A Work Transfer Perspective of Propulsion System Performance

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This paper suggests an approach to analysis of propulsion system performance that focuses entirely on thermodynamic work potential (and loss thereof) as a universal basis for gauging engine performance. This work potential may take a variety of forms, including conventionally known exergy analysis. Emphasis is placed on understanding how work potential initially stored in the chemical bonds of the fuel is manifested as useable work potential in an engine, transferred through a collection of components organized as a propulsion system, and ultimately yields useful thrust work. A model for overall propulsion system efficiency is suggested to facilitate the analysis. Component work transfer functions are introduced as a tool for analyzing work transfer and are used in conjunction with standard methods of block diagram algebra. This analysis reveals the fundamental parameter groupings governing propulsion system thermodynamic performance and makes clear how work is transferred thorough various portions of the engine.

I. Introduction

An engine cycle, viewed from a pure work transfer perspective, is nothing more than a collection of components and processes whose sole purpose is to convert one form of stored work potential (fuel) into thrust work with the greatest efficacy possible. Each component present in a propulsion system can be viewed as a transfer function that takes work potential in one form and yields work potential output in another form. The arrangement and design of components in a propulsion system is fundamentally driven by the need to transfer work potential stored in the chemical bonds of fuel into thrust work as efficiently as possible (subject to a variety of practical constraints on temperatures, pressures, and loadings of various components).

If one begins with the premise that *the fundamental purpose of all propulsion systems is to convert stored fuel work potential into thrust work*, it follows that the quantity of greatest intrinsic interest for propulsion system analysis is work potential. It is therefore natural to ask: how might one quantify thermodynamic performance strictly in terms of the flow of work potential through the various pathways present in the cycle? Work transfer analysis takes this point of view—the emphasis is entirely on the transfer of work potential from fuel into useful work. Temperature and pressure play only an ancillary role in this analysis.

This work builds on previous ideas suggested in Refs. 1 and 2. The goal is to develop a thermodynamic analysis method that: 1) is broadly applicable to any propulsion concept, be it conventional or revolutionary; 2) provides deep insight into the fundamental nature and drivers on propulsion system performance; 3) is readily linked to existing thinking and standards for propulsion performance; 4) need not rely on the availability of component efficiency data in order to get useful analysis results; and 5) requires the fewest possible assumptions to perform the analysis. The role of a propulsion system is not to make heat, pressure, or temperature; it is to make work. Let us therefore focus exclusively on work potential transfer as a performance figure of merit.

II. Work Transfer

Up to this point, the concept of work transfer has been presented in abstract terms. This concept is made more explicit in Fig. 1 by comparing the first and second law perspectives for an arbitrary steady thermodynamic process. This figure assumes that a component can be regarded as a "black box" that takes an arbitrary number of flow input streams and transforms them into an arbitrary number of flow output streams. This is accomplished through thermodynamic interaction with each other and/or an external source of energy (common modes of energy transfer including but not limited to shaft work, fuel heat input, or heat transfer into the component). Each flow stream has a unique thermodynamic state associated with it, defined by pressure, temperature, mass flow rate, and fuel-air ratio

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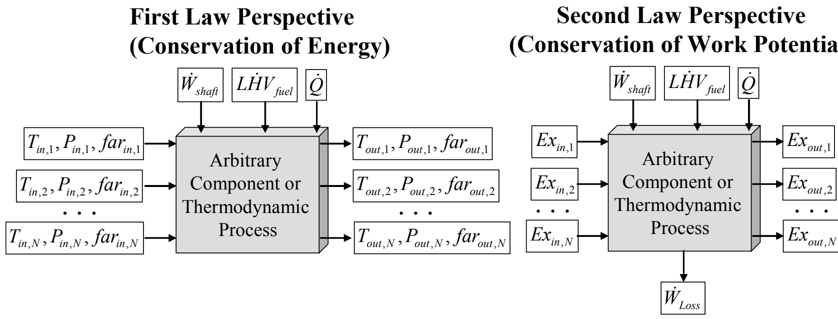


Fig. 1: First and Second Law Perspectives of Thermodynamic Performance.

(or, more generally, chemical composition of the flow stream). It is presumed for now that the kinetic energy of the flow streams is negligible or is book-kept as stagnation pressure and temperature of the streams. The first law of thermodynamics enables the calculation of the output flow stream states given the work, heat, and flow stream inputs (and perhaps some auxiliary assumptions regarding the energy split amongst the output streams).

Whereas the first law perspective is conservation of energy, the second law perspective is conservation of work potential. Specifically, the second law of thermodynamics enables explicit calculation of work potential stored in a substance based on the thermodynamic states of that substance. The work potential figure of merit assumed for illustrative purposes in Fig. 1 is exergy, but other measures of work potential could be used if desired. See Ref. 3 for further discussion on the definition and physical meaning of exergy and work potential. If the state of every flow and energy stream entering and leaving a component is known, one can use this information to calculate loss inside the component. It follows that the ratio of total work potential output to input in a component is a natural measure of the component's performance—a "generalized efficiency."

III. Component Performance in Terms of Work Transfer

Engine component performance is typically defined in terms of component efficiency. Standard definitions for common component efficiencies are a matter of convention. These definitions for component efficiency are somewhat arbitrary, each type of component having its own unique definition for efficiency, usually chosen to facilitate easy calculation of efficiency from test data. Typical examples of commonly used component efficiencies are nozzle thrust coefficient, compressor adiabatic efficiency, turbine adiabatic efficiency, combustion efficiency, pressure drop, inlet pressure recovery, etc.

The definitions for each of these component efficiencies are unique and therefore, efficiencies cannot generally be compared amongst components of different types. For example, a change of one point in nozzle thrust coefficient may have much different impact in terms of work transfer (and loss) than does a change of one point in compressor adiabatic efficiency. As a result, one cannot directly discern which component causes the greatest losses in the engine by inspection of component efficiency alone. Moreover, some measures of efficiency are not directly comparable even within the same class of component. For example, the adiabatic efficiency of a high pressure ratio compressor may considerably lower than a low pressure ratio compressor while still having the same impact in terms of transfer of work potential.

