MODELLING FOR TWO-TIME SCALE FORCE/POSITION CONTROL OF FLEXIBLE ROBOTS

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Abstract - **Distributed flexibility of the links is a severe obstacle for the endpoint position control of lightweight manipulators. In order to accomplish with satisfactory performance certain tasks involving a controlled interaction of the tip of the robot with the worksurfaces, a combined control of the motion and the contact forces can provide some advantages. This paper presents a general and systematic model of a flexible robot interacting with a rigid environment. A force/position control scheme based on this model is also introduced. Results obtained simulating the constrained motion of an existing 2 d.o.f. flexible arm are finally given.**

Keywords - **Flexible manipulators; force control; singular perturbation theory; composite control; robotic simulation.**

1. INTRODUCTION

Lightweight flexible manipulators have lately earned increasing attention from robotic researchers. The demand for high speed and large workspace, coupled with the requirement of a high ratio between payload and arm, have stimulated engineers towards the investigation of new materials and mechanical designs, in an effort to overcome the limitations of rigid industrial robots. A variety of fields where the adoption of lightweight robots prove to be successful can now be characterized, including space robotics, exploration of hazardous environments and nuclear waste retrieval [1].

In certain tasks, requiring the interaction of the tip of the robot with the external environment, even a satisfactory solution for the endpoint position control [2] could however prove to be inadequate. In these cases, it can be beneficial to monitor and control the forces and moments arising at the contact. Though being a very active area of reasearch [3] for rigid robots, only a few works [4]-[13] have been presented so far on the force control of flexible manipulators

The main contribution of this paper is the systematic and general formulation of a dynamic model for the constrained flexible robot. The model is based on a coordinate partitioning procedure, where the dynamic equations of the constrained system are written in terms of a subset of the original generalized coordinates. A singular perturbation version of the model is also given, which shows some interesting differences from previous results on the same subject [10], [11]. A control scheme for the force/position control of flexible robots has also been designed in this research [14]. However, since the focus of this paper is the development of the model, the control is here only briefly introduced to show the utility of the model.

The paper is organized as follows: Section 2 develops the reduced order model of a constrained flexible robot; Section 3 presents the singular perturbation version of the model; a two-time scale force/position controller is briefly presented in Section 4, and is used to validate the model in Section 5, through simulations of a detailed dynamic model of a two d.o.f. flexible robot; finally some concluding remarks are proposed in Section 6.

2. MODEL OF A CONSTRAINED FLEXIBLE ROBOT

2.1 Model of the unconstrained flexible robot

Consider a n d.o.f. robotic arm whose links are affected by distributed elasticity. A finite dimensional model of the robot can be obtained by truncating the modal expansion of the deflection to a finite number of assumed modes [15], [16] under the assumption of small deformation:

$$w_i(x,t)=\sum_{j=1}^{m_i} q_{fij}(t)\psi_{ij}(x) \tag{1}$$

where w_i is the deflection of link i at time t, computed at a distance x from the origin of a suitable reference frame attached to the link, ψ_{ij} is the shape assumed for the j-th mode of link i, while q_{fij} is its time-varying amplitude. The number of modes retained from the asymptotic expansion is denoted by m_i.

Lagrange's equations of motion of the system can be obtained considering as a set of generalized coordinates, the rigid joint coordinates $\boldsymbol{q}_r \in \Re^n$ and the flexible variables $\boldsymbol{q}_f = (q_{f11}, \ldots, q_{f1m_1}; q_{f21}, \ldots, q_{f2m_2}; q_{fn1}, \ldots, q_{fnm_n})^T \in \Re^{N-n}$:

$$
M(q_r,q_f)\begin{bmatrix}\ddot{q}_r\\ \ddot{q}_f\end{bmatrix}+\begin{bmatrix}h_r(q_r,\dot{q}_r,q_f,\dot{q}_f)\\ h_f(q_r,\dot{q}_r,q_f,\dot{q}_f)\end{bmatrix}+\begin{bmatrix}g_r(q_r,q_f)\\ g_f(q_r,q_f)\end{bmatrix}+ \begin{bmatrix}\mathbf{0}_{n,N-n}\\ Kq_f\end{bmatrix}=\begin{bmatrix}B_r^T(q_r,q_f)\\ B_f^T(q_r,q_f)\end{bmatrix}u \tag{2}
$$

where:

$$
M=\begin{bmatrix}M_{rr} & M_{rf}\\ M_{rf}^T & M_{ff}\end{bmatrix}
$$

is the symmetric positive definite inertia matrix of the robot, conveniently partitioned into the matrices $M_{rr} \in \Re^{n\times n}$, $M_{rf} \in \Re^{n\times(N-n)}$, $M_{ff} \in \Re^{(N-n)\times(N-n)}$ and $M_{rf}^T \in \Re^{(N-n)\times n}$; h_r and h_f are the vectors of Coriolis and centrifugal terms, while g_r and g_f the vector of gravitational terms (for the rigid and the flexible parts, respectively); K is the matrix (diagonal and positive definite) of the stiffness constants; $u \in \Re^n$ is the vector of the control inputs (assuming as many control inputs as rigid d.o.f.); B_r and B_f are the matrices that reflect the action of the control inputs u on the equations of the rigid and flexible variables, respectively. Notice that the expressions of matrices B_r and B_f depend on the boundary conditions adopted in the definitions of the mode functions: in case of clamped-free boundary conditions [15], with the links clamped to the actuators hubs, B_r is the identity matrix, while B_f is a null matrix; in case of pinned-pinned or pinned-free boundary conditions, B_f is a constant non null matrix. With complex transmission systems, both the matrices depend, in general, on the values of the rigid and flexible coordinates.

2.2 Constraints

Assume now that the tip of the robot makes contact with a very stiff environment. A convenient way to represent this situation is to write as many constraint equations as the number of d.o.f. inhibited by the interaction with the environment. The constraint equations are easily written in terms of the Cartesian coordinates of the tip of the robot, in a suitable reference frame. However, by way of the direct kinematics of the robot, we can always assume the constraint equations as written in terms of the above defined rigid joint coordinates q_r and flexible variables q_f:

$$
\Phi(q_r,q_f)=\mathbf{0}, \tag{3}
$$

where Φ: $(\Re^n \times \Re^{N-n}) \rightarrow \Re^m$, being m the number of constraints ($m\leq n$).

Defining now the two Jacobian matrices:

$$
A_r=\frac{\partial\Phi}{\partial q_r}, \qquad A_f=\frac{\partial\Phi}{\partial q_f},
$$

where $A_r \in \Re^{m\times n}$, $A_f \in \Re^{m\times(N-n)}$, and recalling that the constraint forces act along the normals to the constraint surfaces, we can rewrite eq.(2), in case of constrained motion, as:

$$
M(q_r,q_f)\begin{bmatrix}\ddot{q}_r\\ \ddot{q}_f\end{bmatrix}+\begin{bmatrix}h_r(q_r,\dot{q}_r,q_f,\dot{q}_f)\\ h_f(q_r,\dot{q}_r,q_f,\dot{q}_f)\end{bmatrix}+\begin{bmatrix}g_r(q_r,q_f)\\ g_f(q_r,q_f)\end{bmatrix}+ \begin{bmatrix}\mathbf{0}_{n,N-n}\\ Kq_f\end{bmatrix}=\begin{bmatrix}B_r^T(q_r,q_f)\\ B_f^T(q_r,q_f)\end{bmatrix}u+\begin{bmatrix}A_r^T(q_r,q_f)\\ A_f^T(q_r,q_f)\end{bmatrix}\lambda \tag{4}
$$

where $\lambda \in \Re^m$ is a vector of Lagrange multipliers.

It is worth noting that the presence of the constraints does not involve any particular concern on the choice of the mode shapes ψ_{ij}, as long as an assumed modes technique is used. The assumed modes technique, in fact, requires that the mode functions satisfy the *geometric* boundary conditions, while *natural* boundary conditions (i.e. boundary conditions involving the balance of force and moments at the ends of the links) are automatically taken into account by the Lagrange formulation of the mathematical model [17]. Since the constraints on the motion of the tip of the arm do not alter the geometric boundary conditions, we will assume the same mode functions for the unconstrained dynamic model and the constrained one.

