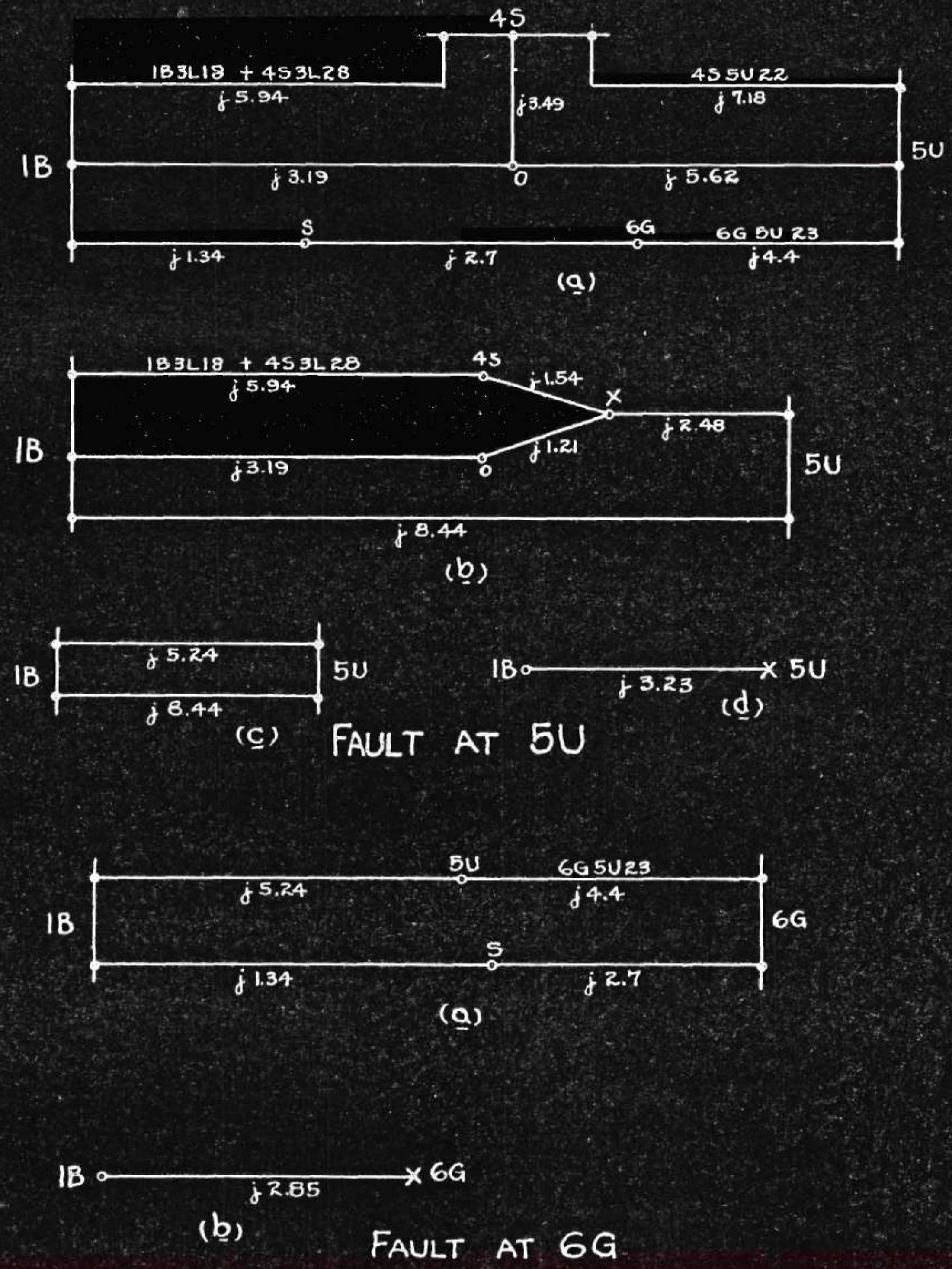


FIG. 14
DISTRIBUTION NETWORK REDUCTION.
ZERO SEQUENCE
FAULT AT 5U BUS; AT 6G BUS.

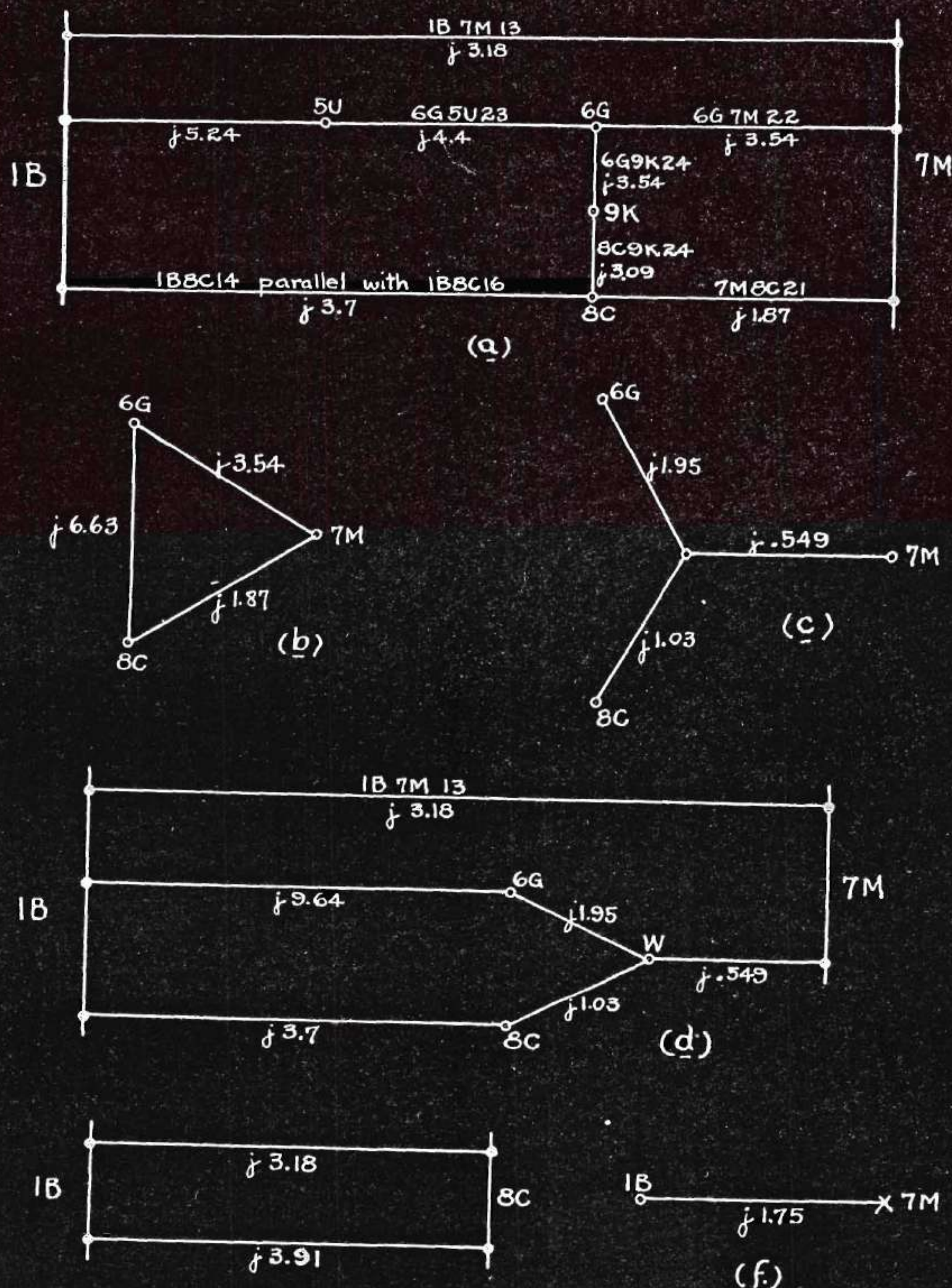


FIG. 15

DISTRIBUTION NETWORK REDUCTION.
ZERO SEQUENCE
FAULT AT 7M BUS.

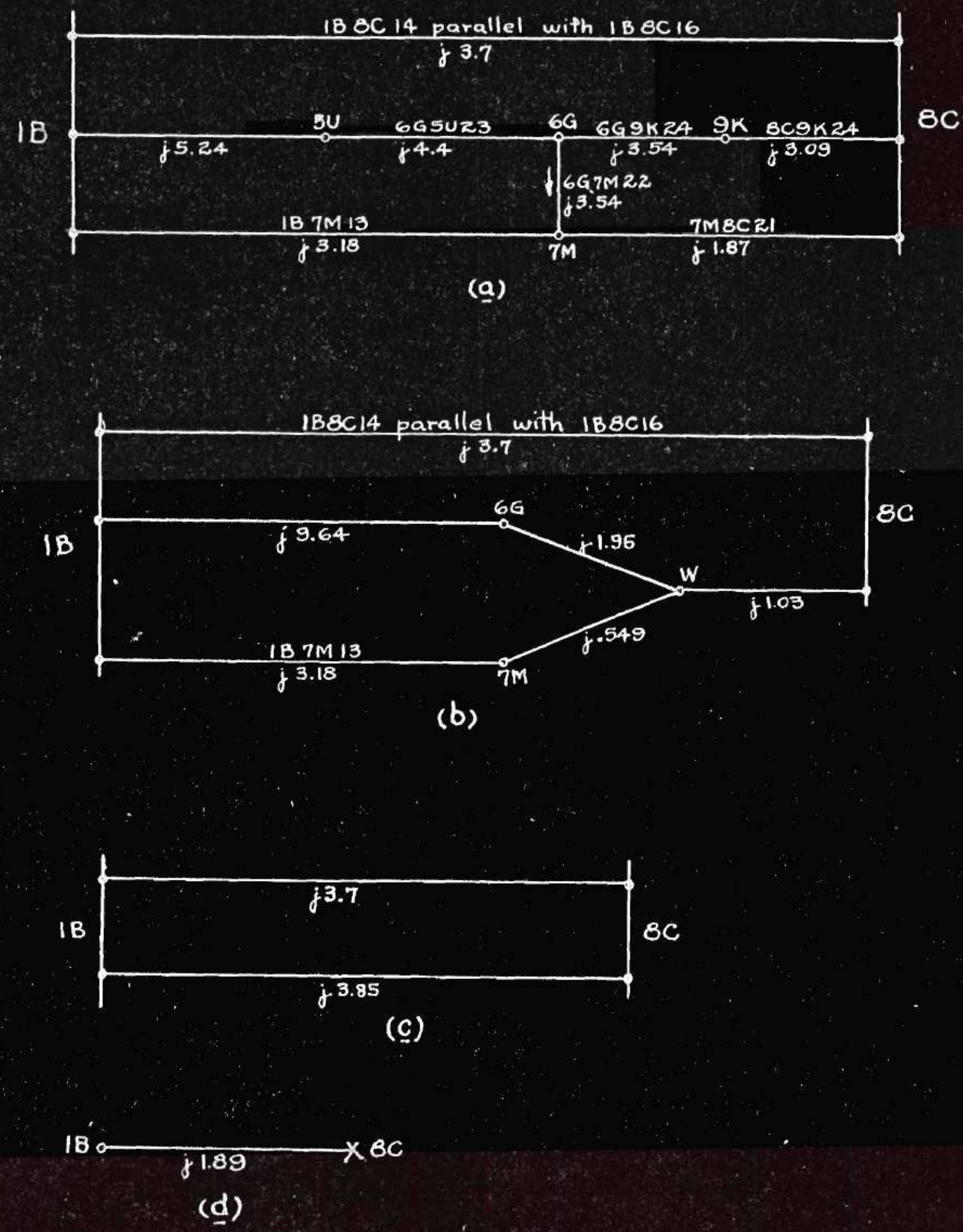


FIG. 16

DISTRIBUTION NETWORK REDUCTION
ZERO SEQUENCE
FAULT AT 8C BUS.

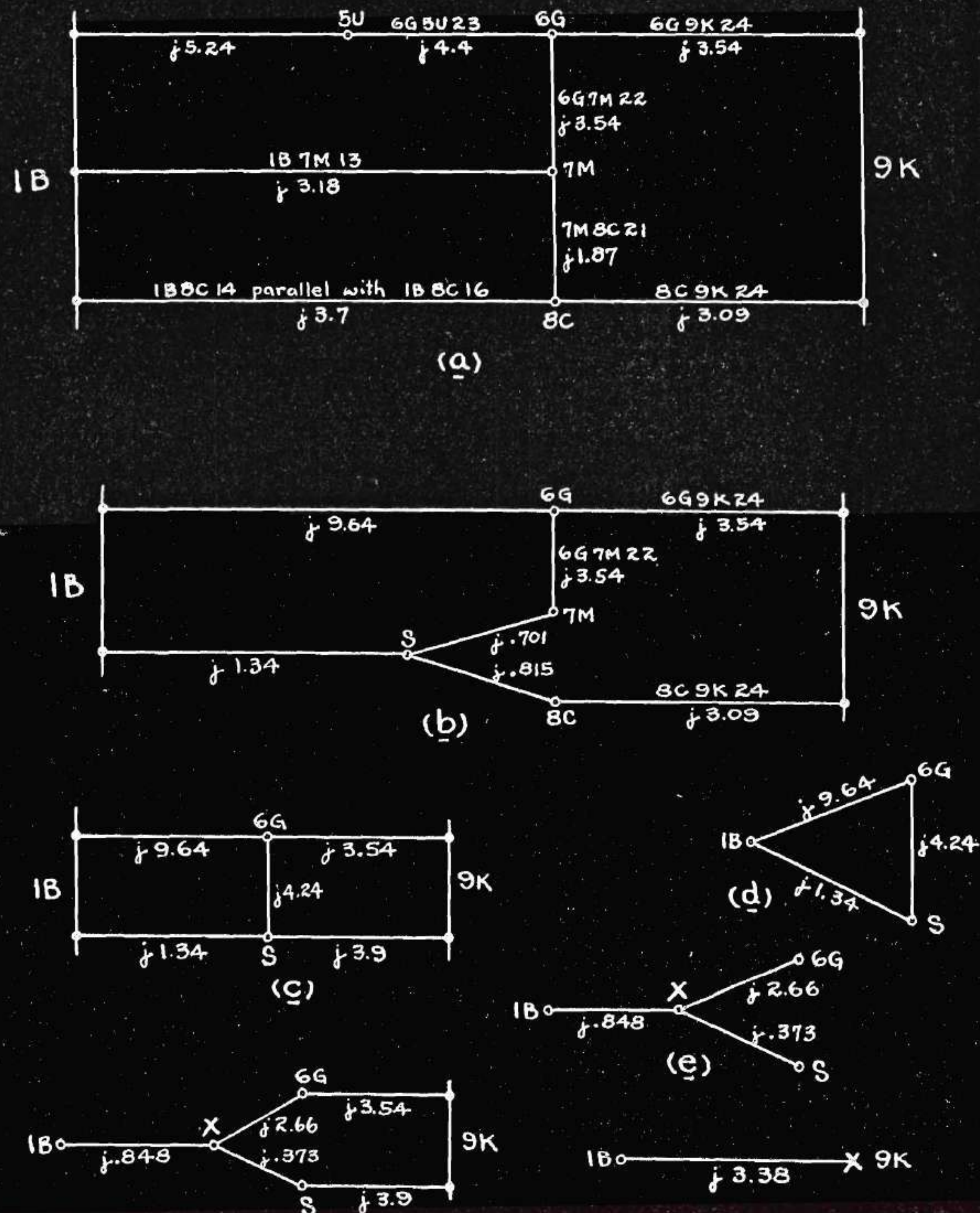


FIG. 17

DISTRIBUTION NETWORK REDUCTION.
ZERO SEQUENCE
FAULT AT 9K BUS.

system for the particular fault location under consideration. For each location, then, the total positive, negative and zero sequence impedances, from supply to fault, are known.

