DEFLECTION EQUATIONS AND LOCATION OF LOCI OF FLEXURAL CENTERS FOR SWEPT PLATES

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OF FLEXURAL CENTERS FOR SWEPT PLATES

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TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

Forces and Moments.

Q_x Q_y	Resultant shearing forces per unit length of plate, normal to the middle surface.
M_x M_y	Resultant bending moments per unit length of plate.
H_{xy}	Resultant twisting moment per unit length of plate.
σ_x	Tensile stress, normal to the yz plane.
τ_{xy}	Shear stress.
P q	Dummy load and twisting couple.

Coordinates and Dimensions.

$x\,y\,z$	Right handed rectangular coordinate system.
w	Elastic displacement in the direction of z.
θ	Angle of sweep.
s	Semi-span of wing or plate, measured at right-angles to the cantilever edge.
c	Chord, measured in the direction of y.
ℓ	Distance of point of convergence of leading and trailing edges from root or cantilever edge.
η	Dummy variable, particular value of x.
μ	Dummy variable of value $(\ell-\eta)$
n	Fraction of chord c.
m	Fraction of semi-span s.
t	Thickness of plate.
λ	Taper Ratio

Material Properties.

E	Young's Modulus of Elasticity.
D	Flexural rigidity of plate $= \dfrac{Et^3}{12(1-\nu^2)}$
ν	Poisson's Ratio.

Subscripts and Special Combinations.

T	Tip of plate or wing.
R	Root of plate or wing.
w_{50}	Deflection at the 50% chord line.
$w_{T.E.}$	Deflection at the trailing edge.
$w_{L.E.}$	Deflection at the leading edge.

DEFLECTION EQUATIONS AND LOCATION OF LOCI OF FLEXURAL CENTERS FOR SWEPT PLATES

I SUMMARY

This paper is devoted to the investigation of the deflections occuring in swept plates under normal loading conditions. It presents a simplified analysis which may be used for an approximate solution of the deflection at points along the span of a given flat plate, rectangular in plan and swept from a cantilever edge.

A new concept, that of a locus of flexural centers, is defined and discussed. Using the basic energy equation, developed in Section I, the locus of flexural centers is located for swept rectangular plates of constant thickness, swept plates tapered in width but constant in thickness, and swept plates tapered in both width and thickness.

Experimental work has been carried out at the Guggenheim Aeronautical Laboratories of California Institute of Technology [1], on the deflections of a flat swept plate

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[1] H.M.DeGroff, Experimental Investigation of the Effect of Sweep Upon the Stress and Deflection Distribution in Cantilever Plates of Constant Chord and Thickness. Air Force Technical Report 5761-3. June 1949.

under various loadings, and these results are used to compare with the theory developed herein.

II INTRODUCTION

The analysis of the unswept wing is a relatively simple matter, loads on the flexural axis cause bending relative to the wing axis without twisting, and loads offset from the flexural axis cause bending and twisting. The method of analysis is familiar and straightforward.

In the case of swept wings, however, the picture is more complex in view of the fact that no point exists where a load may be placed to cause bending without twisting at some other point in the structure. This phenomenan is fully discussed, with respect to a two spar wing, by G. T. R. Hill.[2]

The modern swept wing is usually, though not exclusively, used on high speed aircraft employing airfoil sections of low thickness ratio. This fact, coupled with other aerodynamic requirements of the wing (such as rigid conformity to a specified section contour) has resulted in thin, high density wing structures. The idealized case of such a structure is the thin flat plate, and it is therefore justifiable to open the investigation of the deflections in swept wings by an analysis of the deflection of the thin flat plate under various standard loadings.

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[2] G. T. Hill, "The Nature of the Distortion of Swept Back Wings". Journal of the Royal Aeronautical Society. London, March 1948.

Classical plate theory gives the equilibrium equation, for an infinitesimal element of thin flat plate, in either of the following forms:[3]

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - \frac{2\,\partial^2 H_{xy}}{\partial x \cdot \partial y} = -q$$

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + \frac{2\,\partial^4 w}{\partial x^2 \cdot \partial y^2} = \frac{q}{D}$$

Except for a few isolated cases, that integral of this equation, which also satisfies the condition at the boundary of the plate, is mathematically intractable. The problems of the circular and rectangular plates with simply supported edges, and the rectangular plate with clamped edges were solved in the 19th Century.

Other cases, in particular those of the cantilever rectangular plate and the skew plate, have been solved approximately by Holl[4] and Jensen,[5] respectively, by the use of finite difference equations. Finite difference equations

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[3] Lagrange, 1811, is largely responsible for this equation. It first appears in his "Note Communique aux Commissaires Pour le Prix de la Surface Elastique".

[4] D. H. Holl, "Cantilever Plate with Concentrated Edge Load". <u>Journal of Applied Mechanics,</u> Vol. 4, No. 1, March 1937.

[5] V. P. Jensen, "Analysis of Skew Slabs", <u>University of Illinois, Engineering Experiment Station, Bulletin Series No. 332</u>, Vol. XXXIX, No. 3, September, 1941.

for the analysis of thin rectangular plates with combinations of fixed and free edges have also been set up by Barton.[6]

More recently, the Structures Group at the Guggenheim Aeronautical Laboratories of California Institute of Technology has carried out a program of work on the problem of the deflections in swept plates. A resume of this work was presented by E. E. Sechler, M. L. Williams, and Y. C. Fung to the Institute of the Aeronautical Sciences in 1949.[7] Various approximations have been made to the stress distribution in the swept plate and electrical analogies obtained. All of the work carried out by this group gives results which are very approximate and involve tedious mathematical computation. In certain cases empiricisms have been used which are based on one set of experimental data and consequently there is little justification for their use generally.

This paper sets out to present a simplified analysis of the problem by disregarding the variation of the moments and twisting couples in a chordwise direction. Due to the

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[6] M. V. Barton, Finite Difference Equations for the Analysis of Thin Rectangular Plates with Combinations of Fixed and Free Edges. Defense Research Laboratory, DRL-175, August 1948.

[7] E. E. Sechler, M. L. Williams and Y. C. Fung, "An Initial Approach to the Overall Structural Problems of Swept Wings Under Static Loads." Institute of the Aeronautical Sciences. Preprint No. 257, December, 1949.

nature of this approximation it is not to be expected that the theoretical results will have great practical value, but will give a clear picture of the types of deflection patterns obtained under different loadings. In this respect also, the location of the loci of flexural centers for various types of plate configuration is felt to be useful.

III PART I. ANALYSIS OF THE DEFLECTIONS IN SWEPT PLATES

Figure 3. represents an element of thin flat plate for which the equilibrium equations are:[8]

$$
\begin{aligned}
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q &= 0 \\
\frac{\partial M_x}{\partial x} + \frac{\partial H_{xy}}{\partial y} - Q_x &= 0 \\
\frac{\partial M_y}{\partial y} + \frac{\partial H_{xy}}{\partial x} - Q_y &= 0
\end{aligned}
\tag{2}
$$

Equation group (2) consists of three equations in five unknowns, and hence has an infinitude of solutions. Only one of these, that which satisfies the boundary conditions and the conditions of compatibility, is an admissible solution. In order to arrive at a simplified solution, the equations (2) will be modified.

Let figure 4. represent an element of the swept plate, unit chordwise length by dx elemental distance measured along the span. The element is skew by an amount dy taken in the chordwise direction. Further as a simplification, we will assume that there is no chordwise variation of bending moment, twisting moment or shear. In other words

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[8] S. Timoshenko, Theory of Plates and Shells. (New York: McGraw-Hill, 1940), p. 86.

the element of plate is considered as being subjected to the straight beam moments and forces only. Under such conditions the equations of equilibrium are:

$$\begin{aligned} \frac{\partial Q_x}{\partial x} + q &= 0 \\ \frac{\partial M_x}{\partial x} - Q_x &= 0 \\ \frac{\partial H_{xy}}{\partial x} \cdot dx - Q_x \cdot dy &= 0 \end{aligned} \tag{3}$$

It will be seen that equation group (2) has been reduced to give a group of three equations in three unknowns and is therefore solvable.

