Universality of transport coefficients in the Haldane-Hubbard model

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Joint work with V. Mastropietro, M. Porta and I. Jauslin

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- Model: Haldane-Hubbard, simplest interacting Chern insulator.
  Several approximate and numerical results available.
  Very few (if none) rigorous results.

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- Method: constructive Renormalization Group +
  + lattice symmetries + Ward Identities + Schwinger-Dyson

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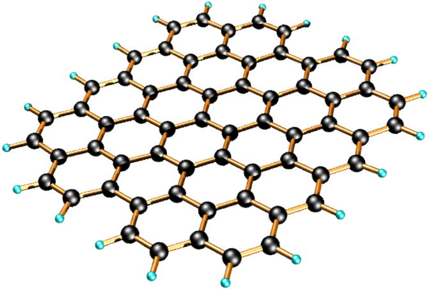
Graphene

Graphene is a 2D allotrope of carbon: single layer of graphite.



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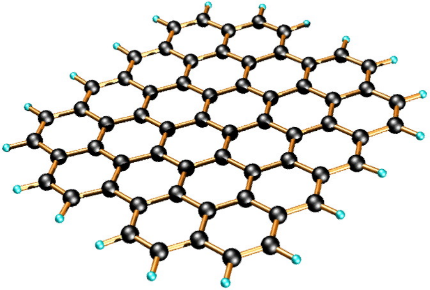

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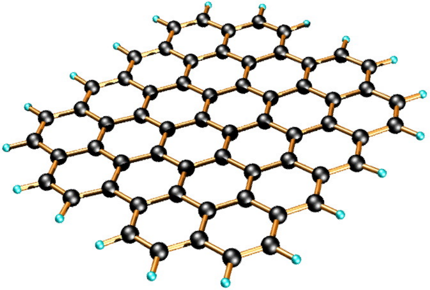

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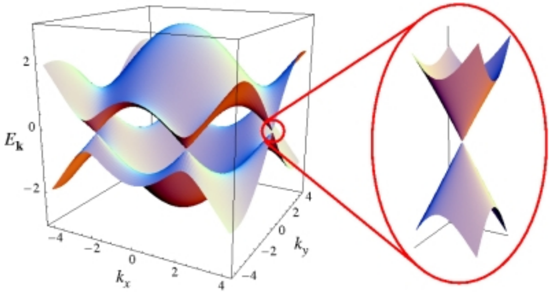
Here we shall focus on its transport properties.

Graphene

Peculiar transport properties due to its unusual band structure:

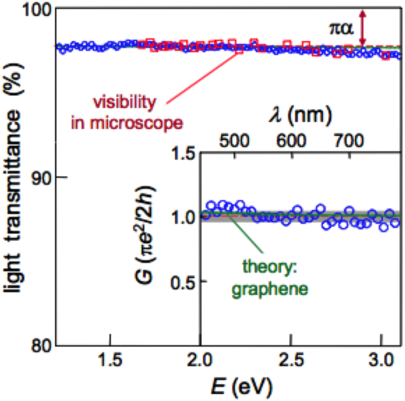
- at half-filling the Fermi surface degenerates into two Fermi points
- Low energy excitations: 2D massless Dirac fermions ($v \simeq c/300$) $\Rightarrow$ 'semi-metallic' QED-like behavior at non-relativistic energies



Minimal conductivity

Signatures of the relativistic nature of quasi-particles:

1. Minimal conductivity at zero charge carriers density.
   Measurable at $T = 20^o$ C from $t(\omega) = \frac{1}{(1+2\pi\sigma(\omega)/c)^2}$

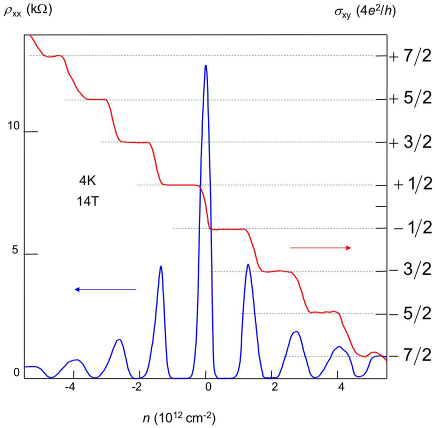


For clean samples and $k_B T \ll \hbar\omega \ll bandwidth$,

$$\sigma(\omega) = \sigma_0 = \frac{\pi}{2}\frac{e^2}{h}$$

Anomalous QHE

2. Constant transverse magnetic field: anomalous IQHE.
   Shifted plateaus: $\sigma_{12} = 4\frac{e^2}{h}(N + \frac{1}{2})$:



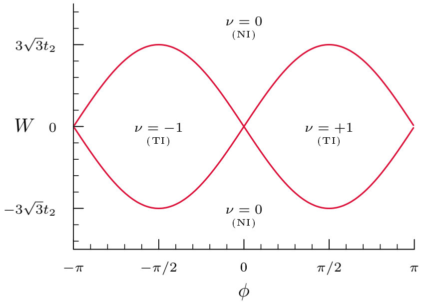
Observable at $T = 20^o$.