Different sequence impedances are connected in a manner to satisfy the conditions of the fault, and current through each of them is computed, using the generator internal voltage, 8700 + j 0 volts, as the E.M.F., and the group impedance of the component sequence networks as the denominator in applying Ohm's law.

It should be noted that, for the phase to phase fault, the positive and negative sequence impedances are connected in series, the sum being the effective impedance to be divided into the internal voltage, but that the negative sequence impedance is reversed, and since all currents are tabulated for direction the same in all component networks, it is necessary to reverse the negative sequence current before recording it in the tables. The same applies to currents through both the negative and zero sequence networks for the double line to ground fault.

Tables 5 through 10 summarize the calculations made in this study. The different tables refer to faults at different stations in the network. Each table gives distribution factors for all lines in the 13.2 KV. distribution system, for fault location as captioned, also positive, negative and zero sequence impedance of the distribution network (from Washington 13.2 KV. bus) to the point of fault, and finally, the various sequence components of current at the fault for the different types of faults.

As stated before, this investigation is primarily with reference to ground currents, but since the data is at hand for calculating phase currents, it may be of some value to indicate how this is done.

If magnitude of current only is required, the following short cut may be used, giving faulted line currents:

(1) From tables 5-10, select that giving values for the fault location concerned.

(2) Read the positive sequence component of current at the fault, for the type fault being considered, and convert value given (positive sequence only) into a scalar magnitude by the ordinary rules of complex quantities.

(3) Multiply by the positive sequence distribution factor, on same table, of the line for which current values are desired.

(4) Multiply by the appropriate one of the following

(a) For 3 phase short ----- 1

(b) For phase to phase short -- $[3]^{1/2}$

(c) For 1 phase to ground ----- 3

(d) For 2 phase to ground ---- $[3]^{1/2}$

(a) and (b) are correct. The method and the (c) and (d) factors are approximations that will give results within 15% of correct for this system only.

If the currents are desired in the complex notation, the following procedure is in order:

(1) From Tables 5-10, select that giving values for the fault location concerned.

(2) Read the positive, negative and zero sequence components of total current at the fault, for the particular type of fault under consideration.

(3) From tabulation at top of the page, select the positive and negative, and zero sequence distribution factors for the line for which currents are desired.

(4) Multiply the distribution factors from (3) by the currents from (2) taking care that only corresponding component values are multiplied.

(5) Vector addition of the three products of (4) is the line current desired, in the reference phase. The direction of the current thus determined is given by the arrow in Fig. 7.

(6) Products determined in (4) are sequence components of currents in the particular branch for which the distribution factors were selected. Setting the positive, negative, and zero sequence products, respectively, equal to I_{a1}, I_{a2} and I_{a0}, and with phase A as reference phase, current in phase B is

$$I_B = a^2 I_{a1} + a I_{a2} + I_{a0}$$

and in C

$$I_C = a I_{a1} + a^2 I_{a2} + I_{a0}$$

where $a = -.5 + j\ .866$; $a^2 = -.5 - j\ .866$.

if line voltages come to a maximum in the order A, B, C.

Total currents at the different fault locations are calculated with respect to the "reference" voltage of 8700 + j 00 volts, so that in making any computations it is necessary to consider the system at the instant the source (positive sequence) voltage in the reference phase is of this value. The "reference" phase is different for the different types of fault:

(1) For a three phase short, the reference phase is, of course, faulted.

(2) For a phase to phase short, the reference phase is the sound phase.

(3) For a single phase to ground fault, the reference phase is the faulted phase.

(4) For a double phase to ground fault, the reference phase is the sound phase.

Means for taking care of change in reference phase is obvious when making any current determinations.

Current magnitudes in the vector notation are required only when voltage vectors are also used, for determining action of watt-element relays during fault.

As this investigation is only concerned with zero sequence voltages, and that is approximately the same throughout the network, no effort is made to compute the various voltage sequence component distribution factors.

The sequence components of line to neutral voltages at any point may be obtained from a knowledge of the distribution of sequence currents and the branch impedance from the system source to the point in question.

The positive sequence voltage at a point is equal to the positive sequence voltage at the source minus the positive sequence IZ drop from source to point.

Similarly for negative and zero sequence components,

Line	Distribution Factors. Pos. & Neg.	Zero
1B 3L 18	.325	.448
4S 3L 28	.325	.448
1B 4S 15	.453	.388
1B 5U 17	.111	.021
1B 7M 13	.0474	.083
1B 8C 14	.0318	.0338
1B 8C 16	.0318	.0338
6G 7M 22	.0685	.096
7M 8C 21	.0211	.0128
8C 9K 24	.0425	.055
9K 6G 24	.0425	.055
6G 5U 23	.111	.151
4S 5U 22	−.222	−.172
Impedance	$.265 + j.524$	$.556 + j2.6$

Component Currents.

Type Fault	Positive	Negative	Zero
Three Phase	$1370 - j6590$	—	—
Phase - Phase	$725 - j3370$	$-725 + j3370$	—
One Phase - Grd.	140	140	140
Two Phase - Grd	$754 - j3360$	$-685 + j3360$	−69

Table 5

Distribution Factors

and

Total Sequence Currents

Fault at 4S Bus.

Line	Distribution Factors Pos. & Neg	Zero
1B 3L 18	.128	.228
4S 3L 28	.128	.228
1B 4S 15	.178	.034
1B 5U 17	.348	.355
1B 7M 13	.147	.212
1B 8C 14	.099	.0857
1B 8C 16	.099	.0857
6G 7M 22	.214	.244
7M 8C 21	.0657	.0326
8C 9K 24	.132	.139
9K 6G 24	.132	.139
6G 5U 23	.346	.383
4S 5U 22	.306	.262
Impedance	.32 + j .635	.65 + j 3.17

Component Currents.

Type Fault	Positive	Negative	Zero
Three Phase	1390 - j 8030	—	—
Phase - Phase	730 - j 3070	-730 + j 3070	—
One Phase - Grd.	139.5	139.5	139.5
Two Phase - Grd.	759 - j 3060	-690 + j 3060	-69

Table 6
Distribution Factors
and
Total Sequence Currents,
Fault at 5U Bus.

Distribution Factors

Line	Pos. & Neg.	Zero
1B 3L 18	.0597	.109
4S 3L 28	.0597	.109
1B 4S 15	.0830	.0163
1B 5U 17	.162	.170
1B 7M 13	.296	.389
1B 8C 14	.199	.158
1B 8C 16	.199	.158
6G 7M 22	.429	.45
7M 8C 21	.132	.06
8C 9K 24	.266	.256
9K 6G 24	.266	.256
6G 5U 23	−.305	−.296
4S 5U 22	.143	.125
Impedance	.329 + j .576	.59 + j 2.79

Component Currents.

Type Fault	Positive	Negative	Zero
Three Phase	1555 − j 6210	—	—
Phase - Phase	810 − j 3160	−810 + j 3160	—
One Phase - Grd.	139	139	139
Two Phase - Grd.	850 − j 3160	−781 + j 3160	−69

Table 7
Distribution Factors
and
Total Sequence Currents,
Fault at 6G Bus.

Line	Distribution Factors Pos. & Neg.	Zero
1B3L18	.0229	.048
4S 3L 28	.0229	.048
1B 4S 15	.0318	.00715
1B 5U 17	.0623	.0748
1B 7M 13	.460	.551
1B 8C 14	.211	.159
1B 8C 16	.211	.159
6G 7M 22	−.149	−.141
7M 8C 21	.389	.307
8C 9K 24	.032	.0113
9K 6G 24	.032	.0113
6G 5U 23	−.117	−.13
4S 5U 22	.0547	.0551
Impedance	$.198 + j.345$	$.35 + j\,1.72$

Component Currents

Type Fault	Positive	Negative	Zero
Three Phase	$1445 - j\,7800$	—	—
Phase - Phase	$755 - j\,3980$	$-755 + j\,3980$	—
One Phase - Gr'd.	141	141	141
Two Phase - Gr'd.	$789 - j\,3980$	$-720 + j\,3980$	−69

Table 8
Distribution Factors
and
Total Sequence Currents,
Fault at 7M Bus.

DISTRIBUTION FACTORS.

LINE	POS. & NEG.	ZERO
1B 3L 18	.0172	.044
4S 3L 28	.0172	.044
1B 4S 15	.0239	.00655
1B 5U 17	.0469	.0685
1B 7M 13	.235	.371
1B 8C 14	.3385	.255
1B 8C 16	.3385	.255
6G 7M 22	−.013	−.00847
7M 8C 21	−.248	−.379
8C 9K 24	−.075	−.111
9K 6G 24	−.075	−.111
6G 5U 23	−.088	−.119
4S 5U 22	.0412	.0505
IMPEDANCE	$.142 + j.288$	$.37 + j1.85$

COMPONENT CURRENTS.

TYPE FAULT	POSITIVE	NEGATIVE	ZERO
THREE PHASE	$1130 - j8300$	—	—
PHASE − PHASE	$599 - j4270$	$-599 + j4270$	—
ONE PHASE − GR'D.	141	141	141
TWO PHASE − GR'D.	$631 - j4270$	$-562 + j4270$	−69

TABLE 9

DISTRIBUTION FACTORS

AND

TOTAL SEQUENCE CURRENTS,

FAULT AT 8C BUS.

Distribution Factors

Line	Pos. & Neg.	Zero
1B 3L 18	.0368	.073
4S 3L 28	.0368	.073
1B 4S 15	.0511	.011
1B 5U 17	.10	.115
1B 7M 13	.255	.38
1B 8C 14	.276	.21
1B 8C 16	.276	.21
6G 7M 22	.192	.208
7M 8C 21	−.067	−.172
8C 9K 24	.62	.592
9K 6G 24	−.38	−.408
6G 5U 23	−.188	−.2
4S 5U 22	.088	.085
Impedance	$.389 + j.666$	$.67 + j3.31$

Component Currents

Type Fault	Positive	Negative	Zero
Three Phase	$1590 - j5750$	—	—
Phase – Phase	$825 - j2920$	$-825 + j2920$	—
One Phase - Gr'd	139	139	139
Two Phase - Gr'd	$860 - j2920$	$-791 + j2920$	-69

TABLE 10

DISTRIBUTION FACTORS
AND
TOTAL SEQUENCE CURRENTS,
FAULT AT 9K BUS.

except that the source E.M.F. in these systems is zero, so the component at the point is the negative of the IZ drop to the point.

As an example of voltage calculations, let it be desired to determine the voltages at the 4S bus for phases B and C faulted to ground at the 5U bus.

Figures 4, 5 and 6 give:

(1) Source positive sequence voltage, phase A,
$= 8700 + j\ 00$ V.

(2) Positive sequence impedance from source to Washington 13.2 KV. bus = $j\ .734$ ohms.

(3) Negative sequence impedance from source to Washington 13.2 KV. bus = $j\ .659$ ohms.

(4) Zero sequence impedance from source to Washington 13.2 KV. bus = 61 ohms.

From Figure 7:

(5) Positive sequence impedance line 1B3L18 plus 4S3L28 (from Washington 13.2 KV. to 4S bus) = $.951 + j\ 1.54$ ohms.

(6) Negative sequence impedance 1B3L18 plus 4S3L28 = $.951 + j\ 1.54$ ohms.

(7) Zero sequence impedance 1B3L18 plus 4S3L28 = $1.01 + j\ 5.85$ ohms.

From Table 6:

(8) Total positive sequence current for a double phase to ground fault at 5U bus = $759 - j\ 3060$ amps.

(9) Total negative sequence current for same = $-690 + j\ 3060$ amps.

(10) Total zero sequence current for same = -69 amps.

(11) Positive sequence distribution factor for 1B3L18 plus 4S3L28 = .128

(12) Negative sequence distribution factor for 1B3L18 plus 4S3L28 = .128

(13) Zero sequence distribution factor for 1B3L18 plus 4S3L28 = .228

The calculation is as follows:

Positive sequence current in the line = (8) x (11)

(14) = (759-j 3060)(.128) = 97-j 392 amps

Negative sequence current in the line = (9) x (12)

(15) = (-690 + j 3060)(.128) = -88.4 + j 392 amps

Zero sequence current in the line = (10) x (13)

(16) = (-69)(.228) = -15.75 amps.

Positive sequence voltage drop from source to Washington 13.2 KV. bus = (2) x (8)

(17) = j.734 (759 -j 3060) = 2245 + j 556 volts

Negative sequence voltage drop from source to Washington 13.2 KV. bus = (3) x (9)

(18) = j .659 (-690 + j 3060) = -2015 -j 455 volts

Zero sequence voltage drop from source to Washington 13.2 KV. bus = (4) x (10)

(19) = 61 (-69) = -4210 volts.

Positive sequence voltage drop from Washington 13.2 KV. bus to 4S bus = (14) x (15)

(20) = (97-j 392)(.951 + j 1.54) = 697 -j 224 volts

Negative sequence voltage drop from Washington 13.2 KV. bus to 4S bus = (15) x (6)

(21) = (-88.4 + j 392)(.951 + j 1.54) = -688 + j 245 volts

Zero sequence voltage drop from Washington

13.2 KV. bus to 4S bus = (16) x (7)

(22) $= (-15.75)(1.01 + j\ 5.85) = -16 - j\ 92$ volts

Total positive sequence drop, source to 4S bus

$= (17) + (20) = (2245 + j\ 556) + (697 - j\ 224)$

(23) $= 2942 + j\ 332$ volts

Total negative sequence drop, source to 4S bus

$= (18) + (21) = (-2015 - j\ 455) + (-688 + j\ 245)$

(24) $= -2703 - j\ 210$ volts

Total zero sequence voltage drop, source to 4S bus

$= (19) + (22) = (-4210) + (-16 - j\ 92)$

(25) $= -4226 - j\ 92$ volts

Positive sequence component of voltage at 4S bus,

$e_{a1} = (1) - (23) = (8700 + j\ 0) - (2942 + j\ 332)$

$= 5758 - j\ 332$ volts

Negative sequence component of voltage at 4S bus,

$e_{a2} = 0-(24) = 0-(-2703-j\ 210) = 2703 + j\ 210$ volts

Zero sequence component of voltage at 4S bus,

$e_{a0} = 0 - (25) = 0-(-4226 - j\ 92) = 4226 + j\ 92$ volts

Also,

$ae_{a1} = (-.5 + j\ .866)(5758 - j\ 332) = -2605 + j\ 5166$ volts

$a^2e_{a1} = (-.5 - j\ .866)(5758 - j\ 332) = -3181 - j\ 4834$ volts

$ae_{a2} = (-.5 + j\ .866)(2703 + j\ 210) = -2034 + j\ 2235$ volts

$a^2e_{a2} = (-.5 - j\ .866)(2703 + j\ 210) = -1670 - j\ 2445$ volts

From the theory of symmetrical components,

$E_a = e_{a1} + e_{a2} + e_{a0}$

$E_b = a^2e_{a1} + ae_{a2} + e_{a0}$

$E_c = ae_{a1} + a^2e_{a2} + e_{a0}$

Therefore, voltages at 4S bus for phases B and C faulted at 5U bus are:

$E_a = (5758 - j\ 332) + (2703 + j\ 210) + (4226 + j\ 92)$

$= 10{,}687$ volts

$E_b = (-3181 - j\ 4834) + (-2034 + j\ 2235) + (4226 + j\ 92)$

$= -989 - j\ 2507$ volts

$E_c = (-2605 + j\ 5166) + (-1670 - j\ 2445) + (4226 + j\ 92)$

$= -41 + j\ 2813$ volts

Line-to-line voltages

$E_{a-b} = [11{,}676^2 + 2507^2]^{1/2}$

$= 11{,}930$ volts

$E_{a-c} = [10{,}728^2 + 2813^2]^{1/2}$

$= 11{,}100$ volts

$E_{b-c} = [948^2 + 5320^2]^{1/2}$

$= 5400$ volts

From the determination of the zero sequence component of voltage at the 4S bus, it is seen that the impedance of the various branches in the distribution network have an inappreciable effect in varying the magnitude of this component, and that for all practical purposes, the zero sequence potential may be considered the same throughout the network. Investigation shows the same to be true for the single line to ground fault, hence, for all stations within the 13.2 KV. network, regardless of fault location so long as it also is within the network, the zero sequence potential will be considered as

For single line to ground faults - 8600 volts

For double line to ground faults 4200 volts

These correspond, respectively, to voltages of -107.5 and + 52.5 across the potential coil of the ground relay, when connected as shown in Appendix B, ratio of potential and auxiliary potential transformers as indicated there.

