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A MATHEMATICAL FORMULATION FOR LOCATING PULPWOOD AND BULK PAPER MILLS

A THESIS

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SUMMARY

This thesis presents a method for selecting pulpwood and paper mill locations which minimizes total product cost* under a set of forest resource and market demand constraints. Although both economic and social factors enter into this location decision, only economic factors are included in the mathematical model.

The three main parts of the thesis are (1) defining the factors pertinent to the location decision, (2) mathematically modeling these factors, and (3) deriving a solution technique for solving the model. Defining the factors provides the groundwork for formulating the model. The resulting model is a mixed zero-one integer programming formulation which includes both fixed and variable operating costs. Benders' partitioning and a modified integer programming procedure are presented as a solution technique. The research concludes with an example application of the model and solution technique.

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*Total product cost refers to financial expenditures for stumpage; harvesting; loading; transporting from forest to mill to market; fixed cost such as plant construction, equipment, and taxes; and variable expenses such as labor, power, and water.

CHAPTER I

INTRODUCTION

Statement of the Problem

This thesis presents a method for selecting pulpwood and paper mill locations which minimizes total product cost* under a set of forest resource and market demand constraints. Many economic and social factors enter into such location decisions; however, economic factors are of primary interest in this research.

The economic aspect of the location problem is one significant reason for its importance. A $40,000,000 (1) initial fixed cost investment in a mill plus an approximate annual transportation cost of $1,000,000 emphasizes the importance of the location decision. Because there are a variety of economic factors to be considered, an analysis raises the question of which costs are fixed and which vary with production. For example, fixed costs may include plant construction, machinery, and taxes. Variable costs, on the other hand, may include such factors as transportation, labor, pulpwood resources, water, and power.

The interrelationships of factors and their significance in the

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*Total product cost refers to financial expenditures for stumpage; harvesting; loading; transporting from forest to mill to market; fixed cost such as plant construction, equipment, and taxes; and variable expenses such as labor, power, and water.

total decision is the next question to be considered. For example, while water and power are two required factors, they are of less economic importance than labor and transportation. It should be noted that many of the factors are strictly dependent on the particular location considered. For example, water, power and labor resources, and transportation facilities are tied more to a particular site than factors such as the pulpwood resources or the market location.

Social as well as economic factors require consideration in the mill location decision. The image which a company develops within the community through its public relations sets the stage for future growth and expansion. Some economic factors are independent of any social effects while others are strictly dependent on the social environment. More specifically, the costs of stumpage, loading, and transportation of the raw materials are usually independent of community or city acceptance since these functions are performed by organizations external to the immediate community or city. However, there are many economic factors which are directly affected by the community's attitudes toward the company. These attitudes may be reflected in such economic factors as (1) cost and availability of labor, (2) local and state taxation and taxation policies, (3) local or state subsidization, (4) costs for water and power, and (5) costs of effluent disposal. These factors alone can determine how successfully a plant operates.

Objective

The objective of this research is to develop a method for analyzing potential pulp and paper mill locations. This objective will be achieved by (1) defining the factors pertinent to the location decision, (2) mathematically modeling these factors, and (3) presenting a solution technique for the model. Meeting this objective should provide the means for making a more comprehensive analysis of potential plant sites.

Defining the factors pertinent to analyzing mill locations is the first step. As the initial step, it provides the groundwork for formulating the mathematical model. A list of important factors provides a checklist to prevent an oversight of significant costs which should be considered. In this research, the economic factors are mathematically related so that an optimum feasible solution can be obtained.

There are advantages of mathematically modeling the problem. First, a mathematical model depicts the problem and reduces its complexity to a workable form. Second, the model provides a means for considering the interrelationships of the factors. Third, a mathematical model can be used as a tool to produce valuable quantitative information which should give the analyst additional insight into the problem.

The final step to develop a method for analyzing pulp and paper mill locations is the presentation of a workable solution technique. In solving mathematical models, it should be recognized that the solution technique used (1) limits the size of the problem that can be solved, (2) determines the time required for a solution, and (3) determines the optimum state of the solution which can be achieved through the model. These three reasons underlie the importance of investigating an appropriate solution technique.

Review of the General Location Problem

The general location problem consists of selecting facility location sites to minimize total cost, subject to a given set of supply and demand constraints. Fixed costs are usually associated with each site if a facility is opened and variable costs are associated with its operations.

This problem arises in many contexts and may be formulated in many ways and solved by different techniques. The location problem is normally formulated as a mixed integer programming problem. Solution procedures include such methods as integer programming, linear programming, and mathematical simulation.

Baumol and Wolfe (2) formulate the warehouse location problem based on strictly concave cost functions with a fixed initial cost. This problem considers the location of the plant as known and fixed with known customer demands, factory to warehouse distances and costs, and warehouse to customer distances and costs. The formulation is similar to the one presented in that both models have (1) capacity limits on the facilities to be located, (2) limitations on the resource supply, and (3) constraints specifying that all customer demands must be met. On the other hand, their formulation differs in that (1) fixed costs associated with opening a warehouse are not explicitly considered in generating solutions, and (2) all production is required to be transported.

The plant location model formulated by M. L. Balinski (3) and the one presented in this paper are mixed integer programming models. Balinski's formulation differs in that (1) plant and resource capacities are not considered, and (2) the components of market demand constraints are expressed as fractions of a particular market demand rather than in units of product. However, both mixed integer programming models use Benders' partitioning scheme in their solution.

The mixed integer problem formulated by Manne (4) considers economies of scale in manufacturing. Both his formulation and the one appearing in this paper are mixed integer formulations. Furthermore both formulations consider manufacturing and finished product transportation costs. There are also differences in formulation. Manne considers economies of scale in manufacturing whereas the model formulated in this paper utilizes typical pulp and paper mill capacities. However, a graph depicting the economy of scale can be derived by varying plant capacities given a set of supply and demand constraints. Secondly, Manne omits cost variations of raw materials while the model herein includes variations in transportation cost as well as raw material cost and supply. Thirdly, Manne omits capacity limitations of existing facilities which are included in this model.

Feldman, Lehrer, and Ray have developed a heuristic approach to a warehouse location problem (5). In their formulation, they allow the economies of scale to affect warehousing costs over the entire range of warehouse sizes. Their approach considers only flow of product from the warehouse to the market. In structure and economic considerations, it differs to a great extent from the model presented in this thesis. Whereas Feldman, et al. consider only cost elements related to warehousing and transportation of finished product, the model presented herein includes cost associated with resources, production, and transportation. The purpose of their approach is to extend the Kuehn-Hamburger (6) results to the case in which the warehousing cost function is concave rather than linear plus a fixed cost. Feldman et al. point out that the basic difference between the linear and concave cases is in the assignment of customers to warehouses that have been opened. As an advantage, they claim that their formulation allows one to deal with a different concave warehousing cost function for each potential warehouse.

In "A Branch-Bound Algorithm for Plant Location," Efroymson and Ray (7) apply the branch-bound technique developed by Land and Doig (8) to Balinski's formulation of the plant location problem. They make minor revisions in Balinski's formulation in order to more easily solve the numerous linear programming problems. The main advantage of their formulation, therefore, is that it reduces the time required to evaluate the nodes in the branch-bound technique.

Spielberg (9) has considered a plant location problem which he solves using a direct-search technique. Similarities to the approach presented in this thesis include (1) consideration of one commodity plants, (2) consideration of fixed and variable costs, and (3) the sum of capacities of potential plants must be greater than or equal to total product demand. Points of dissimilarity include (1) the objective function consisting of fixed cost plus a piecewise linear concave function, (2) quantities transported being expressed as fractions of plant demand, and (3) the omission of purchasing costs, transportation costs, and the consideration of the availability of raw materials. Other differences also arise between the models due to the characteristics of the pulp and paper industry.

Santone and Berlin (10) have developed a simulation model for evaluating existing and proposed fire station locations. Mathematically, the model is structured around a shortest-path algorithm that calculates response time and a weighting function which numerically measures the fire hazard of each structure in a delineated area. After numerically describing the factors using utility values, which were subjectively determined, a network diagram depicting the problem is constructed to evaluate the potential locations.

Review of the Plant Location Problem for the Pulp and Paper Industry

There is only a small quantity of work published on the plant location problem for pulpwood and bulk paper mills. Most of the work in mathematical modeling deals only with transportation costs. Other work has been done on pulp resources and its effects on mill location. The density of pulpwood forests, location of pulpwood, and pulpwood resources in relation to the present pulp mill locations have been studied, for example, by Arias (11).

Mathematical programming applied to pulpwood and paper mill location is scarce. There was no mathematical formulation found which attempted to consider more than transportation costs. For example, Bacon utilized linear programming to consider only railroad transportation costs from woodyard to the mill (12). Also, Dix (13) has developed a mathematical model for determining minimum transportation cost from the forest resource areas to a plant location.

In general, little attention has been given to non-economic location factors in the pulp and paper industry. However, work has been done by Ullman (14) who notes the increasing importance of personal consideration in the location choice. He discusses climate and amenities as attractions to industrialists and the labor they employ. There is increasing evidence that these environmental factors are becoming major locational determinants (15).

Scope and Limitations

This research deals with developing a method for determining the optimum location of pulp and paper mills. The analysis can also be used to evaluate the economic factors of improving or expanding existing facilities. Although the economic factors of locating facilities is of primary concern in this study, some social factors are presented.

Since characteristics of certain factors, such as methods of financing or effluent disposal vary from location to location, it is necessary to omit such detailed information from the analysis. Therefore, it is virtually impossible to present all questions or bits of information relating to each factor.