An alternative definition for component efficiency is available that simplifies this situation by providing a universal definition for efficiency that is applicable to components of all types. This measure of component efficiency is the component work transfer function. *Component work transfer function, X, is defined as the ratio of work potential output to input of a component.* It is directly analogous to the definition of a transfer function used in linear controls theory wherein a transfer function is defined as the ratio of output to input (gain) of a single input, single output system. The generalized "black box" component described in Fig. 1 allowed for any number of input and output streams as well as any number of additional energy interactions with the environment outside the component boundaries. These can be summed to give a total work potential flux in and out of the component, thereby arriving at a single figure of merit characterizing overall component performance.

Because the definition of work transfer is common for all components, a 1 point change in component transfer function has the same impact on loss regardless of the component type. In addition, one can directly compare a change in work transfer of one component to a change in work transfer of another component. Component work transfer functions have the additional benefit of being more intuitive and revealing of true component performance than standard definitions for efficiency. This idea of component work transfer is the basis used herein for development of methods for work transfer analysis of entire systems of components.

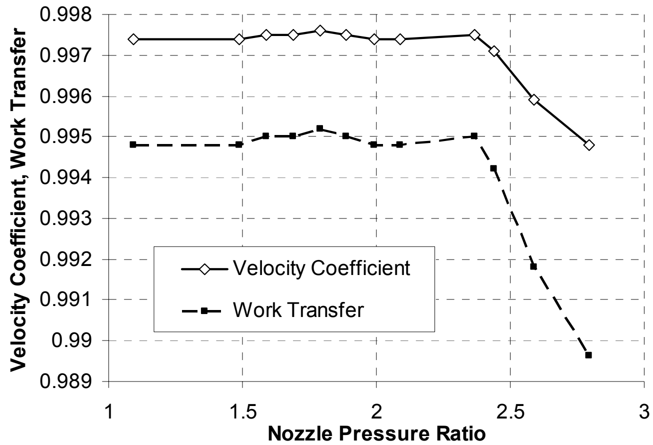


Fig. 2: Comparison of Nozzle Efficiency Versus Nozzle Work Transfer.

Component work transfer functions can be expressed in terms of conventional component efficiencies for those cases where a convention of component efficiency exists. Examples of the relationships between component work transfer and component efficiency for the most common components are derived in detail in Ref. 1. In the case of revolutionary propulsion systems utilizing novel new types of components for which there is no prevailing definition for efficiency, component performance can simply be expressed in terms of component work transfer without the need for any other definition of efficiency.

Consider Fig. 2 as an example of the relationship between work transfer and classic component efficiencies. This figure shows a plot of nozzle thrust coefficient versus nozzle pressure ratio (NPR) for a typical fixed area nozzle used on the core stream of separate flow turbofan engines. Superimposed on this plot is nozzle work transfer as a function of nozzle pressure ratio. The trends for the two curves are similar but the transfer of work potential through the nozzle is less than the transfer of thrust potential. Specifically, the thrust coefficient is roughly 99.7% over much of the operating range, but only about 99.5% of the theoretical gas kinetic energy available at the nozzle inlet is realized as actual exhaust gas kinetic energy at the nozzle exit plane. Also note that work transfer rolls off more quickly than thrust coefficient at high nozzle pressure ratio, suggesting that the disparity between the two figures of merit increases as performance decreases at very high NPR.

Another example of the differences between work transfer and classic component efficiency is illustrated in Fig. 3. This chart shows inlet pressure recovery (dashed lines) and inlet work transfer (solid lines) as a function of mass flow ratio for a series of Mach numbers. Note that inlet pressure recovery is quite high over a relatively wide range of mass flow ratios and Mach numbers. However, inlet *work transfer* is very seriously degraded by seemingly small decreases in inlet pressure recovery. In fact, at very low Mach numbers, even very small reductions in inlet pressure recovery result in destruction of inlet ram work, so much so that the total work transfer readily goes to negative values, meaning that additional work input into the system is required in order to achieve those mass flow

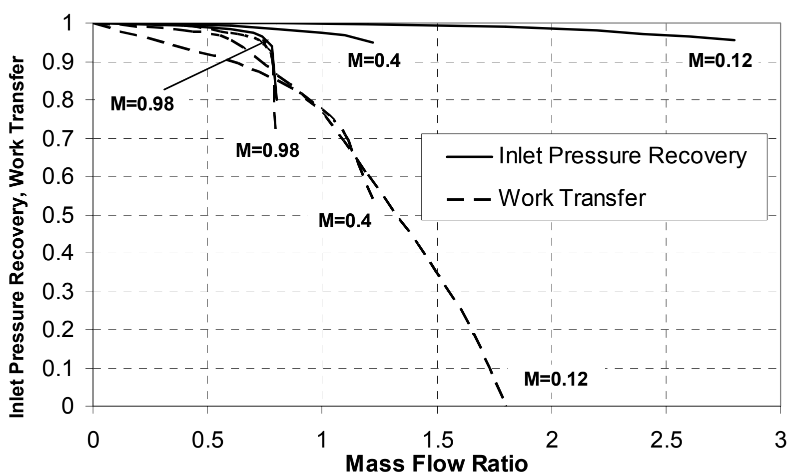


Fig. 3: Comparison of Inlet Pressure Recovery Versus Inlet Work Transfer.

ratio/Mach combinations. In physical terms, a vacuum must be applied to the inlet discharge in order to realize the mass flow ratio/Mach point.

This is an example wherein the standard definition of component efficiency is not particularly revealing of the true cost (loss) in terms of work transfer through the engine, though it is a convenient parameter to measure inlet performance from an experimental or test perspective. Furthermore, the relationship between pressure recovery and work transfer is highly nonlinear and therefore requires considerable experience before one develops an intuitive "feel" for the link between inlet pressure recovery and its consequent impact on overall system performance. This is the case for most commonly used measures of component efficiency.

