A MATHEMATICAL PREDICTIVE MODEL OF ARM STRENGTH

A THESIS

Presented to

The Faculty of the Division of Graduate Studies

By

Robert Spencer Lower

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Operations Research

Georgia Institute of Technology

June, 1976

A MATHEMATICAL PREDICTIVE MODEL OF

ARM STRENGTH

Approved:

R. K. Schutz, Chairman

R. G. Heikes

T. L. Sadosky

H. M. Wadsworth

Date approved by Chairman 9 June 1976

ACKNOWLEDGMENTS

I would like to express my appreciation to the United States Army for providing me the opportunity to attend graduate school and to the Human Engineering Laboratory, Aberdeen Proving Grounds, Maryland for extending contractual support for this research.

My sincerest appreciation goes also to Dr. Rodney K. Schutz, the chairman of my thesis committee, for the many long hours he spend guiding me through the development of this work, and to Dr. Thomas L. Sadosky, Dr. Russell G. Heikes, and Dr. Harrison M. Wadsworth for their assistance as members of my reading committee. My thanks goes also to Dr. Douglas C. Montgomery who, although not on my committee, provided additional guidance on experimental design and response surface methods.

Finally, my deepest gratitude must be extended to Elke, Kristyn, and Tracey Ann, the three women in my life, whose patience and understanding during the many hours I neglected them, are forever appreciated.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

SUMMARY

Arm strength studies have typically been done for arm positions in the frontal and sagittal plane. Little is known concerning arm strength behavior in three dimensions or concerning strength variation in the transverse plane. This research consists of four phases:

1. Design of an arm strength testing apparatus,
2. Collection and statistical analysis of preliminary data on three dimensional arm strength behavior,
3. Examination of transverse plane strength and determination of possible mathematical forms for a predictive equation, and
4. Final development of a useful predictive model for arm strength in the transverse plane.

Two separate studies were conducted, called Study I and Study II. In Study I, tests were made on 18 handle locations in representative portions of the reach sphere, each location uniquely determined by a vertical plane passing through the center of the chair. (sagittal, frontal, or 45 degree planes) and a vertical distance or height above the seat level. (0, 20, or 40 inches) Each of the 18 handle locations were tested at three distances from the body (near, mid-range, far), in three forearm rotations (pronation, mid-position, supination), and in six directions of force (left, right, up, down, push, pull). A total of 972 tests were evaluated for one subject.

Strength was found to be greatest 20 inches above the chair seat, decreasing as the hand is raised or lowered from that level. Greatest

972 tests were evaluated for one subject.

Strength was found to be greatest 20 inches above the chair seat, decreasing as the hand is raised or lowered from that level. Greatest forces can generally be exerted in the right front quadrant of the reach sphere, with lowest forces exerted with the hand directly behind the head. All strength values are greatly dependent upon the direction of force exertion.

Study II consisted of an analysis of push forces exerted in the 20 inch transverse plane, forearm in mid-position. In the transverse plane, strength varies parabolically with the distance of the hand grip from the SRP. A computationally simple mathematical equation that will predict arm strength as a function of spacial coordinates of the hand was derived.

CHAPTER I

INTRODUCTION

Statement of the Problem

The assessment of static muscular strength has been an area of interest to both the military and civilian industrial communities for decades. Static strength data can provide a method of predicting the strength required to operate a hand control, and can help determine if a given physical task can be accomplished. Human strength assessment is also needed to establish population norms for the better design of machines and tools.

Some of the emphasis design engineers are now placing on static strength assessment originated from the requirements of new types of military, aircraft, and electronics equipment and space vehicles. The need to design these systems to optimize their use by human beings has led to wider application of strength data to other types of vehicles and equipment. The application of strength data has been, however, fairly restricted; there are many vehicles and items of equipment in use today by the military and civilian communities that could have benefited from more extensive and systematic attention to the design of human operated controls.

The military and civilian handbooks on equipment design provide arm strength data only for handle locations in the mid-sagittal plane or para-sagittal plane through the shoulder. Data on effects of changing the direction of force are also provided only for sagittal plane exertions. There is, then, a need to provide more extensive strength data for the design handbooks.

There is increasing emphasis in the field of human factors engineering on quantitative methods as a basis for developing principles or guidelines for equipment design. In keeping with this emphasis, it would be helpful to provide the designer with, not only additional data on arm strength, but also a concise mathematical model that will provide reasonably accurate arm strength predictions in situations not yet explored experimentally.

This study, sponsored by the U.S. Army Human Engineering Laboratory, Aberdeen Proving Grounds, Maryland, was initiated with the intention of providing information on human strength assessment that can be of value in the design and construction of vehicles and equipment requiring human operators.

Objectives

There were four objectives to this research.

Objective 1

Design and build an arm strength testing apparatus capable of testing strength in any handle location around the body, any forearm rotation, and any force direction.

Objective 2

Collect preliminary data on arm strength to provide a general understanding of strength variation in three dimensions, and to develop general guidelines and recommendations for use of this data in the design of hand controls.

Objective 3

Determine the mathematical form of any possible strength prediction equations and determine the feasibility of developing a mathematical model. Such models should be simple enough to be useful to the designer, require only hand computations, and yet provide reasonably accurate strength estimates.

Objective 4

If feasible, develop a mathematical model for a subset of the reach sphere data, and assess its applicability in industrial design.

Test Conditions

Two studies were conducted to meet the above objectives, Study I and Study II. For study I, tests were made on 18 handle locations selected in representative portions of the reach sphere. Each handle location is uniquely determined by a vertical plane passing through the center of the chair (sagittal, frontal, or 45 degree planes), and a vertical distance or height above the seat level. (0, 20 or 40 inches) Two exceptions are the overhead position which is determined by the intersection of the frontal and sagittal planes through the SRP, and the location behind the head which is 30 inches above the chair seat. Each location around the body was tested at three distances from the body (near, mid-range, far). At each handle location, three forearm rotations were tested (pronation, mid-position, supination), and six directions of force examined (left, right, up, down, push, pull). These test conditions thus provided 972 (18 · 3 · 3 · 6) data points representing most segments of the reach sphere.

Study II consisted of a more detailed analysis of push forces in the transverse plane 20 inches above the chair seat, with the forearm in mid-position. This test combination was selected because it is a common one which arises frequently in military and industrial equipment operation. A total of 34 models were fit to the data.

In both studies, no harnesses or body restraints were used during testing. The subject was allowed to assume a "normal-natural" position for each force exertion. Tests were conducted on a single subject representing approximately the 84th percentile of static strength scores.

Conclusions

Of the three heights tested, strength was found to be greatest 20 inches above the chair seat, decreasing as the hand is raised to 40 inches or lowered to seat level. The effect of forearm rotation and horizontal position on strength are extremely dependent on the direction in which the force is exerted. In all handle locations, pulling produced the greatest forces, with the lowest forces exerted to the left and right. The preliminary data collected in Study I were used to develop general guidelines for the design of hand controls positioned anywhere in the operator's reach sphere.

The results of Study II indicate that strength varies in the transverse plane parabolically with the distance of the hand grip from the SRP (Seat Reference Point). A computationally simple mathematical equation that will predict arm strength as a function of spacial coordinates of the hand is presented. It also appears feasible to extend the equation to three dimensions without greatly increasing the complexity of the model. The equation which can be used to predict strength in the transverse plane, 20 inches above seat level, forearm in mid-position is:

$$\text{Log}_e(\hat{S}) = 2.837 - .003438(X - 10.5)^2 - .002649(Y - .5)^2 + .137\sqrt{(X-10)^2 + (Y-.1)^2} \qquad (1.1)$$

Order of Reporting

Chapter II provides a review of the current literature on strength testing and compares the results of previous tests of arm strength. The experimental procedures followed and a description of the equipment that was designed and built for use in these tests is discussed in Chapter III. Chapter IV presents the data collected and experimental results, with a more detailed discussion of these results provided in Chapter V. Additional conclusions of this research and recommendations for their application are presented in Chapter VI.

CHAPTER II

LITERATURE REVIEW

An abundance of literature exists today on strength testing. This chapter provides a consolidation of the literature as it relates to testing methods, arm strength data, and the interpretation of arm strength test results.

Terminology

Muscular Exertions

Three types of muscular effort will be referenced: Isometric, concentric, and eccentric. Isometric denotes an effort in which the length of the muscle remains constant as the tension of the muscle varies. Isotonic contraction is constant tension as the length of the muscle varies. Concentric indicates that the muscle shortens actively against an applied resistance; eccentric indicates that the muscle is lengthened by an external force applied to it.

Joint Angle Measurement

Measurement of angles at the joints can be somewhat confusing when it is noted that so many researchers will use procedures which facilitate reporting of their particular experiment, rather than using a standard reference system. For this review, all angle measurements will be referenced from the anatomical position.

The anatomical position is the erect standing position, arms at the sides, palms facing forward. From this position, flexion at any joint will be ventral bending which decreases the angle between the body parts. Thus, for shoulder flexion, 0 degrees will refer to the position of the arm parallel with and held along side of the body; 180 degrees refers to the arm pointing directly overhead. An elbow angle of 180 degrees refers to the arm held straight at the elbow. Extension, conversely, will be an increase in the angle between the body parts. Abduction is the movement of a body segment away from the midline of the body; adduction is movement toward the midline. Horizontal adduction is a similar movement executed in the transverse plane through the shoulder, 90 degrees being the arm directly out to the side, level with the shoulder in the transverse plane. 0 degrees horizontal adduction would thus be the arm level with the shoulder, parallel to the sagittal plane.

Forearm Rotation

Forearm rotations (sometimes referred to as forearm positions) are of three types: pronation, supination, and mid-position (semi-pronation). Pronation is the position with the thumb pointing toward the mid-sagittal plane, supination the position with the thumb pointing away from the mid-sagittal plane, and mid-position the rotation with the thumb pointing parallel to the sagittal plane.

Regardless of the original terminology used, each study referenced herein will be labeled to allow these quite standard terms to be properly applied.

Strength Testing Methods

The simplest technique for objective muscle testing that is widely used in clinical work is the spring balance method developed by Lovett and Martin (1916). Clarke, Elkins, Martin and Wakim (1950), in a study of normal muscles, measured the amount of tension applied to a cable with a specially adapted tensiometer. This instrument worked on the spring principle, but was geared to be sensitive to contractions which are almost isometric. Wakim, Gersten, Martin and Elkins (1950), noted some disadvantages of a spring balance system, such as high inertia or internal resistance, which could cause variations in recorded results due only to the measuring device; such systems also suffer from a low range and often changing level of sensitivity.

Among the many types of pressure systems employed is the myometer. Newman (1949) designed such an instrument consisting of a pressure gauge set in a small cylinder, from the end of which extends a short shaft and a pressure-transmitting device. A built-in hydraulic pressure converter transmits the linear force exerted on the device to a pressure gauge. There are no springs, cams, levers, or cables to transmit this force from the device to the gauge. The myometer could be used to measure resistance of a muscle in isometric contraction and could be used for most muscle groups.

More recent methods involve the use of electrical strain gauges, the advantages of which are low inertia and a wide range of sensitivity. Wakim, et. al. attached strain gauges to a ring attached to a cable. Two gauges were attached on the inner side and two on the outer side of the ring. Tension on the cable produced changes in strain gauge resistance which were recorded on an oscillograph. (Wakim, et. al., 1950) Results obtained were similar to those found by Clarke using a cable tensiometer. (Clarke, et. al., 1950) Another comparison between strain gauge and cable tensiometer methods was done by Kennedy (1965). He found the results to be approximately equal in recorded values.

Schanne (1972) used strain gauges in a manner similar to Wakim, et. al. to conduct tests for muscle strength in three dimensions. He used a technique known as photogrammetry to monitor subject position. Three mutually perpendicular cameras take simultaneous photographs of a subject while he is exerting his maximal isometric strength for a given muscle group. It is thus possible to determine from the photographs the exact spacial coordinates of any point on the subject's body. This technique allows very accurate recording of the subject's position on each trial, a problem which can be even more critical than usual when exertions are in three dimensions.

Strength Data

A number of studies have been conducted to determine the amount of strength which can be applied when the body parts are in different positions or orientations. These studies are herein presented based upon the type of strength tested. Since most researchers have studied more than one strength parameter, Table 1 is provided to facilitate comparisons between studies, and to provide in summary form the basic experimental design and results of these studies. It may also provide a clearer understanding of the test positions used in various experiments. Entries in Table 1 are in alphabetical order. Strength curves from some of the more prominent studies have been plotted on the same axis in Figures 1 through 6 for easy comparison. Thus the strength data surveyed in this chapter may be quickly found either by reference to muscle group in the

Table 1. Tabular Comparison of Major Strength Study Results

SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Asmussen, Heebol-Nielsen, 1962	Average of 25 muscle groups	general test of over-all strength	360 male 250 females ages 15 to 60 years		See Figure 11.
	Arm strength measured separately	General arm strength			See Figure 12.
Campney & Wehr, 1965	Right shoulder	Shoulder flexion	23 males 19 females	Cable-tensiometer supine subject shoulder in 180 degree extension exertions in sagittal plane.	See Figure 3. Also:Study compares men & women, concluding only that strength curves are of same form, but women much lower throughout movement range.
Clarke, et. al, 1950	Right elbow	Elbow extension	64 male college	Same as Campney & Wehr study above	Strongest position at 140 degrees followed by decrease at full extension. Low point of strength at point of maximum possible elbow flexion.
		Elbow flexion			See Figure 1.
	Right shoulder	Shoulder flexion		Supine subject, upper arm in mid-frontal plane, forarm in vertical plane	See Figure 3.

Table 1. Tabular Comparison of Major Strength Study Results (Continued)

SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Clarke, et. al., 1950	Right shoulder	Shoulder extension			See Figure 4.
		Shoulder adduction		Exertions in frontal place while supine. Elbow flexed as much as possible, and kept in same sagittal plane as the humerus	See Figure 5.
		Shoulder abduction		Same as adduction	See Figure 6.
Doss & Karpovich, 1965	Right elbow	Elbow flexion	37 male college juniors	Wall mounted dynamometer using a load cell and oscilograph on standing subject.	See Figure 1. Also: Compares eccentric, concentric and isometric contractions. Concentric force is 23% smaller and eccentric force is 13.5% greater than isometric.
Downer, 1953	Right elbow	Elbow flexion	30 adult women-20 50 years	Beasley Myo-dynamometer, supine subject	See Figure 1. Also: Compares right arm to left. Right values slightly higher. Also compares strength in different forearm rotations. Greatest in mid-position, supinated next, 3.6 lbs less.

Table 1. Tabular Comparison of Major Strength Study Results (Continued)

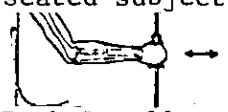
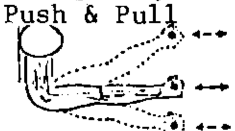
SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Elkins, et. al., 1951	Right elbow	Elbow flexion	14 females 10 males	Strain gauge on supine subject. Upper arm(shoulder) in 180 degree extension-along side of body.	See Figure 1. Also: Compares men & women, & compares 3 forearm rotations. Greater force in mid-position, then supination. Women's scores about 50% of men.
		Elbow extension		All exertions against gravity, i.e. shoulder at 90 degree flexion subject supine.	See Figure 2. Due to gravity effect, readings lower than would have been expected. Also: Same additional comparisons as above.
	Right shoulder	Shoulder abduction	13 male 10 female	Seated subjects	See Figure 6. Also: Same as above
			10 female 7 male	Supine subjects	Force greater with eblow flexed than extended.
Hugh-Jones 1946	Left, right arms	Horizontal push	2 male, 26 & 27 years old	300 lb spring balance with seated subject  Push & Pull  3 Of 4 vertical planes considered	See Figure 7.

Table 1. Tabular Comparison of Major Strength Study Results (Continued)

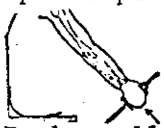
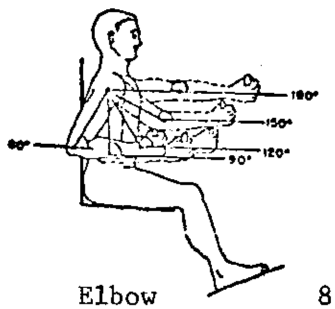
SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Hugh-Jones, 1946	Left, right arms	Horizontal pull			See Figure 8.
		45 degree upward pull 			See Figure 9. Height of hand grip has greater affect than distance from the seat back rest.
Hunsicker, 1955	Right, left arms	Push, pull adduction, abduction, up, down 	55 males	Tensiometer on seated and prone subjects. Exertions in sagittal plane.	See Figure 10. Also: Prone position found to be approximately 71% as strong as sitting position.
Provins & Salter, 1955	Preferred elbow	Elbow flexion	8 males 4 females	Strain gauge dynamometer on seated subject.	See Figure 1. Also: Compares different forearm rotations. Results here are for both flexion and extension.
		Elbow extension			See Figure 2.

Table 1. Tabular Comparison of Major Strength Study Results (Continued)

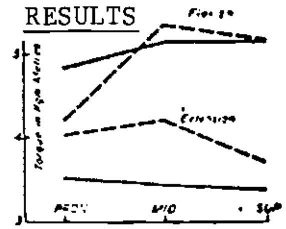
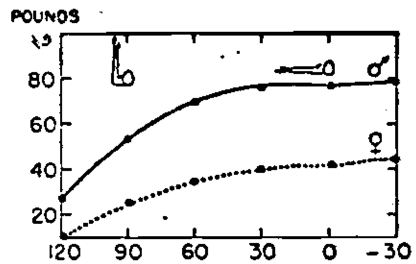
SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Provins & Salter, 1955	Preferred elbow	Elbow flexion & extension			
Schanne, 1972	Right elbow	Elbow flexion	1 male	Orthogonal photogrammetry, strain gauge, seated subject.	See Figure 1. This figure is graph of sagittal plane reduction of: $T_E=336.294+2.088\alpha_E-3.364\alpha_{VS}+.019\alpha_{VS}$. Variables as defined in chapter II. In sagittal plane, $\alpha_{VS}=0$.
		Elbow extension			See Figure 2. Regression equation: $T_E=264.153-.575\alpha_E-.425\alpha_{VS}$ In sagittal plane, $\alpha_{VS}=0$
	Right shoulder	Shoulder flexion			See Figure 3. Regression equation: $T_S=227.338+.525\alpha_E-.372\alpha_{HR}-.296\alpha_{VS}$ In sagittal plane, shoulder flexion is: $T_S=321.9-.296\alpha_{VS}$

Table 1. Tabular Comparison of Major Strength Study Results (Continued)

SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Schanne, 1972	Right shoulder	Shoulder extension			See Figure 4. Regression equation: T_S=208.5 - .0990α_{VS}
		Shoulder abduction			See Figure 6. Regression equation: T_S=227.338 + .525α_E-.372α_{HR}-.296α_{VS}
		Shoulder adduction			See Figure 5. Regression equation: T_S=149.392 - .161α_{HS}+.0086α^2_{HS} - .099α_{VS}
Singh & Karpovich, 1965	Right elbow	Elbow flexion	20 male students	Dynamometer using four SR-4 type strain gauges on a lever arm	See Figure 1. Also: Compares eccentric, concentric and isometric contractions. Isometric force is 41.64% greater for flexors than extensors. Isometric force less than eccentric and greater than concentric. Elbow flexion regression equation: $F=.03+.9087A-.004272A^2$

Table 1. Tabular Comparison of Major Strength Study Results (Continued)

SOURCE	JOINT OR MUSCLE GROUP	TYPE OF EXERTION	SAMPLE SIZE	MEASUREMENT TECHNIQUE	RESULTS
Singh & Harpovich, 1965	Right elbow	Elbow extension			See Figure 2. Regression equation: $F=.1 + .682A -.003926A^2$
Singh & Karpovich, 1968	Right elbow	Elbow flexion	12 male 11 female	Same as 1965 study	See Figure 1. Also Compares eccentric, concentric and isometric contractions. Same result as above. Preferred vs non-preferred found to be non-significant in general. Isometric strength of women was 44% of men.
Williams, et. al. 1959	Right elbow	Elbow flexion	10 adult men. In children's tests, 10 boys, 10 girls	Cable tensiometer, standing subject. Sling at the wrist	See Figure 1.
	Right shoulder	Shoulder flexion			See Figure 3.
		Shoulder extension			See Figure 4.
		Shoulder horizontal adduction			



body of the chapter, or by author in Table 1.

Lumbar Pull

Bedford and Warner (1937) confirmed earlier studies that strength of lumbar pull is strongly influenced by the height of the handle above the ground. These researchers found that strength of a standing subject was at a maximum with the handle at the height of the index finger.

Elbow Flexion (Figure 1)

Clarke, Elkins, Martin and Wakim (1950) found that a plateau existed at the strongest position for pulling at elbow angles of 100 to 140 degrees. The low point of strength was at the point of greatest possible flexion. In Clarke's study, the subject's upper arm was in the mid-frontal plane, his forearm in the vertical plane. All tests were made with the forearm in mid-position. The direction of pull was parallel to the sagittal plane.

Elkins, Leden and Wakim (1951) found muscle power to be greatest during elbow flexion of 80-90 degrees. The strength curves were basically the same as earlier studies. Observations were made under identical conditions as in Clarke's study, at elbow angles of 60, 80, 90, 100, and 120 degrees. A strain gauge was used to measure the strength of muscular contraction.

Williams and Stutsman (1959) corroborate previous studies on elbow flexion. A maximum force of 86 pounds was found at an elbow angle of 90 degrees.

In experiments by Downer (1953), elbow flexion at three points from 10 to 30 degree intervals was determined. The highest mean strength was found to be at 90 degrees of elbow flexion with a decrease in either

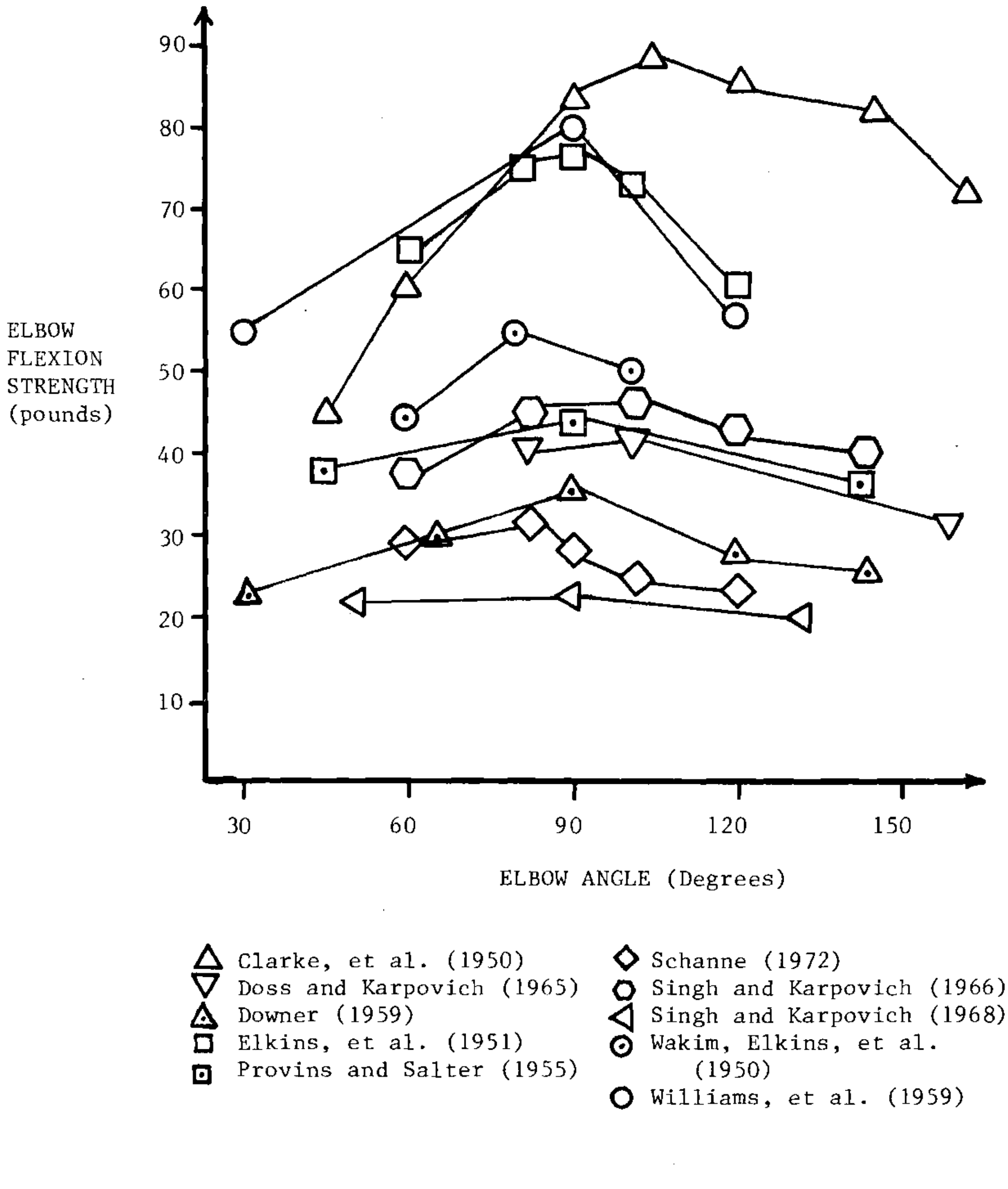


Figure 1. Elbow Flexion Strength Curves

direction away from a right angle. Tests were run on subjects supine on the table, arms at the sides.

Provins and Salter (1955) ran tests to determine the importance of body fixation, posture, and points of force application in limiting the strength of elbow flexion and extension. The strength of flexion was found to be approximately 150 percent that of elbow extension. (Figures 1,2) The maximum force exerted was found to occur at 90 degrees. A strain gauge dynamometer was used with an elbow rest and a single bar attached to the dynamometer. The bar mounted either an adjustable wooden handle of one inch grip diameter, or a steel wrist cuff lined with sorbo-rubber. The subject was in an adjustable seat, his arm at his side. A curved backrest was provided in the lumbar region. In the first experiment a T-handle was used and elbow flexion and extension were tested at three angles. The second experiment with a wrist cuff was identical to the first except only the 90 degree elbow angle was tested. The results of both strain gauge experiments were treated statistically by the analysis of variance method. Five factors were considered: subject, hand (right or left), joint angle, forearm rotation, and direction of force. All of these main effects were found to be significant as well as several of the first order interactions. Excluding interactions involving subject differences, the most significant interaction was that of forearm rotation with direction of force.

Singh and Karpovich (1966) studied the forearm extensors and flexors. Their results led to regression equations to predict each type of force studied. The equation for isometric elbow flexion as a function of elbow angle is:

$$F = .03 + .9087A - .004272A^2$$

Doss and Karpovich (1965) compared the concentric, eccentric and isometric strength of elbow flexors. (Figure 1) They found that for a range of movement between angles of 75–165 degrees eccentric force was always greater than concentric. (Table 1) Comparisons indicated that concentric force was 23 percent smaller and eccentric force 13.5 percent greater than the isometric force. Their results showed a peak of isometric strength at 120 degrees, as compared to a peak at 115 degrees found by Clarke in 1950. The strength values found by Doss and Karpovich were approximately one-half those found by Clarke. This is explained by the fact that the tests by Clarke were made with a sling at the mid-point of the forearm, whereas Doss, et. al. measured force at the hand. With this allowance, their curves are rather close.

Singh and Karpovich (1968) found isometric force of the flexors, on the average, to be 41.64 percent greater than isometric force of the extensors. (Figures 1,2) This result is confirmed by previous findings. (Singh and Karpovich, 1966)

Schanne (1972) developed a three-dimensional hand force capability model using biomechanical methods of joint articulation. Using torque (force multiplied by the distance between the joint and point of attachment of the muscle or tendon to the bone) as a more sensitive expression of muscular force, Schanne developed regression equations for several measures of arm strength. For elbow flexion, the following equation resulted.

$$T_E = 336.294 + 2.088\alpha_E - .015\alpha_E^2 - 3.364\alpha_{VS} + .019\alpha_{VS}^2$$

where T_E = torque at elbow in inch-lbs.

α_E = elbow angle in degrees

α_{VS} = vertical shoulder angle (angle of shoulder flexion) in degrees.

When reduced to the sagittal plane, the equation reduces to:

$$T_E = 336.294 + 2.088\alpha_E - .015\alpha_E^2$$

which agrees in general form with that computed by other researchers. (Figure 1) The maximum value was found at an elbow angle of 69.60 degrees.

Curves found by Williams, et. al. and Elkins, et. al. appear to be typical. Clarke's maximum at 120 degrees rather than the typical 90 degrees could be due to differences in angle measurement. Downer's results may be lower due to the test being run on only women. Other differences in magnitude of forces may be due to differences in measurement techniques and to the variations in sample populations. Clarke took measurements using a strap at the mid-point of the forearm. Other studies used either a handle or wrist strap, providing a longer moment arm and thus values approximately one-half in magnitude.

Elbow Extension (Figure 2)

Clarke, in his 1950 study, found a monotonically decreasing relationship between pounds of force and angle at the elbow when testing the strength of elbow extension. Clarke found that the strongest position

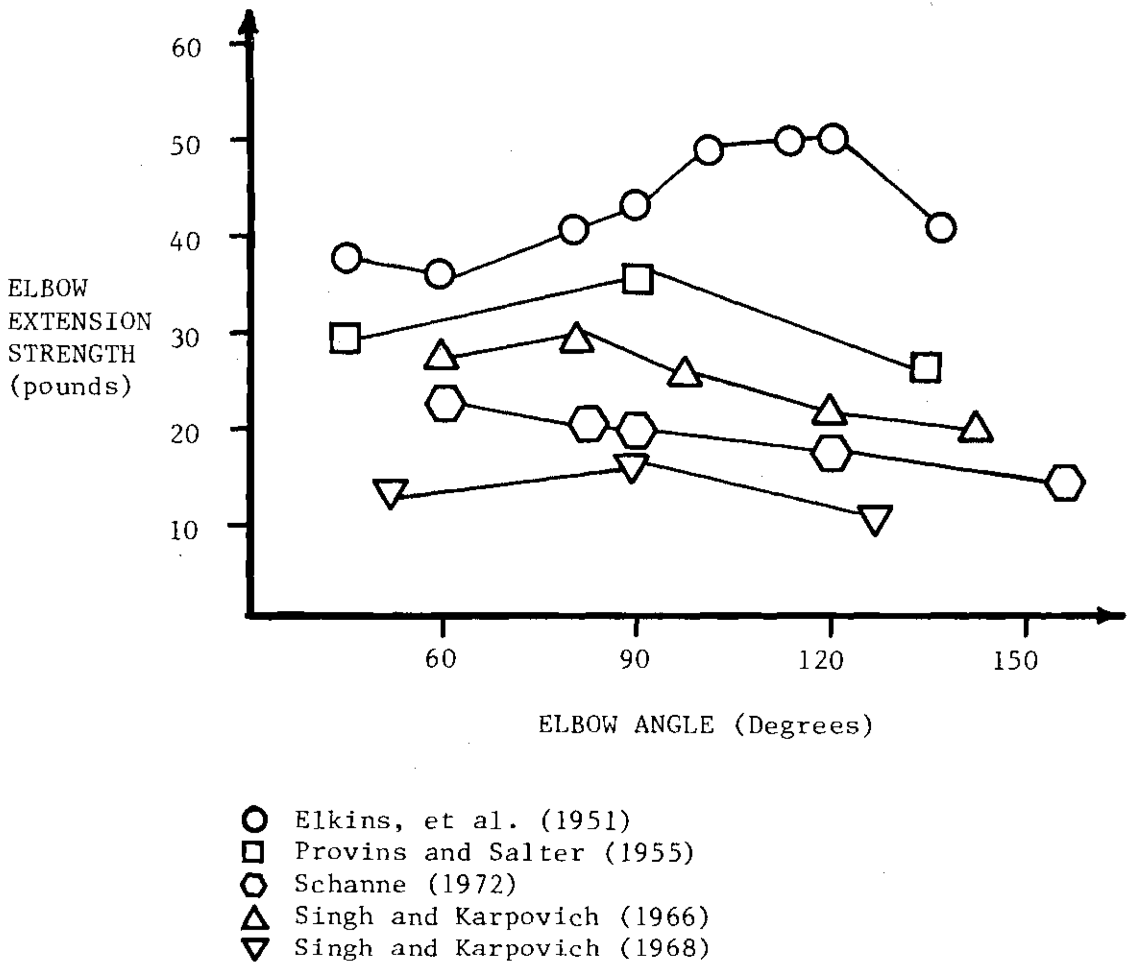


Figure 2. Elbow Extension Strength Curves

for movement of this joint was at 140 degrees, when the muscles were at their greatest stretch, followed by some decrease at full extension due to a loss of leverage. He found a plateau existed between 80 and 140 degrees and a low point of strength at a position of maximum possible elbow flexion.

In the follow-up study by Elkins, Leden, and Wakim (1951), observations of 14 women and 10 men showed a gradual increase in power as the elbow was extended, with a peak occurring at 120 degrees. These tests were designed to eliminate the effect of gravity on the results. This was done because on a supine subject (humerus in frontal plane and parallel to the sagittal plane) at angles greater than 90 degrees, efforts were assisted by gravity, while hindered at acute angles. The subject was, therefore, placed in a recumbent position, his elbow elevated to an angle of 90 degrees flexion at the shoulder (humerus in sagittal plane, perpendicular to frontal plane). Thus, for all angles tested, the force exertions were against a gravity influence and could be compared with each other.