The mathematical model developed is comprehensive insofar as it takes into account most economic factors encountered in setting up and operating a pulp or paper mill. The inclusion of fixed and variable costs make it possible for the analyst to consider most expenses included in the total cost of the product. Locational factors may be deleted from or added to the model with minimum effort required. Although the model is designed for locating pulp and paper mills, it may be modified to apply to location problems in general.

CHAPTER II

PROCEDURE

Description of the Approach

This study approaches the problem of locating pulp and paper mills through the formulation of a mixed integer program. Although intangible factors such as recreational and educational facilities and community attitudes were not included in this model, their importance is not denied. The mill location decision should be based upon the optimum solution of the model as well as these intangible factors. A brief summary of the factors considered is presented in this chapter with a more detailed outline analysis given in Appendix A.

The model is designed to accommodate economic changes which occur in the industry. Future market or product cost trends may be considered by manipulating coefficients of the defined variables. Constraint equations may also be added to or deleted from the model to better define a particular problem.

Discussion of the Factors

This section of the study defines and discusses factors pertinent to the location of pulp and paper mills. The discussion of each factor is on a general level and does not attempt to present a detailed factor analysis. Following is a presentation of pertinent locational factors.

Water

Approximately 38,000 gallons of water per ton of finished product are required in the average pulp or paper mill (16). This water is of two standards. The higher grade of water required is potable or drinkable water while mill water is that which is used only in the manufacturing process of pulp and paper.

The purity of the potable water is analyzed by local or state health authorities using criteria published by the American Public Health Association. Mill water tests, however, are for a different purpose. These tests are run to determine the following characteristics of the water: turbidity, color, solids content, hydrogen ion concentration, alkalinity, acidity, dissolved oxygen and biochemical oxygen demand, oil content, and hardness. Detailed test procedures for these characteristics are published by the Standards Committee of the Technical Association of the Pulp and Paper Industry (17).

Utilities

There are several utilities which need to be considered as sources of power. Included are electricity, steam, gas, coal, oil, and wood bark (18). Availability, dependability, purchase agreements, and cost per unit are important considerations to the pulp or paper mill locational decision.

Effluent Disposal

The problem of plant wastes is of particular importance to the analyst. He must consider the community's health as well as the company's image. Local, state, and federal laws pertaining to the disposal of wastes must be adhered to. The analyst should examine possible connections with a municiple sewage system and investigate various means of filtering wastes to prevent water and air pollution.

The Availability of Raw Materials

As noted in the literature search, the cost and availability of raw materials are essential factors in selecting the mill site. The analyst must consider the resource availability, the type of pulpwood, and especially the cost of pulpwood.

Transportation

Raw materials must enter the mill and finished products must be exported. This may be achieved economically through a variety of transportation systems. The availability of railroads, water carriers, highway vehicles, pipelines (19), and aircraft should be investigated.

Marketing

Marketing a product is usually very significant in the successful operation of most production facilities. Existing and projected supplies must be analyzed to determine the product demand. Various questions must also be answered concerning the proposed mill's competitive position.

Financing

Financing the construction costs of a given mill is usually one of the first steps in the decision to invest in a production plant. An analysis of this factor, therefore, occurs in the first planning stages when potential sites are selected. The type of financing, either internal (financing from a source within the company) or external (financing from a source outside the company), should normally be decided before the selection of potential locations. Because external financing may be available only to mills in a given region, potential locations may be restricted to that region. However, this restriction may not exist if the financing is internal.

Taxation

The influence of taxes as a factor affecting pulp and paper mill location ranges from 1 to 2 per cent of the operating costs (20). This cost becomes a fixed cost of operation regardless of the rate of output. The interstate cost differential is usually negligible, whereas the intrastate differential is often significant. Therefore, the tax consideration is essentially a regional problem (see Appendix B).

Labor

Knowledge of the socio-economic conditions in the surrounding community is critical in evaluating a plant site. The cost of labor in the production of pulp and paper is a significant portion of the total product cost. This cost may be considered to vary directly with production when operating within the designed limits of the mill. The output per production man-hour has shown an average annual increase of 3.7 per cent from 1947 to 1960 due to greater mechanization in the industry (21).

Community or Living Environments

The community environment for the plant personnel is becoming an increasingly important factor. Due to the intangible nature of an "environment," it is difficult to measure its importance. The interrelationship between this factor and the labor factor is significant.

A favorable environment can be a strong attraction for recruiting competent employees. Such items as the following should be considered in the evaluation of a community: population, population trends, civic advantages, schools, domestic housing, climate, and the cost of living.

CHAPTER III

DEVELOPMENT OF THE MODEL

General Description

A mixed zero-one integer model is developed to define quantitatively those economic factors which should affect the location decision for pulp and paper mills. The objective of the mathematical model is to minimize total product cost subject to resource and requirement constraints.

Assumptions in the Model

Specific assumptions underlie the formulation and solution of the mixed integer model. The first three assumptions which follow are concerned with the entire problem; the remaining five deal with the factors included in the model.

1. A potential supply of pulpwood is equal to or greater than the demand for an equivalent amount of product in a delineated area.

2. A subset of production facilities can produce an amount equal to or greater than the calculated demand.

3. The economic factors can be effectively quantified and integrated into a mixed integer programming model.

4. The location analyst determines the period of time considered by the model. He can set the model up, for example, to calculate monthly or annual total product costs.

5. The location analyst can determine a minimum cost coefficient for variables in the objective function. For example, if a forest resource can be transported to the mill by either rail or truck, the analyst would determine the means of minimum cost and calculate the respective coefficients.

6. Each market can be regarded as a discrete point. This is usually the case for the market of bulk pulp and paper products where the market consists primarily of converting or finishing plants.

7. Sufficient manpower can be obtained to manufacture the product demand.

8. All variable operating expenses, including labor, are linear and can be expressed in dollars per ton of output.

Formulation of the Model

The model is composed of five basic sets of constraints plus an objective function. The five constraint sets are (1) forest resource, (2) plant capacity, (3) market demand, (4) resource input-product output, and (5) softwood-hardwood input distribution. These sets describe the flow of materials through the network. The objective function describes the cost of the product flow through the network.

The definitions for the variables used in the model formulation are given below:

Definition of Variables

S_{ij} - cords of softwood transported from forest "i" to mill "j."

H_{ij} - cords of hardwood transported from forest "i" to mill "j."

P_{jk} - tons of finished product transported from mill "j" to market "k."

s - number of softwood forest resources.

h - number of hardwood forest resources.

p - number of mill sites to be considered.

m - number of markets to be served.

R_i - maximum supply of softwood (measured in cords) transported from forest "i" in a given period of time.

T_i - maximum supply of hardwood (measured in cords) transported from forest "i" in a given period of time.

Y_j - plant capacity in tons of finished product of mill "j."

D_k - demand of market "k" in tons of finished product.

E_j - tons of finished product per cord of softwood in mill "j."

G_j - tons of finished product per cord of hardwood in mill "j."

K_j - per cent of hardwood in total pulpwood resource for plant "j" (measured in tons of finished product).

X_j - zero or one variable indicating that mill "j" will not or will be built at location "j."

F_j - fixed cost required to set up mill "j."

a_{ij} - a summation of stumpage, harvesting, loading, and transportation cost per cord of softwood from forest "i" to mill "j."

b_{ij} - a summation of stumpage, harvesting, loading, and transportation cost per cord of hardwood from forest "i" to mill "j."

c_{jk} - variable operating expenses (including labor) plus transportation cost per ton of product produced in mill "j" and transported to market "k."

Forest Resource Constraints

Wood required for the manufacturing process is supplied by both softwood and hardwood forests. However, it is common to find both soft and hard woods mixed in a given forest area. This mixture may range from predominantly softwood to predominantly hardwood. A distinction is made between the two classes of wood because each has its own cost and demand. Specifically, hardwood yields more tons of output per dollar cost than softwood (22). This is primarily because hardwood has a lower stumpage cost and a greater weight per unit volume. However, due to the characteristics of hardwood fibers, its use is normally limited to a maximum of 15 per cent of the total wood used.

The basic relationship for each forest and its output states that the summation of all resources shipped from each forest is less than or equal to the amount of resources which that forest can produce during a given time period. The typical equations for softwood and hardwood forests are given below:

Softwood:

$$\sum_{j=1}^{p} S_{ij} \leq R_i$$

Hardwood:

$$\sum_{j=1}^{p} H_{ij} \leq T_i$$

The first equation specifically states that the summation of all softwoods transported out of forest "i" to "p" mills is less than or equal to the amount of "r" cords of softwood which forest "i" can produce in a given time period (for example, one year).

The second equation states that the summation of all hardwoods transported out of forest "i" to "p" mills is equal to or less than an amount of "t" cords of hardwood which forest "i" can produce in a given time period. Note that a given resource area may contain both softwood and hardwood.

Mill Capacity Constraints

Each proposed mill site was assumed to have a given production capacity expressed in finished tons of product per time period. Shipments of a product are made to any number of the "m" markets. The typical equation for this section is:

$$\sum_{k=1}^{m} P_{jk} \leq Y_j X_j$$

In this equation a maximum of "Y" tons of finished product from mill "j" is being shipped to any number of the "m" markets. The value of "X_j" is either zero or one. If "X_j" is equal to one, there will be product flow through the mill. If "X_j" is set equal to zero, no product will flow through the mill. In analyzing the objective function, it is apparent that an "X_j" value of one would initiate the predetermined fixed cost associated with mill "j."

Market Demand Constraints

The market for pulpwood and paper products is generally limited primarily to conversion and finishing plants. Therefore, any particular market "k" may be treated as a point demand. The typical market demand equation used in this model is:

$$\sum_{j=1}^{p} P_{jk} \geq D_k$$

This equation states that the summation of tons of bulk product shipped from all "p" mills to market "k" is equal to or greater than the demand for bulk product at market "k." The objective function for minimizing total cost prevents an "overshipment" to market "k" since total cost increases with added shipments.