Table 11 is a tabulation of the ground currents (three times zero sequence current) existing in all the lines for the two types of fault involving ground currents, and for fault locations at the different stations. Of course it is the zero sequence current that is in the overhead conductors, but current transformer connection is such that ground relays are energized by a current equal to three times the zero sequence current, divided by the ratio of the current transformers. Table 11 should be prepared in terms of C. T. secondary, or rather, of relay current, but current transformer ratios are not at hand.

The minus signs indicate current in the direction reverse to that shown by the arrows in Figure 7.

With respect to the "reference" voltage to which all calculated currents are referred, the zero sequence current for a double line to ground fault is in a direction reverse of that of the zero sequence current for a single line to ground fault. This is indicated by the signs of this component current for the two types of faults, as shown in Tables 5-10. The direction of the zero sequence voltage also reverses, so that, with respect to zero sequence voltage, zero sequence current direction is the same for both types of faults. In using the "CR" relay for ground fault protection, the zero sequence voltage is the reference voltage, so that, in preparing Table 11, no cognizance is taken of the reversal of zero sequence current with respect to the system reference voltage. The minus signs in the

Line	4S	5U	6G	7M	8C	9K
1B 3L 18	188	96	45.5	20	19	30
4S 3L 28	188	96	45.5	20	19	30
1B 4S 15	163	14	7	3	3	5
1B 5U 17	9	148	71	31	29	48
1B 7M 13	35	89	162	234	157	159
1B 8C 14	14	36	70	67	108	88
1B 8C 16	14	36	70	67	108	88
6G 7M 22	40	102	188	-60	-35	87
7M 8C 21	6	14	25	130	-160	-72
8C 9K 24	23	58	107	5	-47	247
9K 6G 24	23	58	107	5	-47	-170
6G 5U 23	63	160	-124	-124	-51	-84
4S 5U 22	-72	110	52	23	21	35

Single Phase to Ground.

1B 3L 18	94	48	23	10	9	15
4S 3L 28	94	48	23	10	9	15
1B 4S 15	82	7	3	1	1	2
1B 5U 17	5	74	35	15	14	24
1B 7M 13	17	44	81	117	78	79
1B 8C 14	7	18	35	33	54	44
1B 8C 16	7	18	35	33	54	44
6G 7M 22	20	51	94	-30	-2	43
7M 8C 21	3	7	12	65	-80	-36
8C 9K 24	11	29	53	2	-23	123
9K 6G 24	11	29	53	2	-23	-85
6G 5U 23	31	80	-62	-27	-25	-42
4S 5U 22	-36	55	26	11	10	14

Double Phase to Ground

Table II

Ground Currents
in network branches for
different fault locations.

table indicate direction of the current reverse to that shown by the arrows in Figure 7, and not phase relation with respect to any reference voltage.

The purpose of protective relays is to cut out faulty sections of systems, allowing the sound portion to remain in service. Discrimination by relays between faulty and sound sections may be made upon one or more of the following bases:

(1) Current magnitude

(2) Current direction

(3) Distance (from relay to fault)

(4) Distance and direction

(5) Voltage

(6) Current balance (for parallel lines)

(7) Current balance (differential - for one line)

(8) Time

There are other bases of discrimination, but they are not customarily employed for transmission line protection. Of the above, current and voltage provide the only means by which relays may recognize faults; all the others may be regarded as conditions preventing the relay from operating in recognition of the fault, until they are satisfied.

Current magnitude is practically universally used as a basis for the detection of faults, and almost as general is the use of the time factor in preventing more than one group of relays, through which the same current may be passing, from operating.

The simplest type of induction relay is therefore the overcurrent relay, with various time settings. The relay may be set to operate at various values of current transformer secondary current, and up to a certain percentage overcurrent, the time required for the relay to close contacts and trip the breaker is inversely proportional to the current magnitude. This feature permits temporary overloads, which would do no harm, from tripping the breaker. However, for currents of about 400 or 500 percent that for which the relay is set, and above, the tripping time is definite, and no matter what the current, the relay will not trip in less than the predetermined time.

This relay is therefore well suited to protection of radial systems, if the different spokes do not have so many sections that the time increments in the settings of successive relays build up to an unsafe value. Three and a half seconds is about the maximum time considered safe for the clearing of faults, even on small systems. Hence, with increments of 0.5 seconds, the overcurrent relay is suitable for radial systems having as many as six sections to a spoke. Beyond that, the overcurrent relays with distance as well as time discrimination must be used.

Distance discrimination is in reality a form of voltage discrimination, the principle being that the voltage at successive points away from a fault increases, the amount of increase being the IZ drop in the line. For a constant Z per mile, the increase is proportional to the distance to the fault. The relay takes care of different fault currents, in that a lesser IZ drop is required when current is not so great.

For the loop system, directional discrimination is required, since fault current may flow in either direction in the line. The directional feature, as explained in Appendix B, is a watthour meter element, prevented from rotating when power is in one direction, but operating to close contacts when in the other direction. For small loop systems, this feature is added to the overcurrent relay, the combination being adjustable for current magnitude, current direction, and time. Application of directional relays to loop systems is made by starting at the source station, considering one path around the loop, installing relays on the "outgoing" feeders at each substation, connected to function for outgoing excess power, just as though the path being followed were from generating source out to end of a dead end feeder. The other path around the loop is then considered in the same light, and relays installed accordingly. The result is that all lines at all substations are equipped with relays set to trip for excess power away from the substation bus. Since power may flow only away from the generating source bus, outgoing feeders from this station may be protected by the simple overcurrent relay. The same limitations in applying the directional overcurrent relay exist as for the plain overcurrent relay, and the solution is the same - addition of the distance discriminating feature. The distance directional relay is the same as the distance relay except for the addition of the directional feature, and needs no discussion.

Network systems require more attention in the application of relays, and each must be given individual attention in determining the basis of selective action. In general, with two or more power sources feeding the network, the overcurrent relay with time, distance and directional features is required.

The system at hand is not very complex, and with the exception of four lines, is a loop system with one power source. The changes are that the overcurrent relay with time and directional discrimination will be suitable. As a matter of fact, this is the type protection used.

The same general principles which must be considered in applying line relays hold for ground relays also. In considering the protection against ground faults for the system at hand, the following should be noted:

First, the values of fault current as determined are maximum values for faults at station busses. Impedance of the fault has been omitted from the calculations. This item is of particular importance in the faults involving ground currents, as any impedance in the return path is put into the zero sequence network at a value of three times its actual ohms. A majority of ground faults are high resistance leaks, and if the ohms at the fault is as low as twenty, the ground current will be halved. Again, the faults actually occur on the line instead of the station busses, and if the lines were considerably longer it would be necessary to determine currents for the faults located midway between the stations. The total ground current would not be particularly different from that at the station bus fault, but distribution throughout the network might be. However, the determination of ground currents is for maximum values only, and the current for a fault just outside the station will be same as that for the station.

Second, in the case of double phase to ground faults, the zero sequence component is very small in comparison with total fault current. Referring to Tables 5-10, ground currents for this type fault are approximately 200 amperes, whereas line current is of the order of 6000 amperes. Minimum current setting of ground relays is 1/2 ampere, corresponding to 20 ground amperes if the current transformer ratio is 200/5. If line relays are set at 10 amperes secondary (100 percent overload on the line) corresponding line amperes is 400. Hence, despite the fact that the ground relays operate at a current 1/20 that of the line relays it is seen the current in the ground relay during fault is 1000 percent of its setting, whereas that through line relays is 1500 percent its setting, if the relay is in a line carrying total fault current. This will seldom be the case, and on the assumption positive, negative, and zero sequence distribution factors of the line in question are .4, the percentages will be, respectively, 400 and 600. Unless the line relays are set at a definite minimum time greater than the ground relay time setting, they will operate to close contacts and clear the fault before ground relays may do so. Of course the fault is cleared either way.

Actual setting of relays at the different stations cannot be determined without knowledge of the current transformer ratios.

Appendix A

Resolution of Unbalanced Three Phase Vectors into Symmetrical Components

Any unbalanced "----system of three co-planar vectors is completely defined by six parameters; the system possesses six degrees of freedom. When, however, one applies the restriction that the system be symmetrical, the added restraint reduces the system to one of two degrees of freedom. Going a step further, it is quite conceivable that a system of three co-planar vectors with six degrees of freedom can be defined in terms of three symmetrical systems of vectors, each having two degrees of freedom."

Such decomposition would considerably simplify, if not actually make possible in a number of instances, the determination of system magnitudes under unbalanced fault conditions. The development and adaptation of these components to the solution of such problems was first publicly demonstrated by C. L. Fortescue, in "Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks," a paper presented before the A.I.E.E., June, 1918.

This paper and the discussion appertaining are highly mathematical, and the applications deal chiefly with problems associated with rotating machinery. In discussing the paper, Prof. Karapetoff suggested "the expression 'Symmetrical Components' is a more correct and descriptive term of the method than the term 'Symmetrical Co-ordinates'," and his suggestion has been followed generally.

A number of articles and papers have appeared since that mentioned, some of which are included in the Bibliography. Probably the most thorough are the "Symmetrical Components" articles, by Wagner and Evans, from the first of which is taken the quotation constituting the first paragraph of this Appendix.

The method of Symmetrical Components was developed as a tool, but it does not follow that these components are abstract mathematical conceptions. The existance and value of any component of current and voltage may be determined by properly connecting instrument transformers to the primary system, and the impedance to flow of any component current may be measured. Furthermore, standard equipment, operating in response to a selected component, or groups of components, has been on the market some time, witness the positive phase sequence voltage regulator. Ground relays are actuated by zero sequence currents, or the interaction of zero sequence currents and voltages.

A system study by this method has in many instances resulted in changes in method of protection, setting of relays, application of circuit breakers and in determination of characteristics of new generating and transforming devices.

The discussion of the theory in this Appendix is limited to that portion necessary to explain the application to the specific problem at hand. This limitation does not particularly curtail exposition of the theory itself, but limits discussion of equipment characteristics to such apparatus as is encountered in this particular system.

The value of the method as a means of solving systems is dependent upon the condition that the impedances of the three phases of the system are equal. It is important to note that, in the following discussion, all vectors rotate in the positive (counter-clockwise) direction; root mean square values are used throughout; capital letters indicate vector quantities for the customary magnitudes.

The vector operator a is used extensively, and is defined:

$$a = 1.0 \angle 120^\circ = -.5 + j\ .866 = \varepsilon^{j\frac{2\pi}{3}} = \varepsilon^{j\,120^\circ}$$

from which it may be shown that:

$$a^2 = 1.0 \angle 240^\circ = -.5 - j\ .866 = \varepsilon^{j\frac{4\pi}{3}} = \varepsilon^{j\,240^\circ}$$

$$a^3 = 1.0 \qquad a + a^2 + 1 = 0$$

$$a^4 = a^7,\ \text{etc.} = a \qquad a + a^2 = -1$$

$$a^5 = a^8,\ \text{etc.} = a^2 \qquad a - a^2 = j\,[3]^{1/2}$$

$$a^6 = a^9,\ \text{etc.} = a^3 = 1 \qquad a^2 - a = -j[3]^{1/2}$$

$$1-a = j\,a^2[3]^{1/2} = 1.5 - j\ .866 \qquad a - 1 = -j\,a^2[3]^{1/2}$$

$$1-a^2 = -j\,a\,[3]^{1/2} = 1.5 + j\ .866 \qquad a^2 - 1 = j\,a\,[3]^{1/2}$$

The three component systems are termed the positive phase sequence system, the negative phase sequence system, and the zero phase sequence system.

Assume an original unbalanced system E_a, E_b, E_c, reaching a maximum in the order given.

The positive phase sequence component system is denoted by the subscript 1, and is defined:

$$E_{a1} = E_{a1} \qquad E_{b1} = a^2 E_{a1} \qquad E_{c1} = a\,E_{a1} \tag{1}$$

The negative phase sequence component system is denoted by the subscript 2, and is defined:

$$E_{a2} = E_{a2} \qquad E_{b2} = a\,E_{a2} \qquad E_{c2} = a^2 E_{a2} \tag{2}$$

The zero phase sequence component system is denoted by the subscript 0, and is defined:

$$E_{a0} = E_{a0} \qquad E_{b0} = E_{a0} \qquad E_{c0} = E_{a0} \tag{3}$$

In reality, therefore, only two of the component systems, the positive and negative, are symmetrical, the three vectors of the zero sequence system being in phase. The magnitudes of the three vectors in any component system are equal.

From the definitions given, the nomenclature of the component systems may be understood: the vectors of the positive sequence system reach a maximum in the same order as the original system, a, b, c; the vectors of the negative sequence reach a maximum in the order reverse to that of the original system, c, b, a; the vectors of the zero sequence system reach a maximum at the same time (i.e., there is no sequence of vectors in this system). Note the term negative applies to the sequence in which the vectors reach a maximum, and not to the rotation. As stated before, all rotation is counter-clockwise.

The components must add to the original:

$$E_a = E_{a1} + E_{a2} + E_{a0} = E_{a1} + E_{a2} + E_{a0} \quad (4)$$

$$E_b = E_{b1} + E_{b2} + E_{b0} = a^2E_{a1} + a\,E_{a2} + E_{a0} \quad (5)$$

$$E_c = E_{c1} + E_{c2} + E_{c0} = a\,E_{a1} + a^2E_{a2} + E_{a0} \quad (6)$$

Adding the three equations,

$$E_{a0} = 1/3\ (E_a + E_b + E_c) \quad (7)$$

Multiplying the second by a and the third by a^2, then adding,

$$E_{a1} = 1/3\ (E_a + aE_b + a^2E_c) \quad (8)$$

Multiplying the second by a^2 and the third by a, then adding,

$$E_{a2} = 1/3\ (E_a + a^2E_b + aE_c) \quad (9)$$

E usually represents voltage, but it is obvious the decomposition effected above is appropriate to systems of current, flux or any other complex quantities.