2.3 Model reduction

The mathematical model (4) is made up by N second-order differential equations, for a system that actually presents $N-m$ d.o.f., once the constraints (3) are active. It is however possible to reduce the number of differential equations, by resorting to a coordinate partitioning procedure [18],[19],[20]. Consider the following partition of the vector q_r:

$$
q_r=\begin{bmatrix}q_{r1}\\ q_{r2}\end{bmatrix}, \tag{5}
$$

where $q_{r1} \in \Re^m$, $q_{r2} \in \Re^{n-m}$, and assume that there exist a continuous, twice differentiable function Ω: $(\Theta_{r2}\times \Theta_f) \rightarrow \Re^m$, where Θ_{r2} and Θ_f are two open sets ($\Theta_{r2}\subset\Re^{n-m}$, $\Theta_f\subset\Re^{N-n}$), such that the constraints (3) can be expressed as:

$$
q_{r1}=\Omega(q_{r2},q_f). \tag{6}
$$

A reordering of the rigid variables q_r could be necessary to express the constraints as in (6). Also, note that the dependent variables q_{r1} have been chosen only among the rigid ones, thus implicitly excluding the presence of a constraint acting only on the flexible variables.

Differentiating (5) with respect to time, we have:

$$
\dot{q}_r=T_{rr}\dot{q}_{r2}+T_{rf}\dot{q}_f \tag{7}
$$

where:

$$T_{rr} = \begin{bmatrix} \dfrac{\partial \Omega}{\partial q_{r2}} \\ I_{n-m} \end{bmatrix} = \begin{bmatrix} -A_{r1}^{-1} A_{r2} \\ I_{n-m} \end{bmatrix},\ T_{rf} = \begin{bmatrix} \dfrac{\partial \Omega}{\partial q_f} \\ 0_{n-m,N-n} \end{bmatrix} = \begin{bmatrix} -A_{r1}^{-1} A_f \\ 0_{n-m,N-n} \end{bmatrix}$$

and the Jacobian matrix A_r has been partitioned as:

$$A_r = [A_{r1} \quad A_{r2}] = \begin{bmatrix} \dfrac{\partial \Phi}{\partial q_{r1}} & \dfrac{\partial \Phi}{\partial q_{r2}} \end{bmatrix}.$$

Introducing matrix T:

$$T = \begin{bmatrix} T_{rr} & T_{rf} \\ 0_{N-n,n-m} & I_{N-n} \end{bmatrix},$$

and noting that:

$$[A_r \quad A_f]T \equiv 0 \ , \tag{8}$$

it is possible to eliminate the Lagrange multipliers λ from the dynamic equations (4) by premultiplying the said equations by matrix T^T. Exploting the expressions (7) and its derivative, we finally arrive at the expression of the constrained dynamic system in terms of the independent variables q_{r2} and q_f:

$$\begin{gathered} M_c(q_{r2}, q_f)\begin{bmatrix} \ddot{q}_{r2} \\ \ddot{q}_f \end{bmatrix} + \begin{bmatrix} h_{cr}(q_{r2}, \dot{q}_{r2}, q_f, \dot{q}_f) \\ h_{cf}(q_{r2}, \dot{q}_{r2}, q_f, \dot{q}_f) \end{bmatrix} + \\ \begin{bmatrix} g_{cr}(q_{r2}, q_f) \\ g_{cf}(q_{r2}, q_f) \end{bmatrix} + \begin{bmatrix} 0_{n-m,N-n} \\ Kq_f \end{bmatrix} = \begin{bmatrix} B_{cr}^T(q_{r2}, q_f) \\ B_{cf}^T(q_{r2}, q_f) \end{bmatrix} u \end{gathered} \tag{9}$$

where:

$$M_c = \begin{bmatrix} M_{crr} & M_{crf} \\ M_{crf}^T & M_{cff} \end{bmatrix}$$

and

$$\begin{gathered} M_{crr} = T_{rr}^T M_{rr} T_{rr}, \quad M_{crf} = T_{rr}^T M_{rr} T_{rf} + T_{rr}^T M_{rf}, \\ M_{cff} = M_{ff} + T_{rf}^T M_{rr} T_{rf} + T_{rf}^T M_{rf} + M_{rf}^T T_{rf} \end{gathered},$$

$$h_{cr} = T_{rr}^T h_r + T_{rr}^T M_{rr} \dot{T}_{rr} \dot{q}_{r2} + T_{rr}^T M_{rr} \dot{T}_{rf} \dot{q}_f,$$

$$h_{cf} = T_{rf}^T h_r + h_f + T_{rf}^T M_{rr} \dot{T}_{rr} \dot{q}_{r2} + M_{rf} \dot{T}_{rr} \dot{q}_{r2} + T_{rf}^T M_{rr} \dot{T}_{rf} \dot{q}_f$$