In terms of the stresses:

$$M_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} z \cdot \sigma_x \cdot dz \; ; \quad Q_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \tau_{xy} \cdot dz \; ; \quad H_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} z \cdot \tau_{xy} \cdot dz \tag{4}$$

If the normal stresses are distributed linearly over the thickness of the plate, then the maximum bending and shear stresses at the surface of the plate are given by:

$$\sigma_x = \frac{6\,M_x}{t^2} \quad ; \quad \tau_{xy} = \frac{6\,H_{xy}}{t^2} \tag{5}$$

The strain energy for a plate of thickness t is given by:[9]

$$V = \frac{1}{2E}\iint_R\int_{-\frac{t}{2}}^{\frac{t}{2}}\left\{\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y + 2(1+\nu)\tau_{xy}^2\right\} dx.dy.dz \quad (6)$$

Substituting the expressions (5) and integrating we obtain:

$$V = \frac{1}{2}\iint_R \frac{1}{(1-\nu^2)D}\left\{M_x^2 + 2(1+\nu)H_{xy}^2\right\} dx.dy \quad (7)$$

Since, by the approximation, there is no variation of the moments or twisting couples with respect to y, the first integration may be carried out immeadiately by multiplying by the local chord c_x.

$$\begin{aligned} V &= \frac{1}{2}\int_0^x \frac{c_x}{(1-\nu^2)D}\left\{M_x^2 + 2(1+\nu)H_{xy}^2\right\} dx \\ &= \frac{1}{2}\int_0^x \left\{\frac{M_x^2 c_x}{(1-\nu^2)D} + \frac{2H_{xy}^2 c_x}{(1-\nu)D}\right\} dx \end{aligned} \quad (8)$$

Applying Castigliano's First Theorem, the deflection , at a point (x,y), is obtained from the first derivative of the strain energy with respect to a virtual load P, placed at (x,y).

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[9] S. Timoshenko, Theory of Plates and Shells. (New York: McGraw-Hill) p. 152.

$$\omega_{xy} = \left(\frac{\partial V}{\partial P}\right)_{P=0} = \frac{1}{D}\int_0^x \left\{ \frac{M_x \left(\frac{\partial M_x}{\partial P}\right)_{Cx}}{(1-\nu^2)} + \frac{2H_{xy}\left(\frac{\partial H_{xy}}{\partial P}\right)_{Cx}}{(1-\nu)} \right\} dx \qquad (9)$$

A number of examples for various loading cases will be worked, to show the general method of approach by use of equations (3) and (9). The results will be used for comparison with the experimental results referred to on page 1.

Example. Cantilever Rectangular Swept Plate with Uniform Shear Load along the Tip.

Figure 5. shows a virtual load P placed at a point with coordinates (x,y). For convenience in the handling of the algebra of the derivation, the coordinate y will be replaced by nc, the distance from the center line of the plate, measured positive in the positive direction of y. Then using the relations (3) of page 8:

$$\begin{aligned} Q_x &= Q \quad \text{for the range } x < \eta \leq s \\ &= Q + \frac{P}{c} \quad \text{for the range } 0 \leq \eta \leq x \\ M_x &= \int_\eta^x Q + \frac{P}{c}\,dx + \int_x^s Q\,dx \\ &= \frac{P}{c}(x-\eta) + Q(x-\eta) + Q(s-x) \\ &= \frac{P}{c}(x-\eta) + Q(s-\eta) \\ \frac{\partial M_x}{\partial P} &= \frac{(x-\eta)}{c} \end{aligned}$$

$$\frac{\partial H_{xy}}{\partial \eta} = Q_x \frac{dy}{dx} = Q_x \tan\theta$$

$$H_{xy} = \int_{\eta}^{x} \left(Q + \frac{P}{c}\right) \tan\theta \, dx + \int_{x}^{s} Q \tan\theta \, dx + \frac{P}{c} \cdot nc$$

$$= \frac{P}{c}(x-\eta)\tan\theta + Q(s-\eta)\tan\theta + \frac{P}{c} \cdot nc$$

$$\frac{\partial H_{xy}}{\partial P} = \frac{(x-\eta)\tan\theta + nc}{c}$$

Substituting in equation (9) and letting P = 0:

$$w_{x,nc} = \frac{1}{D} \int_0^x \left\{ \frac{Q(s-\eta)(x-\eta)}{(1-\nu^2)} + \frac{2Q(s-\eta)\tan\theta}{(1-\nu)} \Big[(x-\eta)\tan\theta + nc \Big] \right\} d\eta$$

Integrating and simplifying:

$$w_{x,nc} = \frac{Q}{D(1-\nu^2)} \left(\frac{sx^2}{2} - \frac{x^3}{6} \right) + \frac{2Q\tan\theta}{D(1-\nu)} \left[\left(\frac{sx^2}{2} - \frac{x^3}{6} \right) \tan\theta + ncsx - \frac{ncx^2}{2} \right]$$

and writing x = ms:

$$w_{ms,nc} = \frac{Qs^3}{D(1-\nu^2)} \left(\frac{m^2}{2} - \frac{m^3}{6} \right) + \frac{2Q\tan\theta s^2}{D(1-\nu)} \left[\left(\frac{m^2}{2} - \frac{m^3}{6} \right) s\tan\theta + mnc - \frac{m^2nc}{2} \right] \qquad (10)$$

Equation (10) gives the deflection at a point (ms, nc) on the plate loaded as specified, where m is the fraction of the semi-span s, outboard of the cantilever edge, and n is the fraction of the chord c from the center line of the plate.

Example. Cantilever Swept Plate with Uniformly Distributed Loading.

Figure 6. shows a virtual load P placed at a point with coordinates (x, nc) as in the previous example.

Using relations (3):

$$Q_x = q(s-x) \qquad \text{for the range } x \leqslant \eta \leqslant s$$
$$\quad = q(s-x) + \frac{P}{c} \qquad \text{for the range } 0 \leqslant \eta \leqslant x$$

$$M_x = \int_\eta^x q(s-x) + \frac{P}{c}\, dx + \int_x^s q(s-x)\, dx$$
$$= \frac{q}{2}(s-\eta)^2 + \frac{P}{c}(x-\eta)$$
$$\frac{\partial M_x}{\partial P} = \frac{(x-\eta)}{c}$$

$$H_{xy} = \int Q_x \tan\theta\, dx$$
$$= \int_\eta^x q(s-x)\tan\theta + \frac{P}{c}\, dx + \int_x^s q(s-x)\, dx + \frac{P}{c}.nc$$
$$= \tan\theta\left[\frac{q}{2}(s-\eta)^2 + \frac{P}{c}(x-\eta)\right] + \frac{P}{c}.nc$$
$$\frac{\partial H_{xy}}{\partial P} = \frac{(x-\eta)\tan\theta + nc}{c}$$

Substituting in equation (9):

$$w_{x,nc} = \frac{1}{D}\int_0^x \left\{\frac{q}{2}\frac{(s-\eta)^2(x-\eta)}{(1-\nu^2)} + q\frac{\tan\theta}{(1-\nu)}(s-\eta)^2\left[(x-\eta)\tan\theta + nc\right]\right\} d\eta$$

Integrating and simplifying as in the previous example:

$$w_{ms,nc} = \frac{q\, s^4}{2D(1-\nu^2)}\left(\frac{m^2}{2} - \frac{m^3}{3} + \frac{m^4}{12}\right) \tag{11}$$
$$+ \frac{q}{D}\frac{\tan\theta\, s^3}{(1-\nu)}\left[\left(\frac{m^2}{2} - \frac{m^3}{3} + \frac{m^4}{12}\right)s\tan\theta + nc\left(m - m^2 + \frac{m^3}{3}\right)\right]$$

Equation (11) gives the deflection at a point (ms, nc) on the plate loaded as specified, where m is the fraction of the semi-span s outboard of the cantilever edge and n is the fraction of the chord c from the center line of the plate.

IV COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS.

The Guggenheim Aeronautical Laboratories of the California Institute of Technology has tested a number of specimens of thin flat plates.[10] A rectangular plate, 1"thick 10" chord and 40" length, made of 24-st aluminum alloy was used for these experiments. It was swept at various angles from a cantilever edge and subjected to a number of different loadings.