These equations suggest a method for the graphical determination of the various components. When solving by this means, a little time may be saved by remembering that the vector from the origin to the center of gravity of any triangle equals one-third the vector sum of the three vectors from the origin to the vertices of the triangle.

Equations (7), (8), (9) give the three components of the vector representing phase a. Corresponding components of the other two phases are determined by the expressions in the group of equations (1), (2), (3). Phase a is arbitrarily chosen as the reference phase, but calculations may be carried out with any phase as reference phase.

Consider a three phase line in which the currents are I_a, I_b, I_c. The zero phase sequence component is $1/3\ (I_a + I_b + I_c)$, and this component exists in each of the phases. Since these currents cannot return over a line wire, they must do so through the ground circuit, so the ground, or residual current equals $(I_a + I_b + I_c)$ or $3\ I_{a0}$.

Refer to the circuit represented in group (a) of Figure 18. The three phases offer: equal self impedance, Z_{aa}; equal mutual impedances between phases, Z_{ab}; equal mutual impedance between phase and return, Z_{an}; the self impedance of the return path Z_{nn}. The circuit may represent any usual static machinery, such as transformer bank, transmission lines, etc.

It is desired to determine the relations between the impedances and the various sequence components of current and voltage:

By inspection:

$$E_a = I_a Z_{aa} + (I_b + I_c)\ Z_{ab} + I_n Z_{an} - I_n Z_{nn} - (I_a + I_b + I_c)\ Z_{an}$$

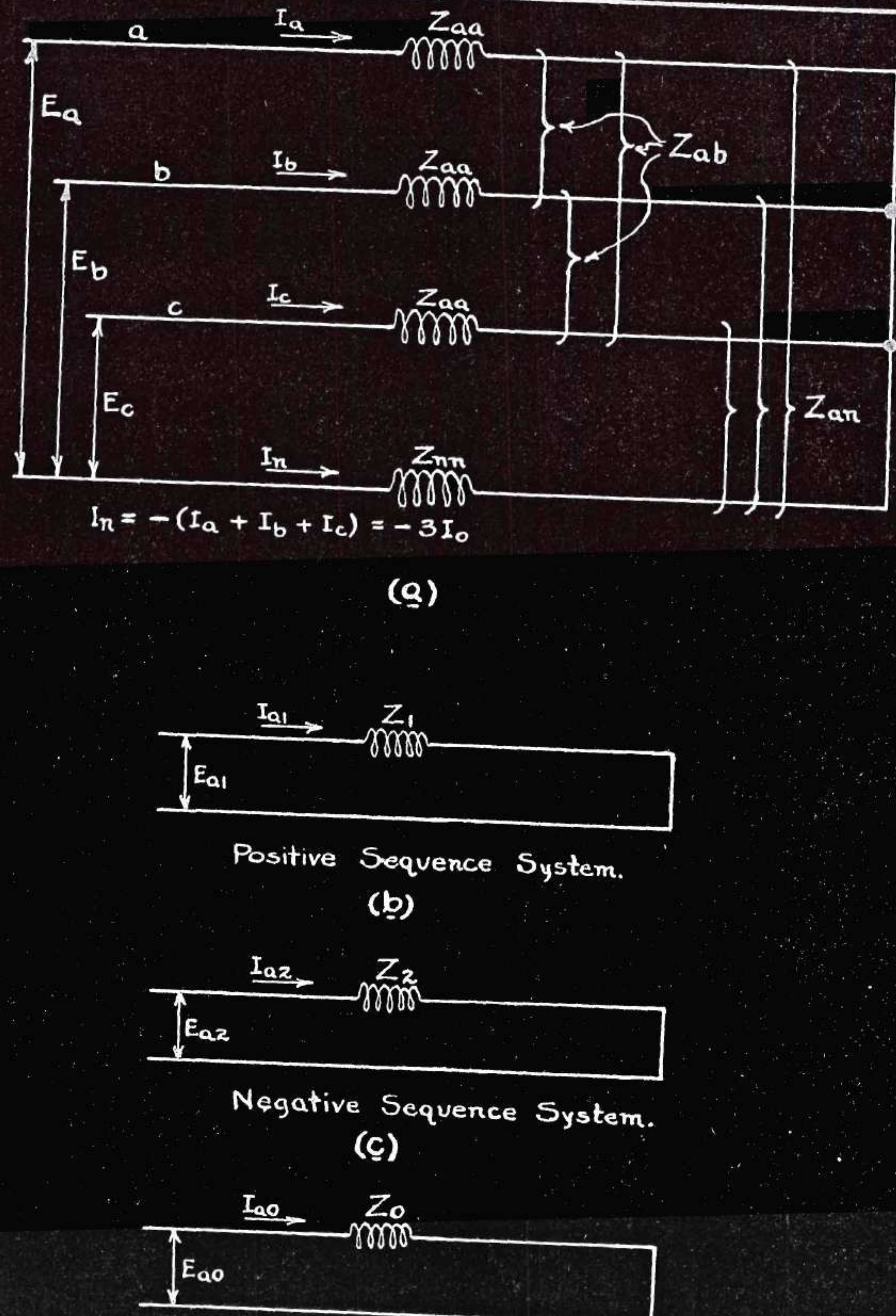


FIG. 18
ORIGINAL AND COMPONENT SEQUENCE SYSTEMS.

$$E_a = I_a Z_{aa} + (I_b + I_c) Z_{ab} + 3 I_0 Z_{nn} - 6 I_0 Z_{an}$$

Similarly:

$$E_b = I_b Z_{aa} + (I_a + I_c) Z_{ab} + 3 I_0 Z_{nn} - 6 I_0 Z_{an}$$

$$E_c = I_c Z_{aa} + (I_a + I_b) Z_{ab} + 3 I_0 Z_{nn} - 6 I_0 Z_{an}$$

Therefore, the positive sequence:

$$E_{a1} = 1/3 (E_a + aE_b + a^2E_c) = 1/3 (I_a + aI_b + a^2I_c) Z_{aa}$$
$$+ 1/3 (I_b + I_c + aI_a + aI_c + a^2I_a + a^2I_b) Z_{ab}$$
$$+ 1/3 (1 + a + a^2) 3 I_0 Z_{nn} - 1/3 (1 + a + a^2) 6 I_0 Z_{an}$$

remembering that $1 + a + a^2 = 0$ and $I_{a1} = 1/3(I_a + aI_b + a^2I_c)$

$$E_{a1} = I_{a1} (Z_{aa} - Z_{ab})$$

The negative phase sequence

$$E_{a2} = 1/3 (E_a + a^2E_b + aE_c) = 1/3 (I_a + a^2I_b + aI_c) Z_{aa}$$
$$+ 1/3 (I_b + I_c + a^2I_a + a^2I_c + aI_a + aI_b) Z_{ab}$$
$$+ 1/3 (1 + a^2 + a) 3 I_0 Z_{nn} - (1 + a^2 + a) 6 I_0 Z_{an}$$

$$E_{a2} = I_{a2} (Z_{aa} - Z_{ab})$$

The zero phase sequence

$$E_{a0} = 1/3 (E_a + E_b + E_c) = 1/3 (I_a + I_b + I_c) Z_{aa}$$
$$+ 1/3 (I_a + I_b + I_c) 2 Z_{ab} + 1/3 \times 9 I_0 Z_{nn} - 1/3 \times 18 I_0 Z_{an}$$

$$E_{a0} = I_0 (Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6 Z_{an})$$

Recapitulating and transferring

$$I_{a1} = \frac{E_{a1}}{Z_{aa} - Z_{ab}} = \frac{E_{a1}}{Z_1} \qquad (10)$$

$$I_{a2} = \frac{E_{a2}}{Z_{aa} - Z_{ab}} = \frac{E_{a2}}{Z_2} \qquad (11)$$

$$I_{a0} = \frac{E_{a0}}{Z_{aa} + 2 Z_{ab} + 3 Z_{nn} - 6 Z_{an}} = \frac{E_{a0}}{Z_0} \qquad (12)$$

These three equations indicate the property largely responsible for the importance of symmetrical components; i.e. the independence of the different sequence systems when applied to an original system having equal impedances per phase. In such an original system, positive sequence currents produce only positive sequence voltage drops, negative sequence currents produce only negative sequence voltage drops and zero sequence currents only zero sequence drops. For this reason, the polyphase system represented in Figure 18, group (a) may be replaced by three independent single phase systems, shown in groups (b), (c), (d), which are individually much easier to handle.

Since the return current conductors of the single phase system contain no impedance, the impedances Z, Z_2, Z_0 may be used directly in single line diagrams.

These equations also give expressions for the determination of the impedance offered by the circuit to current of the several phase sequence components. Z_1, Z_2, Z_0 are known respectively as "impedance to positive sequence current", "impedance to negative sequence current" and "impedance to zero sequence current." Impedances are discussed more fully in the section giving constants of equipment.

As the conditions assumed above apply only to static machinery, the deductions are subject to the same limitations. However, in so far as independence of the component systems is concerned, this property is also true of rotating machinery. The only condition necessary for separation is that the impedances offered by the three phases be equal - a requirement inherently met by practically all modern equipment.

Solution of a network is made by reducing the network to three sequence "equivalent impedances" similar to those of groups (b), (c), (d), Figure 18, connecting these in a manner to satisfy the conditions of the fault, solving for the sequence component currents at the fault, then in the branches, and finally adding the three components in each branch to obtain phase magnitudes of the original system. Voltage at any point is determined by adding IZ drops in each sequence network to the point in question.

The remainder of this appendix is therefore devoted to a discussion of the method of determining the impedance to sequence currents of the different types of equipment encountered in this study, the proper connections of the three sequence networks for the different types of faults, and methods for reducing original networks to "equivalent impedances."

Constants of equipment.

From equations (10), (11) and (12):

$$Z_1 = Z_{aa} - Z_{ab}$$

$$Z_2 = Z_{aa} - Z_{ab}$$

$$Z_0 = Z_{aa} + 2\ Z_{ab} + 3\ Z_{nn} - 6\ Z_{an}$$

in which Z_1, Z_2, Z_0 are the impedances, respectively, to positive sequence current, negative sequence current, and zero sequence current.

Impedances, being vector quantities, may be broken into phase sequence components in the same manner as voltage and current vectors, and these components, following the nomenclature of similar components of the other quantities, would be termed "positive sequence impedance", "negative sequence impedance" and "zero sequence impedance." It is only necessary to resolve impedances into sequence components when the actual impedances of the three phases of the original system are unequal.

Practically all electrical power equipment, motors, generators, transformers, transmission lines, etc., have equal phase impedances, and for this customary condition, the equations (10), (11), (12) show it is not necessary to determine the sequence components of impedances in order to arrive at the impedances limiting the magnitude of component currents. Of course, at the fault, conditions are different for the different line phases, but in general, from supply to fault, system impedances are equal in all phases.

This has led to the general adoption of an inexact terminology, in that the impedance offered to positive sequence current is called "positive sequence impedance", that to negative sequence current is called "negative sequence impedance", and, similarly, "zero sequence impedance." The general practice is followed throughout this study, so the expressions Z_1, Z_2 and Z_0 are termed, respectively, positive, negative and zero sequence impedance.

This section is devoted to a discussion of the impedances of different types of equipment encountered in this investigation.

(a) Transmission Lines. A device frequently used to facilitate calculation of inductance and capacitance of transmission lines is known as the geometric mean distance. This enters from a determination of an equivalent simple conductor that will have the same inductive (and capacitive) effect on itself and on other conductors, as the one, two, or more actual parallel conductors that it replaces.

Replacing the actual conductor or group of conductors by an imaginary tube of infinitesimal thickness, all the current may be considered as flowing at the surface, and therefore no flux within the conductor. This replacement is of particular use in calculating the zero sequence impedance of a polyphase line, or of parallel lines.

The geometric mean radius of a conductor is the radius of the imaginary replacing tube. For uniform current distribution, non-magnetic material, this radius is equal to the geometric mean distance from the conductor to itself. The G.M.D. of a conductor to itself has a mathematical definition, whereas the G.M.R. is based upon inductive effect, so that in instances where current distribution is not uniform, or the conductor is of magnetic material, or for other reasons, it is necessary to correct the G.M.R., and it no longer is the equivalent of the G.M.D. of the conductor to itself.

The formula for the inductance of one wire of a line, return at distance D, solid conductor, radius a, unit permeability, is

$$L = 2 \log_{\varepsilon} \left(\frac{D}{a}\right) + 1/2, \text{ abhenries per cm.}$$

The inductance of the imaginary tube, radius r, is to be

$$L = 2 \log_{\varepsilon} \left(\frac{D}{r}\right), \text{ same units}$$

since the 1/2 is due to flux inside the wire. The two expressions are equal:

$$2 \log_\epsilon \left(\frac{D}{r}\right) = 2 \log_\epsilon \left(\frac{D}{a}\right) + 2 \log_\epsilon \epsilon^{.25}$$
$$= 2 \log_\epsilon \frac{D \epsilon^{.25}}{a}$$

or

$$r = a \epsilon^{-.25} = .779 \text{ a.}$$

Other conductor G.M.R.:

Material	Stranding	G.M.R.
Non-magnetic	Solid	.779 a
" "	7	.724 a
" "	19	.759 a
" "	37	.767 a
" "	61	.771 a
A.C.S.R.	all	.95 a

With the same limitations:

The geometric mean distance between two circular conductors is the physical distance between their centers.

The geometric mean radius of a group of three circular conductors is:

$$\text{Group G.M.R.} = [\text{G.M.R.} \times D^2]^{1/3}$$

in which

G.M.R. = conductor geometric mean radius

and D = equivalent spacing = $[d_1\ d_2\ d_3]^{1/3}$

The geometric mean distance from a group of three circular conductors to another group of three circular conductors, is:

$$\text{G.M.D.} = [d_{1a}\ d_{1b}\ d_{1c}\ d_{2a}\ d_{2b}\ d_{2c}\ d_{3a}\ d_{3b}\ d_{3c}]^{1/9}$$

Stated in words, this is the ninth root of the ninefold product of the distances from each conductor in one group to each of the conductors in the other group.

Positive sequence impedance.

$$Z_1 = Z_{aa} - Z_{ab}$$

Z_{aa} is the self impedance of the conductor, and is the vector sum of the resistance of the wire and the reactance due to its own current, with return at a great distance.

Z_{ab} contains no resistance component; it is the reactance in conductor a due to current in the other conductors distant D from a, where D is line spacing.

X_{aa}, reactance component of Z_{aa}, is a function of all the flux caused by current in a, integrated from zero to P, the return, $\phi_{a\,(0 \to P)}$, where zero is the radius of the imaginary replacing tube.

