MULTI-OBJECTIVE OPTIMIZATION OF PUMPING RATES AND WELL PLACEMENT IN COASTAL AQUIFERS

by

Chan-Hee Park and Mustafa M. Aral

Multimedia Environmental Simulations Laboratory

School of Civil and Environmental Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

MESL-01-03 **March 2003**

MULTI-OBJECTIVE OPTIMIZATION OF PUMPING RATES AND WELL PLACEMENT IN COASTAL AQUIFERS

by

Chan-Hee Park and Mustafa M. Aral

Multimedia Environmental Simulations Laboratory

School of Civil and Environmental Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

The work on which this report is based was supported by the School of Civil and Environmental Engineering, Georgia Institute of Technology, and by the Agency for Toxic Substances and Disease Registry, United States Department of Health and Human Services. The project was administered through the Multimedia Environmental Simulations Laboratory of the School of Civil and Environmental Engineering, Georgia Institute of Technology.

MESL-01-03 **March 2003**

Authors

Mr. Chan-Hee Park is a graduate student in the Multimedia Environmental Simulations Laboratory, School of Civil and Environmental Engineering, Georgia Institute of Technology.
Dr. Mustafa M. Aral is a Professor and the Director of Multimedia Environmental Simulations Laboratory, School of Civil and Environmental Engineering, Georgia Institute of Technology.

Acknowledgments

This report is based on the work conducted in the Multimedia Environmental Simulations Laboratory at the School of Civil and Environmental Engineering, Georgia Institute of Technology. Partial financial support for this work was provided by ATSDR, USDHHS and the Georgia Institute of Technology. Authors are grateful for the continued support of the Multimedia Environmental Simulations Laboratory efforts in the area of environmental health studies by these two agencies. Computational services were provided by the Multimedia Environmental Simulations Laboratory facilities.

Chan-Hee Park and Mustafa M. Aral

Multi-Objective Optimization of Pumping Rates and Well Placement in Coastal Aquifers

Research report No.: MESL-01-03; Project No.: 20020306; March 2003.

Keywords – Saltwater intrusion, analytic solution, genetic algorithm, optimization.

Table of Contents

List of Figures

List of Tables

Multi-Objective Optimization of Pumping Rates and Well Placement in Coastal Aquifers

Chan-Hee Park and Mustafa M. Aral
Multimedia Environmental Simulations Laboratory
School of Civil and Environmental Engineering
Georgia Institute of Technology
Atlanta, Georgia, 30332

Abstract

Saltwater intrusion, which is commonly associated with extensive groundwater extraction, is an important problem for coastal regions. To avoid saltwater intrusion and maintain the integrity of coastal aquifers, proper management of groundwater supplies is necessary. In this study, which includes the research results of phase one of our two phase research program, we present a multi-objective optimization approach to determine pumping rates and well locations to prevent saltwater intrusion, while satisfying desired extraction rates in coastal aquifers. The proposed method is an iterative sub-domain method, in which the algorithm searches for the optimal solution by perturbing the well locations and pumping rates simultaneously. The decision variables of the optimization problem are modeled as continuous independent variables. In the proposed approach, sharp interface solution for homogenous steady state problem is used along with the Dupuit and Ghyben-Herzberg assumptions. The analytical solution developed follows the single-potential theory concept introduced by Strack [1976]. Using this approach, the direct method of searching for saltwater intrusion points is formulated by comparing the location of the stagnation points of flow fields, and the saltwater intrusion profiles obtained from the single-potential theory solution. These critical conditions are incorporated into the formulation as the constraints of the problem. The search for the optimal solution, within each sub-domain, is conducted using Genetic Algorithm (GA). The multi-objective problem is formulated to maximize pumping rates while minimizing the distance between critical stagnation point and the reference coastline location, such that the wells are placed as closely to the coast as possible. The efficiency of the optimization process is improved by solving the problem through a sub-domain perturbation approach. Several numerical experiments are conducted to evaluate the effectiveness of the proposed method. As a case study, the numerical results obtained from the proposed method are compared with the work of *Cheng et al.*, [2000], with the proposed approach yielding higher pumping rates than was reported in their study. The sequential use of multi-objective criteria, with pre-selected weights, successfully demonstrates the capability of the model to achieve two objectives simultaneously. This approach provides cost effective solutions to an important management problem in coastal aquifers. In phase two of the proposed research program, 3-D density dependent models will be solved to evaluate saltwater intrusion problem. The optimization algorithm developed in this study will be coupled to the 3-D models to determine the optimal solutions in a similar manner.

1. Introduction

Saltwater intrusion is an important problem, which may lead to the degradation of freshwater aquifers, especially in areas that are seasonally populated or due to gradual increase of water supply needs of a region, proportional to population increase. Once an aquifer is intruded by saltwater, the damage is very costly to repair [*Frind,* 1982; *Cheng et al.,* 2000]. The two important objectives that are associated with the management of groundwater extraction in coastal aquifers is the maximization of the water supply and the minimization of the cost of this supply, while avoiding saltwater intrusion at all times. In coastal areas, where the groundwater is the major or the only source of freshwater, achieving these goals is of great interest.

In the multi-objective control and management problem, associated with saltwater intrusion in coastal aquifers, various challenging questions are in the minds of scientists, engineers and managers. For example, answers to the following questions are of interest to managers: What is the safe pumping rate for each existing well in a coastal aquifer, before saltwater intrusion becomes a problem? How many wells are needed to supply the required freshwater to a community, before saltwater intrusion becomes a problem? From a more practical point, where should these wells be placed and what should their pumping rates be, before saltwater intrusion becomes a problem? How close can these wells be placed to the coast, before saltwater intrusion becomes a problem? Given some restrictions on aquifer heterogeneity, the mathematical models presented in this report and the software tool developed can be used to answer these questions. In this sense, the software tool included to this report, provide a useful and practical tool to analyze these problems.

The use of optimization approach in the solution of saltwater intrusion problems are relatively recent and few [*Shamir et al.,* 1984; *Willis and Finney*, 1988; *Finney et al.*, 1992; *Hallaji and Yazicigil*, 1996; *Emch and Yeh* 1998; *Das and Datta*, 1999a, 1999b; *Cheng et al.,* 2000]. *Cheng et al.* [2000] used an optimization approach to solve for pumping rates for an existing multiple well extraction scenario in a coastal aquifer using a Genetic Algorithm (GA). In their study, the analytical solution of the sharp-interface saltwater intrusion model was used for simplicity. The use of a more complex model is deferred, in part due to the long computation time required by the repetitive use of the model during the optimization process. In their solution, they discretized the pumping rate and used the Structured Messy Genetic Algorithm (SMGA) approach with the pumping rate selected as the decision variable.

Following *Cheng et al.* [2000], the analytical solution of the steady state sharp-interface saltwater intrusion model is also used in this study. This solution is based on the single-potential formulation of *Strack* [1976]. However, in this study we expand the concept of the stagnation point approach to two-dimensions for multiple wells, which leads to the optimization of pumping rates for each well. Eventually, the process is also extended to include the determination of the best location for extraction wells in this solution. The comparison of the location of the stagnation point relative to the saltwater intrusion point is used as a constraint in the optimization model. In this study, we show that this approach can improve the results of the optimized solution significantly.

GA is a robust method, when optimal solutions are searched for nonlinear problems. In the literature, numerous researchers have demonstrated that the GA could yield a significant improvement in computational efficiency for these problems [*McKinney et al.,* 1994; *Huang and Mayer,* 1997]. This approach is used in numerous engineering fields to achieve optimal solutions to complex problems. The GA approach is further extended to progressive GA (PGA) approach to improve efficiency of the solution process. The basic idea in this method may be defined in the following way: search as little as possible, get the trend of the solution in a sub-domain and reach the optimal solution through a sequence of solutions. Often in reality, the number of possible solutions are simply too many to complete an exhaustive search for an optimization problem. Even a search of only one percent of all possible cases is too many in most of the problems, where optimization techniques are required. Therefore, the formulation of the optimization problem is of great importance to reduce the unnecessary simulations, before the search begins. This concept is well incorporated in this study via using the Progressive Genetic Algorithm, which is the combinatorial optimization approach, introduced by *Aral and Guan,* [1997] and *Guan and Aral,* [1999a, 1999b]. With this approach, the pumping rates and the locations of multiple wells can be optimized simultaneously, while allowing the unknown variables to be continuous variables. In this study, the PGA is used along with the stagnation point model in the iterative sub-domain solution of the saltwater intrusion problem.

2. Governing Equations

Analytical solutions used in this study are valid if the following assumptions can be made for the aquifer under study. First, we assume that a sharp interface exists between the saltwater zone and the fresh water zone, rather than a miscible transition zone [*Bear,* 1972, 1979]. Second, the aquifer is assumed to be homogeneous, and steady state conditions are considered. Third, the Dupuit assumption is used to obtain two-dimensional from three-dimensional geometry, by averaging the flow equation in vertical direction [*Bear,* 1972, 1979]. Fourth, the interface location is deduced from the Ghyben-Herzberg assumption [*Bear,* 1972, 1979]. Finally, the single-potential theory approach [*Strack*, 1976] is adopted to use a single governing potential equation across zones (Figure 1).

Following *Strack* [1976] and *Cheng et al.* [1999, 2000], a potential ϕ is defined for both confined and unconfined aquifers as follows:

For confined aquifers:

$$\phi = Bh_f + \frac{(s-1)B^2}{2} - sBd \qquad zone1 \tag{1}$$

$$\phi = \frac{1}{2(s-1)}\left[h_f + (s-1)B - sd\right]^2 \qquad zone2 \tag{2}$$

For unconfined aquifers:

$$\phi = \frac{1}{2}\left[h_f^2 - sd^2\right] \qquad zone1 \tag{3}$$

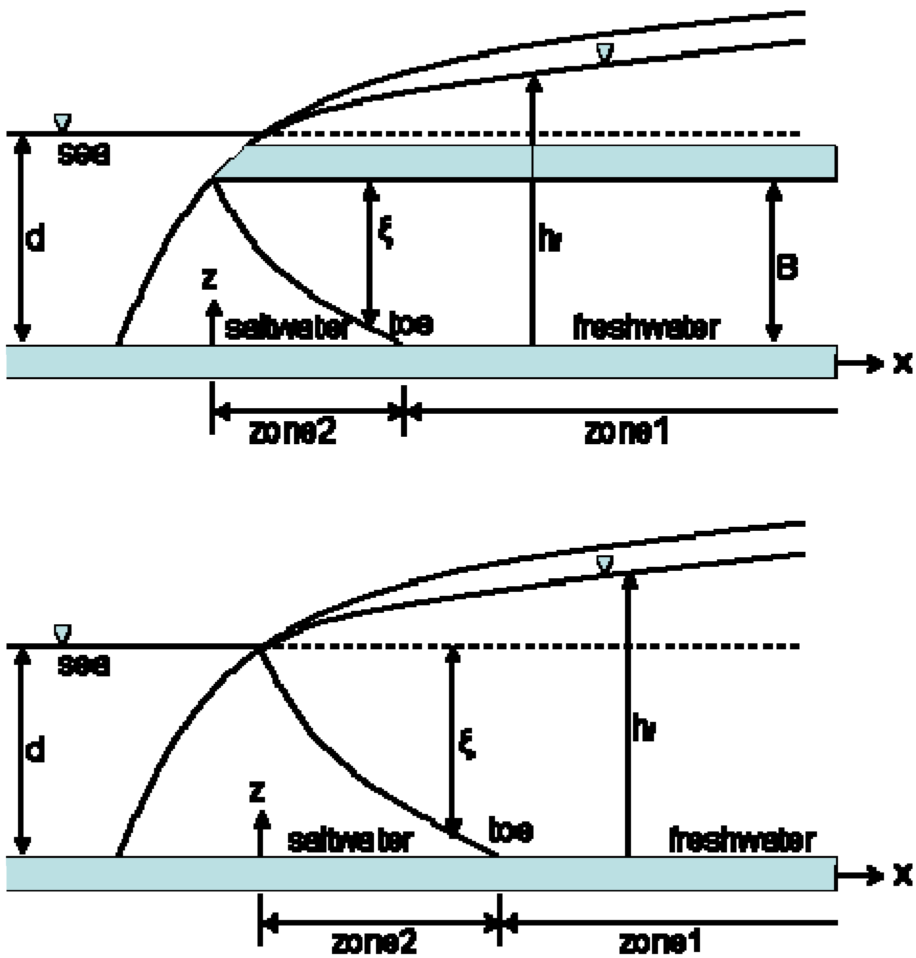


Figure 1. Cross-sections of a coastal aquifer in a confined and unconfined aquifer.

$$\phi = \frac{s}{2(s-1)}\left(h_f - d\right)^2 \qquad zone\,2 \tag{4}$$

where h_f is the freshwater head, d is the elevation of mean sea level above the datum, and B is the confined aquifer thickness. The density ratio of the saltwater and freshwater is,

$$s = \frac{\rho_s}{\rho_f} \tag{5}$$

where ρ_s and ρ_f are the saltwater and freshwater densities respectively.

Since the previously defined potential function satisfies the Laplace equation $\nabla^2\phi = 0$, the interface location ξ can be obtained by solving for the interface location with proper boundary conditions:

For a confined aquifer:

$$\xi = \sqrt{\frac{2\phi}{s-1}} + d - B \tag{6}$$

For an unconfined aquifer:

$$\xi = \sqrt{\frac{2\phi}{s(s-1)}} \tag{7}$$

From Figure 1, the toe of saltwater can be evaluated at $\xi = d$. Hence, the potential at the toe can be calculated from (6) and (7).

