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CALCULATION OF AIR FLOW AND
FUEL CONTROL CHARACTERISTICS FOR AN
OTTO CYCLE ENGINE

A THESIS

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CALCULATION OF AIR FLOW AND

FUEL CONTROL CHARACTERISTICS FOR AN

OTTO CYCLE ENGINE

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SUMMARY

A mathematical analysis of the intake and exhaust processes for a four stroke cycle engine was made so that the air flow into the engine could be calculated. The particular engine under consideration was a 283 cubic inch Chevrolet V-8, and the results of the air flow calculation are presented in tabular and graphical form. These results agree favorably with the expected trend and an approximate comparison of the calculated data with measured data showed that the calculated data was 13.5% less than the measured air flow at an engine speed of 1000 rpm over a wide range of intake pressures.

In addition to the air flow calculation, records of the cylinder pressure versus crank angle for the intake process were also obtained. These pressure records agree closely with the expected results and exhibited the correct trend throughout the range of engine operating conditions covered.

CHAPTER I

INTRODUCTION

For any engine, tests may be conducted to determine the most desirable air-fuel ratios for optimum operation. Once the air-fuel ratios have been determined, it is necessary to know the amount of air flow so that the fuel can be properly metered into the engine. However, it is not always possible to predict this air flow in a new engine design. It is because of this problem that attempts have been made to calculate the air flow into an engine.

Tsu (1)[*] calculated the air flow into a single cylinder engine. This work is an attempt to extend Tsu's method to calculate the air flow into a multicylinder automotive engine. The objective here is to calculate the air flow into each cylinder of the engine during that cylinder's intake process. The engine specifically being studied is a 283 cubic inch Chevrolet V-8. The measured data used are from the same engine, but fitted with a "Ram Manifold" for use with a full injection system.

In the process of developing the equations for calculating the air flow, it is necessary to examine both the intake and exhaust processes and to find the cylinder pressure as a function of the crank angle for the period that the intake valve is open. During part of the time when the intake valve is open, the exhaust valve is also open. The effect that this

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[*]Numbers in parenthesis refer to references in the Bibliography.

valve overlap has on the cylinder pressure during the intake process is included in this pressure record as a function of crank angle for the intake process.

CHAPTER II

DERIVATION OF EQUATION OF INTAKE PROCESS

If the contents of an internal combustion engine cylinder are considered as a perfect gas, their weight at any time can be written as

$$W = \frac{PV}{RT}\ ,^{*} \tag{1}$$

their internal energy at any time is

$$I = K_V WT\ , \tag{2}$$

and the change in internal energy during a time interval dt is

$$dI = K_V(W\ dT + T\ dW)\ , \tag{3}$$

where K_V is assumed to be constant.

During most of the intake process the air-fuel mixture flows from the inlet port into the cylinder, and in time dt the weight of fresh charge drawn into the cylinder through the inlet valve is

$$dW = \rho_1 g A_1 u_1 dt\ , \tag{4}$$

where section "1" is a section through the inlet valve. In addition, during the time dt, the change in cylinder volume is dV and the work done

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*Symbols are defined in the Nomenclature in Appendix B.

on the piston by the gaseous mixture in the cylinder is

$$P\,dV\ . \tag{5}$$

The interval energy per unit weight of fresh charge just outside the inlet valve is

$$\frac{u_1^{\,2}}{2g} + K_V\,T_1\ . \tag{6}$$

But owing to the flow work, sometimes referred to as "PV" work, done upon it, the energy brought into the cylinder by a unit weight of fresh charge is

$$\frac{u_1^{\,2}}{2g} + K_P\,T_1\ . \tag{7}$$

Therefore the energy brought into the cylinder by the fresh charge in time dt is

$$dW\left(\frac{u_1^{\,2}}{2g} + K_P\,T_1\right)\ . \tag{8}$$

From the First Law of Thermodynamics, it can be seen that

$$dI = K_V\Big(W\,dT + T\,dW\Big) = dW\left(\frac{u_1^{\,2}}{2g} + K_P\,T_1\right) - PdV + dQ\ , \tag{9}$$

where dQ is the heat energy transferred into the cylinder in time dt.

Let section "i" be a section distant enough from the inlet port so that the velocity of flow of the fluid is negligible. The pressure and temperature at section "i" are also constant. The energy relation between sections "i" and "l" is then

$$K_P\ T_i + \frac{dQ'}{dW} = K_P\ T_1 + \frac{u_1^{\,2}}{2g} , \tag{10}$$

where dQ' is the heat transferred to the fresh charge dW on its way from section "i" to section "l".

Substituting equation 10 in 9 we get

$$K_V \left(W\ dT + T\ dW \right) = dW \left(K_P\ T_i + \frac{dQ'}{dW} \right) + dQ - P\ dV , \tag{11}$$

which is the general differential equation of the intake process. The effect of heat transfer is now neglected because Tsu (1) indicated that unless the mean piston speed of the engine is very low, the effect of heat transfer on volumetric efficiency is negligible. Dropping the terms dQ and dQ' from equation 11, we get

$$K_V(W\ dT + T\ dW) = K_P\ T_i\ dW - PdV . \tag{12}$$

From the perfect gas equation,

$$pV = WRT , \tag{13}$$

we observe that

$$d(PV) = R\ d(WT) = R(W\ dT + T\ dW) . \tag{14}$$

Combining equations 12 and 14,

$$\frac{K_V}{R}\ d(PV) = K_P\ T_i\ dW - PdV . \tag{15}$$

Dividing through by K_V/R, transposing terms and substituting

$$\frac{K_P}{K_V} = \gamma \tag{16}$$

$$\frac{R}{K_V} = \gamma - 1 \tag{17}$$

we get

$$d(PV) + (\gamma - 1)PdV = \gamma RT_i \, dW \tag{18}$$

Integrating equation 18 from time t = 0 to time t = t, we get,

$$PV - P_oV_o + (\gamma - 1)\int_{V_o}^{V} PdV = \gamma RT_i \int_{W_o}^{W} dW \ . \tag{19}$$