Input-Output Constraints

The input-output equations set the input flow of pulpwood equal to the output flow of product. The input flow in cords is adjusted by a coefficient to make it equivalent to the output in tons. The following is the typical input-output equation and its explanation:

$$\sum_{j=1}^{h} G_j H_{ij} + \sum_{i=1}^{s} E_j S_{ij} = \sum_{k=1}^{m} P_{jk}$$

The equation consists of three components: (1) tons of bulk product from hardwood, (2) tons of bulk product from softwood, and (3) total tons of product transported from mill "j" during a given time period. The hardwood input component is the summation of cords of hardwood from all hardwood forests multiplied by a conversion factor for tons of finished product per cord of hardwood used in mill "j." In the same manner, the softwood input component is the summation of cords of softwood from all softwood forests multiplied by a conversion factor for tons of finished product per cord of softwood used in mill "j." The remaining component is the total tons of bulk product transported from mill "j" to all markets "k" during a given time.

Softwood-Hardwood Input Distribution Constraints

This set of equations defines the maximum per cent of hardwood that can be used in a manufacturing process. The following is the typical equation describing the per cent of softwood and hardwood used:

$$\sum_{i=1}^{h} G_j H_{ij} \leq K_j \left[\sum_{i=1}^{h} G_j H_{ij} + \sum_{i=1}^{s} E_j S_{ij} \right]$$

This equation states that the total amount of hardwood used, measured in tons of bulk product, is less than or equal to a per cent "K" of the total wood resource input, measured in tons of bulk product. As previously cited, the use of hardwood is often favored over the use of softwood due to the lower cost per ton of bulk product produced.

The Objective Function

The objective of the model is to minimize total product cost subject to the given constraints. Many factors which determine the cost per unit may be included in the coefficients which precede the variables. A set of fixed costs is included in the linear equation to account for the fixed cost associated with each potential mill site. Three basic cost components compose the cost equation. These components are: (1) wood resource cost (delivered to the mill), (2) product cost (delivered to the market), and (3) fixed costs associated with the mill.

The wood resource cost, including delivery to the mill, is composed of two different sets of equations, one for softwood and another for hardwood. As previously described, S_{ij} and H_{ij} are the number of cords of softwood and hardwood, respectively, transported from forest "i" to mill "j." The cost components a_{ij} and b_{ij} are a summation of stumpage, harvesting, loading, and transportation costs per cord from forest "i" to mill "j." By knowing the resource (forest) location, the proposed mill location, available labor, transportation facilities, and stumpage costs for a particular site, it is possible to calculate the minimum a_{ij}'s and b_{ij}'s for the respective variables.

The second type of cost which appears in the objective function pertains to production and transportation from the mill to the market. The cost coefficient c_{jk} is calculated in dollars per ton of bulk product produced in mill "j" and shipped to market "k." This coefficient includes variable operating expenses (such as water and power), labor cost, and the minimum transportation cost from mill "j" to market "k." All such costs are based on an operating scale as determined by the predetermined capacity of the mill.

The third type of cost included in the objective function is fixed costs associated with the mill. As previously explained, the fixed costs "F_j" for a mill "j" will be included in the function if mill "j" is selected to be built. If not, the fixed costs for plant "j" will equal zero. The fixed costs may include costs of constructing the mill, mill equipment, taxation, and other lesser costs which are determined once the production capacity is set for the mill.

The basic costs named above comprise the economics of the mill location decision. Cost coefficients, as discussed, must be determined for each forest, mill, and market location. The following is a presentation of the objective function with the complete constraint set:

$$\text{MIN } Z = \sum_{j=1}^{p} \sum_{i=1}^{s} a_{ij}S_{ij} + \sum_{j=1}^{p} \sum_{i=1}^{h} b_{ij}H_{ij} + \sum_{k=1}^{m} \sum_{j=1}^{p} C_{jk}P_{jk} + \sum_{j=1}^{p} X_jF_j$$

subject to

$$\sum_{j=1}^{p} S_{ij} \le R_i, \quad \text{for } i = 1 \text{ to } s$$

$$\sum_{j=1}^{p} H_{ij} \le T_i, \quad \text{for } i = 1 \text{ to } h$$

$$\sum_{k=1}^{m} P_{jk} \le Y_jX_j, \quad \text{for } j = 1 \text{ to } p$$

$$\sum_{j=1}^{p} P_{jk} \ge D_k, \quad \text{for } k = 1 \text{ to } m$$

$$\sum_{j=1}^{h} G_jH_{ij} + \sum_{i=1}^{s} E_jS_{ij} = \sum_{k=1}^{m} P_{jk}, \quad \text{for } j = 1 \text{ to } p$$

$$\sum_{i=1}^{h} G_jH_{ij} \le K_j\left[\sum_{i=j}^{h} G_jH_{ij} + \sum_{i=1}^{s} E_jS_{ij}\right], \quad \text{for } j = 1 \text{ to } p$$

$$S_{ij},\ H_{ij},\ P_{jk} \ \ge\ 0$$

$$X_j = 0 \text{ or } 1$$

CHAPTER IV

SOLUTION TECHNIQUE

Introduction

This chapter describes a technique for solving mixed zero-one integer programming problems. The formulated problem falls into this category because it has continuous variables describing variable costs and zero-one variables describing fixed costs. In the past, linear programming has been widely used to solve similar problems having all continuous variables, whereas methods of implicit enumeration have been used to solve zero-one integer problems. An algorithm to combine these techniques is presented in this chapter.

Restating the Problem

The algorithm presented to solve the formulated model is based on Benders' partitioning procedure (23), Balas' additive algorithm (24), and the special structure of the model. In choosing a solution technique for solving a particular location problem, the analyst might decide to use only part of the approach presented.

The mixed zero-one integer problem formulated in Chapter III can be stated in matrix notation. Inequality constraints of the problem may be written as equality constraints by addition of slack and surplus variables. The following is the problem formulated in matrix notation with the equality constraints:

$$\text{Maximize} \qquad F'X + W'U \qquad [1]$$

subject to

$$YX + BU = E$$

$$X, U \geq 0$$

$$X = 0 \text{ or } 1$$

F = n_1 component column vector which is the negative of the fixed cost associated with each plant; n_1 = p for p plants

X = n_1 component column vector of integer variables

W = n_2 component column vector; represents the negative of the cost coefficients of all non-integer variables

U = n_2 component column vector; represents all non-integer variables

Y = m component column vector; y_j represents the production capacity of plant "j" measured in tons of bulk product

B = mxn_2 matrix; represents the non-negative coefficients of the non-integer variables

E = m component column vector.

Now, summing over the market demand constraints gives

$$\sum_{k=1}^{m} \sum_{j=1}^{p} P_{jk} \geq \sum_{k=1}^{m} D_k \qquad [2]$$

and summing over the mill capacity constraints gives

$$\sum_{k=1}^{m} \sum_{j=1}^{p} P_{jk} \leq \sum_{j=1}^{p} Y_j X_j. \qquad [3]$$

Thus,

$$\sum_{j=1}^{p} Y_j X_j \geq \sum_{k=1}^{m} D_k. \qquad [4]$$

Equation [4] is simply a non-negative linear combination of the existing equations, and therefore any feasible solution that satisfies the constraint set of problem [1] also satisfies Equation [4]. Conversely, given any X vector, say X^+, satisfying constraint [4], one can obtain a feasible solution to the linear programming problem

$$\text{Maximize} \qquad W'U \qquad [5]$$

subject to

$$BU = [E-YX^+)$$

$$U \geq 0$$

Proof by Construction

Definitions of variables are as follows:

A_T = total cords of wood available for use

A'_T = total tons of product available to satisfy market demand

A_S = total cords of softwood available for use

A_H = total cords of hardwood available for use

G = tons of finished product per cord of hardwood

E = tons of finished product per cord of softwood

K = per cent of hardwood in total pulpwood resource (measured in tons of product)

D_K = demand of market K in tons of product

H = number of cords of hardwood

S = number of cords of softwood

Y_j = capacity of mill j (measured in tons of product)

The total cords of wood available for use, A_T, is

$$A_T = A_S + \min\left(A_H, \frac{E}{G} \cdot \frac{KS}{(1-K)}^{*}\right). \qquad [6]$$

Converting [6] into tons of product gives

$$A'_T = E \cdot A_S + \min\left(G \cdot A_H, G \cdot \frac{E}{G} \cdot \frac{KS}{(1-K)}\right). \qquad [7]$$

Recall that assumption (1) in Chapter III states

$$A'_T \geq \sum_k D_k. \qquad [8]$$

Now, re-index the X vector, X^+, so that X_j for j=1 to r has the value one, and X_j for j = r + 1 to p has the value zero.

Step 1

Consider the capacity of the plant having the lowest index, j = 1, and transport the following amount:

---

*Derivation. Total hardwood used is limited by softwood used. This may be stated as follows:

$$GH = K(ES+GH) \Rightarrow GH = KES + KGH \Rightarrow H(G-GK) = KES \Rightarrow H = \frac{E}{G} \cdot \frac{KS}{(1-K)}$$

$$\text{Maximum}\{Y_1, A_T'\}.$$

Case 1. $A_T' \le Y_1$.

Since $A_T' \ge \sum_k D_k$, then $Y_1 \ge \sum_k D_k$.

Thus, total market demand is satisfied and there exists a feasible solution to [5], since product can be transported from any plant "j" to any market "k." Therefore, the proof is completed.

Case 2. $A_T' > Y_1$.

If $Y_1 \ge \sum_k D_k$, then there exists a feasible solution to [5] and the proof is completed.

