APPLICATION OF WATER-CHANNEL COMPRESSIBLE GAS ANALOGIES TO A PROBLEM OF SUPERSONIC WIND TUNNEL DESIGN

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APPLICATION OF WATER-CHANNEL COMPRESSIBLE

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LIST OF SYMBOLS

W	Velocity of flow in air
V	Velocity of flow in water
a	Velocity of sound
a^*	Critical speed of sound
M	Mach number W/a, V/a
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
γ	Ratio of specific heats c_p/c_v
A	Area
A^*	Nozzle throat area
θ	Prandtl-Meyer angle
ρ	Density
p	Pressure
β	Wave angle
T	Absolute temperature of gas
d	Water depth
g	Acceleration of gravity

Subscripts

No subscript	Any value of variable
0	Value at stagnation
L	Local value of variable
t	Value at nozzle throat
1	Conditions before the hydraulic jump
2	Conditions after the hydraulic jump

APPLICATION OF WATER CHANNEL COMPRESSIBLE GAS ANALOGIES TO PROBLEMS OF SUPERSONIC WIND TUNNEL DESIGN

SUMMARY

A small supersonic wind tunnel of the blowdown type and a water channel were used to determine the applicability of the compressible gas analogy to one phase of the design of supersonic nozzles.

A model of the two-dimensional supersonic nozzle was tested in the water channel; and by applying the hydraulic analogy, the pressure distribution along the nozzle was obtained and compared with pressures measured in the wind tunnel. Pictures were taken of the water flow in the nozzle and compared with schlieren photographs of the air flow in the wind tunnel.

The results obtained show good agreement between the water channel flow and that of the wind tunnel. Local variations of contour resulted in disturbance of the flow, this was observed in both the water channel and the wind tunnel. Upon modification of the contours, similar and improved flow was observed in both. This demonstrated the application of water channel testing of wind tunnel shapes as a preliminary step in supersonic wind tunnel design.

INTRODUCTION

Supersonic wind tunnels of today are very complex mechanisms and are expensive to construct and modify; therefore, just as models of airplanes are tested, modified, and retested, it appears to be feasible to test models of supersonic wind tunnels and modify the design before constructing the actual tunnel. A first step in this problem is consideration of the supersonic nozzle.

It may be first thought desirable to test a three-dimensional model of a supersonic wind tunnel; however, since a well-designed wind tunnel has essentially one-dimensional flow,[1] it is sufficient to test a two-dimensional model. Now, to visualize the flow in the wind tunnel, a schlieren system can be used. The pressure distribution may be obtained by connecting openings along the tunnel to a large group of manometers. However, an excellent schlieren system is a very delicate piece of apparatus and quite costly. Manometers tend to fluctuate rapidly, and are known to have a considerable lag if an appreciable length of connecting tubing is necessary.

Considering these points, it appeared that a model of the supersonic wind tunnel design or any portion of the design could be placed in a water channel where the flow could be easily visualized and photographed. Also, the pressures could be determined by the hydraulic analogy which was presented first on a mathematical basis in 1932 by Riabouchinsky[2] and used to investigate the flow in a Laval nozzle.

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[1] Harold Sibert, High-Speed Aerodynamics (New York, Prentice-Hall, Inc., 1948) p.84

[2] Riabouchinsky, D., Mecanigno des fluides. Comptes Rendrs, t. 195, No.22, November 28, 1932, pp. 998-999

Binnie and Hooker[3] further investigated the hydraulic analogy, and Ernst Preiswerk[4] conclusively proved that water flow with a free surface can be applied to the theory of gas dynamics.

North American Aviation, Incorporated, recognized the possibilities of extending experimental compressibility research by use of the hydraulic abalogy and ran numerous tests.[5] On the basis of their recommendations, Hatch[6] built a water channel at the Georgia Institute of Technology and conducted tests on supersonic airfoil profiles. His work has been extended by Catchpole.[7] The water channel tests for this thesis were conducted in the equipment mentioned above.

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[3] Binnie, A.M., and S. G. Hooker, "The Flow Under Gravity of an Incompressible and Inviscid Fluid Through a Constriction in a Horizontal Channel," Proceedings of the Royal Society. Vol. 159, (London, England: 1937), pp. 592-608.

[4] Preiswerk, Ernst., "Application of the Methods of Gas Dynamics to Water Flows with Free Surface."
Part I. "Flows with No Energy Dissipation." NACA TM. No.934, 1940.
Part II."Flows with Momentum Discontinuities." NACA TM.No.935, 1940.

[5] Bruman, J.R., "Application of the Water Channel Compressible Gas Analogy." (North American Aviation, Incorporated, Engineering Report, No. NA-47-48, 1947).

[6] Hatch,J.E., "The Application of the Hydraulic Analogies to Problems of Two-Dimensional Compressible Gas Flow," (Unpublished Master's Thesis, Georgia Institute of Technology, Atlanta,1949).

[7] Catchpole, J.E., "Application of the Hydraulic Analogy to Study the Performance of Several Airfoils in Compressible Flow," (Unpublished Master's Thesis, Georgia Institute of Technology, Atlanta, 1949).

THEORY

The problem of the motion of fluids which is already sufficiently involved even when considered as incompressible, becomes still further complicated and more difficult when the property of compressibility is taken into account. In the majority of cases, therefore, when compressibility is to be allowed for, it is necessary to make simplifying assumptions in other directions. Therefore, in the air flow of this paper, viscosity has been neglected and the fluid considered to be frictionless and compressible. The further assumptions are made that the density of the compressible fluid depends on the pressure only and such inhomogeneities resulting from the heat conducted from the fluid by the surroundings are excluded. A rigorous development considering these aerothermodynamic problems has been given by Bailey.[8] The flow is considered to be irrotational, since the state of rest is a special case of irrotational motion and any compressible fluid flow starting from rest is an irrotational flow.[9]

The "method of characteristics", first applied to supersonic flows by Prandtl and Buseman,[10] provides a means of computing the flow through a given two-dimensional supersonic nozzle. It also provides a means of

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[8] Bailey, N.P., "The Thermodynamics of Air at High Velocities," Journal of the Aeronautical Sciences, Vol. II, No.3, pp.227-238, July, 1944.