In order to put equation 19 into a more easily handled form let the right hand member be written as

$$\gamma RT_i \int_{W_o}^{W} dW = \gamma RT_i \int_{t_o}^{t} \rho_1 g A_i u_1 dt = \gamma RT_i \rho_i g C \int_{t_o}^{t} A_i \frac{\rho_1}{\rho_i} \frac{u_1}{C} dt \ . \tag{20}$$

Substituting

$$dt = \frac{d\theta}{6N} \tag{21}$$

and letting

$$F_i = \sqrt{\frac{\gamma_1}{2}} \ \frac{\rho_1}{\rho_i} \ \frac{u_1}{C_i} \tag{22}$$

the subscript "i" being used to indicate that the symbols are for the intake process, equation 20 becomes

$$\gamma_i R_i T_i \int_{W_o}^{W} dW = \frac{\sqrt{2\gamma_i} \quad R_i T_i \rho_i g C_i}{6N} \int_{\theta_o}^{\theta} A_i F_i d\theta \ .$$

Since

$$\frac{P_i}{\rho_i g} = R_i T_i \ ,$$

we get

$$\gamma_i R_i T_i \int_{W_o}^{W} dW = \frac{\sqrt{2\gamma_i} \quad C_i P_i}{6N} \int_{\theta_o}^{\theta} A_i R_i d\theta \ . \tag{23}$$

Collecting terms in equation 23 and letting

$$K_i = \frac{\sqrt{2\gamma_i} \quad C_i P_i}{6N} \ , \tag{24}$$

and combining equations 19, 23, and 24 we get the equation

$$(PV - P_o V_o) + (\gamma_i - 1) \int_{V_o}^{V} PdV - K_i \int_{\theta_o}^{\theta} A_i F_i d\theta = 0 \ , \tag{25}$$

which is the final form of the equation of the intake process.

In order that equation 25 may be solved, the flow factor F_i must be evaluated. For a reversible adiabatic process,

$$\left(\frac{\rho_1}{\rho_i} \right) = \left(\frac{P}{P_i} \right)^{1/\gamma} , \tag{20}$$

and for steady flow,

$$\frac{u_1^{\;2}}{2g} + \int_S \frac{dP}{\rho} = \text{constant} \tag{27}$$

where the integration is carried out along a streamline S. Equation 27 is Bernoulli's equation but since the flow of the air-fuel mixture through the inlet part is unsteady, Euler's equation should be used instead of Bernoulli's equation. Euler's equation is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{dP}{dx} . \tag{28}$$

Integrating along a streamline S we get

$$\int_S \frac{\partial u}{\partial t} \, dx + \frac{u^2}{2} + \int_S \frac{dP}{\rho} = \text{constant} . \tag{29}$$

Using the adiabatic relation

$$\frac{P}{(\rho)^\gamma} = \text{constant}$$

equation 29 becomes

$$\int_S \frac{\partial u}{\partial t} \, dx + \frac{u^2}{2} + \frac{\gamma}{\gamma - 1} \frac{P}{\rho} = \text{constant} , \tag{30}$$

which has one more term, the first, than Bernoulli's equation. However, in dealing with general hydrodynamic problems where a gas, not a liquid, is involved, the inertia force due to the unsteadiness of flow is usually small compared with other forces. A proof of this statement in the particular case of the flow through an engine valve is given by Tsu (1). Therefore, the first term in equation 30 is dropped and we get

$$\frac{\gamma}{\gamma-1}\frac{P_i}{\rho_i} = \frac{u_1^{\,2}}{2} + \frac{\gamma}{\gamma-1}\frac{P}{\rho_1}, \tag{31}$$

or

$$u_1^{\,2} = \frac{2\gamma}{\gamma-1}\left[\frac{P_i}{\rho_i} - \frac{P}{\rho_1}\right]. \tag{32}$$

Recalling that

$$C_i = \sqrt{\gamma_i \frac{P_i}{\rho_i}}$$

and

$$\frac{\rho_1}{\rho_i} = \left(\frac{P}{P_i}\right)^{1/\gamma}$$

we get

$$\frac{u_1}{C_i} = \sqrt{\frac{2}{\gamma-1}\left[1 - \frac{P}{P_i}\right]^{\frac{\gamma-1}{\gamma}}} \tag{33}$$

and

$$F_i = \frac{\gamma_i}{2}\frac{\rho_1}{\rho_i}\frac{u_1}{C_i} = \sqrt{\frac{\gamma}{\gamma-1}\left[\left(\frac{P}{P_i}\right)^{2/\gamma} - \left(\frac{P}{P_i}\right)^{\frac{\gamma+1}{\gamma}}\right]}\,. \tag{34}$$

A curve of F_i versus (P/P_m) may now be plotted. Notice that the pressure ratio (P/P_m) is used. This is because the above definition of F is also applicable to small portions of some exhaust processes where gases flow back into the cylinder through the exhaust port. In such cases F is a function of (P/P_e), P_e being the exhaust pressure. Therefore the symbol P_m is used to indicate P_i or P_e, as conditions may dictate.