If $Y_1 < \sum_k D_k$, then since [4] $\sum_j Y_j X_j \ge \sum_k D_k$, at least x_2 must equal one. Therefore, go to step 2.

In general, if Equation [4] is not satisfied in step t-1, go to step t.

<u>Step t (General Step)</u>

The unsatisfied demand is $D_t = \sum_k D_k - \sum_{j=1}^{t-1} Y_j$. The wood resource available to plant "t" is expressed in tons of product as $A_t' = A_T' - \sum_{j=1}^{t-1} Y_j$.

Case 1. $A_t' \le Y_t$.

Since $A_t' \le Y_t$, $A_T' \ge \sum_k D_k$, and $\sum_j Y_j X_j \ge \sum_k D_k$, then $\sum_{j=1}^{t} Y_j X_j \ge \sum_{k=1}^{m} D_k$. Therefore the demand is satisfied and a feasible solution exists to [5]. Proof is completed.

Case 2. $A_t' > Y_t$.

Since $A_t' > Y_t$ and $\sum_j Y_j X_j \ge \sum_k D_k$, if $\sum_{j=1}^{t} Y_j \ge \sum_k D_k$ there exists a feasible solution to [5] and the proof is completed. However, if

$\sum_{j=1}^{t} Y_j < \sum_k D_k$, then since [4] $\sum_j Y_j X_j \geq \sum_k D_k$, at least X_{t+1} must equal one. Therefore, go to step t+1.

Continue this procedure until either Equation [4] is satisfied or j=r. If Equation [4] is satisfied for some j less than r, stop the process since a feasible solution has been found. If Equation [4] is not satisfied until "j" equals "r," go to Step r.

Step r

The unsatisfied demand is $D_r = \sum_k D_k - \sum_{j=1}^{r-1} Y_j$. The wood resource, expressed in tons of product available to plant "r" is $A'_r = A'_T - \sum_{j=1}^{r-1} Y_j$. Since X_r is the final component of X^+ which has the value one, $A'_T \geq \sum_k D_k$, and $\sum_j Y_j X_j \geq \sum_k D_k$, then $\sum_{j=1}^{r} Y_j \geq \sum_k D_k$. Therefore there exists a possible solution to problem [5], and the proof is completed.

Now, consider the problem

$$\text{Maximize} \qquad z_o \qquad [9]$$

subject to

$$z_o \leq F'X + \min \lambda^k(E-YX^+)$$

$$\sum_j Y_j X_j \geq \sum_k D_k$$

$$X = 0 \text{ or } 1$$

where K is the set of extreme points of the convex polyhedral set $R = \{\lambda | \bar{\lambda}' B \geq W', \lambda \text{ unrestricted}\}$, and $\bar{\lambda}$ a T-component column vector with elements λ_t.

THEOREM 1

If (z_o^*, X^*) is an optimal solution to [9], then there exists a vector Y^* such that (X^*, Y^*) is optimal to [1] with value $F'X^* + W'U^* = z_o^*$.

Proof

Setting $X = X^*$ in [1] given the linear programming problem

$$\text{Maximize} \qquad W'U + F'X^+ \tag{10}$$

subject to

$$BU = (E - YX^+)$$

$$U \geq 0$$

Omitting the constant term $F'X^+$, problem [10] may be written as

$$\text{Maximize} \qquad W'U \tag{11}$$

subject to

$$BU = (E - YX^+)$$

$$U \geq 0$$

with dual

$$\text{Minimize} \qquad \bar{\lambda}'(E - YX^+) \tag{12}$$

subject to

$$\bar{\lambda}' B \geq W'$$

$$\lambda \text{ unrestricted}$$

It is obvious that $\bar{\lambda} = \bar{0}$ is a feasible solution to [12] since all elements of $W' \leq 0$ and all dual constraints are of $\geq$ form. Recall that problem [11] also has a feasible solution as proved by construction. Therefore, since both [11] and [12] have feasible solutions, they must have finite optimum solutions. Let Y^* be the optimal solution to [11] for $X^+ = X^*$. Then, by duality theory $W'U^* = \min\limits_{\lambda^k \epsilon} \lambda^{k'} (E-YX^*)$. Adding the constant $F'X^*$ to both sides of the above equation gives

$$F'X^* + W'U^* = \min_{\lambda^k \epsilon} \lambda^{k'} (E-YX^*) + F'X^*,$$

and hence

$$z_o^* = F'X^* + W'U^*.$$

Clearly, since Y^* is a feasible solution to [10] for $X^+ = X^*$, (Y^*,X^*) is a feasible solution to [1]. To show that it is optimal to [1], assume the contrary; i.e., assume (z_o^*,X^*) is optimal to [9], but there exists a solution to [1], say (X^{**},Y^{**}), with $F'X^{**} + W'U^{**} > z_o^*$. Clearly for $X^+ = X^{**}$ must be an optimal solution to [10]. If not, Y^{**} could not be optimal to [1]. Thus, for $X^+ = X^{**}$, $z_o^{**} = F'X^{**} + \min\limits_{\lambda^k \epsilon K} \lambda^{k'}(E-YX^{**})$.

Now since X^{**} is a feasible X vector for [1], it is also feasible for [9], and thus a solution can be obtained to [9] with value $z_o^{**} = F'X^{**} + \min_{\lambda^k \epsilon K} \lambda^{k'}(E-YX^{**}) = F'X^{**} + W'U^{**} > z_o^*$.

This contradicts the assumption that (z_o^*, X^*) is the optimal solution to [9]. Thus, given an optimal solution (z_o^*, X^*) to [9], there exists an optimal solution to [1] with value z_o^*.

One may solve problem [1] by first solving problem [9] and then given X^*, the optimal X vector for [9], set $X^+ = X^*$ and solve problem [10] for Y^*. Then, (X^*, Y^*) will be the optimal solution to [1].

Solution Procedure

Benders (25) suggests the following iterative scheme for solving problem [9].

Maximize z_o

subject to

$$z_o \leq F'X + \min \lambda^k(E-YX^+) \text{ for all } \lambda^k \epsilon K$$

$$\sum_j Y_j X_j \geq \sum_k D_k$$

$$X = 0 \text{ or } 1.$$

Step (a)

Given a subset K' of K, solve the integer programming problem [9] replacing K by K'. If no feasible solution exists then [1] is not feasible. Otherwise let the optimal solution be $\bar{X}$ with value $\bar{z}_o$.

Step (b)

Now, determine whether or not an optimal solution to [9] has been found by solving [11] for the $\bar{X}$ obtained in Step (a). Let the solution vector be $\bar{\lambda}$ with corresponding solution vector $\bar{Y}$ to [10], then $(\bar{X},\bar{Y})$ is a feasible solution to [1]. Now z_o is maximized over a restricted set of constraints and if

$$\bar{z}_o \le F'\bar{X} + W'\bar{U}$$

since

$$W'\bar{U} = \min_{\lambda^k \epsilon K} \lambda^k \, [E-Y\bar{X}]$$

condition [9] is met for all $\lambda^k \epsilon K$ and hence $(\bar{X},\bar{Y})$ is the optimal solution. If, however, $\bar{z}_o > F'\bar{X} + W'\bar{U}$, an extreme point $\bar{\lambda}$ has been located such that $\bar{z}_o \le F'\bar{X} + \bar{\lambda}(E-Y\bar{X})$ is violated. Therefore, add $\bar{\lambda}$ to the set K' and return to Step (a).

The integer programming problem may be solved using Unger's modification (26) to Geoffrion's implicit enumeration (27). The following technique is based on Unger's procedure.

Letting

$$c_j^i = F_j - \sum_{t=1}^{T} \lambda_t^i a_{tj}$$

$$b^i = \sum_{t=1}^{T} \lambda_t^i E_t$$

$$D = \sum_k D_k$$

the reduced problem [9] may be restated as

$$\text{Maximize} \qquad z_o \qquad [13]$$

subject to

$$z_o \le b^i + \sum_{j=1}^{p} c_j^i x_j, \qquad i=1,\ldots,m$$

$$\sum_j y_j x_j \ge D$$

$$x_j = 0 \text{ or } 1$$

where m is the number of elements in the set K'. Any binary vector X will be called a solution to [13]. A solution satisfying the constraint set $\sum_j y_j x_j \ge D$ will be called a feasible solution. A feasible solution that maximizes z_o over all feasible solutions for the complete problem will be called an optimal feasible solution. Constraints of the form

$$z_o \le b^i + \sum_{j=1}^{p} c_j^i x_j$$

will be referred to as objective function constraints since they limit the maximum value of z_o but do not affect feasibility. A partial solution S will be an assignment of binary values to a subset of the p binary variables. The variables not assigned values by S can take on either the value 0 or 1 and are called free. A completion of a partial solution S is defined as the binary vector X^S determined by S and an assignment of binary values to the free variables.

As a bookkeeping procedure, the symbol j denotes $x_j = 1$, and the symbol -j denotes $x_j = 0$. The procedure is:

Given a partial solution S,

Step 1

Find the best completion X^{S_i} of each objective function constraint independently. Let $z_i = b^i + \sum_{j=1}^{p} c_j^i x_j^{S_i}$.

Step 2

Find $z_k = \min_i z_i$. Let $z = z_k$ and $X^S = X^{S_k}$.

If $z \leq \bar{z}$ (the current best feasible solution) S is fathomed. Go to Step 3. Otherwise, go to Step 4.

Step 3

Locate the rightmost element of S that is not underlined. If none exists, terminate. Otherwise, replace it by its complement, underline it, and delete all underlined elements to its right. Return to Step 1.

Step 4

Make the completion X^S on each of the objective function constraints. Let the value of each constraint under this completion be z_i' and let $\min_i z_i' = z'$.