Four sets of these results are used for comparison with the theory developed in part I. They are:

(a) 20 sweep, with a load of 600 lbs. uniformly distributed along the chord at 100% semi-span.

(b) 40 sweep, loaded as in (a).

(c) 20 sweep, load of 1200 lbs. uniformly distributed over the plate, 3 lbs. per square inch.

(d) 40 sweep, loaded as in (c).

Specimen calculations are given in Appendix II, and the theoretical curves together with the experimental

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[10] H. M. DeGroff, Air Force Technical Report 5761-3. See ref. 1.

results are plotted in figures 7, 8, 9 and 10. the comparison shows that there is fairly good agreement between the theory and the experimental results. In all cases the theory gives high deflection values on the trailing edge and low values on the leading edge. This may be accounted for, in part, by the fact that the theory does not allow for the chordwise variation of stress and the consequent warping of the section.

As would be expected, from the nature of the approximation, there is slightly better agreement between the theory and experiment for the smaller angles of sweep. The theory shows a slight negative deflection along the leading edge up to 20% in the case of the 20 swept plate, and up to 25% in the case of the 40 swept plate. The experimental results record only one point with a negative deflection at the leading edge.

The effect of the approximation has already been noted, and with respect to the negative deflection on the leading edge, it must be borne in mind that in addition the experimental cantilever edge is not of the type which the theory assumes, i.e. no provision is made for the linear variation of stress across the thickness of the plate.

V PART II THE LOCUS OF FLEXURAL CENTERS

The locus of flexural centers will be defined as the locus of all points p, such that when a vertical load of any magnitude (consistent with small deflections), is placed at that point, then the section of the plate, parallel to the root section and containing the point p, will deflect without twisting relative to the root section.

Three special cases of plate will be considered:

(a) Rectangular plate of constant thickness.

(b) Plate of constant thickness tapered in width.

(c) Plate tapered in thickness and width.

(a) Rectangular plate of constant thickness.

A point load P is placed at p with coordinates (x, nc) and a dummy torsional couple q is placed at the same section. The values of M_x and H_{xy} are the same as those worked out on page 10 with the exception that there is now a zero load at the tip and the dummy torsional couple $\frac{q}{c}$ must be added to the expression for H_{xy}.

This gives:

$$M_x = \frac{P}{c}(x-\eta)$$
$$H_{xy} = \frac{P}{c}\tan\theta\,(x-\eta) + \frac{P}{c}.nc + \frac{q}{c}$$

$$\frac{\partial M_x}{\partial q} = 0$$

$$\frac{\partial H_{xy}}{\partial q} = \frac{1}{c}$$

For the condition of no twist at the section (x, nc):

$$\left(\frac{\partial V}{\partial q}\right)_{q=0} = 0 = \frac{1}{D}\int_0^x \left\{ \frac{M_x \frac{\partial M_x}{\partial q}}{(1-\nu^2)} + \frac{2 H_{xy} \frac{\partial H_{xy}}{\partial q}}{(1-\nu)} \right\} d\eta$$

Substituting the values obtained:

$$0 = \frac{2P}{cD(1-\nu)}\int_0^x \left[\tan\theta(x-\eta) + nc\right] d\eta$$

Integrating and simplifying:

$$nc = -\frac{x}{2}\tan\theta \qquad (12)$$

Equation (12) is the locus of flexural centers in terms of n, the fraction of the chord from the center line of the plate, and x, the distance from the root of the swept plate. This equation represents a staight line, forward of the center line and falling mid-way between the normal to the cantilever edge and the center line.

<u>(b) Plate of constant thickness, tapered in width.</u>

Referring to figure 2.

$$\text{Taper ratio} = \frac{c_T}{c_R} = \lambda$$

$$\frac{\ell}{c_R} = \frac{\ell - s}{c_T} \ ; \quad \lambda = \frac{\ell - s}{\ell} \ ; \quad \ell = \frac{s}{1-\lambda}$$

Loading in the same manner as in case (a), the expressions for the bending moment and twisting moment are the same, with the exception that they are now averaged out over the local chord c_η. n is now the fraction of the root chord c_R.

$$H_{xy} = \tan\theta \frac{P}{c_\eta}(x-\eta) + \frac{q}{c_\eta} + \frac{P}{c_\eta} n c_R$$
$$M_x = \frac{P}{c_\eta}(x-\eta)$$

By the same condition for no twist at (x, nc):

$$0 = \frac{2}{D(1-\nu)} \int_0^x \left[\tan\theta \frac{P}{c_\eta}(x-\eta) + \frac{P}{c_\eta} n c_R\right] d\eta$$

$$c_\eta = \frac{\mu c_R}{\ell} \qquad \text{where } \mu + \eta = \ell$$

$$\therefore \ 0 = \int_\ell^{\ell - x} \left[\frac{\ell \tan\theta}{\mu c_R}(x + \mu - \ell) + \frac{\ell n c_R}{\mu c_R}\right] d\mu$$

Which simplifies to:

$$n c_R = - \frac{\left[(x-\ell)\ln\frac{\ell}{\ell - x} + x\right]}{\ln\frac{\ell}{\ell - x}} \tan\theta \qquad (13)$$

Equation (13) is the locus of flexural centers in terms of n, the fraction of the root chord c_R from the center line of the plate, and x the distance from the root of the swept plate.

(c) Plate tapered in thickness and in width.

In both of the foregoing cases, D, the flexural rigidity of the plate, has been a constant. Since D is proportional to the cube of the plate thickness, it follows that for a plate which is tapered in thickness in the same ratio as the width that:

$$\frac{D_\eta}{D_R} = \left(\frac{c_\eta}{c_R}\right)^3$$

D_R being the root rigidity and D_η being the local rigidity.

Then using the same loading and plan dimensions as were used in the last case, and applying the condition for zero twist at (x, nc):

$$0 = \frac{2P}{(1-\nu)}\int_0^x \frac{1}{D_\eta}\left[\frac{\tan\theta}{c_\eta}(x-\eta) + \frac{nc_R}{c_\eta}\right]d\eta$$
$$= \frac{2Pc_R^3}{(1-\nu)D_R}\int_0^x \frac{1}{c_\eta^3}\left[\frac{\tan\theta\,(x-\eta) + nc_R}{c_\eta}\right]d\eta$$

Making the same substitution as in the last case:

$$0 = \frac{2Pc_R^3}{(1-\nu)D_R c_R^4}\int_e^{\ell-x}\frac{\ell^4}{\mu^4}\left[\tan\theta(x+\mu-\ell) + nc_R\right]d\mu$$

which simplifies to:

$$nc_R = -\frac{(5\ell x^2 - 3\ell^2 x - 2x^3)}{2(3\ell x - 3\ell^2 - x^2)}\tan\theta \qquad (14)$$

Equation (14) is the locus of flexural centers for case (c) in terms of n, the fraction of the root chord c , from the center line of the plate, and x, the distance from the root of the swept plate.

It is noted in cases (b) and (c) that when $\lambda \rightarrow 1$ i.e. a rectangular wing, that $nc_R = -\frac{x}{2} \tan \theta$. This agrees with the result of case (a).

Curves of the non-dimensional factor $-\frac{nc_R}{s} c^{4} \tan\theta$ are plotted in figures 11. and 12. Figure 11. is for the case of plates of constant thickness and figure 12. is for plates tapered in thickness. In both families of curves, the value $\lambda = 1$ gives a straight line.

At present there is no experimental data which could be used to check the results of this section.

CONCLUSION

This paper has presented a simplified analysis for the deflections of swept plates, and a number of conclusions are to be drawn from the results and comparison with experimental data. In the first place, the comparison with the experiments carried out at the California Institute of Technology is fairly good. The agreement obtained is as close as that obtained by Yuan-Chen Fung[11] using a theory involving computations with matrix algebra. The specimen calculations of Appendix I demonstrate the relative simplicity of the computations necessary when the equations developed in this paper are used.

There is no apparent application for the locus of flexural centers since a load placed on it will cause no twist at one section only. However, the location of this line, and a full understanding of its meaning, enables a rapid visualization of the resistance of a given swept plate to changes in angle of attack under load. The

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[11] Yuan-Chen Fung, "Stress and Deflection Analysis of Swept Plates". Air Force Technical Report No. 5761-2, February 1950.

greater the divergence of the locus of flexural centers from the line of loading, the greater will be the twist of the plate compared to its bending deflections.