Provins and Salter (1955) found the maximum force exerted in elbow extension occurred at the 90 degree angle. This, of course, differs from the results previously mentioned of the Elkins study. Provins and Salter explain the disparity as being due to the fact that at 60 degrees, extension is hindered by gravity and at 120 degrees is gravity assisted. Elkins' experiment, however, was conducted in a manner which made the gravity effect constant throughout (see previous paragraph).

Singh and Karpovich (1966) found a regression equation for isometric elbow extension force prediction as a function of elbow angle:

$$F = .1 + .682A - .00392A^2$$

In a follow-up study on the forearm extensors and flexors Singh and Karpovich (1968) found, contrary to Elkins, et. al. (1951), that extensors showed greatest strength not at full extension, but when they were stretched, the elbow angle being 90 degrees or less. (Figure 2) These findings agree with previous results obtained by these investigators. (Singh and Karpovich, 1968)

For elbow extension Schanne's equation is:

$$T_E = 264.153 - .575\alpha_E - .425\alpha_{VS}$$

where T_E = torque at the elbow in inch-lbs. Sagittal plane form merely causes the vertical shoulder angle term to drop out. It can be seen that the minimum strength for elbow extension is obviously at the maximum elbow angle. (Figure 2) The strength curve exhibits a monotone decreasing property. A parabolic relationship with a maximum at approximately 90 degrees is typical. Shanne felt that the portion of the parabola within the range of motion of the elbow is the strictly decreasing section. Hence, simplicity dictated a linear (first degree polynomial) regression model would suffice.

Shoulder Flexion (Figure 3)

Clarke (1950) found that in shoulder flexion, a plateau was reached for elbow angles of from 45 to 90 degrees. The greatest power was exhibited with the arm at the side.

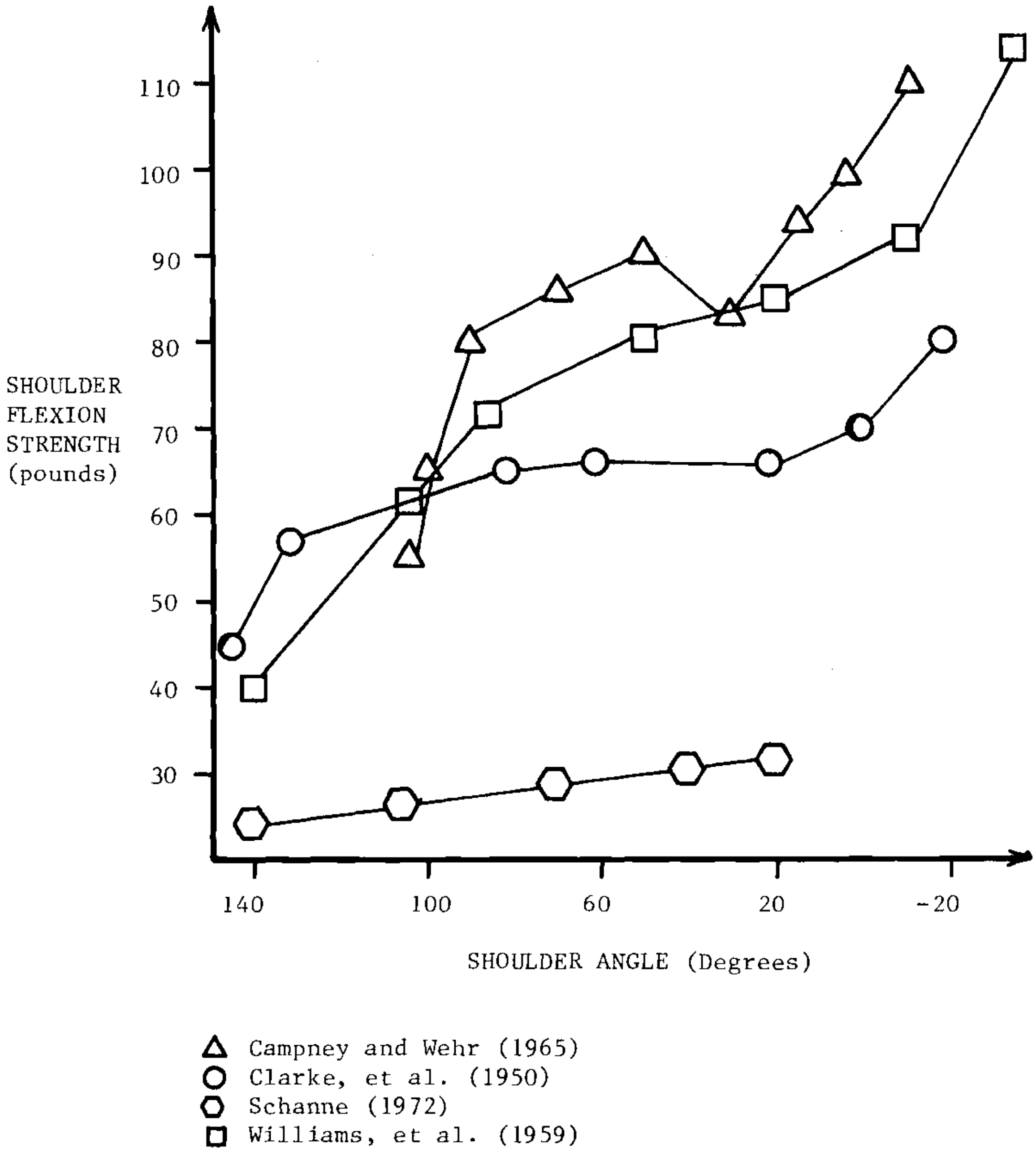


Figure 3. Shoulder Flexion Strength Curves

Campney and Wehr (1965) studied shoulder flexion with a cable tensiometer. A linear relationship was hypothesized, as strength increased with decreasing joint angles.

Williams, et. al. (1959) also studied shoulder flexion. Although little can be drawn from the Williams study because neither the position of the subject nor the means of stabilization are discussed, results of the study do indicate that in shoulder flexion, isometric contractions increase in strength as the shoulder angle decreases, with a relative plateau found around 60 degrees.

Schanne (1972) found a regression equation for should flexion as:

$$T_S = 321.9 - .296\alpha_{VS}$$

where T_S = torque at the shoulder in inch-lbs.

Shoulder Extension (Figure 4)

Clarke, et. al. (1950) found that in shoulder extension, only small changes occurred throughout the range of motion. Williams and Stutzman (1959) found a plateau around 60 degrees in shoulder extension. Their results agree well with Clarke (1950).

Schanne's equation for shoulder extension was found to be:

$$T_S = 208.5 - .099\alpha_{VS} \qquad \text{(Schanne, 1972)}$$

This equation disagrees somewhat with other researchers in that the data most commonly reported appears parabolic. Schanne believes that his single subject used for development of the regression equations may have

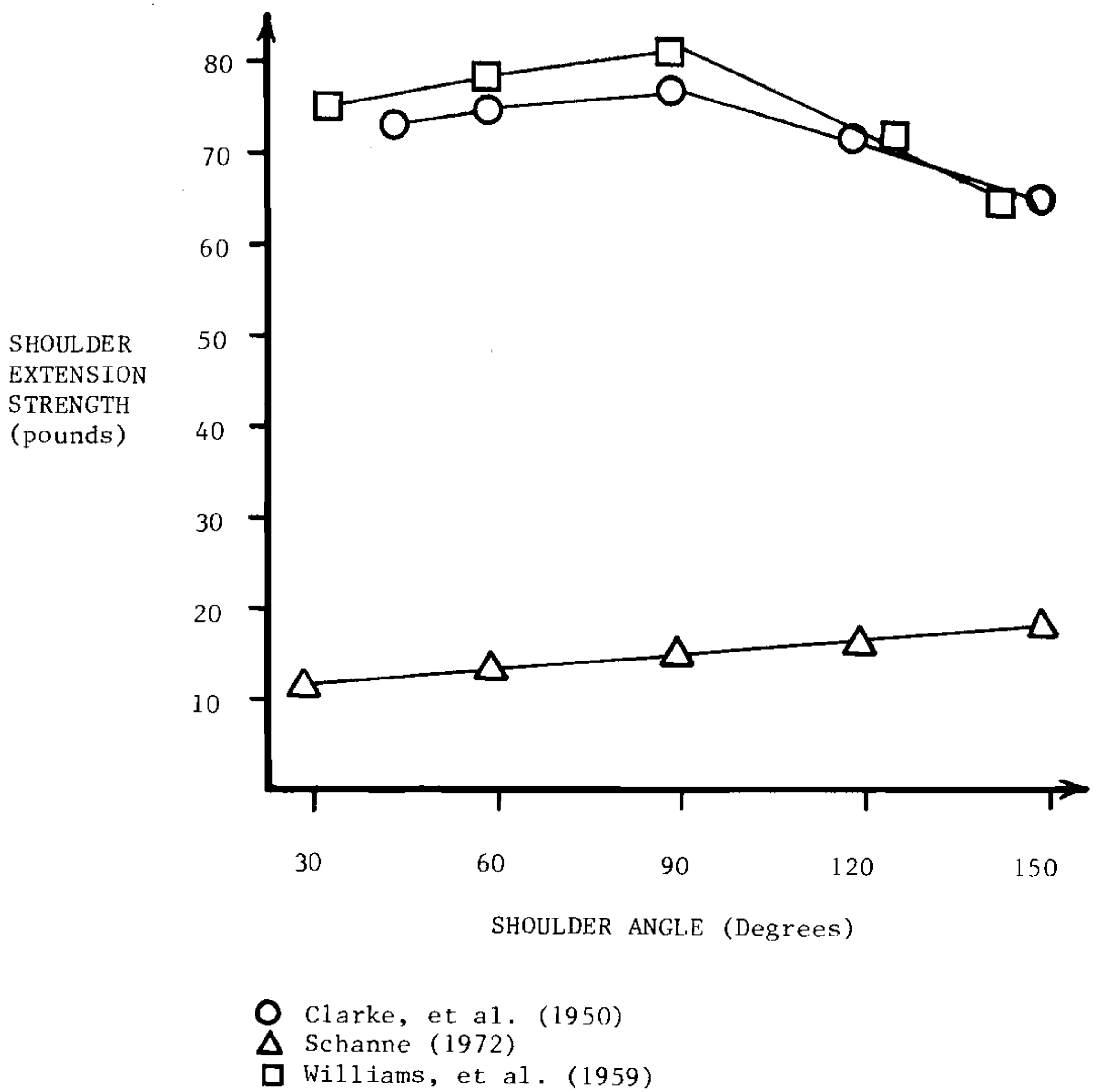


Figure 4. Shoulder Extension Strength Curves

had some unknown peculiarity for this muscle group when the data is considered in the sagittal plane.

<u>Shoulder Adduction (Figure 5)</u>

Clarke (1950) found that the graph representing shoulder adduction strength was nearly symmetrical from 25 to 180 degrees with the greatest power exhibited with the arm at right angles to the shoulder.

Schanne found the following regression equation for shoulder adduction:

$$T_S = 149.392 - .161\alpha_{HS} + .0086\alpha_{HS}^2 - .099\alpha_{VS}$$

where α_{HS} = angle of shoulder abduction. (Schanne, 1972)

It can be seen that Schanne found the vertical shoulder angle (angle of flexion) had little effect on adduction strength. This, again, does not agree with published data.

<u>Shoulder Abduction (Figure 6)</u>

In shoulder abduction, Clarke found that the height of the strength curve was at 180 degrees; the plateau was from 90 to 135 degrees. (Clarke, 1950)

With subjects seated on a table, feet resting on a chair, the back supported and a sling attached immediately above the elbow, Elkins and his co-workers observed 13 women and 10 men. He found that the force obtained in shoulder abduction was slightly greater with the elbow extended than with the elbow flexed. (Elkins, Leden, Wakim, 1951)

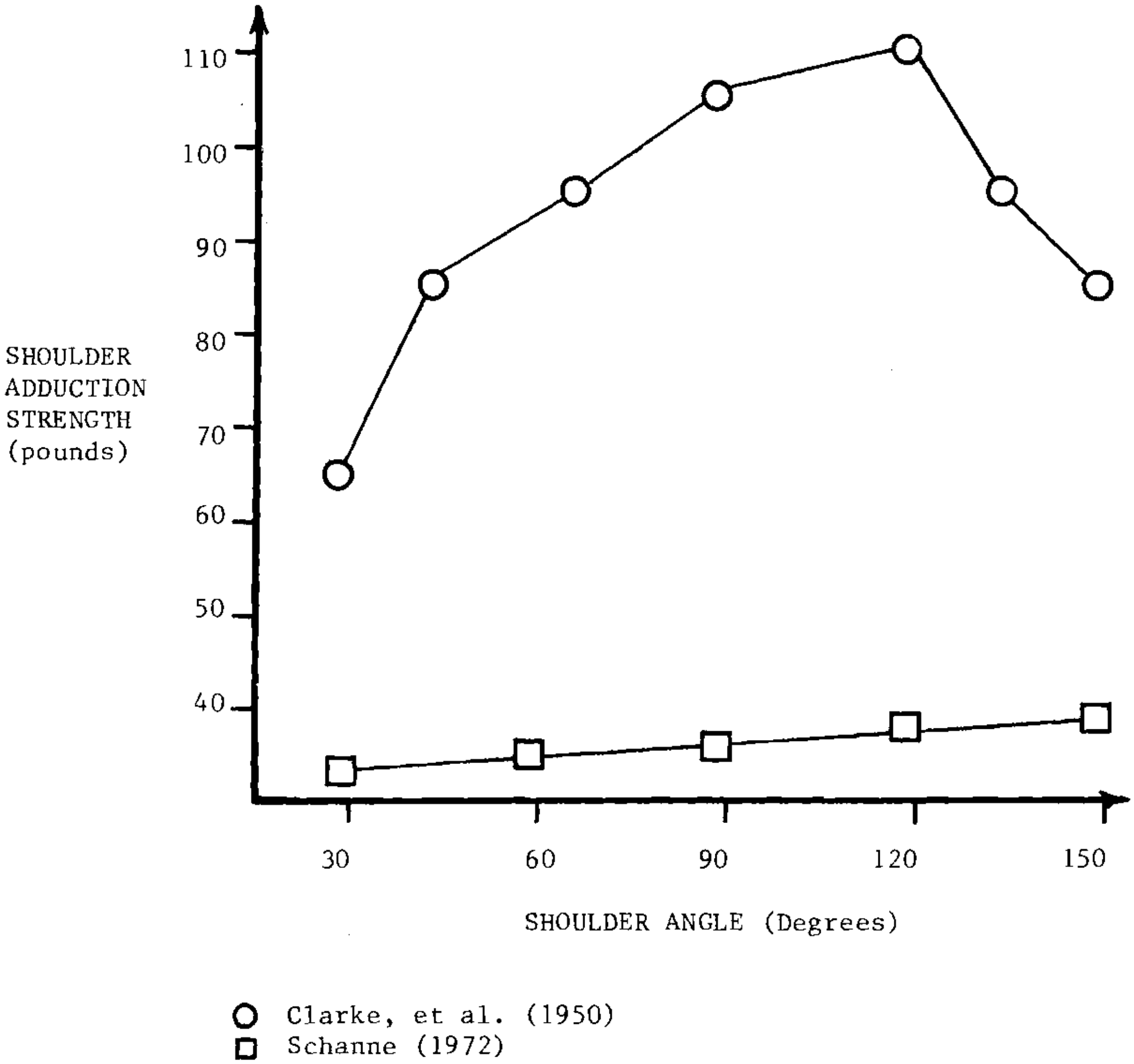


Figure 5. Shoulder Adduction Strength Curves

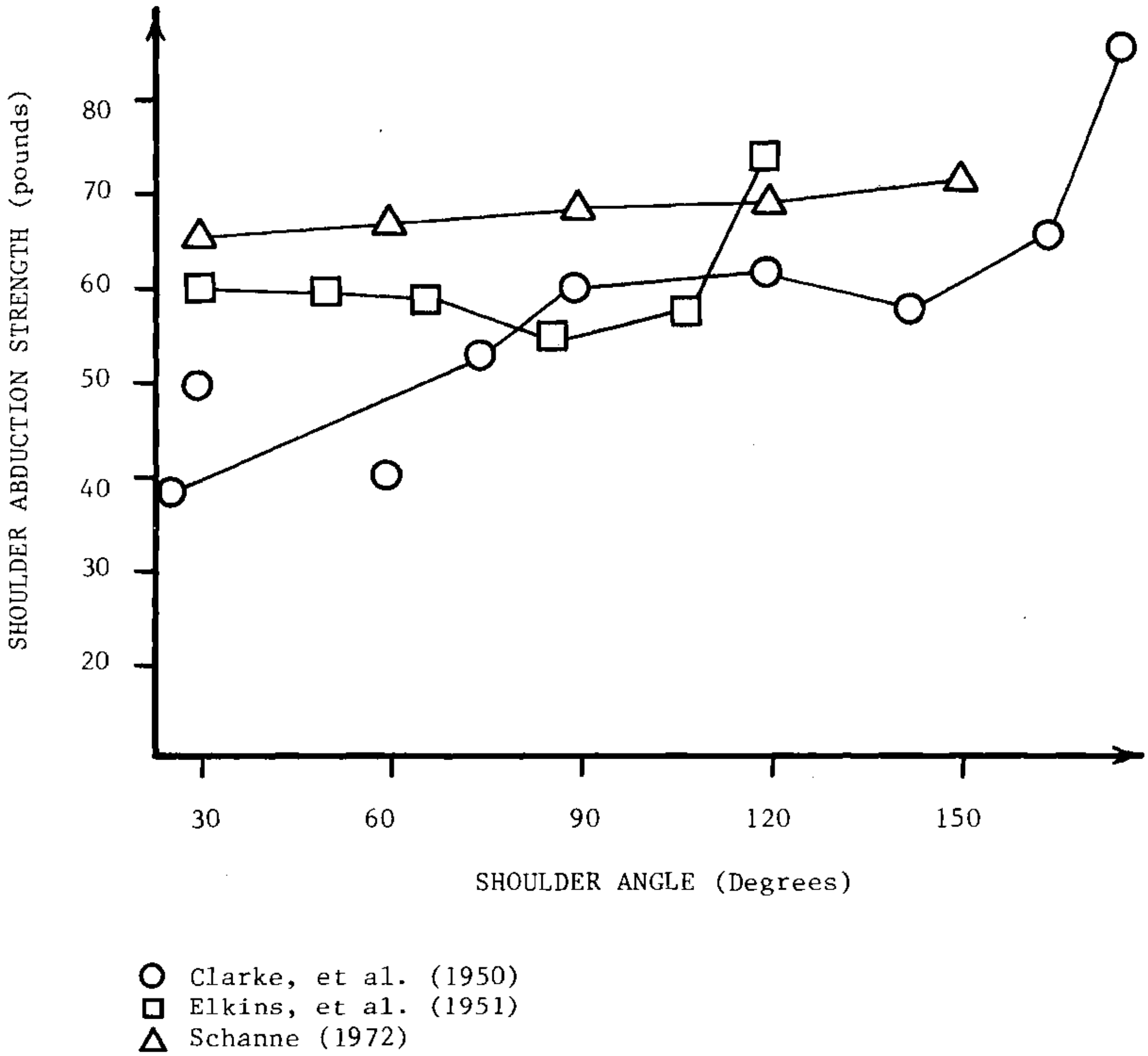


Figure 6. Shoulder Abduction Strength Curves

The equation Schanne developed to explain shoulder abduction strength is:

$$T_S = 227.338 + .525\alpha_E - .372\alpha_{HR} - .296\alpha_{VS}$$

where α_{HR} = angle of humeral rotation (rotation of the upper arm) (Schanne, 1972). This equation also disagrees with other researchers who found a parabolic form to the data.

Effects of Varying Force Directions

In an early study, Hugh-Jones (1947) was concerned with the relationship of exerted force to the angle at the joint for the purpose of studying control placement optimization. He compared the maximum push and pull on an isometric hand lever for different heights and distances in relation to a seat reference point (SRP). Results of the horizontal push tests showed an increase in the force exerted as the distance of the handle from the SRP increased, allowing greater elbow joint extension, and reached a maximum just prior to full extension. (Figure 7) It was found that there was no significant difference between force exerted in the mid-line of the body or force exerted in the plane of the shoulder, but that strength is less when the plane of effort is outside the shoulder.

Horizontal pull forces were found to be dependent on the angle of elbow extension but, rather than decreasing at full extension, these forces tend to increase gradually up to full extension, showing no limiting angle. (Figure 8)

Hugh-Jones also measured the maximum pull in a direction 45 degrees upwards for a hand grip at different distances and heights in relation

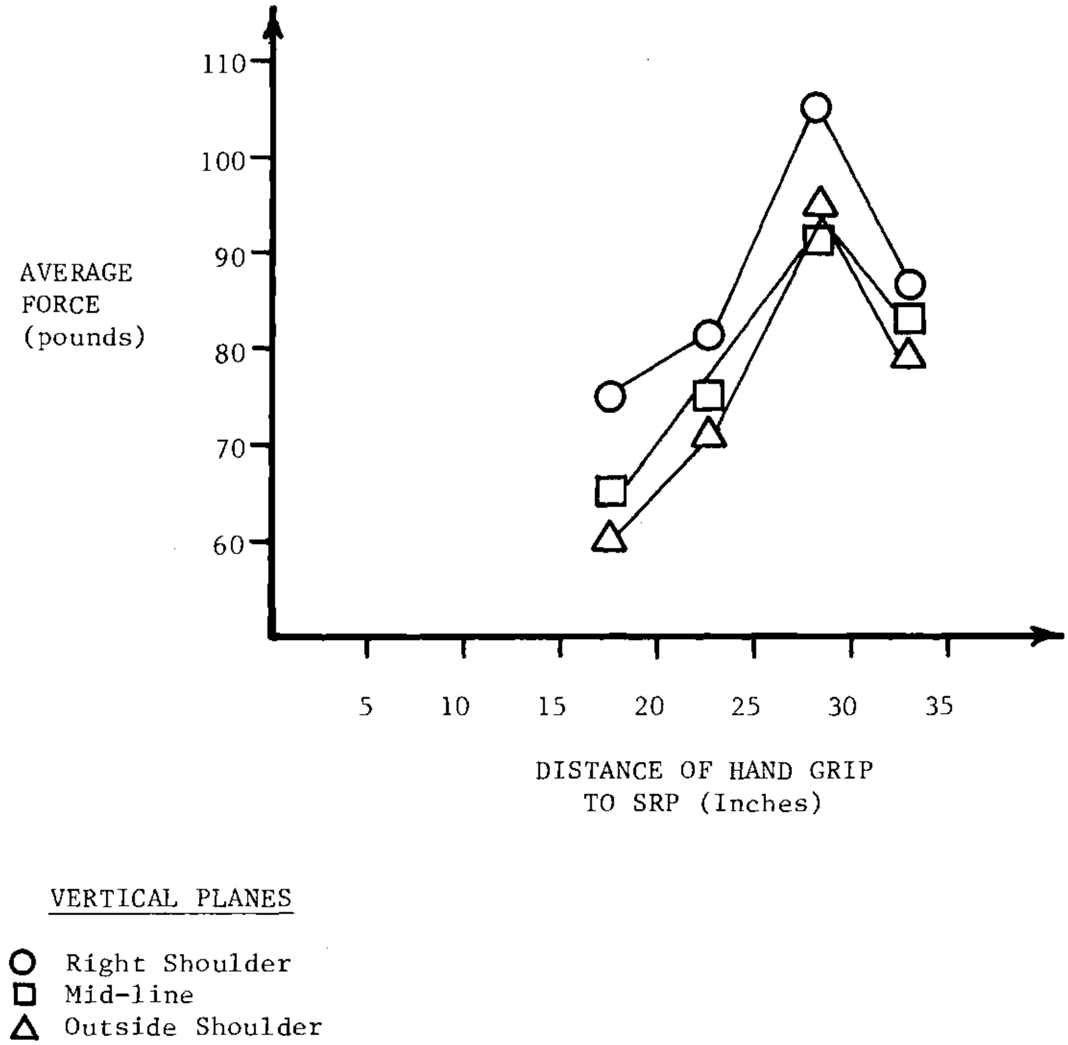


Figure 7. Horizontal Push Forces By Seated Subjects in Varying Vertical Planes, Hugh-Jones, 1946 (All tests were run at a constant height of 15 inches above the SRP.)

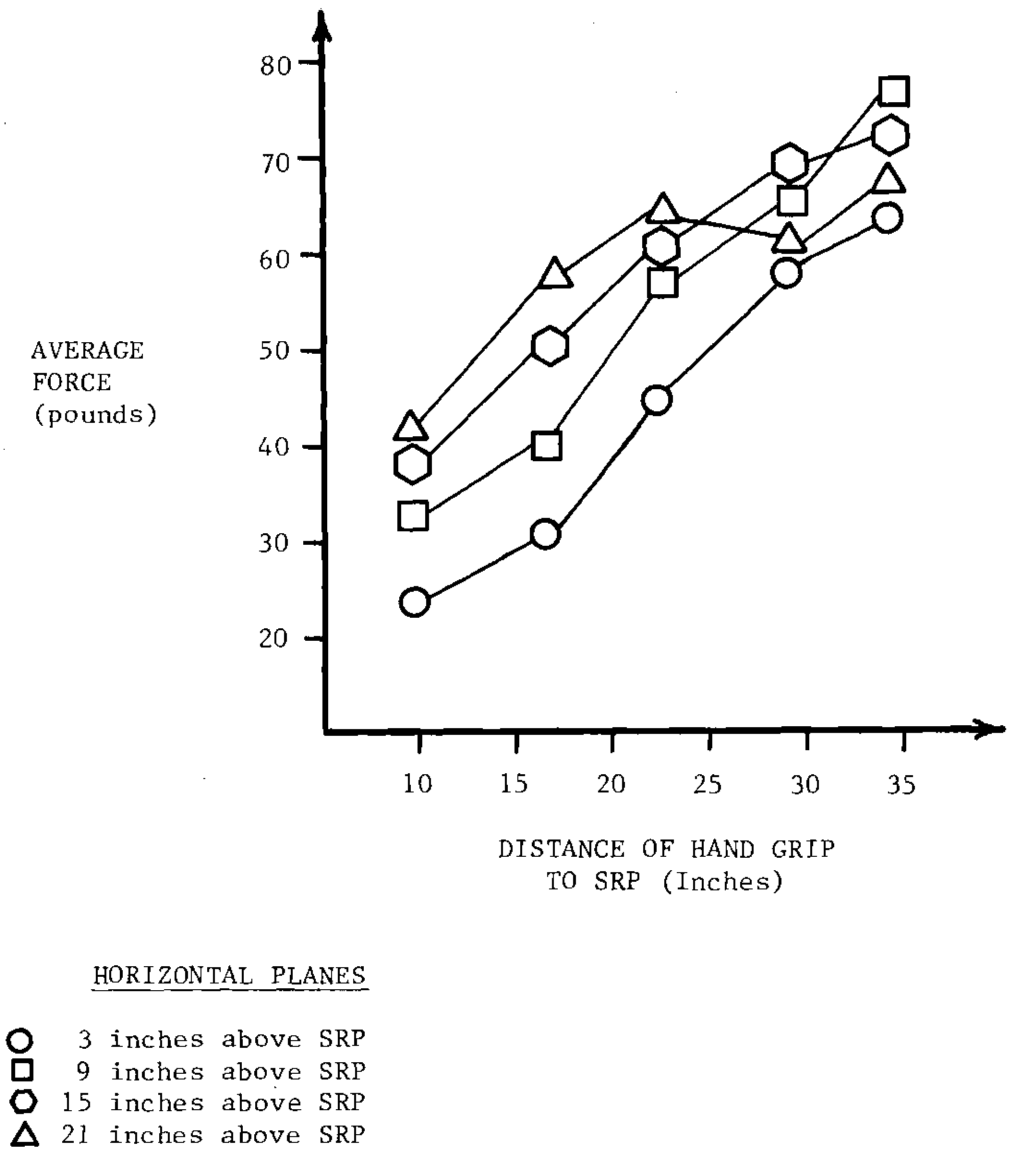


Figure 8. Horizontal Pull Forces By a Seated Subject in Varying Horizontal Planes, Hugh-Jones, 1946

to the seat reference point. (Hugh-Jones, 1947) (Table 1) He found that the height of the hand grip in relation to the SRP has a greater effect on exertable pull than the distance from the backrest. Pull strength continually decreased as the height of the hand grip increased above waist height. (Figure 9)

Hunsicker (1955) investigated the amount of force a subject could exert with the arms at selected elbow angles. (Figure 10) Six directions of movement were tested at each angle: pull, push, lift up, down, abduction and adduction, in both sitting and prone positions. In the sitting position, the order in which greatest forces could be applied was pull, push, up, down, adduction, abduction. In the prone position the order was pull, push, up adduction, down and abduction; in both the sitting and prone positions, the weakest action was approximately one third of the strongest. The prone position was generally found to be 71 percent as strong as the sitting position.