For a confined aquifer:

$$\phi_{toe} = \frac{s-1}{2}B^2 \tag{8}$$

For an unconfined aquifer:

$$\phi_{toe} = \frac{s(s-1)}{2}d^2 \tag{9}$$

Since the Laplace equation is linear, the freshwater potential for multiple pumping wells, in an aquifer with uniform flow, can be obtained using the method of superposition [*Strack*, 1976 and *Cheng et al.* 2000].

$$\phi = \frac{q}{K}x + \sum_{i=1}^{n} \frac{Q_i}{4\pi K} \ln\left[\frac{(x-x_i)^2 + (y-y_i)^2}{(x+x_i)^2 + (y-y_i)^2}\right] \tag{10}$$

Using either (8) or (9) in (10), the toe location for the multiple wells can be solved.

The location of multiple stagnation points of the flow field is important to define the maximum pumping rate for the pumping wells. The locations of the stagnation points can be obtained from the following relation [*Strack*, 1972],

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = 0 \tag{11}$$

Differentiating (10):

$$\frac{\partial \phi}{\partial x} = \frac{q}{K} + \sum_{i=1}^{n} \frac{Q_i}{4\pi K} \frac{(x+x_i)^2 + (y-y_i)^2}{(x-x_i)^2 + (y-y_i)^2}\left[\frac{2(x-x_i)}{(x+x_i)^2 + (y-y_i)^2} - 2(x+x_i)\frac{(x-x_i)^2 + (y-y_i)^2}{\left\{(x+x_i)^2 + (y-y_i)^2\right\}^2}\right] \tag{12}$$

$$\frac{\partial \phi}{\partial y} = \sum_{i=1}^{n} \frac{Q_i}{4\pi K} \frac{(x+x_i)^2 + (y-y_i)^2}{(x-x_i)^2 + (y-y_i)^2}\left[\frac{2(y-y_i)}{(x+x_i)^2 + (y-y_i)^2} - 2(y+y_i)\frac{(x-x_i)^2 + (y-y_i)^2}{\left\{(x+x_i)^2 + (y-y_i)^2\right\}^2}\right] \tag{13}$$

Equations (12) and (13) form a set of nonlinear equations. Newton-Raphson method was used to solve the stagnation points from these equations. These equations need to be further differentiated following the Newton-Raphson method. The derivatives of these equations are provided in Appendix A. The stagnation point location is used to detect the well-intrusion, by comparing the location of this point relative to the toe location on the coastline. In Figure 2, we describe various cases of the well intrusion condition for one- and two pumping well conditions.

3. Formulation of saltwater intrusion problem

In most cases, one of the objectives in the optimal solution of a saltwater intrusion problem is maximizing the pumping rate. The optimization of the saltwater intrusion problem becomes unique, if the extraction wells are forced to be placed as closely as possible to the coast, since the development region is selected close to the coastline. These situations, in practice, cause saltwater intrusion problems. If the extraction wells can be placed further away from the coastline, all the extraction wells may be allowed to pump higher rates of fresh water with no

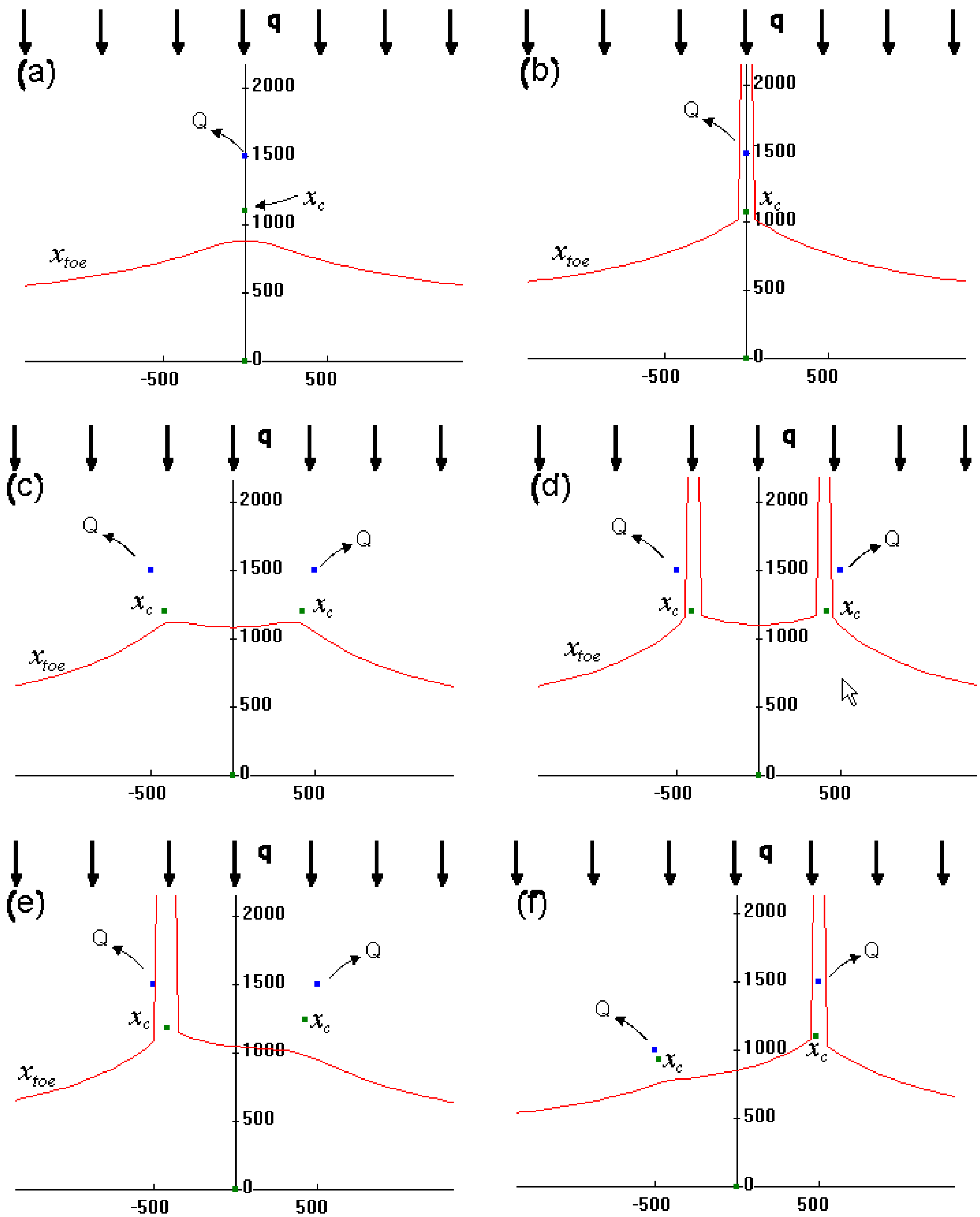


Figure 2. Stagnation points: (a) One pumping well un-intruded case; (b) One pumping well intruded case; (c) Two pumping well un-intruded case; (d) Two pumping well intruded case; (e) Symmetric two pumping well intruded case; (f) Asymmetric two pumping well intruded case.

limitations. In order to integrate this constraint into the saltwater intrusion problem, the authors have introduced a second objective. That is, all extraction wells should be placed as close to the coastline as possible. This, in turn, restricts the first objective, that is maximizing the pumping rate. Researchers working on these multi-objective problems have attempted to solve the multi-objective optimization problem using various techniques. *Fonseca and Fleming* [1993, 1995] used the GA for solving multi-objective problems and introduced the concept of dominated and non-dominated populations. There are other techniques associated with the solution of multi-objective problems, such as fitness sharing, niche approach, etc. GA uses the survival of the fittest idea for selecting the competitive populations. This selection process is linear in the scalar objective value of each population. However, the main difficulty originates from the multi-objective nature of the problem. There is no simple way to differentiate the relative importance of each objective in the selection of the fittest. The easiest way to deal with multiple objectives is to use the summation of each objective function to form a single scalar objective function. However, this brings disadvantages such as the difficulty of adjusting the weight of each objective. This is basically because a single scalar objective function generated is not capable of representing the vector tendency of each objective. Nonetheless, in this work we use the single scalar objective function approach for simplicity. The two objectives of the optimization problem are given as follows:

The first objective: Maximizing the pumping rate.

$$Max \sum_{i=1}^{n} Q_i \tag{14}$$

The second objective: Minimizing the distance between the stagnation points and the reference coastline location.

$$Min \sum_{i=1}^{n} (x_c^i - x_{ref}) \tag{15}$$

Equation (15) is modified to convert the second objective to be a maximization problem,

$$Max\left(-\sum_{i=1}^{n} (x_c^i - x_{ref}) \right) \tag{16}$$

The combined and normalized objective function can be written as,

$$f\left(Q, x_{ref}, x_c^i\right) = \sum_{i=1}^{n} \left(\alpha \frac{Q_i}{Q_i^{\max}} + \beta \left(\frac{x_{ref}}{x_c^i} - 1 \right) \right) \tag{17}$$

Subject to the following conditions,

$$x_{toe}^{i}(Q,X,Y) < x_{c}^{i}(Q,X,Y) \tag{18}$$

$$Q_i^{\min} < Q_i < Q_i^{\max},\ x_i^{\min} < x_i < x_i^{\max},\ y_i^{\min} < y_i < y_i^{\max}\ \ i = 1,\ldots,n \tag{19}$$

where Q_i is the pumping rate of well *i*, x_c^i is the stagnation point associated with the pumping well Q_i, x_{ref} is the toe location determined only by hydraulic parameters such as freshwater discharge, hydraulic conductivity, saltwater depth, density difference between saltwater and freshwater, or aquifer thickness for the confined aquifer when there is no pumping, x_{toe}^i is the toe location, and α and β are the objective function weighting parameters. The independent variable vectors, Q, X, Y, will be re-defined in the modified optimization formulation for the progressive genetic algorithm application.

4. The modified optimization model

Cheng et al. [2000] used the discretized pumping rate as independent variable for optimizing pumping rates of fifteen fixed wells in a case study. This independent variable itself requires a large number of simulations for the solution of the optimization problem. For a typical application, it is almost impossible to complete an exhaustive search for the global solution. To avoid this computationally intensive approach, we propose to change the decision variables of the optimization problem. Rather than having pumping rates and well locations as decision variables, we chose the perturbations of these variables as decision variables. In this way, GA solution can eliminate a considerable amount of unnecessary simulations at each step of the iterative solution. To achieve this, for each optimization step of each perturbation, a sub-domain should be defined, and the search is conducted within this sub-domain. No matter where the simulation starts, there is a path from this starting point to the optimal solution. The direction from the current to the next location on the correct path may be determined easily by GA within the pre-determined sub-domain, even if it is far away from an optimal solution. This approach allows the consequent sub-domains to move in the optimal direction as well, which leads the solutions closer to the global optimum. Thus, within each step the solution gets closer to the optimal solution by effectively eliminating the unnecessary paths, as shown in Figure 3a.

The saltwater intrusion problem is complex, due to the dependence of the pumping rates and well locations on each other. The sub-domain concept handles this dependence issue effectively and provides good feedback as the simulation proceeds. This approach filters out the unnecessary runs by taking the direction of the moving wells or increasing and decreasing the pumping rates during iteration. One other advantage of this method is that the independent variables are continuous variables rather than discrete. Figure 3a illustrates the concept of the perturbation solution approach.

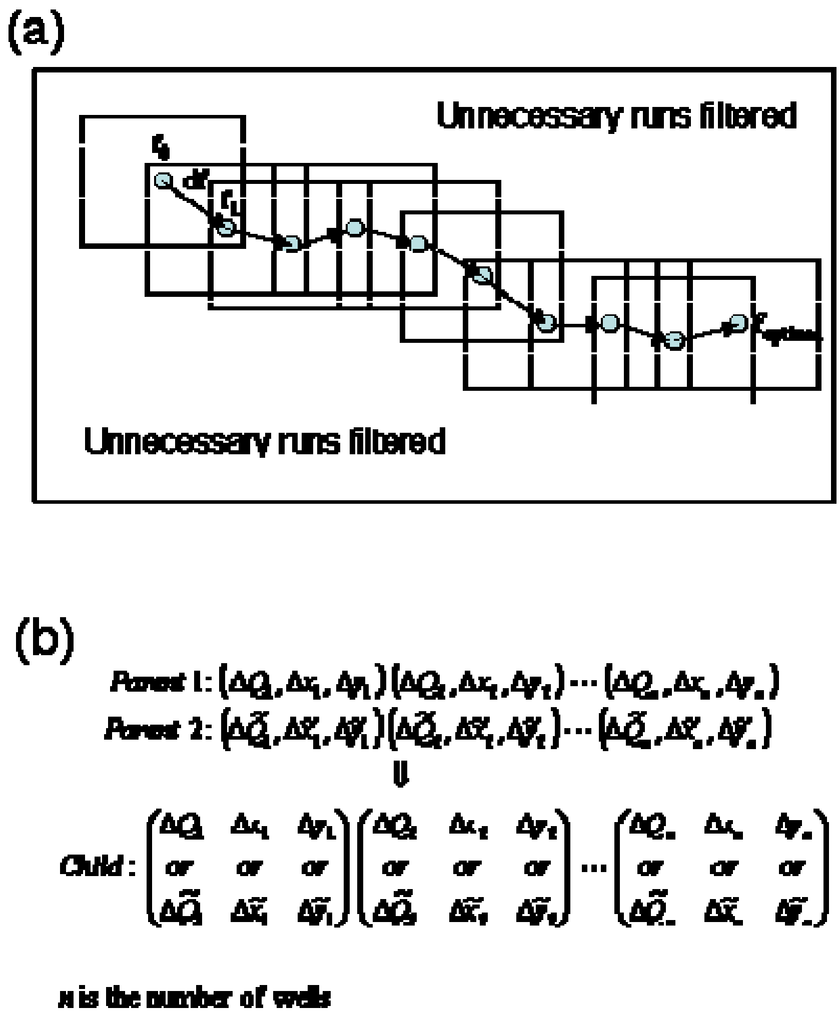


Figure 3. (a) The concept of eliminating unnecessary runs using perturbation method; (b) Coding and crossover of design variables.