Step 5

Case 1. $z' = z$: Here, the best possible completion of S has been reached.

i) If this solution is feasible, S is fathomed. Store the solution, replace $\bar{z}'$ by z', and backtrack (Step 3).

ii) If the solution is not feasible, augment S by a new variable so as to drive towards feasibility, Step 6.

Case 2. $z' < z$, but $z' > \bar{z}$.

i) If feasible, replace $\bar{z}$ with z' and store the solution. Augment S by a new variable so as to increase z', Step 7.

ii) If not feasible, augment S by a new variable so as to drive towards feasibility, Step 6.

Case 3. $z' < z$ and $z' \leq \bar{z}$.

i) If feasible, augment S by a new variable so as to increase z', Step 7.

ii) If not feasible, augment S by a new variable so as to drive towards feasibility, Step 6.

Step 6

S is to be augmented as to drive towards feasibility. It is necessary to consider only those variables in the set T^S where T^S is defined as follows:

If in X^S, $x_j = 0$, j free, j is included in the set T^S if in the ith objective function constraint c_j^i was < 0 and $z_i + c_j^i > \bar{z}$. If the set T^S is empty, S is fathomed since there is no way to obtain a feasible solution, go to Step 3.

Next, check to see if it is possible to satisfy each of the violated constraints. In order to satisfy the feasibility constraint

$$V^S + \sum_{j \in T^S} y_j x_j \geq 0$$

where $V^S = \sum_j y_j x_j^S - D$. If the feasibility constraint cannot be satisfied, S is fathomed. Go to Step 3. In the event S cannot be fathomed, augment S by adding j_o, where j_o is the j that makes $V^S + y_j$ a maximum. Set $x_{j_o} = 1$ and return to Step 1.

Step 7

S is to be augmented in order to increase z'. Proceed as follows:

Find the constraint that limited z_o to z' (if more than one such constraint, arbitrarily select one). To obtain an increase in z' change the values assigned by X^S to at least one of the free variables in this objective function constraint, denoted as the kth constraint. If in X^S, $x_j = 1$, j free, and $c_j^k < 0$, consider setting $x_j = 0$. Consider this change only if $z_i - c_j^i > \bar{z}$, all i. Note this check need only be made for $c_j^i > 0$, since if $c_j \leq 0$ this condition is met. Each j meeting the above conditions is included in the set T^{S+}. If on the other hand in X^S, $x_j = 0$, j free, include j in the set T^{S-} if $c_j^k > 0$ and $z_i + c_j^i > \bar{z}$, all i. Let the set $T^S = T^{S+} \cup T^{S-}$.

If the set T^S is empty, S is fathomed. Therefore go to Step 3. Furthermore, if

$$z' - \sum_{j \in T^{S+}} c_j^k + \sum_{j \in T^{S-}} c_j^k \leq \bar{z}$$

it is not possible to make $z' > \bar{z}$ and S is fathomed. Therefore, go to Step 3. If T^S is not empty and the above test is passed, select j_0 so as to yield a maximum increase in z'. The best possible value that can be obtained for z' is z. Now, select as j_0 that j that gets the result as close as possible to z in the sense that

$$\sum_i \max \{(z-(z_i'-c_j^i)),0\},\ j\epsilon T^{S+}$$

or

$$\sum_i \max \{(z-(z_i'+c_j^i)),0),\ j\epsilon T^{S-}$$

is a minimum. If $j_0 \in T^{S+}$, set $x_j = 0$ in the augmented partial solution. If $j_0 \in T^{S-}$, set $x_j = 1$. Go to Step 1.

CHAPTER V

APPLICATION OF THE MODEL

Introduction

This chapter presents an example application of the mixed zero-one integer model to the location of bulk paper mills. All economic factors discussed in Chapter II are included in this example. Several preliminary decisions must be made before the model can be applied. The following are decisions made for presenting this example problem:

1. There is sufficient mill capacity and pulpwood resource to meet market demand.

2. Minimum resource, production, and transportation cost coefficients can be determined.

3. All costs are expressed as annual costs.

4. Each of the two markets are finishing mills which are treated as discrete points.

5. There are four forest resources, three potential plant sites, and two markets considered in this problem.

Presentation of Data

Figure 1 is a network representation of the problem with variable costs shown on the arcs.

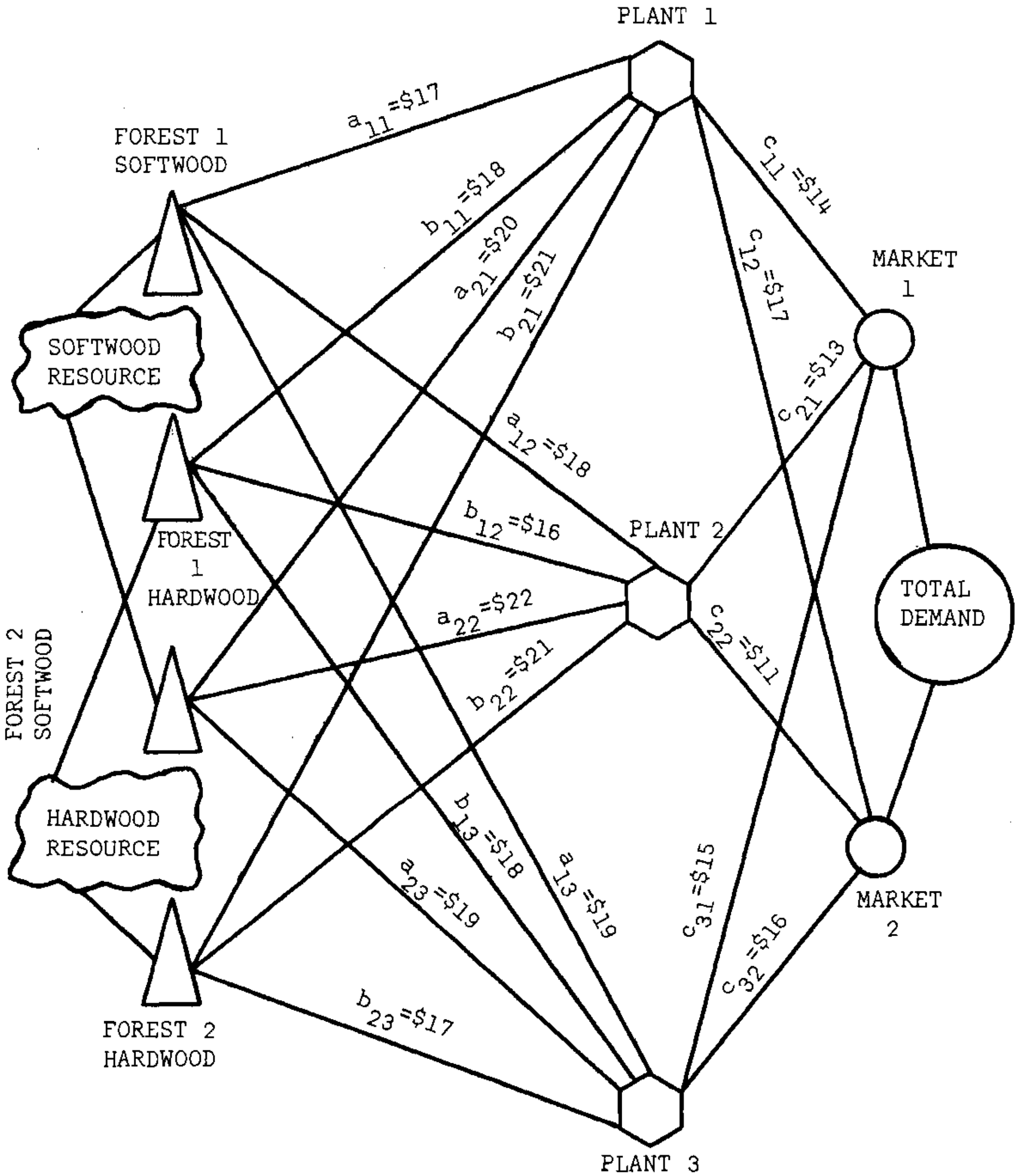

Figure 1. Network Representation of the Problem

The following data present the market demand, production capacity, fixed costs, and forest resource for the example problem.

MARKET DEMAND

	Market 1	140,000 Tons of Product per Year
	Market 2	200,000 Tons of Product per Year
Total Market Demand		340,000 Tons of Product per Year

POTENTIAL PRODUCTION CAPACITIES

	Mill 1	180,000 Tons of Product per Year
	Mill 2	180,000 Tons of Product per Year
	Mill 3	180,000 Tons of Product per Year
Total Capacity		540,000 Tons of Product per Year

FIXED COSTS OF MILLS (Includes Overhead, Taxes, and Amortized Construction Costs)

Mill 1 F_1 = \$6,600,000 per Year

Mill 2 F_2 = 6,900,000 per Year

Mill 3 F_3 = 7,400,000 per Year

FOREST RESOURCES

	Softwood	Hardwood	
Forest 1	400,000	25,000	Cords per Year
Forest 2	300,000	60,000	Cords per Year
Totals	700,000	85,000	Cords per Year

Conversion Factors: $E_j = 1/2$ for all j

$G_j = 1/1.5$ for all j.

Maximum Per Cent of Hardwood That May Be Used in Production:

$K = 0.10$

Problem Formulation

The following is the example problem formulated by the model.