In another more recent study conducted primarily for use by the military, Thordsen, Kroemer and Laubach (1972) examined human force exertions in aircraft control locations. They were concerned with the maximum isometric forces male subjects could exert at six different control locations in two vertical and four to eight horizontal directions of force. In these tests, some use of the shoulder and body was allowed during the exertions. There was no attempt to isolate a particular muscle group. Thordsen, et. al. found that the amount of force exertable depends decidedly upon the location of the aircraft control and on the directions of force exertion. In all of the locations, smaller forces were generally exerted in directions perpendicular to the line from the handle to the

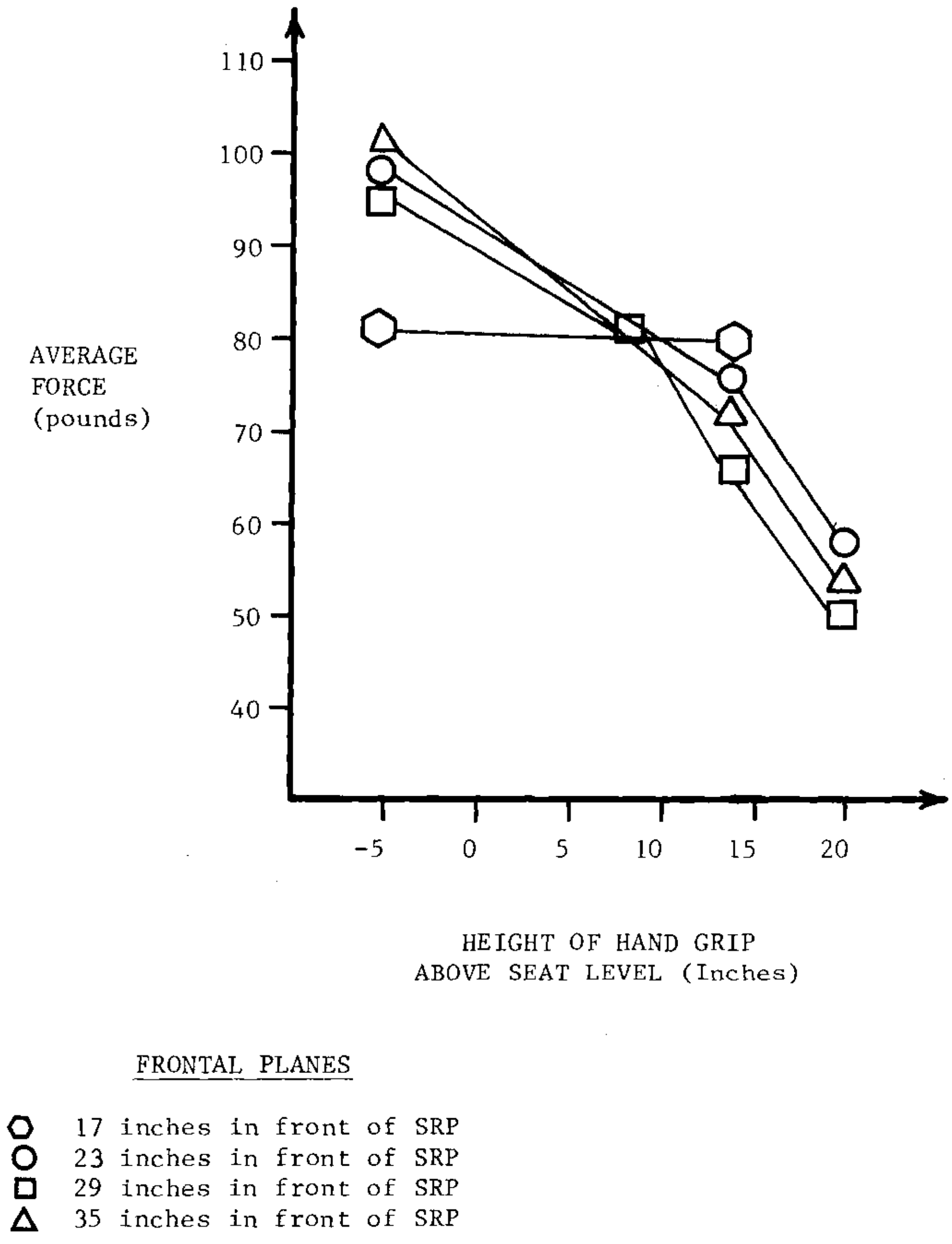


Figure 9. 45 Degree Upward Pull Forces By a Seated Subject in Varying Frontal Planes, Hugh-Jones, 1946

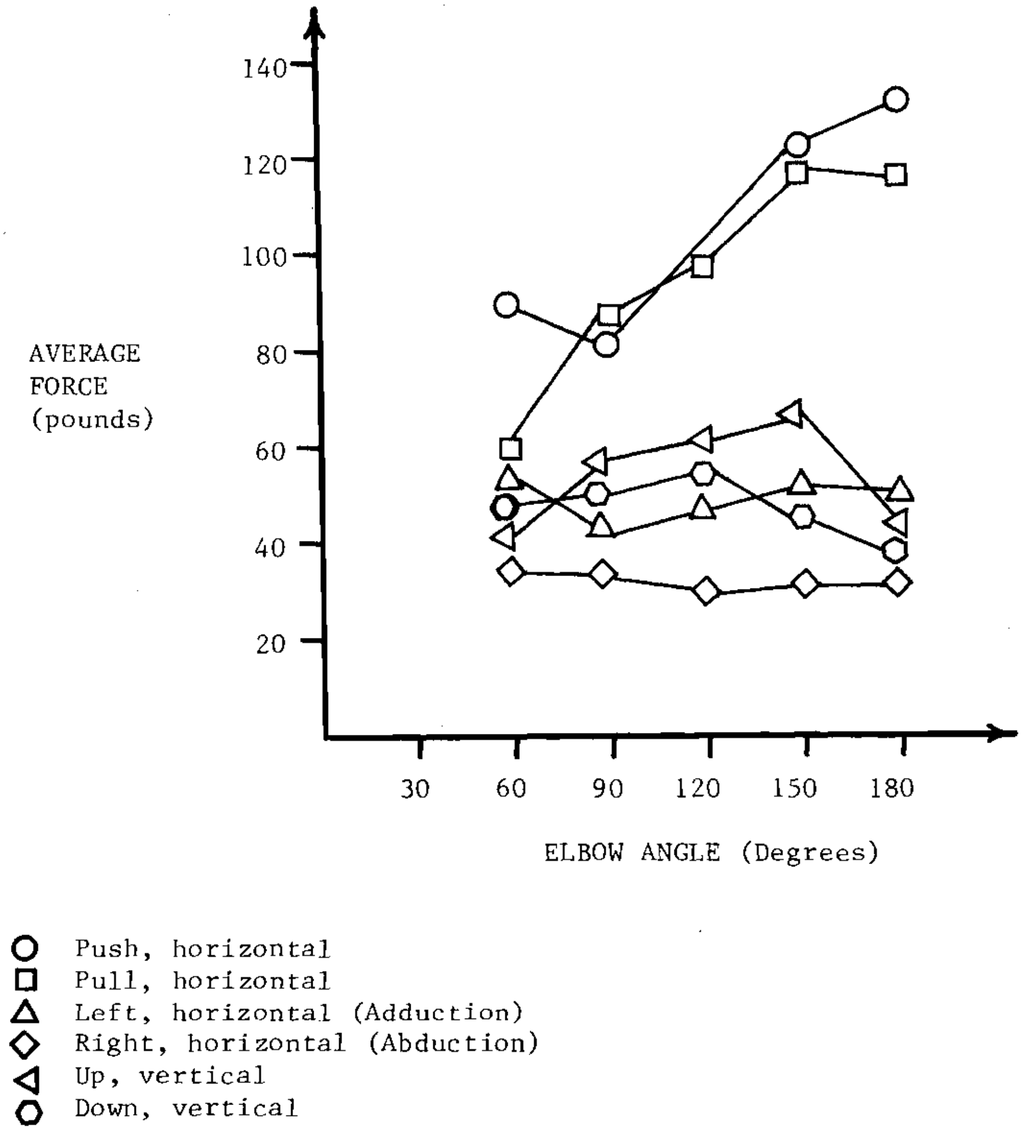


Figure 10. Maximal Static Hand Forces at Various Elbow Angles, Exerted on a Vertical Handgrip, Hunsicker, 1955 (All values are mean right hand values, recorded on a seated subject.)

shoulder joint, while larger forces were usually recorded in directions along that line.

Forearm Rotation

In the Elkins study (1951) comparisons of forearm rotations showed an average difference in power between the three rotations was not greater than 2 pounds. (Figure 11) It was concluded that, on the average, there is little difference between the elbow flexion force exerted in supination, pronation, or mid-position, but that mid-position appears to be most favorable for normal subjects.

In comparing strengths at different forearm rotations, Downer (1953) found the greatest strength of elbow flexion occurred with the forearm in mid-position. Supination was next with an average of 3.6 pounds less than mid-position. She found pronation to be the weakest position for elbow flexion. Her subjects were 30 adult women; she used a Beasley Myodynamometer. Her results compare favorably to those found by Elkins, et. al. (1951).

In testing forearm flexion and extension using a T-handle strain gauge dynamometer previously described, Provins and Salter found significant differences in force occurring with varying forearm rotations and directions of force. (Table 1) That this was not found by Elkins is probably because their study was done using wrist straps. Provins and Salter conclude there is a significant contributing influence of the hand to strength in various forearm rotations. (Provins and Salter, 1955)

Rasch (1956) conducted tests to compare the effects of forearm rotations in elbow flexion. Measurements were made for supination, pronation and mid-position. Rasch found that mean elbow flexion strength

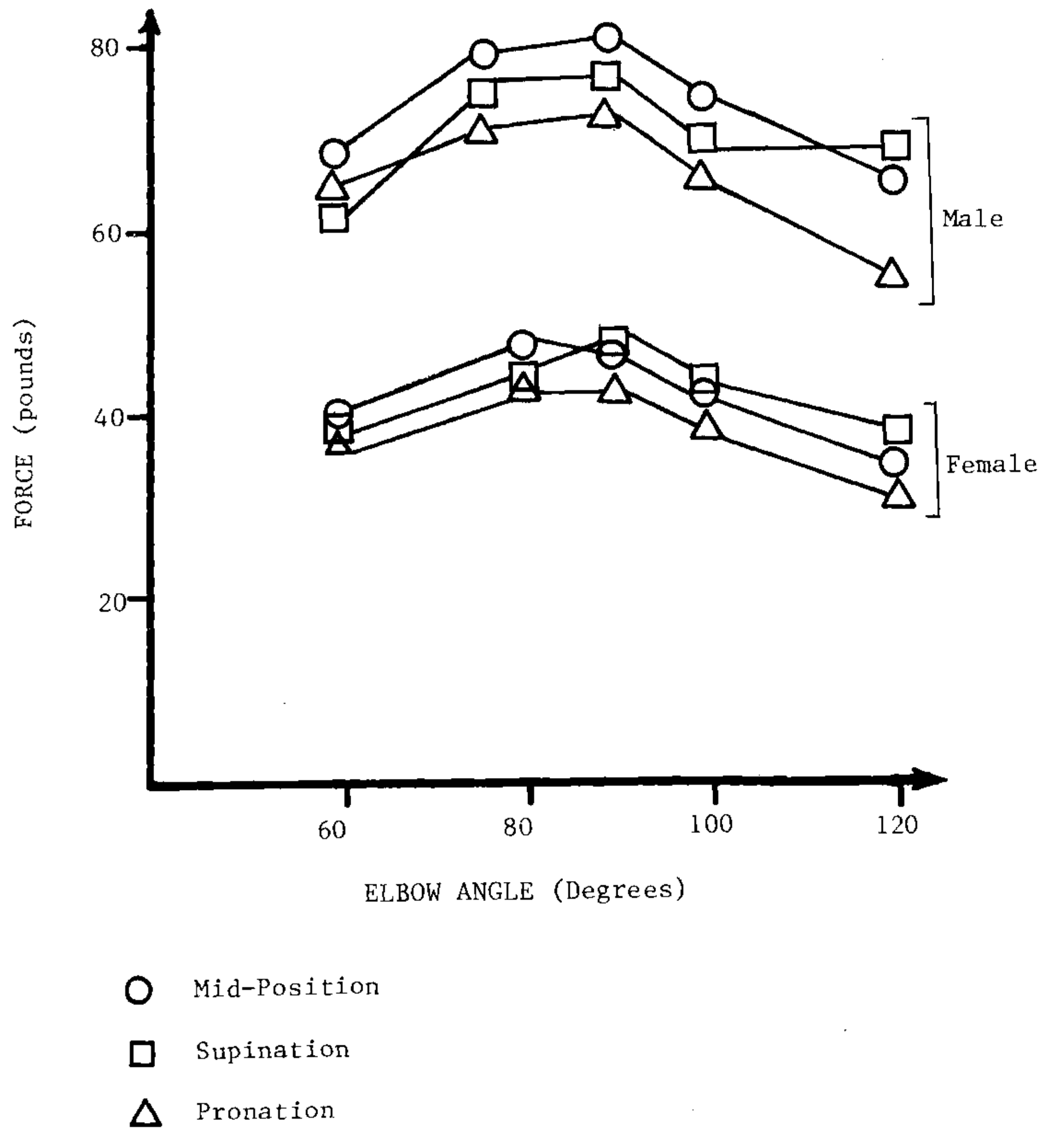


Figure 11. Forearm Rotation and Effects of Sex Differences, Elkins, et al., 1951

was greater (47.8 lbs.) in mid-position, with supination (43.2 lbs) somewhat less. Strength with the forearm pronated was measured at 27.5 pounds. Tests were conducted on 24 adult male students ages from 23 to 25. His tests were run on a standing subject using a cable attached to a floor pulley. Tests were made with each subject's forearm placed in an extended position 100 degrees from his humerus, i.e. in flexion of 100 degrees.

Effects of Body Support

Clarke, et. al. (1950) tested the forearm flexors and extensors both with and without the use of a footboard for the subject's stability. The average pull of forearm flexors with the footboard was 105.2 pounds, while without the footboard it was only 77.5 pounds. Muscle power was greatest with the footboard in every instance. This indicates the need for careful documentation when reporting subject's position and stabilization to enable different studies to be compared.

Caldwell (1962) studied the effect of a back rest and elbow angle on the strength of push (arm extension). He found that with no back support, elbow angle had little effect on the strength of the response. When a backrest was provided, strength increased regularly up to angles of 135 to 160 degrees. As well, he found that an increase from a 20 percent to an 80 percent back rest height resulted in a 24 percent increase in output.

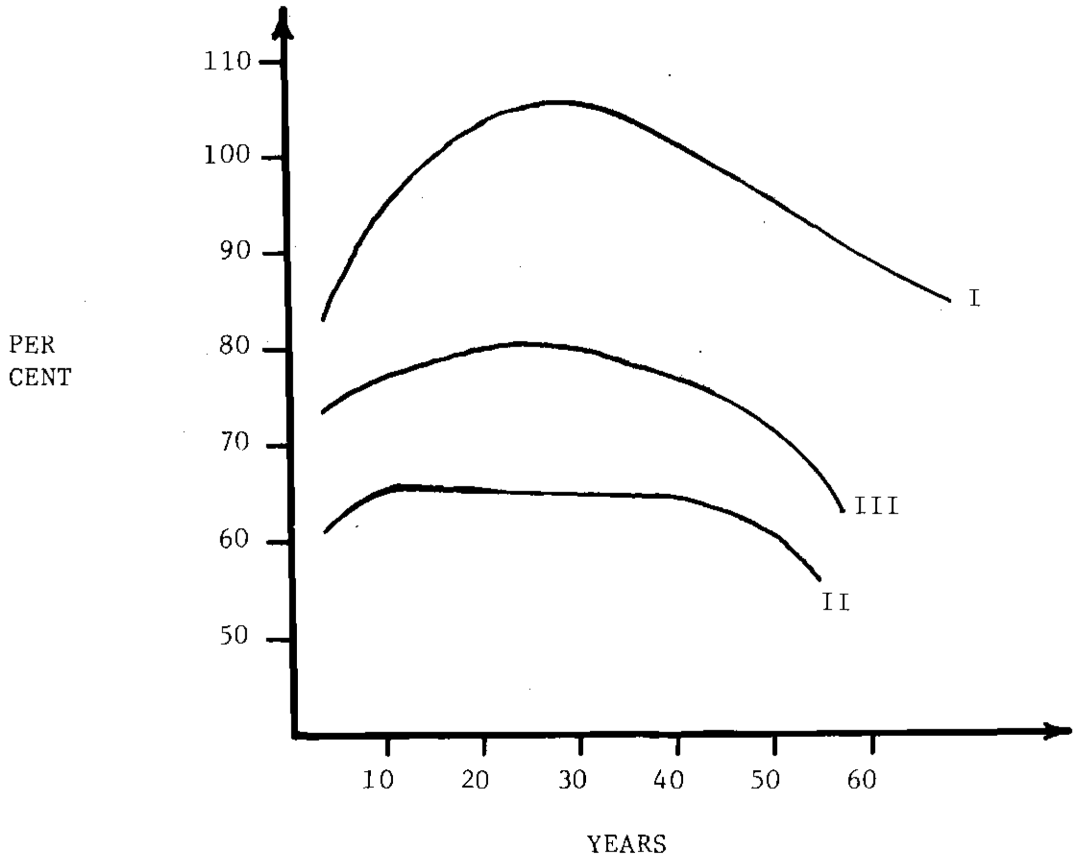
Sex and Age Differences

While testing men and women in shoulder abduction, (Figure 11) Elkins and his associates (1951) found that in male subjects, there was stronger pull at angles of 10 to 40 degrees and at the larger angle of

110 degrees with slightly weaker pull in between. The women's strength was almost uniform throughout the entire range tested, with only a slight rise at greater angles. These researchers gave no reason for this last result.

Singh and Karpovich (1968) made comparisons of the strength values of forearm flexion and extension between the sexes and between preferred and non-preferred arms. Angles tested ranged between 50 and 140 degrees elbow angle. Isometric strength of women was found to be, on the average, 44 percent that of men. (Table 1)

Asmussen and Heebol-Nielsen (1962) tested muscle strength on 360 men and 250 women, ages 15 to 60 years, in 25 different muscle groups. To graph the over-all changes in isometric muscle strength, all values for the different muscle groups were expressed as percentages of the strength of the same muscle group in the 20-22 year old age group for men and averaged. Asmussen found that the strength of man increased to about the age of 30, at which time it is about 104 percent of the strength at age 20-22 years. (Figure 12) It then decreases gradually so that at age 60, it is about 80 percent of the 20-22 year strength. Women were found to have 65 percent the strength of men the same age, decreasing to 63 percent at age 30. At age 55, strength has declined more rapidly for females and is about 54 percent of the isometric strength of men in the same age group. Arm strength in both men and women measured separately from other muscle groups (Figure 13) seems to reach a peak at age 20 and stays constant at age 40, at which time a decrease occurs.

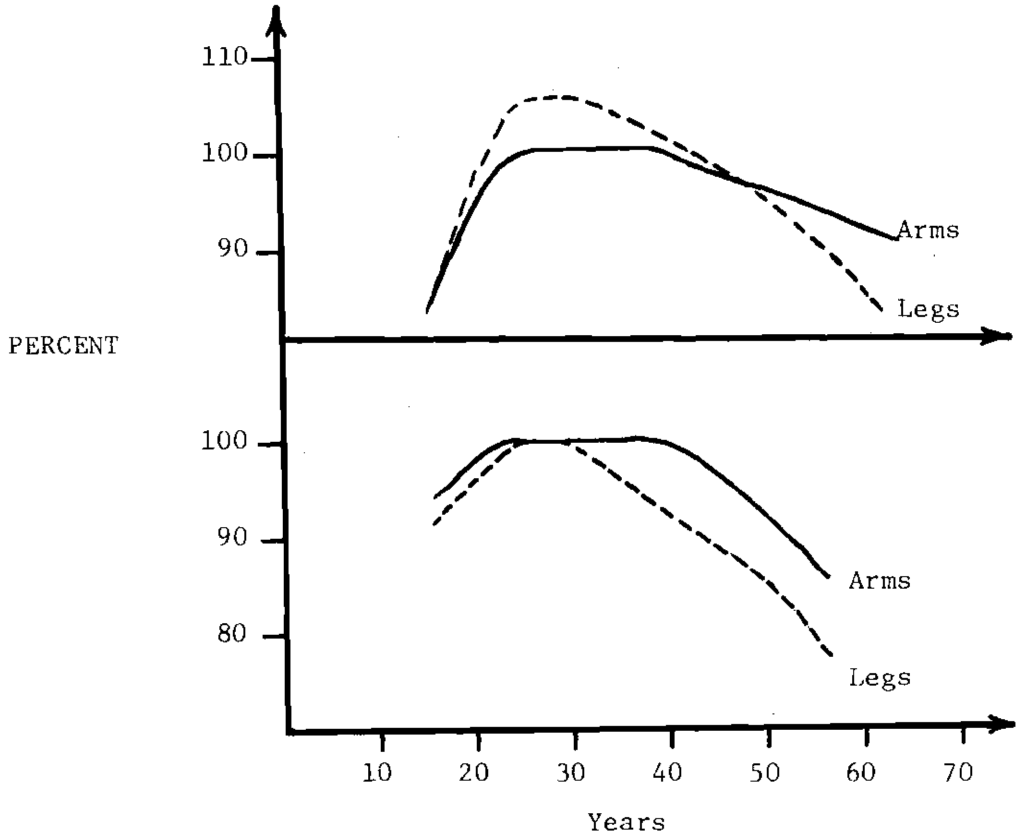


I - Males

II - Females uncorrected

III - Females, corrected for height

Figure 12. Isometric Strength (All Muscles) in Per Cent of Strength of 20-22 Year Old Men in Relation to Age (Asmussen, Heebol-Nielsen, 1962)



Top Curve - Men

Bottom Curve - Women

Figure 13. Isometric Strength (Arms, Legs) in Percent of Strength at 20-22 Years, in Relation to Age (Asmussen, Heebol-Nielsen, 1962)

Antropometric Relationships

Early in the history of strength testing it became of interest to know which factors actually contributed to strength. Of particular interest has always been which anthropometric measurements of a subject can be used most accurately to predict the strength of that individual.

Martin (1918) ran studies of children ranging in ages from 5 to 18 years. His data shows the calculations of over-all strength based on testing only part of the body's muscle groups can be useful in evaluating body strength without the need for a large number of tests. Martin also demonstrated that strength varies directly from year to year during this period of growth with both weight and height.

In 1954, studies were done by Clarke to determine the relationship of five anthropometric measurements to arm strength. The five anthropometric measurements used were: length of the upper arm; standing height; circumference (girth) of upper arm relaxed, flexed and "tension flexed". For purposes of analysis, several arm strength criteria were employed to determine relationships. These criteria are defined as follows:

Raw score composite - total score obtained by adding the raw strength scores obtained on nine isometric strength tests. (This method had the effect of weighting the tests, as small muscle movements with low strength scores were combined directly with large movements with high scores.)

Hull-score composite - Hull score values obtained on each test are added. (Hull scores are based on the standard deviation distances of scores from their respective means.)

McCloy's Index - Pull ups and push ups (from parallel bars) were utilized by solving the equation:

$$\text{Arm strength} = 1.77\text{Wgt} + 3.42 \text{ (number of pull-ups)} - 46$$

The same equation was used for push ups; combined pull-up and push-up strength was obtained by adding both.

Roger's Index - Arm strength = (Push ups + pull ups)(Wgt/10 + Hgt - 60)

Initial studies relating measurements to cable tension strength tests showed correlations of .28 and .55 with girth of the upper arm relaxed and flexed respectively. (Table 2) In subsequent tests, fairly high correlations were found between three of the anthropometric measurements and the strength scores. McCloy's arm strength test was found to be positively correlated (.73) with the girth of the upper arm flexed. The correlations between the three anthropometric measurements and tests were significant, the highest being .52 with shoulder flexion. Subjects in these studies were predominantly mesomorphic (athletic build) with some tendency toward meso-ectomorphy (slender-athletic build) and had better than average musculatures. (Clarke, 1954)

Walkey and Cowan (1963) considered the possible relationship of strength and age, height and weight, on grip strength. The following regression equation resulted:

$$Y = .19578025X_1 - .29481108X_2 + 21.834137X_3 - 10.902322$$

Table 2. Intercorrelations of Experimental Variables and Their Correlations with the Various Arm Strength Criteria. (Clarke, 1954)

Experimental Variables	1	2	3	4	5	6	7	8	Rogers' Arm Strength C1	McCloy's Arm Strength (pull-ups) C2	McCloy's Arm Strength (push-ups) C3	McCloy's (push-ups + pull-ups) C-4
1. Shoulder adduction		.78	.60	.62	.53	.34	.28	.34	.17	.51	.47	.51
2. Shoulder extension			.66	.60	.47	.22	.34	.47	.07	.55	.49	.53
3. Shoulder flexion				.56	.47	.19	.11	.52	.17	.53	.56	.59
4. Shoulder inward rotation					.48	.29	.19	.41	.22	.51	.53	.58
5. Elbow flexion						.18	.41	.42	.29	.39	.48	.49
6. Standing height							.54	.07	.14	.34	.35	.25
7. Length of upper arm								.30	.04	.47	.42	.32
8. Girth, flexed upper arm									.04	.74	.74	.73

where Y = grip pressure (kg)

X_1 = height (cm)

X_2 = age (completed years)

X_3 = body weight (log kg)

This analysis is based on 362 men and 394 women, aged 60 to 89 years. Hand grip pressure was measured with a "Meredith" dynamometer. Each individual was permitted three tries with each hand, with the highest reading recorded. Body weight and height were measured with men and women wearing a minimum of clothing and no footwear.

It was found that body weight was insignificant in contributing to the explained variation of strength, and was therefore eliminated from the equation. In tests on both men and women, prediction equations for right hand grip strength as a function of age and height were found:

$$\text{Men:} \quad Y = 7.674334 + .3161221X_1 - 28500889X_2$$

$$\text{Women:} \quad Y = .21375040X_1 - .17020288X_2 + 9.416400$$

The coefficients of determination, R^2, for these two prediction equations are:

$$\text{Men:} \quad R^2 = .3750$$

$$\text{Women:} \quad R^2 = .3231$$

Laubach and McConville (1969) ran a rather extensive study to determine the relationship of strength to body size and typology. The primary purpose of the report was to examine the correlations between muscle strength and various body size characteristics. Eleven cable tension strength tests were run on 77 male subjects. Thirteen anthropometric measurements were examined plus three types of body types proposed by Sheldon: endomorphic (fat build), mesomorphic (muscular), and ectomorphic (lean). (Sheldon, 1954) The highest zero order correlations for elbow flexion were found with weight (.50), lean body mass (.58), and mesomorphy (.37). For shoulder flexion the highest correlations were also with lean body mass (.49), mesomorphy (.49) and weight (.40). Other correlations were found significantly different from zero, but were of much lower value. Arm length produced only three significant correlations with muscle strength: elbow flexion, hand grip strength, and ankle dorsi-flexion strength. These researchers were forced to conclude, however, that for most practical purposes, the measures of body size and typology used in this analysis were not effective predictors of static strength. (Laubach and McConville, 1969)

Summary

Much can be found in the literature regarding the proper methods of conducting strength tests, (10, 11, 14, 18, 27, 35, 39, 40, 52) to name but a few. Strength variations in the working population, both civilian and military, are very large. Because effective muscle strength may vary with sex, age, body size, physical conditions and many other factors, no one body of data can be considered universally valid. It is easily seen from Hunsicker's data (Figure 10 or Hunsicker, 1955), e.g. on static hand forces for a seated male, that arm strength varies greatly. Using a sample of only 30 men, Hunsicker found that the 95th percentile scores were anywhere from 300 to 600 percent of the 5th percentile scores in all six tested directions, with standard deviations reaching as much as 50 percent of the mean strength score. In addition to this great variability between subjects, no one muscle strength or anthropometric dimensions has yet to be shown to be a good predictor of the strength of a second muscle group. (Laubach and McConville, 1969)

CHAPTER III

METHODS AND PROCEDURES

This study was conducted in two phases, referred to here as Study I and Study II, respectively. Study I consisted of tests of arm strength as it is affected by four variables: location of the handle, distance of the handle from a seat reference point (SRP), forearm rotation, and direction of force. The purpose of this study is to develop a data base for strength as it varies with these four variables, and to analyze the experimental data for trends in arm strength behavior that can provide valuable equipment design information.

Study II was a detailed study of push forces exerted in the transverse plane 20 inches above the SRP, with the forearm in mid-position. A total of 65 test positions were evaluated and used in deriving a strength prediction equation. Procedures for both studies are presented in this chapter.

Study I

Apparatus and Instrumentation

A picture of the strength apparatus is shown in Figure 14. The critical dimensions are as follows:

Chair seat: Width = 17 inches
Length = 14.5 inches

Backrest: Height = 14 inches

Height of SRP: 15.75 inches above ground

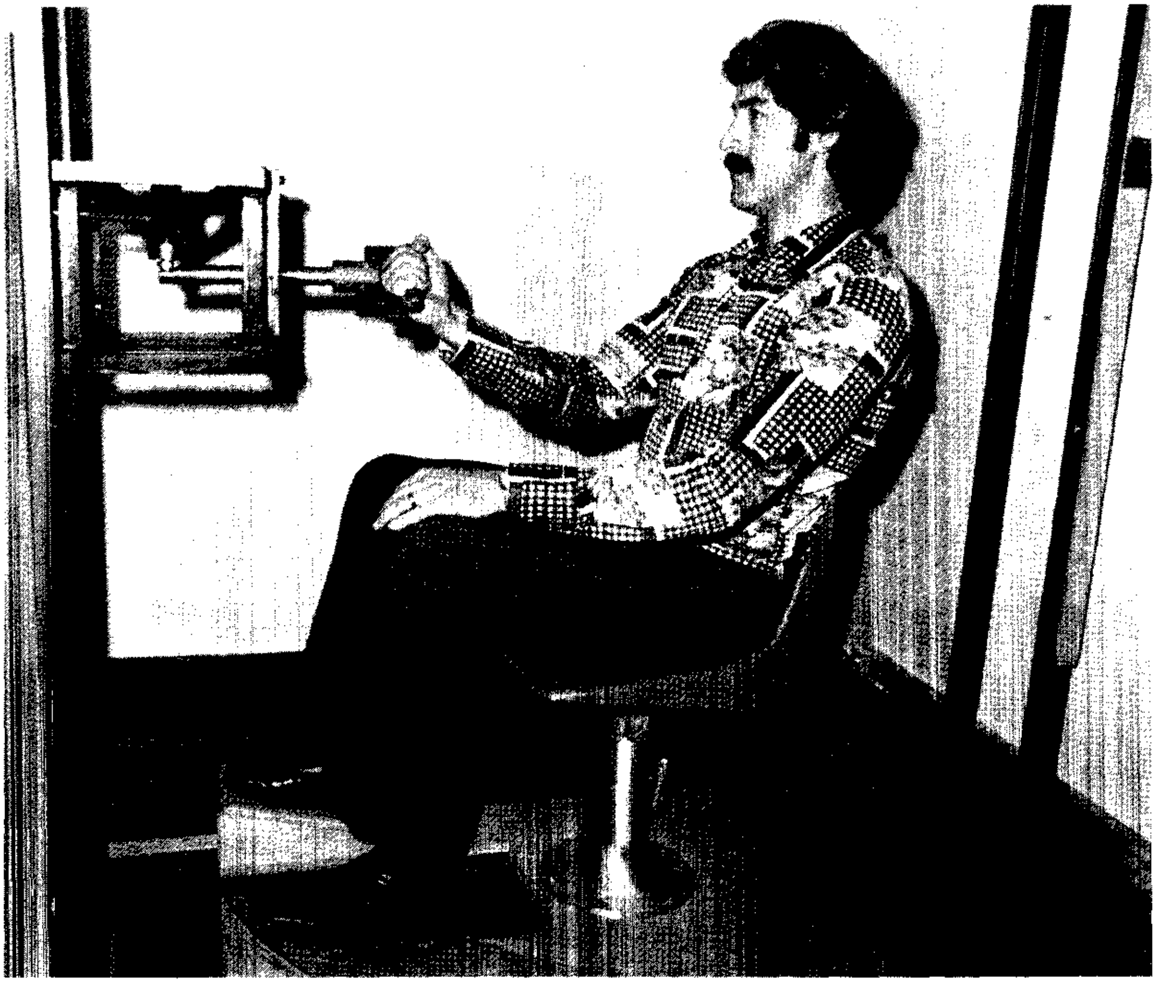


Figure 14. Strength Testing Apparatus

Chair base: Width = 3 feet
Length = 3 feet

Front wall to real wall = 5 feet, 4 inches

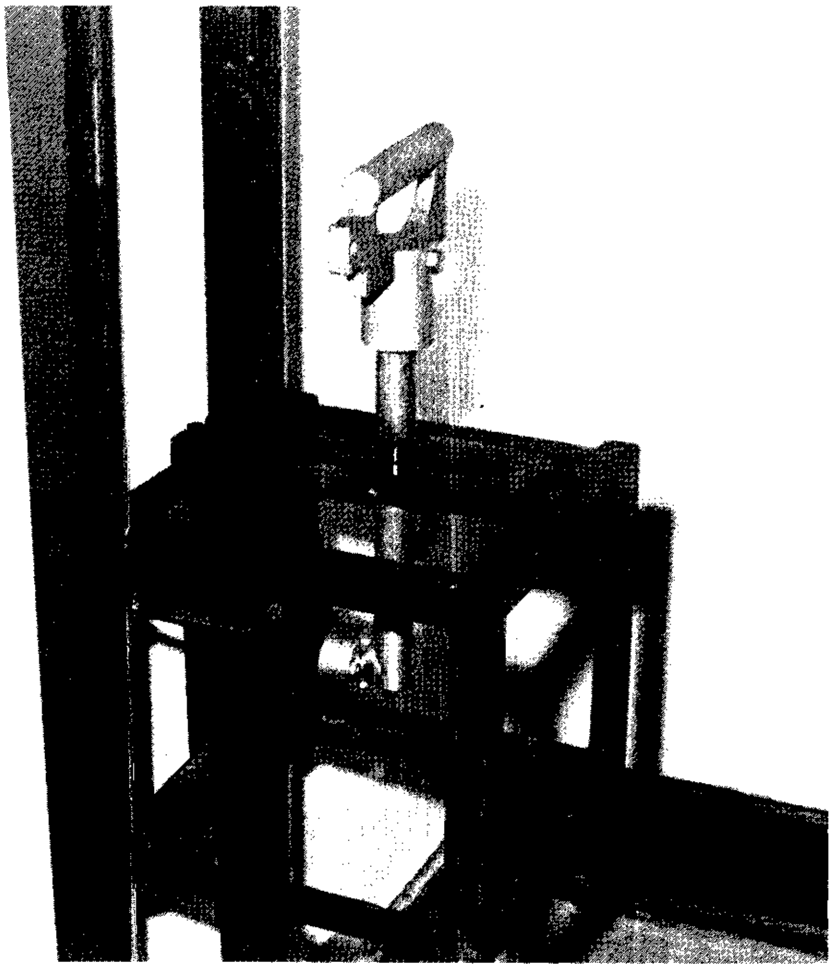
Vertical wall track: Height: = 89 inches
Width : = adjustable

Floor track: Width = 23 inches center to center

Angle of foot support: Approximately 26 degrees

Strength measurements were recorded using a load cell force transducer (Piezotronics 208A03). The transducer was installed as shown in Figure 15 between the base of the handle shaft and a fixed plate attached to the box frame. Both mounting surfaces contacting the load cell were precision machined, and attached with elastic berrylium copper studs. The force transducer is capable of measuring dynamic and short term static forces from one to five hundred pounds. It is calibrated in both tension and compression with a linearity of one percent. The transducer is connected to a Piezotronics 484B line power unit. Sensitivity of the force transducer is 10 mv/lb. The output is recorded on a Honeywell X-Y recorder. A time based readout in pounds is permanently recorded on paper. The handle is a one inch diameter aluminum bar. Since it was possible to obtain only one transducer, forces orthogonal to the desired direction could not be recorded.