Due to the nature of the approximations introduced, it is considered that the method developed in this paper would be especially useful in obtaining the deflections of wings in which the chordwise sections are kept rigid by the action of wing ribs.

Finally, it should be stated that there is a need for considerable experimentation on swept wing models of all types before a completely reliable theoretical or empirical method of analysis can be made for swept structures under load.

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APPENDIX I

SPECIMEN CALCULATIONS

The following calculations are for the deflection pattern of the 20 swept plate, loaded with 600 lbs. across the tip, used by the Guggenheim Aeronautical Laboratories of the California Institute of Technology. The main dimensions of the plate were:

Length, measured along the center line....... 40"

Chord, at right angles to the center line.... 10"

Thickness .... 1"

Material - 24 ST, aluminum alloy.

$E = 10.4 \times 10^6$ lbs. per square inch.

Poisson's Ratio = 0.32

In terms of the coordinate system used in this paper:

$$s = 40 \times \cos 20^\circ = 40 \times .9397 = 37.588''$$

$$c = 10 \times \sec 20^\circ = 20 \times 1.064 = 10.640''$$

$$D = \frac{Et^3}{12(1-\nu^2)} = \frac{10.4 \times 10^6 \times 1^3}{12(1-.103)} = .988 \times 10^6$$

$$Q = \frac{P}{c} = \frac{600.0}{10.64}$$

Using equation (10), page 11:

$$\frac{Qs^3}{D(1-\nu^2)} = \frac{600 \times (37.56)^3}{10.64 \times .988 \times 10^6 \times .680} = 3.38$$

$$\frac{2Q\tan\Theta s^2}{D(1-\nu)} \times \frac{2 \times 600 \times (37.56)^2 \times .364}{10.64 \times .988 \times 10^6 \times .680} = .0864$$

Substituting in equation (10), page 11:

$$\omega = 3.38\left(\frac{m^2}{2}-\frac{m^3}{6}\right) + .0864\left[\left(\frac{m^2}{2}-\frac{m^3}{6}\right)37.59 \times .364 + 10.64n\left(m-\frac{m^2}{2}\right)\right]$$

Now calling the part which is dependent on n, w_n and the remainder w_{50}, so that $w_{50} + w_n = w$, then:

$$w_{50} = 2.281m^2 - .761m^3$$

$$w_n = \pm(.460m - .230m^2)$$

the sign of which depends on whether the value is required at the trailing or leading edge, i.e. $n = \pm\frac{1}{2}$

EVALUATION OF w_{50}

m	m^2	m^3	$2.281m^2$	$.761m^3$	w_{50}
.1	.01	.001	.028	-	.028
.2	.04	.008	.091	.006	.085
.4	.16	.064	.365	.049	.316
.6	.36	.216	.821	.164	.657
.8	.64	.512	1.460	.390	1.070
1.0	1.00	1.000	2.281	.761	1.520

EVALUATION OF w_n, $w_{T.E.}$ and $w_{L.E.}$

m	.460m	$.230m^2$	w_n	$w_{T.E.}$	$w_{L.E.}$
.1	.046	.002	.044	.067	-.021
.2	.092	.009	.083	.168	.002
.4	.184	.037	.147	.463	.169
.6	.276	.083	.193	.850	.464
.8	.368	.147	.221	1.291	.849
1.0	.460	.230	.230	1.750	1.290

APPENDIX II

Figures

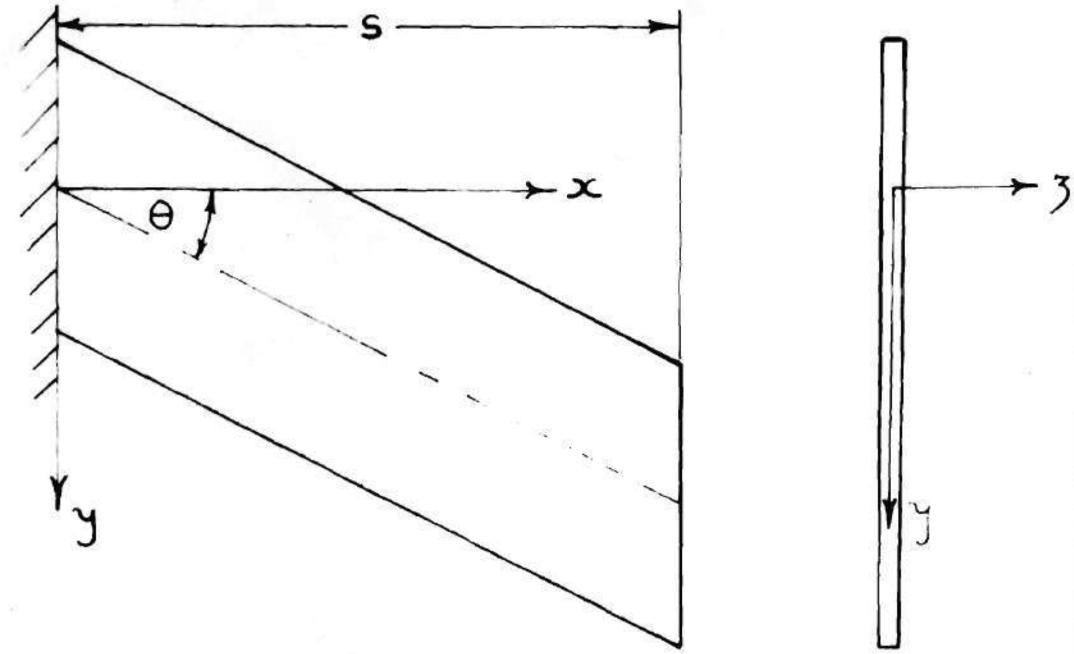

FIG. 1 COORDINATE SYSTEM OF THE RECTANGULAR SWEPT PLATE.

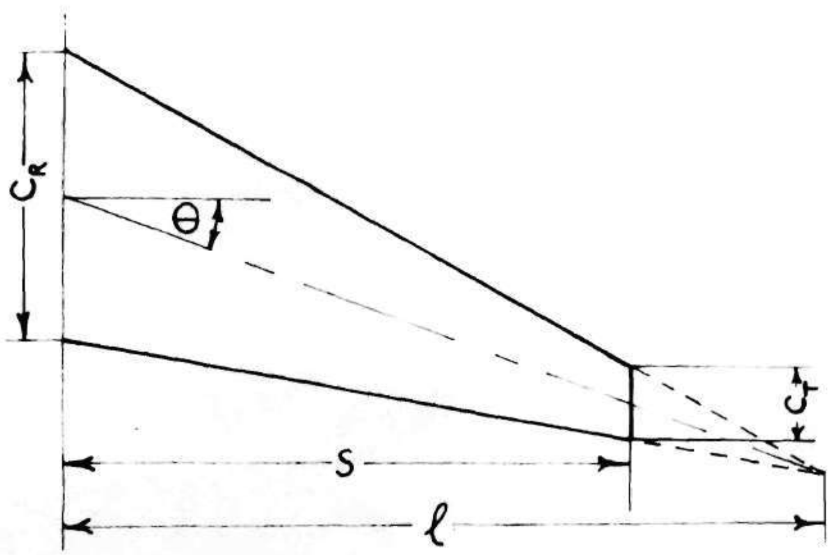

FIG. 2 DIMENSIONS OF THE TAPERED PLATE.