As can be seen in equation 34, F is a function of both γ, the ratio of specific heats and the ratio P/P_i. Fortunately, by defining the flow factor F as in equation 34, F becomes practically independent of γ for the range $1.25 < \gamma < 1.40$. Therefore, for all values of γ in that range, which covers fuel-air mixtures of all fuels and all fuel-air ratios in practical internal combustion engine operation, a single curve of F versus P/P_m will be sufficient. Although the flow factor increases as the pressure ratio (P/P_m) decreases, there is a critical pressure ratio (P/P_m) at which the flow factor becomes constant and does not increase further with a decrease in the pressure ratio. This is due to the choking of flow through the valve. The ratio at which choking occurs is in the neighborhood of 0.55.

Returning to equation 25, it can be seen that if V and A_i are known functions of θ, the equation is solvable because γ_i and K_i are constants that can be evaluated and the flow factor is a known function of P/P_i. The actual solution of this equation will be treated later.

From equations 23 and 24, we see that

$$K_i \int_{\theta_o}^{\theta} A_i F_i \, d\theta = \gamma_i R_i T_i \int_{W_o}^{W} dW \, .$$

If the integration is taken over the entire intake process, that is from the crank angle θ_o where the inlet valve opens to crank angle θ_c where the inlet valve closes, we have

$$K_i \int_{\theta_o}^{\theta_c} A_i F_i d\theta = \gamma_i R_i T_i \int_{W_o}^{W_c} dW ,$$

and

$$W_t = \int_{W_o}^{W_c} dW = \frac{K_i}{\gamma_i R_i T_i} \int_{\theta_o}^{\theta_c} A_i F_i d\theta , \tag{35}$$

where W_t is the total weight of fresh charge drawn into the cylinder during the intake process. But W_t is the total weight of air plus fuel, and the weight of air alone is

$$W_a = \frac{W_t}{1+f} , \tag{36}$$

where f is the fuel-air ratio.

CHAPTER III

DERIVATION OF EQUATION OF EXHAUST PROCESS

Proceeding in a manner similar to the development of the equation of the intake process, the equation for the exhaust process is now developed.

At time t the cylinder volume is V, the cylinder pressure is P and the absolute temperature of the cylinder contents is T. Considering the cylinder contents as a perfect gas, their weight is

$$W = \frac{PV}{RT}$$

and their internal energy is

$$I = K_V WT \ .$$

Then

$$I = K_V WT = K_V T \left(\frac{PV}{RT}\right) = K_V \frac{PV}{R} \ . \qquad (37)$$

After an infinitesimal length of time dt, the time is t + dt, the cylinder volume is V + dV, the pressure is P + dP and the temperature T + dT. The weight of the cylinder contents becomes W + dW and its internal energy is then

$$I + dI = K_V (W + dW)(T + dT) \ , \qquad (38)$$

where K_V is considered to be constant. Using the perfect gas relation, equation 38 may be written as

$$I + dI = K_V(W + dW)\ (T + dT) = K_V \frac{(P + dP)\ (V + dV)}{R(T + dT)}\ (T + dT)$$

and

$$I + dI = K_V \frac{(P + dP)\ (V + dV)}{R}\ . \tag{39}$$

Subtracting equation 37 from equation 39 and neglecting infinitesimals of the second order,

$$dI = \frac{K_V}{R}\ (PdV + VdP) = \frac{K_V}{R}\ d(PV)\ . \tag{40}$$

Neglecting heat transfer, this change in internal energy in the cylinder occurs because (1) work is done on the piston by the cylinder contents and (2) energy is carried out by the gases passing out through the exhaust valve.

The internal energy per unit weight of gas inside the cylinder is $K_V T$, but owing to the flow work done upon it by the gas remaining in the cylinder, the energy carried out by a unit weight of gas flowing through the exhaust valve is

$$(K_V T + PV) = K_P T\ .$$

Therefore, from the First Law of Thermodynamics,

$$dI = (K_V R)d(PV) = -\ K_V T(dW) - PdV\ . \tag{41}$$

Dividing equation 41 by K_V/R and substituting

$$K_P/K_V = \gamma$$

$$K_P - K_V = R$$

$$R/K_V = \gamma - 1 ,$$

we get

$$d(PV) + (\gamma - 1)PdV = - \gamma RTdW . \tag{42}$$

Now

$$dW = (\frac{dW}{dt})dt$$

and

$$dt = \frac{d\theta}{6N}$$

so that

$$\frac{dW}{dt} = A_e P \sqrt{\frac{2g}{RT}\left(\frac{\gamma}{\gamma-1} \right)\left[\left(\frac{P_e}{P}\right)^{2/\gamma} - \left(\frac{P_e}{P} \right)^{\frac{\gamma+1}{\gamma}} \right]} . \tag{43}$$

Therefore

$$\gamma_e RTdW = \gamma_e RT \; (\frac{dW}{dt}) \; dt$$

and

$$\gamma_e RTdW = \gamma_e RTA_e P \sqrt{\frac{2g}{RT}\left(\frac{\gamma}{\gamma-1} \right)\left[\left(\frac{P_e}{P} \right)^{2/\gamma} - \left(\frac{P_e}{P} \right)^{\frac{\gamma+1}{\gamma}} \right]} \frac{d\theta}{6N}$$

so that

$$\gamma_e RTdW = \gamma_e R_e P_e \sqrt{\left(\frac{2\gamma}{\gamma-1}\right) gRT \left(\frac{P}{P_e}\right)^2 \left[\left(\frac{P_e}{P}\right)^{2/\gamma} - \left(\frac{P_e}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]} \frac{d\theta}{6N} . \quad (45)$$

Since heat transfer is neglected, we may assume that the gas flows adiabatically and reversibly from the cylinders through the valve into the exhaust port. Hence

$$T = T_e\left(\frac{P}{P_e}\right)^{\frac{\gamma-1}{\gamma}} . \quad (46)$$

Here T_e is the exhaust temperature in its usual sense, and is considered to be constant. The exhaust pressure P_e is also constant.