[9] Prandtl, L., "General Considerations on the Flow of Compressible Fluids," U.S.National Advisory Committee for Aeronautics, Technical Memorandum No.805, October 1936, 25 pp.

[10] Prandtl, L. and A. Buseman, "Nahrungsverfahren zur zeichnerische Ermittlung von ebenen Stromungen mit Uberschall geschiwindigkeit," Stodola Festschrift, Zurich, 1929, p.499.

determining the shape of the walls of a supersonic nozzle in such a manner that the velocity at the end of the nozzle is uniform and parallel across the entire section. A similar but more complicated method has been developed for three-dimensional flows.[11]

A general supersonic flow field can be imagined to consist of an infinite number of small disturbance waves (see Figure 5). The fundamental idea of the method of characteristics is to replace the infinite number of waves by a finite number which can then be treated separately. In the areas between waves, the velocity magnitude and direction are considered constant. Therefore, it is necessary to replace the actual boundaries of the flow by approximate boundaries with a finite number of disturbances. That is, a curved boundary is replaced by a number of straight line segments with a definite angle between each.

The effect of the wave produced by the wall bend in Figure 13 is to deflect the flow through an angle d . It also produces a change in the speed and pressure which are determined only by d . The mathematical treatment necessary in the computation of the flow changes produced by a single disturbance wave has been given by Puckett.[12] These equations may be computed from the equations of motion for the fluid in differential form. If W_n is the component of W_1 normal to the wave, then conservation of mass for the flow across the wave requires that

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[11] Sauer, Edward, "Einfuhrung in die Gasdynamik," Ann Arbor, Michigan, 1945, p. 137.

[12] Puckett, A.E., "Supersonic Nozzle Design," Journal of Applied Mechanics, Vol. 13, No.4, December 1946. P. A-265.

$$\rho W_n = (\rho + d\rho)(W_n + dW_n) \tag{1}$$

or

$$W_n d\rho + \rho dW_n = 0$$

Let W_t be the component of W_1 parallel to the wave front; the conservation of momentum parallel to the wave requires that

$$\rho W_n W_t = (\rho + d\rho)(W_n + dW_n)(W_t + dW_t) \tag{2}$$

or

$$dW_t = 0$$

That is, the only velocity increment produced by the wave is normal to it. The momentum component normal to the wave is conserved if

$$\rho W_n dW_n + dp = 0 \tag{3}$$

Eliminating dW_n between Equations (1) and (3) gives

$$W_n^2 = \frac{dp}{d\rho} = a^2 \tag{4}$$

where "a" is the local speed of sound.

Thus, the velocity component normal to the wave is the speed of sound. Use of the energy equation will show that the process across the wave is isentropic, and $dp/d\rho$ is computed using the isentropic law

$$\frac{p}{\rho^\gamma} = constant \tag{5}$$

Since $W_n = W \sin\beta$ where β is the wave angle, we have

$$\sin\beta = \frac{a}{W} = \frac{1}{M} \tag{6}$$

where M is the local Mach number. The wave angle β is then called the "Mach angle". The increase in the velocity W_1 is given by

$$dW = dW_n \sin\beta \tag{7}$$

while the change in flow direction must be equal to

$$d\theta = \frac{dW_n \cos\beta}{W} \tag{8}$$

From Equations(7) and (8), the relation between increase in speed and change in flow direction is immediately found

$$\frac{dW}{W} = \cancel{\frac{d\theta}{\sqrt{M^2-1}}} = \tan\beta\, d\theta$$

or using Equation (6)

$$\frac{dW}{W} = \frac{d\theta}{\sqrt{M^2-1}} \tag{9}$$

This Equation is the only result necessary to perform the step-by-step computation through any number of waves. It must be observed, however, that M is a function of W which must include the variation in the speed of sound, a, with W. The Mach number and W are related by[13]

$$\left(\frac{W}{a^*}\right)^2 = \frac{\gamma+1}{\gamma-1+\frac{2}{M^2}} \tag{10}$$

where a* is the "critical speed of sound", a constant for the flow. This may be deduced from the energy equation for the fluid.[14] If this relation is used in Equation (9) with $\overline{W} = W/a^*$, then

$$d\theta = \sqrt{\frac{\overline{W}^2-1}{1-\frac{\gamma-1}{\gamma+1}\cdot\overline{W}^2}}\cdot\frac{d\overline{W}}{\overline{W}} \tag{11}$$

With Equation (11) the successive flow changes through a series of bends in a wall may be computed. If W_1 is given, the increment

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13 Sibert, H.W., "High-Speed Aerodynamics," Prentice-Hall, Inc., New York, N. Y. 1948, p. 23

14 Liepmann, H.W. and Puckett, A.E., "Introduction to Aerodynamics of a Compressible Fluid," John Wiley & Sons, Inc. New York, N.Y. 1946, Chapter 3.

producing W_2 and W_3 are obtained from Equation (11); it is necessary to compute only the Mach numbers corresponding to W_1, W_2, and W_3 in order to draw the Mach wave at the proper angles, as given by Equation (6). It should be pointed out that the waves described so far are "expansion waves," since the d shown will, according to Equations (9) and (3), produce a drop in pressure.

The evaluation of Equation (11) for every wave in the flow becomes a tedious computation, but may be avoided by noticing that Equation (11) may be integrated, resulting in a function $= f(W)$. Since W is also a function of M, an expression of as a function of M may be obtained; the explicit relation becomes

$$\theta = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left[\sqrt{\frac{\gamma-1}{\gamma+1}} \cdot \sqrt{M^2-1}\right] - \tan^{-1}\sqrt{M^2-1} \qquad 12$$

This is the relation obtained by Prandtl and Meyer, in computing the flow around a single sharp convex corner. The θ so defined is sometimes called the Prandtl-Meyer expansion angle. The function has been tabulated for small integral increments by Puckett[15] and by the staff of the Ames Aeronautical Laboratory.[16]

When two expansion waves intersect, they continue to have their same strength and increase the flow by $d\theta$. The angle which the wave makes with the flow is changed due to the fact that the wave has passed into a region of increased flow. The angle of the wave has to be re-

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15 Puckett, loc. cit.

16 "Notes and Tables for Use in the Analysis of Supersonic Flow", U.S. National Advisory Committee for Aeronautics, Technical Memorandum No. 1428, December 1947, 86 pp.

computed using the angles of the increased flow.