Forest Resource Constraints

Softwood

$$S_{11} + S_{12} + S_{13} \leq 400{,}000$$

$$S_{21} + S_{22} + S_{23} \leq 300{,}000$$

Hardwood

$$H_{11} + H_{12} + H_{13} \leq 25{,}000$$

$$H_{21} + H_{22} + H_{23} \leq 60{,}000$$

Mill Capacity Constraints

Mill 1 $\quad P_{11} + P_{12} \leq 180{,}000\ X_1$

Mill 2 $\quad P_{21} + P_{22} \leq 180{,}000\ X_2$

Mill 3 $\quad P_{31} + P_{32} \leq 180{,}000\ X_3$

Market Demand Constraints

Market 1 $\quad P_{11} + P_{21} + P_{31} \geq 140{,}000$

Market 2 $\quad P_{12} + P_{22} + P_{32} \geq 200{,}000$

Input-Output Constraints

Mill 1

$$\frac{1}{1.5}(H_{11}+H_{21}) + \frac{1}{2}(S_{11}+S_{21}) = P_{11} + P_{12}$$

Rewritten as

$$4(H_{11}+H_{21}) + 3(S_{11}+S_{21}) = 6P_{11} + 6P_{12}$$

Mill 2

$$4(H_{12}+H_{22}) + 3(S_{12}+S_{22}) = 6P_{21} + 6P_{22}$$

Mill 3

$$4(H_{13}+H_{23}) + 3(S_{13}+S_{23}) = 6P_{31} + 6P_{32}$$

Softwood-Hardwood Input Distribution Constraints

Mill 1

$$H_{11} + H_{21} \leq \frac{1}{10} [H_{11}+H_{21}+S_{11}+S_{21}]$$

$$9H_{11} + 9H_{21} - S_{11} - S_{21} \leq 0$$

Mill 2

$$9H_{12} + 9H_{22} - S_{12} - S_{22} \leq 0$$

Mill 3

$$9H_{13} + 9H_{23} - S_{13} - S_{23} \leq 0$$

Objective Function

$$\text{Min } Z = 17S_{11} + 18S_{12} + 19S_{13} + 20S_{21} + 22S_{22} + 19S_{23}$$

$$+ 18H_{11} + 16H_{12} + 18H_{13} + 19H_{21} + 21H_{22} + 17H_{23}$$

$$+ 14P_{11} + 17P_{12} + 13P_{21} + 11P_{22} + 15P_{31} + 16P_{32}$$

$$+ 6{,}600{,}000X_1 + 6{,}900{,}000X_2 + 7{,}400{,}000X_3$$

Solution to the Example Problem

Partitioning the problem is the first step in the solution technique. The general form for partitioning the problem is

$$\text{Maximize} \qquad z_o$$

subject to

$$z_o \le F'X + \min_{\lambda^k \in R} \lambda^k \, (E-YX)$$

$$\sum_{j=1}^{p} Y_j X_j \ge \sum_{k=1}^{m} D_k$$

$$X = 0 \text{ or } 1$$

Consider the example problem [1] in Figure 2. The non-integer part of the partitioned problem is the linear programming problem [2] shown in Figure 3. The dual to the problem, shown in Figure 3, is problem [3] shown in Figure 4. Rewriting the dual to eliminate unrestricted variables gives problem [4] shown in Figure 5.

Now the integer part of the original formulation, problem [1], is

$$\text{Maximize} \qquad z_o \qquad [5]$$

subject to

	S_{11}	S_{12}	S_{13}	S_{21}	S_{22}	S_{23}	H_{11}	H_{12}	H_{13}	H_{21}	H_{22}	H_{23}	P_{11}	P_{12}	P_{21}	P_{22}	P_{31}	P_{32}	X_1	X_2	X_3		
MAX =	-17	-18	-19	-20	-22	-19	-18	-16	-18	-19	-21	-17	-14	-17	-13	-11	-15	-16	$-F_1$	$-F_2$	$-F_3$		
	1	1	1																			≤	400,000
				1	1	1																≤	300,000
							1	1	1													≤	25,000
										1	1	1										≤	60,000
													1	1								≤	$180{,}000X_1$
															1	1						≤	$180{,}000X_2$
																	1	1				≤	$180{,}000X_3$
													1		1		1					≥	140,000
														1		1		1				≥	200,000
	3			3			4			4			-6	-6								=	0
		3			3			4			4				-6	-6						=	0
			3			3			4			4					-6	-6				=	0
	-1			-1			9			9												≤	0
		-1			-1			9			9											≤	0
			-1			-1			9			9										≤	0

$F_j \equiv$ FIXED COST OF PLANT J $S_{ij} \geq 0$

$F_1 = 6{,}600{,}000$ $H_{ij} \geq 0$

$F_2 = 6{,}900{,}000$ $P_{ij} \geq 0$

$F_3 = 7{,}400{,}000$ $_j = 0$ or 1

Figure 2. Problem 1, Mixed Integer Example Problem

	S_{11}	S_{12}	S_{13}	S_{21}	S_{22}	S_{23}	H_{11}	H_{12}	H_{13}	H_{21}	H_{22}	H_{23}	P_{11}	P_{12}	P_{21}	P_{22}	P_{31}	P_{32}		
MAX Z=	-17	-18	-19	-20	-22	-19	-18	-16	-18	-19	-21	-17	-14	-17	-13	-11	-13	-16		
	1	1	1																≤	400,000
				1	1	1													≤	300,000
							1	1	1										≤	25,000
										1	1	1							≤	60,000
													1	1					≤	$180{,}000X_1$
															1	1			≤	$180{,}000X_2$
																	1	1	≤	$180{,}000X_3$
													1		1		1		≥	140,000
														1		1		1	≥	200,000
	3			3			4			4			-6	-6					=	0
		3			3			4			4				-6	-6			=	0
			3			3			4			4					-6	-6	=	0
	-1			-1			9			9									≤	0
		-1			-1			9			9								≤	0
			-1			-1			9			9							≤	0

$$S_{ij} \geq 0$$
$$H_{ij} \geq 0$$
$$P_{ij} \geq 0$$

Figure 3. Problem 2, Primal Linear Programming Problem

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}		
1									3			-1			≥	-17
1										3			-1		≥	-18
1											3			-1	≥	-19
	1								3			-1			≥	-20
	1									3			-1		≥	-22
	1										3			-1	≥	-19
		1							4			9			≥	-18
		1								4			9		≥	-16
		1									4			9	≥	-18
			1						4			9			≥	-19
			1							4			9		≥	-21
			1								4			9	≥	-17
				1			-1		-6						≥	-14
				1				-1	-6						≥	-17
					1		-1			-6					≥	-13
					1			-1		-6					≥	-11
						1	-1				-6				≥	-15
						1		-1			-6				≥	-16

$\lambda_i \geq 0$ for $i=1,\ldots,9$

λ_i Unrestricted for $i=10,11,12$

$\lambda_i \geq 0$ for $i=13,14,15$

$$\text{Minimize } Z = 400{,}000\lambda_1 + 300{,}000\lambda_2 + 25{,}000\lambda_3 + 60{,}000\lambda_4 + 180{,}000\lambda_5 X_1 + 180{,}000\lambda_6 X_2 + 180{,}000\lambda_7 X_3 - 140{,}000\lambda_8 - 200{,}000\lambda_9$$

Figure 4. Problem 3, Dual Linear Programming Problem

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ^*_{10}	λ^*_{11}	λ^*_{12}	λ^*_{13}	λ^*_{14}	λ^*_{15}	λ^*_{16}	λ^*_{17}	λ^*_{18}		
-1									-3	3					1			≤	17
-1											-3	3				1		≤	18
-1													-3	3			1	≤	19
	-1								-3	3					1			≤	20
	-1										-3	3				1		≤	22
	-1												-3	3			1	≤	19
		-1							-4	4					-9			≤	18
		-1									-4	4				-9		≤	16
		-1											-4	4			-9	≤	18
			-1						-4	4					-9			≤	19
			-1								-4	4				-9		≤	21
			-1										-4	4			-9	≤	17
				-1			1		6	-6								≤	14
				-1				1	6	-6					-4	4		≤	17
					-1		1				6	-6						≤	13
					-1			1			6	-6						≤	11
						-1	1						6	-6				≤	15
						-1		1					6	-6				≤	16

$\lambda, \lambda^* \geq 0$; $\lambda_{10} = \lambda^*_{10} - \lambda^*_{11}$; $\lambda_{11} = \lambda^*_{12} - \lambda^*_{13}$; $\lambda^*_{12} = \lambda^*_{14} - \lambda^*_{15}$;
$\lambda_{13} = \lambda^*_{16}$; $\lambda_{14} = \lambda^*_{17}$; $\lambda_{15} = \lambda^*_{18}$

$$\text{Minimize } Z = 400{,}000\lambda_1 + 300{,}000\lambda_2 + 25{,}000\lambda_3 + 60{,}000\lambda_4 + 180{,}000\lambda_5 X_1 + 180{,}000\lambda_6 X_2 + 180{,}000\lambda_7 X_3 - 140{,}000\lambda_8 - 200{,}000\lambda_9$$

Figure 5. Problem 4, Dual Problem with Added Variables

$$z_o \le -6{,}600{,}000X_1 - 6{,}900{,}000X_2 - 7{,}400{,}000X_3$$

$$\sum_j Y_j X_j \ge \sum_k D_k$$

$$X_j = 0 \text{ or } 1 \text{ for all } j$$

The solution begins with a solution for the dual linear programming problem, $\bar{\lambda}_i = \bar{0}$ for i=1 to i=15.

Step 1-a

Recall from Chapter IV that $\lambda'^1(E-YX)$ is as follows:

$$= (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)\begin{pmatrix} 400{,}000 \\ 300{,}000 \\ 25{,}000 \\ 60{,}000 \\ 180{,}000X_1 \\ 180{,}000X_2 \\ 180{,}000X_3 \\ -140{,}000 \\ -200{,}000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= 0 = W'\bar{U}^1.$$

This gives the integer problem

Maximize z_o [5]

subject to

$$z_o \leq -6,600,000X_1 - 6,900,000X_2 - 7,400,000X_3 + 0$$

$$180,000X_1 + 180,000X_2 + 180,000X_3 \geq 340,000$$

$$X_j = 0 \text{ or } 1$$

Applying the procedure presented in Chapter IV gives the integer solution $\bar{X}^1 = [X_1=1, X_2=1, X_3=0]$ and $\bar{z}_o^1 = -13,500,000$.