Putting equation 45 in equation 44 and simplifying,

$$\gamma_e RTdW = \gamma_e RT\left(\frac{dW}{dt}\right)dt$$

and

$$\gamma_e RTdW = \frac{\sqrt{2\gamma_e}\sqrt{\gamma_e gRT_e}\ P_e}{6N} A_e \sqrt{\left(\frac{\gamma}{\gamma-1}\right)\left[\left(\frac{P}{P_e}\right)^{\frac{3\gamma-3}{\gamma}} - \left(\frac{P}{P_e}\right)^{\frac{2\gamma-2}{\gamma}}\right]}\, d\theta .$$

But

$$\sqrt{\gamma_e gR_e T_e} = C_e$$

and defining

$$K_e = \frac{\sqrt{2\gamma_e}\ C_e P_e}{6N} \quad (47)$$

$$\overline{F_e} = - \sqrt{\left(\frac{\gamma}{\gamma-1}\right)\left[\left(\frac{P}{P_e}\right)^{\frac{3\gamma-3}{\gamma}} - \left(\frac{P}{P_e}\right)^{\frac{2\gamma-2}{2}}\right]} \, , \tag{48}$$

we get

$$- \gamma_e R_e T dW = K_e A_e \overline{F}_e d\theta \, . \tag{49}$$

Combining equations 42 and 49,

$$d(PV) + (\gamma_e - 1)PdV - K_e A_e \overline{F}_e d\theta = 0 \, ,$$

and integrating from $\theta = 0_o$ to $\theta = \theta$, we get

$$(PV - P_o V_o) + (\gamma_e - 1)\int_{V_o}^{V} PdV - K_e \int_{\theta_o}^{\theta} A_e F_e d\theta = 0 \, . \tag{50}$$

Equation 50, the equation for the exhaust process, is exactly similar to equation 25, the equation for the intake process. The important difference between the equations is in the flow factor $\overline{F}$, about which the following may be said:

(1) Theoretically, equation 43 is valid for steady flow only. The flow through the exhaust valve, like the intake valve, is unsteady. To be rigorous, the equation for the flow factor should have been derived from Euler's equation, not Bernoulli's equation. But as was indicated in the development of the equation for the flow factor for the intake process, the effect of unsteadiness in this application is negligible. Therefore, the equation for steady flow was used immediately in deriving the equation for the flow factor for outward flow.

(2) Although the form of the flow factor for outward flow, equation 48, is quite similar to that of the flow factor for inward flow, equation 34, their physical meanings are different. The flow factor for inward flow is proportional to the rate of flow per unit area, that is, proportional to $(\frac{dW}{dt})\ (\frac{1}{A})$, the mass velocity. From equations 46 to 49, it can be seen that the flow factor for outward flow is a function of the absolute temperature of the cylinder contents and the mass velocity, that is, proportional to $T(\frac{dW}{dt})(\frac{1}{A})$.

(3) The flow factor for outward flow may also be plotted as a function of the pressure ratio (P/P_m) because the flow factor is applicable whenever the flow is outward. That is, it may be used for flow from the cylinder out through the exhaust valve in the exhaust process or for flow from the cylinder out through the intake valve as may occur in some portions of the intake process.

CHAPTER IV

EQUATION FOR THE INTAKE PROCESS, INCLUDING THE EFFECT OF VALVE OVERLAP BUT NEGLECTING HEAT TRANSFER

The equation for the intake process when valve overlap and heat transfer are neglected has been developed (Equation 25). Heat transfer will still be neglected but valve overlap will now be accounted for.

Heat transfer is neglected on the basis of the earlier statement that unless the mean piston speed of the engine is very low, the effect of heat transfer is negligible. Valve overlap will be considered because in modern high speed internal combustion engines, and in particular in the engine being studied in this work, the valve overlap is great. Failure to account for it would lead to errors in both the pressure and mass flow calculations.

Looking at equations 25 and 50, it follows that during the period when both the intake and exhaust valves are open that both of the terms

$$K_i \int_{\theta_o}^{\theta} A_i F_i d\theta$$

and

$$K_e \int_{\theta_o}^{\theta} A_e \overline{F}_e d\theta$$

should be included in the equation for the intake process. Assuming that the gases in the cylinder are well mixed and that there is no short circuit flow between the intake and exhaust ports when both valves are open, we have as the equation for the intake process,

$$(PV - P_oV_o) + (\gamma_i - 1)\int_{V_o}^{V} PdV - K_i\int_{\theta_o}^{\theta} A_i\mathcal{F}_i d\theta - K_e\int_{\theta_o}^{\theta} A_e\mathcal{F}_e d\theta = 0 .$$

The approach used here in this equation is not exactly vigorous since the effects are not strictly additive but the error introduced is small. The flow factor for the intake and exhaust processes are designated by the general flow factor $\mathcal{F}$ to show that the flow may be either inward or outward through either valve, and the particular flow factor, F or $\overline{F}$ would be accordingly used. The subscripts i or e on the general flow factor $\mathcal{F}$ are used to designate that the flow factor is a function of the pressure ratio P/P_i or P/P_e respectively.

Examining further the case of backflow through the intake valve, it is evident that backflow occurs whenever the pressure in the cylinder is greater than the pressure in the intake reservoir. This is likely to occur at two different times during the intake process; first, when the intake valve just begins to open and, second, just before the intake valve closes. The pressure build up and consequent backflow just before the intake valve closes occurs when the intake valve closes late so that it may still be partly open during the upward compression stroke of the piston. If backflow occurs when the inlet valve first opens, what flows out is not fresh charge but residual exhaust gas. If backflow occurs just before the inlet valve closes, some of the fresh charge just drawn into the cylinder is forced back into the intake port.