When a wave strikes a straight wall, it must be reflected in a manner so that the flow will again be parallel to the wall; hence, the reflected wave has the same strength as the wave striking the wall. Had the wall been deflected against the flow by an angle equal to the angle producing the wave, no reflection would occur, and the flow would be parallel to the tunnel wall.

A supersonic nozzle such as shown in Figure 14 can now be designed. The nozzle is considered to commence at the throat, across which it is assumed that the Mach number is uniformly 1; the validity of this assumption depends on the shape of the subsonic contraction section which must precede the throat. An expanding section follows and then a return to a parallel section at the end of the nozzle. It is the object of the nozzle design to produce a flow with no disturbance waves present at the exit.

The final nozzle shape should actually consist of a smooth curve faired through all the points of intersection of waves with the wall. This can be accomplished best on a plot with exaggerated vertical ordinates. The nozzle may be drawn symmetrically about the center line or the center line may be replaced by a solid wall.

A check on the overall graphical construction may be obtained by computing the ratio of the exit area A, produced by the construction, to the throat area A*, and comparing this with the theoretical area ratio deduced for one-dimensional flow, which is given by

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (13)$$

Since the flow in both throat and exit section is one-dimensional, these two area ratios should agree within the error of the graphic construction.

An approximation method for designing the nozzle may be employed which will reduce the labor involved. It will be noted from Figure 14 that the flow pattern at a section through the maximum point of expansion consists of nearly uniformly spaced fields with constant Mach number. Thus, the flow is almost uniformly radial at that section. Therefore, it appears feasible to design a nozzle by starting at the section of maximum expansion and assuming a uniform radial flow. This amounts to assuming the existence of uniformly spaced wave quadrilaterals with constant Mach number and flow direction varying uniformly. The number of waves which must be present is determined by the expansion angle used and is given by

$$m = \frac{\theta_o}{d\theta} \tag{14}$$

The design for the nozzle in Figure 14, starting at the section of maximum expansion and assuming uniform radial flow at that section, is shown in Figure 15. Using this design procedure, it is necessary to connect the maximum expansion point with a throat of the correct size by a smooth curve representing the expansion section. The correct throat size must be computed in this case from Equation (13).

If boundary layer growth and friction losses are likely to be objectionable, it is advantageous to use the shortest possible nozzle design. This point of view was predominant in the German supersonic wind tunnel designs. However, the longer a design, the less critical is its design insofar as producing a uniform final velocity is concerned. If the expansion angle is small enough, almost any smooth curve connecting the throat to exit section will produce a reasonably uniform final velocity. Therefore, it is conservative to use an expansion angle much smaller than the maximum.

The real flow consists of a core of nonviscous flow and a thin layer of retarded flow along the walls. It can be shown that the perfect fluid core behaves as though it were floating between walls which lie somewhere in the boundary layer inside the physical walls.[17]

The boundary layer in a supersonic nozzle gives the effect of reducing the area of the nozzle as the boundary builds up. Since the boundary layer builds up on all four sides of the nozzle, for the two-dimensional case, it is necessary to expand the top and bottom a sufficient amount to correct for the boundary layer on the four sides.

The Guggenheim Aeronautical Laboratory, California Institute of Technology, has made some theoretical estimates of the rate of boundary-layer growth and compared them with boundary-layer profiles measured in their 2.5 inch supersonic wind tunnel. They recommend that the rate of increase through a square test section is such that an expansion of the top and bottom walls by 0.007 to 0.010 inch per inch of length on each wall will roughly provide the necessary compensation for all four sides. These figures were found to be useful for Reynolds numbers from 5×10^5

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[17] Durand, W.F., "Aerodynamic Theory", Julius Springer, Berlin, 1935, Vol. 3, p. 89.

to 5×10^6, and for Mach numbers on the order of 1.5 to 2.5.[18]

For a nozzle having a cross-section other than square, the boundary layer correction has to be calculated on the basis of the perimeter of the cross-section compared to the perimeter of a square cross-section with sides equal to the width of the top and bottom walls.

If the Mach number and pressure are known at any point in the flow or if the pressure is known in the reservoir when M= 0, then the pressure at any point in the nozzle can be determined by

$$\frac{p_1}{p_2} = \left[\frac{2+(\gamma-1)M_2^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}} \qquad (15)$$

Therefore, a nozzle cam be designed to give speed greater than sound, and the Mach number and pressure at any point can be calculated.

The physical basis of the water channel compressible-gas analogy has been very thoroughly described by Preiswerk[19]. A summary of the analogous relationships between a two-dimensional compressible gas flow (γ=2) and incompressible liquid flow (water), is given below

Two-Dimensional Compressible Gas Flow, $\gamma = 2$	Incompressible Liquid Flow, Water
Temperature ratio, $\frac{T}{T_o}$	Water depth ratio, $\frac{d}{d_o}$
Density ratio, $\frac{\rho}{\rho_o}$	Water depth ratio, $\frac{d}{d_o}$
Pressure ratio, $\frac{p}{p_o}$	Square of depth ratio, $\left(\frac{d}{d_o}\right)^2$
Velocity of sound, $a = \sqrt{p\gamma}$	Wave velocity, $\sqrt{gd}$
Mach number, $\frac{V}{a}$	number, $\frac{V}{\sqrt{gd}}$
Shock wave	aulic jump

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[18] Puckett, lo

[19] Preiswerk,

Two-Dimensional Compressible Gas Flow $\gamma = 2$	Incompressible Liquid Flow, Water
Expansion	Level drop
Compression	Level rise
Expansion wave	Depression wave
Subsonic flow	Streaming water
Supersonic flow	Shooting water

Preiswerk has shown that the "method of characteristics" is applicable to water flow.[20] He also developed the relationships between conditions in front of the shock wave and behind the shock wave for the hydraulic analogy. This is known as the hydraulic jump and may be defined as an unsteady motion in which the velocity may strongly decrease for short distances and the water depth suddenly increase.

In developing the theory, the assumption is made that the water motion is entirely unsteady which indicates that the water jumps suddenly along a line from the lower water level to the level after the jump. An additional assumption to those previously made is that the vertical accelerations at the free surface are negligible compared to the acceleration of gravity. By this assumption, the pressures at any point in the fluid depend on only the height of the free surface above that point.