IV. Work Transfer and Overall Propulsion System Efficiency

The previous sections examined the definition of work transfer as a measure of performance for a single component in isolation. The objective of this work, however, is to analyze propulsion *systems* that typically consist of collections of components arranged in a predefined way. Let us therefore consider work transfer in an engine cycle and develop a generic model expressing the transfer of fuel work potential into thrust work.

Lewis[2] describes a very useful and intuitive framework for defining overall propulsion system efficiency. This model is commonly used today and consists of three terms: thermal efficiency, η_{Th}; transfer efficiency, η_{Tr}; and propulsive efficiency, η_P. These are defined as and related to overall efficiency through the equation:

$$w_{Thrust} = \underbrace{\frac{w_{Thrust}}{w_{KE}}}_{\eta_P}\underbrace{\frac{w_{KE}}{w_{Cycle}}}_{\eta_{Tr}}\underbrace{\frac{w_{Cycle}}{LHV}}_{\eta_{Th}}LHV \Rightarrow \eta_o = \eta_p \eta_{Tr} \eta_{Th} \tag{1}$$

where: LHV = Heat content (lower heating value) of the fuel per unit mass fuel
w_{Cycle} = Net work produced by the engine cycle (gas generator available energy) per unit mass flow
w_{KE} = Net change in kinetic energy of the propulsive flow per unit mass flow
w_{Thrust} = Net thrust work produced per unit mass flow
η_o = Overall propulsion system efficiency (defined as quotient of thrust work out to heat input).

Thermal efficiency measures how much of the fuel's energy (the heat released from combustion) is realized as net cycle work. Transfer efficiency measures how efficiently the available energy produced by the cycle is converted into a change in propulsive stream kinetic energy. Propulsive efficiency measures the transfer of propulsive stream kinetic energy into thrust work done on the engine.

This equation is almost adequate as a model for transfer of fuel work potential into net thrust work. It captures the transfer of cycle work into change in propulsive stream kinetic energy as well as the transfer of propulsive stream kinetic energy into thrust work. The only shortcoming of this model from a work transfer point of view is that the thermal efficiency term measures conversion of heat energy into cycle work, not the transfer of *fuel work potential* into cycle work.

As the objective of this method is to treat the entire analysis in terms of transfer of work potential, consider a slightly modified version of the above equation. Specifically let us express the thermal efficiency in terms more consistent with the work transfer ideas discussed previously:

$$w_{Thrust} = \underbrace{\frac{w_{Thrust}}{w_{KE}}}_{\eta_P}\underbrace{\frac{w_{KE}}{w_{Cycle}}}_{\eta_{Tr}}\underbrace{\overbrace{\frac{w_{Cycle}}{w_{Fuel}}}^{Component}\overbrace{\frac{w_{Fuel}}{LHV}}^{Cycle}}_{\eta_{Th}}LHV \tag{2}$$

where w_{fuel} = Maximum cycle work theoretically available in the fuel per unit mass flow.

The thermal efficiency in this model now consists of two terms, one that is principally a function of component efficiencies and another that is mainly a function of the thermodynamic cycle. The "cycle" term of the thermal efficiency is effectively the ideal cycle efficiency for the nominal cycle if there were no component losses present in the thermal cycle. It measures how effectively the heat energy of the fuel is converted into available energy in the cycle. The "component" term of thermal efficiency accounts for the impact of component losses on total cycle output. In effect, this model separates the nominal engine cycle from the cycle component losses in like fashion to the way propulsive efficiency and transfer efficiency are separated. Before proceeding further, let us name the two new system efficiency terms:

$$w_{Thrust} = \underbrace{\frac{w_{Thrust}}{w_{KE}}}_{\eta_P}\underbrace{\frac{w_{KE}}{w_{Cycle}}}_{\eta_{Tr}}\underbrace{\frac{w_{Cycle}}{w_{fuel}}}_{\eta_{CT}}\underbrace{\frac{w_{fuel}}{LHV}}_{\eta_{CA}}LHV \tag{3}$$

where: η_{CT} = Cycle transfer efficiency
η_{CA} = Cycle availability efficiency.

It should be apparent that the model proposed in Eq. (3) is, in fact, a series of four successive work transfer functions, each capturing the impact of a particular type of loss mechanism, as depicted in Fig. 4. The heating value of the fuel is a property of the fuel itself and is independent of both the cycle and specific component implementation used in the propulsion system. The cycle availability efficiency measures how efficiently the work potential contained in the fuel is transferred into work potential accessible to the cycle (expressed as a fraction of fuel LHV). For example, if work potential is added to the cycle in a gas turbine combustor, then η_{CA} is only a function of the how efficiently fuel energy is converted into available energy in the combustor. This is in turn a function of the conditions in the combustor (i.e. combustion pressure, temperature, temperature rise, etc.). The cycle transfer efficiency measures how efficiently the available energy from the cycle is transferred into net cycle work. It is a function of component efficiency and arrangement. Net cycle work is then converted into a change in propulsive stream kinetic energy through the transfer efficiency (which is also a function of the engine component arrangement and efficiency). Finally, the change in propulsive stream kinetic energy is transferred into thrust work via the propulsive efficiency (a function of flight condition and specific power output).

Note that Eq. (2) assumes nothing about how the fuel work potential, w_{fuel}, is measured. Rather, w_{fuel} was simply defined as the work potential available from the cycle, and it is left to the analyst to define what should be considered "available' or "useable" energy. For internal combustion and gas turbine engines, the logical definition for w_{fuel} is the gross gas specific power added to the working fluid through combustion. Strictly speaking, if Eq. (3) were a pure work potential model, then the LVH terms in the equation should be replaced by the fuel exergy. LHV was retained in the present model for the sake of maintaining compatibility with existing definitions of overall efficiency, all of which use fuel heating value as the denominator.