At low temperatures: plateaus measured at $\sim 5 \times 10^{-11}$ precision.

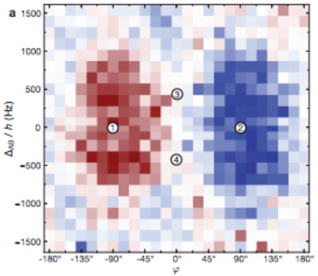
QHE without net magnetic flux

3. Another unusual setting for IQHE with zero net magnetic flux: proposal by Haldane in 1988 (Nobel prize 2016). Main ingredients:

- dipolar magnetic field ⇒ n-n-n hopping t_2 acquires complex phase
- staggered potential on the sites of the two sub-lattices



Phase diagram (predicted...)



(... and measured, Esslinger et al. '14)

Theoretical understanding

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- QHE: let $P_\mu = \chi(H \le \mu) =$ Fermi proj. If $\mathbf{E}|P_\mu(x;y)| \le Ce^{-c|x-y|}$, i.e., $\mu \in$ spectral gap, or $\mu \in$ mobility gap:

$$\sigma_{12} = \frac{ie^2}{\hbar} \mathrm{Tr}\, P_\mu[[X_1, P_\mu], [X_2, P_\mu]] \in \frac{e^2}{h} \cdot \mathbb{Z}$$

(Thouless-Kohmoto-Nightingale-Den Nijs '82, Avron-Seiler-Simon '83, '94, Bellissard-van Elst-Schulz Baldes '94, Aizenman-Graf '98...)

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- Minimal conductivity: gapless, semi-metallic, ground state. Exact computation in a model of free Dirac fermions (Ludwig-Fisher-Shankar-Grinstein '94), or in tight binding model (Stauber-Peres-Geim '08).

Effects of interactions?

What are the effects of electron-electron interactions? In graphene, interaction strength is intermediate/large:

$$\alpha = \frac{e^2}{\hbar v} \sim 2.2$$

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- Minimal longitudinal conductivity: no geometrical interpretation. Cancellations due to Ward Identities? Big debate in the graphene community, still ongoing (Mishchenko, Herbut-Juričić-Vafek, Sheehy-Schmalian, Katsnelson et al., Rosenstein-Lewkowicz-Maniv ...)

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Today: Universality of σ_{12} (up to the critical line) and of σ_{11} (on the critical line) in the weakly interacting Haldane-Hubbard model.

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  Note: our method: no topology/geometry, no assumption on gap: decay of interacting correlations + cancellations from WI and SD.

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The lattice and the Hamiltonian

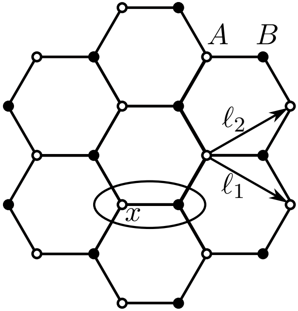

Figure: Dimer $\rightsquigarrow (a^{\pm}_{x,\sigma},\, b^{\pm}_{x,\sigma})$.

- Hamiltonian: $\mathcal{H} = \mathcal{H}_0 + U\mathcal{V}$, where

$$\mathcal{H}_0 = \text{n.n.} + \text{complex n.n.n. hopping} + \text{ staggered potential} - \mu\mathcal{N}$$

$$\mathcal{V} = \sum_x (n^A_{x,\uparrow} n^A_{x,\downarrow} + n^B_{x,\uparrow} n^B_{x,\downarrow})$$

Conductivity

- Finite temperature, finite volume Gibbs state (eventually, $\beta, L \to \infty$):

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- Kubo formula: linear response at $t = 0$, after having switched on adiabatically a weak external field $e^{\eta t}E$ at $t = -\infty$

The non-interacting Hamiltonian (Haldane model)

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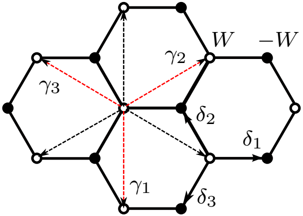