In this approach, the modified objective function, the constraints and the independent variables of the perturbations yield the following formulation:

$$f\left(Q, x_{ref}, x_c^i\right) = \sum_{i=1}^{n}\left(\alpha \frac{Q_i^0 + \Delta Q_i}{Q_i^{\max}} + \beta\left(\frac{x_{ref}}{x_c^i} - 1\right)\right) \tag{20}$$

$$\Delta Q_i^{\min} < \Delta Q_i < \Delta Q_i^{\max},\ \Delta x_i^{\min} < \Delta x_i < \Delta x_i^{\max},\ \Delta y_i^{\min} < \Delta y_i < \Delta y_i^{\max} \quad i = 1,\ldots,n \tag{21}$$

where

$$\Delta Q_i^{\min} = Q_i^{\min} - Q_i^0;\ \Delta Q_i^{\max} = Q_i^{\max} - Q_i^0 \tag{22a}$$

$$\Delta x_i^{\min} = x_i^{\min} - x_i^0;\ \Delta x_i^{\max} = x_i^{\max} - x_i^0 \tag{22b}$$

$$\Delta y_i^{\min} = y_i^{\min} - y_i^0;\ \Delta y_i^{\max} = y_i^{\max} - y_i^0 \tag{22c}$$

$Q_i^0, x_i^0, and\ y_i^0$ are initial starting points, and the bounding sub-domain is determined to be polyhedron having the starting points at the center of the polyhedron.

In order to handle the inequality constraint of Equation (18) properly in the objective function of GA, the slack vector and penalty functions are introduced. Thus, the modified objective function and constraints can be given as follows,

$$f\left(Q, x_{ref}, x_c^i\right) = \sum_{i=1}^{n}\left(\alpha \frac{Q_i^0 + \Delta Q_i}{Q_i^{\max}} + \beta\left(\frac{x_{ref}}{x_c^i} - 1\right)\right) - \sum_{\forall_i} \gamma s^2 \tag{23}$$

$$x_{toe}^i(Q, X, Y) + S = x_c^i(Q, X, Y) \qquad S \geq 0 \tag{24}$$

where $\gamma = \begin{cases} c_1 \ if\ s_i < 0 \\ 0\ otherwise \end{cases}$ and c_1 is any large constant.

It should be noted that the independent variables of this formulation are the perturbations $\Delta Q_i, \Delta x_i, \Delta y_i$.

5. Genetic Algorithm

In GA's [*Holland,* 1975; *Goldberg,* 1989; *Davis,* 1991], the parameter set of the optimization problem is coded as a finite-length string. Traditionally, the binary numbers are used to represent such a string. Thus, each bit of a string can be either 0 or 1. Owing to the dramatic growth of computer technology, the use of a binary representation is somewhat cumbersome these days. Since the vector representation of real numbers for real function optimization is more natural and of no difficulty, the real-number coding is used in this study [*Obayashi et al.,* 2000]. Thus, the length of the real-number string corresponds to the number of design variables.

In order for the modified model to work inside the sub-domain, GA search process must be limited to a sub-domain in the neighborhood of $\{Q^0, X^0, Y^0\}$. The search sub-domains of independent variables are evaluated as follows,

$$\delta Q_i = \frac{Q_i^{\max} - Q_i^{\min}}{k_1} \tag{25a}$$

$$\delta x_i = \frac{x_i^{\max} - x_i^{\min}}{k_2} \tag{25b}$$

$$\delta y_i = \frac{y_i^{\max} - y_i^{\min}}{k_3} \tag{25c}$$

In the neighborhood of $\{Q^0, X^0, Y^0\}$, the intervals, $\lfloor \Delta Q_{lower}, \Delta Q_{upper} \rfloor$, $\lfloor \Delta X_{lower}, \Delta X_{upper} \rfloor$, and $\lfloor \Delta Y_{lower}, \Delta Y_{upper} \rfloor$ are determined by,

$$\Delta Q_i^{lower} = \max\{\Delta Q_i^{\min}, -0.5 e^{-\eta k} \delta Q_i\} \tag{26a}$$

$$\Delta Q_i^{upper} = \min\{\Delta Q_i^{\max}, 0.5 e^{-\eta k} \delta Q_i\} \tag{26b}$$

$$\Delta x_i^{lower} = \max\{\Delta x_i^{\min}, -0.5 e^{-\eta k} \delta x_i\} \tag{26c}$$

$$\Delta x_i^{upper} = \min\{\Delta x_i^{\max}, 0.5 e^{-\eta k} \delta x_i\} \tag{26d}$$

$$\Delta y_i^{lower} = \max\{\Delta y_i^{\min}, -0.5 e^{-\eta k} \delta y_i\} \tag{26e}$$

$$\Delta y_i^{upper} = \min\{\Delta y_i^{\max}, 0.5 e^{-\eta k} \delta y_i\} \tag{26f}$$

where k_1, k_2, and k_3 are positive integers (e.g. $k_1 = k_2 = k_3 = 5$), k is the index of iteration, η is a positive coefficient (e.g. $\eta = 0.001$), $e^{-\eta k}$ is a contraction coefficient for the sub-domain. The sub-domain defined by (26) forms a regular polyhedron, with the center located at $\{Q^0, X^0, Y^0\}$. The volume of the polyhedron decreases by the contraction function as the number of iterations increases. The idea of decreasing sub-domain size makes sense, especially when the solution approaches to a local (sub-domain) or to the global optimal solution. In this way, a more precise populations generated in the reproduction of GA. Obviously, the sub-domain defined should satisfy,

$$\left\lfloor \Delta Q_{lower},\, \Delta Q_{upper} \right\rfloor \subseteq \left[\Delta Q_{\min},\, \Delta Q_{\max}\right] \tag{27a}$$
$$\left\lfloor \Delta X_{lower},\, \Delta X_{upper} \right\rfloor \subseteq \left[\Delta X_{\min},\, \Delta X_{\max}\right] \tag{27b}$$
$$\left\lfloor \Delta Y_{lower},\, \Delta Y_{upper} \right\rfloor \subseteq \left[\Delta Y_{\min},\, \Delta Y_{\max}\right] \tag{27c}$$

The GA approach consists of generating populations, applying genetic operators such as crossover and mutation. Normally, the populations are generated in the discretized domain randomly. Since each optimization step works within its sub-domain, populations generated for that step should satisfy the sub-domain constraints. The other technique introduced here, is the contraction function used in defining the sub-domain. For this purpose, various contraction functions, including the one used in this study, can be defined. This approach would be similar to the definition of the energy used in the *Boltzmann* probability distribution, when simulated annealing approach is employed. This process reduces the number of iterations and thus the computational time [*Dougherty and Marryott,* 1991; *Shonkwiler,* 2000].

The crossover of GA is designed such that the offspring inherits the gene from either parent with equal chance. Figure 3b illustrates the coding of the crossover.

The purpose of mutation operator in GA is to give diversity to the genes in the population. This diversity is hoped to seek better optimum, when the solution is stuck in a lower local optimum. In this work, the mutation is designed to reassign different pumping rates to the selected population within the allowed bounds previously defined. Generally, the probability of the mutation is set much lower than that of the crossover.

6. Convergence

Since it is impossible to know if the solution has reached the global optimum, the relative error of the objective function was used to escape the GA loop. Thus, the convergence criterion is defined as,

$$\frac{\left|f_1 - f_0\right|}{f_0} \le \varepsilon \tag{28}$$

ε is a predefined tolerance for the convergence of iterations. If two consecutive objective values satisfy the criterion given in Equation (28), then $\left\{Q^1, X^1, Y^1\right\}$ is taken as a final solution in the corresponding sub-domain. Otherwise, the solution sequence continues with the new starting point as $Q_i^0 = Q_i^1$, $x_i^0 = x_i^1$, $y_i^0 = y_i^1$, $k = k+1$, $i = 1,...n$.

7. Numerical Experiments

Several numerical experiments are performed to test the accuracy of the proposed model and the solution algorithm. The numerical examples included here are single- and multi-objective problems, where well locations and pumping rates are selected as independent variables. For single objective problems, maximization of the pumping rate is considered as the objective. For one- and two-well cases, the global solution can be deduced from the heuristic observation that the pumping well needs to be placed farther inland, limited by the boundaries of the selected solution domain. This way, the pumping rates can be increased to a maximum value, given the constraint that the pumping wells are not to be intruded by saltwater. This constraint is evaluated by comparing the position of the stagnation point and the location of each well. Table 1 lists the physical parameters used in the examples included here, and the GA parameters used for the simulation of one- and two- pumping well cases for an unconfined aquifer. In these applications, the aquifer domain setting and physical parameters selected are similar to that used in *Cheng et al.* [2000].

Example 1: In this example, a one-well and two-well cases are considered. The aquifer domain is 4,000 m by 7,000 m. A uniform freshwater flow of 0.6 m^2/d exists in the negative x-axis direction. The aquifer is an unconfined aquifer with a hydraulic conductivity of 100 m/d. The saltwater depth at the coastline is 14 m. The minimum and maximum allowable pumping rates are selected as 100 m^3/d to 6,000 m^3/d for the one-well case and 100 m^3/d to 5,000 m^3/d for the two-well case. The starting position of the well (x, y) is at coordinates (1,000, 0) for the one-well case and (1,000, -1,500); (1,000, 0) for the two-well case. Other parameters of this problem can be found in Table 1.

As shown in Figure 4a, as the solution progresses, the extraction well moves farther inland for the one-well case, while the pumping rate increases, reaching a maximum value 6,000 m^3/d. Since there is no well interaction in the one-well case, there are an infinite number of solutions for this problem. That is, a well placed on any y-coordinate position would yield the same results at all iteration steps as well as the final solution. In all figures, the dotted line shows the location of the saltwater interface, the cross marks show the location of the stagnation point and the plus sign shows the location of the pumping well. Unlike with the one-well case, there is only one global optimum for the two-well case. This is because of the interaction between the two wells. Obviously, in the two-well application, the extraction wells spread as far as possible from each other to reduce the well interaction, which also leads to the maximization of the pumping rates. In Figure 4, we show the trace of the well positions for these two cases, including the global optimum positions after the solution is completed.

Example 2: With the same single objective of maximizing the pumping rate, the results of three-, two of four- and five-well cases are shown in Figure 5. In these cases, the aquifer domain and aquifer parameters are the same as in the previous example. Other selected modeling parameters are given Table 2. Although the number of pumping wells for each of these four cases is different, the optimal total pumping rates are close to each other. This result indicates that, for a single objective problem, the optimal total pumping rate for a finite domain can be achieved by a finite number of wells, and adding more wells to the domain will not improve the maximum pumping rate significantly. This can be seen in Figure 6. The maximum number of

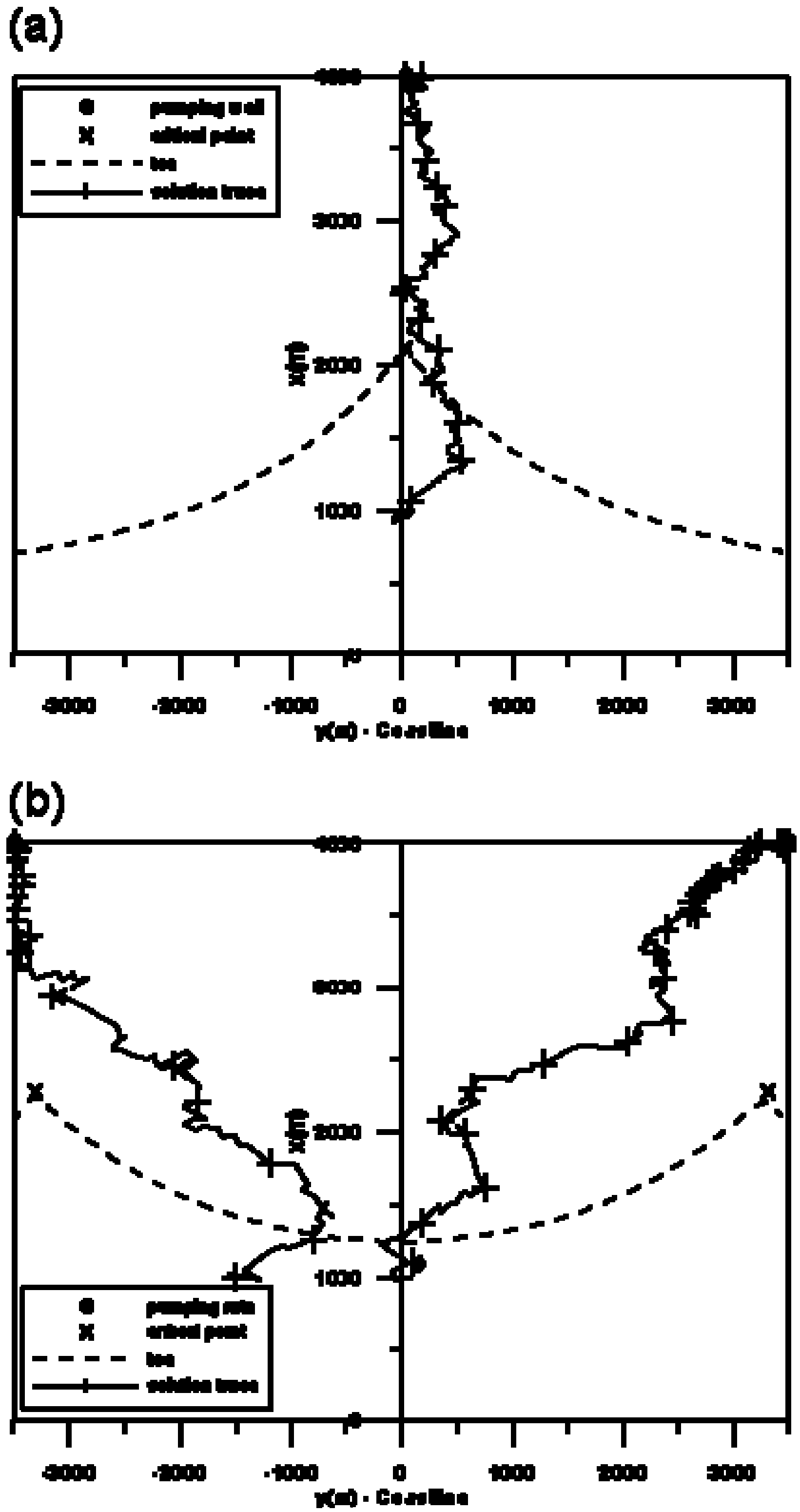


Figure 4. (a) One pumping well placement optimization path; (b) Two pumping well placement optimization path.