A. Transfer of Fuel Energy into Available Energy

Let us consider the cycle availability component of Fig. 4 in more detail, with emphasis on how chemical work potential of the fuel relates to usable work potential in the cycle. It was previously mentioned that for most propulsive cycles, available energy is manifested as gas specific power. Gas specific power is defined in detail in Ref. 3 and physically corresponds to the work output obtainable by adiabatic expansion of a flow to ambient pressure. Modern engines work by converting chemical work potential stored in the fuel into usable work potential through combustion at high-pressure and subsequent expansion to ambient pressure. This conversion process is far from perfect, and the reduction from chemical potential to useable gas-specific power is captured by the cycle availability efficiency of Eq. 4.

One can gain useful insight into the nature of cycle availability efficiency by examining the equation for gas specific power of an ideal gas:

$$\Delta gsp = c_p \Delta T \left[1 - \left(P_{comb} / P_{amb} \right)^{\frac{1-\gamma}{\gamma}} \right] = w_{fuel} \tag{4}$$

where: Δgsp = Change in gas specific power due to combustion
c_P = Constant pressure ratio of specific heats
ΔT = Temperature rise due to combustion
P_{comb} = Pressure at which combustion occurs

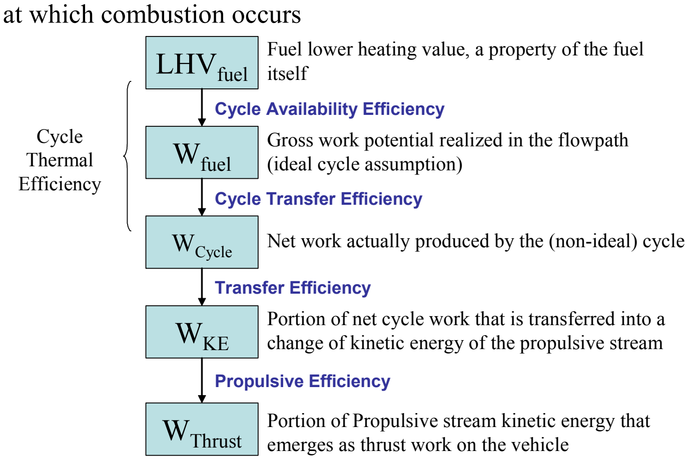


Fig. 4: Model for Work Potential Transfer from Fuel to Thrust Work.

P_{amb} = Ambient reference pressure to which products of combustion are expanded.

One can readily see by examination of this equation that gas specific power added to a stream through combustion is a function of the enthalpy rise of combustion (given by the fuel lower heating value) and the ratio of combustion pressure to ambient pressure. Eq. (4) can therefore be thought of as consisting of an energy term ($c_P\Delta T$) and an "availability coefficient" that is a function of the combustion pressure ratio. This availability coefficient is bounded by 0 and 1, and approaches 1 as the combustion pressure ratio increases. Assuming that w_{fuel} is given by the gas specific power input into the system and substituting Eq. (4) into the definition of cycle availability efficiency yields:

$$\eta_{CA} \equiv \frac{w_{fuel}}{LHV} = \frac{\Delta gsp}{LHV} \approx \frac{c_p \Delta T\left[1-\left(P_{comb}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}}\right]}{LHV} \tag{5}$$

and assuming that LHV is equal to the change in enthalpy of the products of combustion, we get the following.

$$\eta_{CA} \equiv \frac{w_{fuel}}{LHV} \approx \frac{LHV\left[1-\left(P_{comb}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}}\right]}{LHV} = 1-\left(P_{comb}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}} \tag{6}$$

Thus, it is apparent that cycle availability efficiency is in fact the previously defined "availability coefficient" evaluated at the combustion pressure ratio.

It is of obvious importance to raise the combustion pressure ratio to as high of a level as is practical and is the fundamental reason why it is necessary to have a work feedback paths that act to transfer work from downstream portions of the engine cycle to upstream portions (as, for example, in the transfer of shaft work from a turbine to a compressor a typical turbojet engine. The work feedback loops themselves contribute only loss to the total system output. However, the work transferred in these feedback loops is used to increase the combustion pressure ratio, thereby increasing the transfer of fuel energy into usable work potential. The optimum cycle occurs when the incremental losses in the work feedback loops exactly equal the incremental gain in gas specific power due to increased combustion pressure ratio (see Builder[4] for further discussion and explanation of this result).

The fact that the cycle availability transfer is directly linked to work feedback leads one to ponder the relationship between cycle availability efficiency and the fraction of work fed back upstream of the combustion process. After all, combustion pressure ratio is a function of work input into the working fluid as well as losses in the compression process. Combustion pressure ratio must therefore be a function of the fraction of work fed back into the system as well as the loss function associated with the work feedback loop(s).

B. Propulsive Efficiency Expressed in Terms of Work Transfer

The focus up to this point has been on understanding transfer of fuel energy into cycle work output. However, the ultimate aim is to understand how all portions of Eq. (3) relate to work transfer in the propulsion system. Therefore, let us explore this relationship between propulsive efficiency and work transfer.

It is intuitively apparent that propulsive efficiency is intimately related to both ram compression work (kinetic energy of the incoming flow) and kinetic energy production in the propulsive stream. The conventional definition of propulsive efficiency is a function of exit velocity of the propulsion stream as well as flight velocity of the vehicle:

$$\eta_P \equiv \frac{\text{Thrust Work}}{\Delta\text{Propulsive Stream KE}} = \frac{\dot{m}(u_e - u_0)u_0}{\dot{m}w_{KE}} \tag{7}$$

where: $\dot{m}$ = Mass flow rate of the propulsive stream

u_e = Exit velocity of propulsive stream relative to the engine

u_0 = Flight speed of engine relative to Earth-fixed reference frame.