$$g_{cr} = T_{rr}^T g_r, \quad g_{cf} = T_{rf}^T g_r + g_f$$

$$B_{cr}^T = T_{rr}^T B_r^T, \quad B_{cf}^T = T_{rf}^T B_r^T + B_f^T.$$

An expression for the Lagrange multipliers λ in terms of the state variables can be obtained by twice differentiating the constraint equations (3) [18]:

$$A_r \ddot{q}_r + \dot{A}_r \dot{q}_r + A_f \ddot{q}_f + \dot{A}_f \dot{q}_f = 0,$$

and eliminating the vector of the acceleration from eq. (4). The result is:

$$\begin{gathered} \lambda = (AM^{-1}A^T)^{-1}\left(-\dot{A}\begin{bmatrix} T_{rr}\dot{q}_{r2} + T_{rf}\dot{q}_f \\ \dot{q}_f \end{bmatrix} + \right. \\ \left. AM^{-1}\left(\begin{bmatrix} h_r \\ h_f \end{bmatrix} + \begin{bmatrix} g_r \\ g_f \end{bmatrix} + \begin{bmatrix} 0_{n,N-n} \\ Kq_f \end{bmatrix} - \begin{bmatrix} B_r^T \\ B_f^T \end{bmatrix} u \right) \right) \end{gathered}, \tag{10}$$

where $A=[A_r \quad A_f]$.

As a result of the coordinate partitioning method, eq. (9) is formally identical to (2): this is in contrast with other reduction methods. In the SVD approach [6] the dynamic model would be expressed in terms of a position vector, combination of the rigid and flexible coordinates. Thus the separation between the rigid and the flexible dynamics, essential in the following developments, would be lost. Moreover, the SVD reduction can be consistently applied only when the constraints are linear or linearized around a reference position.

3. A SINGULARLY PERTURBED VERSION OF THE MODEL

A very reasonable way to approach the control problem of system (9) consists in separating the slow dynamics, associated with the rigid motion of the robot, from the fast dynamics related to the link flexibility. Singular perturbation theory [21] gives the tools to accomplish this task. Following [22], the first step consists in defining the singular perturbation parameter as $\mu = 1/k$, where k is a common factor among the stiffness constants of the arm (elements of matrix K), say the smallest stiffness constants. New variables are then introduced as:

$$\zeta = Kq_f = k\hat{K}q_f \ ,$$

with $\hat{K} = K/k$. Defining the inverse of the inertia matrix of the constrained system as:

$$H_c = M_c^{-1} = \begin{bmatrix} H_{crr} & H_{crf} \\ H_{cfr} & H_{cff} \end{bmatrix},$$

where H_{crr}, H_{crf}, H_{cff} and $H_{cfr} = H_{crf}^T$ have the same dimensions as M_{crr}, M_{crf}, M_{cff} and M_{crf}^T respectively, it is possible to rewrite system (9) in the following singularly perturbed form:

$$\ddot{q}_{r2} = -H_{crr}[h_{cr} + g_{cr}] - H_{crf}[h_{cf} + g_{cf} + \zeta] + [H_{crr}B_{cr}^T + H_{crf}B_{cf}^T]u \tag{11}$$

$$\mu\ddot{\zeta} = -\hat{H}_{cfr}[h_{cr} + g_{cr}] - \hat{H}_{cff}[h_{cf} + g_{cf} + \zeta] + [\hat{H}_{cfr}B_{cr}^T + \hat{H}_{cff}B_{cf}^T]u \tag{12}$$

where $\hat{H}_{cfr} = \hat{K}H_{cfr}$ and $\hat{H}_{cff} = \hat{K}H_{cff}$. Observe that in this paper the singularly perturbed model is derived based upon the reduced order model (9) of the constrained mechanical system, rather than on the original constrained model (4). This will lead to some interesting consequences in the following developments

In the limit as $\mu \rightarrow 0$, eq. (12) collapses to the following algebraic equation:

$$\begin{gathered} \bar{\zeta} = \hat{H}_{cff}^{-1}(\bar{q}_{r2}, 0)\left[-\hat{H}_{cfr}(\bar{q}_{r2}, 0)\left[h_{cr}(\bar{q}_{r2}, \dot{\bar{q}}_{r2}, 0, 0) + \right.\right. \\ \left.\left. + g_{cr}(\bar{q}_{r2}, 0) - B_{cr}^T(\bar{q}_{r2}, 0)\bar{u} \right] \right] - \\ h_{cf}(\bar{q}_{r2}, \dot{\bar{q}}_{r2}, 0, 0) - g_{cf}(\bar{q}_{r2}, 0) + B_{cf}^T(\bar{q}_{r2}, 0)\bar{u} \end{gathered} \tag{13}$$

(the overbars denote that all the variables are evaluated in the special case μ=0). By plugging this equation into (11), with μ=0, the equations of the rigid robot model are obtained, as it can be proven: thus the slow dynamics of the system is readily identified as the dynamics of the rigid system.