Table 1. Summary of modeling parameters for one and two pumping well cases.

Aquifer parameters	Value	MOGA parameters	Value
Aquifer type	unconfined	α	1
Saltwater density	1.025 g/cm^3	β	0
Uniform flow rate	0.6 m^2/d	**Population size**	40
Saltwater depth	14 m	**Mating probability**	0.9
Hydraulic conductivity	100 m/d	**Mutation probability**	0.1
		Convergence	0.001

Sub-domain and bounding parameters (for all wells)

	Single well	Double wells
k_1	250	250
k_2	150	150
k_3	150	150
η	0.00001	0.00001
Q_{min}, Q_{max} m^3/d	100, 6000	100, 5000
x_{min}, x_{max} m	0, 4000	0, 4000
y_{min}, y_{max} m	-3500, 3500	-3500, 3500

Starting Points			Optimal Points		
Q, m^3/d	***x*, m**	**y, m**	**Q, m^3/d**	***x*, m**	**y, m**
One well					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	5433.74	3999.7	33.9
Two wells					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	4474.88	3998.5	3497.1
$Q_2 = 300$	$x_2 = 1000$	$y_2 = -1500$	4477.56	3998.3	-3497.4

wells for this domain are 3, since the total pumping rate does not increase significantly, if we add more wells to the solution. In this figure, we also show the average pumping rate for the wells. Based on the heuristic stated previously, the global maximum pumping rates for this simple application can be obtained, when we perturb the well locations and the pumping rates simultaneously. It should be noted that once the critical number of wells is established within the problem domain, adding more wells often creates a similar shape of toe delineation to that of the critical number of wells case. This can be observed from Figure 5d, i.e. there are three peaks even though the number of wells is four. We note that this figure represents an intermediate solution obtained from the four well solution. This indicates that the same optimal pumping rate from three pumping wells can be obtained from four pumping wells by formulating a similar pattern of toe delineation. Since Figure 5d is an intermediate solution, continuing the simulation will reach to the results shown in Figure 5b. Obviously, the improvement achieved is minimal as shown in Figure 6, since the four pumping wells are already more than the critical number of wells for this case.

Example 3: Another practical application of the procedure developed in this study is the solution to the saltwater intrusion problem for fixed well locations. In a sequential approach, the model developed can first be used to optimize the pumping rates for the existing wells to prevent the saltwater intrusion. In such a case, the independent variable vector consists of only the pumping rates of each well. With the current model, this situation can be simulated very easily by not allowing perturbations for well locations. To test the current model for fixed well location case, a comparison between *Cheng et al.*'s work [2000] and the model developed in this study is presented for two cases. In Table 3, we provide the physical parameters of the aquifer and all other related pumping well information. After the evaluation on the physical parameters used by *Cheng et al.*, the authors have reached to the conclusion that there was a typing error in their fresh water discharge of 40 m^2/d, since this freshwater discharge is too large. For this problem, a value of 0.4015 m^2/d is used as the fresh water discharge to duplicate their results.

The first case is the Case 1 of the example problems presented in *Cheng et al.* [2000] which consists of eight wells. The second is the Case 3 of the same work which contains seven wells. In their work, the authors initially started with a total of fifteen wells and eventually screened down to eight and seven. Here, we compared our results with their results for only eight and seven wells cases. The pumping rates of all wells are initially set to the minimum pumping rate, 150 m^3/d, as suggested by *Cheng et al.* [2000]. While this solution was obtained by *Cheng et al.* [2000] on a Pentium 450-MHz microcomputer using about 6 hours of CPU time, we have used less than 30 minutes of simulation time on a compatible machine to obtain our results. Both runs obtained better results than those of *Cheng et al.* One of the reasons for this may be that the independent variable in this study is continuous, while in their case the independent variable, that is the pumping rate, is discretized. The continuous variable approach provides a flexible design, and it leads to a better result. Comparison of the results is given in Figure 7 and Table 3. We note here that the optimal pumping rates are still far less than the global pumping rates, estimated from the moving well simulations, even though more wells are used for the fixed location case. It is obvious from this result that the locations of the wells are not optimized. Hence, the existing wells may be moved to new locations to further increase the total pumping rate, while avoiding saltwater intrusion. This, for practical purposes, means that the old wells must be abandoned and new wells must be placed at the different locations with new pumping rates. The model

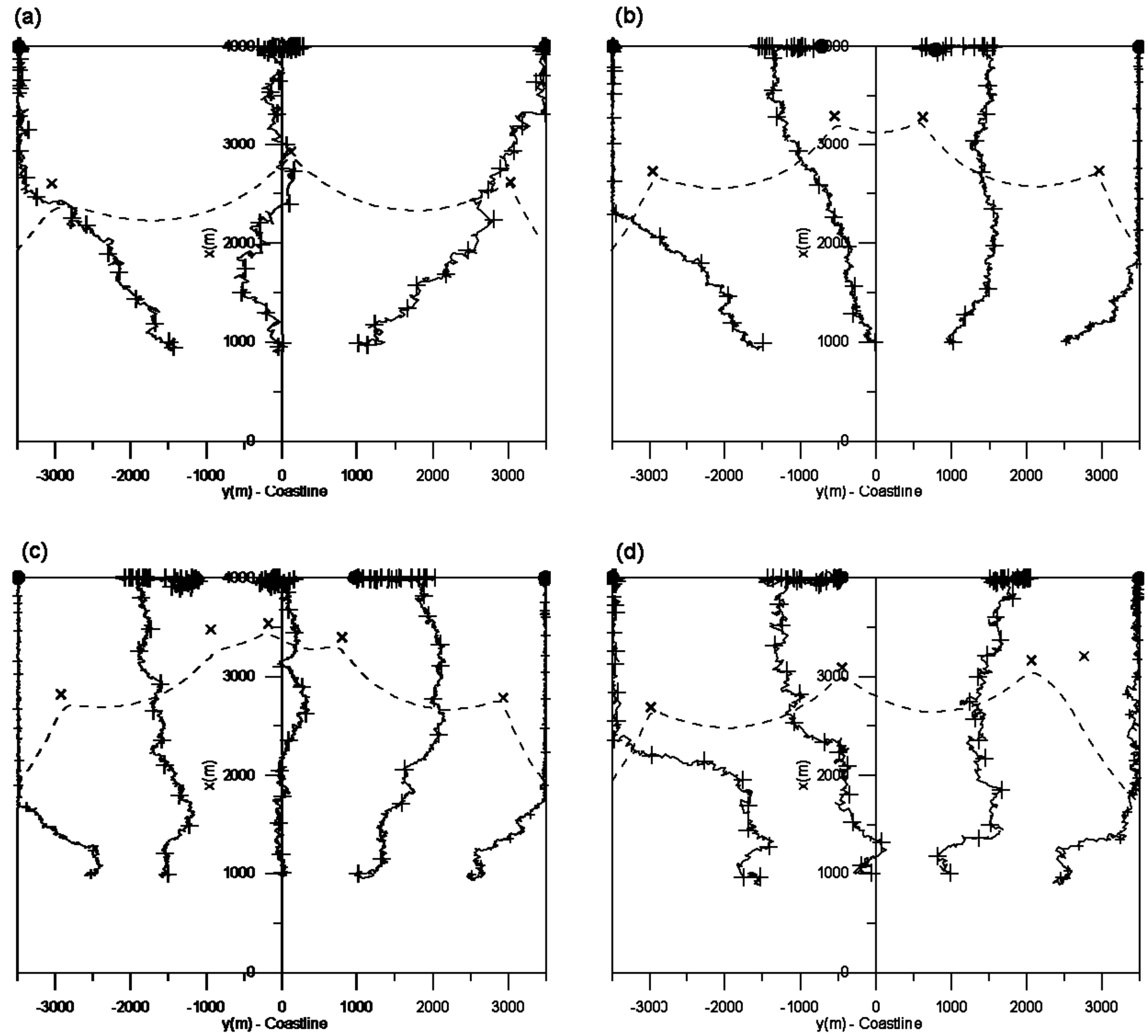


Figure 5. (a) Three pumping well placement optimization path; (b) Four pumping well placement optimization path; (c) Five pumping well placement optimization path; (d) Four pumping well placement optimization path (an intermediate solution – similar toe pattern to that given in (a))

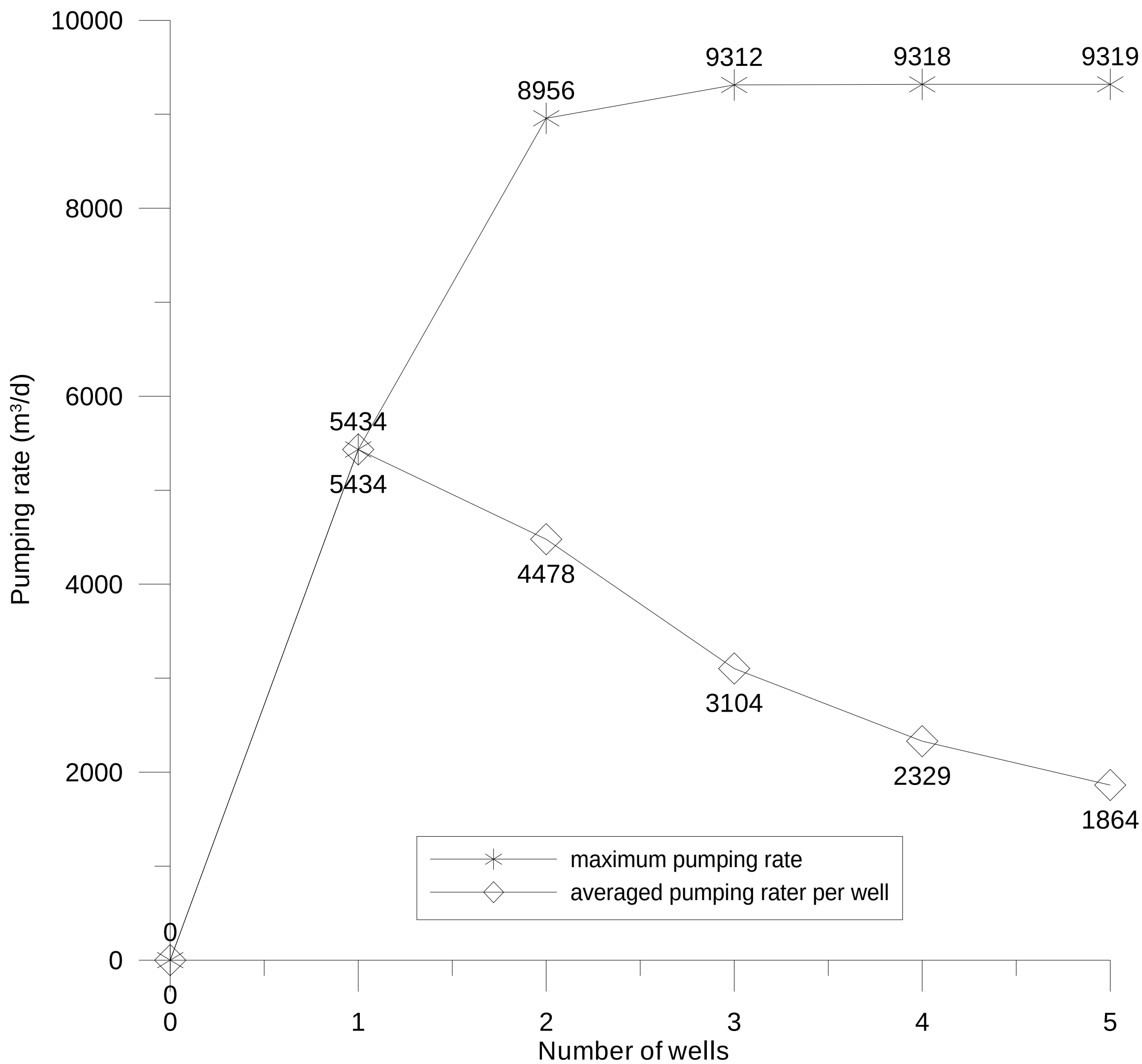


Figure 6. The relationship between the number of wells and the maximum pumping rates and the well efficiency.