The exit velocity of the propulsive stream is in turn a function of the net specific power output of the cycle after transfer losses (i.e. the change in propulsive flow kinetic energy produced by the engine acting on the propulsive stream):

$$u_e = \sqrt{2w_{KE}} \tag{8}$$

and likewise for u_0:

$$u_0 = \sqrt{2w_{ram}} \tag{9}$$

where w_{ram} is the mass-specific ram work done by the engine on the incoming flow and is equal to the kinetic energy of the incoming flow per unit mass. Substituting into Eq. (7) yields the following expression.

$$\eta_P = \frac{\left(\sqrt{2w_{KE}} - \sqrt{2w_{ram}}\right)\sqrt{2w_{ram}}}{w_{KE}} \tag{10}$$

This expression gives propulsive efficiency as a function of the ratio of ram work transfer to net cycle work output. Note that propulsive efficiency is a highly nonlinear function of the ram to net work fraction. For the simplified case of a turbojet engine, we can use the well-known result for propulsive efficiency.

$$\eta_P = \frac{2}{1 + {u_e}/{u_0}} = \frac{2}{1 + \sqrt{{w_{KE}}/{w_{ram}}}} \tag{11}$$

This equation is plotted in Fig. 5 as a function of the ratio of ram work to kinetic energy production. As one would expect, propulsive efficiency approaches zero as ram work vanishes. In the other extreme, propulsive efficiency approaches unity as the ratio of ram work to exit kinetic energy approaches unity.

C. Cycle Work Transfer Versus Second Law Efficiency

A great deal of prior work has been done with regards to second law analysis of thermodynamic systems, of which, Ref. 5 is a very comprehensive example. It may be evident to the reader familiar with current literature in this field that what is referred to here as a "work transfer function" is essentially equivalent to what is more commonly known as second law efficiency. One might naturally wonder how the concept of work transfer is different from second law efficiency? Furthermore, when referring to component work transfer, why use the term "work transfer function" instead of the more commonly accepted term, "second law efficiency?"

The primary difference between cycle efficiency and work transfer function is that the former is essentially a conventional view of efficiency packaged in terms of the second law of thermodynamics. Work transfer analysis is a wholly work potential-centric point of view. It applies not only to systems, but also to components, assemblies of components, and even sub-components. The term "work transfer function" is used herein to emphasize that the concept of work transfer as developed herein is not based on any one measure of work potential, but rather admits a variety of measures for work potential including exergy, gas specific power, thrust work potential, and so on. The choice of which is most appropriate depends on the analysis objectives and the analyst's point of view. This is in contrast to second law efficiencies, which are almost always defined in terms of exergy. Furthermore the term "work transfer" is more reflective of the true nature of the quantity that is the term "second law efficiency," and is intended to emphasize the philosophical distinction between the approach advocated herein and other approaches.

At the component level, second law efficiency is the same as work transfer. The concept of work transfer analysis is essentially a means to link component-level second law efficiencies to overall system level efficiencies. Ultimately, work transfer analysis attempts to express system work transfer in terms of component work transfer, hopefully revealing useful insights regarding how work is transferred through the system.

V. Block Diagram Representations of Work Transfer

A very appealing benefit of defining system performance in terms of transfer functions is the fact that all the standard rules of block diagram algebra and system reduction can be applied to the analysis of a propulsion system. Block diagrams are used in a variety of engineering fields, most notably controls engineering, and consist of standard rules and formulas that enable the simplification of complex sets of components into simpler subsets.[6] For

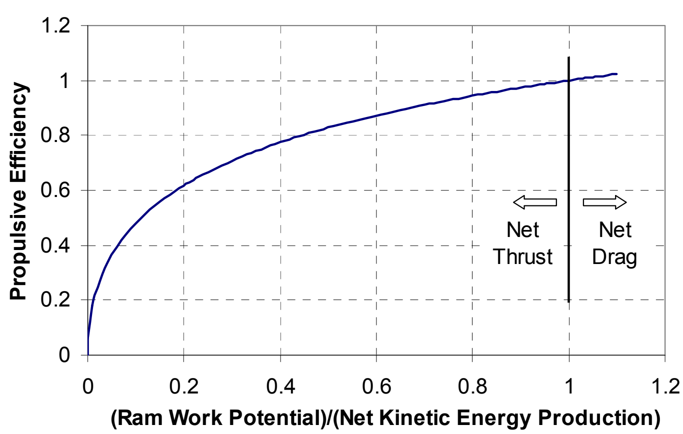


Fig. 5: Propulsive Efficiency as a Function of Work Transfer.

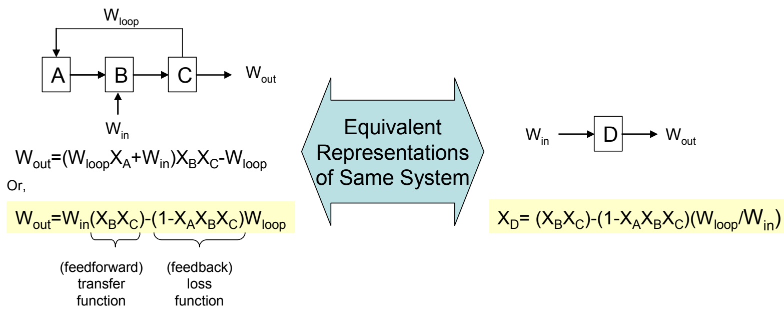


Fig. 6: Typical Work Feedback Loop.

example, one of the most common arrangements of engine components is a feedback loop having a single input and output of the form shown at left in Fig. 6. This block diagram configuration corresponds to a simple gas generator, with A as the compressor, B as the combustor, and C as the turbine.

In the conventional view, each component of the gas generator takes mass flow in and puts mass flow out. In the work potential view, each component of this gas generator is a transfer function that takes work potential in and yields work potential out. If the system has only a single input and output then the transfer function for this group of blocks is equivalent to a single "black box" having a transfer function given by the expression on the right of Fig. 6. This enables considerable simplification in the process of developing work transfer representations for groupings of components. In fact, with the application of block diagram algebra, a few simple rules, and some practice, it is usually possible to develop an expression for work transfer by inspection of the block diagram schematic.