To reveal the fast dynamics, we first introduce the so called fast variables:

$$\eta_1 = \zeta - \bar{\zeta}, \quad \eta_2 = \varepsilon\dot{\zeta}$$

where $\varepsilon = \sqrt{\mu}$, and then the fast time scale $\tau = t/\varepsilon$. Rewriting the system in this time scale, and examining it for ε=0, it is easy to conclude that system (11) confirms that q_{r2} and $\dot{q}_{r2}$ are constant on the boundary layer, while the expression of the fast dynamics can be obtained by combining eq. (12) and (13). The result is:

$$\frac{d\eta_1}{d\tau} = \eta_2$$

$$\frac{d\eta_2}{d\tau} = -\hat{H}_{cff}(\bar{q}_{r2},0)\eta_1 + \qquad .(14)$$

$$\left[\hat{H}_{cfr}(\bar{q}_{r2},0)B_{cr}^T(\bar{q}_{r2},0) + \hat{H}_{cff}(\bar{q}_{r2},0)B_{cf}^T(\bar{q}_{r2},0)\right](u - \bar{u})$$

The expression (14) found for the fast dynamic system differs from the corresponding expression for the unconstrained flexible robot, as reported in [22], where B_r=I and B_f=0. Note, in fact, that all the matrices appearing in (14) depend on the constraint equations through the relations worked out in the previous Section. In other works dealing with singularly perturbed models of constrained flexible robots, such as [10], [11] the same expression is found for constrained as for unconstrained motion. This is due to the fact that, in these works, the singularly perturbed model is derived directly from the constrained equations (2), still containing the Lagrange multipliers. However, when computing the fast dynamic system, the fact that the Lagrange multipliers depend on both the slow and the flexible variables, as it is apparent from eq. (10), is ignored. Taking into account this essential fact, an additional term, related to difference between the expression of λ computed at a generic ζ and the expression of λ computed at $\zeta = \bar{\zeta}$, would appear, which, in turn, would lead to the expression (14). However, the same expression can be derived in a much more straightforward way as shown above, i.e. by first deriving the motion equations of the constrained system in the residual d.o.f. and then applying the singular perturbation decomposition.

4. A TWO-TIME SCALE FORCE/POSITION CONTROLLER

The model developped in the previous Sections is suitable for the design of a force/position control law. The separation between the slow (rigid) system and the fast system actually suggests a similar separation in the control action. The composite control strategy pursues this goal by splitting the control action as:

$$u = \bar{u}(\bar{q}_{r2}, \dot{\bar{q}}_{r2}) + u_f(\bar{q}_{r2}, \eta_1, \eta_2),$$

with $u_f(\bar{q}_{r2}, 0, 0) = 0$. The signal $\bar{u}$ is responsible for the control of the slow subsystem, while the remaining part of the control vector u_f is designed to control the fast system dynamics, while being inactive along the solutions of the slow subsystem.

The goal of the slow control action $\bar{u}$ is to make the tip of the robot track a prescribed trajectory while maintaining a desired force contact with the environment. The controller is designed based on the model of the rigid robot. The position controller is a decentralized PD controller plus a gravity compensation term, the same used in unconstrained motion:

$$\bar{\tau} = K_P(q_{dr} - \bar{q}_r) + K_D(\dot{q}_{dr} - \dot{\bar{q}}_r) + \bar{g}_r(\bar{q}_r) \qquad (15)$$

where K_P and K_D are the matrices of the proportional and derivative gains of the PD controllers, respectively, $q_{dr} \in \Re^n$ is the vector of the position set-points, $\bar{g}_r(\bar{q}_r) = g_r(\bar{q}_r, 0)$, and $\bar{\tau} = B_r^T(\bar{q}_r, 0)\bar{u}$.