Table 2. Summary of modeling parameters for three-, four- and five pumping well cases.

Aquifer parameters	**Value**		**MOGA parameters**	**Value**	
Same with Table 1.					
Sub-domain and bounding parameters (for all wells)					
	Three wells (a)	**Four wells (b)**	**Five wells (c)**	**Four wells (d)**	
k_1	250				
k_2	150				
k_3	150				
η	0.00001				
Q_{min}, Q_{max} m^3/d	100, 4000				
x_{min}, x_{max} m	0, 4000				
y_{min}, y_{max} m	-3500, 3500				
Starting Points			**Optimal Points**		
Q, m^3/d	***x*, m**	**y, m**	**Q, m^3/d**	***x*, m**	**y, m**
Three wells (a)					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	2195.6	3987.1	89.7
$Q_2 = 300$	$x_2 = 1000$	$y_2 = -1500$	3561.5	3994.3	-3491.4
$Q_3 = 300$	$x_3 = 1000$	$y_3 = 1000$	3530.3	3997.3	3499.4
Four wells (b)					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	1447.4	3997.1	-596.4
$Q_2 = 300$	$x_2 = 1000$	$y_2 = -1500$	3371.9	3995.0	-3497.6
$Q_3 = 300$	$x_3 = 1000$	$y_3 = 1000$	1293.3	3975.4	993.9
$Q_4 = 300$	$x_4 = 1000$	$y_4 = 2500$	3207.5	3998.4	3495.0
Five wells (c)					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	725.1	3966.2	-161.6
$Q_2 = 300$	$x_2 = 1000$	$y_2 = -1500$	1031.2	3981.4	-1136.0
$Q_3 = 300$	$x_3 = 1000$	$y_3 = 1000$	1206.4	3997.0	978.7
$Q_4 = 300$	$x_4 = 1000$	$y_4 = 2500$	3200.2	3995.6	3492.0
$Q_5 = 300$	$x_5 = 1000$	$y_5 = -2500$	3155.6	3999.9	-3498.8
Four wells (d)					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	1817.5	3997.6	-474.9
$Q_2 = 300$	$x_2 = 1000$	$y_2 = -1500$	3385.2	3996.9	-3491.9
$Q_3 = 300$	$x_3 = 1000$	$y_3 = 1000$	1402.5	3994.3	1902.6
$Q_4 = 300$	$x_4 = 1000$	$y_4 = 2500$	2567.9	3989.9	3490.5

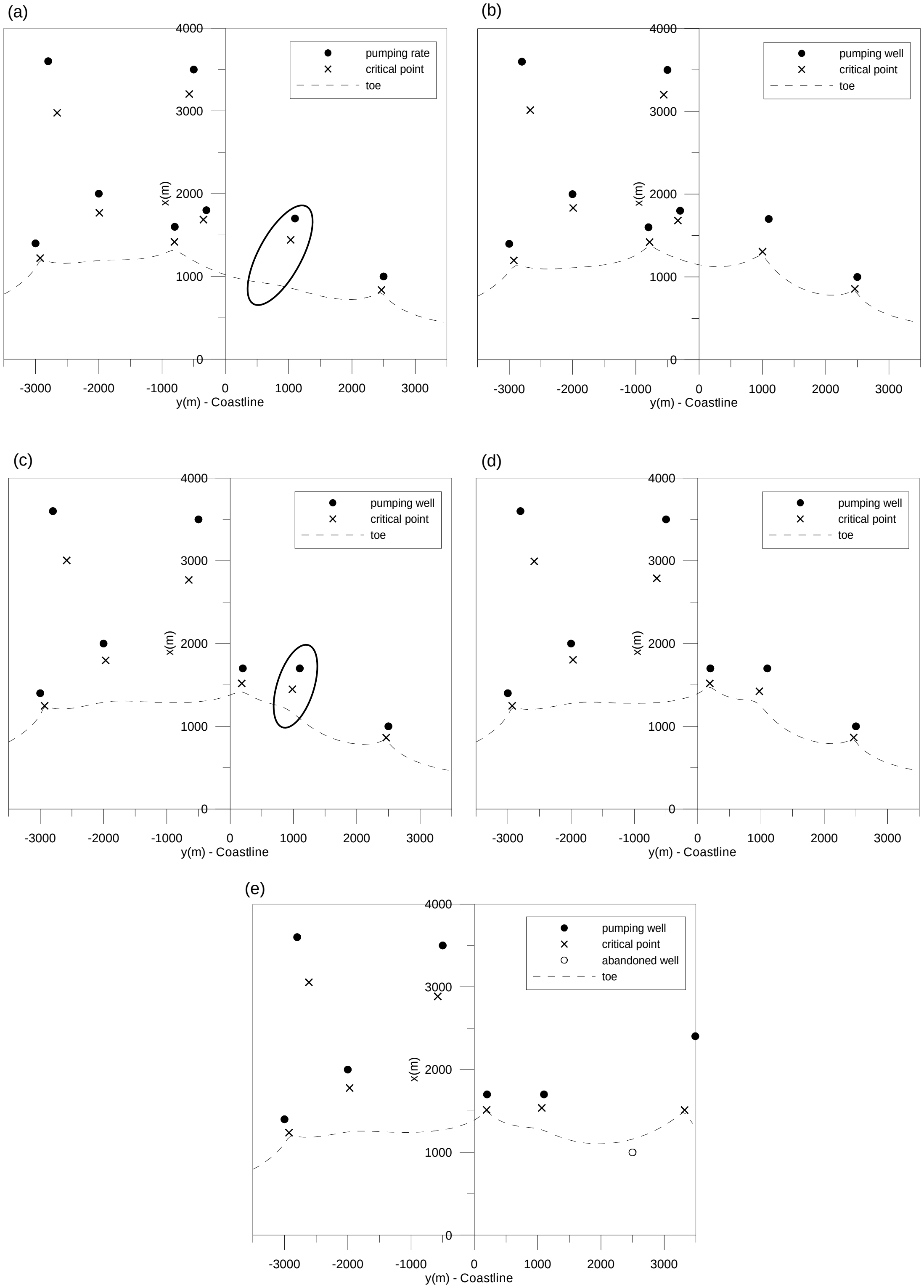


Figure 7. Comparison of results between *Cheng et al.* [2000] and MOGA application; (a) Cheng et al. Case 1 results; (b) MOGA Case 1 results; (c) Cheng et al. Case 3 results; (d) MOGA Case 3 results. (e) MOGA results obtained after abandoning one well given in Case 3.

Table 3. Summary of modeling parameters and the comparison of results with *Cheng et al.* [2000] fixed well optimization example.

Aquifer parameters	Value	MOGA parameters	Value
Aquifer type	unconfined	α	1
Saltwater density	1.025 g/cm^3	β	0
Uniform flow rate	0.4015 m^2/d	**Population size**	20
Saltwater depth	15 m	**Mating probability**	0.9
Hydraulic conductivity	40 m/d	**Mutation probability**	0.1
		Convergence	0.1

Sub-domain and bounding parameters (for all wells)

	Case 1 (eight wells)	Case 3 (seven wells)
k_1	150	250
k_2	0 (fixed-well)	150
k_3	0 (fixed-well)	150
η	0.000001	0.000001
Q_{min}, Q_{max} m^3/d	150, 1500	100, 5000
x_{min}, x_{max} m	0, 4000	0, 4000
y_{min}, y_{max} m	-3500, 3500	-3500, 3500

Starting Points			Optimal Pumping Rate	
Q, m^3/d	***x*, m**	**y, m**	**Q, m^3/d**	
Case 1 (eight wells) fixed-well				
			Cheng et al.'s	This work
$Q_1 = 150$	$x_1 = 1000$	$y_1 = 2500$	$Q_1 = 255$	$Q_1 = 221.7$
$Q_2 = 150$	$x_2 = 1700$	$y_2 = 1100$	$Q_2 = 402$	$Q_2 = 579.8$
$Q_3 = 150$	$x_3 = 1800$	$y_3 = -300$	$Q_3 = 158$	$Q_3 = 154.4$
$Q_4 = 150$	$x_4 = 3500$	$y_4 = -500$	$Q_4 = 728$	$Q_4 = 733.2$
$Q_5 = 150$	$x_5 = 1600$	$y_5 = -800$	$Q_5 = 150$	$Q_5 = 151.1$
$Q_6 = 150$	$x_6 = 3600$	$y_6 = -2800$	$Q_6 = 1500$	$Q_6 = 1402.9$
$Q_7 = 150$	$x_7 = 1400$	$y_7 = -3000$	$Q_7 = 185$	$Q_7 = 215.9$
$Q_8 = 150$	$x_8 = 2000$	$y_8 = -2000$	$Q_8 = 232$	$Q_8 = 178.4$
			$Q_{total} = 3610$	$Q_{total} = 3637.4$
Case 3 (seven wells) fixed-well				
			Cheng et al.'s	This work
$Q_1 = 150$	$x_1 = 1000$	$y_1 = 2500$	$Q_1 = 201$	$Q_1 = 198.1$
$Q_2 = 150$	$x_2 = 1700$	$y_2 = 1100$	$Q_2 = 351$	$Q_2 = 380.0$
$Q_3 = 150$	$x_3 = 1700$	$y_3 = 200$	$Q_3 = 150$	$Q_3 = 150.1$
$Q_4 = 150$	$x_4 = 3500$	$y_4 = -500$	$Q_4 = 1497$	$Q_4 = 1462.0$
$Q_5 = 150$	$x_5 = 2000$	$y_5 = -2000$	$Q_5 = 155$	$Q_5 = 150.0$
$Q_6 = 150$	$x_6 = 3600$	$y_6 = -2800$	$Q_6 = 1387$	$Q_6 = 1406.6$
$Q_7 = 150$	$x_7 = 1400$	$y_7 = -3000$	$Q_7 = 150$	$Q_7 = 150.2$
			$Q_{total} = 3891$	$Q_{total} = 3897.0$

developed is a very useful tool for such purposes. Based on the previous results, another simulation is conducted to see if optimization of the well locations can improve the solution. The model and the software developed is capable of assigning a moving well condition to any number of wells (i.e., none, some or all of the wells), the simulation was conducted by only moving one well which is the first well of Case 3 in Table 3. In this case, the total pumping rate increased to 4642.8 m^3/d from 3897 m^3/d (Figure 7e). This suggests that optimization of the well locations together with the pumping rates improves the results.

Example 4: Separate simulations of the two-well case, each with one component of the multi-objective criteria were conducted, before experimenting with a full multi-objective problem. The results for these cases are given in Figures 8a and 8b. As shown in Figure 8a and 8b, each objective successfully forces the solution in opposite but expected directions. The first objective forces the wells to be placed farther inland, while the second objective forces them to move closer to the coastline. As seen in Figure 8b, the optimal solution is obtained when the pumping rate of both wells are decreased to the minimum pumping rate, so that the distance between the stagnation point and the reference location of the initial saltwater interface is minimized. To demonstrate the use of the model for the multi-objective formulation, simulation starts with the parameter set, given in Table 5 ($\beta = 0$, to maximize pumping rate only), and as the pumping rate reaches the maximum allowable pumping, the pumping wells stay around the location identified by the circles in Figure 8c. Then, the second objective, which is intended to minimize the distance between the stagnation point and the reference saltwater interface location is turned on ($\beta = 1$, to minimize the distance between the stagnation point and the reference location together with maximizing the pumping rate). The objective weighting coefficient was determined from previously conducted numerical experiments. When the model works with multi-objective criteria, the wells position themselves in a way to minimize the distance between their stagnation points and the reference saltwater interface location, while maintaining the maximum pumping rates. Figure 8c illustrates this multi-objective tendency, and the results obtained for this case. As the final solution, the two wells are located at extreme y-axis locations and at around x-axis location 2000 m.

8. Conclusions

The problems we have tested and presented here have shown that the proposed mathematical formulation have produced optimal solution in an efficient manner as indicated in Example 3. The efficiency of this method makes it possible to include well locations as continuous independent variables. The formulation of the optimization problem combined with GA is straightforward and can be applied to the homogenous coastal aquifers under steady state freshwater flow. While the previous work often focuses on indirect methods or pumping rate optimization only, in this paper we present a new formulation in which the perturbations of pumping rates and well locations are used as continuous independent variables explicitly [*Cheng et al.,* 2000]. This approach produces the optimal solutions not only for the fixed-well cases, but it also handles the optimization of well location cases as well. The proposed model perturbs the well location and pumping rate within the sub-domain, and GA is applied to simultaneously find the optimal well location and pumping rate in the defined problem domain. The advantage is a significant reduction in model runs which consequently reduces the computational time and cost.

The constraint for detecting the well intrusion uses the stagnation point concept. This is a more relaxed constraint for the well-intrusion detection when compared to the studies which uses the well location instead. This in turn may have improved the optimal solution we have obtained when compared to the results obtained earlier.