With this in mind, let us develop a series of useful block diagram equivalencies that are helpful in conducting work transfer analyses, starting with the simple feedback loop discussed previously. Note that when the gas generator performance is expressed in terms of work transfer, the resulting equivalent transfer function contains two terms: a feedforward term (X_BX_C) and a loss function ($1-X_AX_BX_C$). The feedforward term describes the path transferring work potential form the input to output of the system. The loss function describes what fraction work fed back into the system (w_{loop}) is lost when traversing the feedback path. Two variations on the basic feedback loop work transfer system are shown in Fig. 7. Notice that the feedback loss function is the same in all cases, and the feedforward transfer function changes in accordance with the components through which the input work potential must pass to reach the system output node.

Further variations on the basic work feedback arrangement are shown in Fig. 8. If there is an additional loss in the work feedback transfer, this appears in the feedback loss function but does not impact the work feedforward function. The addition of multiple work potential inputs (as shown at the bottom of Fig. 8) still results in a single feedback loss function as before, but each work potential source will have a feedforward transfer function associated with it. Note also that the equivalent block representation of such a system would not be a single block, but would instead consist of three separate block equivalents: one describing the transfer of work from source 1 to output, another for transfer of work from source 2 to output, and so on.

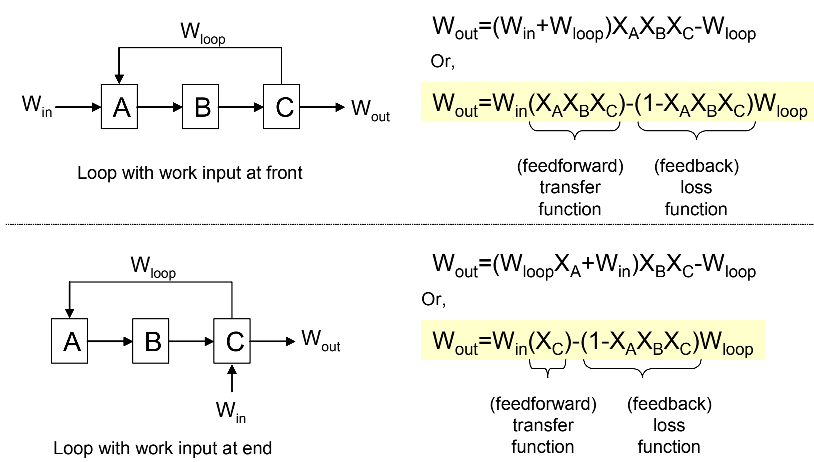
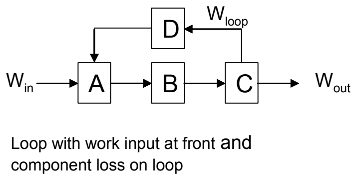


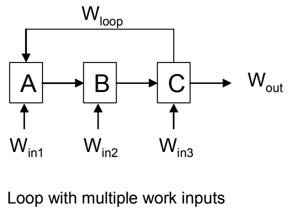
Fig. 7: Variations on the Basic Feedback Loop Work Transfer.



$$W_{out}=(W_{in}+X_DW_{loop})X_AX_BX_C-W_{loop}$$

Or,

$$W_{out}=W_{in}\underbrace{(X_AX_BX_C)}_{\text{(feedforward) transfer function}}-\underbrace{(1-X_AX_BX_CX_D)}_{\text{(feedback) loss function}}W_{loop}$$



$$W_{out}=(((W_{loop}+W_{in1})X_A+W_{in2})X_B+W_{in3})X_C-W_{loop}$$

Or,

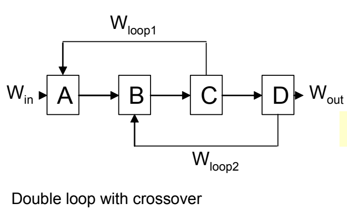
$$W_{out}=\underbrace{W_{in1}(X_AX_BX_C)+W_{in2}(X_BX_C)+W_{in3}(X_C)}_{\text{(feedforward) transfer functions}}-\underbrace{(1-X_AX_BX_C)}_{\text{(feedback) loss function}}W_{loop}$$

Fig. 8: Further Variations on the Basic Work Feedback Loop.

More sophisticated variations on the basic work feedback loop are shown in Fig. 9. This shows two commonly arising arrangements of work feedback systems: the nested loop (used in dual-spool turbine engines) and the nested loop with crossover. As before, each system has a single feedback loss function for each loop, and the feedback loss functions consist of those components that are directly in the periphery of the feedback loop. However, one additional term appears in the feedback expressions for those loops that are not directly adjacent to the work output node. Specifically, an additional feedforward work transfer term is appended to the feedback loss function, and this feedforward term is the transfer path of work from the loop edge to the system output. The feedforward work transfer function is the same as in previous cases, consisting of the transfer path from the work source (system input) to the system output.

Aside from feedback loops, another common configuration is the feedforward transfer function, as shown at the top of Fig. 10. In this configuration, a portion of work potential is split off and fed back into the system at a downstream location. The feedforward transfer function through the primary path is unchanged from what it was previously. However, the secondary path is now adding work to the total output. Thus, instead of resulting in a loss function as in the case of the feedback loop, the feedforward loop results in a gain function of the form shown in the figure. Note that once again, the gain function is multiplied by a feedforward work transfer term. A more general variation of the same feedforward loop is shown at the bottom of Fig. 10. This is a configuration commonly encountered in mixed flow turbofan engines and consists of a splitter, two separate work transfer functions, followed by a mixer. The resultant equations consist of a feedforward transfer function as before in addition to what can best be described as a feedforward difference function. A significant difference in this block arrangement from previous cases is that the feedforward and difference functions are dependent on which branch is chosen as the primary and which is the secondary work transfer path. That is to say that there are two equivalent transfer function representations available to model parallel work transfer streams.