For the water flow without a jump, the energy equation holds

$$V^2 = 2g(d_{o_1} - d) \qquad (16)$$

where d_o the total head is a constant. After a hydraulic jump has taken place, the total head d_{o_2} is smaller than d_{o_1}, since part of the kinetic

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20 Ibid, I, 57-63

energy of the water is converted into heat and is treated as lost energy. For the flow after the jump, the energy equation again applies and now becomes

$$V_2^2 = 2g(d_{o2} - d) \tag{17}$$

To solve this equation for d_{o_2}, the new total head, use is made of the shock polar diagram to determine V_2, the water velocity behind the shock wave.

The equation for the shock polar for the hydraulic analogy is

$$\bar{V}_2 = \left[\bar{u}_1 - \bar{u}_2\right]\left[\bar{u}_2 - u_1 \cdot \sqrt{\frac{3-\bar{u}_1^2}{3-4\bar{u}_1\bar{u}_2+3\bar{u}_1^2}}\right] \tag{18}$$

By employing the shock polar diagrams and the equations given by Preiswerk, conditions across a shock wave may be determinded.

When the analogy is applied to the study of air flow, accurate quantitative results will not be obtained because, for strict agreement between water flow and gas flow, γ must equal 2.0, whereas for air, γ is 1.4. A given pressure ratio corresponds to a high mach number in air than in the fictitious gas with γ = 2.0. The differences are not very large, however, and the flow phenomena observed in the water flow should be qualitatively the same as those occurring in the two dimensional compressible flow of air.

EQUIPMENT

The equipment used for this project was a supersonic wind tunnel of the blow-down type and a water channel in which the model was moved through stationary water.

A general view of the supersonic wind tunnel is shown in Figure 1.

The supersonic wind tunnel was supplied with air by a "Worthington" ten horsepower compressor operating at a working pressure of 125 psi which was connected through approximately 150 feet of pipe to the storage tanks. Although there was a head loss due to considerable length of pipe, it gave the advantage of cooling the air heated by the compressor and allowing the moisture to condense out of the cooled air.

The storage tanks consisted of a 4.2 cubic foot tank and a 29.5 cubic foot tank. Before entering the small tank, the air from the supply line passed through an oil filter to remove any oil which was picked up from the compressor. The small tank provided a reservoir for water which had been condensed out of the air, and the water could be drained between operations by a valve at the bottom of the tank. The small tank also provided additional air storage to be used during the operation. The large tank had a spigot at the bottom for the purpose of draining any water which might condense.

The large tank was connected by a four-inch pipe to a high pressure balanced regulator valve which had a small regulator attached to it for the purpose of adjusting the balance pressure across the diaphragm.

The regulator could set from 10 psi to 100 psi with an accuracy of ± 1 psi.

A quick-opening gate valve was attached to the exit of the pressure regulator so that the four inch line could be opened fully or closed instantaneously. This gave full utilization of the stored air at the beginning of the run and prevented the loss of excess air at the conclusion of a run.

Immediately aft of the valve the wind tunnel commenced with a contraction section of relatively short length in order to prevent excessive boundary layer growth. This construction section reduced the four-inch circular section to a minimum section of 0.8 inch X 0.5 inch in a distance of 3.5 inches. This contraction was designed to give an approximately uniform velocity distribution with no boundary layer separation at the throat.[21]

Starting with the above mentioned minimum section, the supersonic wind tunnel or nozzle (shown in Figure 2 and Figure 4) was designed by the method of characteristics, described in the theory section, to give a maximum wall angle of seven degrees and produce a flow of M = 1.99. The expansion section was three inches aft of which was added a straight section one and one-half inches in length. After the nozzle was designed, the boundary layer correction as explained in the theory section was applied from the minimum section to the end of the test section. At the end of the test section, the air is expelled into the room.

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[21] Tsien, H. S., "On the Design of the Contraction Cone for a Wind Tunnel," Journal of the Aeronautical Sciences. Vol. 10, February, 1943, pp. 68-70.

For the purpose of calibrating the flow speed and also observing the flow, three cones were constructed, having semi-apex angles of 7.5, 10, and 15 degrees. These cones had a maximum diameter of 0.25 inch and were mounted on stings of 0.125 inch diameter. The cones and stings were turned down on a lathe out of quarter inch cold rolled steel. The sting was mounted in a wooden holder which could be adjusted up and down and could be moved fore or aft on a metal channel running parallel to the tunnel center line and five inches below the center line.

A schlieren system was used for visualizing the flow in the plexiglass enclosed test section. The light source used for the schlieren system was a 400 watt BH1 mercury vapor lamp, which received its power from an auto-transformer having a rated open secondary voltage of 270 volts. This lamp produced a line of light approximately a quarter inch wide and five inches long. The lamp was mounted on its side in the center of an aluminum box ten inches square and eighteen inches long. A slit of 3/16 inch width and 6.5 inches in length was cut in one side of the box for the passage of light. This box was mounted on an adjustable stand to permit proper alignment of the light and slit to the optical axis of the system.

The light then passed through two condensing lenses which narrowed the original beam of light down to a width of 1/16 inch and a length of 1.625 inches. At this point a very narrow slit one and one-half inches long was introduced to cut out all except the most intense light in the center. This slit was located at the focal point of a war surplus Kohad Aero Ektar, 13.5 inch, f/3.5 camera lens. The light which passed through the slit expanded to cover the lens and pass through it. At approximately one focal length of this lens, as can be seen in Figure 1, the test section was located perpendicular to the optical axis. The light rays passed through the test section and continued to another lens identical with the one described above but reversed in direction. This lens was located two focal lengths aft of the test section as this was found to be the minimum distance that would give a clear image of the test section. This lens caused the light rays to converge again so that at the focal length on the other side of the lens, the light had converged to a line the same size as the original slit. At this point a knife edge was introduced to cut out the bottom half of the line of light. The top half continued to a "Speed Graphic" camera which was mounted at a point where a clear image could be obtained with the lens removed. The camera used cut-film and the lens was removed so that the light shone directly on the film.

The whole schlieren system, light, lenses, slit, knife edge, and camera were mounted on a channel iron base so that the system could be moved in order to view any point along the tunnel.