Positive sequence currents are symmetrical three phase currents, so if the system is considered at the instant I_{a1} is a maximum, then the currents in each of the other conductors are equal, and equal $I_{a1/2}$; flux caused by each of these currents is half that due to I_{a1}. Reactance in a is due to only that portion of flux from other conductors which cuts a, $1/2\,\phi_{a(D \to P)}$, from each.

The effects of the currents in the other two conductors are additive, so

$$Z_1 = R + j(f)\,\phi_{a(0 \to P)} - j\,(f)\phi_{a(D \to P)}$$

$$= R + j(f)\,\phi_{a[(0 \to P) - (D \to P)]}$$, since the same function applies to both fluxes.

$$= R + j(f)\,\phi_{a(0 \to D)}$$

From this it is seen that the positive sequence impedance of a transmission line is the same as the impedance customarily used, for which tables are prepared giving resistance and reactance per mile for different sizes and spacings of conductors.

The above is correct for equal spacings between conductors. In the case of irregular configuration, it is correct if transpositions are made, but when this is not done, the impedances of the phases are no longer equal and considerable complication results. In general, the method is sufficiently accurate lines with conductors assymmetrically arranged, even without transpositions; in the particular system at hand, conductor spacings are all fairly equal.

Positive sequence impedance may be obtained from tables or from the following:

$$Z_1 = R + j\ .05248 \log_{10} \frac{D}{G.M.R.}, \text{ ohms per 1000 ft., 60 cycles.}$$

D is the equivalent spacing, and G.M.R. is the conductor geometric mean radius, in the same units as D.

The positive sequence impedance of two identical parallel lines is taken as one-half that of one line. This is not exact, as even with transpositions to eliminate mutual induction between the two, there is a further reduction of approximately three percent, but neglecting this three percent does not cause a corresponding error in the fault current.

Negative sequence impedance is the same as positive sequence impedance for all non-rotating apparatus.

Zero sequence impedance is the impedance offered to equal and in-phase currents in each of the overhead line conductors, with return through the ground circuit. There are no ground wires within the distribution system at hand, and all return is through the ground.

Zero sequence impedance is therefore a special case of a single phase circuit, in which one conductor consists of the three line wires in parallel, the other conductor being the earth.

The expression for the inductance of one wire of a single phase circuit is

$$L = 2 \log_{\varepsilon} \frac{P}{G.M.R.}, \text{ abhenries per cm.}$$

P is the spacing of the two conductors.

When both conductors are same size and same material, the inductances of the two are equal, and the reactance of the complete circuit is a function of twice the expression given.

From a consideration of the geometrical configuration of an overhead transmission line with ground return, it is apparent the go and return conductors are of entirely different properties, and consequently their inductances are not equal. In computing the reactance of such a circuit, it is necessary to make separate calculations for each side. The same expression applies to the ground circuit as to the overhead conductor, the only difference being that G.M.R. in the case of the line is the geometric mean radius of the group of three conductors, whereas it is the geometric mean radius of the earth path when applying the expression to the return side of the circuit.

Applying the expression $L = 2 \log_{\varepsilon}(D/G.M.R.)$ to the overhead line results in a quantitative value for the inductance of the overhead conductor. This conductor is in reality three parallel wires, and the inductance of any one is therefore three times that of the group.

Now consider the determination of the inductance of the ground return path by the same method. G.M.R. is obviously very nearly equal to D, and the resultant inductance is quite small, depending upon the geometric mean radius of the earth return path.

The main value of considering the expression in relation to the earth return path is the conclusion that inductance for this part of the circuit should be determined by test rather than attempting to theorize about probable earth G.M.R. It is easier to make tests including both sides of the circuit.

Considerable work has been done both in this country and abroad on the problem of zero sequence impedance. It is apparent the distribution of return current in the earth is dependent upon condition of the soil, dry or damp, upon geological formations, conducting veins, etc.

The expressions generally used for zero sequence impedance of transmission lines result from emperical formulae presented by John R. Carson in "Wave Propagation in Overhead Circuits with Ground Return," see bibliography. While the article concerns only single phase circuits, the conclusions may be corrected to cover the condition of the power transmission line, and the results are important in that they afford a means for evaluating peculiar local conditions of the earth, and incorporating them in the expression for zero sequence impedance.

For the usual transmission line, with ordinary spacing, the Carson formula may be changed to give the following expression for the zero sequence impedance:

$$Z_0 = R + .0542 + j\ .1587 \log_{10} \frac{D_e}{G.M.R.}, \text{ ohms per 1000 ft., per phase, 60 cycles.}$$

in which

R = resistance per 1000 ft. of one overhead conductor

D_e = equivalent depth of earth return.

G.M.R. = group geometric mean radius of overhead conductors, in same units as D_e.

The 0.0542 represents resistance per 1000 ft., of earth, at 60 cycles, and is independent of earth conductivity. Wagner and Evans explain this by pointing out that at high conductivity the current path is restricted to a small section area below the line, whereas at low conductivity the current spreads out over considerable area. Test and experience indicate the return path follows very closely the overhead line, rather than cutting across country to return by the shortest physical path, and it is for this reason the expression for earth return resistance and reactance may be incorporated with that for the line. As a matter of fact, 0.0542 is not the actual resistance of the ground per 1000 ft. Zero sequence currents exist in each of the line (when at all), but computations are made for only one phase. The total current returns through the earth, hence total resistance (and reactance) drop in the return path in three times what it would be were only the zero sequence current of one phase returning. In order to refer this drop to a single phase circuit with one-third the ground current, it is necessary to multiply any actual neutral path resistance (or reactance) by three. This also applies to resistance of arcs to ground, transformer grounding resistors, etc.

The equivalent depth of earth return, D_e, is the depth at which the return current may be considered as concentrated upon a conductor. It is in the selection of this distance that

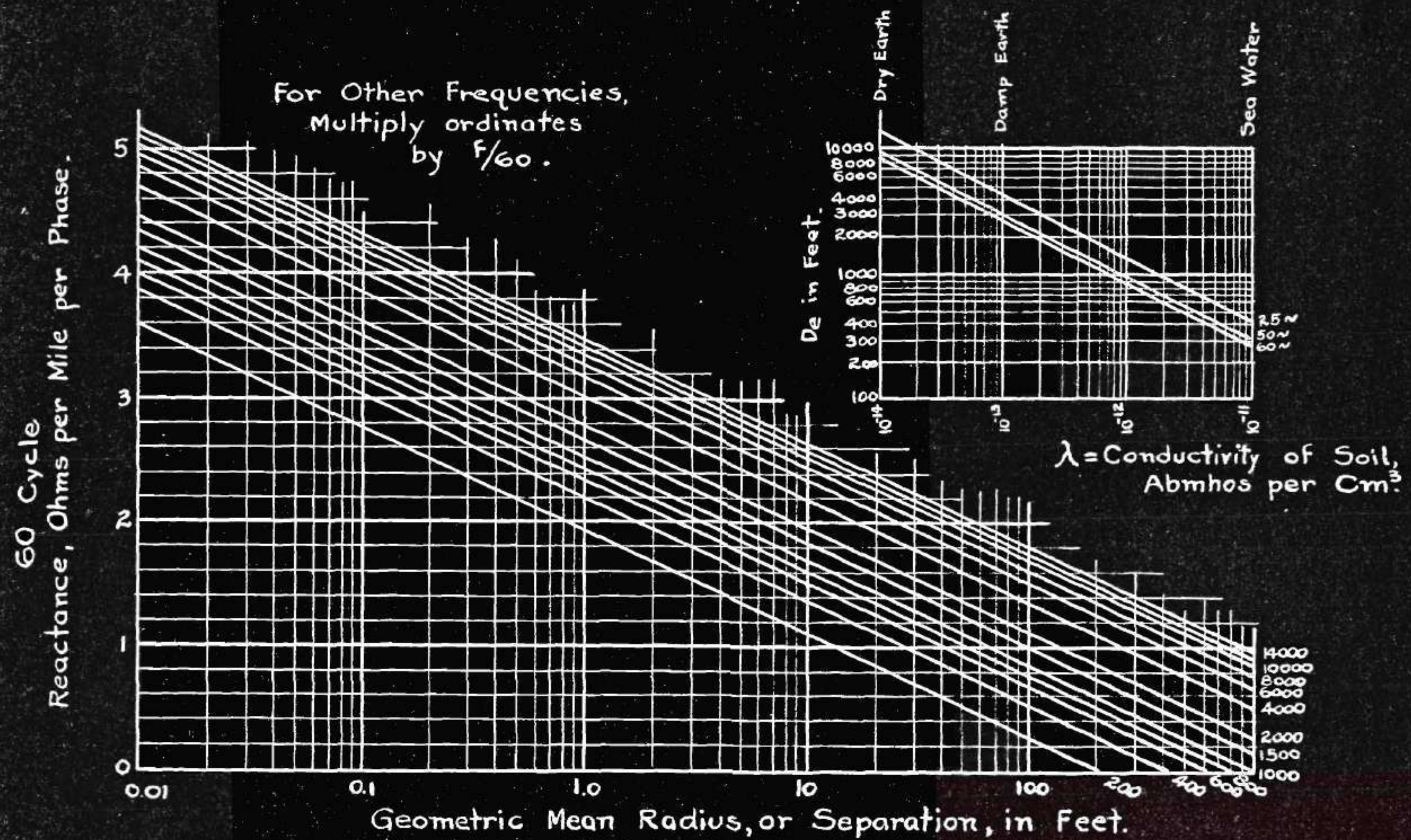


FIG. 19

ZERO SEQUENCE SELF & MUTUAL REACTANCE. TRANSMISSION LINES.

local peculiarities of the earth may be taken into account. Carson brings out that the equivalent depth is dependent upon earth conductivity, and Figure 19 expresses this relation. This figure also gives a graph for determining the zero sequence reactances (self and mutual) for various group G.M.Rs. and equivalent depth of return.

Earth conductivity for Tennessee, as found by test, is 10^{-14} abmhos per cm^3, corresponding to a depth of 8800 ft., the depth for very dry earth, and this value is used in the study for the high tension line calculations. However, Knoxville is situated at the confluence of the Holston and French Broad Rivers, marking the beginning of the Tennessee River, so it is reasonable to assume the earth around Knoxville as damp. Earth conductivity for the distribution network is therefore assumed as 10^{-13}, corresponding to an equivalent depth of 2800 ft. The reactance at this depth is about 10 percent less than at the deeper.

Reactance component of zero sequence impedance of parallel lines is not one-half that for one line, as is the case in the positive and negative sequence networks. The inductive effect of all conductors are additive on each, different from the positive sequence condition.

In short, the zero sequence impedance of two identical parallel transmission lines is:

$$Z_0 = \frac{R}{2} + 0.0542 + j\, 0.1587 \log_{10} \frac{D_e}{[(G.M.R.)(G.M.D.)]^{1/2}} \quad \text{ohms per 1000 ft. 60 cycles.}$$

Here D_e is as before, G.M.R. and G.M.D. are respectively the geometric mean radius of one of the line groups, and the geometric mean distance between the two groups.

It is sometime preferable to use the self and mutual impedances of parallel lines separately. The self impedance of one line is that already given for the zero sequence impedance of a three phase line. The mutual zero sequence impedance for one line is

$$Z_{0m} = .0542 + j\ .1587\ \log_{10}\frac{D_e}{G.M.D.},\ \text{ohms per 1000 ft. 60 cycles}$$

(b) Transformers. Transformers, like transmission lines, offer equal impedance to negative and positive sequence currents, because reversal of phase sequence does not cause reversed rotation of any mechanical parts such as is the case with generators and motors.

It is not necessary to make any particular calculations to determine the various impedances of transformers. It could be done, and is done by manufacturers in designing transformers to have a specified reactance. The nameplate of most transformers is stamped with percent reactance on rated KVA. base, and this is the value to be used. If reactance is desired in ohms on a voltage base instead of percent on a reference KVA. base, the conversion is made by

$$\text{Ohms} = \frac{\%\ \text{impedance} \times (\text{KV.})^2 \times 10}{\text{KVA.}}$$

in which the % impedance is expressed as percentage and not as a decimal, KV. is line reference voltage, and KVA. machine rating.

The impedance of transformers may be considered as all reactive.

The "equivalent circuit" of a transformer is used in the network diagrams, the admittance branch, representing exciting circuit, is omitted.

Hence a two winding transformer is represented on the positive and negative sequence diagrams by an inductance of the value determined from the nameplate reading. The zero sequence reactance of a two winding bank is either the same as the positive and negative reactances, or it is infinity, depending upon whether or not there is a path for zero sequence currents.

From a consideration of the nature of zero sequence currents, it may be appreciated that the only means by which such can get into and pass through the transformer windings is a connection from neutral of the bank to the return path; in other words, the windings on the fault side of the bank must be grounded.

Compensating currents must then exist in the other winding, which, if delta connected, provides a free path, the zero sequence compensating currents, being in-phase in the delta windings, circulate in the same manner as third harmonic currents, and the impedance of the bank is the same as positive and negative sequence impedance. Since the compensating currents circulate in the delta, they cannot pass into the system connected to the delta side, therefore representation on the zero sequence diagram is by an inductance connected on one side (Y) to the fault portion of the system, the other side connected to the bus of zero potential, but disconnected from the system of the delta side.

If the other winding is also wye, grounded, a path is provided for compensating currents, which, however, must pass over the lines of the system to which the winding is connected, and return through some other wye connected equipment to the ground, through which the circuit is completed. Representation on the network diagram is by an inductance connecting the primary and secondary systems, same as for the positive and negative sequence networks.

The three winding transformer representation is somewhat different. In the first place it is necessary to determine the reactances of the individual windings. The positive sequence reactance between each pair of windings is given on the nameplate. Designate the quantities associated with the primary, secondary and tertiary windings by the subscripts a, b, c, respectively. The reactance encountered in the transfer of energy from primary to secondary is the sum of the reactances of the primary and secondary windings.

$$Z_{ab} = Z_a + Z_b$$

also $$Z_{ac} = Z_a + Z_c$$

and $$Z_{bc} = Z_b + Z_c$$

So the impedances (or reactance) of the individual windings are:

$$Z_a = \frac{Z_{ab} + Z_{ac} - Z_{bc}}{2}$$

$$Z_b = \frac{Z_{ab} + Z_{bc} - Z_{ac}}{2}$$

$$Z_c = \frac{Z_{ac} + Z_{bc} - Z_{ab}}{2}$$

Representation on the positive and negative sequence networks for three winding transformers is made by a group of three impedances, connected in wye, each branch having a value of impedance (considered wholly reactive) equal to that determined above, the three branches being connected to the three lines representing the original connections to the transformer. In the system at hand, the tertiaries of transformers, when connected, are considered as shunt branches and are omitted. Hence representation on the positive and negative sequence networks is the same as for two winding transformers.

The same considerations in the matter of zero sequence currents apply to three winding transformers as to two winding. First it is necessary for the zero sequence currents to get into the bank, second it is necessary that there be a path for compensating currents.

There are now two possibilities for compensating currents, either the other two windings, or both, may provide a path. Each winding and the system connected thereto, is considered separately, and the connections made for each as though it were part of a two winding bank.

From this it is seen that a delta connected tertiary may, in a sense, short out all the zero sequence network connected to the primary, since its path is directly to the bus of zero potential, whereas the other is through the complete zero sequence network to the same bus. The currents through the primary and tertiary are inversely as their reactances to the zero potential bus, when fault is on the secondary side of the bank.