The fulcrum of the handle is a pin running through the handle shaft. (See Figure 16) This pin(a) and the pin mounts(b) restrict handle movement to one direction only. Distance (A), the distance from the center of the hand grip to the fulcrum pin is exactly equal to (B), the distance from the fulcrum pin to the point of attachment of the



Dimensions: 10 inches square outside to outside
Lever arm (from fulcrum pin to handle center) = 5.5 inches
Handle: Height = 2.5 inches
Width = 4.5 inches

Figure 15. Handle Control Box and Transducer

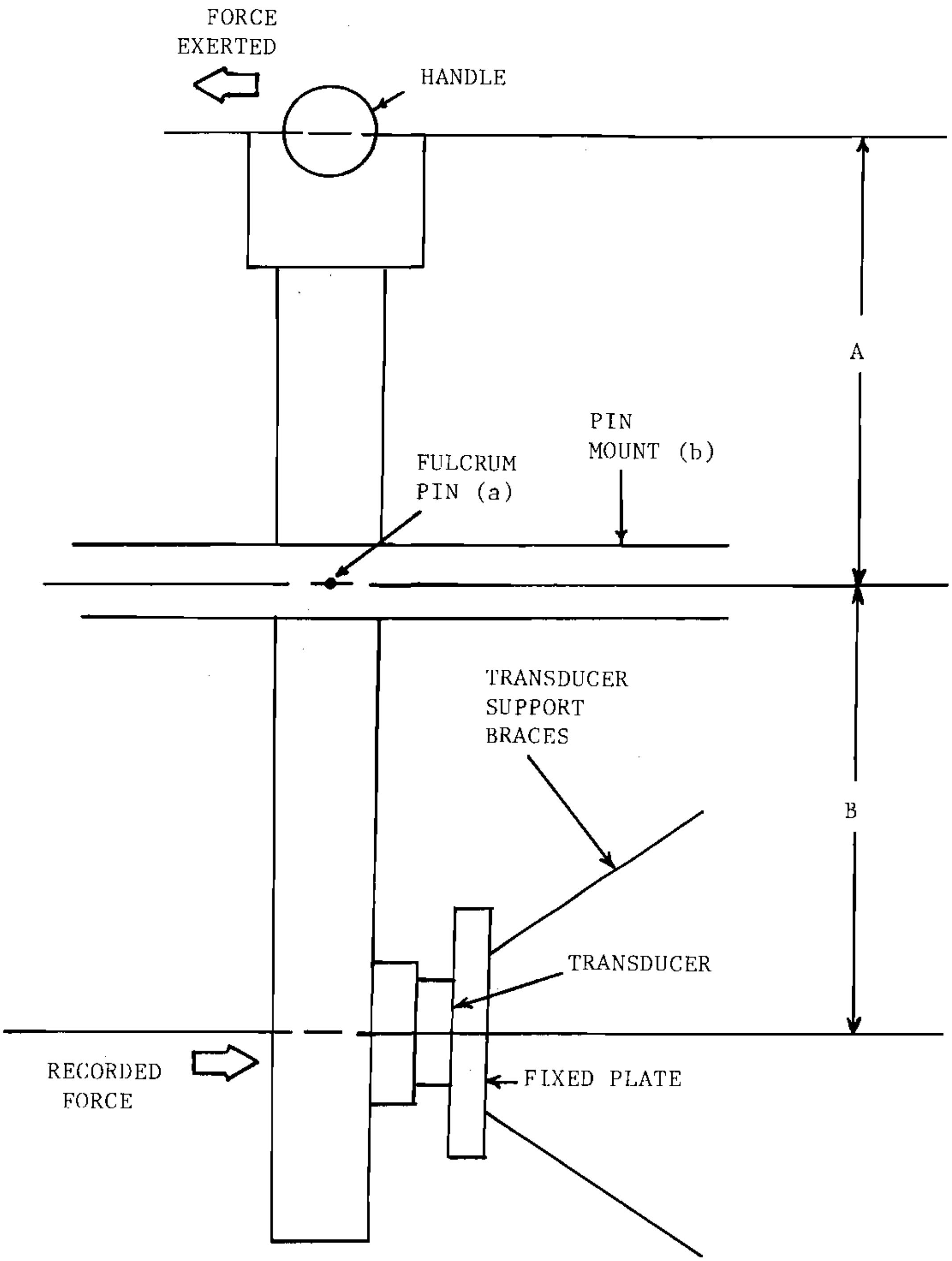


Figure 16. Schematic Diagram of Handle and Transducer Describing Mechanical Principle of Force Calculations

transducer. Based on mechanics, the force exerted on the load cell will be exactly equal to the force applied to the hand grip. $(F_1d_1 + F_2d_2)$ Calibration of the transducer with the recorder was accomplished using known static weights in fifteen pound increments. To achieve a given handle location anywhere in a 360 degree circle around the body, the subject's chair was rotated to the required angle and locked into position. To vary the distance from the seat reference point, the chair, which was mounted on a ball bearing trolley, was moved to any desired distance from the handle. The base of the chair was provided with a locking mechanism. Vertical distances were varied by sliding the handle control vertically on the metal wall frame. Scales in inches were provided along the floor and wall tracks so that distances could be set accurately in each direction. A foot support was provided for subject stabilization. Up-down strength was recorded by orientating the handle as shown in Figure 17a, left-right strength as shown in Figure 17b, and push-pull strength as in Figure 17c. The overhead handle location is an exception to the rule given in Figure 17. In this position, the six directions are defined in a different manner. With the handle oriented as in Figure 17a or Figure 17b (with the handle directly above the SRP and the subjects back facing the wall frame), directions are redefined in Figure 17 as follows:

PUSH (Overhead) $\equiv$ UP (Figure 17)

PULL (Overhead) $\equiv$ DOWN (Figure 17)

RIGHT (Overhead) $\equiv$ LEFT (Figure 17)

LEFT (Overhead) $\equiv$ RIGHT (Figure 17)

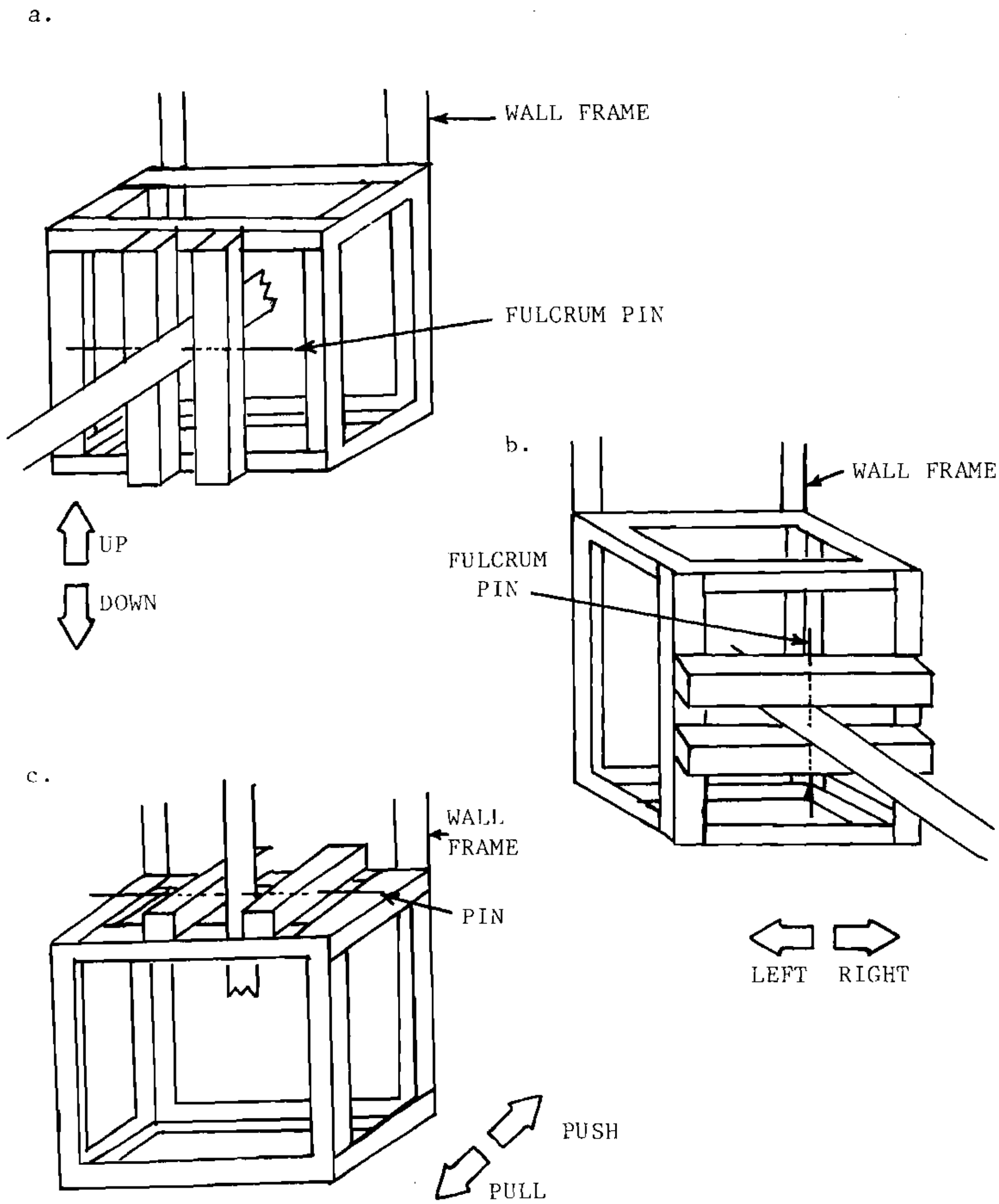


Figure 17. Schematic Diagram of Handle Control Box
Describing Force Directions

With the handle pointing down toward the SRP (the reverse of the position shown in Figure 17c), the directions of Figure 17c are redefined as:

UP (Overhead) $\equiv$ PUSH (Figure 17)

DOWN (Overhead) $\equiv$ PULL (Figure 17)

The vertical wall frame, the track for moving the chair, and the handle control box were constructed from 1 5/8 inch iron channel beams (Unistrut).

Changes in forearm rotation were obtained by adjusting the handle to the subject's hand. The handle itself was attached to the handle shaft on bearings allowing 360 degree rotation around the shaft, as well as approximately 250 degree movement in a plane through the long axis of the hand grip, and parallel to the long axis of the handle shaft. (Figure 18) A locking set screw is provided on the handle base to allow the desired handle orientation to be locked in place.

Experimental Procedures

Pilot Study. A two week pilot study was conducted prior to Study I for subject training, with emphasis on testing all handle locations. The subject practiced exerting forces in the specified directions. This training period reduced the inter-subject variability.

Experimental Design. Handle location was evaluated at 18 positions around the reach sphere. These positions are depicted pictorially in Figure 19. Each handle location is uniquely determined by a vertical plane passing through the center of the chair (sagittal, frontal, or 45 degree planes), and a vertical distance or height above seat level (0, 20, Or 40 inches). Two exceptions are the overhead handle location which is

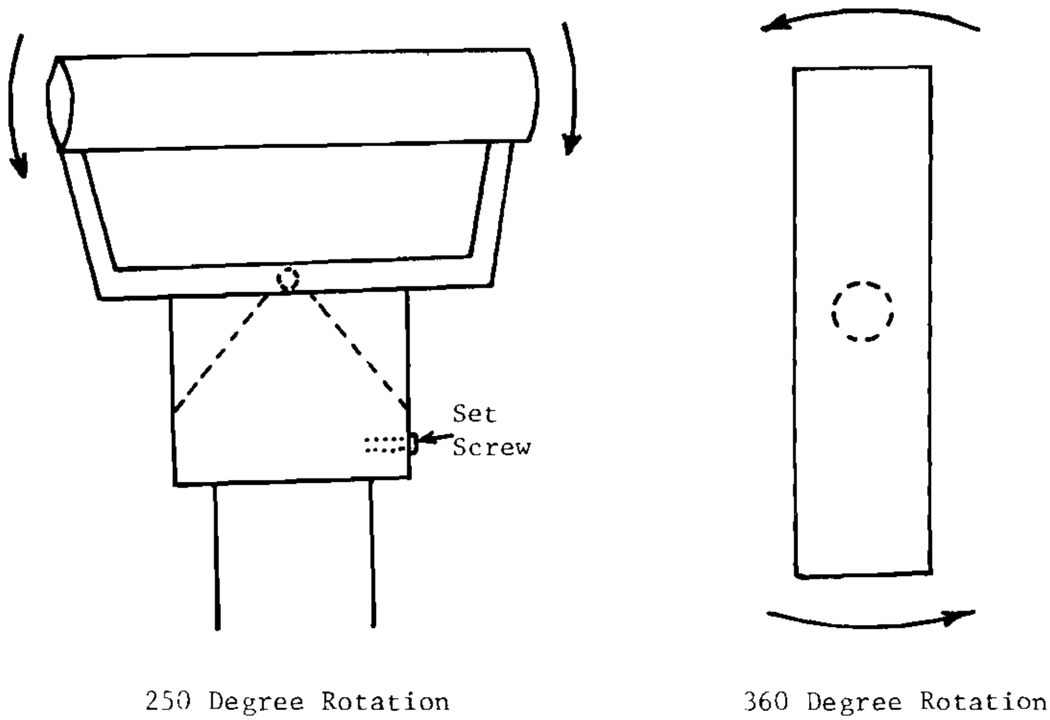


Figure 18. Schematic Diagram of Handle Showing Ranges of Movement

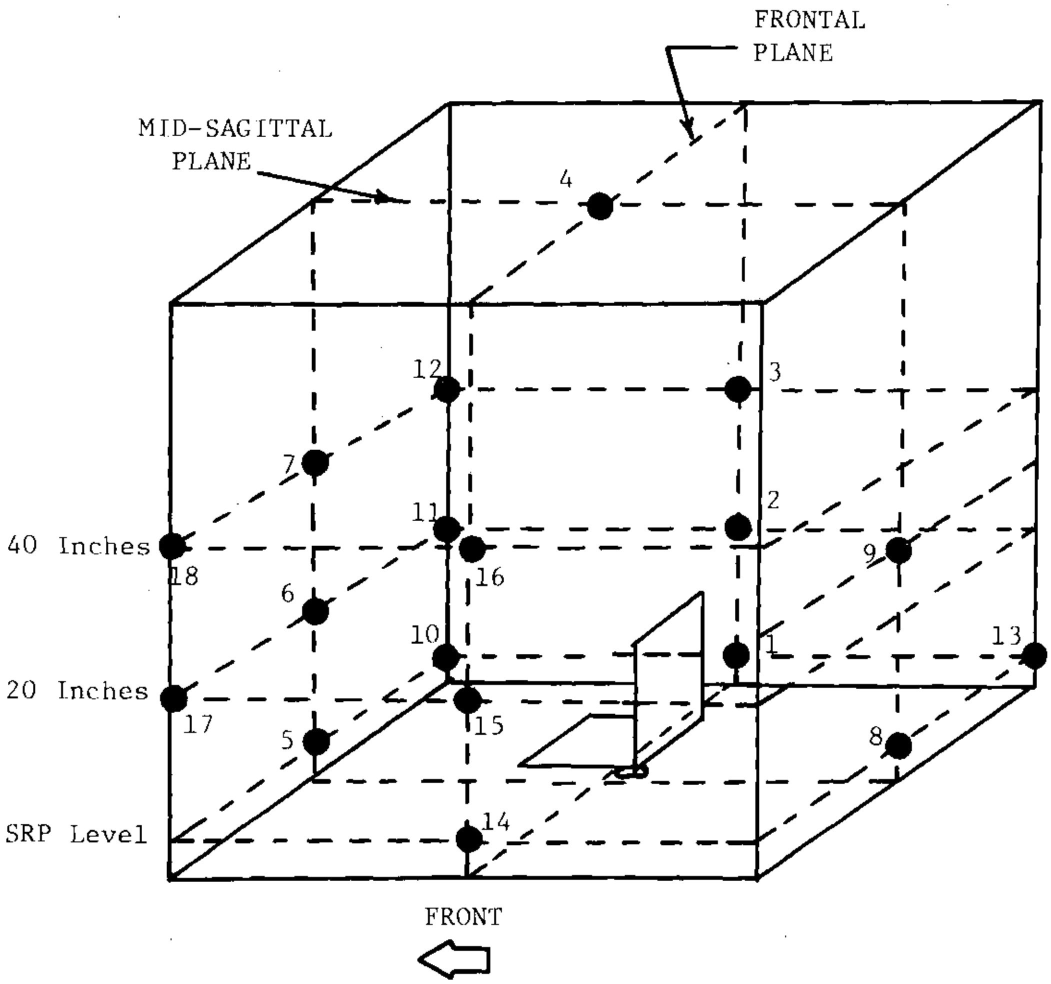


Figure 19. Pictorial Description of the 18 Handle Locations Tested in Study I

determined by the intersection of the frontal and sagittal planes through the SRP, and the handle location behind the head which is 30 inches above the chair seat. A description of each handle location is presented in Table 3. The coordinates of the center of the hand grip at each location are also provided in Table 3, where

Positive X = distance right of the mid-sagittal plane

Positive Y = distance anterior to the mid-frontal plane

Positive Z = distance above seat level

At each handle location, tests were made at three distance levels: near, mid-range, and far. In each handle location except the overhead position, these distance levels are measures of horizontal distance measured radially out from the seat reference point to the center of the hand grip. In the overhead location, distance levels are vertical distances from the SRP. At each handle location, forearm rotation was evaluated at three levels: pronation, mid-position, and supination. Six orthogonal force directions were examined, each direction determined with reference to the handle control box. These six directions are left, right, up, down, push, pull. Figure 17 depicts these directions as they are defined in this study. For the definition of force directions for the handle location directly overhead, which are somewhat different, see page 54. A total of 972 test combinations were performed (18 handle locations · 3 distance levels · 3 forearm rotations · 6 directions), precluding the ability to replicate.

Tests were run in three test periods each day. Period one was from 8 AM to 10 AM, period two from 11:30 AM to 1:30 PM, and period three from 3 PM to 5 PM. Actual testing time per period was approximately one hour and 45 minutes, consisting of 18 tests each period for a total of 54

Table 3. Description of the 18 Test Positions and Cartesian Coordinates of Each Handle Location

POSITION	DESCRIPTION	DISTANCE LEVEL	COORDINATES(x,y,z)
1	Right side, in frontal plane, level with SRP	Near Mid-range Far	(13,6,0) (18,6,0) (23,6,0)
2	Same as 1, 20 inches above SRP	Near Mid-range Far	(15,6,20) (21,6,20) (27,6,20)
3	Same as 1, 40 inches above SRP	Near Mid-range Far	(19,6,40) (23,6,40) (27,6,40)
4	Overhead	Near Mid-range Far	(0,0,40) (0,0,45) (0,0,50)
5	Front of body, in mid-sagittal plane, level with SRP	Near Mid-range Far	(0,17,0) (0,20,0) (0,23,0)
6	Same as 5, 20 inches above SRP	Near Mid-range Far	(0,17,20) (0,22,20) (0,27,20)
7	Same as 5, 40 inches above SRP	Near Mid-range Far	(0,19,40) (0,22,40) (0,25,40)
8	Behind body, in mid-sagittal plane, level with SRP	Near Mid-range Far	(0,-5,0) (0,-10,0) (0,-15,0)
9	Directly behind the head, 30 inches above the SRP	Near Mid-range Far	(0,-4,30) (0,-8,30) (0,-12,30)

Table 3. Description of the 18 Test Positions and Cartesian Coordinates of Each Handle Location (Continued)

POSITION	DESCRIPTION	DISTANCE LEVEL	COORDINATES(x,y,z)
10	Front of body, 45 degrees clockwise from mid-sagittal plane, level with SRP	Near Mid-range Far	(9,12,0) (11.5,15,0) (14,18.5,0)
11	Same as 10, 20 inches above SRP	Near Mid-range Far	(14,18.5,20) (17,22.5,20) (20,26.5,20)
12	Same as 10, 40 inches above SRP	Near Mid-range Far	(11.5,15,40) (14,18.5,40) (16,21.5,40)
13	Behind body, 45 degrees clockwise from mid-frontal plane, level with SRP	Near Mid-range Far	(6.5,-5,0) (10.5,-8,0) (14.5,-11,0)
14	Directly across body, in frontal plane, level with SRP	Near Mid-range Far	(-10,6,0) (-12,6,0) (-14,6,0)
15	Same as 14, 20 inches above SRP	Near Mid-range Far	(-7,6,20) (-12,6,20) (-17,6,20)
16	Same as 14, 40 inches above SRP	Near Mid-range Far	(-9,6,40) (-13,6,40) (-17,6,40)
17	Across body, 45 degrees clockwise from mid-frontal plane, 20 inches above SRP	Near Mid-range Far	(-7,9,20) (-10,13.5,20) (-14,18.5,20)
18	Same as 17, 40 above SRP	Near Mid-range Far	(-5.5,7,40) (-8,10.5,40) (-10,13.5,40)

tests per day. A completely randomized design was used to establish the sequence of test positions. The experiment was analyzed as a four factor nested-factorial experiment. The distance factor is nested in the handle location factor because the three distance levels are not identical for each handle location. (This can be verified by examining Table 3, coordinate values for each distance level.)

Instructions and Test Procedures. The subject was first seated in the chair. The equipment was adjusted for the handle height and chair distance combination for the required test. The subject placed his hand on the handle. He was allowed to make minor adjustments in body position to achieve a "normal-natural" position. Allowing the subject to adjust his arm to its most natural position for each test more closely typifies the position of a seated operator in actual practice. No attempt was made to isolate a particular muscle group. The handle was then adjusted to the subject so that the required forearm rotation was obtained. He was instructed to reach his maximum exertion in two to three seconds, hold for two seconds, then release. This time period of five seconds was determined sufficient time to reach a maximum effort, yet minimizing fatigue. (5, 6, 10, 11, 12, 13, 33, 35) Two or three exertions were made for each test. The maximum values for each exertion were recorded and averaged. This averaging method was necessary because some of the test positions required a relatively uncomfortable body position, which caused larger variations in strength scores.

Rest periods after each exertion ranged from four to six minutes. This was partly due to the time required to reset the apparatus for the next test position. It is generally felt that this amount of rest is sufficient to avoid undue fatigue. (10, 14, 37, 52) The subject was allowed to rest longer if he desired.

Control of Subject Variability

Physical stabilization consisted of a 14 inch backrest and a foot support. The subject was required and so instructed to exert his efforts in the required direction using only arm strength aided by what stabilization was available. The subject was instructed to keep at least one point of the back in contact with the chair back at all times. No portion of the buttocks was to lose contact with the chair seat, and he was not allowed to use his legs to assist in any effort other than in stabilizing his body position. Chest harnesses or seat belts were not used. Detailed guidelines were provided, and the subject was constantly monitored during testing to insure that exertions were accomplished only with the arm, and only in the required direction.

Residual Effects Sub-Study

One of two test positions was always performed at the beginning and end of each period of 18 tests. These tests were used to assess the effects of fatigue, residual learning, and diurnal effects which may exist, but were not isolated by the experimental design. One necessary assumption made throughout the study was that there was no day of the week effect.

The two selected test positions, labelled A and B, were defined as:

A = Hand out to the side, six inches in front of the mid-frontal plane, 20 inches above and 21 inches to the right of the SRP

Forearm rotation - pronation

Direction of force - left

B = Hand between the legs in the mid-sagittal plane, 17 inches in front of the mid-frontal plane, 4 inches below the SRP

Forearm rotation - pronation

Direction of force - up

The scheduling of the tests is shown in Table 4. The experiment was run daily, with every four day cycle being considered as one learning period.

Subject Variability Sub-Study

To obtain an estimate of the variation exhibited by the single subject during the course of the testing period, eight positions were randomly chosen and replicated five times each. These replicated positions were used to test homoscedasticity, which is assumed when performing analysis of variance and regression analysis. Table 5 describes these eight test positions.

Study II

The same apparatus, instructions, and test procedures were used as in Study I.

Experimental Design

Within the reach sphere of the subject in the 20 inch transverse plane, positions around the body were defined by a set of X,Y coordinates. Positive X was defined as right of the origin (SRP), positive Y as anterior to the mid-frontal plane. A total of 65 positions were selected to cover the full range of possible hand positions from closest to farthest from the body in representative sections of the subject's reach sphere in this transverse plane. The X,Y coordinates are given in Table 13,

Table 4. Experimental Design for Residual Effects Sub-Study

TEST PERIOD						
Day	1		2		3	
	Begin	End	Begin	End	Begin	End
1	A		B		A	
2		B		A		B
3	B		A		B	
4		A		B		A

Table 5. Eight Test Positions Used in Subject Variability Sub-Study

REPLICATED TREATMENT NUMBER	SPACIAL POSITION OF HAND COORDINATES	FOREARM ROTATION/ FORCE DIRECTION
1	Front of body, in mid-sagittal plane, 17 inches in front of SRP, in transverse plans through SRP./(0,17,0)	Pronation/ Up
2	Front of body, right front quadrant 11.5 inches right of SRP, 15 inches in front of SRP, in transverse plane 40 inches above the SRP./(11.5,15,40)	Mid-posn/ Pull
3	Between legs, 17 inches in front of SRP, in mid-sagittal plane, 3 inches above the SRP./(0,17,3)	Pronation/ Left
4	Front of body, in mid-sagittal plane, 22 inches in front of SRP, in transverse plane 40 inches above the SRP./ (0,22,40)	Supination/ Right
5	Side of chair, 12 inches right of SRP, 6 inches in front of SRP, in transverse plane 4 inches below the SRP./(12,6,-4)	Pronation/ Down
6	Front of body, in mid-sagittal plane, 23 inches in front of SRP, in transverse plane through SRP./(0,23,0)	Supination/ Down
7	Front of body, left front quadrant, 8 inches left of SRP, 10.5 inches in front of SRP, in transverse plane 40 inches above the SRP./(-8,10.5,40)	Mid-posn/ Up
8	Side of body, 23 inches right of SRP, 6 inches in front of SRP, in transverse plane 40 inches above the SRP./(23,6,40)	Pronation/ Down

Chapter IV. These 65 test positions were run in random order over a two day test period. A total of 39 tests were run on day one, 26 tests on day two. Each day's testing was done in three test periods identical to those used in Study I. However, only 13 tests were required per period in Study II, and only two periods were needed on the second day.

Subjects

The age and demographic information on the single subject used for both Study I and Study II are given in Table 6. The subject was right handed and used the preferred hand in all exertions. Only one subject was used because of the large number of tests required. The time required daily for testing dictated that the author act as his own subject.

The subject was in excellent physical condition. He engaged in weight training three to four times each week, played basketball and tennis frequently, and was accustomed to running eight to ten miles weekly. The test subject represents approximately the 82-85th percentile of static strength with both preferred and non-preferred hands. (See Table 15, Chapter IV)

Table 6. Anthropometric Description of Test Subject

Age	30 years
Height	6 feet 2 inches
Weight	192 pounds
Neck	16 1/2 inches
Chest	44 inches
Forearm	12 inches
Wrist	6 3/4 inches
Waist	33 1/2 inches
Bicep (relaxed)	14 inches
Bicep (flexed)	15 1/4 inches
Hips	36 inches
Thigh	23 1/2 inches
Shoulder to elbow	15 1/2 inches
Shoulder width	20 1/2 inches
Somatotype	Mesomorphic

CHAPTER IV

RESULTS

This chapter presents the results of both Study I and Study II. All raw data collected during this research is presented in the body of the chapter, with the exception of data collected for Study I, which may be found in Table 19, Appendix A.

Residual Effects Sub-Study

Additional tests were performed on two representative test conditions identified as A and B (see Table 4) to determine if the data contained any residual learning, fatigue or training effects which may bias the results. The data is presented in Table 7. A four way Analysis of Variance was run on the following model:

$$\hat{S} = \mu + P_i + T_j + F_k + L_l + \{\text{First-Order Interactions}\} + \varepsilon_{m(ijkl)} \tag{4.1}$$

where S = Strength

P_i = Test period during the Day [i=1:morning, 2:mid-day, 3:afternoon]

T_j = Test position [j=1:A, 2:B]

F_k = Fatigue effect from beginning to end of each period [k=1: beginning, 2:end]

Table 7. Data for Residual Effects Sub-Study

		PERIOD (P) MORNING 1		MID-DAY 2		AFTERNOON 3	
LEARNING PERIOD (L)	FATIGUE EFFECT (F)	POSITION (T) 1	2	POSITION (T) 1	2	POSITION (T) 1	2
First four days (1)	Begin (1)	43.23	83.48	50.13	82.54	42.65	88.88
	End (2)	43.23	80.95	42.97	80.9	46.76	78.53
2nd four days (2)	Begin (1)	45.93	83.25	46.4	84.83	46.25	83.8
	End (2)	43.37	83.67	43.95	78.4	45.2	78.65
3rd four days (3)	Begin (1)	44.53	80.8	43.33	83.07	48.43	87.77
	End (2)	46.57	82.37	46.4	80.5	48.83	84.5
4th four days (4)	Begin (1)	45.93	86.23	46.33	82.33	51.0	84.7
	End (2)	43.77	84.07	46.33	79.53	50.17	84.83
5th four days (5)	Begin (1)	45.75	86.5	43.27	83.7	45.3	85.07
	End (2)	46.53	86.03	46.5	83.83	45.83	85.1

Table 8. Analysis of Variance for Residual Effects Sub-Study

SOURCE	SUM OF SQUARES	DF	MS	F
Daily Test Period (p_i)	36.121	2	18.06	4.92**
Test Position (T_j)	21052.15	1	21052.15	5730.6*
Fatigue Effect (F_k)	22.93	1	22.93	6.24**
Four-Day Learning Period (L_1)	35.12	4	8.78	2.39
Two-way Interactions				
PT_{ij}	15.71	2	7.86	2.14
PF_{ik}	4.08	2	2.04	.56
PL_{il}	52.77	8	6.60	1.80
TL_{jl}	18.52	4	4.63	1.26
TF_{jk}	18.25	1	18.25	4.97**
LF_{kl}	34.08	4	8.52	2.32
Residual	110.21	30	3.67	
Total	21399.9	59		

Three and Four way interactions pooled as error.

*Significant at .1%   **Significant at 5%

L_1 = Learning effect over the 21 test days [1=1:first four days, 2:second four days,...,5:last four days]

All three and four way interactions were assumed to be insignificant and were pooled with the error term. The ANOVA table is given in Table 8.

The ANOVA provides an indication that there are differences in strength due to all four variables in the model. Table 9 gives each factor, its mean, expressed as a deviation from the grand mean, and a measure of association labelled r. The term r cannot be strictly considered as a correlation coefficient beacuse the variables in the sub-study are not random variables in the statistical sense. However, it does give an indication of the amount of variability explained by that particular variable in the ANOVA model. The multiple R, listed at the bottom of Table 9, is a measure of association between strength and all the independent variables. The R^2 term is called the coefficient of multiple determination and is a measure of the usefulness of the terms in equation 4.1. The R^2 term measures the percentage of the variability in strength explained by all the variables in the ANOVA model. (Draper and Smith, 1966) The R^2 term indicates here that the main effects account for almost 99 percent of the data variability.

Subject Variability Sub-Study

Table 5, Chapter III presents a description of eight randomly chosen test positions which were replicated five times each at random intervals throughout the testing period. These replications were used to estimate the subject's overall variability as well as to test for variance homoscedasticity.

Table 9. Deviations from Grand Mean - Residual Effects Sub-Study

Grand Mean = 64.56

VARIABLE		DEVIATION FROM GRAND MEAN	r
Daily Test Period(P)			
	1	-.25	
	2	-.80	
	3	1.05	.04
Test Position (T)			
	1	-18.73	
	2	18.73	.99
Four Day Learning Period(L)	1	-.87	
	2	-.92	
	3	.20	
	4	.87	
	5	.72	
		.62	.04
Fatigue Effect(F)			
	1	.62	
	2	-.62	.03

First-Order Interactions

	POSITION(T)	
PERIOD(P)	1	2
1	-19.67	19.18
2	-19.00	17.40
3	-19.52	19.62

	FATIGUE(F)	
PERIOD(P)	1	2
1	0.0	-.50
2	.03	-1.63
3	1.83	.28

Table 9. Deviations from Grand Mean - Residual Effects Sub-Study (Continued)

	LEARNING(L)				
PERIOD(P)	1	2	3	4	5
1	-1.82	-2.50	-.99	.44	1.64
2	-.42	-1.17	-1.23	-.93	-.23
3	-.35	-1.08	2.82	3.12	.77

	LEARNING(L)				
POSITION(T)	1	2	3	4	5
1	-19.72	-19.38	-18.21	-17.30	-19.03
2	17.99	17.54	18.61	19.06	20.48

	FATIGUE(F)	
POSITION(T)	1	2
1	-18.66	-18.80
2	19.90	17.56

	LEARNING(L)				
FATIGUE(F)	1	2	3	4	5
1	.59	.52	.10	1.53	.37
2	-2.33	-2.35	.30	.22	1.08

Multiple R = .994

Multiple R^2 = .988

Sample variances were calculated as:

$$S^2 = 1/N-1 \sum_{i=1}^{N} (X_i - \overline{X})^2 \tag{4.3}$$

where S^2 = sample variance

N = sample size

X_i = dependent variable, sample i

$\overline{X} = 1/N \sum_i X_i$

The random variation within each of the eight test positions is assumed to follow the normal distribution. Table 10 presents the variances, standard deviations, and the coefficients of variation for each replicated position.