In order to obtain pressure measurements, openings one-half inch apart were placed in the lower wall from the minimum section to the exit of the tunnel. These openings were connected to a multiple mercury manometer. The pressures were read from pictures of the manometer taken while the tunnel was running.

The water channel used for this project was of the type in which the water remained stationary and the model moved through the water. A general view of the water channel is shown in Figure 3.

The water channel is four feet wide and twenty feet long, having a framework made of structural steel bolted together. The bottom of the channel is plate glass one quarter of an inch thick and is supported every thirty inches by transverse members. The advantage of a glass bottom is for the purpose of underwater lighting as well as providing a smooth surface for the models to slide upon. The legs of the channel were made adjustable; and by the use of a surveyor's transit, the bottom was leveled to ± 0.01 inch. A small portion at one end of the channel had a metal bottom which contained a drain.

The model carriage is made of steel tubing welded together and has two support arms bolted to it which fit into holes in the model and hold it against the bottom of the channel preventing any flow beneath the model. The carriage has four rubber tire ball bearing wheels which restrain the carriage from lateral movement.

The carriage is pulled by a 3/32 inch diameter conventional steel aircraft cable. This cable is operated from the pulley of a "Speed-Ranger" constant speed device powered by one-quarter horse power, single phase, alternating current motor. The "Speed-Ranger" device provides speeds from 0.5 feet per second to 3.5 feet per second. A D.C. motor provides an alternate drive for higher speeds and extends the range to 5.5 feet per second.

An observation platform with a control panel mounted on it for operating the water channel is located at the channel test section. The observation platform gives access to the cameras as well as the controls so that any number of tests can be run without leaving the stand except to change the model.

For the purpose of photographing the flow through the model, a camera was mounted directly over the center of the channel in front of the observation platform. The best lighting for photographing the overhead pictures was to have a uniform white background, achieved by placing tracing paper immediately under the glass bottom and then placing a white surface on the floor underneath this section, lighting this white surface uniformly with photoflood lights. Also for this set-up, a uniform overhead lighting was achieved by placing a screen overhead and shining photofloods on to it. A single photospot was then used to shine directly at the center of the model and eliminate the shadows created by the sides of the model.

For photographing the local water depth along the model, a camera was mounted on the observation platform to an angle to prevent obstruction of the view by the near side of the model. The lighting for this arrangement consisted of a photoflood located on each side of the camera directed towards the center of the model.

The shutters of both the cameras were operated electrically at the correct time by a cam device mounted on the model carriage. The cam tripped a microswitch which operated solonoids connected to the camera shutters.

The exact speed of the carriage was determined by a timing device consisting of a cam of known length tripping a microswitch connected to an electric clock.

The model used for this project consisted of an exact reproduction of the wind tunnel section beginning with a portion just ahead of the contraction and continuing to the exit. The contours are shown in Figure 4 and Figure 9. The model was made in two pieces and joined overhead at the ends by aluminum supports. One side of the model was made considerably lower than the other to permit photographing of the local water heights.

The model was painted a flat black except for the insides which were painted a flat yellow to aid photographing. On the surface to be photographed a horizontal line was inked one-half inch from the bottom of the model for a reference line since the local water heights are scaled directly from a photograph, Vertical lines were also inked on the model at various positions from the minimum section to the exit.

PROCEDURE

The following procedure was used to obtain data from the supersonic wind tunnel: First, the 400 watt mercury vapor lamp was turned on and allowed to warm for approximately twenty minutes. At the same time, the valves were opened from the compressor line to allow the tanks to fill up with air. This required fifteen minutes if the tanks were at zero gauge pressure.

When the light had warmed sufficiently, the camera was focussed and set at the proper speed. Then as soon as the tanks reached the required pressure, the gate valve was opened; and then allowing 3 to 5 seconds for the flow to develop, the camera of the schlieren system was tripped. The camera photographing the manometer bank was tripped when the mercury became steady and the static tubes returned to their previous height.

Since the whole wind tunnel could not be photographed at one time by the schlieren system, it was then necessary to move the system so that the other half of the tunnel could be viewed.

In obtaining data from the water channel, the water height in the channel was adjusted first. The model was then attached to the carriage so that it moved at a zero angle of attack. By varying the control on the "Speed-Ranger", the proper speed was set. The appropriate lights were then turned on (depending on the pictures being overheads or side shots) as described in the equipment section. By placing the carriage in a position so that its attached cam just depressed the camera microswitch, the camera could then be focussed.

The carriage was moved down the channel and the camera was tripped automatically. The time was also recorded automatically. The lights were then turned on for the overhead picture and the camera set. The same switches tripped the overhead camera and recorded the time.

DISCUSSION

Considering the flow through the water channel model, it can be seen from Figure 9 that in the reduction section no distinct waves appear. There is, however, a uniform disturbance resulting from either vibration of the model or capillary waves which have to be disregarded. At the minimum section, the water definitely appears to become shooting water (M 1). As the expansion begins, expansion waves are seen to come from each side of the model and form a crisscross pattern. The crisscross pattern continues as the curvature is changed and the model wall begins to straighten out. The waves in this portion simulate shock waves and result from the surface turning against the flow. It is seen that the shock and expansion waves begin to cancel each other and there is very little of the crisscross pattern in the straight section at the end of the nozzle. Expansion waves are seen to exist in the small visible region of flow at the exit. This results because the water at the exit is higher than the water in the channel, thus simulating flow from a nozzle into a region of lower pressure.

On the basis of the analogy presented in the theory section, the ratio of local pressure to pressure at the minimum section was determined. This was calculated directly, since for a compressible gas

$$\frac{\rho}{\rho_t} \cdot \frac{T}{T_t} = \frac{p}{p_t}$$

therefore, for water

$$\frac{d}{d_t} \cdot \frac{d}{d_t} = \left(\frac{d}{d_t}\right)^2 = \frac{p}{p_t}$$

The eights of the water along the model were measured from Figure 10; and using t e above relation, the pressure ratios were calculated and plotted in Figure 23.

A theoretical pressure ratio curve for the nozzle is also shown in Figure 23. This was calculated by first determining the theoretical Mach number at any point on the basis of the area at that point. The Mach number vs area is plotted in Figure 21. After determining the Mach number, the pressure ratio could be determined by Equation 15[22] of the theory section. A plot of pressure vs Mach number is shown in Figure 22.