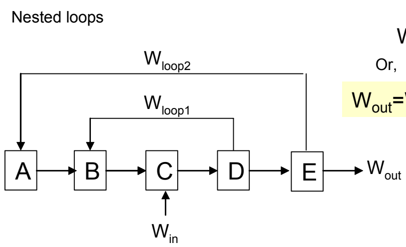
Still another set of common block configurations are those having multiple output streams, such as those shown



$$W_{out}=(((W_{in}+W_{loop1})X_A+W_{loop2})X_BX_C-W_{loop1})X_D-W_{loop2}$$

Or,

$$W_{out}=W_{in}\underbrace{(X_AX_BX_CX_D)}_{\text{(feedforward) transfer function}}-\underbrace{(1-X_AX_BX_C)}_{\text{(feedback) loss function}}\underbrace{X_D}_{\text{(feedforward) transfer function}}W_{loop1}-\underbrace{(1-X_AX_BX_CX_D)}_{\text{(feedback) loss function}}W_{loop2}$$



$$W_{out}=(((W_{loop2}X_A+W_{loop1})X_B+W_{in})X_cX_D-W_{loop1})X_E-W_{loop2}$$

Or,

$$W_{out}=W_{in1}\underbrace{(X_CX_DX_E)}_{\text{(feedforward) transfer function}}-\underbrace{(1-X_AX_BX_CX_DX_E)}_{\text{(feedback) loss function}}W_{loop2}-\underbrace{(1-X_BX_CX_D)}_{\text{(feedback) loss function}}\underbrace{X_E}_{\text{(feedforward) transfer function}}W_{loop1}$$

Fig. 9: Multiple Work Feedback Loop Arrangements.

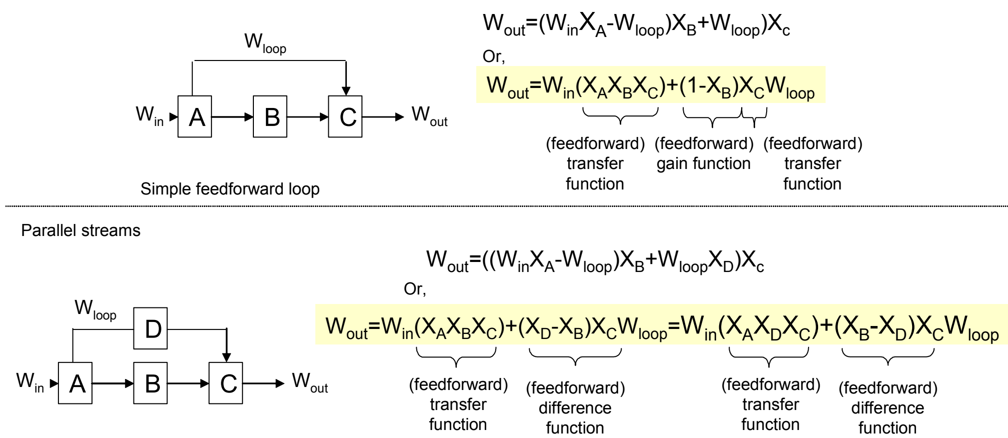


Fig. 10: Feedforward and Split Transfer Functions.

in Fig. 11. The top portion of this figure shows a configuration having a simple secondary work output stream. One can write a work transfer equation for both feedforward paths, as before. However, these equations both contain terms with a "loss-like" term to subtract out the portion of work transferred through the alternate path. If a total work transfer function is taken to be the sum of the two work output streams, then algebraic substitution yields a work transfer function having a feedforward transfer term as before in addition to a gain function term applied to one of the work outputs. Once again, there are two equivalent expressions for the total work transfer, the differences arising depending on which work transfer path is chosen as the primary. The symmetric case for this same situation is shown at the bottom of Fig. 11. Inspection of this arrangement reveals that the gain function terms present in the previous case are actually ratioed in proportion to the work transfer through the two paths.

A. General Rules for Application of Work Transfer Analysis

The work potential transfer block diagrams discussed in this section are but a few of many possible configurations. Any arbitrary configuration and combination of blocks can be represented using a work transfer function by application of the same basic rules of block diagram algebra used here. If one examines the previous examples, a pattern is discernible in the mathematics. These patterns lead to several useful generalizations regarding the application of work transfer analysis. Based on this discussion, it should be evident that the following general rules apply when employing work transfer analysis:

1) Each work potential input stream results in a single feedforward work transfer function.
2) The feedforward work transfer functions approach 1.0 as the cycle approaches perfection.
3) Each work potential output stream is described with its own work transfer equation.
4) The equations for multiple input, multiple output work transfer systems are typically manifested as a set of coupled nonlinear equations.
5) Every 'loop' wherein work potential is extracted from one point in the cycle and fed back into an upstream