Step 1-b

With $\bar{X} = \bar{X}^1$, the objective function of the dual linear programming problem [4] becomes

$$\text{Minimize } Z = 400,000\lambda_1 + 300,000\lambda_2 + 25,000\lambda_3 + 60,000\lambda_4$$

$$+ 180,000\lambda_5(X_1=1) + 180,000\lambda_6(X_2=1) + 180,000\lambda_7(X_3=0)$$

$$- 140,000\lambda_8 - 200,000\lambda_9.$$

Minimizing the equation above with respect to the constraint set of problem [4] yields the optimal solution

$\lambda_1 = 3$	$\lambda_6 = 3.87$	$\lambda_{11} = -6.77$
$\lambda_2 = 0$	$\lambda_7 = 39.52$	$\lambda_{12} = 0$
$\lambda_3 = 5$	$\lambda_8 = 52.52$	$\lambda_{13} = 0.74$
$\lambda_4 = 0$	$\lambda_9 = 55.52$	$\lambda_{14} = 0.13$
$\lambda_5 = 0$	$\lambda_{10} = 6.42$	$\lambda_{15} = 0$

Thus,

$$\lambda'^{2}(E-YX^{+})$$

$$= (3,0,5,0,0,3.87,39.52,52.52,55.52,$$

$$6.42,-6.77,0,0.74,0.13,0)\begin{pmatrix} 400{,}000 \\ 300{,}000 \\ 25{,}000 \\ 60{,}000 \\ 180{,}000X_1 \\ 180{,}000X_2 \\ 180{,}000X_3 \\ -140{,}000 \\ -200{,}000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= 696{,}780X_2 + 7{,}112{,}880X_3 - 17{,}130{,}440$$

For $\bar{X}^1$,

$$\min \lambda'^{2}(E-YX^{+}) = \max W'\bar{U} = -16{,}433{,}660$$

Since $z_o' = -13{,}500{,}000 \not< -6{,}600{,}000(1) - 6{,}900{,}000(1) -16{,}433{,}660$, add the constraint $z_o \le -6{,}600{,}000X_1 -6{,}203{,}220X_2 -287{,}120X_3 -17{,}130{,}440$ to the integer programming problem.

Step 2-a

The revised integer problem is:

Maximize z_o [6]

subject to

$$z_o \leq -6,600,000X_1 - 6,900,000X_2 - 7,400,000X_3 + 0$$

$$z_o \leq -6,600,000X_1 - 6,203,220X_2 - 287,120X_3 - 17,130,440$$

$$180,000X_1 + 180,000X_2 + 180,000X_3 \geq 340,000$$

$$X_1,X_2,X_3 = 0 \text{ or } 1$$

Applying the procedure presented in Chapter IV gives the integer solution $\bar{X}^2 = [X_1=0,X_2=1,X_3=1]$ with $\bar{z}_o^2 = -23,620,780$.

Step 2-b

Now using $\bar{X}^2 = [X_1=0,X_2=1,X_3=1]$ the new dual objective function is as follows:

$$\text{minimize } Z = 400,000\lambda_1 + 300,000\lambda_2 + 25,000\lambda_3 + 60,000\lambda_4$$

$$+ 180,000\lambda_5(X_1=0) + 180,000\lambda_6(X_2=1) + 180,000\lambda_7(X_3=1)$$

$$- 140,000\lambda_8 - 200,000\lambda_9$$

Minimizing the equation above with respect to the constraint set of problem [4] yields

$\lambda_1 = 0$	$\lambda_6 = 5.97$	$\lambda_{11} = 5.90$
$\lambda_2 = 0$	$\lambda_7 = 0$	$\lambda_{12} = 6.06$
$\lambda_3 = 5$	$\lambda_8 = 51.39$	$\lambda_{13} = 0$
$\lambda_4 = 0$	$\lambda_9 = 52.39$	$\lambda_{14} = 0.29$
$\lambda_5 = 8.89$	$\lambda_{10} = 4.75$	$\lambda_{15} = 0.81$

Therefore,

$$\lambda'^{3}(E-YX^{+})$$

is

$$= (0,0,5,0,8.89,5.97,0,51.39,52.39,4.75,5.90,6.06,$$

$$0,0.29,0.81)\begin{pmatrix} 400{,}000 \\ 300{,}000 \\ 25{,}000 \\ 60{,}000 \\ 180{,}000X_1 \\ 180{,}000X_2 \\ 180{,}000X_3 \\ -140{,}000 \\ -200{,}000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= 1{,}600{,}000X_1 + 1{,}074{,}000X_2 - 17{,}547{,}600$$

For $\bar{X}^2$, min $\lambda'^3(E-YX)$ = max $W'\bar{U} = -14{,}873{,}600$. Since $z_o^2 = -23{,}620{,}780 \nless -6{,}900{,}000 - 7{,}400{,}000 - 14{,}873{,}000$, add the constraint $z_o \leq 5{,}000{,}000X_1 - 5{,}826{,}000X_2 - 7{,}400{,}000X_3 - 17{,}547{,}600$ to the integer programming problem.

Now, the revised integer programming problem is as follows:

$$\text{Maximize} \qquad z_o \tag{7}$$

$$z_o \leq -6{,}600{,}000X_1 - 6{,}900{,}000X_2 - 7{,}400{,}000X_3 + 0$$

$$z_o \leq -6{,}600{,}000X_1 - 6{,}203{,}220X_2 - 287{,}120X_3 - 17{,}130{,}440$$

$$z_o \leq -5{,}000{,}000X_1 - 5{,}826{,}000X_2 - 7{,}400{,}000X_3 - 17{,}547{,}600$$

$$180{,}000X_1 + 180{,}000X_2 + 180{,}000X_3 \geq 340{,}000$$

$$X_1, X_2, X_3 = 0 \text{ or } 1$$

Step 3-a

Solving the integer programming problem [7] gives the solution $\bar{X}^3 = [X_1=1, X_2=1, X_3=0]$ which is the same integer solution as obtained in Step 1-a. Since $\bar{X}^3$ will give the same dual solution as $\bar{X}^1$, it is apparent that $\bar{z}_o^3 = F'\bar{X}^3 + W'\bar{U}^3$. Therefore, the optimal solution $(\bar{X},\bar{Y})$ is found.

The optimum solution to the problem is to build plants one and two having a total annual product cost of \$29,933,660. Either solving the primal problem [2] using $X_1=X_2=1$, $X_3=0$, or inspecting the final dual tableau will give the following quantities produced and transported:

S_{11} = 86,452	H_{21} = 30,968
S_{12} = 313,548	H_{22} = 9,839
S_{13} = 0	H_{23} = 0
S_{21} = 192,258	P_{11} = 140,000
S_{22} = 0	P_{12} = 20,000
S_{23} = 0	P_{21} = 0
H_{11} = 0	P_{22} = 180,000
H_{12} = 25,000	P_{31} = 0
H_{13} = 0	P_{32} = 0

CHAPTER VI

DISCUSSION OF RESULTS AND CONCLUSIONS

The objective of this research was to develop a method for analyzing potential pulp and paper mill locations. This objective was met by (1) defining the factors pertinent to the location decision, (2) mathematically modeling these factors using mixed zero-one integer programming, and (3) presenting a solution technique developed by Benders. The sample problem presented in Chapter V exemplified the solution technique applied to the formulated model.

Defining the factors pertinent to the location decision served as the basis for developing the model. A differentiation was made between social and economic factors and the importance of each was recognized. Researching the economic factors indicated that the cost of labor, transportation, pulpwood resource, and initial investment are generally the major economic factors important to plant location.

A mixed zero-one integer program was used to model the location problem. The formulated model did include all economic factors defined in Chapter II. The developed model is versatile for it considers both fixed and variable costs. It also can easily be modified to include additional economic factors which may arise in a particular location problem.

The solution technique was based on Benders' partitioning procedure. Partitioning the problem allowed developed solution algorithms to be used. Computerizing Unger's modified Geoffrion technique would make the solution of the integer part of the partitioned problem much more convenient as well as more practical for solving large problems. Rather than solving alternate integer and linear programming problems, as in Benders' procedure, Unger developed another type procedure which requires the solution of only one integer and multiple linear programming problems (28). This latter procedure merits recognition as an alternate solution technique for the formulated problem.

The number of calculations involved in applying the model was indicated by the example problem in Chapter V. The need for computer facilities is apparent, even for small problems.

The value of the developed model as a decision tool for plant location depends upon the discretion used by the analyst in delineating the region to be analyzed, obtaining data, and calculating the minimum cost coefficients. Obtaining the necessary information is usually not a complicated procedure. However, the large number of factors involved requires a thorough, time consuming investigation.

In summary, the formulated model can minimize the total product cost for pulp and bulk paper mills.

APPENDICES

APPENDIX A

CHECKLIST OF VARIABLES

Presented below is an outlined checklist of variables which the analyst may consider in the location of pulp and paper mills. The checklist is by no means exhaustive and should be supplemented by information required for a specific site (29).