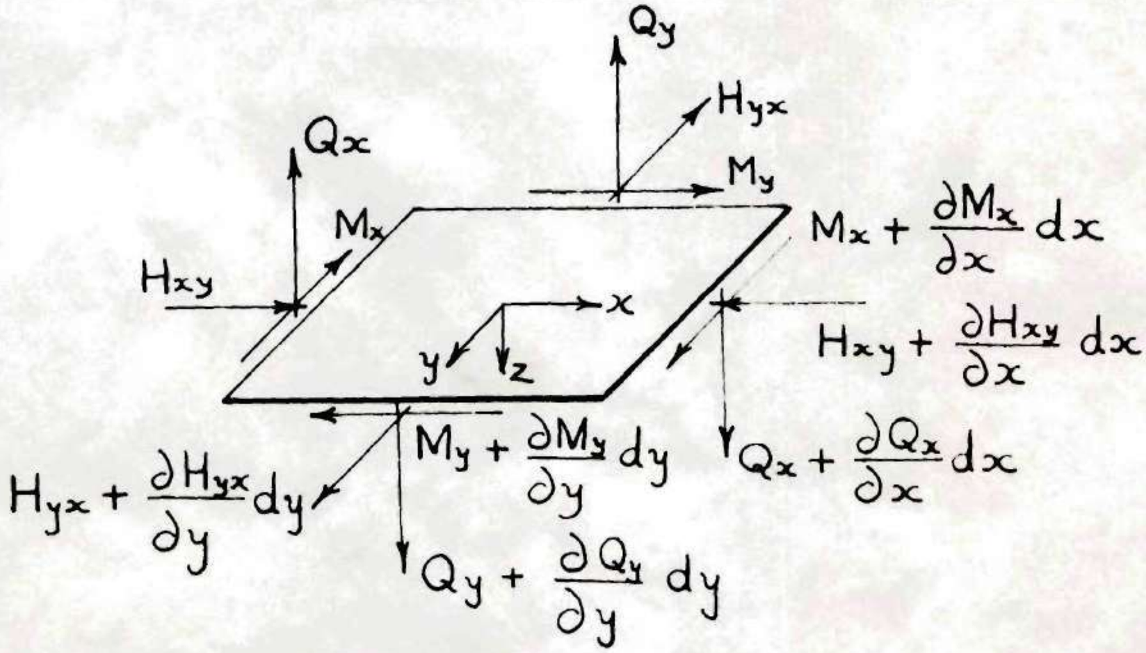

FIG. 3 CONVENTIONAL FLAT PLATE ELEMENT.

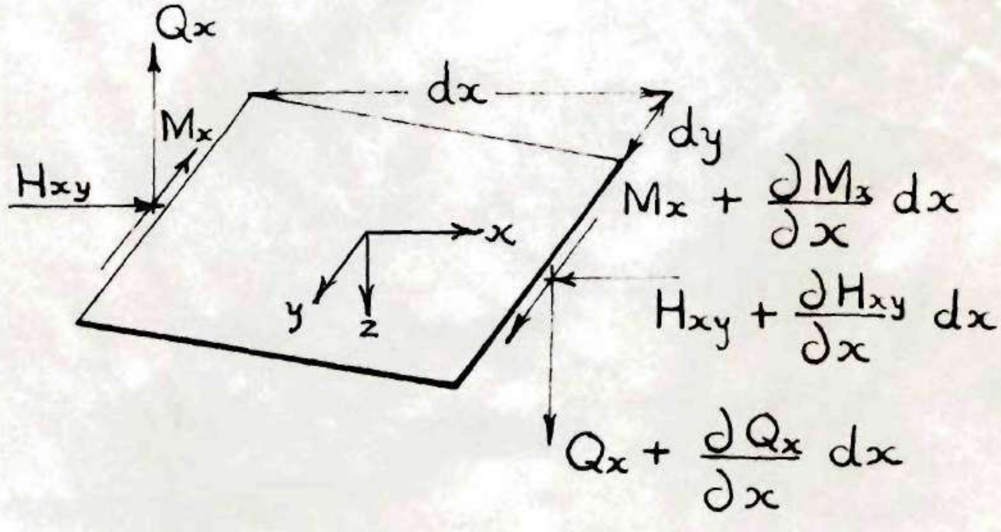

FIG. 4 MODIFIED FLAT PLATE ELEMENT.

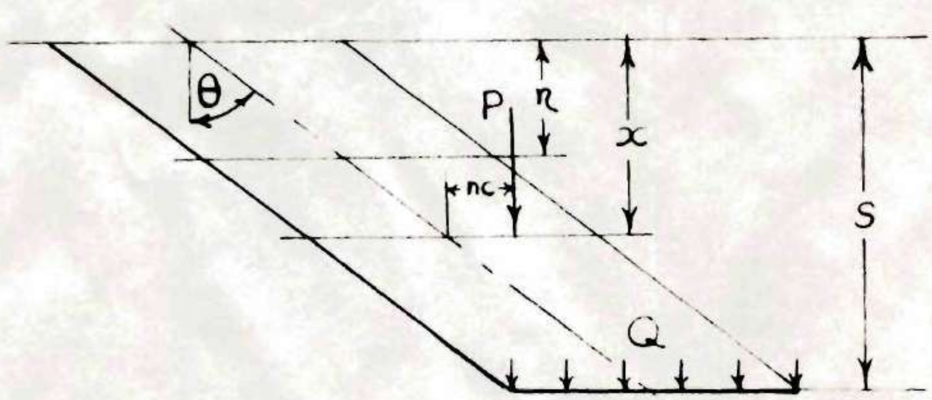

FIG. 5 CANTILEVER PLATE UNDER TIP LOAD Q.

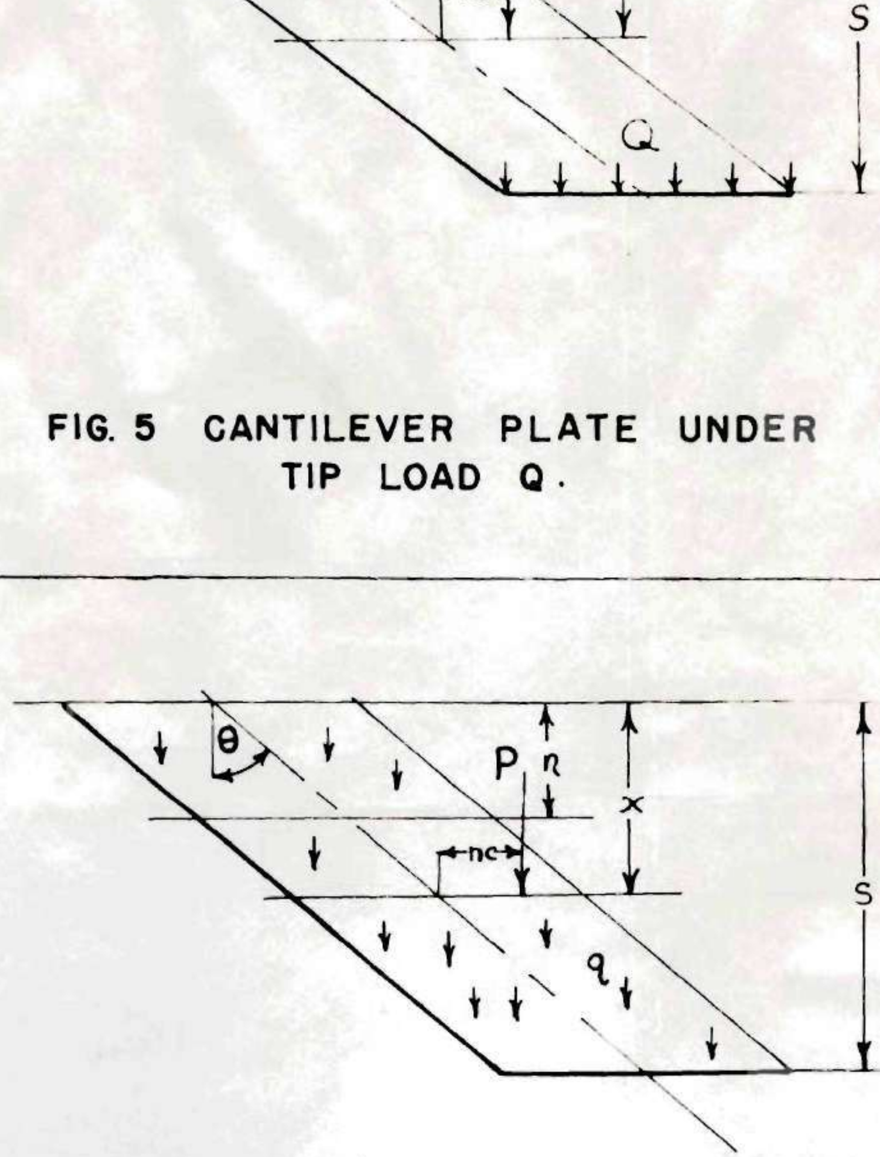

FIG. 6 CANTILEVER PLATE UNDER DISTRIBUTED LOAD q.