To produce results for fixed-well cases, perturbation of well locations are turned off. The results generated are compared with the best solution by *Cheng et al.*'s. Our model produced slightly better results than theirs, indicating that both models solve the problem of pumping rate optimization successfully.

The proposed model is also run for several moving well cases. The results indicated that the total pumping rate, from a selected finite domain, can be improved significantly by introducing well locations as independent variables to the model. For practical purposes, being able to use well locations as independent variables is of great importance in the design stage.

In water management problem for coastal aquifers, the two main objectives are maximizing the pumping while placing the pumping wells as close to the coast as possible. Here, the possible restriction considered is the pumping site boundaries, which may be close to the shore line. In this work, these two objectives are combined into a single scalar objective function. In order to test the proposed single objective, a hypothetical coastal aquifer is set up (see Table 5). First, the impact of each separate objective is demonstrated. The results for maximizing pumping rates and minimizing the distance between the stagnation points and the reference location are shown in Figures 8a and 8b respectively. Based on the heuristic provided previously, both cases produced expected results. The wells move apart from each other in order to maximize pumping rates and they move closer to the coastline to minimize the distance. As can be seen from Figure 8a and 8b, these two objectives conflict with each other. In our formulation, the decision maker must assign weights for each of these objectives to control the preference weight of each objective in the solution. There are other approaches to solve multi-objective problems. The details can be found in *Fonseca and Fleming*, 1993; *Fleming and Fonseca* 1995; *Obayashi et al.,* 2000.

In the approach presented here, to find the optimum solution for the multi-objective case, first the model is run only maximizing the pumping rates. Once the wells reach the allowed maximum pumping rates, the second objective is activated so that they can be placed close to the coastline as much as possible. This is achieved by reducing the interaction between the wells by moving them further apart from each other. However, this does not imply that the model should run on a sequential two-step process to satisfy multi-objective criteria. The proposed model is capable of performing multi-objective analysis without using the sequential approach. In such cases, the weighing of objective weighing parameters is of importance and should be determined appropriately by the user. Most of the time this process is empirical, and the choice of relative importance of the multi-objective criteria depends on the preferences of the user.

The saltwater intrusion detection algorithm works efficiently when small perturbations in independent variables are used, especially when the search continues in near optimal solutions. When the saltwater toe location moves further inland than the current pumping well location, oscillations in toe delineation start. These oscillations feed wrong information about the well intrusion to the GA resulting in failure of the optimization procedure. However, this problem can be prevented by using small sub-domains which will not let the toe location to move over the well location. For instance, as is the case in the problems discussed here, the model can start the simulation with a large sub-domain so that each perturbation can be long without feeding wrong information to the GA due to failure of well-intrusion detection. Later the solution can continue with a smaller sub-domain as the solution reaches the optimal solution.

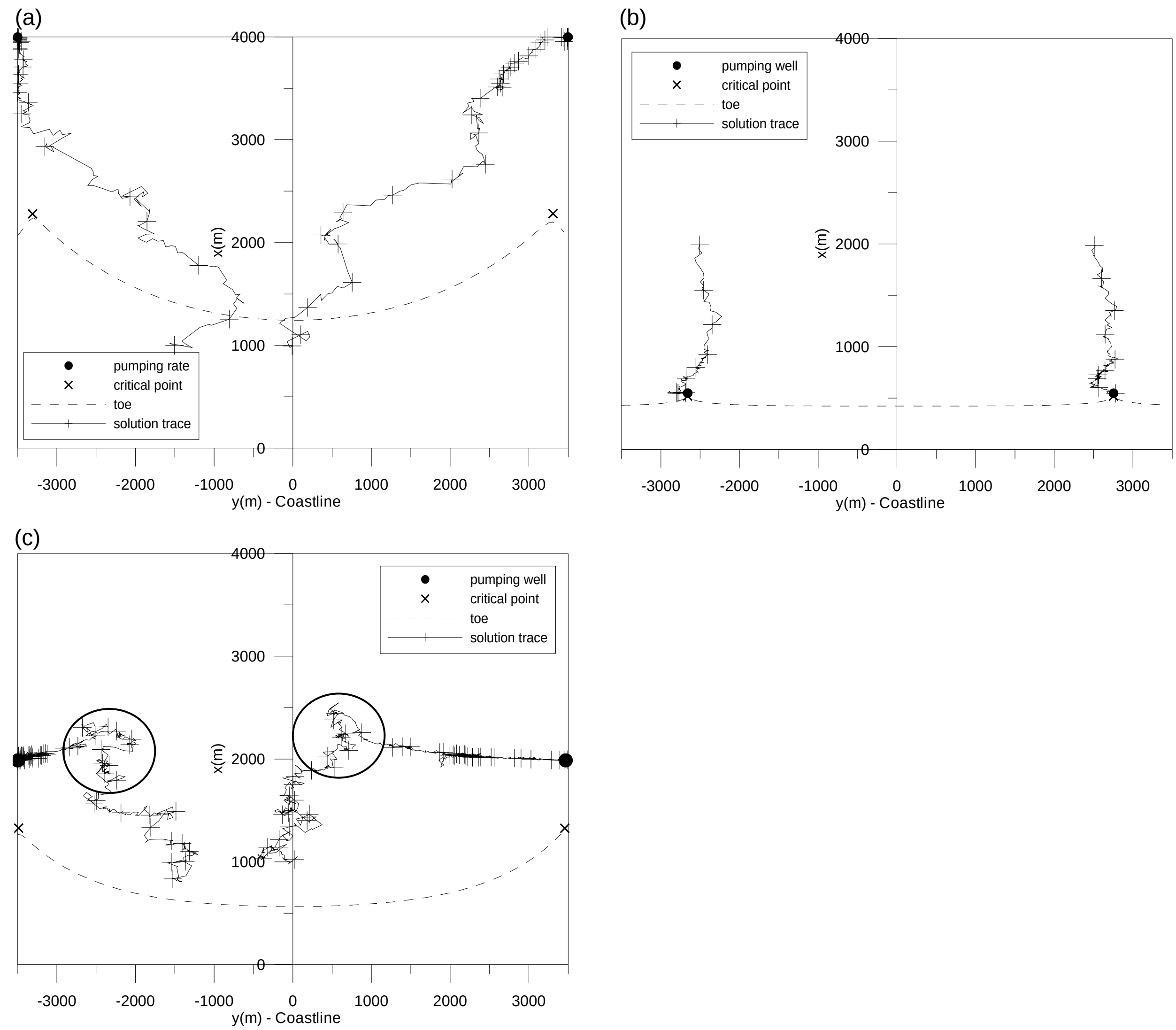


Figure 8. (a) The pumping rate maximization objective for two pumping well case; (b) Placement of the well as close as possible to the shoreline objective for two pumping well case; (c) Use of multi-objective criteria for two well case.

Table 4. Summary of modeling parameters and the results of pumping well placement - Case 3 of *Cheng et al.* [2000].

Aquifer parameters	**Value**	**MOGA parameters**	**Value**
Same with Table 3			
Sub-domain and bounding parameters – Case 3 Well #1 moving			
Same as Table 3 except for well number 1 k_1= 150, k_2 = 75, k_3 = 75.			

Starting Points			**Optimal Pumping Rate**
Q, m^3/d	***x*, m**	**y, m**	**Q, m^3/d**
Q_1 = 150	x_1 = 1000	y_1 = 2500	Q_1 = 1477.4 x_1 = 2403.4 y_1 = 3492
Q_2 = 150	x_2 = 1700	y_2 = 1100	Q_2 = 166.4
Q_3 = 150	x_3 = 1800	y_3 = -300	Q_3 = 150.1
Q_4 = 150	x_4 = 3500	y_4 = -500	Q_4 = 1220.7
Q_5 = 150	x_5 = 1600	y_5 = -800	Q_5 = 186.9
Q_6 = 150	x_6 = 3600	y_6 = -2800	Q_6 = 1275.1
Q_7 = 150	x_7 = 1400	y_7 = -3000	Q_7 = 166.2
			Q_{total} = 4642.8

Table 5. Summary of modeling parameters and the results of MOGA simulation.

Aquifer parameters	**Value**	**MOGA parameters**	**Value**		
Same as Table 1 initially. β is changed to be 1 when the pumping reaches the maximum pumping					
Sub-domain and bounding parameters (for all wells)					
		Two wells for multi-objective analysis			
k_1		150			
k_2		75			
k_3		75			
η		0.00001			
Q_{min}, Q_{max} m^3/d		100, 2000			
x_{min}, x_{max} m		0, 4000			
y_{min}, y_{max} m		-3500, 3500			
Starting Points			**Optimal Points**		
Q, m^3/d	***x*, m**	**y, m**	**Q, m^3/d**	***x*, m**	**y, m**
Double wells					
$Q_1 = 300$	$x_1 = 1000$	$y_1 = 0$	1999.2	1986.8	3467.9
$Q_2 = 300$	$x_2 = 1000$	$y_2 = -1500$	1996.4	1986.6	-3495.8

References

Aral, M. M. and Guan, J. (1997) *Optimal groundwater remediation system design with well locations selected as decision variables*, Multimedia Environmental Simulations Laboratory, publication No. MESL-01-97, Georgia Tech, Atlanta, Georiga.

Bear, J. (1972) *Dynamics of Fluids in Porous Media*, Dover Publications, Inc., New York., 557p.

Bear, J. (1979) *Hydraulics of Groundwater*, Mc-Graw Hill, New York, 379p.

Cheng, A. H.-D., and Ouazar, D. (1999) Analytical solutions, in *Saltwater Intrusion in Coastal Aquifers – Concepts, Methods, and Practices*, edited by J. Bear et al., Kluwer Acad., Norwell, Mass., chap. 6, pp. 163-191.

Cheng, A. H. –D., Halhal, D., Naji, A., and Ouazar, D. (2000) "Pumping optimization in saltwater-intruded coastal aquifers", *Water Resources. Research*, 36(8), 2155-2165.

Das, A. and Datta, B., (1999a) "Development of multiobjective management models for coastal aquifers", *J. Water Resour. Plann. Manage.*, 125, 76-87.

Das, A. and Datta, B., (1999b) "Development of management models for sustainable use of coastal aquifers", *J. Irrig. Drain. Eng.*, 125, 112-121.

Davis, L. (1991) *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, New York, 385p.

Dougherty, D. E. and Marryott, R. A., (1991) "Optimal groundwater management – 1. Simulated annealing", *Water Resources. Research*, 27(10), 2493-2508.

Emch, P. G., and Yeh, W. W. G., (1998) "Management model for conjunctive use of coastal surface water and groundwater", *J. Water Resour. Plann. Manage.*, 124, 129-139.

Finney, B. A., Samsuhadi, and R. Willis, (1992) "Quasi-3-dimensional optimization model of Jakarta Basin", *J. Water Resour. Plann. Manage.*, 118, 18-31.

Frind, E. O. (1982) "Saltwater intrusion in continuous coastal aquifer-aquitard systems", *Advanced Water Resources*, 5, 89-97.

Fonseca, C. M. and Fleming, P. J. (1993) "Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization", in *Genetic Algorithms: Proc. Fifth International Conference* (S. Forest, ed.), Morgan Kaufmann, pp. 416-423.

Fleming, P. J. and Fonseca, C. M. (1995) "An overview of evolutionary algorithms in multiobjective optimization", *Evolutionary Computation*, vol. 3, Spring.

Goldberg, D. E. (1989) *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley-Longman, Reading, Mass.

Guan, J. and Aral, M. M. (1999a) “Optimal remediation with well locations and pumping rates selected as continuous decision variables”, *Journal of Hydrology*, 211, 20-42.

Guan, J. and Aral, M. M. (1999b) “Progressive genetic algorithm for solution of optimization problems with nonlinear equality and inequality constraints”, *Applied Mathematical Modelling*, 23, 329-343.

Holland, J. H. (1975) *Adaptation in natural and artificial systems*, University of Michigan Press, Ann Arbor, 183p.

Huang, C. and Mayer, A. S. (1997) “Pump-and-treat optimization using well locations and pumping rates as decision variables”, *Water Resources. Research*, 33(5), 1001-1012.

Hallaji, K., and Yazicigil, H. (1996) “Optimal management of coastal aquifer in southern Turkey”, *J. Water Resour. Plann. Manage.*, 122, 233-244.

McKinney, D. C., and Lin, M. (1994) “Genetic algorithms solution of groundwater management models”, *Water Resources. Research*, 30(6), 1897-1906.

Obayashi, S., Tsukahara, T., and Nakamura, T. (2000). “Multiobjective genetic algorithm applied to aerodynamic design of cascade airfoils”, *IEEE Transactions on Industrial Electronics*, 47(1), 211-216.

Shamir, U., Bear, J., and Gamliel, A. (1984). “Optimal annual operation of a coastal aquifer”, *Water Resources. Research*, 20, 435-444.

Shonkwiler R. (2000) *Monte Carlo Methods*, Math 4225 Class notes, School of Mathematics, Georgia Tech, Atlanta, Georgia.

Strack, O. D. L. (1976) “A single-potential solution for regional interface problems in coastal aquifers”, *Water Resources. Research*, 12(6), 1165-1174.

Willis, R., and Finney, B. A. (1988) “Planning model for optimal control of saltwater intrusion, *J. Water Resour. Plann. Manage.*, 114, 333-347.