I. WATER

A. River

1. Flow--maximum, average, minimum
2. Analysis for specific use
3. Riparian rights
4. Restrictions

B. Wells

1. Depth
2. Diameter size, capacity
3. Analysis of water for specific use
4. History of ground water level, source in rocks or sand

C. Public System

1. Dependability
2. Unit cost
3. Future expansion plans
4. Adequate capacity for mill growth
5. Water supply contract
6. Analysis of water for specific use

II. UTILITIES

A. Electricity

1. Unit cost
2. Dependability
3. Future expansion plans
4. Adequate supply for mill expansion
5. Purchase agreement

II. UTILITIES (Continued)

B. Steam

1. Unit cost
2. Dependability
3. Future expansion plans
4. Adequate supply for growth
5. Purchase agreement
6. Purchase available

C. Gas--Natural or L.P.

1. Unit cost
2. Dependability
3. Future expansion plans
4. Adequate supply for growth
5. Purchase agreement

D. Coal

1. Cost delivered
2. Qualities
3. Quantity of supply--maximum and minimum per time period
4. Time required for delivery
5. Alternate sources
6. Purchase agreement

E. Oil

1. Cost delivered
2. Qualities
3. Quantity of supply--maximum and minimum per time period
4. Time required for delivery
5. Alternate sources
6. Purchase agreement

F. Wood Bark

1. Quantity of supply
2. Incurred savings

III. EFFLUENT DISPOSAL

A. Public Disposal

1. Cost
2. Capacity for growth
3. Sample disposal contract

III. EFFLUENT DISPOSAL (Continued)

B. River

1. Location
2. Flow--maximum, average, minimum
3. Required local and state standards
4. Experience of other industries in area

C. Air Pollution Requirements

1. Present local, state, and federal requirements
2. Future requirements

IV. AVAILABILITY OF RAW MATERIALS

A. Wood

1. Available species and quantities
2. Possession status of wood limits or grants (owned, leased, cutting rights)
3. Rate of growth to pulpwood size
4. Weight per green cord (128 cubic feet piled)
5. Specific gravity
6. Barking characteristics (easy, medium, or hard)
7. Delivery distance to plant site
8. Seasonal delivery? Which months?
9. Method of delivery (rail, truck, river, barge, boom)
10. Long-term storage (insects, climate)
11. Length of logs to be delivered
12. Maximum and minimum diameter of logs
13. Cost per delivered cord (various species)
14. Chipped wood
    a. Cost
    b. Availability
    c. Advantages

B. Other Raw Materials (Bagasse, Straw, Esparto Grass, Old Papers, Etc.)

1. Type used
2. Annual quantities available within economic proximity
3. Cost per unit
4. Mode of transportation (truck, rail, etc.)
5. Form in which delivered (bulk, bundled, baled, etc.)
6. Fiber content per ton

V. TRANSPORTATION

A. Rails

1. Costs
    a. Rates on pulpwood per ton mile
    b. Rates on major items to and from major cities in areas to be served
    c. Cost of installing service; who pays?
2. Frequency of service
3. Names of companies serving

B. Trucks (Common Carriers)

1. Rates on major items to and from major cities in area to be served
2. Frequency of service
3. Time to and from major cities in area to be served
4. Names of companies serving

C. Airline Service

1. Freight rates
2. Schedules
3. Distance to ports

D. Bus Service or Other Commuter Service

E. Highways

1. Number of roads existing to site
2. Condition of roads
3. Nearby interstates
4. Distance to major cities

F. Water

1. Barge
    a. Rates
    b. Schedules
2. Ship
    a. Rates
    b. Schedules
3. Wharf or docks existing at site

G. Pipelines

1. Any existing?
2. Feasibility?

VI. MARKETING

A. Market Demand (Local or Regional Market)

1. Local or regional market trends
2. Present production capacity
3. Sales volume and value of each product
4. Estimated production of each major product

B. Competitors for New Products

1. General information
   a. Names
   b. Location
   c. Present and future output
   d. Production costs
   e. Selling prices
2. Changes in competition
   a. Expansion
   b. Modernization
   c. New plants
   d. New competing products
3. Foreign competition
   a. Tariffs
   b. Other laws affecting volume (health, agricultural, etc.)
4. Competitive position
   a. Selling prices
   b. Estimated transportation costs
   c. Maximum competitive selling prices
   d. Competitive advantages of proposed mill

VII. LABOR

A. Mobility

1. Local labor trend
2. Local population trend
3. Farm to urban migration

B. Availability

1. Union (closed and open shops)
   a. Availability of each skill, trade
   b. Sample labor contracts in area
   c. Average rates
2. Non-Union
   a. Availability of each skill, trade
   b. Sample labor contracts in area
   c. Wage rates

VII. LABOR (Continued)

3. Education
   a. Level
   b. Abilities
   c. Available training facilities

VIII. COMMUNITY OR LIVING ENVIRONMENT

A. Relative Size

1. City or town
   a. Population trends
   b. Land development trends
2. Size of surrounding communities

B. Schools

1. Graded
2. Vocational
3. Universities

C. Civic Advantages

1. Synagogues; churches--denominations
2. Entertainment

D. Cross Section of Population

1. Income
2. Age
3. Sex
4. Race

E. Climate

1. Temperature--maximum, minimum, median
2. Rainfall--annual, daily

F. Housing

1. Rental
2. Owned

G. Cost of Living

H. Hotel and Motel Facilities

I. Fire Protection

VIII. COMMUNITY OR LIVING ENVIRONMENT (Continued)

J. Insurance Rates

K. Water Supply

L. Local Taxes

APPENDIX B

TAXATION AND INDUSTRIAL LOCATION

The influence of taxes as a factor affecting industrial location ranges from 1 to 5 per cent of the operating costs for different types of manufacturing (30). This percentage also varies because of the differences in tax situations in particular counties and states. Therefore, in considering a particular location, the manufacturer must examine not only the current tax stipulations but also the tax program projected for the future.

A tax becomes a fixed cost regardless of rate of output. A general property tax has about the same effect as a higher interest rate. It penalizes localities where plants and equipment are less fully utilized and sharpens the producer's incentive to find a location where less capital investment is required per unit of output. Thus, a combination of tight restrictions on overtime work, night work, and speed-up procedures with a large degree of reliance on general property taxes for local revenues can be doubly burdensome to industry. General property taxes are likewise a threat to solvency in periods of poor business and may contribute to a cumulative weakening of a producer's competitive position.

Between distant states, tax differentials appear to exercise little plant location influence. The manufacturer will ordinarily select a region on the basis of economic conditions in general, rather than on the tax structure in particular. However, on a regional level, the tax factor may be influencial in the decision. It can be used by the various communities to compete for the desired industry. Specifically, within a state and more particularly within a metropolitan area such as Atlanta, significant local property tax rate variations can and do become swing factors in plant location decisions.

Personal Property Tax Influence

The general property tax as a state and/or local levy appears to have the greatest influence on managerial decisions (31). In jurisdictions where this tax is levied upon business inventories it is possible to discern a clear interrelationship between the property tax costs and decisions made by management. The interrelationship exists for large national concerns operating in several states where the tax treatment accorded such inventories differs sharply. Textile firms operating in the two Carolinas, for example, have the opportunity to minimize taxes by concentrating inventories in South Carolina, where they are largely tax exempt, even though the firms maintain manufacturing facilities in both states (32). Good highways and truck transportation permit this type of tax minimizing.

High city property tax rates on inventories encourage erection of warehouses outside the city not only by manufacturers but also by merchandisers, such as supermarket operators. Even minor differences in tax procedure may be used to advantage by business. If one government taxes on year-end values while the other applies average values over the year, shifting of inventories at strategic times reduces tax liability in the former case. It is not always possible for business to minimize taxes by these means. If the product is highly perishable, for example, the total inventory carried will be small and the potential tax saving available by movement of inventory is hardly worth the trouble. Nevertheless, it appears to be a general rule that whenever sizable liabilities accrue from property taxes on mobile property such as business inventories, alert management takes steps to minimize that liability.

Influence of Tax Differentials

In the appraisal of tax considerations, it is the size of the tax differential rather than the size of the total tax bill that is significant; a fact that sharply limits the value of Federal tax deductibility as a "neutralizing" force.

As pointed out earlier, tax costs vary both between different sites in the same state and between states. The intrastate variation is largely attributable to the effective general property tax rates at alternative sites. On the other hand, interstate variations reflect both the types of taxes used by the several states and the bases and rates of the taxes. Intrastate variations, however, may be as large, if not larger, than tax burden variations between states.

Individual firms are concerned with the type of taxes levied, depending upon the nature of their operation. For example, a firm with a large labor force relative to its capital investment is concerned with payroll levies, while firms with a large capital investment and a comparatively small labor force are concerned with the property tax burdens.

Another similar problem which often concerns management is the number of taxes levied and the large amount of paper work required. The utilization of electronic data processing has counteracted this problem in many cases, however, and this point is therefore demanding less attention and consideration by industry's management.

The Influence of Intrastate and Metropolitan Tax Differentials

High property tax rates in the central cities appear to be an important factor in explaining the movement of industry to suburban and non-metropolitan areas. The case for this inference rests on the fact that (1) property tax rates are generally lower in suburban areas than in the central cities; (2) the presence of industrial tax havens in these areas; and (3) the fact that certain cities noted for their high property tax rates have had to grant special property tax concessions in order to attract new industry. Following is a list of suburban property tax rates as a percentage of central city rates for various metropolitan areas (33):

City	Per Cent	City	Per Cent
Memphis	4	Chicago	76
San Antonio	8	Cleveland	83
Fort Worth	36	Detroit	85
Omaha	44	Buffalo	86
Oklahoma City	49	Atlanta	87
Baltimore	53	Oakland	87
Cincinnati	54	Toledo	88
Newark	58	San Diego	90
Rochester	58	Washington, D. C.	101
Portland	61	New York City	102
Denver	64	San Francisco	105
Philadelphia	64	Saint Louis	109
Milwaukee	66	Birmingham	117
Los Angeles	67	Columbus	117
Louisville	68	Kansas City	119
Seattle	73	Phoenix	141

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