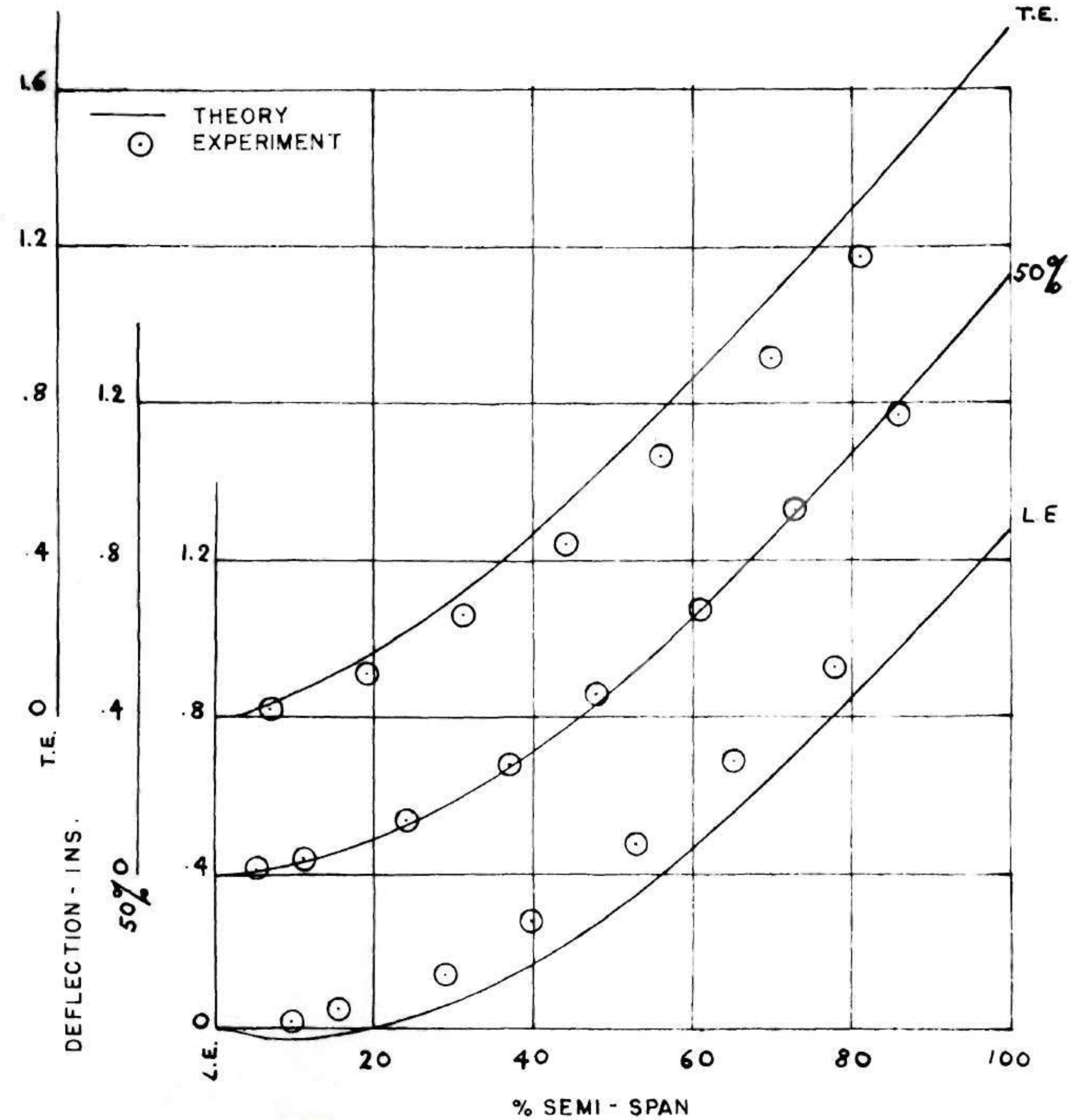

FIG. 7 DEFLECTION OF SWEPT PLATE
($\theta = 20°$, LOAD OF 600 LBS. DISTRIBUTED ALONG THE TIP)

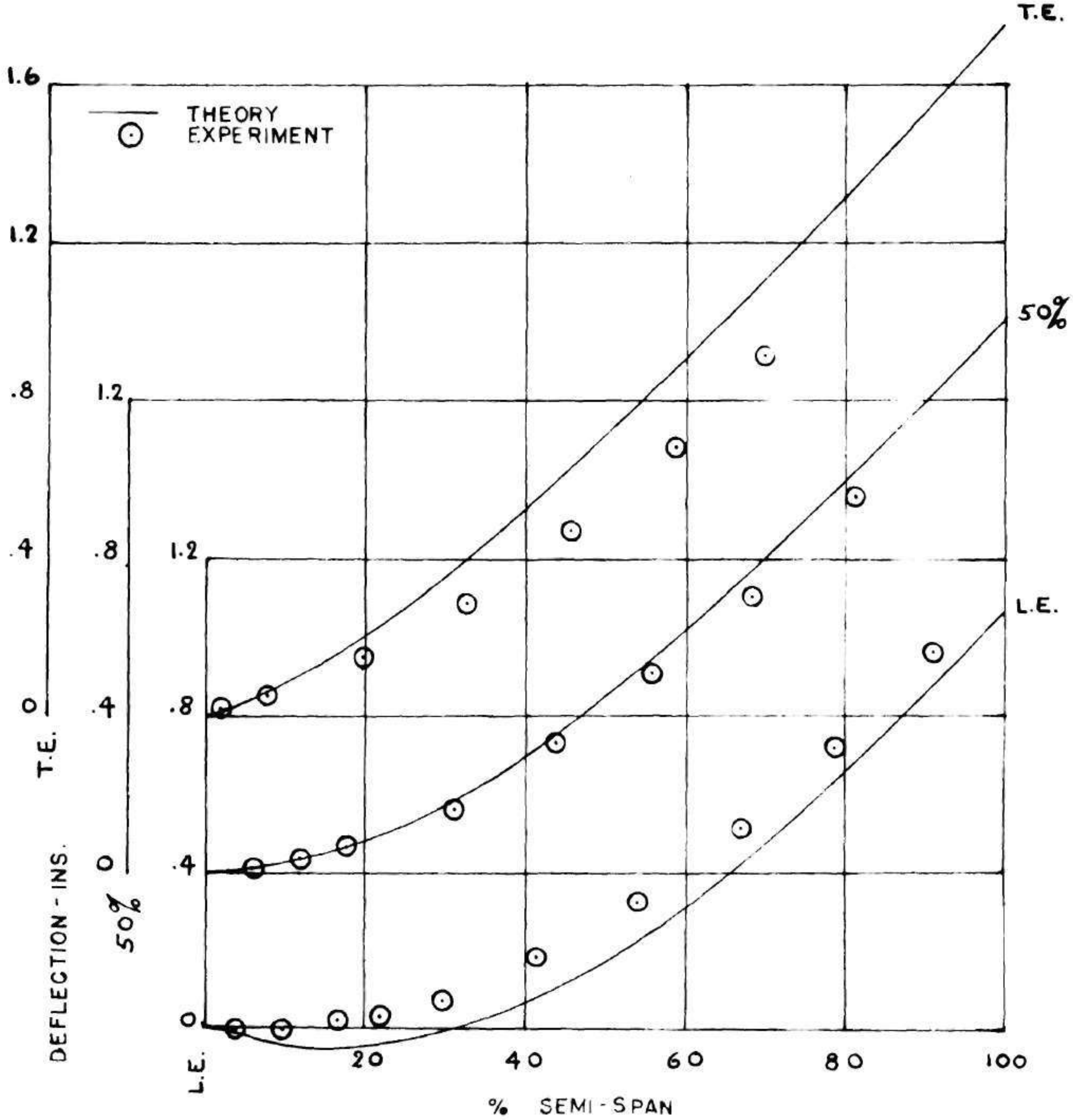

FIG. 8 DEFLECTION OF SWEPT PLATE.
(θ = 40°, LOAD OF 600 LBS. DISTRIBUTED ALONG TIP.)