To test whether the set of eight variances is homogeneous, the Bartlett test of variance homoscedasticity was used. (Duncan, 1965) The Bartlett test is based on the statistic

$$M = 2.3026 \left[v\log\left(\sum_i v_i S_i^2 / v\right) - \sum_i v_i \log S_i^2 \right]$$

where S_i = the independent variance estimates to be compared

v_i = number of degrees of freedom, sample i

Table 10. Within Subject Variation Estimates

REPLICATED TEST POSITION	MEAN $\overline{X}_i$	VARIANCE S_i^2	STD. DEV. S_i	COEFF. OF VARIATION $S_i/\overline{X}_i$
1	124.12	10.58	3.25	.0262
2	50.54	8.643	2.94	.0582
3	53.05	13.26	3.64	.0686
4	22.0	5.28	2.30	.1044
5	62.60	7.148	2.67	.0427
6	35.12	11.75	3.43	.0976
7	20.34	7.824	2.80	.1375
8	73.53	6.16	2.48	.0338
Mean	55.16	8.83	2.94	.0711
Minimum	20.34	5.28	2.30	.0262
Maximum	124.12	13.26	3.64	.1375

$$v = \sum_i X_i$$

M is approximately distributed as χ^2 with g-1 degrees of freedom where g is the number of samples. Bartlett has shown that if C is defined as

$$C = 1 + 1/3(g-1)\left[\sum_i 1/v_i - 1/v\right]$$

then M/C is more closely approximated by a χ^2 distribution than M alone.

The values of M, C, and M/C for this study are:

$$M = 1.4224$$

$$C = 1.094$$

$$M/C = 1.300$$

Testing against the ratio M/C, $\chi^2_{.05,7}$ equals 14.1, which is greater than the sample value of 1.3. The null hypothesis is, therefore, not rejected, and we conclude that the variances are homogeneous.

Study I

The ANOVA model for Study I consists of four variables. The ANOVA table is given in Table 11. The analysis of variance model tested was:

$$\hat{S}_T = H_i + S_{j(i)} + F_k + D_l + \{\text{First-Order Interactions}\} + \varepsilon_{m(ijkl)} \qquad (4.4)$$

Table 11. Analysis of Variance for Study I

SOURCE	SUM OF SQUARES	DF	MS	F
Handle Location (H)	29679.4	17	1745.85	17.75*
Distance from SRP (S)	2962.09	2	3376.07	34.32*
Forearm Rotation (F)	2849.87	2	1424.93	14.48*
Direction of Force (D)	163376	5	32675.37	332.14*
<u>1st Order Interactions</u>				
HF_{ik}	7529.33	34	221.45	2.251*
HD_{il}	273034.3	85	3212.17	32.65*
$SF_{jk(i)}$	5149.9	4	1287.5	13.09*
FD_{kl}	18088.93	10	1808.89	18.39*
$SD_{jl(i)}$	30540.75	10	3054.08	31.04*
Residual	75,553.33	768	98.38	
Total	612,554.5	971		

*Significant at .1%

where S_T = Strength

H_i = Handle location as defined in Table 3 and Figure 19, $i=1,\ldots,18$

$S_{j(i)}$ = Distance from the SRP (j=1:far, 2:mid-range, 3:near)

F_k = Forearm rotation (k=1:pronation, 2:mid-position, 3:supination)

D_l = Direction of force (l=1:left, 2:right, 3:up, 4:down, 5:push, 6:pull)

All second and higher order interactions were assumed to be insignificant and were pooled into the error term. Although the error mean square is high, any second order or higher interaction that may prove statistically significant would be difficult to interpret. The best estimates of all main effects and first order interactions are given in Table 12. The parameter estimates are calculated from the ANOVA model. The data for this study may be found in Table 19, Appendix A.

Study II

Table 13 presents the data from the second phase of data analysis, the study of strength variation in the transverse plane 20 inches above the SRP. A more meaningful display of the data in Table 13 is a plot of the estimated strength contours on the X-Y plane. (See Figure 20)

The data was used to develop several potential mathematical models. All models fit to the data will be presented in Chapter V. Of all models that were fit, however, three were selected as being of particular interest. Table 14 lists these three models, their regression coefficients, the mean square error, multiple correlation coefficient R, and the coefficient of determination R^2. Although some of these models yield a poor fit, they are useful in analyzing the data. Contours generated by

Table 12. Parameter Estimates for Main Effects and First Order Interactions, Study I

Grand Mean = 48.12
Main Effects

EFFECT	CELL MEAN	PARAMETER ESTIMATES	EFFECT	CELL MEAN	PARAMETER ESTIMATES
H1	45.33	-2.79	S1	45.99	-2.13
H2	51.38	3.66	S2	47.85	-.27
H3	47.99	-.13	S3	50.37	2.25
H4	53.65	5.53			
H5	54.33	6.21	F1	50.96	2.84
H6	57.61	9.49	F2	47.71	-.41
H7	49.68	1.56	F3	46.26	-1.86
H8	44.56	-3.56			
H9	34.73	-13.39	D1	31.49	-16.63
H10	50.55	2.43	D2	34.57	-13.55
H11	54.83	6.71	D3	46.11	-2.01
H12	49.73	1.61	D4	60.63	12.51
H13	44.57	-3.55	D5	47.53	-.59
H14	40.68	-7.44	D6	68.01	19.89
H15	46.34	-1.78			
H16	41.76	-6.36			
H17	52.26	4.14			
H18	46.49	-1.63			

First Order Interactions

	DISTANCE (S)					
	1		2		3	
HANDLE LOCATION(H)	CELL MEAN	PAR. EST.	CELL MEAN	PAR. EST.	CELL MEAN	PAR. EST.
1	40.63	-6.83	44.36	-.70	51.01	3.43
2	50.88	1.63	51.10	-.01	52.16	-1.47
3	43.77	-2.09	48.32	.60	51.86	1.62
4	58.70	7.18	52.64	-.74	49.62	-6.28
5	52.92	-8.88	53.55	-.51	56.52	-.06
6	57.78	2.30	55.04	-2.30	59.88	.02
7	49.02	1.47	47.89	.038	52.13	.20
8	41.47	-.96	46.82	2.53	45.39	-1.42
9	33.69	1.09	34.76	.30	35.74	-1.24
10	47.44	-.98	50.26	-.02	53.96	1.16
11	50.99	-1.73	56.86	2.30	56.66	-.42
12	49.72	2.12	49.82	.36	49.65	-2.33
13	42.76	.32	43.84	-.46	47.12	.30

Table 12. Parameter Estimates for Main Effects and First Order Interactions, Study I (Continued)

First Order Interactions

DISTANCE (S)

HANDLE LOCATION(H)	1 CELL MEAN	1 PAR. EST.	2 CELL MEAN	2 PAR. EST.	3 CELL MEAN	3 PAR. EST.
14	36.78	-1.77	40.55	.14	44.71	1.78
15	43.44	-.77	44.92	-1.15	50.67	2.08
16	37.12	-2.51	41.2	-.29	46.94	2.93
17	50.46	.33	54.09	2.10	52.23	-2.28
18	43.27	-1.09	45.71	-.51	50.49	1.75

FOREARM ROTATION (F)

HANDLE LOCATION(H)	1 CELL MEAN	1 PAR. EST.	2 CELL MEAN	2 PAR. EST.	3 CELL MEAN	3 PAR. EST.
1	48.40	.23	46.68	1.76	40.91	-2.56
2	52.94	-1.28	54.72	3.75	46.48	.24
3	52.85	2.02	44.01	-3.57	47.1	.97
4	51.55	-4.94	50.81	-2.43	58.6	6.81
5	57.73	.56	52.58	-1.34	52.68	.21
6	59.53	-.92	59.24	2.04	54.28	-1.47
7	54.47	1.95	48.68	-.59	45.89	-1.93
8	49.47	2.07	41.64	-2.51	42.57	-.13
9	34.78	-2.79	34.55	.23	34.86	1.99
10	53.44	.05	51.89	1.75	46.33	-2.36
11	57.07	-.60	57.32	2.90	50.11	-2.86
12	53.12	.55	47.85	-1.47	48.22	.35
13	42.65	-4.76	42.14	1.87	48.93	6.22
14	43.83	.31	39.77	-.50	38.43	-.39
15	49.59	.41	47.69	1.76	41.74	-2.74
16	43.12	-1.48	42.63	1.28	39.52	-.38
17	54.76	-.34	51.92	.07	50.09	-.31
18	48.81	-.52	44.74	-1.34	45.92	1.29

Table 12. Parameter Estimates for Main Effects and First Order Interactions, Study I (Continued)

DIRECTION OF FORCE (D)

HANDLE LOCATION(H)	1 MEAN/ EST.	2 MEAN/ EST.	3 MEAN/ EST.	4 MEAN/ EST.	5 MEAN/ EST.	6 MEAN/ EST.
1	27.12 -1.58	48.17 16.39	65.43 22.11	49.41 4.08	29.32 -15.42	52.54 -12.68
2	44.43 9.68	44.88 7.05	35.35 -14.02	53.10 -10.79	59.91 9.12	70.61 -.66
3	32.34 .98	32.47 -1.97	36.53 -9.45	93.97 33.47	36.43 -10.97	56.12 -11.70
4	22.06 -14.96	32.93 -7.17	24.55 -27.09	41.72 -24.44	80.89 27.83	119.78 46.24
5	28.51 30.03	41.98 1.20	89.24 36.92	45.23 -21.61	46.45 -7.29	74.57 .35
6	31.30 -9.68	35.86 -8.20	35.65 -19.95	56.84 -13.28	79.55 22.53	106.48 28.98
7	29.97 -3.08	26.17 -9.96	24.12 -23.55	91.81 29.62	48.63 -.46	77.38 7.81
8	35.74 7.81	19.52 -11.49	70.34 27.79	53.71 -3.36	33.82 -10.15	54.21 -10.24
9	33.08 14.98	29.01 7.83	34.66 1.94	21.34 -25.84	29.0 -5.14	61.29 6.67
10	29.28 -4.64	53.46 16.46	75.51 26.97	44.10 -18.96	33.10 -16.86	67.88 -2.56
11	29.67 -8.53	40.86 -.42	34.01 -18.81	50.9 -16.44	86.39 32.15	87.16 -8.59
12	29.19 -22.84	34.53 -1.65	32.22 -15.39	94.23 31.99	47.06 -2.08	61.03 -8.59
13	26.71 -1.23	29.23 -1.79	71.14 28.58	59.44 2.36	35.66 -8.32	45.26 -19.20
14	17.98 -6.07	32.52 5.39	87.54 45.87	32.54 -20.65	28.11 -11.98	48.37 -12.20

Table 12. Parameter Estimates for Main Effects and First Order Interactions, Study I (Continued)

DIRECTION OF FORCE (D)

HANDLE LOCATION(H)	1 MEAN/ EST.	2 MEAN/ EST.	3 MEAN/ EST.	4 MEAN/ EST.	5 MEAN/ EST.	6 MEAN/ EST.
15	40.16	28.52	31.38	52.82	52.06	73.12
	10.39	-4.27	-12.95	-6.03	6.31	6.89
16	37.56	30.89	23.78	89.08	32.46	36.78
	12.43	2.68	-15.97	34.81	-8.71	-24.90
17	38.27	29.76	37.28	59.58	60.18	88.49
	2.64	-8.95	-12.97	-5.19	8.51	16.34
18	36.2	32.29	24.2	101.61	41.3	43.34
	6.34	-.65	-20.28	-4.60	-4.60	-23.04

FOREARM ROTATION (F)

DISTANCE (S)	1 CELL MEAN	1 PAR. EST.	2 CELL MEAN	2 PAR. EST.	3 CELL MEAN	3 PAR. EST.
1	48.75	-.08	44.97	-.61	44.67	.54
2	49.52	-1.17	47.90	.46	46.16	.17
3	52.89	-.32	50.28	.32	47.95	-.56

DIRECTION OF FORCE (D)

DISTANCE (S)	1 MEAN/ EST.	2 MEAN/ EST.	3 MEAN/ EST.	4 MEAN/ EST.	5 MEAN/ EST.	6 MEAN/ EST.
1	27.09	29.20	37.46	48.33	55.39	78.49
	-2.27	-3.24	-6.52	-10.17	9.99	12.61
2	30.96	34.91	45.24	60.76	46.96	68.24
	-.26	.61	-.60	.40	-.30	.50
3	36.42	39.61	55.64	72.81	40.43	57.3
	2.68	2.79	7.28	9.93	-9.35	-12.96

Table 12. Parameter Estimates for Main Effects and First Order Interactions, Study (Continued)

DIRECTION OF FORCE (D)

FOREARM ROTATION(F)	1 MEAN/ EST.	2 MEAN/ EST.	3 MEAN/ EST.	4 MEAN/ EST.	5 MEAN/ EST.	6 MEAN/ EST.
1	36.73	37.65	47.52	65.53	51.58	66.74
	2.40	.24	-1.43	2.06	1.21	-4.11
2	32.17	35.95	44.14	58.16	49.48	66.40
	1.09	1.79	-1.56	-2.06	2.36	-1.20
3	29.64	30.28	46.68	58.22	41.80	70.94
	.01	-2.43	2.43	-.55	-3.87	4.79

Table 13. Data for Study II

X	Y	STRENGTH	X	Y	STRENGTH
-5.0	6.0	61.5	12.0	6.0	38.8
-7.0	6.0	64.8	15.0	6.0	43.1
-8.0	6.0	75.6	18.0	6.0	54.0
-9.0	6.0	77.7	21.0	6.0	68.0
-11.0	6.0	66.2	24.0	6.0	68.1
-12.0	6.0	63.1	27.0	6.0	87.0
-15.0	6.0	60.0	30.0	6.0	71.9
-17.0	6.0	53.2	6.0	-8.0	42.5
-5.5	7.0	63.1	7.0	-9.5	45.0
-7.0	9.0	68.2	9.0	-12.0	59.9
-8.5	11.0	74.0	10.0	-13.5	75.8
-9.0	12.0	82.0	12.0	-16.0	84.5
-9.5	13.0	70.3	13.0	-17.5	86.9
-10.0	13.5	69.1	15.0	-20.0	92.0
-11.0	14.5	66.9	16.5	-21.5	95.0
-14.0	18.5	56.6	18.0	-24.0	81.4
0	7.0	51.0	-6.5	-5.0	59.9
0	12.0	53.0	-9.0	-7.0	64.0
0	17.0	69.9	-10.5	-8.0	65.2
0	22.0	84.5	-12.0	-9.0	57.0
0	27.0	104.5	-14.5	-11.0	51.6
0	32.0	101.2	-17.0	-13.0	45.4
0	33.0	88.7	-20.0	-15.0	39.8
0	35.0	63.5	-22.5	-17.0	28.5
5.5	7.0	48.1	6.0	3.5	31.0
8.0	10.0	56.0	10.5	6.0	34.0
11.0	14.5	77.2	13.0	7.5	43.0
14.0	18.0	87.1	16.5	9.5	57.3
15.0	20.0	109.0	20.0	11.5	68.2
16.5	21.5	121.0	22.5	13.0	91.0
18.0	24.0	102.0	25.0	14.5	73.2
20.0	26.5	88.0	27.0	15.5	71.1
10.0	6.0	33.0			

FOREARM ROTATION - Mid-Position

DIRECTION OF FORCE - Push

HORIZONTAL PLANE - Z = 20 inches above SRP

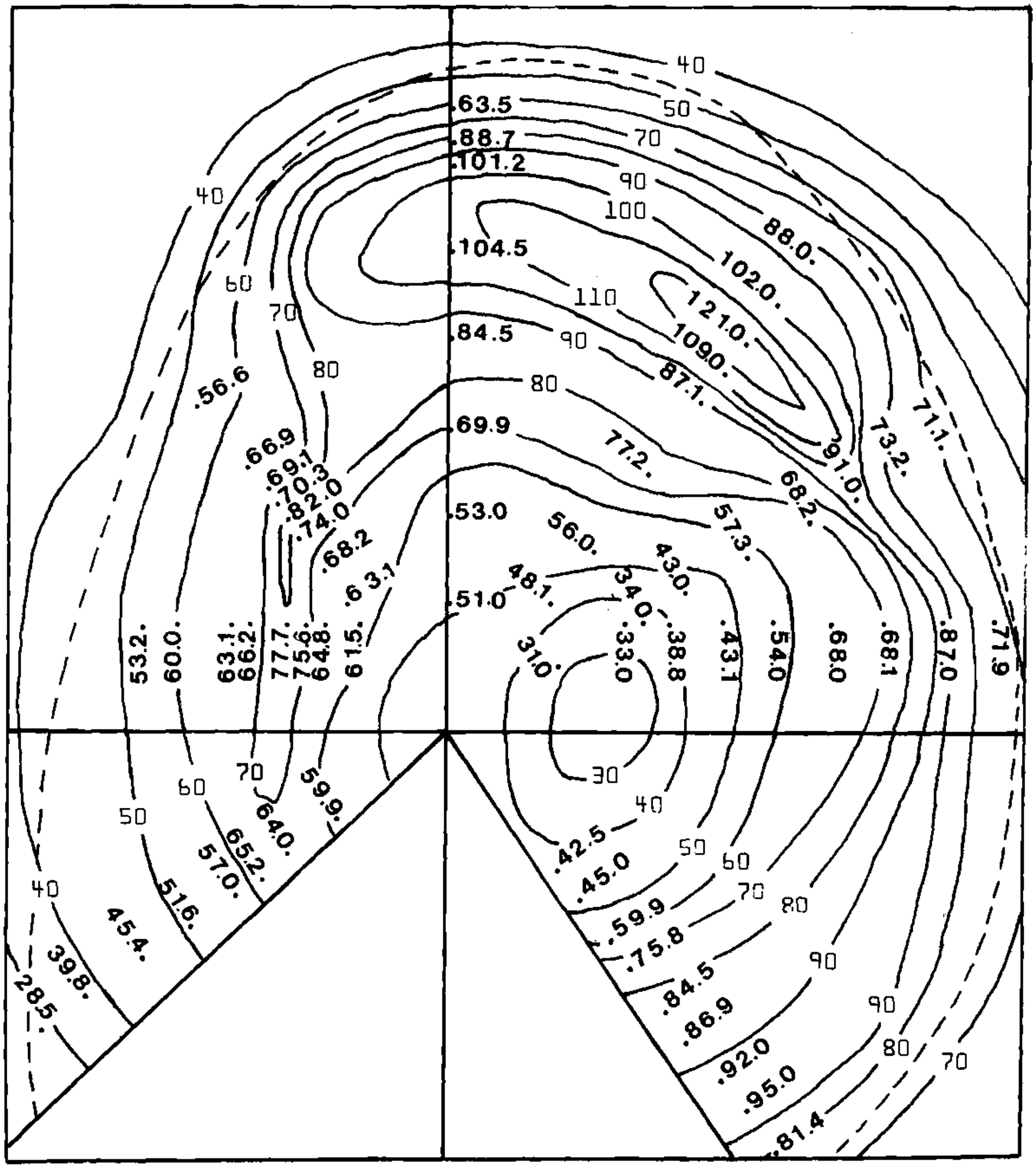


Figure 20. Equal Strength Contours Generated by Data For Study II
(Dotted Line Indicates Reach Sphere of the Subject)

Table 14. Three Selected Mathematical Models

MODEL: Step-wise Regression Model

$$\hat{S} = b_0 + b_1X + b_2X^2 + b_3X^3 + b_4X^5 + b_5Y + b_6Y^2 + b_7Y^3$$

$$+ b_8Y^4 + b_9XY + b_{10}XY^2 + b_{11}XY^3 + b_{12}XY^4 + b_{13}YX^4$$

COEFFICIENT	VALUE	COEFFICIENT	VALUE
b_0	47.643	b_7	.00456
b_1	-2.811	b_8	-.0001890
b_2	.00035	b_9	-.00071
b_3	1.0351	b_{10}	.0152
b_4	-.00000522	b_{11}	.000128
b_5	-.388	b_{12}	-.00001967
b_6	.1025	b_{13}	-.00016193

Mean Square error = 34.387
Multiple R = .9649
Multiple R^2 = .9311

Table 14. Three Selected Mathematical Models (Continued)

MODEL: One term parabolic model

$$\mathrm{Log}_e(\hat{S}) = b_0 + b_1(R^*-R_0)^2$$

COEFFICIENT	VALUE
b_0	4.3464
b_1	-.00261

Where $R^* = \sqrt{(X-9.6)^2+(Y-2.9)^2}$

$R_0 = 21.1$

Mean square error = .0416
Multiple R = .7656
Multiple R^2 = .5861

One term parabolic model in exponential form

$$\hat{S} = b_0 + b_1 e^{\alpha(R^*-R_0)^2}$$

COEFFICIENT	VALUE
b_0	10.1361
b_1	69.15

Where $R^* = \sqrt{(X-8.6)^2+(Y-2.7)^2}$

$R_0 = 22.1$

$\alpha = -.0031$

Mean square error = 216.971
Multiple R = .6806
Multiple R^2 = .4632

Table 14. Three Selected Mathematical Models (Continued)

MODEL: Predictive Model

$$\text{Log}_e(\hat{S}) = b_0 + b_1(X-10.5)^2 + b_2(Y-.5)^2 + b_3\sqrt{(X-10)^2+(Y-.1)^2}$$

COEFFICIENT	VALUE
b_0	2.837
b_1	-.003438
b_2	-.002649
b_3	.137

With Residuals Calculated in Terms of $\text{Log}_e(S)$.

Mean square error	=	.010
Multiple R	=	.9508
Multiple R^2	=	.9040

With Residuals Calculated in Terms of S.

Multiple R	=	.9224
Multiple R^2	=	.8508

these three models are presented in Figures 21 through 24 for comparison with the data contours of Figure 20.

Calculation of Subject Percentile Data

Calculation of the subject's percentile rating was done by duplicating test positions of previous studies. Test positions were selected from Hunsicker's 1955 data, and from Thordsen and Kroemer (1972). These studies provided percentile data which were compared to the results of the duplicate tests. Table 15 presents the source of each selected test position, a description of each position, the force in pounds exerted by the subject of this study at each position, and the estimated percentile rating. An overall subject percentile rating was calculated by averaging the five estimates.

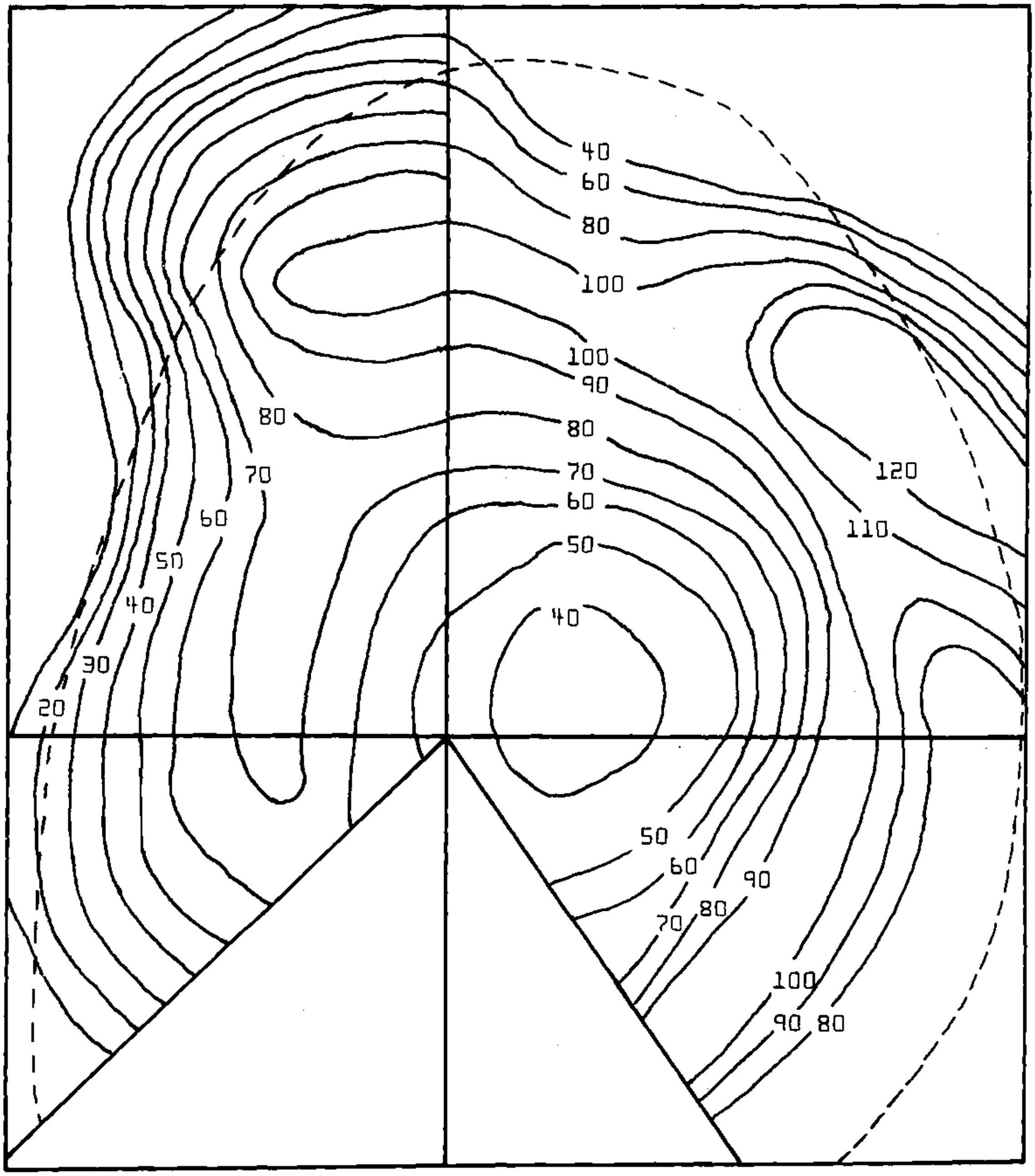


Figure 21. Equal Strength Contours Generated by the Step-wise Regression Model (Dotted Line Indicates Reach Sphere)

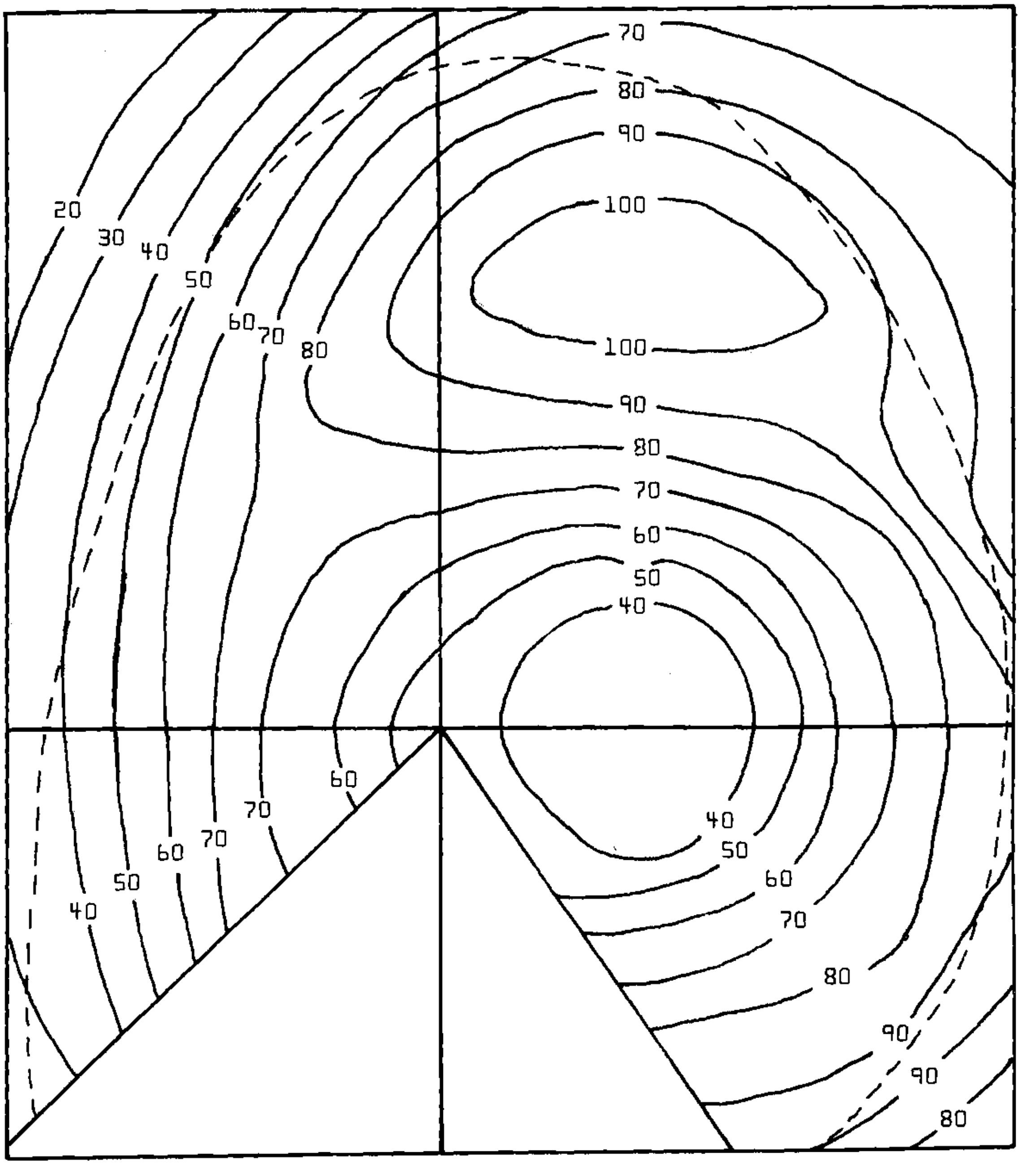


Figure 22. Equal Strength Contours Generated by the Final Predictive Model (Dotted Line Indicates Reach Sphere)

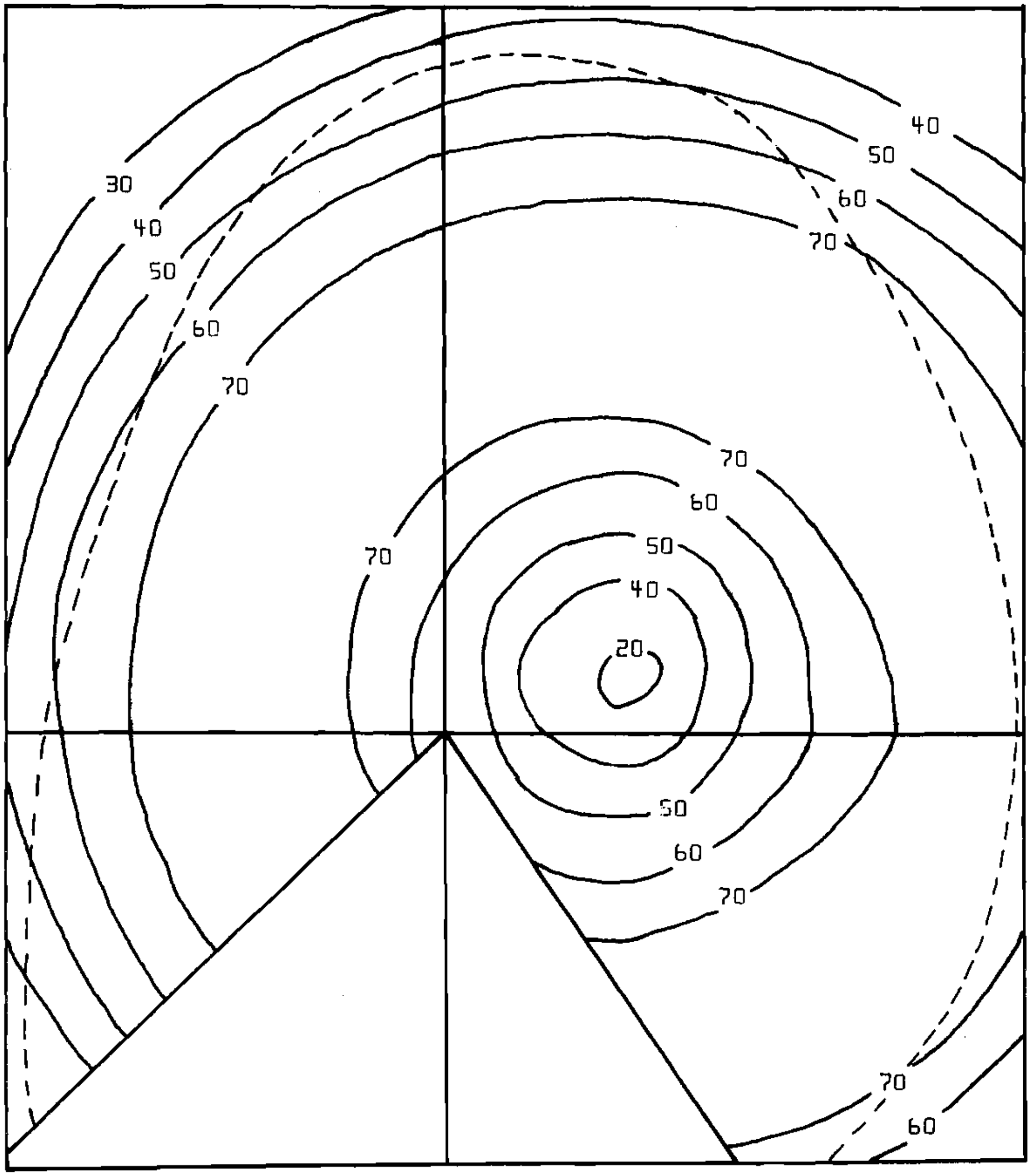


Figure 23. Equal Strength Contours Generated by Parabolic Model
(Dotted Line Indicates Reach Sphere)