Considering now the measured pressure ratio curve for the water channel model in Figure 23, as compared to the theoretical curve, it is seen that the pressure falls very rapidly aft of the minimum section and is a little below the predicted value. At the point where the wall curvature begins to reverse, a disturbance is indicated such as would be caused by a weak shock or Mach wave. Another disturbance is indicated farther down the model and is evidently the result of the upstream Mach wave from the other wall striking at this point since in this region the wall is straight and could not cause a disturbance.

In order to determine the exact effect of the speed in the case of the water channel model while still maintaining a quarter inch of water at the minimum section, the model was operated at a speed of approximately M = 0.99. The flow of the water at this speed is shown in Figure 11. There is no indication of expansion waves and Mach waves. The pressure ratios calculated at this speed from Figure 12 are also shown in Figure 23. From the plot, there is seen to be an indication of a local disturbance 1.25 in. aft of throat even at this slow speed. This disturbance also causes a small disturbance downstream, this must be a reflection since

[22] Cf. Ante, p.11

the wall is straight at that point and could not originate a disturbance. The second disturbance is not as far down-stream as when the model was run faster. This is the result of the wave propagating at a lesser angle in the slower flow.

Figure 5 shows a schlieren picture of the flor in the wind tunnel. The expansion waves and Mach waves can be easily visualized. A shock is seen to be produced on each wall at the point where the wall begins to back against the flow. These shocks cross and are reflected from the opposite wall at a distance of 0.8 inch from the exit. This is as was predicted from the water channel model data.

The ratio of the pressures measured along the wind tunnel are plotted in Figure 23. By comparing Figure 5 with the plotted pressure ratios, it is seen that the first discontinuity is caused by the shock coming from the wall and the second discontinuity occurs where the shock is reflected from the wall and affects the flow aft of that point.

By comparing the pressure ratio curves of Figure 23 for both the water channel model and wind tunnel model, it is seen that the trends of the two curves are the same, thus indicating that very good qualitative data can be obtained from the water channel.

In an effort to obtain more quantitative data, red coloring was added to the water and a picture was taken of the water channel model at rest.. This is shown in Figure 8. From this a meniscus of 0.04 inch was measured. From photographs of the model moving, a meniscus of 0.02 inch was measured at the throat of the model. It was also observed that with the model moving there was less meniscus along a wall with rising water than along a wall with level flow and a greater meniscus along a wall with descending water than one with level flow. There was also less meniscus along a wall turning away from the flow and more along a wall turning against the flow than existed along a straight wall. On the basis of these observations and from study of the photographs where a white band indicates the meniscus which was measured, the following meniscus corrections were decided upon and used; In the minimum section where the walls are rather straight and the water has been rising, a value of 0.02 inch, which was the same as the measured value. In the expansion section where the water is falling and the wall is turning away from the flow, 0.03 inch. In the expansion section where the wall is turning against the flow and water is falling and in the test section, 0.04 inch.

From Figure 6, it is seen that the large shock wave striking the lower wall is reflected from the top of the boundary layer. This confirms the procedure of using the "methods of characteristics" to design the section and then applying the boundary layer correction. Measuring the distance from Figure 6 between the reflected shock and the wall gives 0.027 inch for the boundary layer thickness at that point. The expansion added at that point when designing the section was 0.06 inch; however, this was to take account of the boundary layer on one of the side walls as well as that on the bottom wall. If the boundary layer, measured at the bottom wall, is considered to have the same magnitude on all sides at that section, then the required expansion of the bottom wall would be 0.064 inch. This is a very close agreement with the amount applied. It is probable, considering that three surfaces are affecting the boundary layer along the bottom and top walls, that the boundary layer is slightly greater along these surfaces than along the side walls; hence, the boundary layer correction applied is more than sufficient.

A small change from the contour designed by the "method of characteristics" resulted from the entire boundary layer correction being applied to the top and bottom walls. This change resulted in too much expansion on the basis of the one-dimensional design procedure. In the basis of the theoretical area ratio, however, the proper expansion area was maintained. The wave produced by the cone in Figure 7 indicates a flow of approximately $M = 2.0$, as desired; therefore, no error has resulted by using this procedure in the design.

The reasons for the discontinuity of the flow resulted from attempting to obtain a very short section to give $M = 1.99$; and as stated in the theory, the shorter a section, the more critical it becomes. Also, when using the appropriate method to design the section, too abrupt a curvature occurred immediately at the point of reversed curvature.

On the basis of the data from both the water channel and the wind tunnel showing a local disturbance at 1.25 inches aft of the minimum section, the two models were modified. The modification consisted of filling with a very thin layer of "Proxylin" putty at a point where the curvature appeared to be too great. The two sides of each model were clamped together during the modification to assure symmetry of the walls.

The flow in the water channel model after the modification is shown im Figure 16. The heights of the water along the model were taken from Figure 17, and the meniscus corrections as discussed were applied. A plot of the pressure ratios found is shown in Figure 24.

The flow in the wind tunnel after the modification is shown in Figures 18 and 19. The variation of pressure ratios measured from the wind tunnel is also shown in Figure 24.

Considering Figure 24, it is seen that very close agreement has again been obtained, the difference in most cases being only the very small γ difference which is shown by the two theoretical curves.

Moving the points obtained from the water channel up or down a distance equal to the distance between the two theoretical curves will bring a majority of the points to coincide. It is thus seen that very good quantitative data can be obtained from the water channel model.

It is seen that in all cases the phenomena observed from the wind tunnel model and the water channel model was identical. At first when the two models were run, the same disturbance occurred in both models. Repeatedly the results were identical when the models were modified, showing that each of the models was as sensitive to changes as the other. This demonstrated the application of water channel testing of wind tunnel shapes as a preliminary step in supersonic wind tunnel design.

On the basis of the analyzed data, it is seen that quantitative as well as qualitative data can be obtained from a wind tunnel section model placed in the water channel by the following procedure:

(1) Obtain the heights of the water along the model.

(2) Apply the proper meniscus corrections as described.

(3) Calculate the pressure ratio from the resulting heights.

(4) Calculate the theoretical pressure ratios for $\gamma = 1.4$ and $\gamma = 2.0$.

(5) Move the measured pressure ratios up or down a distance equal to the spacing between the two theoretical curves, depending upon the theoretical curve for $\gamma = 2.0$ being above or below the theoretical curve for $\gamma = 1.4$.