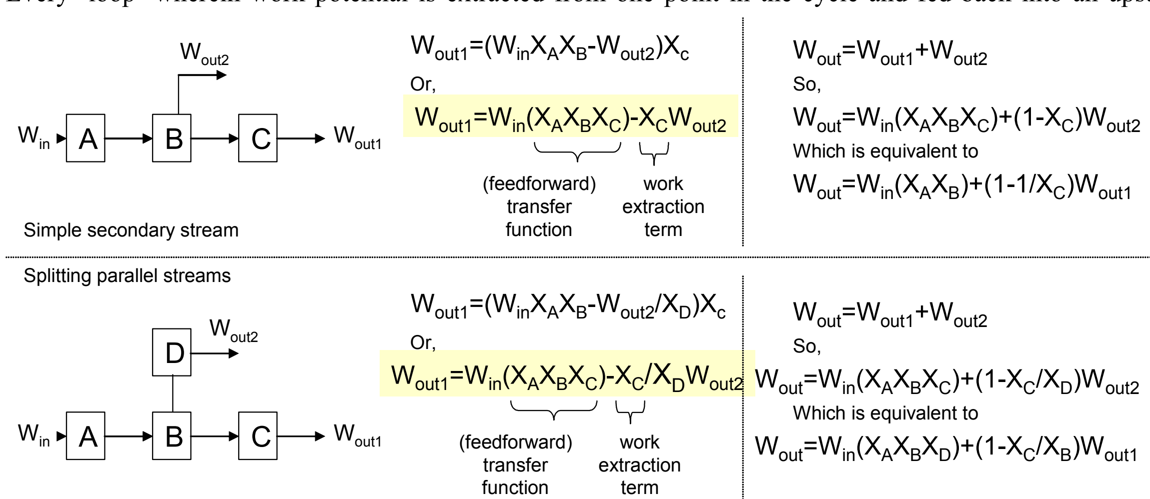


Fig. 11: Typical Multiple-Output Block Diagram Arrangements.

location will manifest itself as a term containing a loss function. There will be one work loss function for each feedback loop. The loss function approaches 0.0 as the cycle approaches thermodynamic perfection.

6) The loss function terms in a feedback loop consist of the components on the periphery of the loop.
7) The feedforward terms in a feedback loop consist of the work transfer functions of the components from the loop to the system output.
8) All block diagrams having a single input and output can be represented by a single equivalent component, no matter how complex the specific configuration may be.
9) Conversely, any block having a single input and output can be broken into multiple subcomponents in any way convenient to secure accurate representation of the loss mechanisms present inside the component.
10) Any conventional cycle schematic representation can be expressed in terms of a work transfer block diagram, the key being to focus on mechanisms of work transfer (which may or may not resemble the order and physical arrangement of the engine flowpath).
11) Even components with multiple input and output streams can be broken into smaller subcomponents in any way likely to secure an accurate representation of the underlying physics. Once again, the key is to divide the component processes according to the processes impacting work transfer through the component and not just mass transfer.
12) It cannot be emphasized enough that work transfer representations must account for all work transfer mechanisms present. This may or may not correspond to a standard cycle block diagram representation familiar to propulsion engineers. The two are distinctly different: the work transfer block diagram emphasizes the flow of work potential through the system. The conventional component diagram emphasizes mass transfer through the system. Therefore, some work transfer paths (such as the ram work feedback loop) don't ordinarily appear on a conventional cycle schematic but do appear in a work transfer representation and vice-versa.

B. Additional Notes and Useful Equations

One should intuitively expect that the sum of the work potential output from the cycle plus the work potential losses internal to the engine should sum to the work potential transferred into the cycle through the fuel. Expressed mathematically:

$$w_{fuel} = w_{net} + w_{lost} \tag{12}$$

and therefore, the total loss in the engine cycle can be expressed as a function of the work transfer function:

$$\frac{w_{lost}}{w_{fuel}} = 1 - \frac{w_{net}}{w_{fuel}} \,. \tag{13}$$

The reader should be aware that the concepts presented herein are not new, though they are expressed in a different form than has been used in the past. Specifically, Work Transfer Analysis is related to the "Entropy Method" described by Foa.[7] The difference is primarily that the Entropy Method quantifies losses in terms of entropy increments—not an intuitive measure of loss—while work transfer quantifies losses in terms of work transfer functions. The equivalence of methods can be expressed symbolically:

$$\frac{w_2}{w_1}\frac{w_3}{w_2}\frac{w_4}{w_3}\ldots \Leftrightarrow \exp\left(\Delta s_{1-2} + \Delta s_{2-3} + \Delta s_{3-4} + \ldots\right) \tag{43}$$

Another difference is that Foa uses entropy as the one and only measure of work potential whereas the present model suggests that work potential can be defined various ways, depending on the assumptions and boundary conditions imposed on a particular problem.

VI. Conclusions

The work transfer analysis method suggested herein is based on the idea that the quantity of fundamental interest from a thermodynamic standpoint is work potential. The fundamental strength of the work transfer analysis approach is that it requires very little prior knowledge of propulsion system or component performance. It is not necessary to have an accepted definition for component efficiency, as all components are viewed in terms of work transfer function (which is a universal efficiency). All that is needed to obtain useful work transfer analysis results is knowledge of how the components are connected together.

This is not intended to suggest that the work transfer point of view replaces classical cycle analysis. On the contrary, results from standard cycle analysis *are a necessary prerequisite* to performing work transfer analysis. Work transfer results *compliment* classical cycle analysis, yielding additional insights that are not ordinarily evident from standard (first law-based) cycle analysis.

Work transfer analysis is applicable to any propulsion system, conventional and revolutionary, and this is a key feature that makes this method particularly useful. Work transfer analysis is relatively simple to implement, and most cycle configurations can be analyzed by inspection of their component block diagrams. The method is particularly useful for revealing natural groupings of component efficiency parameters governing system performance, and does so without the need to explicitly define component performance parameters. Finally, the method suggested herein is not tied to a single measure of work potential, but is equally applicable regardless of how one chooses to define "available energy." Additional strengths of work transfer analysis are:

- It lays bare the fundamental quantities of interest for the thermodynamic design of prime movers, this being transfer and loss of work potential.
- It identifies specific work feedback and feedforward streams and quantifies a performance of each stream as a whole (this is not done today with present methods).
- It enables clear and easy quantification of work losses occurring in each work transfer path.
- The analysis is based entirely on quantities having physical and tangible meaning; entropy never directly enters the analysis. The method is therefore a good pedagogical tool.
- It can be related to conventional component efficiencies (therefore couched in terms of current thinking, historic data can be readily adapted for use in this analysis method).
- Although Work Transfer Analysis was only discussed in the context of propulsion system analysis, the idea is very general and applies equally well to analysis of an entire vehicle or power plant (in fact, any thermal system).

Acknowledgments

The author would like to thank NASA Glenn Research Center for their support under contract NAS-3-03115.

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