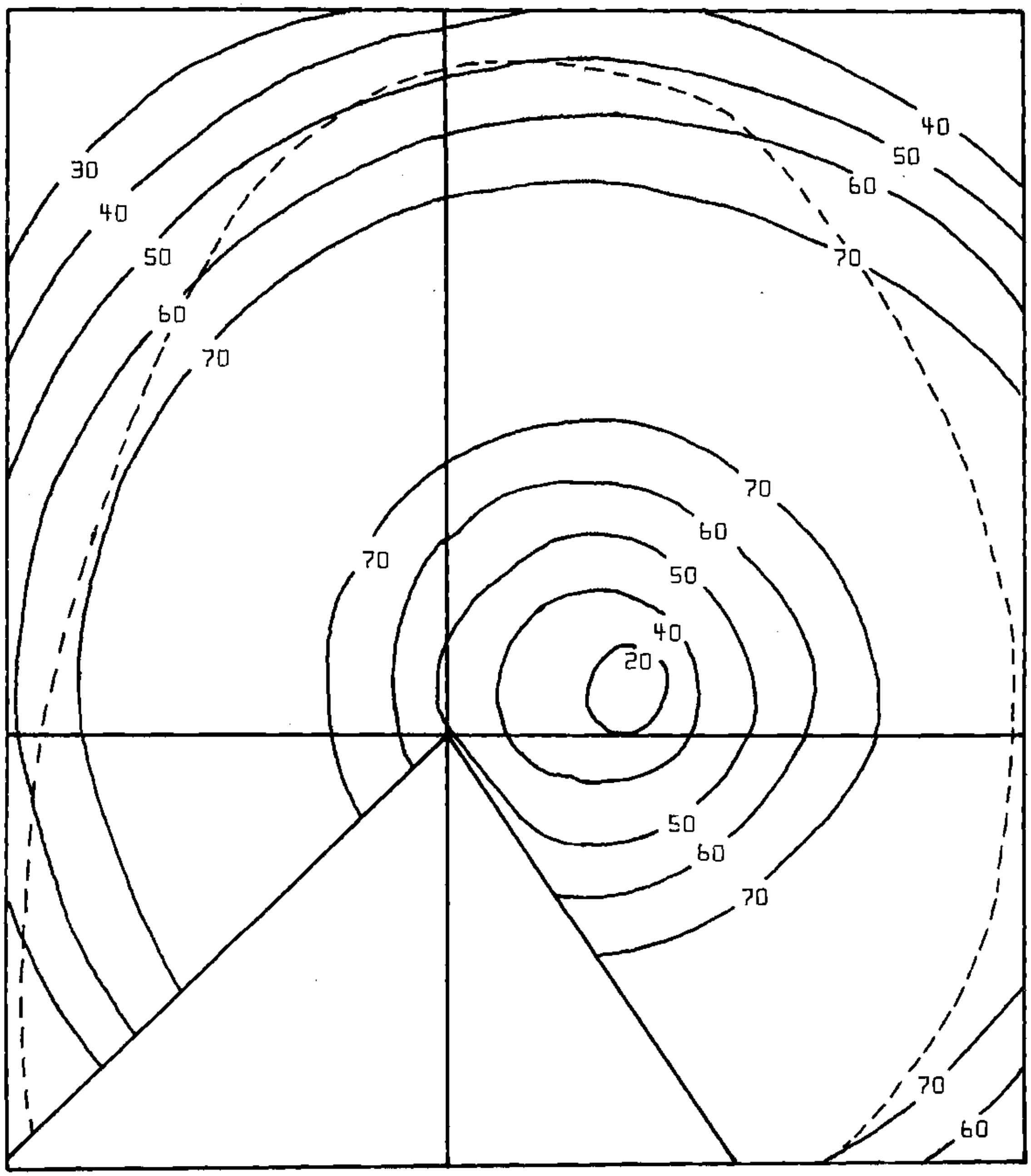


Figure 24. Equal Strength Controus Generated by the Parabolic Model in Exponential Form (Dotted Line Indicates Reach Sphere)

Table 15. Subject Percentile Data

POSITION NUMBER	SOURCE	DESCRIPTION	STRENGTH	PERCENTILE
1	Hunsicker, 1955	Horizontal push, 90 degree elbow angle, exertion in sagittal plan of shoulder, forearm in mid-position, right hand.	133.2	84
2	Hunsicker, 1955	Horizontal right, 90 degree elbow angle, sagittal plane of shoulder, forearm in mid-position, right hand.	61.33	85
3	Hunsicker, 1955	Up force, eblow angle 180 degrees, in sagittal plan of shoulder forearm in mid-position, right hand.	73.00	83
4	Thordsen Kroemer, 1972	Downward force (named forward force by Thordsen), overhead, in mid-frontal plane, 10 inches left of SRP, 47.3 inches above SRP, forearm in pronation, left hand.	49.6	88
5	Thordsen Kroemer, 1972	Left forces, in mid-sagittal plane, 12.4 inches above SRP, 13 inches right of SRP, forearm in mid-position, left hand.	60.41	71

Estimated overall subject percentile for both hands = 84

CHAPTER V

DISCUSSION

This chapter provides a discussion of all results presented in Chapter IV. Following a discussion of the sub-studies, general guidelines are presented from the analysis of Study I on strength variation in three dimensions. The models developed in Study II are evaluated. A description of the techniques used to derive these models and to determine values of the various parameters may be found in Appendices C and D.

Residual Effects Sub-Study

The assumption of no day of the week effect effect seems to be a valid one; without it the analysis of strength test results would certainly be more difficult. Residual effects in this research were found to be relatively insignificant; diurnal variations, fatigue, and learning effects are discussed and are graphically represented in Figure 25.

Diurnal Effects

Recorded strength seems to decrease from morning to mid-day, then increase to a maximum in period three. This may be because the subject ate the noon meal between periods two and three. His energy level was, therefore, higher during period three. The differences, however, are less than two pounds, as is shown in Figure 25a.

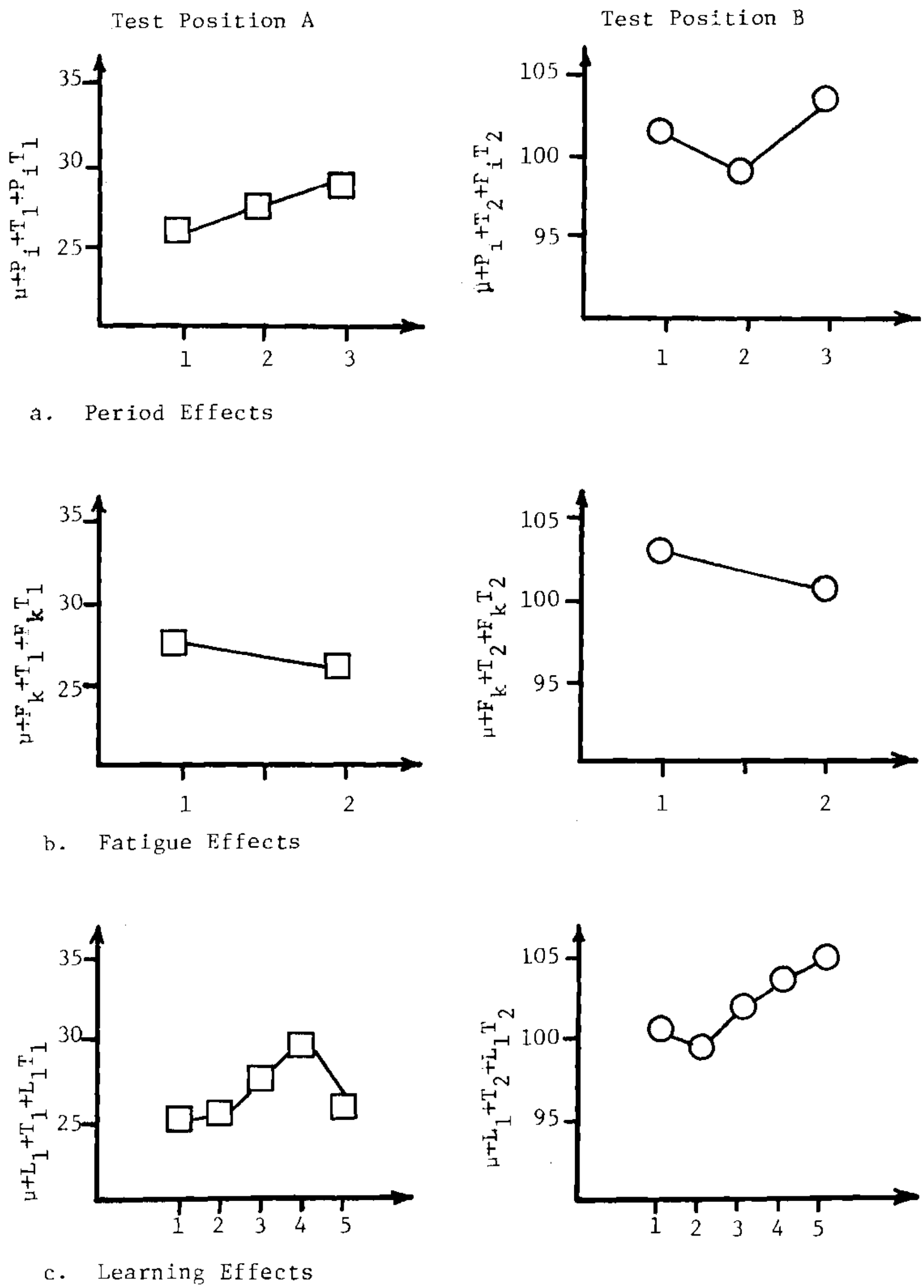
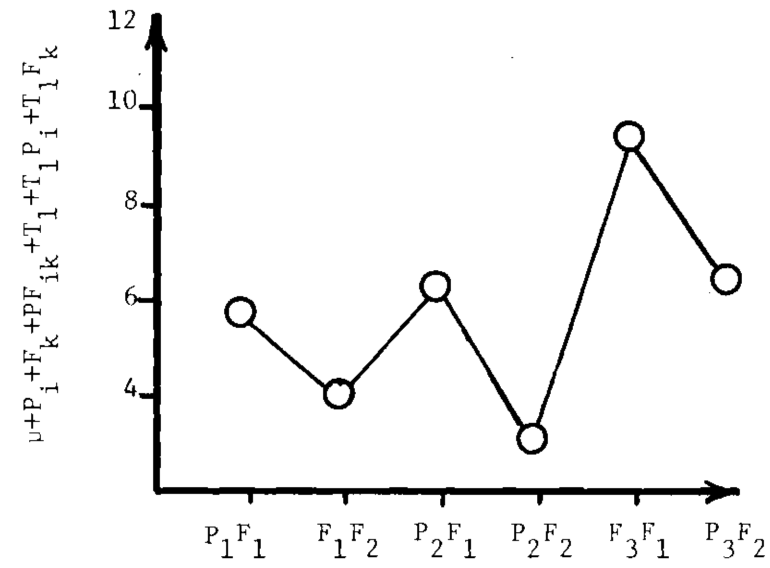
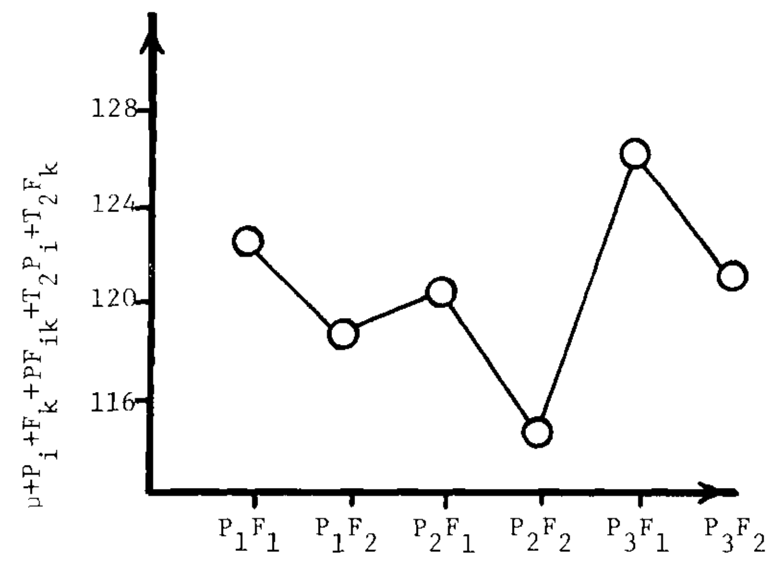


Figure 25. Residual Effects Plotted Against the Partial ANOVA Models



d. Fatigue during the day - Test Position 1



e. Fatigue during the day - Test Position 2

Figure 25. Residual Effects Plotted Against the Partial ANOVA Models (Continued)

Fatigue Effects

The ANOVA in Table 8 indicates a statistically significant fatigue effect from the beginning to the end of a test period. However, from Figure 25b. it can be seen that the average strength decrease was less than 1.5 pounds. In fact, in the third and fifth learning periods, there were actually increases in average strength scores from beginning to end of the periods. The fatigue effect was not consistent throughout the testing period.

Learning Effects

Maximum strength variations due to residual learning were less than five pounds (see Figure 25c.). The test position allowing the greatest strength exertions (test position B - see chapter III) showed a continual learning trend (strength increase) from the fifth day throughout the entire 21 day test period. In the test position with the lower average strength scores (test position A), learning effects stopped after 16 days of testing, and actually began to show signs of negative learning (strength decreases) in the last four test days (Figure 25c.).

Subject Variability Sub-Study

Training

The most important factor affecting subject variability was practice at the task. When the pilot study was begun, there were some test positions that produced as much as 25 pound strength differences from test to test. At the end of the training period, recorded strength in any test position varied no more than five to eight pounds from one test to another. With training, the subject was able to determine his "normal-natural" position for each handle location, and learned to duplicate the position more accurately with practice.

Comparison to Other Studies

From Table 10, the coefficient of variation for the eight replicated test positions (Table 5) ranged from .0262 to .1375. Comparison of these values with those found in the literature indicates that the subject variability is comparable to that found by other researchers. Wakim (1950) reports ranges in the coefficient of variation for males of from .053 to .093 when testing elbow flexion. Darcus (1951) reports these coefficients for forearm pronation and supination strengths. For supination, he reports a range of .023 to .091, for pronation a range of .042 to .134. Values found in this sub-study are considerably less than the values reported by Schanne (1972), whose values ranged from .067 to .369.

Study I

Evaluation of the ANOVA Model

The residual mean square for the ANOVA model is large compared to the estimated experimental variability. That is, from Table 10, the average standard deviation is 2.94, and the ANOVA estimate of this value, $\hat{\sigma}_e = \sqrt{MS_e} = 9.91$. However, the ANOVA error term is a combination of experimental error, second and higher order interactions, and residual effects such as fatigue, learning, and diurnal variations. Since the estimate of experimental variability was calculated from a sample of eight test positions, the sample size may be too small to provide a true measure of $\hat{\sigma}_e$. It is, therefore, believed that restricting the model to first order interactions which can be interpreted was justified. In addition, the inclusion of any second order interactions would only inflate the values of the F statistics, leading to greater indicated significance levels than is already evident in the present model. Since all factors are now significant at the .1% level, it would accomplish nothing to include additional interaction terms.

Comparison with the Results of Other Studies

Studies by Hunsicker (1955) and (1957) and Thordsen, et al. (1972) are similar in concept to this study. It is difficult, however, to compare results for the following reasons:

1. Hunsicker's maximal force was achieved by taking three readings of three force components (X,Y,Z directions of which one was the required direction), during a five second exertion period. The resultant forces were calculated and the largest reported.
2. The positions Hunsicker examined were designed to achieve specific elbow angles, while the positions in this study were concerned with a specific handle location and distance from the SRP.
3. Thordsen and Kroemer (1972) conducted their study on force exertions on aircraft controls. Their method of defining force directions in certain handle locations differs from the method used in this study. All directions used in this study were defined in relation to the handle control box, regardless of spacial location of the hand. Thordsen, et al. defined directions generally in relation to the body of the

subject. For example, Thordsen's overhead control, up force would be defined in this study as overhead, push.

4. Thordsen's study was conducted on left hand exertions, while this study was on right hand exertions only.

General Guidelines - Study I

Effects of Handle Location

Forces exerted vary as the location of the handle is changed. This section provides general guidelines on locations of greatest and lowest strength, and on the way in which strength varies throughout the reach sphere.

Maximum Strength. Overall, the greatest strength is exerted in positions directly in front of the body, in the mid-sagittal plane, specifically, 20 inches above the SRP. In that 20 inch transverse plane, comparable strength values were also found in positions to the side of the body in the frontal plane six inches in front of the SRP, and also in the right front quadrant of the body, 45 degrees from the mid-frontal plane. There have been very few studies reported in the literature that can be referenced for comparison. In sagittal and frontal plane studies dealing with elbow angle variation, several researchers have found that both shoulder and elbow strengths in flexion, extension, adduction and abduction reach a maximum at or near the 90-120 degree elbow angle (7, 16, 24, 49, 54, 65). In this study, tests run at the near and mid-range distance levels in the 20 inch transverse plane cause elbow angles to range from 60 to no more than 120-130 degrees in handle locations to the front and side of the body. Thus, most handle locations in this plane place the elbow at an angle allowing greater strength to be exerted.

Minimum Strength. The weakest position around the body seems to be with the handle directly behind the head, the average strength value there being six pounds less than the next lowest value, and over 13 pounds less than the grand mean. Other weak areas include positions directly across the body in the left quadrant, both in the plane through the SRP and 40 inches above it. This may be because many positions across the body are awkward to maintain, especially in supination, thus causing lower strength scores in general.

Effects of Varying Vertical Position of the Handle. In all cases, strength is greatest in the transverse plane, 20 inches above the SRP (with little significant difference noted between strength scores in the low and high transverse planes). Strength in the 20 inch plane is approximately six pounds greater in all handle locations, the two extreme planes varying less than two pounds from each other.

Effects of Varying Horizontal Position of the Handle

For the range of handle locations tested in Study I, in all directions perpendicular to the long axis of the arm, strength tends to decrease as the hand moves away from the body. In directions parallel to the long axis, namely push and pull, strength increases as the hand moves away from the body. (See first order interactions of distance level and direction, Table 12.)

Forearm Rotation Effects

As can be seen from the ANOVA, the effect of forearm rotation alone seems less significant than other effects. This appears to be due to the fact that the effect of forearm rotation is extremely dependent on both the handle location and the direction in which the force is exerted. Note, the elbow position is not fixed. By allowing the subject to assume a "natural-normal" position, the subject adjusts his arm position to counteract as much as possible any awkardness of a given forearm rotation. In general, strength is greatest in pronation, followed by mid-position, with supination being the weakest orientation.

Direction of Force Effects

Even for a given location of the handle, the forces measured in different directions varied greatly. For example, the pulling force exerted in the overhead position at the far distance level (50 inches above the SRP) was extremely large (137.3 lbs.) while the upward force (see page 54, and Figure 17) exerted at the same handle location and distance level was one of the smallest observed (16.4 lbs.). This shows that no handle location can be generally labelled as being well (or poorly) suited for large force exertions without first specifying the required direction of force.

In general, greatest strength values occur when pulling forces are exerted, followed by downwardly directed forces. The fact that "down" forces were so great disagrees with results obtained by Thordsen and Kroemer (1972) who found that forces throught the long axis of the arm (push, pull) tend to be greater than those in directions perpendicular to that axis.

Effects of Forearm Rotation When Handle Location is Changed

In most handle locations anterior to the mid-frontal plane, pronation gave greatest average strength scores. In locations posterior to the mid-frontal plane, supination allowed the greatest force exertions. The only exception is directly behind the body at seat level, where pronation was by far the strongest rotation. This could be because supination in that position is uncomfortable and unnatural.

Effects of Forearm Rotation When Force Direction is Changed

The only previous studies found dealing with forearm rotation effects for different force directions dealt primarily with comparisons of elbow flexion and extension. Provins and Salter (1955) reported that the orientation of the hand influenced strength scores; pronation was strongest in elbow extension, and supination greatest in elbow flexion, the direction of force in each case being in the direction of the palm of the hand.

Results of Study I indicate that forearm rotation has little effect on strength when the direction of force is either "up" or "pull." The greatest force difference occurs when pushing forces are exerted. For pushing, the strength values obtained when the arm is pronated are almost 10 pounds greater than when supinated. Pronation of the forearm allows the greatest forces to be exerted in all tested directions except "pull," when largest values of strength occur under supination.

Study II

The purpose of Study II was to obtain more detailed information on the strength of push forces in the 20 inch transverse plane, forearm in mid-position, and to develop a mathematical model capable of predicting these forces. It is now felt that such detailed knowledge of particular variable combinations is prerequisite to the proper development of a general mathematical model for arm strength. A total of 34 models were fit to the data and are summarized in Table 16. The final predictive model and other models of particular interest are discussed in the text. The three selected models will be discussed briefly and evaluated. The procedures followed in deriving the models and in determining their parameter values are presented in Appendices C and D.

Criteria for Model Evaluation

There were two criteria for evaluating each model: prediction accuracy and computational simplicity. It was important to keep the model as simple as possible so that it could be used easily for design purposes. Yet, the model must be capable of predicting arm strength with an accuracy as good as the accuracy of the experimental procedures employed.

Selection of Variables

To define the spacial location of the hand grip, the variables used were X and Y, defined as follows:

Positive X = distance right of the mid-sagittal plane

Positive Y = distance anterior to mid-frontal plane

Origin = SRP

During the analysis of various models, it was useful to consider hand locations as a function of one other variable R*, defined as:

$$R^* = \sqrt{(X-X_0)^2 + (Y-Y_0)^2}$$

This is the radial distance from the point (X_0,Y_0) to the point (X,Y).

Table 16. Mathematical Models - Study II

Model #	Model Form	R^2
1	$\hat{S} = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	.758
2	$\hat{S} = b_0 + b_1(X-8)^2 + b_2^2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4X(X-8)^2 + b_5X(Y-2)^2 + b_6X\sqrt{(X-8)^2+(Y-2)^2}$	.787
3	$\hat{S} = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4Y(X-8)^2 + b_5Y(Y-2)^2 + b_6Y\sqrt{(X-8)^2+(Y-2)^2}$	.8218
4	$\hat{S} = b_0 + b_1(X-8)^4 + b_2(X-8)^2(Y-2)^2 + b_3(X-8)^2\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4(X-8)^3 + b_5Y(X-8)^2 + b_6Y(Y-2)^2 + b_7Y\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_8Y + b_9(X-8)^2 + b_{10}(Y-2)^2 + b_{11}\sqrt{(X-8)^2+(Y-2)^2}$	.8448
5	$\hat{S} = b_0 + b_1(7/5X - 16)^2 + b_2(Y-2)^2 + b_3\sqrt{(7/5X-16)^2+(Y-2)^2}$	.6910
6	$\hat{S} = b_0 + b_1(7/5X-16)^2 + b_2(Y-2)^2 + b_3\sqrt{(7/5X-16)^2+(Y-2)^2}$	
	$+ b_4X(7/5X-16)^2 + b_5X(Y-2)^2 + b_6X\sqrt{(7/5X-16)^2+(Y-2)^2}$	.7582

Table 16. Mathematical Models - Study II (Continued)

Model #	Model Form	R^2
7	$\hat{S} = b_0 + b_1[X-10(X-8)^2+3]^2 + b_2(Y-2)^2$	
	$+ b_3\sqrt{[X-10(X-8)^2+3]^2+(Y-2)^2} + b_4Y[X-10(X-8)^2+3]^2$	
	$+ b_5Y(Y-2)^2 + b_6Y\sqrt{[X-10(X-8)^2+3]^2+(Y-2)^2}$	.5654
8	$\hat{S} = b_0 + b_1[X-10(X-8)^2+3]^2 + b_2(Y-2)^2 + b_3\sqrt{[X-10(X-8)^2+3]^2+(Y-2)^2}$	
	$+ b_4[X-10(X-8)^2+3]^4 + b_5[X-10(X-8)^2+3] +$	
	$+ b_6[X-10(X-8)^2+3]^2(Y-2)^2 + b_7[X-10(X-8)^2+3]^2$	
	$\cdot \sqrt{[X-10(X-8)^2+3]^2+(Y-2)^2} + b_8(Y-10)^2[X-10(X-8)^2+3]^2$	
	$+ b_9(Y-10)^2(Y-2)^2 + b_{10}(Y-10)^2\sqrt{[X-10(X-8)^2+3]^2+(Y-2)^2}$	
	$+ b_{11}(Y-10)^2$	.6908
9	$\hat{S} = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4 Log_e(Y)$	.7962

Table 16. Mathematical Models - Study II (Continued)

Model #	Model Form	R^2
10	$\hat{S}^{1/3} = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4 \mathrm{Log}_e(Y)$	.8542
11	$\hat{S}^{2/3}$ = Same as #10	.8269
12	$\hat{S}^{.20}$ = Same as #10	.5404
13	$\hat{S}^{.30}$ = Same as #10	.5463
14	$\hat{S}^{.32}$ = Same as #10	.5474
15	$\hat{S}^{1/3}$ = Same as #3	.8669
16	$\hat{S} = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4 \mathrm{Log}_e(X)$	.7801
17	$\hat{S}^2 = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	.6555
18	$\hat{S}^{1/2}$ = Same as #17	.8019

Table 16. Mathematical Models - Study II (Continued)

Model #	Model Form	R^2
19	$\hat{S}^{1/2}$ = Same as #9	.8410
20	$\hat{S} = b_0 + b_1X + b_2X^2 + b_3X^3 + b_4X^5 + b_5Y^2 + b_6Y^3 + b_7Y^4$	
	$+ b_8XY + b_9XY^2 + b_{10}XY^3 + b_{11}XY^4 + b_{12}YX^4 + b_{13}Y$	.9311
	(Model yielding the best predictions and highest R^2)	
21	$\hat{S} = b_0 + b_1X + b_2X^3 + b_3Y^2 + b_4Y^3 + b_5Y^4 + b_6XY^2$	
	$+ b_7XY^4 + b_8YX^4$	.8557
	(Stepwise regression model using forward stepwise procedure)	
22	$\text{Log}_e(\hat{S}) = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4\text{Log}_e(Y)$	.8772
23	$\text{Log}_e(\hat{S})$ = Same as #4	.8998
24	$\text{Log}_e(\hat{S}) \doteq b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4\text{Log}_e(Y) + b_5\text{Log}_e\left(\sqrt{X^2+Y^2}\right)$	.8825

Table 16. Mathematical Models - Study II (Continued)

Model #	Model Form	R^2
25	$\text{Log}_e(\hat{S}) = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4\text{Log}_e(Y) + b_5(X-8)^3 + b_6(Y-2)^3 + b_7X(Y-2)^3$	
	$+ b_8Y(X-8)^3$	.8937
26	$\text{Log}_e(\hat{S}) = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4\text{Log}_e(Y) + b_5(1.1)^{R^*}(Y-2)^2 + (1.1)^{R^*}(X-8)^2$	
	$+ (1.1)^{R^*}\sqrt{(X-8)^2+(Y-2)^2}$ where $R^* = \sqrt{X^2 + Y^2}$	.8965
27	$\text{Log}_e(\hat{S}) = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4\sqrt{X^2 + Y^2}$	.8624
28	$\text{Log}_e(\hat{S}) = b_0 + b_1(X-8)^2\sqrt{X^2+Y^2} + b_2(Y-2)^2\sqrt{X^2+Y^2}$	
	$+ b_3\text{Log}_e(Y) + b_4\sqrt{(X-8)^2+(Y-2)^2}\cdot\sqrt{X^2+Y^2}$	.7195

Table 16. Mathematical Models - Study II (Continued)

Model #	Model Form	R^2
29	$Log_e(\hat{S}) = b_0 + b_1(X-8)^2 + b_2(Y-2)^2 + b_3\sqrt{(X-8)^2+(Y-2)^2}$	
	$+ b_4 Log_e(Y) + b_5(X_2+Y_2)^{3/2}$	.8884
30	$\hat{S} = b_0 + b_1X + b_2X^2 + b_3Y + b_4X\ Log_e(X) + b_5Y\ Log_e(Y)$	.5922
31	$\hat{S} = b_0 + b_1\sqrt{X^2+Y^2}$	.2922
32	$\hat{S} = b_0 + b_1e^{b_2(R^*-R_0)^2}$	
	where $R^* = \sqrt{(X-8.6)^2+(Y-2.7)^2}$	.4632
33	$Log_e(\hat{S}) = b_0 + b_1(R^*-R_0)$ where $R^* = \sqrt{(X-9.6)^2+(Y-2.9)^2}$	.5861
34	$Log_e(\hat{S}) = b_0 + b_1(X-10.5)^2 + b_2(Y-.5)^2$	
	$+ b_3\sqrt{(X-10)^2+(Y-.1)^2}$	.9040
	(Model selected as best overall model)	

The point (X_0, Y_0) was approximately the joint center of rotation for the right shoulder.

The three models that will be discussed in detail are referred to as the polynomial regression model, the parabolic model (in two forms), and the final predictive model.

Polynomial Regression Model

Regression Equation. The first model fit to the data was a polynomial in X and Y. These variables, their second through fifth order functions, and all cross products were used in the model and a stepwise regression performed. A backward elimination procedure was used in the regression analysis (Draper and Smith, 1966). All possible variables were first fit to the data. The variable displaying the lowest partial F-test value was eliminated from the equation, and the regression run again. If the value of R^2 decreased less than one percent when this variable was eliminated, the process was repeated. If the R^2 decreased more than one percent, the variable was replaced into the model and the procedure terminated. The resulting equation was:

$$\begin{aligned}\hat{S} = {} & 47.643 - 2.811X + .00035X^2 + 1.035X^3 - .00000522X^5 \\ & - .388Y + .1025Y^2 + .00456Y^3 - .0001890Y^4 - .00071XY \\ & + .0152XY^2 + .000128XY^3 - .0001967XY^4 - .0001619YX^4\end{aligned} \tag{5.1}$$

with $R^2 = .9311$. A list of values for strength predicted by this equation is presented in Table 17. The residual plots of error vs. X,Y, and actual recorded strength produced no clearly defined pattern.

Table 17. Predicted Values - Stepwise Regression Model

RECORDED STRENGTH	PREDICTED STRENGTH	RECORDED STRENGTH	PREDICTED STRENGTH
61.5	62.70	38.8	37.09
64.8	66.54	43.1	43.33
75.6	67.93	54.0	52.96
77.7	68.85	68.0	64.30
66.2	69.13	68.1	74.57
63.1	68.37	87.0	79.72
68.2	68.18	71.9	74.30
74.0	70.38	42.5	49.31
82.0	70.89	45.0	53.42
70.3	71.15	59.9	68.10
69.1	70.78	75.8	69.06
66.9	69.20	84.5	80.85
56.6	57.59	86.9	85.68
51.0	51.06	92.0	92.69
53.0	61.71	95.0	97.51
69.9	77.29	81.4	81.26
84.5	92.99	59.9	63.97
104.5	101.20	64.0	64.63
101.2	91.42	65.2	63.16
88.7	86.18	57.0	60.27
63.5	71.49	51.6	51.86
48.1	39.60	45.4	41.52
56.0	46.85	39.8	33.28
77.2	70.56	28.5	34.34
87.1	95.28	31.0	33.86
109.0	105.67	34.0	35.51
121.0	112.84	43.0	42.61
102.0	109.82	57.3	58.11
88.0	86.22	68.2	75.35
33.0	35.22	91.0	83.61
60.0	61.95	73.2	81.98
53.2	53.88	71.1	68.55
63.1	64.41		

Contours generated by equation 5.1 are shown in Figure 21.

A polynomial model using a forward step-wise regression procedure was also fit to the data and is listed as model number 21 in Table 16.

Advantages. This polynomial model describes the data well as can be seen by comparing Figures 20 and 21. At all but one point predicted values are less than 10 pounds from actual recorded values and for 60% of the coordinate locations, the strength values are within 5%. Sample variances obtained from the subject-variability sub-study (see Table 10) range from 5.28 to 11.75 pounds, values that are generally of the same magnitude as the prediction errors at each test position. The relatively high value of $R^2 = .9311$ indicates the potential goodness of fit of the model.

Disadvantages. The model has 13 terms and may be too cumbersome for practical use in industrial design. Also, it is difficult to explain the effectiveness of such high order terms and their interactions. Expansion of this model to include other variables would be guess work, since there is no intuitive understanding of which portions of the variability are explained by each term in the model. This equation, therefore, is probably not suited for general application. The model meets the criterion for predictive ability, but is not simple computationally.

Parabolic Models

Regression Equations. A parabolic relationship between strength and the radial distance variable R* was hypothesized. A parabolic model analyzed in two different forms, was evaluated. The general form of the model is given as:

$$\hat{S} = B + A(R^{*}-R_0)^2 + \varepsilon \tag{5.2}$$

where B = maximum strength obtained (since the coefficient A is negative)

R_0= the distance from (X_0,Y_0) to this point of maximum strength.

(Radial distance to the maximum strength contour)

Least squares regression on the basic model of equation 5.2 indicated that a better fit could be obtained if the transformation

$$\mathrm{Log}_e\ (S) = S$$

was made on the dependent variable. The new form of the parabolic model becomes

$$\mathrm{Log}_e\ (\hat{S}) = B + A(R^{*}-R_0)^2 + \varepsilon \tag{5.3}$$

Written in terms of X and Y, this equation is

$$\mathrm{Log}_e\ (\hat{S}) = B + A\left[\sqrt{(X-X_0)^2 + (Y-Y_0)^2} - R_0\right]^2 + \varepsilon \tag{5.4}$$

After determining the values of all parameters (see Appendices C and D), the final model is:

$$\mathrm{Log}_e\ (\hat{S}) = 4.3464 - .00261(R^{*}-21.1)^2, \tag{5.5}$$

where $R^* = \sqrt{(X-9.6)^2 + (Y-2.9)^2}$ and the value of $R^2 = .5861$.