The force control action is exerted through additional position setpoints q_{dr}^F, and is designed so as to be active along the directions constrained by the environment, without affecting the motion control. Moreover, we will specify a zero steady state error between the setpoints λ_d and the Lagrange multipliers $\bar{\lambda}$. All the above requirements are satisfied if the force controller is designed based on the scheme of Fig. 1 [14]:

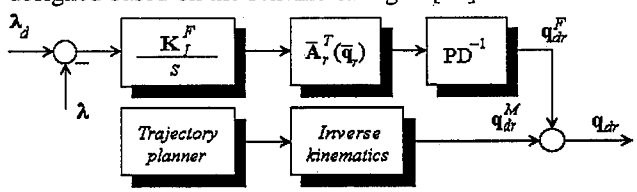

Fig.1: sketch of the force controller

where K_I^F is a diagonal matrix and $\bar{A}_r(\bar{q}_r) = A_r(\bar{q}_r, 0)$.

The fast controller must be designed so as to stabilize the fast system dynamics expressed by (14). A reasonable way to achieve this goal is to design a state-space control law as:

$$u_f(\bar{q}_{r2}, \eta_1, \eta_2) = K_1(\bar{q}_{r2})\eta_1 + K_2(\bar{q}_{r2})\eta_2 \ . \qquad (16)$$

The computation burden, necessary to tune the feedback matrices K_1 and K_2 for every configuration $\bar{q}_{r2}$, can be avoided [22] by tuning the two matrices with reference to a given configuration and using the same matrices throughout the whole task, provided that the closed loop fast system will not go unstable along the slow trajectory.

The overall controller [14] is finally obtained as:

$$u = \left[\bar{B}_r^T(\bar{q}_{r2})\right]^{-1}\bar{\tau}\left(\bar{q}_{r2},\dot{\bar{q}}_{r2}\right)+u_f\left(\bar{q}_{r2},\eta_1,\eta_2\right) ,$$

where $\bar{\tau}$ is given by (15), while u_f is given by (16).

5. SIMULATION RESULTS

The robot modelled in the simulations is RALF (Robotic Arm Large and Flexible), an experimental arm designed and built at the School of Mechanical Engineering of Georgia Tech (Fig.2). Table 1 summarizes the physical parameters used in the simulation of RALF.

	Link 1	Link 2
Length (m)	3.048	3.048
Cross sectional area (m^2)	$2.773\ 10^{-3}$	$2.041\ 10^{-3}$
Density (Kg/m^3)	2707.	2707.
Young's module (N/m^2)	$7.1\ 10^{10}$	$7.1\ 10^{10}$
Moment of inertia (m^4)	$6.308\ 10^{-6}$	$3.002\ 10^{-6}$

Table 1: RALF parameters

Two modes for each link are used to model the flexibility of the arm. Pinned-pinned boundary conditions [1] are considered:

$$\psi_{ij}(x) = \sin\left(\frac{j\pi x}{L_i}\right) ,$$

L_i being the length of each link. The main reason for using pinned-pinned boundary conditions is that the tip position depends only on the joint coordinates and not on the flexible variables, which simplifies the expression of the constraint, in case of contact with the environment. On the other hand, the control vector $\boldsymbol{u}$, see eq. (2), directly affects all the equations of motion, through matrices $\boldsymbol{B}_r$ and $\boldsymbol{B}_f$, while with clamped-free boundary conditions, for example, matrix $\boldsymbol{B}_f$ is null, provided that the actuator torques act along the rigid joint coordinates. Notice, however, that in RALF the control vector $\boldsymbol{u}$ consists in the forces exerted by the hydraulic actuators, which obviously do not act along any joint coordinates. As a consequence, matrices $\boldsymbol{B}_r$ and $\boldsymbol{B}_f$ are full, whatever boundary conditions are chosen for the flexible modes and, in addition, they depend on both the joint and the flexible variables. A possible way to derive this dependence is to first compute the kinematic relation between the actuator lengths and the joint and flexible variables, and then derive $\boldsymbol{B}_r$ and $\boldsymbol{B}_f$ as the Jacobians of this relation with respect to the rigid and flexible variables, respectively. This is done in the model of RALF, and the said kinematic relation is found adopting some simplifying assumptions on the transmission mechanism of the second link.