For the case when residual exhaust gases flow out through the inlet valve, the intake process constant K_i may not be used but must be replaced by a new constant, K_{ei}, the subscript "ei" meaning the expulsion of residual gas into the intake port. Using the same reasoning as in the derivation of the intake and exhaust process constants,

$$K_{ei} = \frac{\sqrt{2\gamma_e}\; C_{ei}\; P_i}{6N} \tag{52}$$

where

$$C_{ei} = \sqrt{\gamma_e g R_e T_{ei}}\;. \tag{53}$$

T_{ei} is the absolute temperature of the residual gas after it has expanded from cylinder pressure P to the inlet pressure P_i. Now

$$\frac{T_{ei}}{T} = \left(\frac{P_i}{P}\right)^{\frac{\gamma-1}{\gamma}} \tag{54}$$

and

$$\frac{T}{T_e} = \left(\frac{P}{P_e}\right)^{\frac{\gamma-1}{\gamma}} \tag{46}$$

so that

$$T_{ei} = T_e\left(\frac{P_i}{P_e}\right)^{\frac{\gamma-1}{\gamma}} \tag{55}$$

Combining equations 52, 53 and 55, simplifying, and using equation 47, we get

$$K_{ei} = K_e \left(\frac{P_i}{P_e}\right)^{\frac{3\gamma-1}{2\gamma}} . \tag{56}$$

Putting this alteration into the equation for the intake process, equation 51, we have

$$(PV - P_oV_o) + (\gamma_i - 1)\int_{V_o}^{V} PdV - \int_{\theta_o}^{\theta} K^*A_i \psi_i d\theta - K_e \int_{\theta_o}^{\theta} A_e \psi_e d\theta = 0$$

where

$$K^* = \begin{cases} K_{ei} & \text{- Used when residual exhaust gas is flowing out through the intake valve} \\ K_i & \text{- Used when fresh charge is flowing in through the intake valve.} \end{cases}$$

K* is now a function of the crank angle and is no longer constant and is therefore included inside the integral sign. Rearranging this equation, we have

$$P = \frac{P_oV_o - (\gamma_i-1)\int_{V_o}^{V} PdV + \int_{\theta_o}^{\theta} K^*A_i \psi_i d\theta + K_e \int_{\theta_o}^{\theta} A_e \psi_e d\theta}{V} , \tag{57}$$

which is the equation for the cylinder pressure P.

In a previous section, equation 35 was developed to calculate the total mass of charge drawn into the cylinder during the intake process. This equation was based on the idealized intake process having no valve overlap and neglecting heat transfer. This equation will now be altered to account for the effect of the residual exhaust gas which may be expelled into the intake port.

Returning to the method to calculate the mass flow in an earlier section, we have, for the period when the residual gas is flowing,

$$K_{ei} A_i F_i d\theta = \gamma_e R_e T dW \ . \tag{58}$$

But

$$T = T_e \left(\frac{P}{P_e}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= T_e \left(\frac{P_i}{P_e}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P}{P_i}\right)^{\frac{\gamma-1}{\gamma}}$$

so that

$$K_{ei} A_i F_i d\theta = \gamma_e R_e T_e \left(\frac{P_i}{P_e}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{P}{P_i}\right)^{\frac{\gamma-1}{\gamma}} dW$$

and

$$dW = \frac{1}{\gamma_e R_e T_e} \ \frac{K_{ei}}{\left(\frac{P_i}{P_e}\right)^{\frac{\gamma-1}{\gamma}}} \ \frac{A_i F_i}{\left(\frac{P}{P_i}\right)^{\frac{\gamma-1}{\gamma}}} \ d\theta \ .$$

Integrating from $\theta = \theta_o$ to $\theta = \theta_x$, where θ_x is the crank angle at which the residual gas stops its outward flow through the intake valve, we have

$$W_r = \int_{\theta_o}^{\theta_x} dW = \frac{1}{\gamma_e R_e T_e} \quad \frac{K_{ei}}{\left(\frac{P_i}{P_e}\right)^{\frac{\gamma-1}{\gamma}}} \int_{\theta_o}^{\theta_x} \frac{A_i \mathcal{F}_i}{\left(\frac{P}{P_i}\right)^{\frac{\gamma-1}{\gamma}}} \, d\theta \, , \tag{59}$$

where W_r is the weight of residual gas that passed out through the intake valve.

Returning to equation 35, we can now calculate the total amount of flow into the cylinder if we integrate from $\theta = \theta_x$ to $\theta = \theta_c$:

$$W_t = \frac{K_i}{\gamma_i R_i T_i} \int_{\theta_x}^{\theta_c} A_i \mathcal{F}_i d\theta \, . \tag{60}$$

Equation 60 represents the total amount of flow into the cylinder, including the residual exhaust gas which was temporarily stored in the intake port. The total weight of fresh charge drawn into the cylinder is then

$$W_f = W_t - W_r \, , \tag{61}$$

and the total weight of fresh air drawn into the cylinder is

$$W_{fa} = \frac{W_f}{1 + f} \, , \tag{62}$$

where f is the fuel-air ratio.

CHAPTER V

PROCEDURE

Equations 57, 59 and 60 will now be solved, but an important restriction placed on these equations earlier is now reiterated. In the derivation of these equations, section "i" was defined to be a section distant enough from the inlet part so that the velocity of flow of the fluid is negligible. On an actual engine, we could not call the intake manifold section "i" because the velocity of the air-fuel mixture in the manifold is quite high and also the pressure may not be constant. In the solution of these equations, the pressure P_i is varied but it should be remembered that this is not the same as varying the intake manifold pressure. P_i may be considered to be the pressure in a very large reservoir from which the engine is drawing air and in which the air velocity is negligible. This reservoir will henceforth be referred to as the intake reservoir.