Appendix A

A typical way to solve a set of nonlinear equations is Newton-Raphson method. The advantage of this method is that it is fast and easy to use. However, it requires the derivatives of a nonlinear equation, and this makes it tricky, since most of the times it is difficult or impossible to obtain these derivatives. For the analytical solution of the saltwater intrusion problem, there are two nonlinear equations to solve for the stagnation point. These equations are the following:

$$\begin{bmatrix} \dfrac{\partial \phi}{\partial x} = 0 \\ \dfrac{\partial \phi}{\partial y} = 0 \end{bmatrix} \Rightarrow \begin{bmatrix} F_1(x,y) = 0 \\ F_2(x,y) = 0 \end{bmatrix} \tag{A1}$$

Taking only the first two terms from the Taylor series expansion of (A1) and simplifying it for x and y, the rearranged equation can be written as,

$$\begin{Bmatrix} x^{k+1} \\ y^{k+1} \end{Bmatrix} = \begin{Bmatrix} x^{k} \\ y^{k} \end{Bmatrix} - \frac{1}{D} \begin{bmatrix} \dfrac{\partial F_2}{\partial y} & -\dfrac{\partial F_1}{\partial y} \\ -\dfrac{\partial F_2}{\partial x} & \dfrac{\partial F_1}{\partial x} \end{bmatrix} \begin{bmatrix} F_1\left(x^k, y^k\right) \\ F_2\left(x^k, y^k\right) \end{bmatrix} \tag{A2}$$

where $D = \begin{vmatrix} \dfrac{\partial F_1}{\partial x} & \dfrac{\partial F_1}{\partial y} \\ \dfrac{\partial F_2}{\partial x} & \dfrac{\partial F_2}{\partial y} \end{vmatrix}$ is the determinant.

The corresponding derivatives for multiple wells of the saltwater intrusion problem can now be evaluated from (10). These are given below,

$$\frac{\partial F_1}{\partial x} = \sum_{i=1}^{n} \frac{Q_i}{4\pi K}\left[T_1 + 4\left\{\frac{(x+x_i)}{R_2} - (x-x_i)\frac{R_1}{R_2^2}\right\}\left\{\frac{(x-x_i)}{R_1} - (x+x_i)\frac{R_2}{R_1^2}\right\}\right] \tag{A3a}$$

$$\frac{\partial F_1}{\partial y} = \sum_{i=1}^{n} \frac{Q_i}{4\pi K}\left[T_2 - 4(y-y_i)\left\{\frac{R_1}{R_2^2} - \frac{1}{R_2}\right\}\left\{\frac{(x-x_i)}{R_1} - (x+x_i)\frac{R_2}{R_1^2}\right\}\right] \tag{A3b}$$

$$\frac{\partial F_2}{\partial x} = \sum_{i=1}^{n} \frac{Q_i}{4\pi K}\left[T_2 - 4\left\{(x-x_i)\frac{R_1}{R_2^2} - \frac{(x+x_i)}{R_2}\right\}\left\{\frac{(y-y_i)}{R_1} - (y-y_i)\frac{R_2}{R_1^2}\right\}\right] \tag{A3c}$$

$$\frac{\partial F_2}{\partial y} = \sum_{i=1}^{n} \frac{Q_i}{4\pi K}\left[T_3 - 4(y-y_i)\left\{\frac{R_1}{R_2^2} - \frac{1}{R_2}\right\}\left\{\frac{(y-y_i)}{R_1} - (y-y_i)\frac{R_2}{R_1^2}\right\}\right] \tag{A3d}$$

where

$$R_1 = (x+x_i)^2 + (y-y_i)^2 \tag{A3e}$$

$$R_2 = (x - x_i)^2 + (y - y_i)^2 \tag{A3f}$$

$$T_1 = \frac{R_1}{R_2}\left\{\frac{2}{R_1} - 8\frac{(x + x_i)(x - x_i)}{R_1^2} + 8(x + x_i)^2\frac{R_2}{R_1^3} - 2\frac{R_2}{R_1^2}\right\} \tag{A3g}$$

$$T_2 = \frac{R_1}{R_2}\left[8\frac{R_2}{R_1^3}(x + x_1)(y - y_1) - \frac{4}{R_1^2}\{(x - x_i)(y - y_i) + (x + x_i)(y - y_i)\}\right] \tag{A3h}$$

$$T_3 = \frac{R_1}{R_2}\left\{\frac{2}{R_1} - 8\frac{(y - y_i)^2}{R_1^2} + 8(y - y_i)^2\frac{R_2}{R_1^3} - 2\frac{R_2}{R_1^2}\right\} \tag{A3i}$$

Appendix B

WinSaltPGA Version 0.6, the simulation software for the saltwater optimization problem discussed in this report, was developed using Microsoft Visual C++. This software is developed as a tool to accompany this report and can be used for practical applications. The software transfers the expert knowledge on optimal saltwater intrusion problem solution to the practitioners. The description and functions of several data input windows of the WinSaltPGA is provided in Appendix B.

B.1 Data configuration

The startup window of WinSaltPGA is shown in Figure 9. The physical problem domain is not set and drawn in the window until either read from the existing file or entered by the user. Any domain size can be set by properly configuring the range of x and y in the "*PGA* (Progressive Genetic Algorithm) *Configuration*".

Once WinSaltPGA is opened, there are two different modes to conduct simulations. User can start with configurations of both the "*Aquifer Properties and Uniform flow*" and the "*PGA Configuration*", or by opening one of the existing sample files in the distribution package. For the novice, it is easier to run the model using the latter method and the user does not have to worry about the model configuration.

Next step is to open the window "*Seawater Optimization*" by clicking either "*WinSaltPGA | Aquifer Configuration*" from the menu or "*C*" icon in the toolbar under the menu. In the "*Seawater Optimization*" window, there are two sub-windows consisting of "*Aquifer Properties and Uniform flow*" and "*PGA Configuration*". As shown in Figure 10, "*Aquifer properties and Uniform flow*" can be configured depending on the aquifer to be modeled. The last two boxes, "*Range for initial guess*" and "*Step of range for initial guess*" are necessary to guarantee that the stagnation points are obtained properly in a smaller domain. The stagnation points are obtained using the nonlinear solver (Newton-Raphson Method). The "*Range for initial guess*" means that the model starts randomly with an initial guess (or a point) within the user-defined square region, having each stagnation point to be centered by default. The second box, "*Step of range for initial guess*" is to determine how dense the initial guess grid will be (i.e., available initial guess locations to start search) in the defined region.

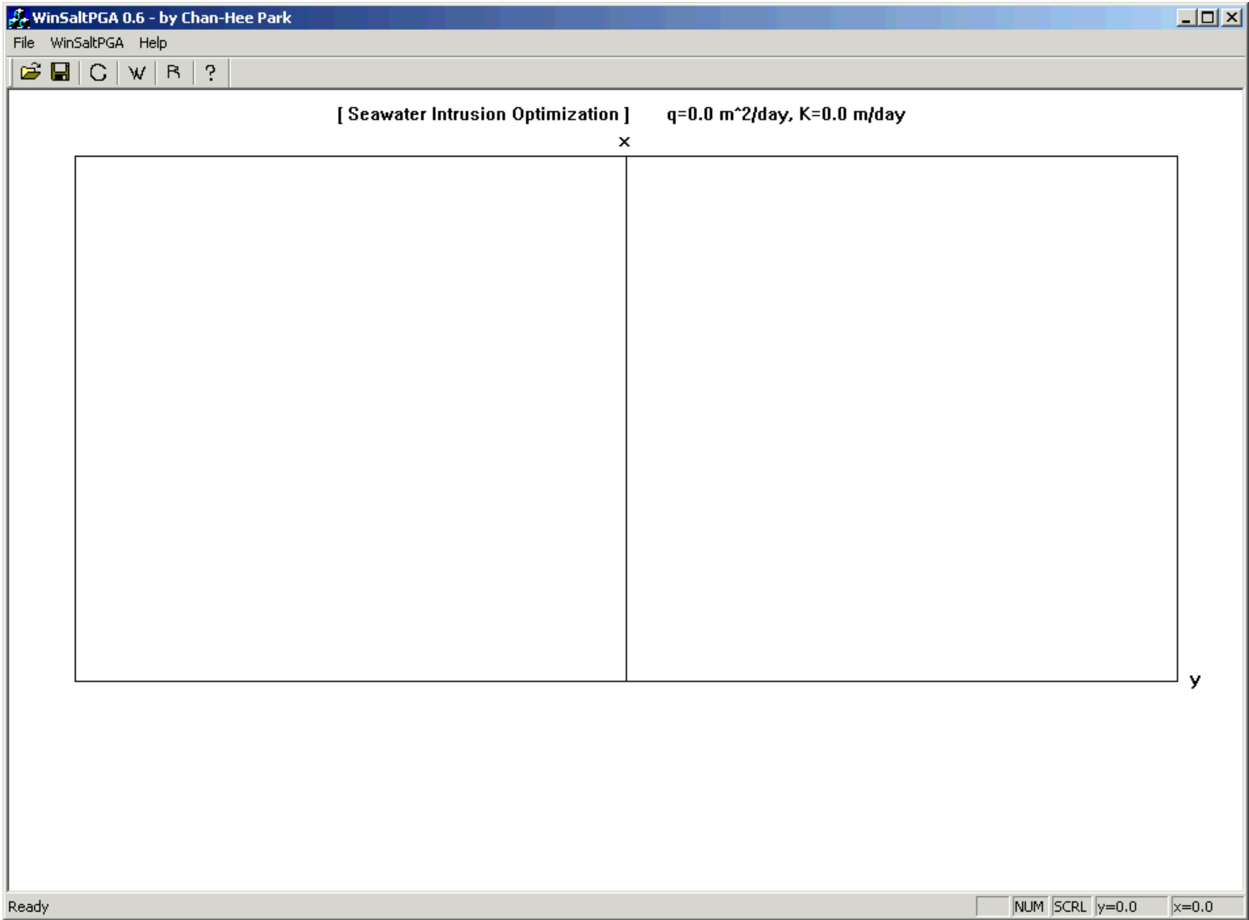


Figure 9. The startup window of WinSaltPGA V 0.6

After configuring the "*Aquifer Properties and Uniform flow*", it is the "*PGA Configuration*" that needs to be set. The "*PGA Configuration*" consists of three sub-sections. The first one, "*Objectives and GA,*" is mainly related to MOGA. Note that the problem may be turned into a single objective problem by setting either "*Objective Weighting Factor (alpha)*" or "*Objective Weighting Factor (beta)*" to be zero. For MOGA as given in Figure 8d, both α and β are assigned to be "1". The second sub-section, "*Population and Convergence*", is to configure the number of populations and tolerance of convergence. Increasing the number of populations gives more genetic divergence while increasing the time of single iteration. The default value is set to 40. Similarly, as the tolerance of convergence decreases, the time of single iteration will be longer. The optimized value for tolerance of convergence is found to be 0.001. The third sub-section, "*Contraction Coefficients and Bounds*" is for configuring the problem domain and the sub-domains for each well. Based on our experience, the proper coefficients are determined and can be found in the sample files and tables presented in this paper. Contraction coefficients, such as k_1, k_2, k_3, can be used directly to set the size of the sub-domain as defined in Equation 25. The other alternative to form the sub-domains is to adjust η so that it decreases the size of the sub-domain gradually as the number of iterations increases. If any of k_1, k_2, or k_3 is set to zero, the model will not make any perturbation on the corresponding variable. In this way, we can optimize the fixed well or the moving well locations. For instance, the user may select the optimization problem for the fixed well locations case (pumping only), by selecting k_2 and k_3 as zero. With this selection, the model does not perturb the Δx and Δy variables, thus the well location is not changed. It should be noted that selecting large k_1, k_2, or k_3 coefficients may lead to convergence problems. If the model fails to solve for the stagnation point, it feeds false information to the GA and this leads to error. Thus, the user should be aware of configuring the parameters conservatively. Authors recommend the user to start with the k_1, k_2, and k_3 values provided in the sample files, then increase them to at least double as the simulation goes on, so that the model will not fail to solve for the stagnation points.

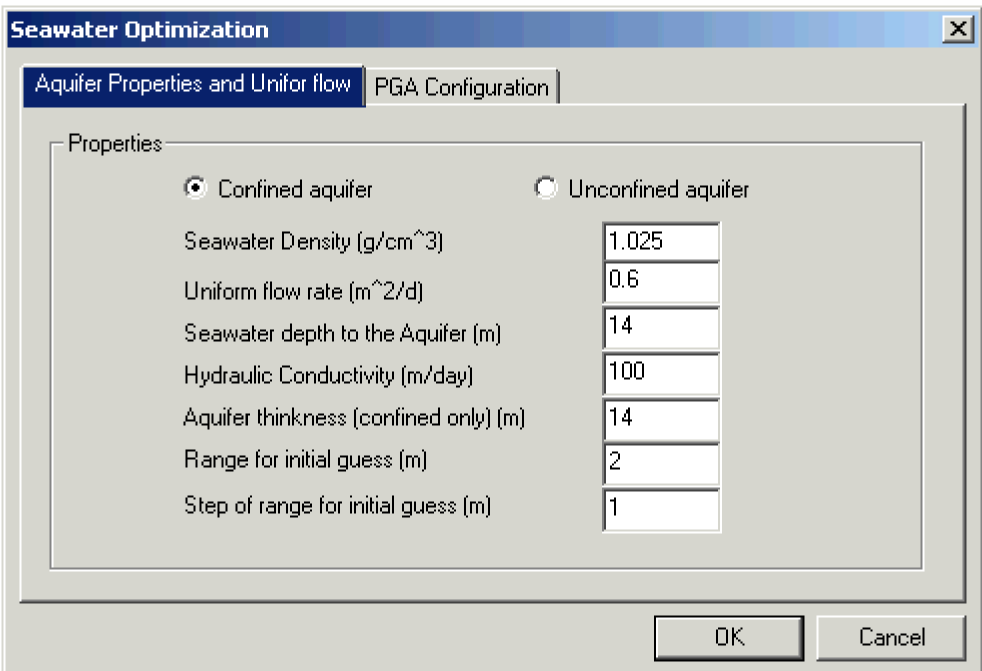


Figure 10. Aquifer properties and uniform flow in saltwater optimization window

Note that each well has its own "*PGA Configuration*" and these can be reconfigured any time during the simulation.