Fig. 2: The experimental arm RALF

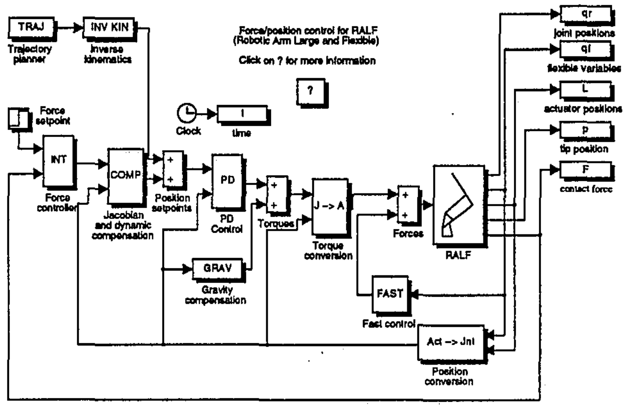

Fig. 3: Simulation plant

Fig. 3 reports the Simulink© layout of a simulation plant obtained closing the model of Section 2 with the controller of Section 4. It is worth noticing that the equations of motion of the constrained system have been directly simulated, by solving the constraint with respect to one joint variable. The robot is considered in contact with a rigid vertical surface, orthogonal to the plane of the robot links. The contact point is ($x = -3.6$ *m*; $y = 2.5$ *m*), (x, y) being a coordinate system in the plane defined by the two links, with origin located at joint 1 and the y axis vertical. In this configuration, the fast system presents four pairs of imaginary eigenvalues at frequencies 288 *rad/s*, 327 *rad/s*, 1070 *rad/s* and 1089 *rad/s* (no damping is introduced in the robot model). These values have been checked by comparison with experimental results.

In a first set of simulations, the robot is initially at rest, in contact with the external surface, with the force control loop open. The initial value of the force is determined by the values assumed by the state variables at this equilibrium point. At time t=0, the force control loop is closed, with the force setpoint set to 50 *N*. Fig. 4 shows the force response with both the slow and the fast control closed, while in the simulation of Fig.5 the fast control has been left open: both the stable response of Fig. 4 (with the initial nonminimum phase inverse response) and the unstable response of Fig. 5 are consistent with the theory.

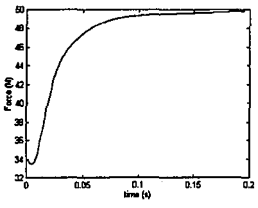

Fig 4: Force response with slow and fast control

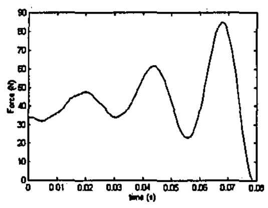

Fig. 5: Force response with only slow control.

In a second set of simulations, the robot moves while in contact with the surface, with the force setpoint kept at the initial value. The commanded trajectory is a vertical downward segment, 0.2 *m* long, with a symmetric trapezoidal velocity profile: the maximum velocity is 0.15 *m/s*, while the acceleration in the initial and final parts is 0.2 m/s^2. Fig. 6 shows the force history during the trajectory tracking. The force controller keeps the error under ± 0.5 *N*, and recovers the errors due to the discontinuities in the acceleration profile. Finally, Fig. 7 shows the trajectory tracking error, computed as the difference between the commanded and the simulated Cartesian positions along the vertical directions (the horizontal one being constrained). The steady-state error (less than 0.4 *mm*) is due to the imperfect compensation of the steady state gravitational disturbance caused by the arm flexibility.

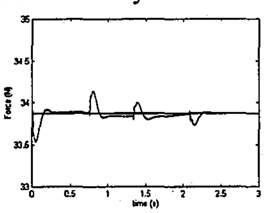

Fig 6: Force history during trajectory tracking

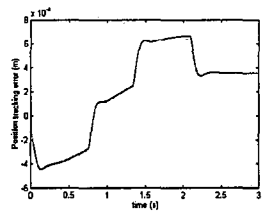

Fig 7: Trajectory tracking error

6. CONCLUSIONS

The model of a flexible robot for force/position control has been addressed and extensively discussed in this paper. A model reduction obtained via coordinate partitioning has allowed to derive a general formulation of the dynamic equations for a constrained flexible manipulator, which had not yet been presented in the literature. Moreover, a correct formulation of the singularly perturbed version of the model, whose fast subsystem has a different expression than in unconstrained motion, has been presented.

ACKNOWLEDGEMENTS

This work was done while Paolo Rocco was a visiting scholar at Georgia Tech, sponsored by the Italian National Research Council (CNR) under NATO Grant No. 215.27/11-216.1865.

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