In order to solve equation 57, the only engine data needed are:

(1) a curve of V versus θ

(2) a curve of the valve lifts versus θ

(3) a curve of A_i versus θ

(4) a curve of A_e versus θ

(5) the diameter of the intake and exhaust valves.

The third and fourth curves listed above were obtained using curves of valve lift versus crank angle and curves of flow coefficients through the valves versus valve lift (3). These flow coefficients can be accurately obtained by means of a model test under steady flow conditions. It is allowable to use flow coefficients obtained under steady flow conditions because several investigators have shown that the effects of unsteadiness are negligible (1, 3, 5).

Although the flow coefficients could have been determined experimentally for the engine under consideration, this procedure was not used in this work. Instead of experimentally determining these values, this author used the average value of several flow coefficient versus the ratio of valve lift to the valve diameter curves (3) to determine the effective inlet and exhaust valve areas for this engine. This represents only an approximation but it is a close approximation as the flow coefficient versus lift/diameter curves were all of the same basic shape and magnitude over a wide range of valve sizes and configurations and an equally wide range of lifts.

Several assumptions were made about the operating conditions of the engine. The engine was constrained to operate at a fixed intake reservoir temperature and exhaust temperature, the values being 100°F and 1540°F respectively. It was also assumed that in the range of engine speed from 1000 rpm to 2000 rpm, the initial cylinder pressure and exhaust pressure were atmospheric. At 3000 rpm these pressures were assumed to be one inch of mercury above atmospheric and at 4000 rpm they were assumed to be two inches of mercury above atmospheric.

Equation 57 was solved on an analog computer at the Georgia Institute of Technology Engineering Experiment Station's Analog Computer Laboratory.

The actual programming was done by the Laboratory personnel. When the equation was programmed, the information necessary for its solution was placed in the machine under three separate classifications. These were:

(1) curves of constant data

(2) values of constants

(3) engine operating parameters.

The curves of constant data applied to all engine operating conditions, that is, over all ranges of speed and manifold pressures. These were the curves of A_i versus θ, A_e versus θ, V versus θ, and ψ versus P/P_m. The constants were the temperatures T_i and T_e, the gas constants γ_i and γ_e, and the ratios of specific heats γ_i and γ_e. The engine operating parameters are the variables which describe the various engine operating conditions together with the variables in the pressure and mass flow computations which vary according to these operating conditions. These parameters are the pressures P_i, P_e and P_o, the engine rpm, and the variables K_i, K_e and K_{ei}, with one set of values of these parameters describing a particular engine operating condition.

As the computer solved the equation, it plotted a curve of the pressure P versus the crank angle   for the set of running conditions as dictated by the given parameter values.

The mass flow calculation was also performed on the analog computer. In the mass flow calculation, equations 59 and 60 were combined to form a separate equation for W as follows:

$$W = \begin{cases} W_r - \theta_o \leq \theta < \theta_x \\ W_t - \theta_x < \theta \leq \theta_c \end{cases}$$

As this calculation was performed, a curve of W versus θ was plotted.

CHAPTER VI

DISCUSSION OF RESULTS

Since the effects of momentum or inertia charging were not included in this analysis, the air flow should decrease with an increase in engine speed when the intake reservoir pressure is constant. The results of the air flow calculation were plotted and it is seen that they follow the expected trend.

As an approximate check of the magnitudes of the calculated air flow, the 1000 rpm line of the calculated air flow was compared with the 1000 rpm line of the measured air flow for the same engine. The 1000 rpm lines were chosen because it is the lowest speed at which the air flow was calculated and in the actual engine there are less momentum charging effects than at a higher engine speed. The calculated air flow at 1000 rpm was approximately 13.5% less than the measured flow at nearly all the intake reservoir pressures. This is considered to be a good correlation.

A reproduction of a plot of the pressure and mass flow records made by the coordinate plotter of the analog computer is also included in Appendix A. Examination of the pressure records as a function of crank angle shows that these curves are in agreement with the desired results. The desired trend was that at a constant intake reservoir pressure, the cylinder pressure in the early stage of the intake process should increase with an increase in engine speed, but that once the cylinder pressure has dropped below the reservoir pressure, the cylinder pressure should decrease with an increase in engine speed.

The results of both the pressure and mass flow calculations agree closely with the desired results.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

It may be concluded that the method used by Tsu (1) to calculate the air flow into a single cylinder engine may be used to calculate the air flow into a multicylinder automotive engine. It is recommended, however, that in any future work of this nature that the effective valve opening areas should be experimentally determined. It is further recommended that the effects of momentum charging be studied and included in the analysis. This would not only improve the accuracy of the calculation but it would also more closely describe the conditions encountered in a modern multicylinder automotive engine.

APPENDIX A

TABLE I

Engine Data

Make and Model:	1959 Chevrolet 283 cubic inch V-8.
Bore X Stroke:	3.875 inches X 3.000 inches.
Compression Ratio:	9.5:1.
Intake Valve Diameter:	1.720 inches.
Exhaust Valva Diameter:	1.500 inches.
Intake Valve Timing:	Open 35° BTDC
	Close 72° ABDC
Exhaust Valve Timing:	Open 76° BBDC
	Close 31° ATDC
Special Equipment:	Equipped with Special Cam and Ram Manifold for Use with a Fuel Injection System.