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$$\begin{aligned}\mathcal{H}_0 &= t_1\sum_{x,\sigma}\left[a^+_{x,\sigma}b^-_{x,\sigma}+a^+_{x,\sigma}b^-_{x-\ell_1,\sigma}+a^+_{x,\sigma}b^-_{x-\ell_2,\sigma}+h.c.\right]\\ &+t_2\sum_{x,\sigma}\sum_{\substack{\alpha=\pm\\ j=1,2,3}}\left[e^{i\alpha\phi}a^+_{x,\sigma}a^-_{x+\alpha\gamma_j,\sigma}+e^{-i\alpha\phi}b^+_{x,\sigma}b^-_{x+\alpha\gamma_j,\sigma}\right]\\ &+W\sum_{x,\sigma}\left[a^+_{x,\sigma}a^-_{x,\sigma}-b^+_{x,\sigma}b^-_{x,\sigma}\right]-\mu\sum_{x,\sigma}\left[a^+_{x,\sigma}a^-_{x,\sigma}+b^+_{x,\sigma}b^-_{x,\sigma}\right]\end{aligned}$$

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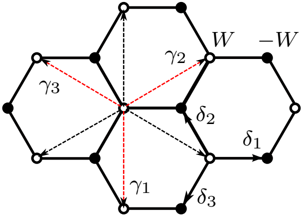

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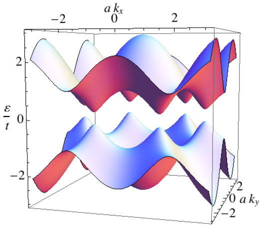
& + t_2 \sum_{x,\sigma} \sum_{\substack{\alpha=\pm \\ j=1,2,3}} \left[e^{i\alpha\phi} a^+_{x,\sigma} a^-_{x+\alpha\gamma_j,\sigma} + e^{-i\alpha\phi} b^+_{x,\sigma} b^-_{x+\alpha\gamma_j,\sigma} \right] \\
& + W \sum_{x,\sigma} \left[a^+_{x,\sigma} a^-_{x,\sigma} - b^+_{x,\sigma} b^-_{x,\sigma} \right] - \mu \sum_{x,\sigma} \left[a^+_{x,\sigma} a^-_{x,\sigma} + b^+_{x,\sigma} b^-_{x,\sigma} \right]
\end{aligned}$$

- Gapped system. Gaps:

$\Delta_\pm = |m_\pm|\,, \quad m_\pm = W \pm 3\sqrt{3} t_2 \sin\phi.$

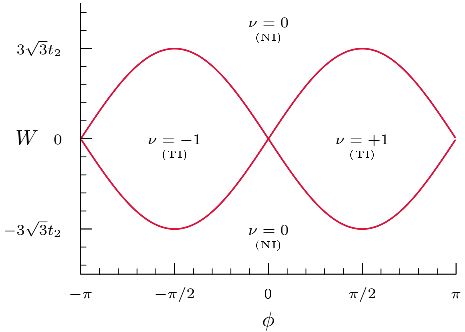
$=$ "mass" of Dirac fermions.



Non-interacting phase diagram

- If $U = 0$, μ is kept in between the two bands, and $m_\pm \neq 0$:

$$\sigma_{12} = \frac{2e^2}{h}\nu\ , \qquad \nu = \frac{1}{2}[\mathrm{sgn}(m_-) - \mathrm{sgn}(m_+)]$$



- Simplest model of topological insulator.

  Building brick for more complex systems (*e.g.* Kane-Mele model).

Phase transitions in the Haldane-Hubbard model

Theorem (Giuliani, Jauslin, Mastropietro, Porta 2016)

There exists $U_0 > 0$ and a function ("renormalized mass")

$$m_{R,\omega} = m_\omega + F_\omega(m_\pm; U) \quad \textit{where} \quad F_\omega = O(U), \quad \omega = \pm$$

such that, for $U \in (-U_0, U_0)$, choosing $\mu = \mu(m_\pm; U)$:

$$\sigma_{12} = \frac{e^2}{h}[\mathrm{sgn}(m_{R,-}) - \mathrm{sgn}(m_{R,+})], \quad \text{if} \quad m_{R,\pm} \neq 0,$$

$$\sigma_{ii}^{cr} := \sigma_{ii}\big|_{m_{R,\omega}=0} = \frac{e^2}{h}\frac{\pi}{4}, \qquad \text{if} \quad m_{R,-\omega} \neq 0\ .$$