The parabolic model was also analyzed in an exponential form obtained by taking anti-logs of both sides of equation 5.3 and adding an intercept term. The general form of the model is:

$$\hat{S} = B + Ae^{\alpha(R^*-R_0)^2} + \varepsilon \tag{5.6}$$

After determining parameters (Appendices C and D), the final model is:

$$\hat{S} = 10.1361 + 69.15^{-.0031(R^*-22.1)^2} \tag{5.7}$$

where $R^* = \sqrt{(X-8.6)^2 + (Y-2.7)^2}$ and the value of $R^2 = .4632$.

The fact that equation 5.5 provides a better fit to the data is due primarily to differences in the calculation of the residuals. Residuals for equation 5.5 are

$$\varepsilon = \mathrm{Log}_e\ (S) - \mathrm{Log}_e\ (\hat{S}),$$

while residuals for equation 5.7 are

$$\varepsilon = S - \hat{S}.$$

Because residuals are calculated for equation 5.5 in terms of natural logarithms, the relative magnitude of the error for large predicted values of S is decreased. Thus, the greater the error for large predicted strength values, the more effective is the log transformation in decreasing the effect of these errors. The increase in R^2 (26%) under the log transformation is substantial. This may provide some reason to believe that the variance is larger for larger predicted S. The fact that the sample variances tested proved to be homogeneous may be an indication that the number of sample variances used in the test (eight) was too small to accurately determine homogeneity of such a large number of test positions. (1972)

Advantages. Although the model has an R^2 value of only .5861, it has much intuitive appeal. The optimal values of X_0 and Y_0 describe a point near the approximate location of the subject's shoulder. The value of R_0 is an average distance from that point to the point of greatest strength. Use of this model for design purposes would be simple in practice, there being only a minimal number of required calculations.

Disadvantages. The coefficients of the X^2 and Y^2 terms in equation 5.4 are the same; from analytic geometry it can be shown that the contours generated by the parabolic model in both forms are concentric circles (Figures 23, 24). This type of surface fails to account for differences in the shape of the parabola for differing sectors of the reach sphere; that is, equal strength contours generated by the data are ellipses, not circles. Since strengths attained across the body left of the SRP are considerably less than those to the front or right front of the body, the regression process averages these values and under estimates for all strength values greater than 80 pounds. This effect can be easily seen by comparing Figures 20, 23, and 24. Both models meet the criterion of simplicity, but fail to be valid predictive equations.

Predictive Model

Regression Equation. From the general parabolic model, a more elaborate form of a parabolic model with more estimation parameters was derived.

$$\text{Log}_e\ (\hat{S}) = b_0 + b_1(X-X_0)^2 + b_2(Y-Y_0)^2 + b_3\sqrt{(X-X_0)^2 + (Y-Y_0)^2} + \varepsilon \qquad (5.10)$$

It can be seen that by allowing the model to expand to three terms, each with a separate coefficient, the model is not limited to circular contours. It can be shown by analytic geometry that by allowing the coefficients of the X^2 and Y^2 terms to differ, the contours generated by the equation will be elliptical. This contour shape better describes the data by accounting for differences in maximum forces exerted in varying quadrants of the reach sphere. Evaluation of this model after determining the values of all parameters (see Appendices C and D) yielded

$$\begin{aligned} \text{Log}_e\ (\hat{S}) &= 2.867 - .00332(X-10.5)^2 - .00269Y^2 \\ &\quad + .134\sqrt{(X-10.5)^2 + Y^2} \end{aligned} \qquad (5.11)$$

with an $R^2 = .8869$. This model is an expanded parabolic model with the contours centering on the point (10.5,0).

Table 18. Predicted Values-Predictive Model

RECORDED STRENGTH	PREDICTED STRENGTH	RECORDED STRENGTH	PREDICTED STRENGTH
61.5	62.75	38.8	36.69
64.8	64.67	43.1	42.38
75.6	65.06	54.0	50.67
77.7	65.05	68.0	59.62
66.2	63.82	68.1	67.47
63.1	62.61	87.0	72.69
60.0	56.85	71.9	74.17
53.2	51.58	42.5	45.31
63.1	64.67	45.0	49.79
68.2	68.14	59.9	59.07
74.0	60.79	75.8	65.37
82.0	70.16	84.5	76.0
70.3	70.18	86.9	81.71
69.1	69.59	92.0	89.28
66.9	67.83	95.0	91.96
56.6	57.87	81.4	92.98
51.0	55.18	59.9	62.13
53.0	69.21	64.0	64.04
69.9	83.68	65.2	63.39
84.5	92.92	57.0	61.50
104.5	92.70	51.6	55.86
101.2	82.23	45.4	47.73
88.7	79.11	39.8	36.73
63.5	72.15	28.5	27.47
48.1	43.28	31.0	31.90
56.0	52.46	34.0	35.45
77.2	73.29	43.0	43.80
87.1	89.69	57.3	58.23
109.0	96.65	68.2	72.51
121.0	100.39	91.0	80.55
102.0	102.88	73.2	85.08
88.0	99.84	71.1	85.41
33.0	35.32	71.1	85.41

Further flexibility is gained by allowing the parameters X_0, and Y_0 to differ inside the radical in equation 5.10 from the values in the other two terms, resulting in

$$\text{Log}_e\ (\hat{S}) = b_0 + b_1(X-X_0)^2 + (Y-Y_0)^2 + b_3\sqrt{(X-X_0')^2 + (Y-Y_0')^2} + \varepsilon \qquad (5.12)$$

After determining the values of all parameters (see Appendices C and D), the final model is

$$\text{Log}_e\ (\hat{S}) = 2.837 - .003438(X-10.5)^2 - .002649(Y-.5)^2 + .137\sqrt{(X-10)^2 + (Y-.1)^2} \qquad (5.13)$$

with an $R^2 = .9040$. Table 18 provides actual strength scores and the corresponding values predicted by equation 5.13.

Analysis of Residuals. A plot of residuals for the predictive model given in equation 5.13 against actual strength data is presented in Figure 26. A plot of residuals on the X-Y plane is shown in Figure 27. Both plots indicate a tendency to over-predict at intermediate strength ranges and under-predict for larger values of recorded strength. The plot against actual strength gives some indication that the variances are possibly not homogeneous; variance appears larger for large values of strength. The contour error plot in Figure 27 also indicates the largest residuals along the surface ridge (at the points of maximum strength) and at points directly in front of the body. However, the shape of the residual contours in unclear. The X-Y graph would

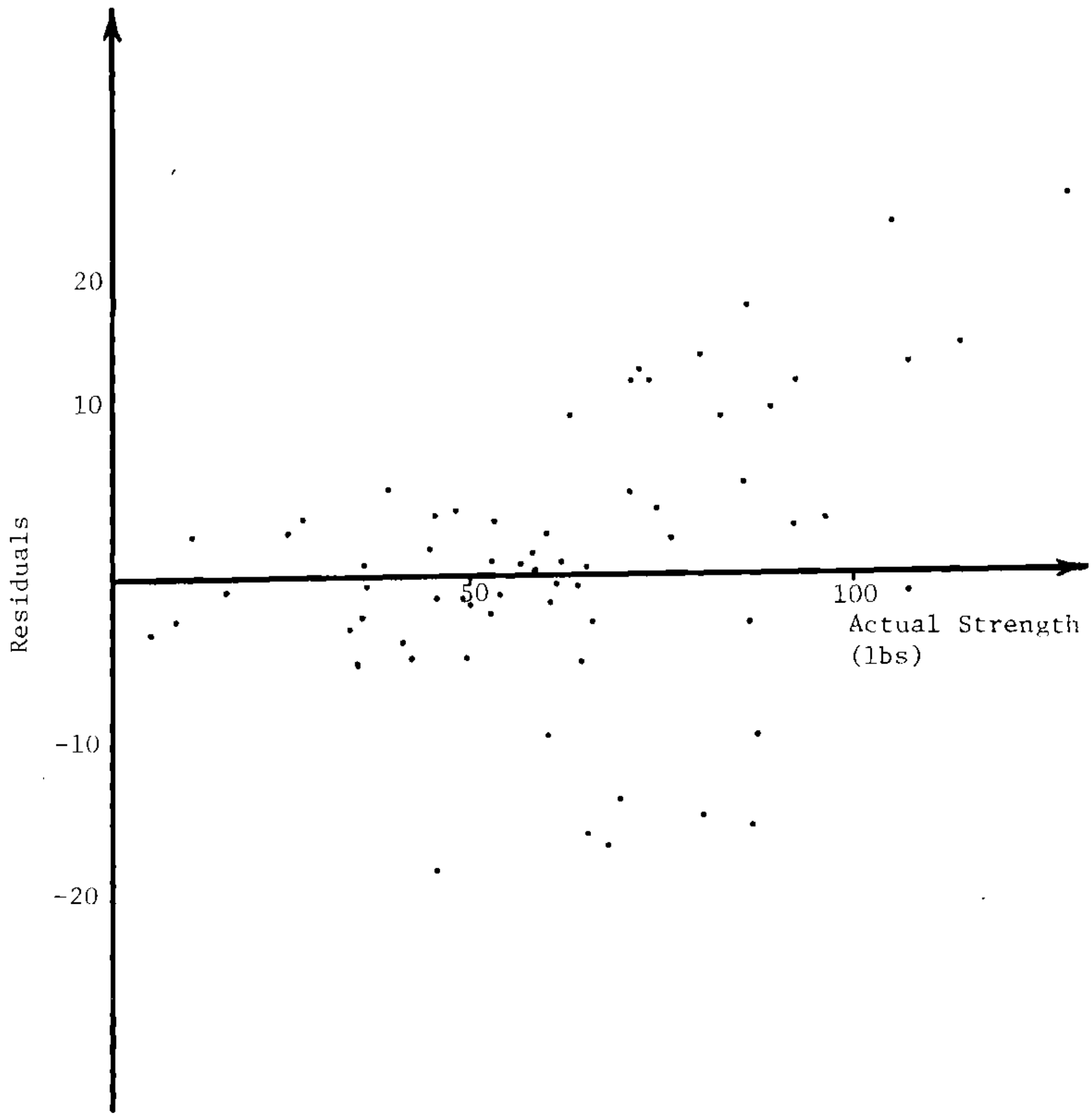


Figure 26. Residuals of Predictive Model vs Actual Strength Values

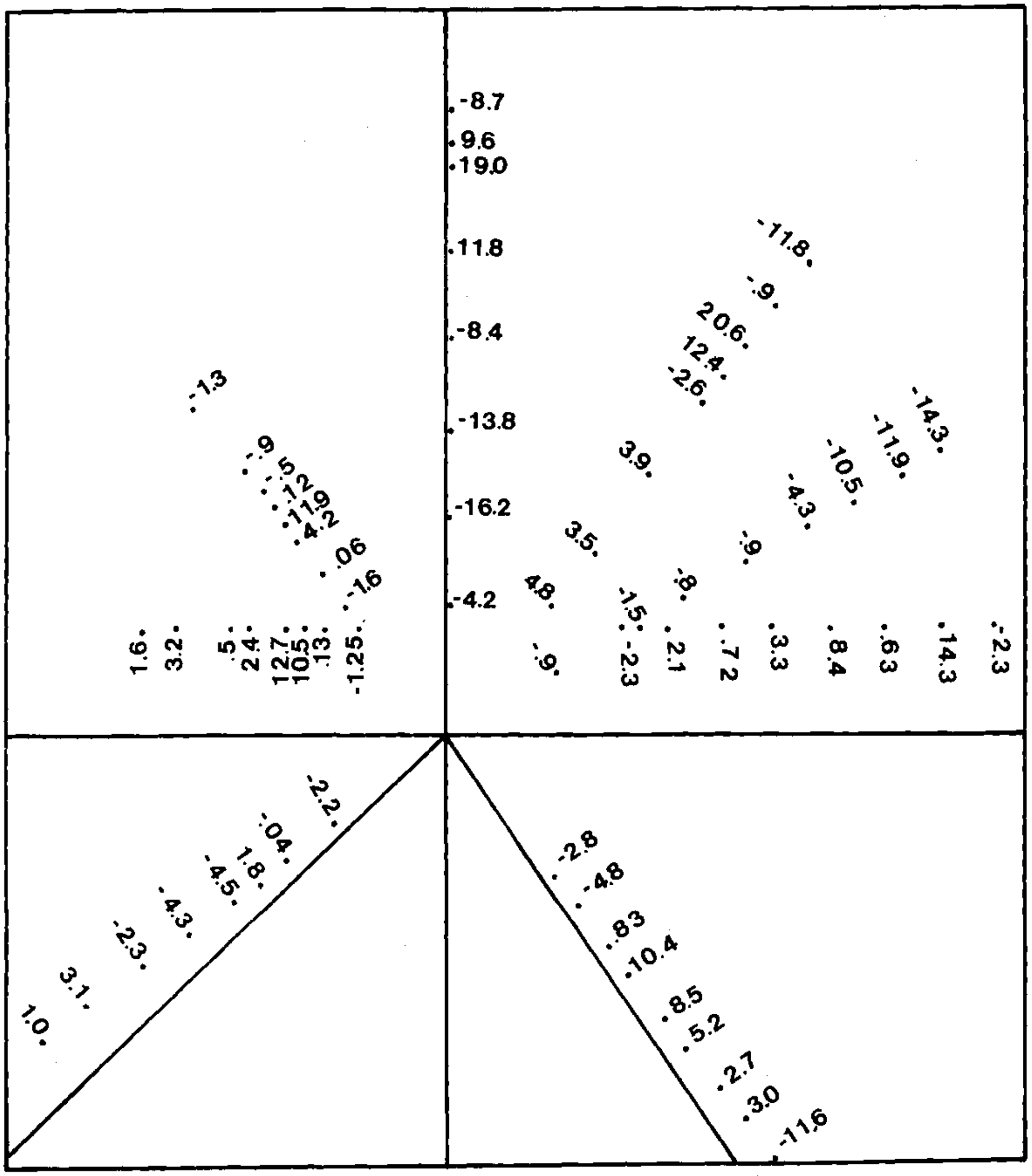


Figure 27. Residuals of Predictive Model vs X and Y

suggest a possible additional term as a function of R. However, several such functions were tried with no appreciable increase in model efficiency.

Evaluation. To further evaluate the final predictive model, the contours generated by the model have been superimposed on the actual data from Study II and presented in Figure 28. The major problem with the model seems to be its inability to predict strength greater than 105 pounds. As a result, large error exists in the right front quadrant of the body, where strength ability is greatest. Note, points of high prediction error are, in many cases, only four to six inches from the correct contour line generated by equation 5.13. The prediction error, therefore, may not be as great as when only magnitude of the residual is considered. It is easily possible for the location of the hand to vary four to six inches in relation to the body. Minor variations in posture, or differences in body position in the chair can all contribute minor differences in relative spacial location of the hand. Strength predictions are thought to be within the limits of experimental variability.

Advantages. The model is simple computationally and could be used for design purposes with relative ease. There is good intuitive appeal for the model and for the values of its parameters. Strength predictions are within the limits of experimental variability. The model meets both selection criteria.

Disadvantages. The model over-predicts in areas directly in front of the body and under-predicts at points of maximum strength in most areas to the right of the mid-sagittal plane.

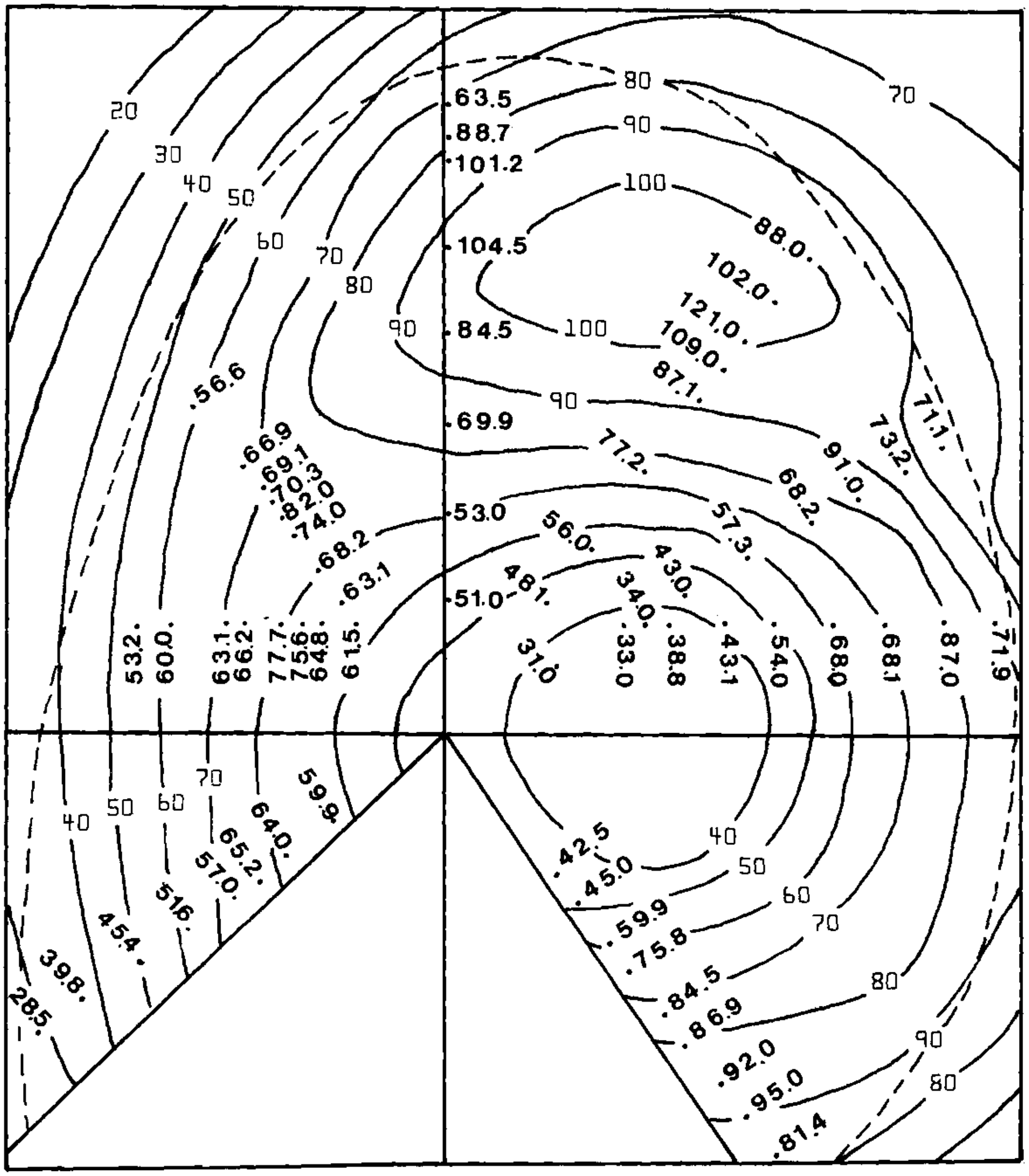


Figure 28. Contours of Final Predictive Model With Data From Study II

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Study I

Strength was found to be greatest in the front and right front quadrants of the reach sphere. Smallest forces were exerted in the handle location directly behind the head. For a given force direction and hand grip location, the effect of forearm rotation is minimal, in that the effect is extremely dependent on handle location and direction of force. Strength was greatest in all handle locations when tested 20 inches above the SRP, decreasing as the hand is raised to 40 inches or lowered to seat level.

The most significant effect on strength variation is the direction of force. For any given handle location, forces exerted in different directions varied greatly. None of the handle locations or forearm rotations is uniformly suited for exertions of very large forces in all directions. In general, forces exerted in directions perpendicular to the long axis of the arm will decrease as the hand moves away from the body. For push and pull forces, strength increases as the radial distance of the hand from the body increases. In all locations, smaller forces were exerted in the left or right directions while, generally, the greatest forces were exerted in all positions when pulling.

The data tables and the statistical analysis provided should aid the design engineer in the arrangement and design of hand operated controls. These data may be used as general guidelines for such designs. Although the data is on a single subject only, the results may be used to assign lower limit specifications on the strength required to operate hand controls.

Study II

Strength varies in the transverse plane parabolically with the distance of the hand grip from the SRP. It is possible to develop a simple mathematical model of arm strength in the transverse plane in a form which is useful to the design engineer. For the plane 20 inches above seat level, an equation that can be used to predict pushing forces with the forearm in mid-position is:

$$\begin{aligned} \text{Log}_e(\hat{S}) = 2.837 - .003438(X - 10.5)^2 - .002649(Y - .5)^2 \\ + .137\sqrt{(X-10)^2 + (Y-.1)^2} \end{aligned} \quad (6.1)$$

The methodology used to develop this predictive model is a useful method by which to analyze strength data. It may be effectively used to provide other design equations that can significantly increase the flexibility of the design engineer in determining specifications for man-operated equipment.

Recommendations

Future Research

Immediate Extensions. Further work may be done which follows directly from this research. This work could be based on a similar but possibly more extensive data base.

1. Collect data on push forces in the transverse planes through the SRP and 40 inches above it, and extend the predictive model of equation 6.1 to three dimensions.
2. Develop such a model for each of the six directions considered in this research.
3. Investigate the possibility of developing a single predictive equation by incorporating the equation for each direction of force. This could be done by using one or two angles to define the directions.

Such a study would require several months of data collection prior to initiating the analysis. Study I data collection lasted 42 days. It would require at least four times this number of days to collect enough data to properly develop a predictive model for Study I.

Further Study of the "Normal-Natural" Position. Since the "normal-natural" position is the one most commonly found in practice, further investigation of strength variations in this position should be conducted. It is felt that more useful information can be obtained from the study of strength exertions as they actually occur in practice than can be obtained from controlled, restricted single plane studies. The knowledge now available on sagittal and frontal plane strength should be used as a basis for extending this knowledge to three dimensions. The standardization of such studies should be an immediate goal.

Studies on a Standing Subject. Many hand controls are operated in the standing position. Studies with objectives similar in scope to those of this study should be made on arm strength for the standing subject.

Gather Percentile Data. There is a need to extend this study to more than one subject. Proper use of the data would be facilitated by knowledge of population norms. The use of equations such as equation 6.1 could be extended by developing such an equation for each of several selected percentile standings. Once the subject's percentile rating is known, the appropriate equation could be applied.

Sex and Age Differences. Strength studies similar to Studies I and II should be made on more subjects of both sexes to examine strength variation attributable to sex and age differences.

Examination of Anthropometric Relationships. Although there has been little success in the past in determining high correlations between strength and body dimensions, it may be possible to conduct studies similar to this one in order to find correlations high enough to be included in a prediction equation. The feasibility of such a model should be investigated further.

Applications of this Research

It is recommended that the data for Study I be incorporated into the existing arm strength data base. The final recommendation is that the mathematical model from Study II, equation 6.1, be implemented for use as a basis for future study, and as a predictive tool in determining strength requirements in equipment design problems for which the model applies.

A P P E N D I C E S

APPENDIX A

This appendix contains a complete table of data obtained from Study I. Each data point occupies a cell made up of a combination of four factors: handle location, distance level, forearm rotation, and direction of force. All data is recorded in pounds. A description of the 18 handle locations may be found in Table 3, and Figure 19, Chapter III.

Table 19. Data From Study I

Handle Location 1

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	27.08	39.6	53.5	41.25	31.97	46.43
	2 Mid-Pos'n	20.75	39.0	44.77	41.5	38.6	63.5
	3 Sup	22.6	33.23	56.67	26.17	31.6	73.07
2 Mid-range	1 Pron	34.0	49.75	71.9	59.55	28.55	44.8
	2 Mid-pos'n	22.2	48.0	51.7	58.2	31.93	62.9
	3 Sup	23.87	40.0	61.1	34.0	26.05	40.53
3 Near	1 Pron	34.9	69.24	97.64	74.47	26.0	40.53
	2 Mid-pos'n	24.1	62.87	81.20	71.37	30.0	47.7
	3 Sup	34.6	51.87	70.37	38.2	19.15	44.0

Handle Location 2

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	42.4	39.0	24.5	48.43	93.2	80.43
	2 Mid-pos'n	34.58	39.0	27.95	35.33	87.0	95.95
	3 Sup	35.85	35.17	28.97	36.13	47.77	83.75
2 Mid-range	1 Pron	46.57	49.4	27.2	60.03	63.83	76.67
	2 Mid-pos'n	48.33	46.27	37.67	56.5	68.0	66.53
	3 Sup	36.65	36.7	31.47	41.1	46.5	80.43
3 Near	1 Pron	48.13	56.67	32.6	62.13	57.3	44.05
	2 Mid-pos'n	61.33	61.33	39.5	81.67	43.0	55.0
	3 Sup	46.0	40.37	68.33	56.17	32.57	52.65

Table 19. Data From Study I (Continued)

Handle Location 3

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	30.9	31.87	43.05	71.83	47.5	70.87
	2 Mid-pos'n	23.6	34.03	21.83	69.75	40.5	48.65
	3 Sup	24.9	16.43	20.47	76.33	40.63	74.8
2 Mid-range	1 Pron	32.3	38.6	44.17	103.0	45.7	58.45
	2 Mid-pos'n	37.33	36.5	22.83	99.23	28.33	45.03
	3 Sup	36.2	25.67	34.43	80.6	31.43	64.0
3 Near	1 Pron	33.95	39.33	49.33	126.0	31.0	53.47
	2 Mid-pos'n	40.37	37.7	26.4	108.45	28.13	43.5
	3 Sup	31.5	32.1	66.25	110.5	28.63	46.85

Handle Location 4

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	20.5	39.03	20.5	35.2	134.9	126.73
	2 Mid-pos'n	15.03	28.87	22.4	28.5	142.4	101.9
	3 Sup	18.75	29.6	16.4	28.55	110.05	137.27
2 Mid-range	1 Pron	21.0	31.58	20.75	40.17	47.0	102.8
	2 Mid-pos'n	22.33	28.0	26.5	49.85	58.37	125.5
	3 Sup	20.0	33.1	24.5	31.7	117.0	146.8
3 Near	1 Pron	21.75	36.5	24.13	68.5	29.27	107.55
	2 Mid-pos'n	29.47	26.55	32.9	34.77	34.03	106.75
	3 Sup	29.73	43.1	32.83	58.2	54.5	122.7

Table 19. Data From Study I (Continued)

Handle Location 5

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	24.0	43.8	69.83	42.33	57.5	95.03
	2 Mid-pos'n	30.2	42.77	76.25	38.45	42.75	80.75
	3 Sup	21.5	34.25	67.17	37.0	70.1	78.95
2 Mid-range	1 Pron	29.17	37.16	103.73	45.4	51.95	83.4
	2 Mid-pos'n	31.45	43.1	82.33	37.7	32.43	72.7
	3 Sup	25.33	41.75	78.33	37.33	60.03	70.57
3 Near	1 Pron	29.33	32.77	122.9	60.2	48.2	62.5
	2 Mid-pos'n	40.17	53.4	103.83	56.17	24.75	57.25
	3 Sup	25.47	48.83	98.8	52.5	30.33	69.98

Handle Location 6

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	27.45	31.77	27.35	46.23	123.5	115.5
	2 Mid-pos'n	19.23	29.5	18.67	29.6	104.5	125.53
	3 Sup	28.0	21.83	29.0	44.35	67.0	125.0
2 Mid-range	1 Pron	28.25	43.47	27.7	49.75	102.5	100.8
	2 Mid-pos'n	22.0	41.83	41.47	69.5	84.5	112.9
	3 Sup	36.2	26.55	32.57	59.45	61.0	97.6
3 Near	1 Pron	46.8	48.35	34.33	54.67	82.5	95.67
	2 Mid-pos'n	28.77	46.33	51.6	85.2	69.9	89.2
	3 Sup	45.0	29.0	58.13	72.67	47.5	96.13

Table 19. Data From Study I (Continued)

Handle Location 7

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	24.6	24.87	19.35	103.27	54.65	95.5
	2 Mid-pos'n	19.75	21.08	18.83	69.6	62.95	89.5
	3 Sup	28.2	22.0	21.7	63.0	63.85	79.43
2 Mid-range	1 Pron	30.17	28.75	24.25	105.27	51.6	75.9
	2 Mid-pos'n	20.26	25.83	22.25	85.9	53.3	74.1
	3 Sup	30.53	22.73	26.2	76.37	32.1	76.25
3 Near	1 Pron	47.0	31.83	28.5	131.45	34.63	68.5
	2 Mid-pos'n	31.83	27.55	25.57	104.58	53.0	70.2
	3 Sup	37.33	30.77	30.33	86.65	31.53	66.95

Handle Location 8

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	52.8	12.47	67.03	33.93	52.33	75.37
	2 Mid-pos'n	32.5	13.2	38.5	25.05	49.03	68.43
	3 Sup	27.7	13.28	55.4	41.03	32.75	55.67
2 Mid-range	1 Pron	49.2	26.33	72.9	62.0	42.2	61.45
	2 Mid-pos'n	34.0	17.27	76.7	55.25	33.73	49.25
	3 Sup	28.8	17.5	77.87	66.93	24.9	46.27
3 Near	1 Pron	39.9	28.67	75.03	66.97	23.35	48.53
	2 Mid-pos'n	34.2	22.87	82.47	57.73	22.05	37.05
	3 Sup	22.55	24.0	87.2	74.5	24.0	45.67

Table 19. Data From Study I (Continued)

Handle Location 9

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	24.6	27.0	36.27	25.83	15.33	85.05
	2 Mid-pos'n	32.8	26.75	33.5	14.0	25.25	13.27
	3 Sup	33.8	27.0	38.0	12.3	58.17	77.3
2 Mid-range	1 Pron	31.0	24.1	24.0	22.77	18.13	71.75
	2 Mid-pos'n	34.83	31.33	51.3	32.77	24.65	71.5
	3 Sup	14.75	27.0	36.3	9.7	29.0	70.65
3 Near	1 Pron	40.33	35.13	32.65	27.5	35.0	49.5
	2 Mid-pos'n	48.35	39.77	33.8	25.5	20.5	61.78
	3 Sup	37.17	23.03	26.0	21.7	34.9	50.55

Handle Location 10

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	24.06	41.87	70.63	52.9	47.03	60.67
	2 Mid-pos'n	21.9	45.83	54.47	41.25	31.0	88.0
	3 Sup	23.45	36.8	63.33	28.9	35.5	86.33
2 Mid-range	1 Pron	25.0	59.55	84.1	55.17	36.55	54.5
	2 Mid-pos'n	37.17	60.5	72.77	39.97	30.5	74.5
	3 Sup	24.07	45.83	67.5	31.5	33.83	71.41
3 Near	1 Pron	35.83	62.0	102.2	63.1	31.25	55.25
	2 Mid-pos'n	41.5	69.15	80.83	51.06	26.37	67.07
	3 Sup	30.43	59.5	83.75	32.9	25.83	53.1

Table 19. Data From Study I (Continued)

Handle Location 11

DISTANCE LEVEL	FOREARM ROTATION	Left	Right	Up	Down	Push	Pull
1 Far	1 Pron	24.5	41.7	27.25	40.6	94.3	90.83
	2 Mid-pos'n	23.8	35.33	29.6	42.7	88.0	97.85
	3 Sup	27.47	29.55	29.4	41.5	51.13	101.77
2 Mid-range	1 Pron	25.33	43.73	27.5	40.92	130.11	77.77
	2 Mid-pos'n	24.0	45.75	36.0	60.4	121.0	80.37
	3 Sup	31.25	33.53	31.95	53.3	63.6	96.9
3 Near	1 Pron	37.35	52.75	47.7	56.0	91.5	77.30
	2 Mid-pos'n	36.2	50.43	38.75	67.83	87.1	66.53
	3 Sup	37.0	34.9	37.8	54.9	50.83	95.0

Handle Location 12

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	22.63	33.4	35.5	88.87	54.67	79.0
	2 Mid-pos'n	20.5	35.55	17.6	70.1	71.35	72.3
	3 Sup	30.77	21.73	30.33	88.03	47.05	75.43
2 Mid-range	1 Pron	23.33	42.68	44.88	96.5	51.63	67.4
	2 Mid-pos'n	23.7	37.37	22.5	87.03	52.45	64.37
	3 Sup	33.83	27.17	30.3	94.8	37.6	59.2
3 Near	1 Pron	36.97	44.75	45.73	119.75	33.7	34.5
	2 Mid-pos'n	35.45	40.03	26.05	91.5	44.65	48.5
	3 Sup	35.53	28.0	38.0	111.5	30.15	48.6

Table 19. Data From Study I (Continued)

Handle Location 13

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	27.67	17.83	39.63	32.43	66.23	52.93
	2 Mid-pos'n	29.5	28.7	58.3	40.0	45.15	54.83
	3 Sup	32.0	23.67	77.67	52.8	33.83	56.55
2 Mid-range	1 Pron	23.5	41.9	59.61	63.4	31.45	32.8
	2 Mid-pos'n	28.3	29.45	59.55	42.77	31.75	46.6
	3 Sup	30.55	30.27	80.3	77.6	29.95	49.4
3 Near	1 Pron	22.1	34.67	88.7	74.57	28.0	30.25
	2 Mid-pos'n	27.5	26.5	83.38	61.5	27.4	37.53
	3 Sup	19.2	30.0	93.13	90.33	26.95	46.43

Handle Location 14

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	21.45	23.6	65.3	33.83	21.0	68.8
	2 Mid-pos'n	12.33	27.0	69.0	33.57	21.8	52.3
	3 Sup	12.5	28.17	74.57	22.5	26.0	48.15
2 Mid-range	1 Pron	22.0	26.35	90.75	40.0	23.83	60.3
	2 Mid-pos'n	13.93	39.87	73.8	32.43	24.6	42.25
3 Near	1 Pron	26.0	27.23	108.0	42.33	37.8	50.28
	2 Mid-pos'n	21.37	40.0	103.55	31.37	37.87	38.6
	3 Sup	17.75	40.3	91.8	32.55	32.18	26.53

Table 19. Data From Study I (Continued)

Handle Location 15

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	35.63	28.47	19.35	40.63	56.0	88.0
	2 Mid-pos'n	41.3	24.23	21.3	39.07	53.15	87.75
	3 Sup	26.5	28.43	27.17	41.55	27.5	95.75
2 Mid-range	1 Pron	35.6	38.5	28.97	52.25	67.71	62.53
	2 Mid-pos'n	45.5	26.65	25.97	41.13	63.1	67.9
	3 Sup	27.97	20.0	35.7	62.03	29.13	77.75
3 Near	1 Pron	50.5	40.97	37.43	83.3	69.5	57.25
	2 Mid-pos'n	69.5	31.1	48.03	51.9	64.8	56.0
	3 Sup	28.87	18.25	38.43	63.5	37.55	64.97

Handle Location 16

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	27.07	31.33	24.25	76.0	31.17	44.95
	2 Mid-pos'n	37.67	23.7	15.1	61.27	27.9	58.0
	3 Sup	31.03	20.4	19.28	73.05	27.5	38.27
2 Mid-range	1 Pron	33.83	34.33	26.17	99.0	32.0	37.0
	2 Mid-pos'n	43.23	30.8	21.27	75.7	37.87	31.0
	3 Sup	38.25	27.0	22.27	89.0	27.65	35.07
3 Near	1 Pron	40.67	42.2	33.45	100.0	36.75	25.68
	2 Mid-pos'n	43.83	37.37	28.67	128.25	40.6	25.0
	3 Sup	42.4	30.9	23.25	99.33	30.5	35.93

Table 19. Data From Study I (Continued)

Handle Location 17

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	35.85	27.5	28.43	54.95	61.18	97.6
	2 Mid-pos'n	36.07	26.8	38.55	43.67	56.6	104.33
	3 Sup	22.77	23.5	29.63	46.33	43.75	130.5
2 Mid-range	1 Pron	48.1	34.9	33.97	68.83	78.14	130.5
	2 Mid-pos'n	41.83	29.6	40.73	49.1	69.1	95.0
	3 Sup	47.95	24.2	38.23	65.45	46.35	93.75
3 Near	1 Pron	51.6	44.67	36.67	71.55	75.0	68.25
	2 Mid-pos'n	28.5	30.13	47.65	64.43	68.2	64.17
	3 Sup	31.6	26.5	41.6	71.75	43.23	74.43

Handle Location 18

DISTANCE LEVEL	FOREARM ROTATION	1 Left	2 Right	3 Up	4 Down	5 Push	6 Pull
1 Far	1 Pron	34.25	30.03	24.7	84.47	42.0	46.85
	2 Mid-pos'n	33.53	31.5	19.4	83.55	48.5	45.75
	3 Sup	26.73	16.3	14.8	89.85	55.54	51.0
2 Mid-range	1 Pron	38.2	36.93	25.25	113.0	40.27	41.0
	2 Mid-pos'n	33.95	33.0	27.5	88.8	31.95	41.2
	3 Sup	30.4	26.67	17.87	106.97	40.1	49.5
3 Near	1 Pron	41.3	46.1	34.43	125.37	37.95	36.17
	2 Mid-pos'n	45.0	33.75	30.1	98.33	38.33	41.07
	3 Sup	42.35	36.33	23.73	124.0	37.0	37.35

APPENDIX B

This appendix contains a complete FORTRAN IV listing of a program which minimizes the error sum of squares from a simple linear regression. The subroutine performs regression on the basic parabolic model of (5.3). The main program performs the optimization of parameters.