TABLE II

Operating Condition Constants

Intake Reservoir Temperature:	100°F.
Exhaust Temperature:	1540°F.
Gas Constant, Fuel-Air Mixture:	609 $\frac{in}{°R}$.
Gas Constant, Exhaust Gas:	650 $\frac{in}{°R}$.
Ratio of Specific Heats, Fuel-Air Mixture:	1.322
Ratio of Specific Heats, Exhaust Gas:	1.300

Table III

Calculated Air Flow

Intake Reservoir Pressure ("Hg)	Engine Speed (rpm)	$W_r \times 10^{-4}$ (Lb.)	$W_t \times 10^{-4}$ (Lb.)	$W_f \times 10^{-4}$ (Lb.)	$W_{fa} \times 10^{-4}$ (Lb.)
10	1000	0.30	3.18	2.88	2.70
10	2000	0.28	3.08	2.80	2.63
10	3000	0.26	2.84	2.58	2.42
10	4000	0.30	2.48	2.18	2.05
15	1000	0.25	4.92	4.67	4.38
15	2000	0.18	4.73	4.55	4.27
15	3000	0.20	4.36	4.16	3.90
15	4000	0.25	3.67	3.42	3.21
20	1000	0.16	6.48	6.32	5.93
20	2000	0.14	6.26	6.12	5.74
20	3000	0.16	5.85	5.69	5.34
20	4000	0.19	5.00	4.81	4.51

Table III (Continued)

Calculated Air Flow

Intake Reservoir Pressure ("Hg)	Engine Speed (rpm)	$W_r \times 10^{-4}$ (Lb.)	$W_t \times 10^{-4}$ (Lb.)	$W_f \times 10^{-4}$ (Lb.)	$W_{fa} \times 10^{-4}$ (Lb.)
25	1000	0.11	8.62	8.51	7.98
25	2000	0.10	8.44	8.34	7.82
25	3000	0.12	7.90	7.78	7.30
25	4000	0.14	6.38	6.24	5.85
28	1000	0.06	9.44	9.38	8.80
28	2000	0.07	9.41	9.34	8.75
28	3000	0.11	8.53	8.42	7.90
28	4000	0.12	7.17	7.05	6.61