(c) Machines. Motors and condensers supply current to a fault during the first few cycles and before their rotational energy has had time to die down. These machines must therefore be considered along with the generators as a source of fault KVA.

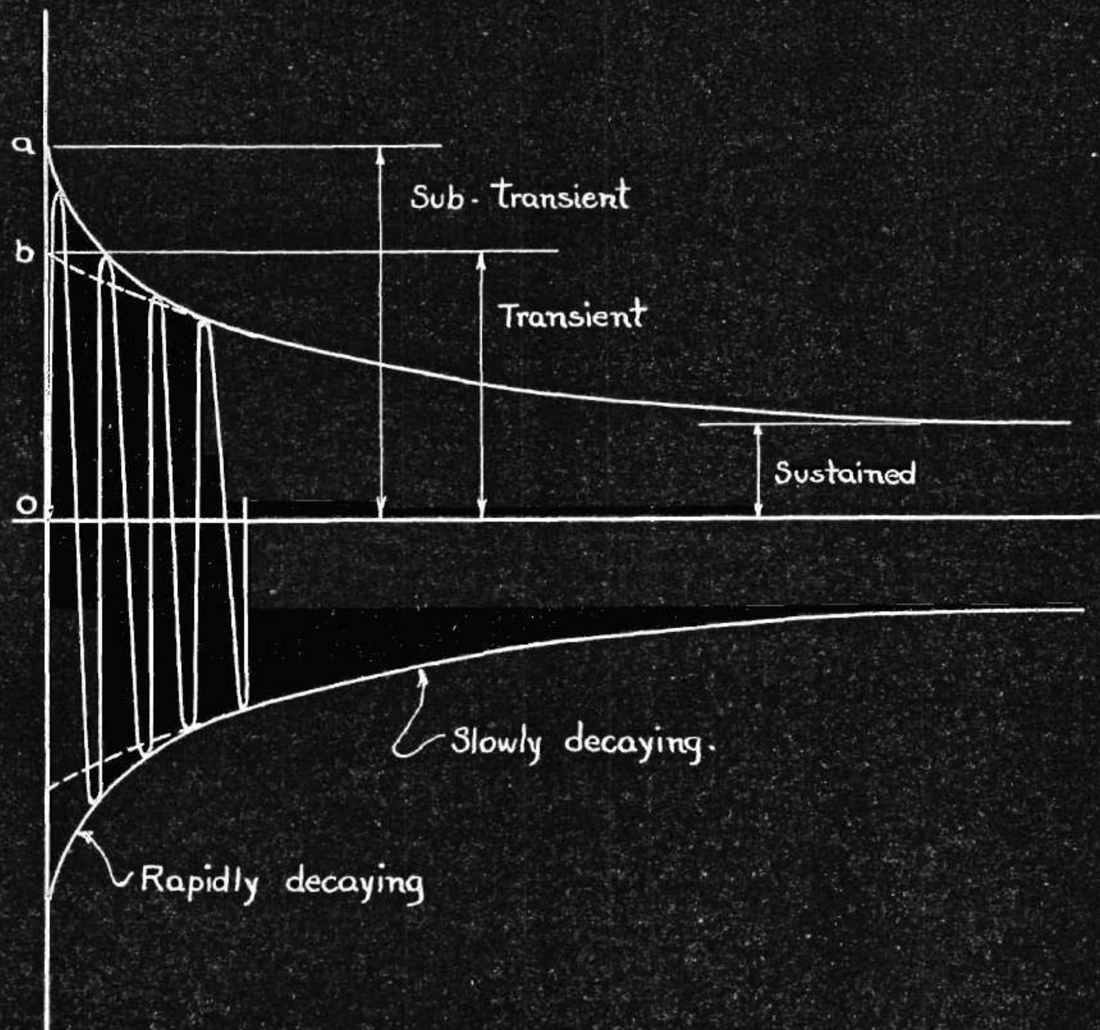
There are no motors to be considered in the system at hand, and the only condenser is small, with consequent high relative reactance. The only generators about which there is any data are the Waterville machines, and the question of machine constants will therefore be limited to them. No attempt is made to go into the theory thoroughly, both because it is not necessary to consider machine characteristics so exactly in the study, and also because of space limitations.

Recently there have been some advances in the theory of synchronous machines, making possible the accurate analysis of unbalanced load and fault conditions. Considerable work is still being done on the subject, and doubtless the next few months will see the development of procedure in dealing with machine problems to a more or less standardized form. Several important papers were presented at the mid-winter convention of the A.I.E.E., January, 1932, and the subject is much more comprehensively covered by them.

New terminology has been created to care for the constants developed in the new theory. Also new decrement curves have been published, superseding the "standard curves" given in the N.E.L.A. Relay Handbook and various manufacturers' pamphlets.

Figure 20 shows short circuit current in a phase having no assymmetrical component. The same envelopes of current could be obtained from a duplicate short circuit in which the wave is assymmetrical, by subtracting the assymmetrical component.

The current wave in any (sudden) short circuit (three phase, line-to-line, etc.) may be resolved into components:



Typical Fault Currents determined by the various positive sequence Machine reactances.

FIG. 20

MACHINE SHORT CIRCUIT CURRENT.

(1) Assymmetrical.

(2) Symmetrical.

(a) Sustained.

(b) Slowly decaying. (This is due to currents induced in the field winding or other high conductivity paths in the rotor.)

(c) Rapidly decaying. (This is due to currents induced in "extra" rotor circuits - damper bars, solid iron field collars, etc.)

The assymmetrical component is the customary D. C. component. Its maximum value at zero time (for short circuits from no load) is equal to the peak A.C. at the same instant. Decay of the assymmetrical component is represented by the D.C. time constant, T_{dc}, the time in seconds from the instant of short circuit for the assymmetrical component to decay to 36.8% of its initial value.

The symmetrical component is as shown in Figure 20. In order to tie this figure in with machine constants, the constants concerned are listed.

Reactances.

(1) Positive sequence.

Sub-transient reactance, X''_d

Transient reactance, X'_d

Synchronous reactance, X_d

(2) Negative sequence reactance

(0) Zero sequence reactance.

Time Constants.

T'_{do} Open circuit transient time constant.

T'_{d} Short circuit transient time constant.

T''_{d} Short circuit subtransient time constant.

The initial symmetrical short circuit current is determined by the sub-transient reactance, in conjunction with the corresponding internal voltage. As shown in Figure 20, this component decays rapidly, T''_{d} being the time in seconds required for the difference between the total symmetrical current and the slowly decaying component of symmetrical current to decay to 36.8 percent of its initial value (difference). Peak value of the initial subtransient current is indicated by a, Figure 20.

Currents determined by the use of the transient and synchronous reactances, in conjunction with appropriate internal voltages, are noted in the figure.

New standard decrement curves, based upon the new theory, were first published by W. C. Hahn and C. F. Wagner in a paper "Standard Decrement Curves" before the A.I.E.E. convention, January, 1932.

In so far as this study is concerned, the old standard curves indicate same as the new, i.e., that for system impedance of over 100%, the initial values may be considered sustained. This is not initial subtransient, but initial transient values.

Connection of the Phase Sequence Networks for Different Types of Faults.

Faults are of the following types:

(a) Three phase short.

(b) Two phase short.

(c) Single phase to ground.

(d) Double phase to ground.

By far the most common of these is the single phase to ground, either direct (low resistance), caused by line breakage, or by insulator flashover, a high resistance ground fault. Probably next in frequency of occurrence is the case of short circuit of two conductors, caused by debris blown into the line, or thrown in by boys. The three phase short is much less frequent than any, and probably next is the double phase to ground. These two are generally due to some serious accident to poles or towers.

It has been stated the only method by which negative and zero sequence voltages (and consequently currents) get into a system is by transformation from positive sequence voltages by the fault. For the different faults, there are different connections of the sequence networks in order to introduce these voltages into the system. There are also other conditions to be met, and it is necessary to analyze each of the faults in order to determine proper connections of the component networks.

The customary treatment at the point of fault is represented in Figure 21 group (a). Dead end jumpers are considered as tied to each of the three line conductors, the fault being assumed to occur at the free ends of these jumpers. I_x, I_y, I_z are total fault currents, in corresponding jumpers; e_x, e_y, e_z are potentials to ground.

<u>Three Phase Short.</u>

No unbalance is caused by this type fault, hence there

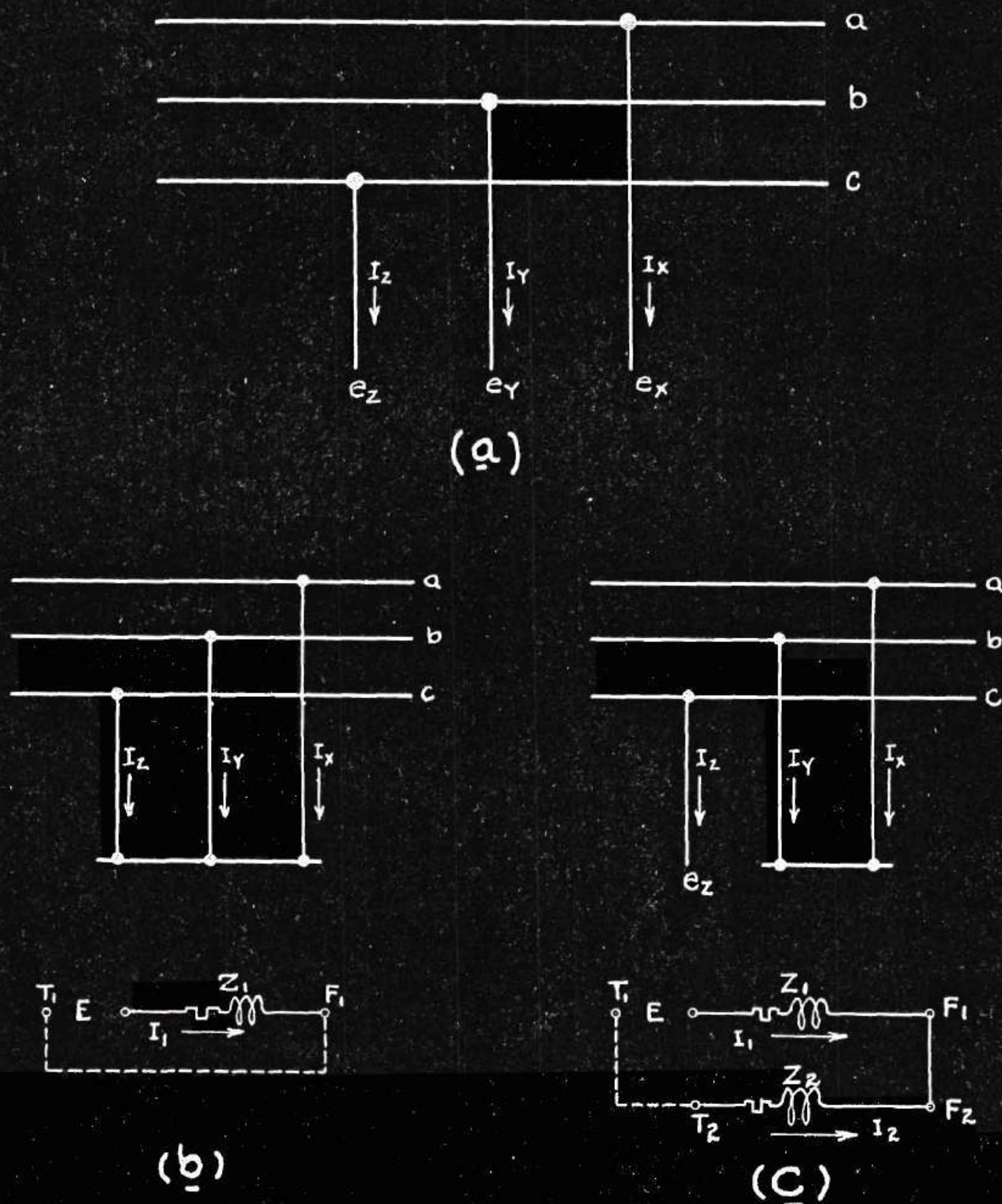


FIG. 21

CONNECTION OF THE SEQUENCE NETWORKS FOR DIFFERENT FAULTS – 1 are no negative or zero sequence components of current or voltage, and the corresponding networks are not considered. Representation is as shown in Figure 21 group (b). The positive sequence current, which in this case is the total current, is

$$I_{x1} = \frac{E}{Z_1}$$

Two phase short.

There is no path for zero sequence currents, and this network is omitted. Since negative and zero sequence voltages are consequences (IZ drops) of corresponding sequence currents, e_0 is also zero.

The conditions of this fault, as determined by inspection of Figure 21 group (c), are:

$$I_z = 0 \qquad\qquad I_x = -I_y \qquad\qquad e_x = e_y$$

Hence, with the unfaulted conductor as reference vector, current components are:

$$\begin{aligned} I_{z1} &= 1/3\ (I_z + aI_x + a^2 I_y) \\ &= 1/3\ (0 + aI_x - a^2 I_x) \\ &= (a - a^2)\ I_x/3 \\ I_{z2} &= 1/3\ (I_z + a^2 I_x + aI_y) \\ &= 1/3\ (0 + a^2 I_x \ - aI_x) \\ &= (a^2 - a)\ I_x/3. \end{aligned}$$

from which $I_{z2} = -I_{z1}$, which suggests the connection of the sequence networks shown in group (c).

It is necessary to check voltage components in order to be certain the arrangement is satisfactory:

$$e_{z1} = 1/3\ (e_z + ae_x + a^2 e_y)$$

$$= 1/3\ [e_z + e_x\ (a + a^2)]$$

$$= 1/3\ (e_z - e_x)$$

$$e_{z2} = 1/3\ (e_z + a^2e_x + ae_y)$$

$$= 1/3\ [e_z + e_x\ (a^2 + a)]$$

$$= 1/3\ (e_z - e_x)$$

from which $e_{z1} = e_{z2}$. Reference to group c will show T_1 and T_2 at the same potential, also F_1 and F_2 at the same potential, so the voltage components conditions are satisfied by the grouping. Component currents are therefore

$$I_{z1} = - I_{z2} = \frac{E}{Z_1 \quad Z_2}$$

Single phase to ground.