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C       THIS PROGRAM IS CALLED REGOPT AND IS DESIGNED
C       TO PERFORM NONLINEAR OPTIMIZATION OF THE
C       RESIDUAL SUM OF SQUARES RESULTING FROM A LINEAR
C       REGRESSION SUBROUTINE CALLED FUNC.  THE
C       SINGLE TERM IN THE MODEL MAY HAVE AS MANY AS
C       100 NONLINEAR PARAMETERS.
      DIMENSION H(100),V(100),S(100),X(100),XSTAR(100),
     *  SHAT(65),
     *T(65)
      COMMON/ONE/A,B,T,SSREG,SSTOT
C       READ IN THE X,Y COORDINATES AND THE ASSOCIATED
C       STRENGTH VALUES
      DO 15 I=1,65
      READ(5,101)H(I),V(I),S(I)
      S(I)=ALOG(S(I))
   15 CONTINUE
C       SET STOPPING CRITERIA AND DESIRED STEP SIZE
C       FOR NONLINEAR PARAMETER OPTIMIZATION
      N=3
      DMIN=.001
      DEL=.001
      AMDA=0
      XSTAR(1)=10
      XSTAR(2)=2
      XSTAR(3)=25
C       BEGIN PARAMETER OPTIMIZATION BY MINIMIZING THE
C       RESIDUAL
C       SUM OF SQUARES OBTAINED FROM REGRESSION
C       ON PARABOLIC MODEL
      DO 60 I=1,N
      X(I)=XSTAR(I)
   60 CONTINUE
      ZSTAR=9999999999999.0
      DISP=0.
      FEAS=0.
      XMOVE=3.
  100 DO 170 I=1,N
  110 X(I)=XSTAR(I)+XMOVE*DEL
C       DO REGRESSION ANALYSIS ON PARABOLIC MODEL
C       USING PRESENT VALUES OF THE PARAMETERS
      CALL FUNC(X,H,V,S,F,R2)
  140 PFN=F+AMDA*FEAS
      IF(PFN.GE.ZSTAR)GOTO 150
      DISP=DISP+(ABS(XMOVE))*DEL
      XSTAR(I)=X(I)
      XMOVE=XMOVE+XMOVE
      ZSTAR=PFN
  145 FEAS=0.
      GOTO 110
```

```
  150 X(I)=XSTAR(I)
      IF(XMOVE.GT.3.3.OR.XMOVE.LT.2.7)GOTO 160
      XMOVE=(-3.)
      GOTO 145
  160 XMOVE=3.
      FEAS=0
  170 CONTINUE
      FEAS=0
      IF(DISP.LT.DMIN)GOTO 190
  180 DISP=0
      XMOVE=3.
      GOTO 100
C       STOPPING CRITERIA HAVE BEEN MET--GET FINAL
C       REGRESSION PARAMETERS AND PRINT THE RESULTS
  190 CALL FUNC(X,H,V,S,F,R2)
      WRITE(6,260)
      WRITE(6,250)(I,X(I),I=1,N)
      WRITE(6,241)F,R2,A,B
      DO 300 K=1,65
      SHAT(K)=B+A*T(K)
      S(K)=EXP(S(K))
      SHAT(K)=EXP(SHAT(K))
      RESID=S(K)-SHAT(K)
      IF(K.NE.1)GOTO 600
      WRITE(6,242)
  600 WRITE(6,243)S(K),SHAT(K),RESID
  300 CONTINUE
  101 FORMAT(3F5.1)
  250 FORMAT(*X(*,I2,*) = *,E14.6)
  260 FORMAT(/*THE OPTIMAL X VECTOR IS*/)
  241 FORMAT(/*THE MINIMUM VALUE OF THE LEAST SQUARES*
     ** OBJECTIVE FUNCTION*,/,*F(X) = *,E14.6,
     *//,*R-SQUARE = *,F8.4,5X,*A = *,F8.5,5X,*B = *,F10.4//
     *  )
  242 FORMAT(*STRENGTH *,5X,*STRENGTH PREDICTED*,5X,
     *  *RESIDUALS*//)
  243 FORMAT(1X,F5.1,14X,F6.2,13X,F6.2)
      STOP
      END




      SUBROUTINE FUNC(X,H,V,S,F,R2)
      DIMENSION H(65),V(65),S(65),T(65),R(65),X(100),SSQ(65)
     *  ,
     *T1(65)
      COMMON/ONE/A,B,T,SSREG,SSTOT
```

```
C         BEGIN LINEAR REGRESSION ON PARABOLIC MODEL
      F=0
      SUMSQ=0
      SUMT=0
      SUMS=0
      SUMTS=0
      SUMTT=0
      SCROS=0
      DO 10 J=1,65
      R(J)=SQRT((H(J)-X(1))**2+(V(J)-X(2))**2)
      T(J)=(R(J)-X(3))**2
      SSQ(J)=S(J)**2
      SUMT=SUMT+T(J)
      SUMS=SUMS+S(J)
      SUMSQ=SUMSQ+SSQ(J)
      CROSS=T(J)*S(J)
      SCROS=SCROS+CROSS
   10 CONTINUE
      SBAR=SUMS/65
      TBAR=SUMT/65
      DO 30 I=1,65
      SUMTS=SCROS-SUMT*SUMS/65
      STT=(T(I)-TBAR)**2
      SUMTT=SUMTT+STT
   30 CONTINUE
      A=SUMTS/SUMTT
      B=SBAR-A*TBAR
      SSREG=A*SUMTS
      SSTOT=SUMSQ-(SUMS**2)/65
      F=SSTOT-SSREG
      R2=SSREG/SSTOT
      RETURN
      END
```

APPENDIX C

This appendix contains a description of the model derivation procedures used in this study. Included are discussions of development of the mathematical form of the model, and of the methods used to determine the optimum values of the model parameters.

Initial Selection of a Parabolic Equation Form

Examination of the data contours in Figure 20 reveals that the strength contours center on a point which approximately corresponds to the location of the subject's shoulder. From this center, strength increases with the radial distance until it peaks, then decreases out to the end of the reach sphere. There seems to be, therefore, a quadratic form to the data surface, the height of the curve occurring at the point of maximum strength. Using the radial distance, R^*, we may view the relationship between strength and R^* as parabolic. The equation of a parabola in these two variables is exactly the same as equation 5.2, the equation for the general form of the parabolic model. (Spiegel, 1968)

Use of Iterative Regression to Examine Parameter Variations

The expanded form, equation 5.4, indicates five parameters, X_0, Y_0, R_0, A and B. It can be seen that this model is linear in the A and B parameters, but non-linear in X_0, Y_0, and R_0. The relationships between these parameters and the variables X and Y were examined using an iterative least squares regression technique. This procedure is explained schematically in Figure 29. A short computer program was written to

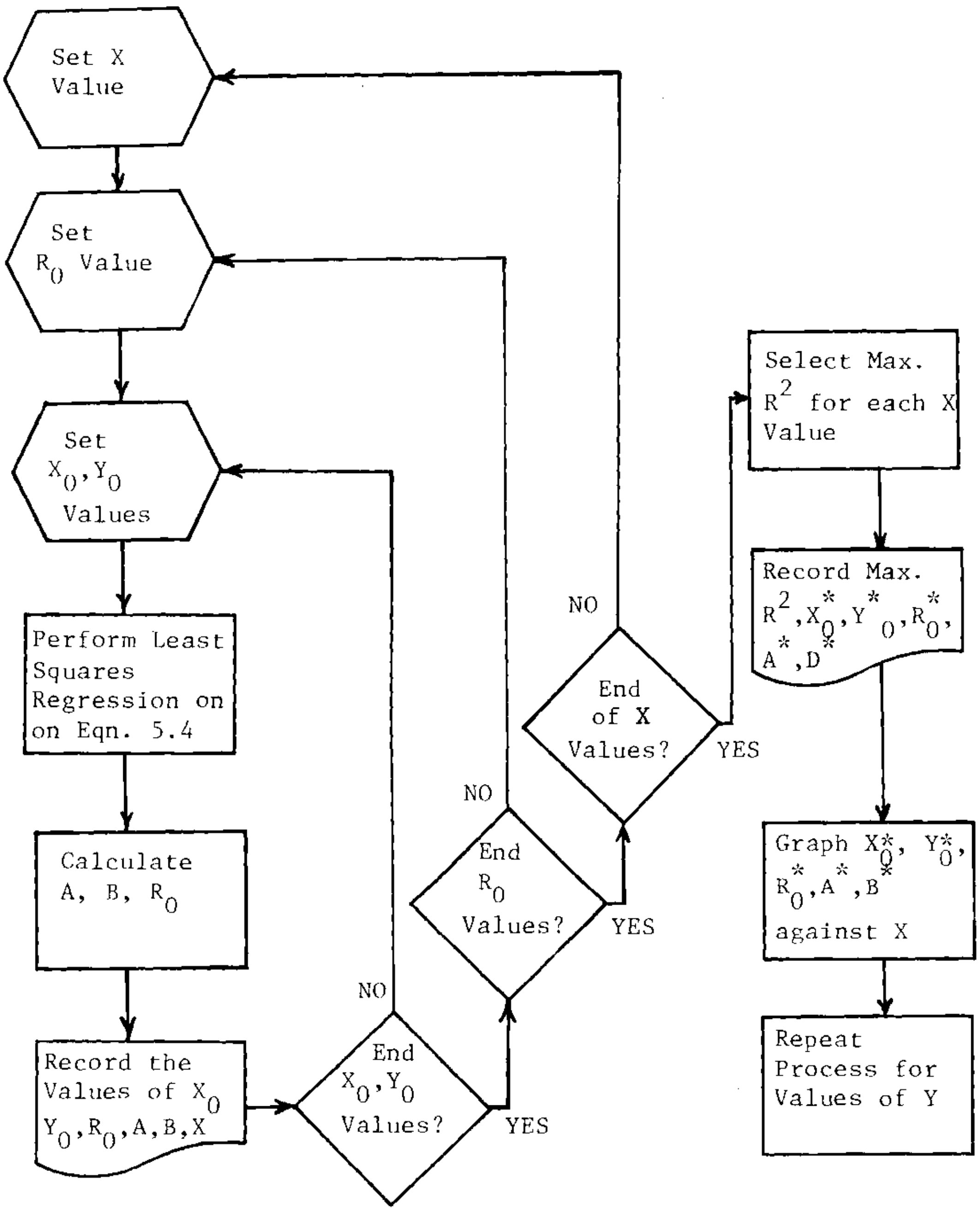


Figure 29. Schematic Diagram of Iterative Least Squares Procedure

accomplish these regressions. From the data of Study II, ten values of X were selected. For a fixed value of X and using Figure 20, ten data points (strength values) were estimated for various values of Y and a least squares regression on equation 5.4 was conducted at each X value using these ten data points. Several hundred different combinations of the parameters X_0, Y_0, and R_0 were sequentially substituted into equation 5.4 for each new regression. The combination yielding the highest R^2 at each X value was recorded along with the corresponding regression coefficients, A and B. The procedure was repeated for the remaining nine values of X. It was then possible to examine the values of the parameters X_0, Y_0, R_0 producing the best fit for each X, and determine possible relationships between the "best" value of a given parameter and the variable X. This process was repeated by selecting ten values of Y, selecting ten data points for each Y value, and conducting another iterative regression. Results of the iterative procedure were not conclusive, but did indicate that the values of X_0 and Y_0 producing the largest R^2 varied little with different values of X and Y, implying that both parameters are probably constants. However, other possible relationships for X_0, A and B could be hypothesized from the results of the iterative regressions. These include, besides being constant valued, the possibility of varying linearly or quadratically with X and Y. All possible relationships were investigated.

Analysis of Possible Parameter Relationships with X and Y

The possible relationships found for each parameter are detailed in Table 20. By substituting the mathematical expressions from Table 20 for

Table 20. Possible Relationships Between Equation Parameters and the Variables X and Y

PARAMETER	VARIABLE	HYPOTHESIZED RELATIONSHIP	MATHEMATICAL EXPRESSION
X_0	---	Constant	$X_0 = 8$
X_0	X	Linear	$X_0 = -2/5X - 16$
X_0	X	Quadratic	$X_0 = 10(X-8)^2 + 3$
Y_0	---	Constant	$Y_0 = 2$
A	---	Constant	A = Constant
A	X	Linear	$A = aX + b$
A	X	Quadratic	$A = a(X-8)^2 + b$
A	Y	Linear	$A = aY + b$
A	Y	Quadratic	$A = a(Y-10)^2 + b$
A	X,Y	Quadratic	$A = a(X-8)^2 + b(Y-10)^2 + c$
B	---	Constant	B = Constant
B	X	Linear	$B = aX + b$
B	X	Quadratic	$B = a(X-15)^2 + b$
B	Y	Quadratic	$B = a(Y+5)^2 + b$
R_0	---	Constant	$20 \leq R_0 \leq 30$

the parameters in equation 5.4, models 1 through 10 in Table 16 resulted. There are more than these ten possible parameter combinations. However, they all become extremely complex and were thus not considered in this research.

Decision on Final Model Form

In light of the two criteria used for model evaluation, simplicity and prediction accuracy, i.e. high R^2 value, the best parameter relationships found from the iterative least squares analysis were:

$X_0 = 8$ $\qquad$ B = Constant

$Y_0 = 2$ $\qquad$ R_0 = Constant, approximately = 25

A = Constant

Although other models yielded higher values of R^2 (see table 16), the additional terms in these models added little to the prediction accuracy, yet quickly increased the model complexity. Iterative least squares provides the following model:

$$\mathrm{Log}_e(\hat{S}) = A\left[\sqrt{(X-8)^2+(Y-2)^2} - 25\right]^2 + B \tag{C.1}$$

with $R^2 = .5301$.

Methods for Optimizing Parameter Values

Once the three models of interest were selected (excluding the polynomial model), the parameters of each as shown in equations 5.4, 5.6, and 5.12 were optimized to provide the best possible fit to the data. Two methods were used to accomplish this.

Nonlinear Programming

Both equations 5.4 and 5.6 are linear in A and B, but nonlinear in the parameters X_0, Y_0, and R_0. To determine the optimum parameter values, linear and nonlinear procedures were combined in a short computer routine (see appendix B). This program performs simple linear regression on the model in equation 5.4 using as initial values, the values of the parameters determined from the iterative procedure. Utilizing a modification of a nonlinear optimization routine due to Bazaraa (1975), the program minimizes the error sum of squares of the regression model until the optimum value is attained. The final results from this procedure produced the parabolic models in equations 5.5 and 5.7.

Steepest Descent

The predictive model of equation 5.12 has eight parameters. The model is linear in b_0, b_1, b_2, and b_3, but nonlinear in X_0, Y_0, X_0', and Y_0'. The method of steepest descent, commonly employed in nonlinear regression and response surface methodology, was used. This method is useful in determining in which direction to vary each parameter in order to continue reducing the error sum of squares. (Draper and Smith, 1966) A description of this technique as it was applied in this study is presented in Appendix D with an example of its use. This technique increased the value of R to .9508 and increased the value of R^2 to .9040.

The final predictive model obtained from this procedure is given in equation 5.13.

APPENDIX D

This appendix contains a description of the method of steepest descent as it was used to optimize the values of the parameters in the final predictive equation. The presentation is limited to a brief description of the procedures followed and does not provide a detailed analysis of response surface methodology or nonlinear regression. The reader desiring further explanation of the steepest descent algorithm is referred to a text on regression or response surface methods.

The steepest descent procedure is a method whereby the experimenter proceeds sequentially along the path of maximum decrease in response. The procedure involves use of an iterative process to find the minimum of the error sum of squares function. A detailed discussion of the elements of the procedure can be found in any good text on response surface methodology or nonlinear regression techniques. It suffices here to provide a brief outline of the process with the use of an example problem.

Example

Step 1. Begin with the response function: Equation 5.12.

Step 2. Using the four parameters that vary nonlinearly with the response function, i.e. X_0, Y_0, X_0', Y_0', select staring values. Set up a 2^{4-1} 1/2 replicate fractional factorial design. Select as the levels of the four factors a high and low value which probably bracket the true value of the variable.

$$9 \le X_0 \le 11 \qquad\qquad 9 \le X_0' \le 11$$

$$-1 \le Y_0 \le 1 \qquad\qquad -1 \le Y_0' \le 1$$

Step 3: Run a multiple linear regression on the response function eight times, producing eight values of the error sum of squares. The design is:

X_0	Y_0	X_0'	Y_0'	SS_E
9	-1	9	-1	.722
11	1	11	1	.949
11	1	9	-1	1.666
9	-1	11	1	3.020
11	-1	9	1	2.003
9	1	11	-1	2.678
11	-1	11	-1	.834
9	1	9	1	.773

Step 4: Using the eight sums of squares as the new response, run a multiple linear regression, regressing the error sums of squares on the four parameters to obtain the equation:

$$\hat{Y} = .861 - .21763X_0 - .06412Y_0 + .28963X_0' + .10563Y_0'$$

Step 5: The estimated coefficients indicate the direction of steepest ascent so the negatives of these indicate the direction of steepest descent. It can be shown that by selecting the desired step size dictated by λ,

$$\Delta X_j = b_j/2\lambda \qquad \text{(D.1)}$$

where ΔX_j denotes a change of the ith variable above. To change X_0 by .5,

$$\Delta X_0 = .5 = b_1/2\lambda = .21763/2 \qquad 2\lambda = -.43526$$

$$\Delta Y_0 = -.06412/-.43526 = .1473$$

$$\Delta X_0' = -.6654$$

$$\Delta Y_0' = -.2427$$

Step 6: Increment the four variables by the amount above. Use the new values to evaluate the response function. Continue until the function no longer decreases.

Step 7: When the function does not decrease, another experimental design is set up and the procedure continued until it converges. This example problem, after several iterations and four directions, yielded the final predictive model of equation 6.1.

BIBLIOGRAPHY

1. Asmussen, E. and Heebol-Nielsen, K., "Isometric Muscle Strength in Relation to Age in Men and Women", Ergonomics, Vol. 5, No. 1, 1962.

2. Astrand, Per-Olag and Rodahl, Kaare. Textbook of Work Physiology, McGraw-Hill Book Company, New York, 1970.

3. Bazaraa, M. S., An Efficient Cyclic Coordinate Method for Optimizing Penalty Functions, Unpublished Report, Georgia Institute of Technology, 1975.

4. Bedford, T. and Warner, C. G., "Strength Tests: Observations on the Effects of Postures on Strength of Pull", The Lancet, Vol. 2, 1937.

5. Caldwell, L. S., "Body Position and the Strength and Endurance of Manual Pull", Human Factors, Vol. 6, No. 5, October 1964.

6. Caldwell, L. S., "Body Stabilization and Strength of Arm Extension", Human Factors, Vol. 4, No. 3, June 1962.

7. Campney, H. K. and Wehr, R. W., "An Interpretation of the Strength Differences Associated with Varying Angles of Pull", Research Quarterly, Vol. 36, No. 4, 1965.

8. Carlson, B. R., "Level of Maximum Isometric Strength and Relative Load Isometric Endurance", Ergonomics, Vol. 12, No. 3, May 1969, pp. 429-435.

9. Carpenter, Aileen, "A Study in the Measurement of the Leg Lift", Research Quarterly, Vol. 9, Oct. 1938, pp. 70-72.

10. Chaffin, Don B., "Ergonomics Guide for the Assessment of Human Static Strength", American Industrial Hygiene Association Journal, July 1975, pp. 505-511.

11. Clarke, H. H. A Manual of Cable Tension Strength Tests, Stuart E. Murphy, Springfield, Mass., 1953.

12. Clarke, H. H., "Improvement of Objective Strength Tests of Muscle Groups by Cable-Tension Methods", Research Quarterly, Vol 21, 1950.

13. Clarke, H. H. Muscular Strength and Endurance in Man, Prentice Hall, 1966.

14. Clarke, H. H., "Recent Advances in Measurement and Understanding of Volitional Muscular Strength", Research Quarterly, Vol. 27, No. 2, May 1956.

BIBLIOGRAPHY (Continued)

15. Clarke, H. H., "Relationship of Strength and Anthropometric Measures to Various Arm Strength Criteria", Research Quarterly, Vol. 25, 1954.

16. Clarke, H. H., Elkins, E. C., Matrin, G. M., Wakim, K. G., "Relationship Between Body Position and the Application of Muscle Power to Movement of the Joints", Archives of Physical Medicine, Vol. 31, Feb. 1950, pp. 81-89.

17. Cuthbert, Daniel and Wood, Fred S. Fitting Equations to Data, Wiley-Interscience, New York, 1971.

18. Daniels, L. M. Muscle Testing, W. A. Saunders Co., Philadelphia, 1966.

19. Darcus, H. D., "The Maximum Torques Developed in Pronation and Supination of the Right Hand", Journal of Anatomy, London, Vol 85, 1951.

20. Doss, W. S. and Karpovich, P. V., "A Comparison of Concentric, Eccentric, and Isometric Strength of the Elbow Flexors", Journal of Applied Physiology, Vol. 20, No. 2, 1965.

21. Downer, A. H., "Strength of the Eblow Flexor Muscles", Physical Therapy Review, Vol. 33, No. 2, Feb. 1953.

22. Draper, N. R. and Smith, H. Applied Regression Analysis, John Wiley and Sons, Inc., New York, 1966.

23. Duncan, Acheson J. Quality Control and Industrial Statistics, Richard D. Irvine, Inc., Homewood, Illinois, 1965.

24. Elkins, E. C., Leden, U. M., Wakim, K. G., "Objective Recording of the Strength of Normal Muscles", Archives of Physical Medicine, Vol. 32, 1951.

25. Goldstein, S. R., "A Servo-Force Balance Isometric Muscle Force Transducer", Journal of Applied Physiology, Vol. 37, July 1974, pp. 134-137.

26. Guest, P. G. Numerical Methods of Curve Fitting, Cambridge University Press, London, 1966.

27. Hall, D. M., "Selection and Standardization of Strength Tests for 4-H Club Members", Research Quarterly, Vol. 27, No. 3, 1956, pp. 285-295.

28. Hicks, Charles R. Fundamental Concepts in the Design of Experiments, Holt, Rinehart and Winston, New York, 1973.

BIBLIOGRAPHY (Continued)

29. Hilton, Harold. Plane Algebraic Curves, Oxford University Press, London, 1920.

30. Hines, William W. and Montgomery, Douglas, C. Probability and Statistics in Engineering and Management Science, The Ronald Press Company, New York, 1972.

31. Houtz, S. J., Lebow, M. J., Beyer, F. R., "Effect of Posture on Strength of the Knee Flexor and Extensor Muscles", Journal of Applied Physiology, Vol. II (3), pp. 475-480.

32. Hugh-Jones, P., "The Effect of Limb Position in Seated Subjects on Their Ability to Utilize the Maximum Contractile Force of the Limb Muscles", Journal of Physiology, Vol. 105, 1947; pp. 332-344.

33. Hunsicker, P., "Arm Strength at Selected Degrees of Elbow Flexion", A.F. Project No. 7214-71727, WADC-TR-54-548, WPAFB, 1955.

34. Hunsicker, P., "A Study of Muscle Forces and Fatigue", WADC TR 57-586, December 1957, AD 131 087.

35. Hunsicker, P. A. and Greey, G., "Studies in Human Strength", Research Quarterly, Vol. 28, 1957.

36. Kennedy, W. R., "The Development and Comparison of an Electrical Strain Gauge Dynamometer and a Cable Tensiometer for Objective Muscle Testing", Archives of Physical Medicine and Rehabilitation, 46, No. 12, 1965.

37. Kroemer, K. H. E., "Human Strength: Terminology, Measurement, and Interpretation of Data", Human Factors, Vol. 12, No. 3, June 1970; pp. 297-313.

38. Kroemer, K. H. E., "Push Forces Exerted in Sixty-five Common Working Positions", AMRL-TR-68-143, Aug. 1969, AD 695 040.

39. Kroemer, K. H. E. and Howard, "Problems in Assessing Muscle Strength", AMRL-TR-68-144, May 1970, AD 708 741.

40. Kroemer, K. H. E. and Howard, "Towards Standardization of Muscle Strength Testing", Medical Science in Sports, Vol. 2, No. 4, 1970; pp. 224-230.

41. Laubach, L. L., Kroemer, K. H. E., Thordsen, M. L., "Relationships Amond Isometric Forces Measured in Aircraft Control Locations", AMRL-72-19, AD 747 190.

BIBLIOGRAPHY (Continued)

42. Laubach, L. L. and McConville, "Muscle Strength, Flexibility and Body Size of Adult Males", Research Quarterly, Vol. 37, No. 3, Oct. 1966; pp. 384-392.

43. Laubach, L. L. and McConville, "The Relationship of Strength to Body Size and Typology", Medical Science in Sports, Vol. 1, No. 4, Dec. 1969; pp. 189-194.

44. Lovett, R. W. and Martin, E. G., "The Spring Balance Muscle Test", American Journal of Orthopaedic Surgery, Vol. 14, 1916.

45. Martin, E. G., "Muscular Strength and Muscular Symmetry in Human Beings", American Journal of Physiology, Vol. 46, May 1918; pp. 67-83.

46. MIL-STD-1472B, dated 15 May 1970, Human Engineering Design Criteria for Military Systems, Equipment and Facilities, Department of the Army, Washington, D.C.

47. Morehouse, L. E., "The Strength of a Man", Human Factors, Vol. 1, No. 2, Apr. 1959.

48. Newman, L. B., "A New Device for Measuring Muscle Strength", Archives of Physical Medicine, Vol. 30, No. 4, 1949.

49. Provins, K. A. and Salter, N., "Maximum Torque Exerted About the Elbow Joint", Journal of Applied Physiology, Vol. 7, No. 4, Jan. 1955.

50. Ramsey, J. D., "The Quantification of Human Effort and Motion for the Upper Limbs", The International Journal of Production Research, Vol. 7, No. 1, 1968; pp. 47-59.

51. Rasch, P. J., "Effect of Forearm Position on Strength of Elbow Flexion", Research Quarterly, Vol. 27, Oct. 1956; pp. 333-337.

52. Roebuck, J. A. Jr., Kroemer, K. H. E., Thomson, W. G. Engineering Anthropometry Methods, John Wiley and Sons, New York, 1975.

53. Salter, N., "Methods of Measurement of Muscle and Joint Function", Journal of Bone and Joint Surgery, Vol. 37B, No. 3, Aug. 1955.

54. Salter, N. and Darcus, H. D., "The Effect of the Degree of Elbow Flexion on the Maximum Torques Developed in Pronation and Supination of the Right Hand", Journal of Anatomy, Vol. 87, Oct. 1953.

BIBLIOGRAPHY (Continued)

55. Schanne, Jr., F., "A Three-Dimensional Hand Force Capability Model for a Seated Person", Unpublished dissertation for Ph.D. in Industrial Engineering, University of Michigan, Ann Arbor, Michigan, 1972. (Dissertation published on demand by University Microfilms, Ltd., High Wycomb, England)

56. Sheldon, W. H., Dupertius, C. W., McDermott, E. Atlas of Men, Harper and Brothers, New York, 1954.

57. Singh, M. and Karpovich, P. V., "Isotonic and Isometric Forces of Forearm Flexors and Extensors", Journal of Applied Physiology, Vol. 21, No. 4, 1966; pp. 1435-1437.

58. Singh, M. and Karpovich, P. V., "Strength of Forearm Flexors and Extensors in Men and Women", Journal of Applied Physiology, Vol. 25, No. 2, Aug. 1968.

59. Spiegel, Murray R. Mathematical Handbook of Formulas and Tables, Schaum's Outline Series, McGraw-Hill Book Company, New York, 1968.

60. Thordsen, M. L., Kroemer, K. H. E., Laubach, L. L., "Human Force Exertions in Aircraft Control Locations", AMRL-TR-71-119, 1972, AD 740 930.

61. Troup, J. D. G. and Chapman, A. E., "The Strength of Flexor and Extensor Muscles of the Trunk", Annal of Biomechanics, Vol. 2, 1969.

62. Van Cott, Harold P., Kinkade, Robert G. Human Engineering Guide to Equipment Design, McGraw-Hill Company, Inc., Washington, D.C., 1963.

63. Wakim, K. G., Gersten, J. W., Martin, G. M., Elkins, E. C., "Objective Recording of Muscle Strength", Archives of Physical Medicine, Vol. 31, 1950.

64. Walkey, F. A. and Cowan, N. R., "Muscle Strength", Gerontologia Clinic, Vol. 9, No. 1, 1967; pp. 30-39.

65. Williams M. and Stutzman, L., "Strength Variation Through the Range of Joint Motion", The Physical Therapy Review, Vol. 39, No. 3, 1959.

66. Williges, R. C., "Predictive Validity of Central-Composite Design Regression Equations", Human Factors, Vol. 15, No. 4, 1973.