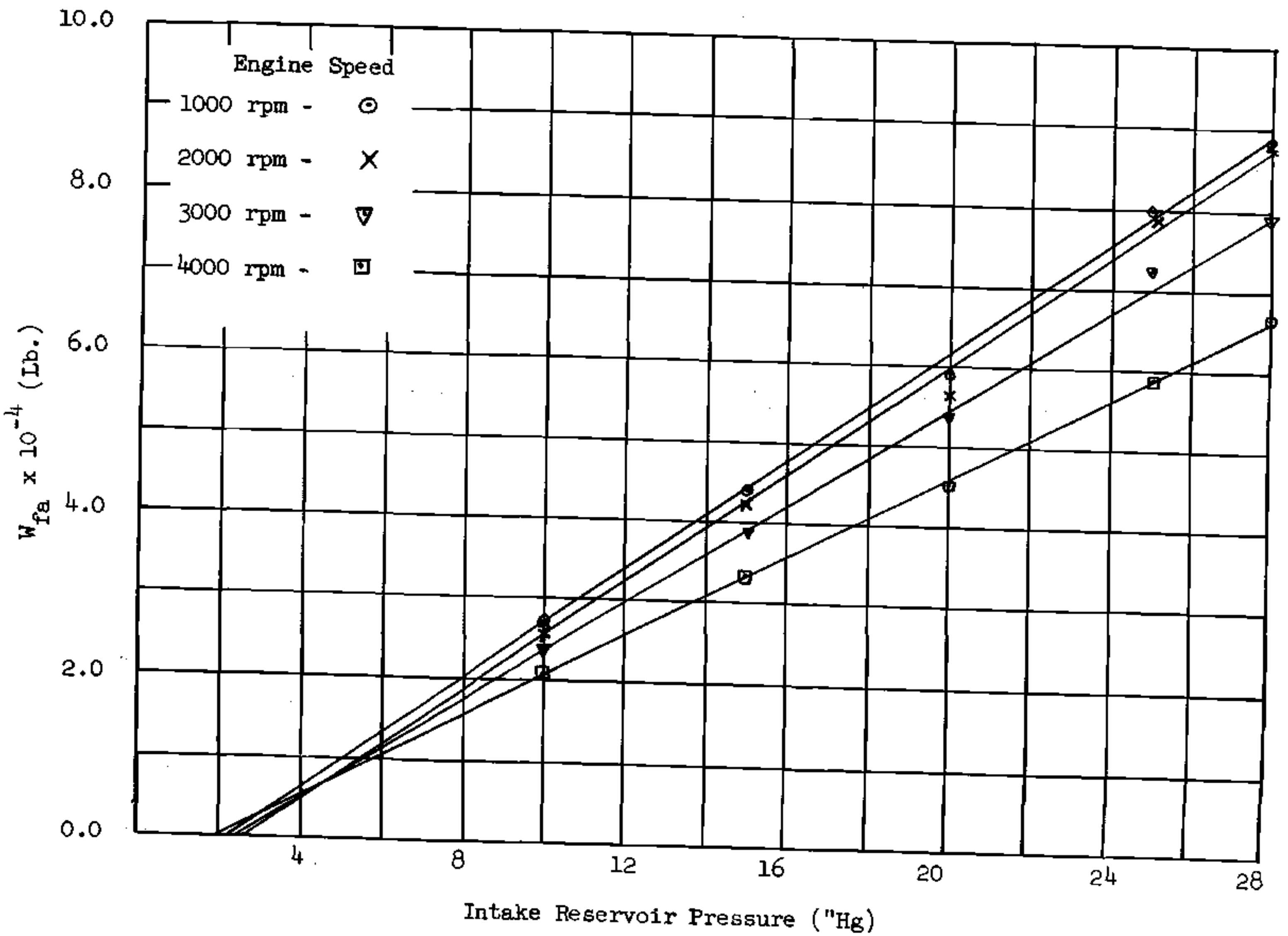

Figure 1. Calculated Air Flow for 283 Cubic Inch Chevrolet V-8 Engine

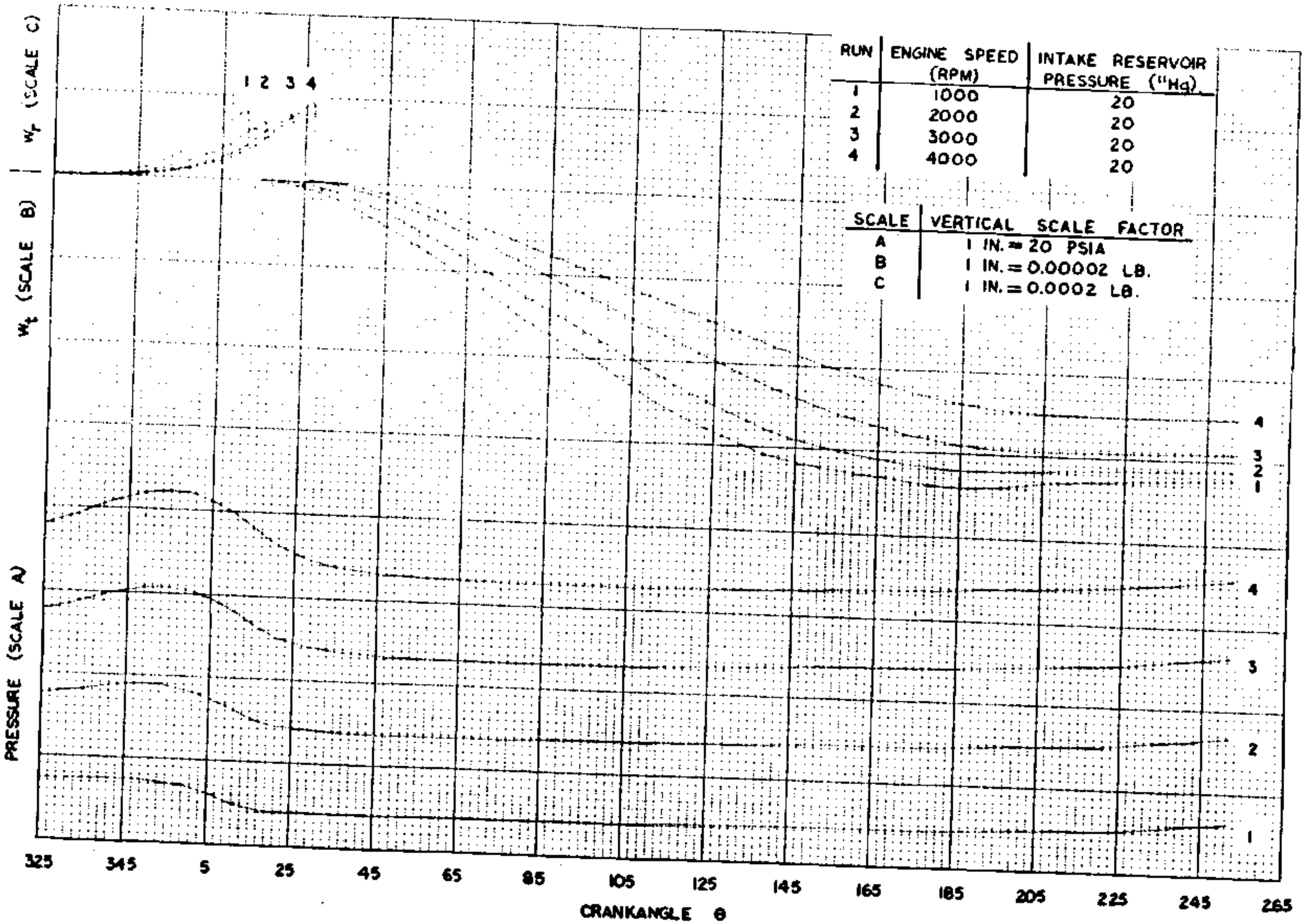

FIGURE 2. CALCULATED PRESSURE AND MASS FLOW VERSUS CRANKANGLE θ

APPENDIX B

NOMENCLATURE

A	Effective valve opening area	$in.^2$
C	Velocity of sound	in./sec.
f	Fuel-air ratio	dimensionless
F	Flow factor for inward flow	dimensionless
$\bar{F}$	Flow factor for outward flow	dimensionless
$\mathcal{F}$	General flow factor	dimensionless
g	Gravitational acceleration	$in./sec.^2$
I	Internal energy	Btu
K_e	Exhaust process constant	lb./in.sec.rpm
K_i	Intake process constant	lb./in.sec.rpm
K*	General intake process constant	lb./in.sec.rpm
K_V	Specific heat at constant volume	Btu/lb.°F.
K_P	Specific heat at constant pressure	Btu/lb.°F.
N	Engine speed	rpm
P	Pressure	psia
R	Gas constant	in./°R
t	Time	sec.
T	Absolute temperature	°R
u	Velocity	in./sec.
V	Cylinder volume	$in.^3$
W	Weight	lb.
x	Distance along streamline	in.

NOMENCLATURE (Concluded)

γ	Ratio of specific heats	dimensionless
θ	Engine crank angle	degrees
ρ	Mass density	lb./in.^{3}

Subscripts

c	Denotes end point, which is taken to be the point where the intake valve closes
e	Denotes exhaust process
ei	Denotes expulsion of residual exhaust gas through the intake valve
i	Denotes intake process
m	Used to designate P_i or P_e, whichever applies
o	Denotes starting point, which is taken to be the point where the intake valve opens

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