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Phase transitions in the Haldane-Hubbard model

Theorem (Giuliani, Jauslin, Mastropietro, Porta 2016)

There exists $U_0 > 0$ and a function ("renormalized mass")

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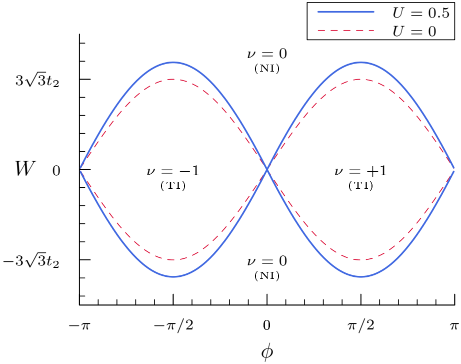
- For each $\omega = \pm$, unique solution to $m_{R,\omega} = 0$ (no bifurcation).

Renormalized transition curves



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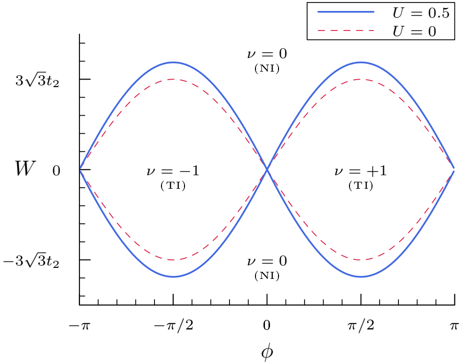

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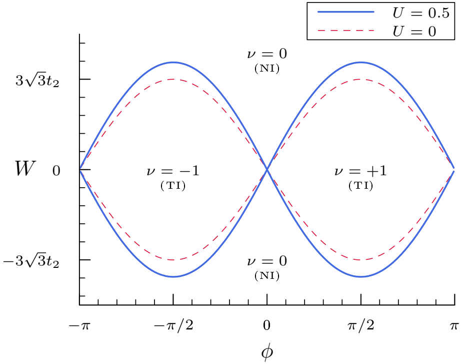

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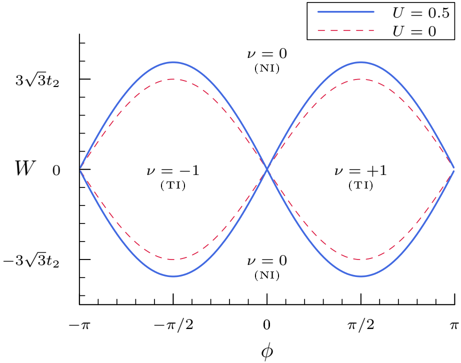

- Away from the blue curve the correlations decay exponentially fast. On the blue curve the decay is algebraic $\Rightarrow$ chiral semi-metal.
- Repulsive interactions enhance the topological insulator phase
- We rigorously exclude the appearance of novel phases in the vicinity of the unperturbed critical lines.

Outline

Main strategy, I

Step 1: We employ constructive field theory methods (fermionic Renormalization Group: determinant expansion, Gram-Hadamard bounds, ...) to prove that:

- the Euclidean correlation functions are analytic in U, uniformly in the renormalized mass, and decay at least like $|\mathbf{x}|^{-2}$ at large space-(imaginary)time separations.

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The result builds upon the theory developed by Gawedski-Kupiainen, Battle-Brydges-Federbush, Lesniewski, Benfatto-Gallavotti, Benfatto-Mastropietro, Feldman-Magnen-Rivasseau-Trubowitz, ..., in the last 30 years or so.

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- $\boxed{Z_{1,R,\omega} \neq Z_{2,R,\omega}}$

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Step 2: Combining the existence of the g.s. euclidean correlations with a priori bounds on the correlation decay at complex times $t \in \mathbb{C}^+$, we infer the analyticity of correlations for $t \in \mathbb{C}^+$ (via Vitali's theorem)

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Next, using the $(\mathrm{Re}\ t)^{-2}$ decay in complex time, we perform a Wick rotation in the time integral entering the definition of $\sigma_{ij}(U)$: the integral along the imaginary time axis is the same as the one along the real line or, better, as the *limit of the integral along a path shadowing from above the real line*. Existence and exchangeability of the limit follows from Lieb-Robinson bounds.

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The general strategy is analogous to [Coleman-Hill '85]: "no corrections beyond 1-loop to the topological mass in QED_{2+1}."

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  - Effect of long-range interactions (e.g., static Coulomb)?

Thank you!