All the components of current exist in this type fault. Referring to Figure 22, group (a), it may be seen that:

$$I_z = 0 \qquad I_y = 0 \qquad e_z = 0$$

Hence, using faulted line as reference:

$$I_{x0} = 1/3\ (I_x + I_y + I_z) = 1/3\ I_x$$

$$I_{x1} = 1/3\ (I_x + aI_y + a^2I_z) = 1/3\ I_x$$

$$I_{x2} = 1/3\ (I_x + a^2I_y + aI_z) = 1/3\ I_x$$

or

$$I_{x1} = I_{x2} = I_{x0}$$

which indicates the three networks should be connected in series. Checking to see the connection shown satisfied voltage conditions:

$$\text{Since } e_x = 0$$

$$e_{x1} + e_{x2} + e_{x0} = 0$$

The lZ drops through the networks add to zero in the

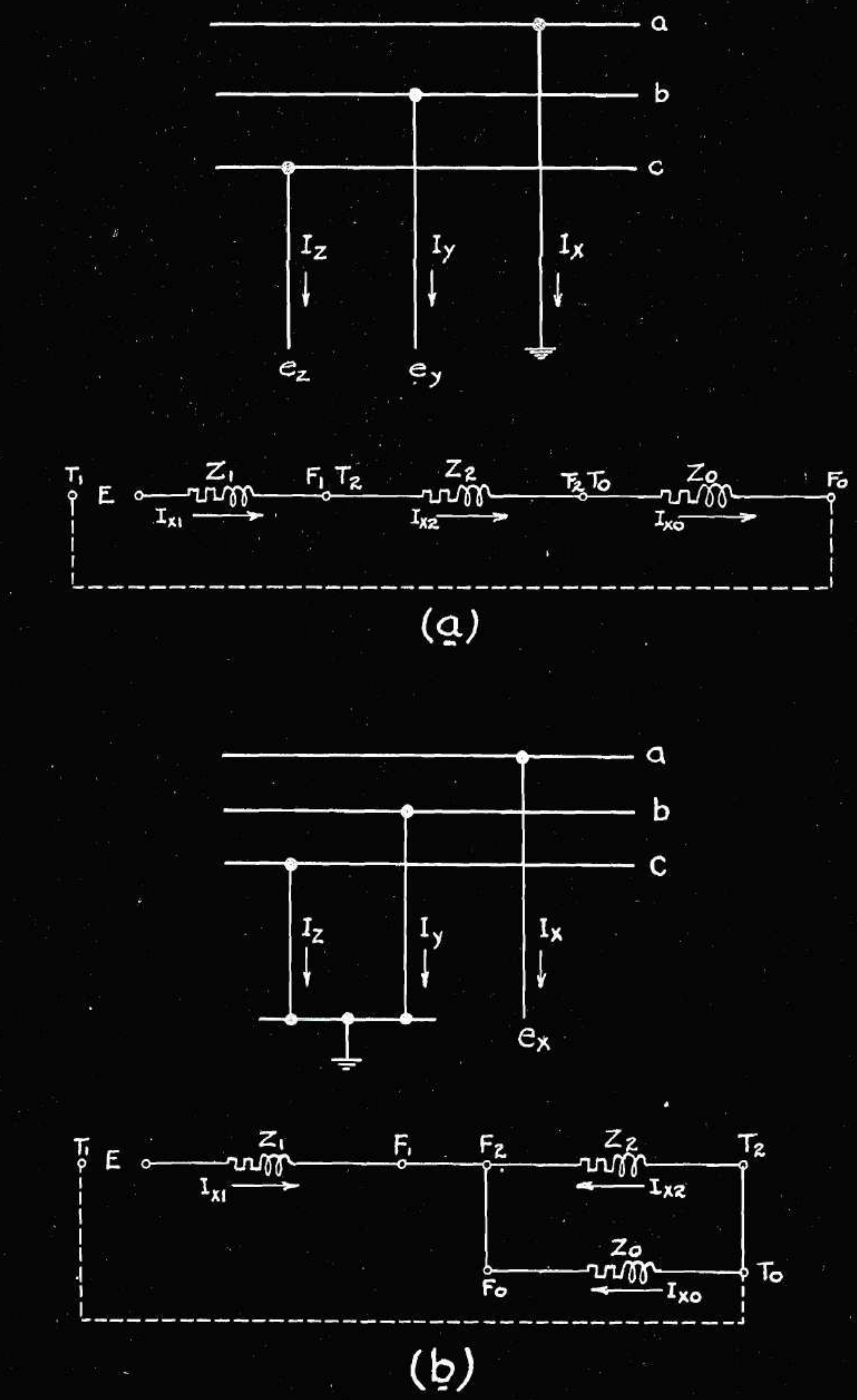


FIG. 22
CONNECTION OF THE SEQUENCE NETWORKS FOR VARIOUS FAULTS – 2.

reference phase. Since the return conductor is without impedance, it may be seen the points F_0 and T_1 are at the same potential.

Component currents:

$$I_{x1} = I_{x2} = I_{x0} = \frac{E}{Z_1 + Z_2 + Z_0}$$

Double phase to ground.

All the components of current exist in this type fault. Referring to Figure 22, group (b), conditions of the fault are:

$$I_x = 0 \qquad e_x = e_y = 0$$

Therefore

$$e_{x1} = e_{x2} = e_{x0} = \frac{e_x}{3}$$

and the sequence networks are connected in parallel.

Checking currents:

$$I_{x1} = 1/3 \ (aI_y + a^2 I_z)$$

$$I_{x2} = 1/3 \ (a^2 I_y + aI_z)$$

$$I_{x0} = 1/3 \ (I_y + I_z)$$

$$\text{so} \quad I_{x1} + I_{x2} + I_{x0} = 1/3 \ (1 + a + a^2) \ (I_y + I_z)$$

$$= 0$$

which condition is satisfied by the arrangement.

The sequence components:

$$I_{x1} = \frac{E}{Z_1 + \frac{Z_0 \ Z_2}{Z_0 + Z_2}}$$

$$I_{x2} = - \frac{Z_0}{Z_0 + Z_2} \ I_1$$

$$I_{x0} = - \frac{Z_2}{Z_0 + Z_2} \ I_1$$

Reduction of Networks.

In reduction of networks to a single "equivalent" impedance, the following relations are of assistance:

Two impedances in parallel:

$$Z = \frac{Z_a \times Z_b}{Z_a + Z_b}$$

$$I_a = \frac{Z_b}{Z_a + Z_b} I$$

$$I_b = \frac{Z_a}{Z_a + Z_b} I$$

Star-delta transformation:

A delta of impedances Z_{AB}, Z_{AC}, Z_{BC}, between points A, B, C, may be replaced by an "equivalent" star network, neutral at N, whose impedances, respectively, are

$$Z_{AN} = \frac{Z_{AB} \; Z_{AC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

$$Z_{BN} = \frac{Z_{BA} \; Z_{BC}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

$$Z_{CN} = \frac{Z_{CA} \; Z_{CB}}{Z_{AB} + Z_{AC} + Z_{BC}}$$

In order to determine currents in the replaced delta, currents in the branches of the equivalent wye are found, and from these the potential difference between each pair of points of the original delta are figured. The potential difference, divided by the original delta impedance between the two points, results in the branch current.

Transformation to eliminate mutual impedance.

Two branches, with self impedance Z_a and Z_b, and mutual between them of M_{ab}, connected at one end, may be replaced by an equivalent star, in which

Impedance from common bus to neutral = M_{ab}

Impedance from a terminal to neutral = $Z_a - M_{ab}$

Impedance from b terminal to neutral = $Z_b - M_{ab}$

Appendix B

Description and Application of Low Energy "CR" Relays to Ground Fault Protection.

The Westinghouse Electric and Manufacturing Company's "CR" relays are known as directional overcurrent relays. Three conditions are necessary before the contacts close: (1) Current in excess of that for which the relay is set, (2) Power flow must be in the direction for which the relay is connected to operate, (3) Conditions (1) and (2) must continue for sufficient time.

The "standard energy" are for use as line protective devices in conjunction with standard instrument transformers.

The "low energy" relays may be used as line relays when it is necessary to energize the current coil from bushing type current transformers. Low energy type are also used as ground relays.

Relays for line protection are generally of the class having a range of current settings from four to fifteen amperes (C.T. secondary current) as this range fits in best with standard practice of selecting current transformer ratios.

Relays for ground protection, however, are so connected that under normal conditions no current is through the relay, and therefore the current setting range may be made much lower, giving greater sensitivity to ground currents. The range is from 0.5 to 2.5 amperes, providing seven taps at which the relay may be set: 0.5, 0.6, 0.8, 1.0, 1.5, 2.0 and 2.5 amperes.

The relay consists of two separate parts, the directional element and the overcurrent element. Both operate upon the induction principle.

Figure 23 is an internal wiring diagram of the "CR" low energy relay, contacts connected for what is known as "directional control," i.e., the overcurrent element is inoperative unless the directional contacts are closed.

As noted on the diagram, the bottom element is the directional portion. In reality this is a watthour meter element, except that characteristics of the circuits are changed to cause maximum torque on the disc when current lags voltage by 15 degrees. Also the relay element requires only about 1% of voltage to operate. A stop prevents the disc from rotating when power is in the normal direction; with reversal of current, disc torque reverses and the element operates to close directional contacts.

At the top is the overcurrent element. Taps are taken off the main current winding up to a terminal block, by means of which the relay may be set to function at different current valves. The overcurrent element operates on the same principle as the directional element (i.e. the production of a shifting flux between the lower and upper pole pieces due to the interaction of the megnetomotive forces produced by the three windings). It is evident from the diagram that, since the auxiliary windings of the ocercurrent element are energized by transformer action at the main pole of the element, and that the circuit to the auxiliary windings is through contacts of the directional element, it is necessary that this latter shall have closed before the overcurrent element begins to function.

The trip circuit is completed by the overcurrent contacts through the operation indicator and the coil of an internal contactor switch. This switch shorts out the overcurrent con-

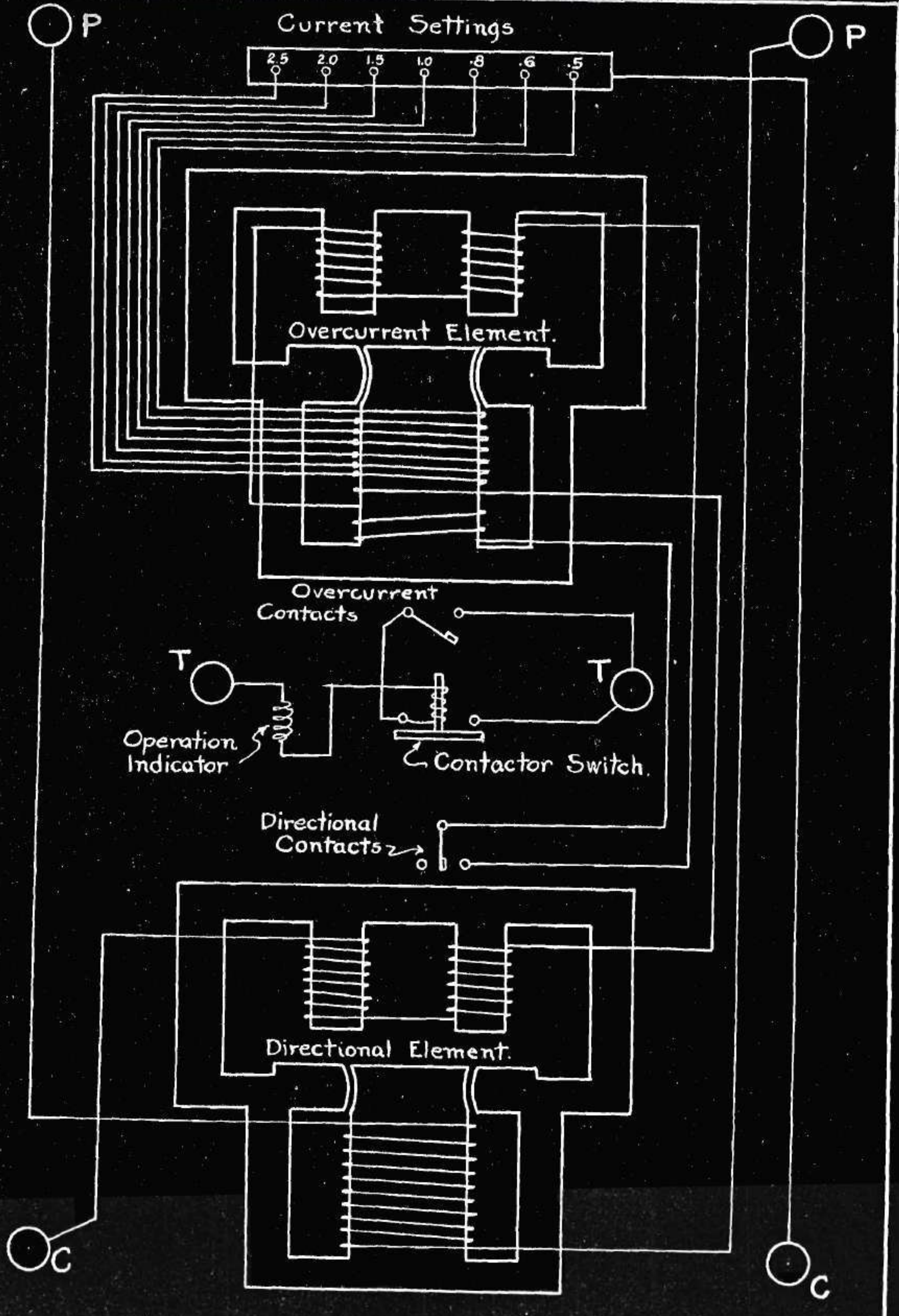


FIG. 23

INTERNAL WIRING

LOW ENERGY "CR" RELAY — DIRECTIONAL CONTROL.

tacts, relieving them of the tripping current duty, and as may be seen, it is necessary to open the tripping circuit before the relay contactor switch will drop out.

The complete assembly is mounted upon a cast base, current and potential studs projecting through the rear, for surface mounting upon a switchboard panel. The case is of metal, with glass front. Dimensions are: 10 3/4" high, 6 3/4" wide, 6 1/4" deep, weight 20 pounds.

Typical Time - Current curves of the low energy "CR" relay are shown by Figure 24. The particular curves are for the even numbered time dial settings, but approximate seconds to trip for any intermediate time setting may be determined by interpolation. Percent amperes is actual amperes through the relay in percent of the current tap setting. Hence, 10 relay amperes with 2.5 ampere setting will cause the relay to close contacts in the same time as would 4 relay amperes on the 1.0 ampere setting - 4.7 seconds for number 10 time dial setting, 1.0 second for #2 time setting.

The "definite minimum time" feature is indicated by the curves in the region of the higher percent amperes. No matter what the percent load for these higher values, the relay requires the same time to operate. Thus relays in several different sections of a line, all carrying the same fault current, may be set with successively higher "minimum time" settings, assuring that only the proper relay will function to clear the fault.