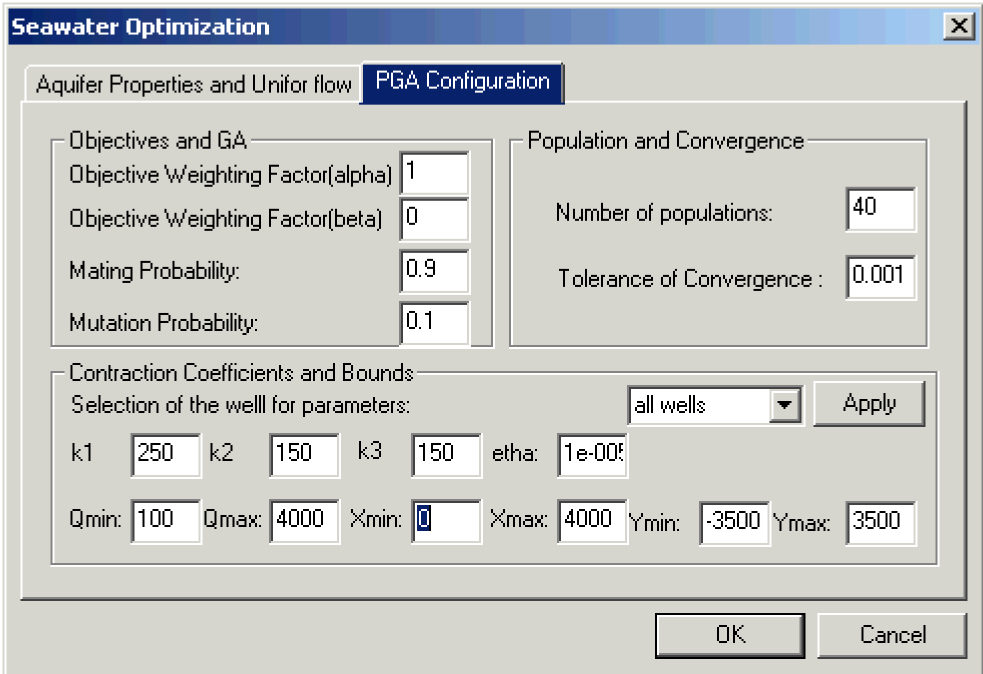


Figure 11. PGA configuration in saltwater optimization window

B.2 Well Initial Conditions and Simulation

After properly configuring "*Aquifer Properties and Uniform flow*" and "*PGA Configuration*" in Seawater Optimization, the model is ready to roll for saltwater optimization. Currently, the maximum number of wells is fixed to be eight. The initial locations and pumping rates of four wells are provided as an example in Figure 12.

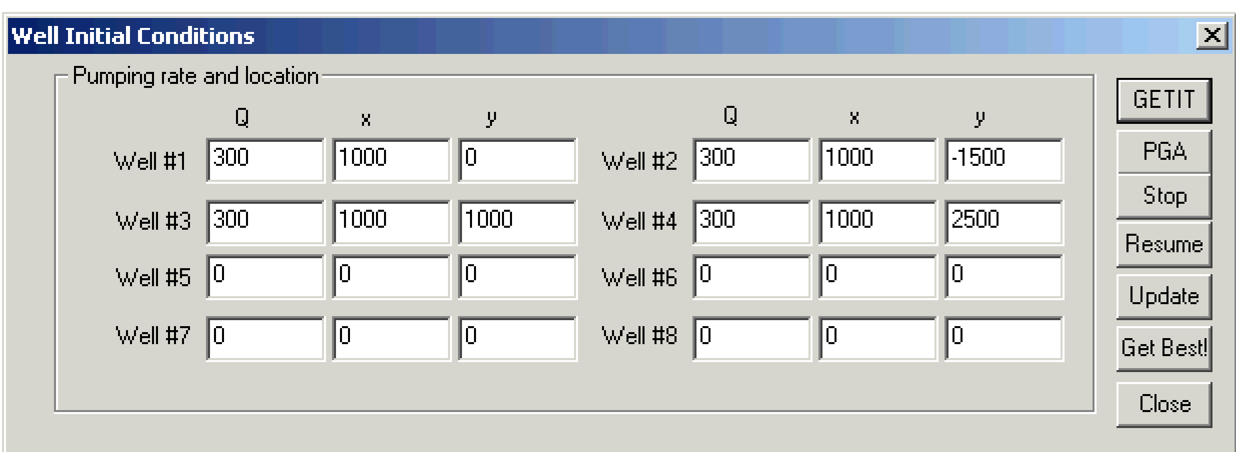


Figure 12. Well initial conditions and the simulation panel

As the optimization goes on by clicking "*PGA*", the model displays the visualization of every optimization step in the main window (see Figure 13) as well as the corresponding values in the "Well Initial Conditions" window. If only stagnation points are of interest, they can be obtained by clicking the "*GET IT*" button. Anytime during the simulation, the parameter set can be modified by clicking the "*Stop*" and revising the parameters which are selected initially. The simulation may then be continued by clicking the "*Resume*" button. After this point, the revised parameter sets will become effective.

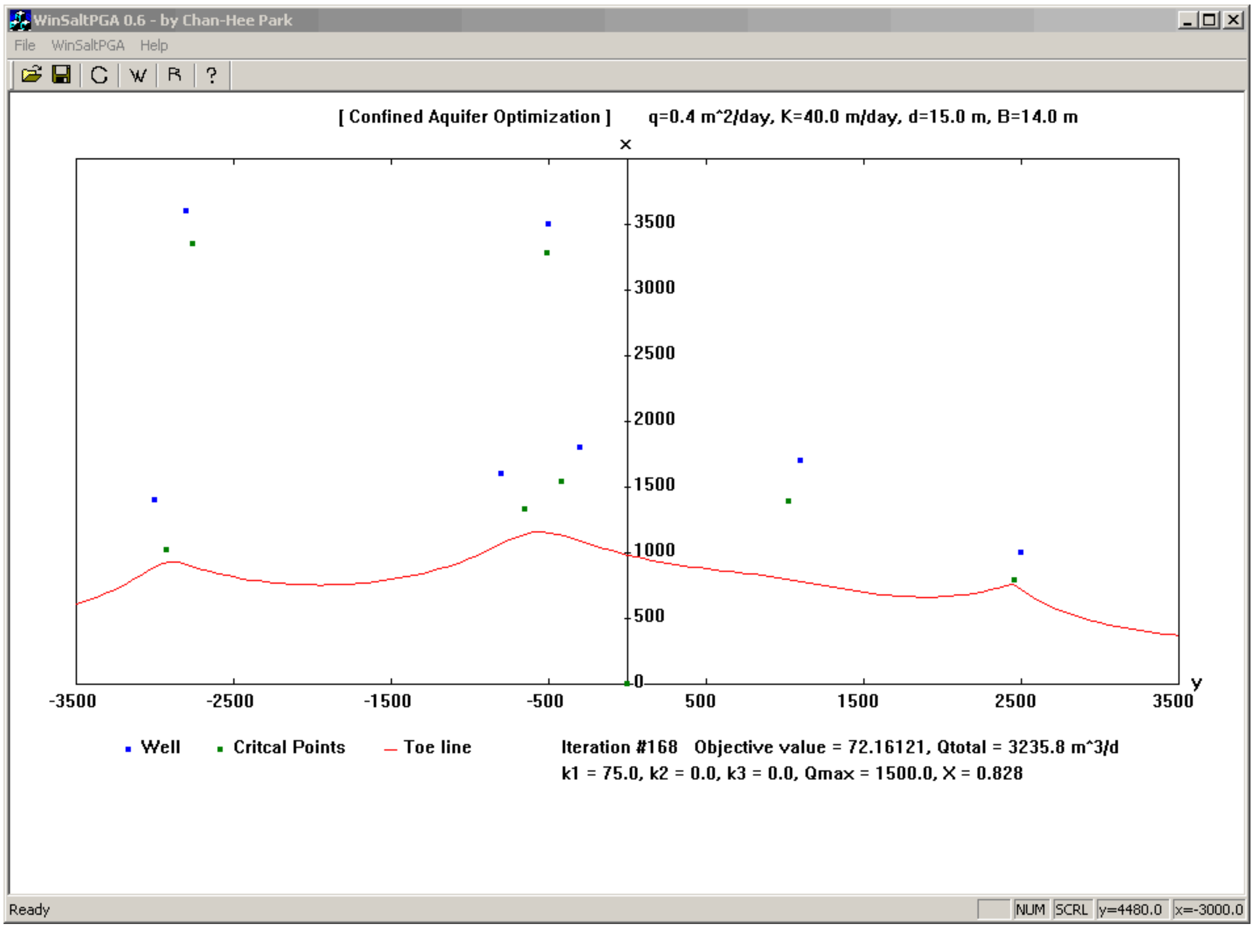
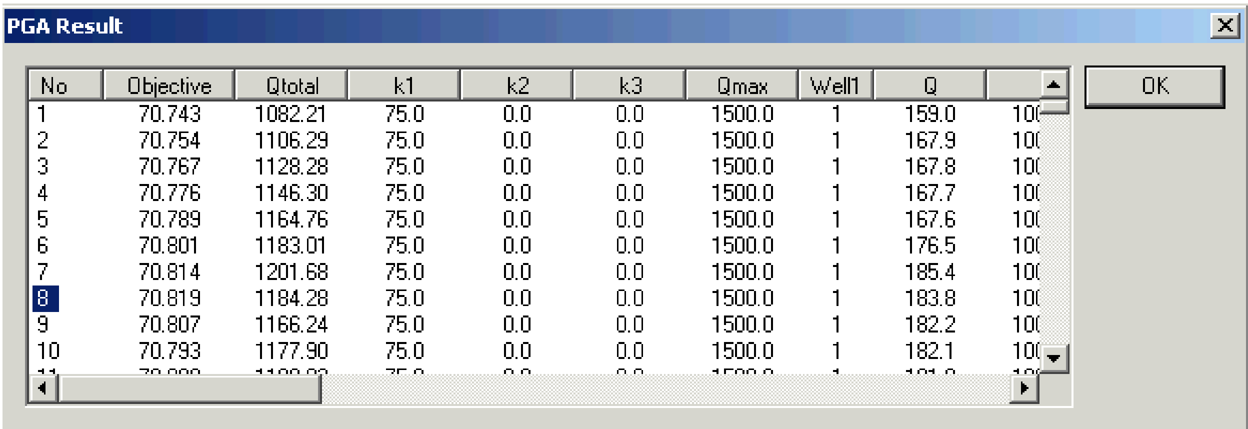


Figure 13. Visualization of the optimization results at each step of simulation

If the solution does not seem to improve considerably and if the user believes the optimal solution is reached, the simulation can be stopped to see this result. The user will find it difficult to stop the simulation at the best result due to the fast update of iteration. The best result of the current simulation can be obtained by clicking the "*Get Best*" button, since the model automatically records all the trace of the simulation. The user may see all records of the simulation, clicking "*WinSaltPGA | PGA Result*" from the menu or the "*R*" button in the toolbar. This will open up the results of the simulation in table format in a new window. In this window, by clicking any sequence of "*No.*" column and then clicking "*Update*" button on the "*Well Initial Conditions*" window, the user can continue optimization starting from the indicated step. When satisfied with the results of the optimization. The results can be saved by clicking either the "*save*" from the menu or the "*disk icon*" in the toolbar. Whenever the user starts the simulation from the previously stored file, the user has to click "*Confined aquifer or Unconfined aquifer*" in "*Aquifer Properties and Uniform flow*" and the "*Apply*" button in "*PGA Configuration*" to properly assign all the parameters.



No	Objective	Qtotal	k1	k2	k3	Qmax	Well1	Q	
1	70.743	1082.21	75.0	0.0	0.0	1500.0	1	159.0	10
2	70.754	1106.29	75.0	0.0	0.0	1500.0	1	167.9	10
3	70.767	1128.28	75.0	0.0	0.0	1500.0	1	167.8	10
4	70.776	1146.30	75.0	0.0	0.0	1500.0	1	167.7	10
5	70.789	1164.76	75.0	0.0	0.0	1500.0	1	167.6	10
6	70.801	1183.01	75.0	0.0	0.0	1500.0	1	176.5	10
7	70.814	1201.68	75.0	0.0	0.0	1500.0	1	185.4	10
8	70.819	1184.28	75.0	0.0	0.0	1500.0	1	183.8	10
9	70.807	1166.24	75.0	0.0	0.0	1500.0	1	182.2	10
10	70.793	1177.90	75.0	0.0	0.0	1500.0	1	182.1	10

Figure 14. Results of the optimization viewed in a text window