CONCLUSIONS

(1) The water channel provides a rapid and inexpensive means of testing the proposed design of a supersonic nozzle. A two-dimensional model of the theoretical design can be constructed and tested to determine if the flow is as desired. The model can then be altered until the desired flow is obtained and then the resulting design used to build the actual tunnel.

(2) The water channel is most applicable to the testing of wind tunnels since good flow in a wind tunnel is one dimensional, and the water channel is most suited for this type of testing.

(3) The difference between the specific heat constants, γ, for the two fluids is of minor importance.

(4) The addition of red coloring aids in photographing the water line.

(5) A schlieren system is a very delicate instrument, difficult to align properly and requires much more equipment than a single overhead camera as used by the water channel.

(6) The schlieren system provides a much better visualization of the flow.

RECOMMENDATIONS

(1) Build the models by constructing the contour for one side and then replacing the center line by a straight wall. This will reduce the problem of obtaining symmetrical walls and make it easier to modify a model.

(2) Apply boundary layer correction to all four walls of the wind tunnel rather than to the top and bottom walls only.

(3) Extend the sides of the wind tunnel so that two-dimensional flow is maintained aft of the exit, as in the water channel.

(4) A future project of interest would be to construct water channel models which will simulate the cross section of a rocket. The external and internal flow of these models could be studied and also the interaction of the two flows at the exit.