Customary external connections of the "CR" ground relay are indicated in Figure 25. The directional element contacts close only when zero sequence current is flowing away from the busbars. If this current is of sufficient magnitude, and

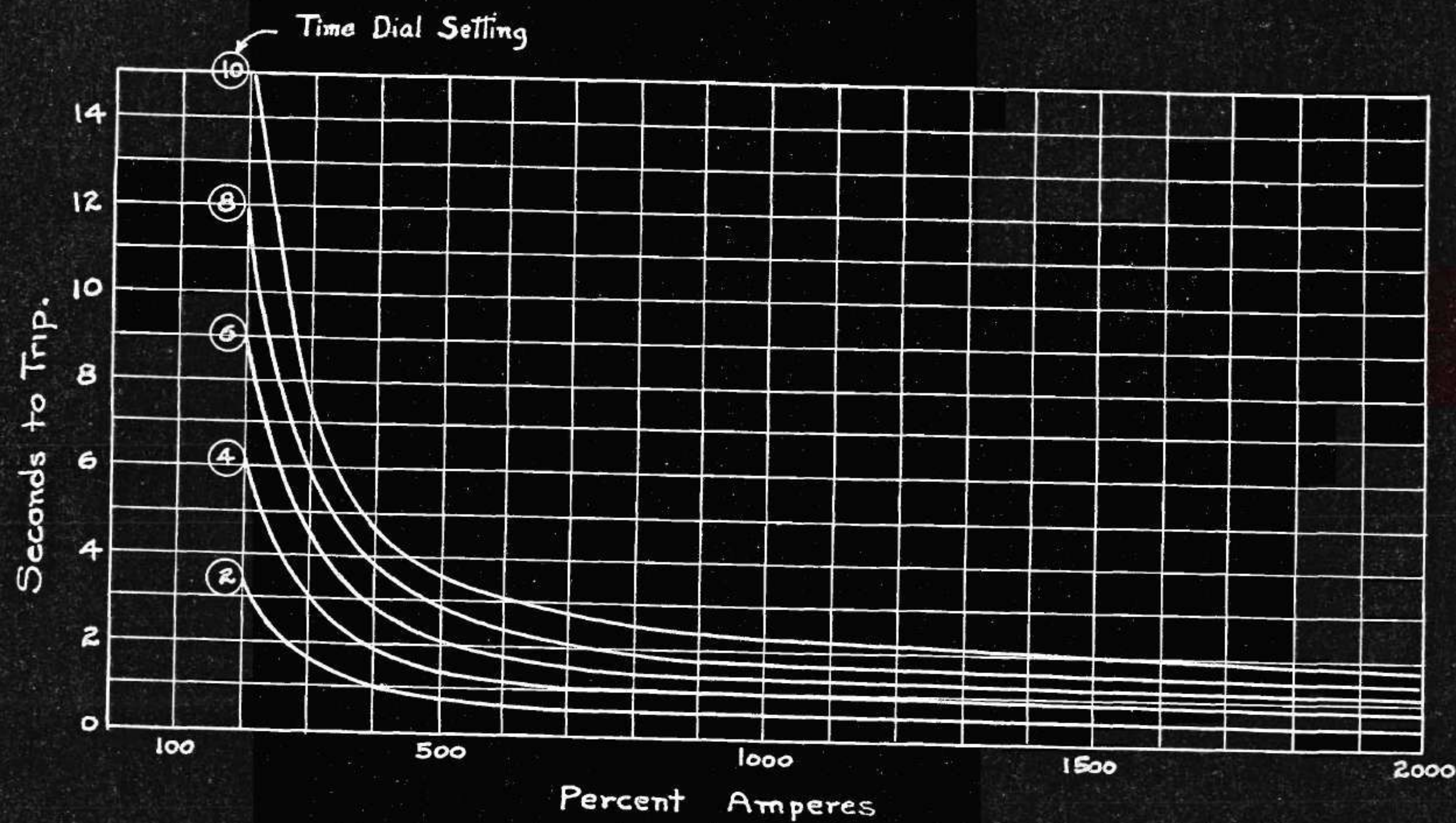


FIG. 24

TYPICAL TIME-CURRENT CURVES OF LOW ENERGY "CR" RELAY.

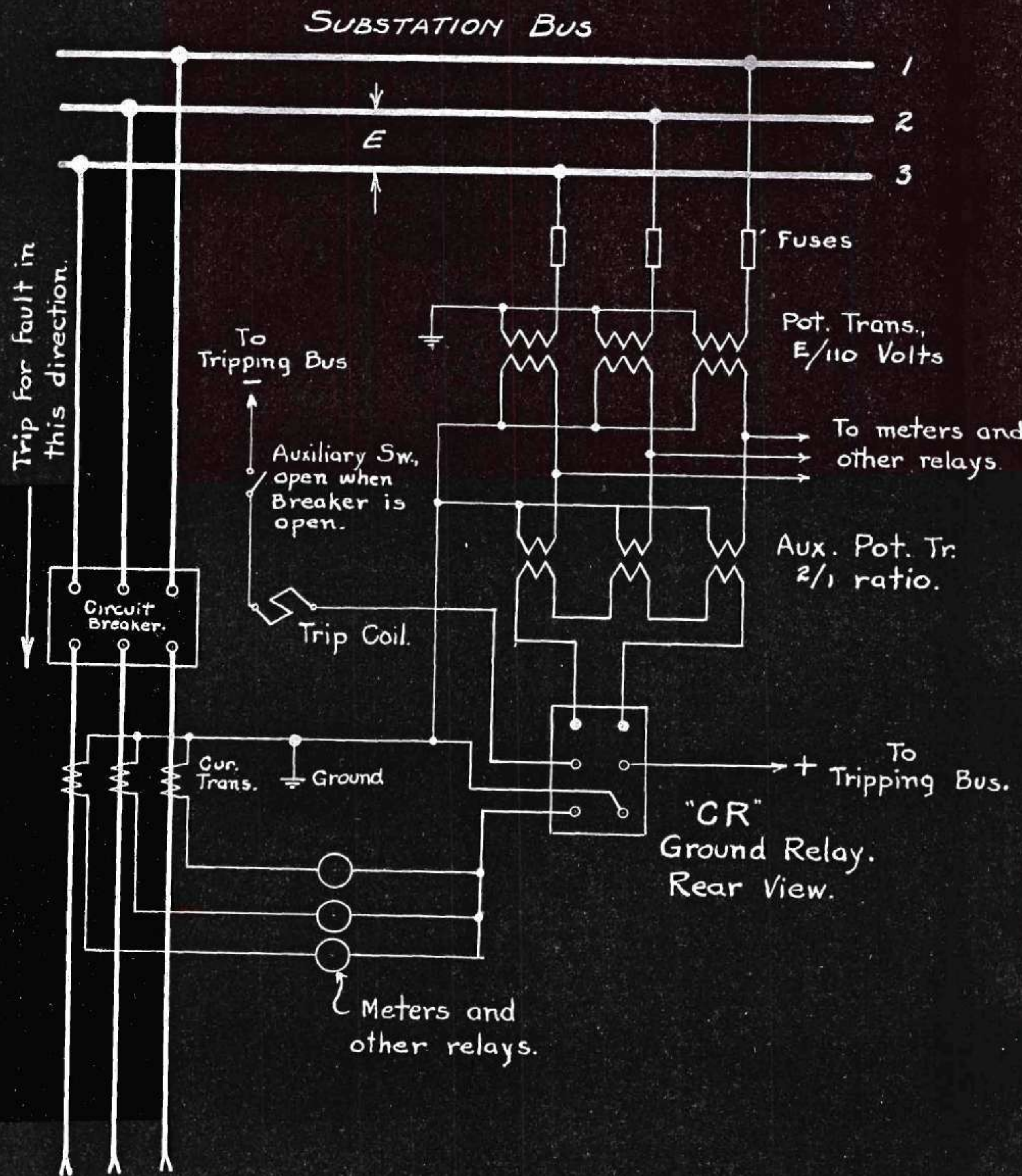


FIG. 25

GROUND FAULT PROTECTION.
WITH
LOW ENERGY "CR" RELAY.

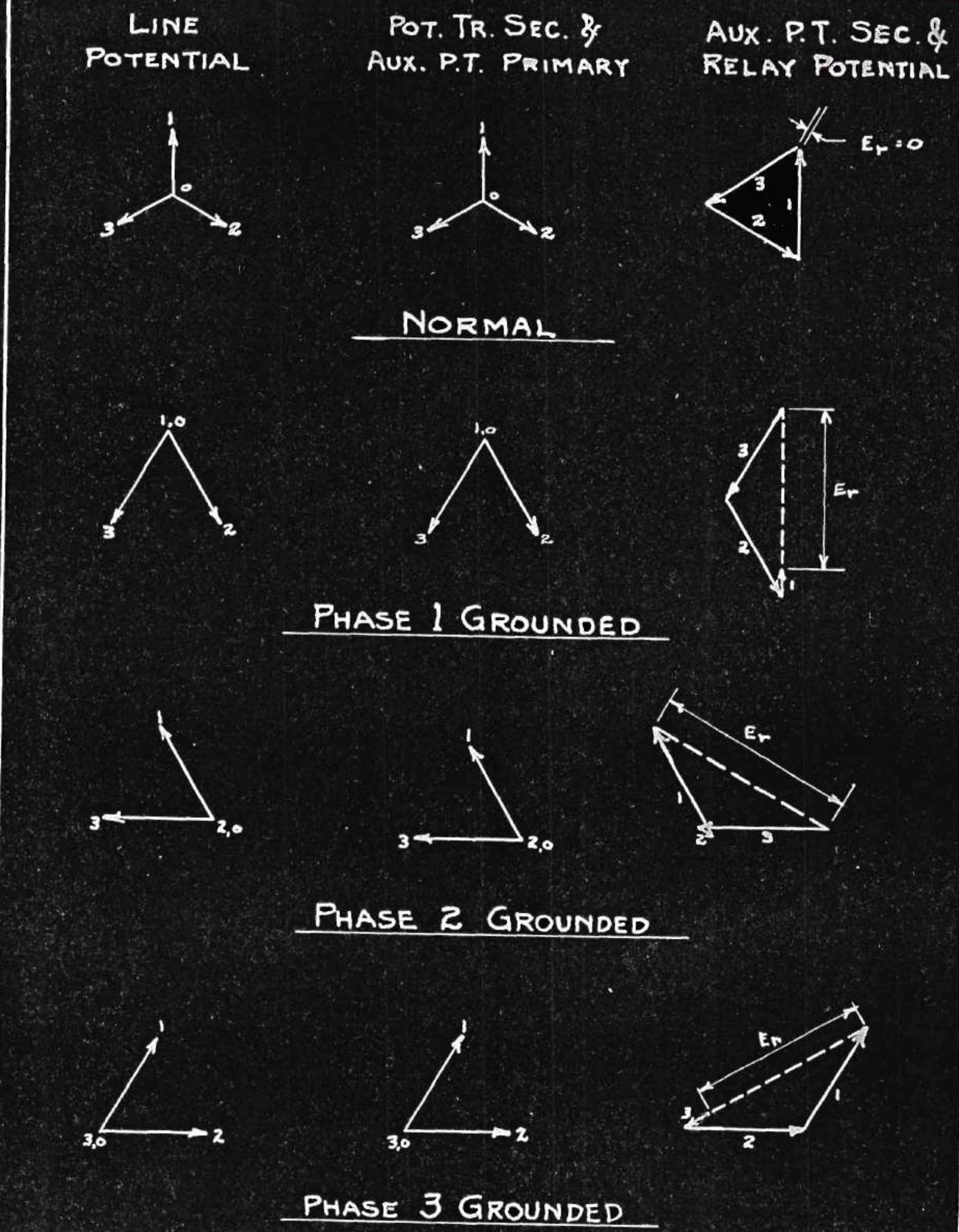


FIG. 26

LINE AND RELAY VOLTAGE VECTORS
FOR
SINGLE PHASE TO GROUND FAULT.

exists for sufficient time, the relay operates to trip the breaker.

The potential coil is connected in one corner of the delta connected secondaries of a set of star-delta connected auxiliary potential transformers. The neutral point of the primaries of these transformers must be grounded, in order to allow zero sequence exciting current to pass. Primaries of the auxiliary transformers are connected to the secondary side of the three main potential transformers, which must be directly on the high tension line, connected wye-wye, both neutrals grounded. Auxiliary potential transformers are necessary because if the main potential transformers were connected wye-delta and the ground relay potential coil connected in one corner of the delta, the effect of the relay impedance drop would prevent use of meters and other relays off the same potential transformers.

Figure 26 shows that a definite relay voltage is produced for a fault on each of the three phases and that each voltage is displaced from the other by 120 degrees. Hence, for any phase faulted, the angle between the residual current and the relay voltage remains the same, and the directional element of the relay will operate properly. The figure is for a system with neutral point grounded through a high resistance, but phase relations are the same for the condition of a solidly grounded neutral, the only difference being in the magnitude of the relay voltage, caused by the neutral not shifting in the manner shown for the high resistance neutral ground.

For this reason, the auxiliary potential transformers are connected step-down when used with high resistance (or reactance) grounded systems. Relay voltage, with line voltage 13,200 is therefore 95 volts, which is quite enough for a coil rated at 110 volts.

Bibliography

"Method of Symmetrical Co-ordinates Applied to the Solution of Pobyphase Networks," C. L. Fortescue, Transactions A.I.E.E., Vol. 37, Part 2, pages 1027-1140, 1918.

"Analytical Solutions," R. D. Evans, Electric Journal, August 1919, p. 345.

"Development of Analytical Solutions," C. L. Fortescue, Electric Journal, August 1919, p. 350.

"Tertiary Windings in Transformers - Their Effect on Short Circuit Currents," J. F. Peters, Electric Journal, Vol. 16, p. 476, 1919.

"General Considerations in Grounding the Neutral of Power Systems," H. H. Dewey, Transactions A.I.E.E., Vol. 42, p. 405, 1923.

"Voltages Induced by Arcing Grounds," J. F. Peters, and J. Slepian, Journal A.I.E.E., August 1923, p. 781.

"Theory of Three Circuit Transformers," A. Boyajian, Transactions A.I.E.E., Vol. 43, p. 508, 1924.

"Calculation of Short Circuit Ground Currents on Three Phase Power Networks Using the Method of Symmetrical Co-ordinates," Sadatoshi Bekku, G.E. Review, July 1925, p. 472.

"Single Phase Short-circuit Calculations," W. W. Lewis, G. E. Review, July 1925, p. 480.

"Calculation of Single Phase Short Circuits by the Method of Symmetrical Components," A. P. Mackerras, G. E. Review, Part 1, April 1926, p. 218, Part 2, July 1926, p. 468.

"Wave Propagation in Overhead Wires with Ground Return," J. R. Carson, Bell System Technical Journal, p. 539, 1926.

"Characteristics of Ground Faults on Three Phase Systems," S. B. Griscom, Electric Journal, April 1927, p. 151.

"How the Phase Sequence Network Works," H. R. Stewart, Electric Journal, June 1929, p. 260.

"Calculation of Ground Currents," Report of the Electric Apparatus Committee, Southeastern Division, N.E.L.A., 1929.

"Symmetrical Components," C. F. Wagner and R. D. Evans, Electric Journal: March, 1928; April, 1928; June, 1928; July, 1928; September, 1929; December, 1929; April, 1931; May, 1931; October, 1931; November, 1931.

"Reactance of Transmission Line with Ground Return," J. E. Clem, presented at A.I.E.E. Convention, Pittsburgh, Pa., March 1931.

"Power System Voltages and Currents Under Fault Conditions," R. D. Evans and S. H. Wright, presented at A.I.E.E. Convention, Pittsburgh, Pa. March 1931.

"Relay Handbook and Supplement," N.E.L.A., 1931.

"Determination of Synchronous Machine Constants by Test," S. H. Wright, presented at summer convention of A.I.E.E., Asheville, N. C., June 1931.

"Calculation of Synchronous Machine Constants," L. A. Kilgore, presented at summer convention of A.I.E.E., Asheville, N. C., June 1931.

"Definition of Terms Used in Power System Studies," H. K. Sels, presented at mid-winter convention of A.I.E.E., New York City, 1931.

"Calculation of Short-circuits on Power Systems," C. F. Wagner and S. H. Wright, presented at winter convention of A.I.E.E., January 1932.

"Standard Decrement Curvesm" W. C. Hahn and C. F.

Wagner, presented at winter convention of A.I.E.E., New York City, January 1932.

"Decrement Curves for Specific Systems," W. C. Hahn, presented at winter convention of A.I.E.E., New York City, January 1932.