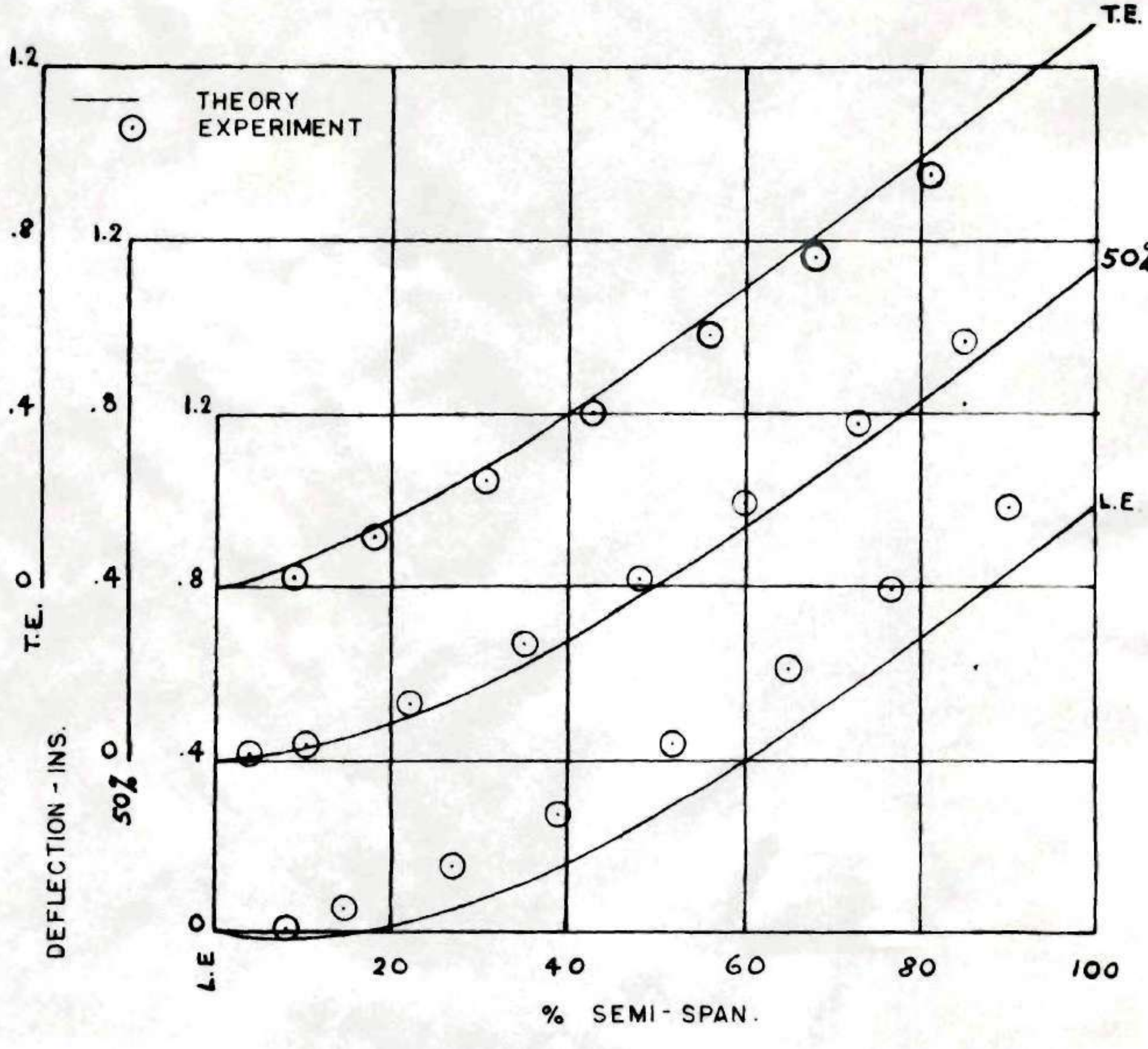

FIG. 9 DEFLECTION OF SWEPT PLATE.
(θ = 20°, LOAD OF 1,200 LBS. DISTRIBUTED 3 LBS/SQU. IN.)

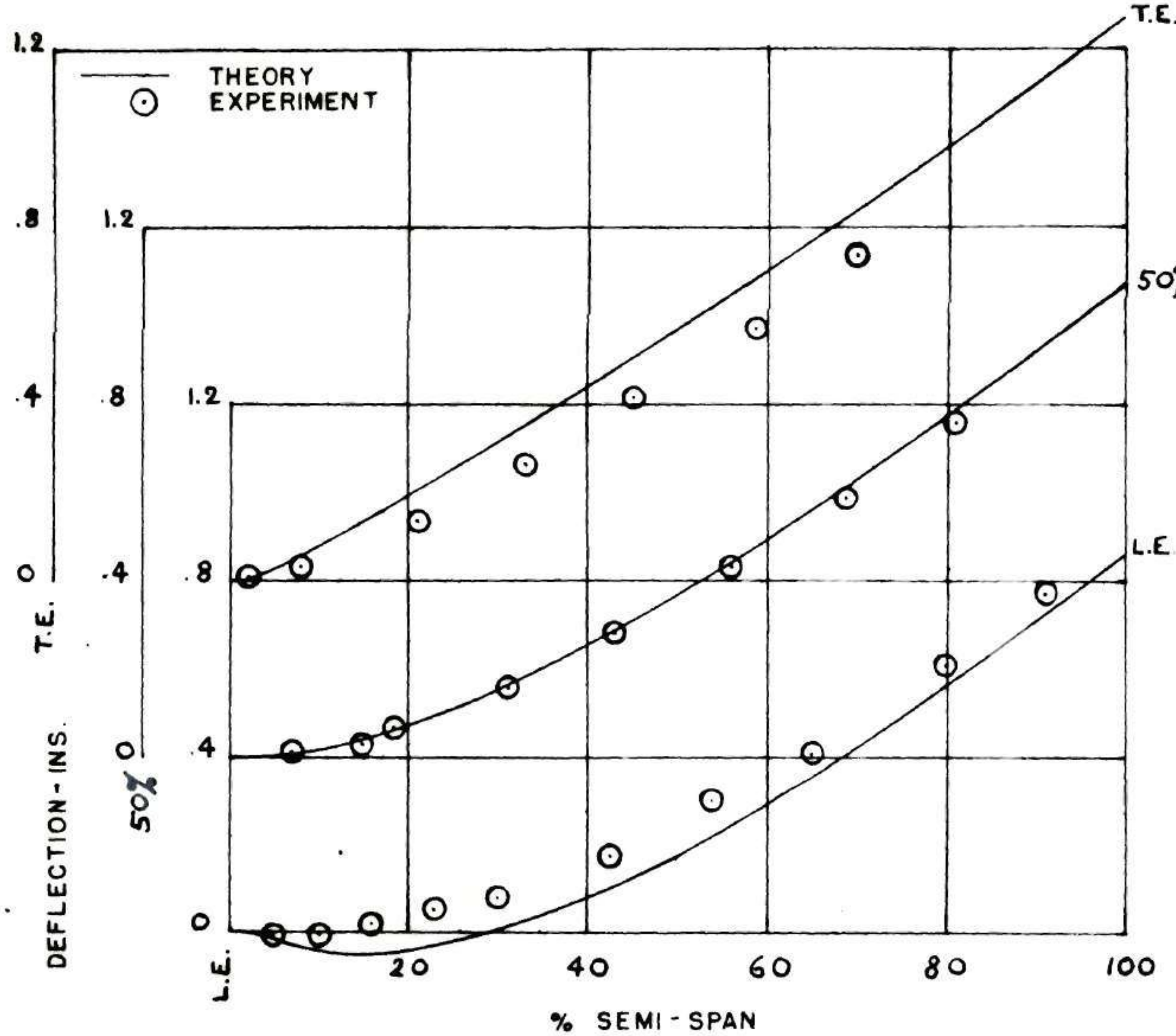

FIG. 10 DEFLECTION OF SWEPT PLATE.
(θ = 40°, LOAD OF 1,200 LBS. DISTRIBUTED 3 LBS/SQU. IN.)

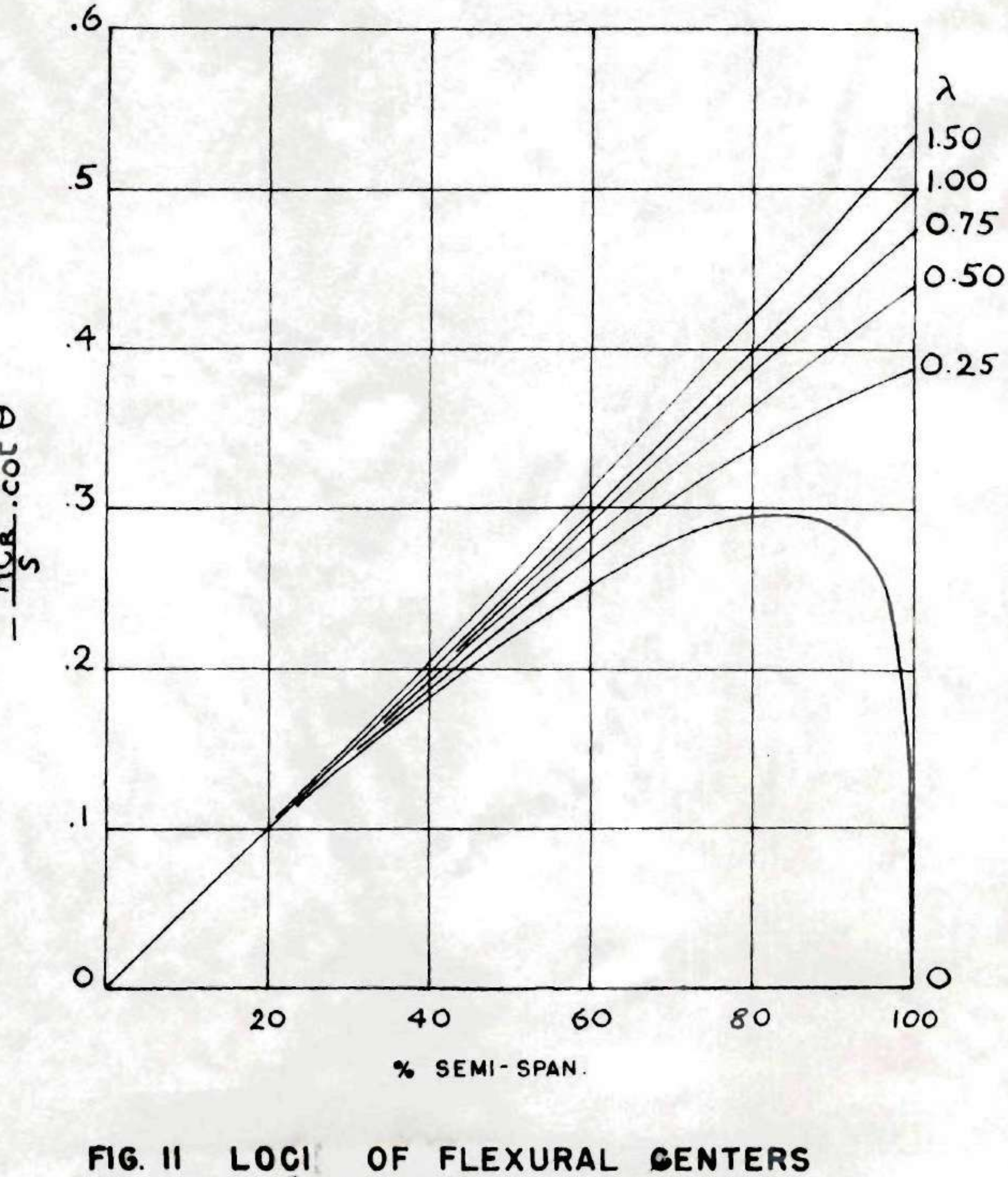

FIG. II LOCI OF FLEXURAL CENTERS FOR PLATES OF CONSTANT THICKNESS.

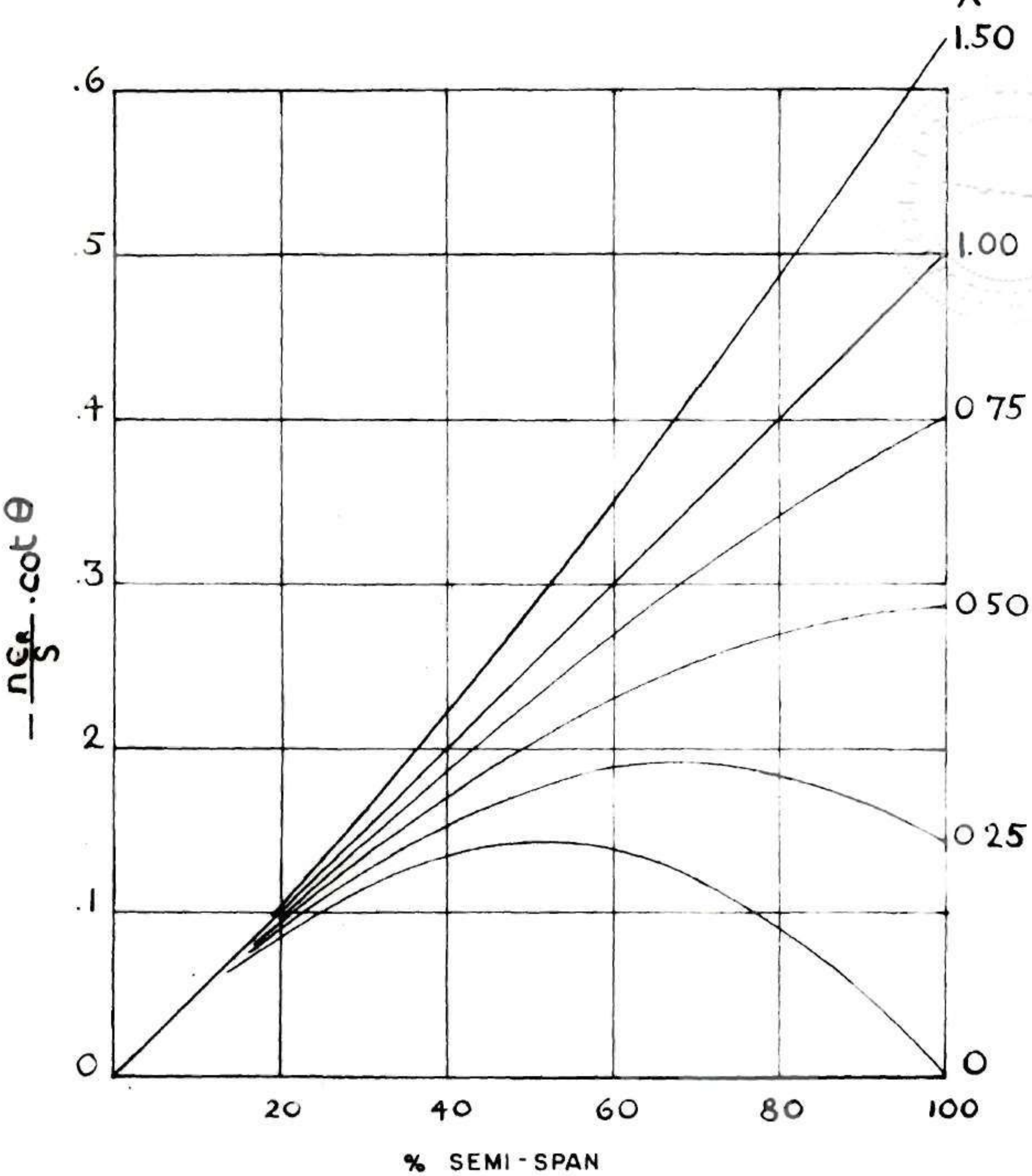

FIG. 12 LOCI OF FLEXURAL CENTERS FOR PLATES TAPERED IN THICKNESS AND WIDTH.