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APPENDIX I

FIGURES

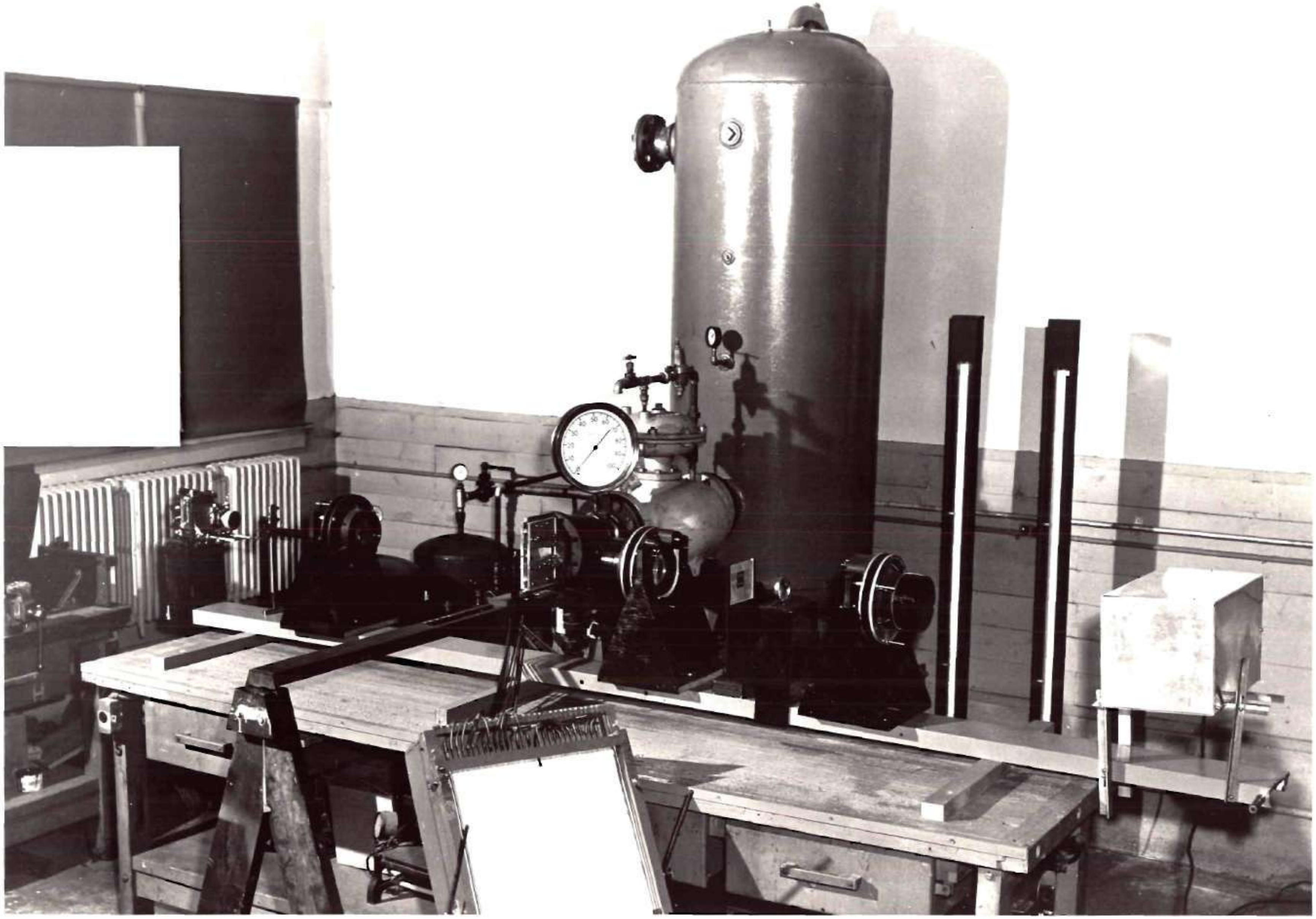


FIGURE 1. GENERAL VIEW OF WIND TUNNEL

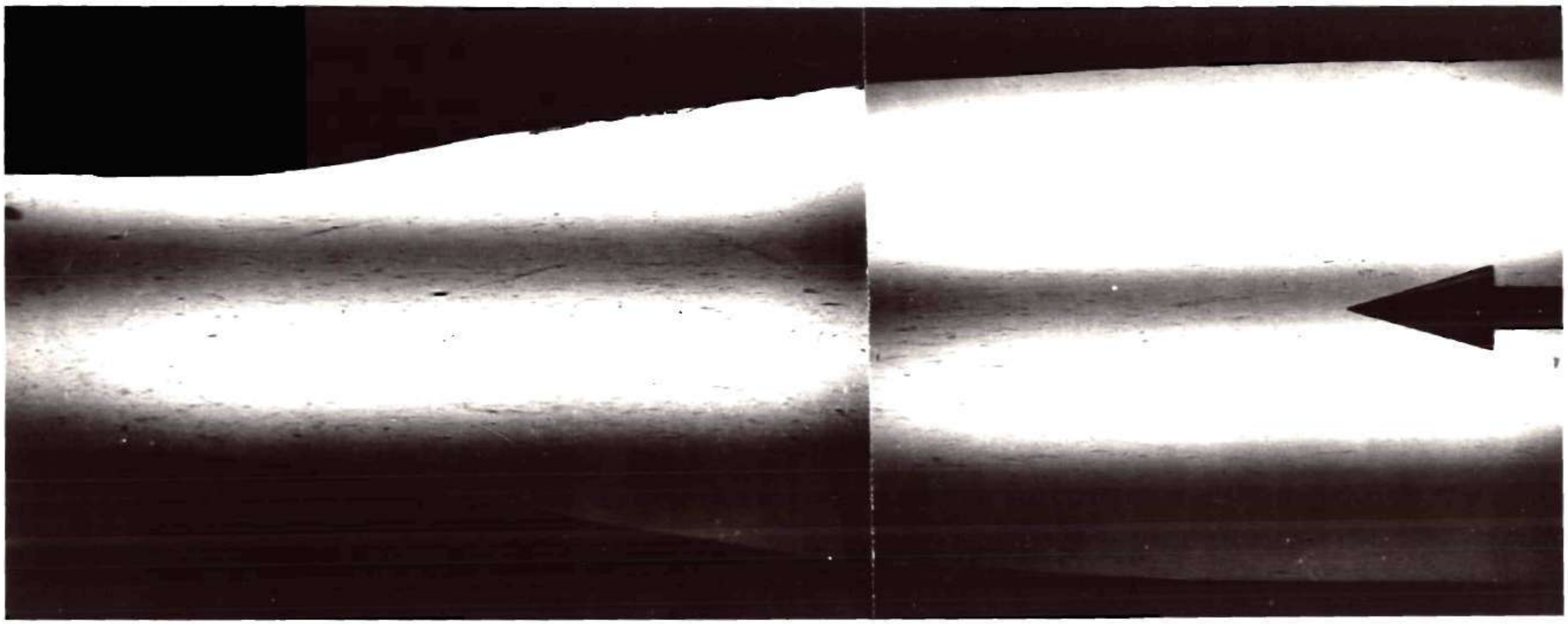


FIGURE 2. WIND TUNNEL NOZZLE

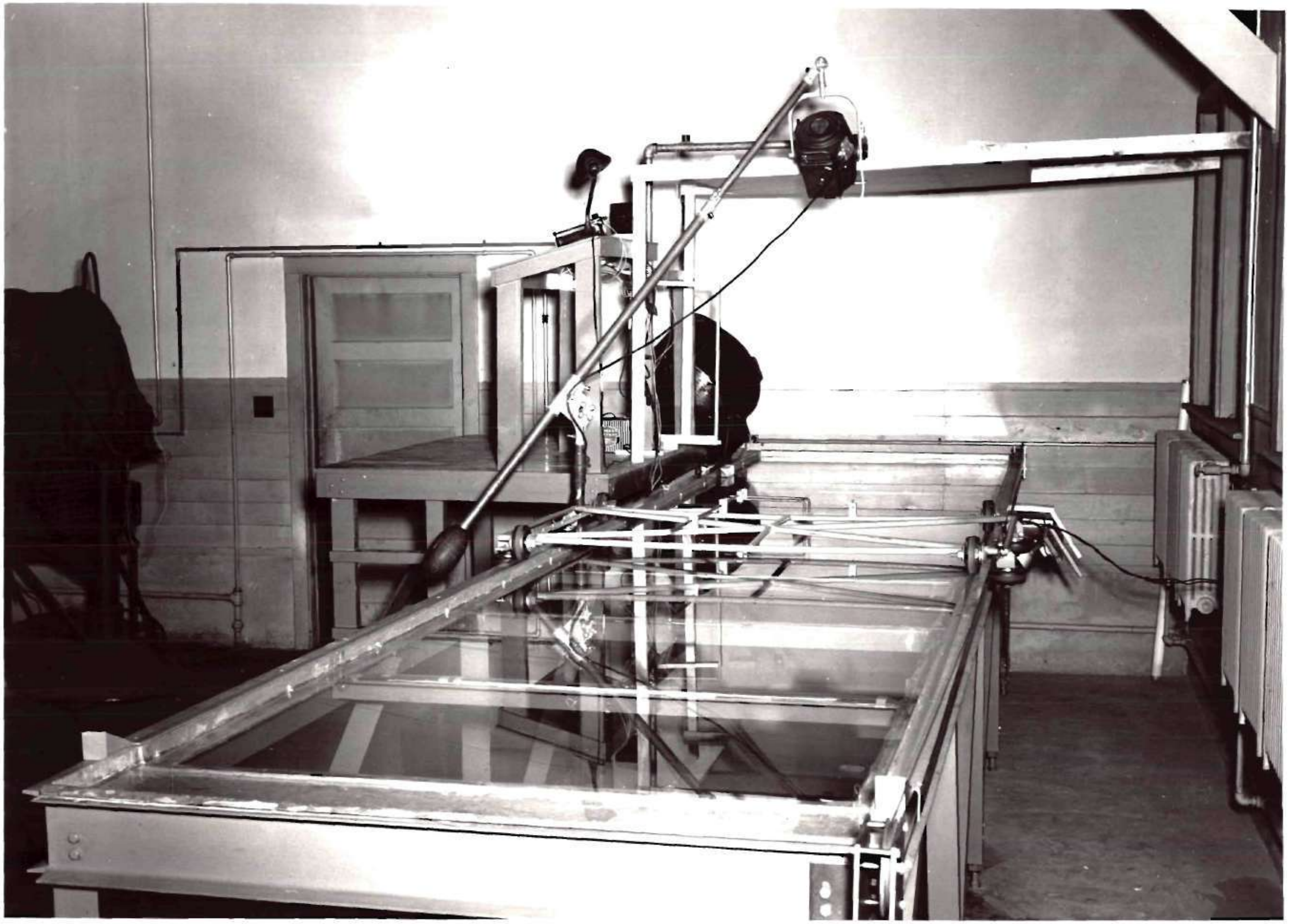
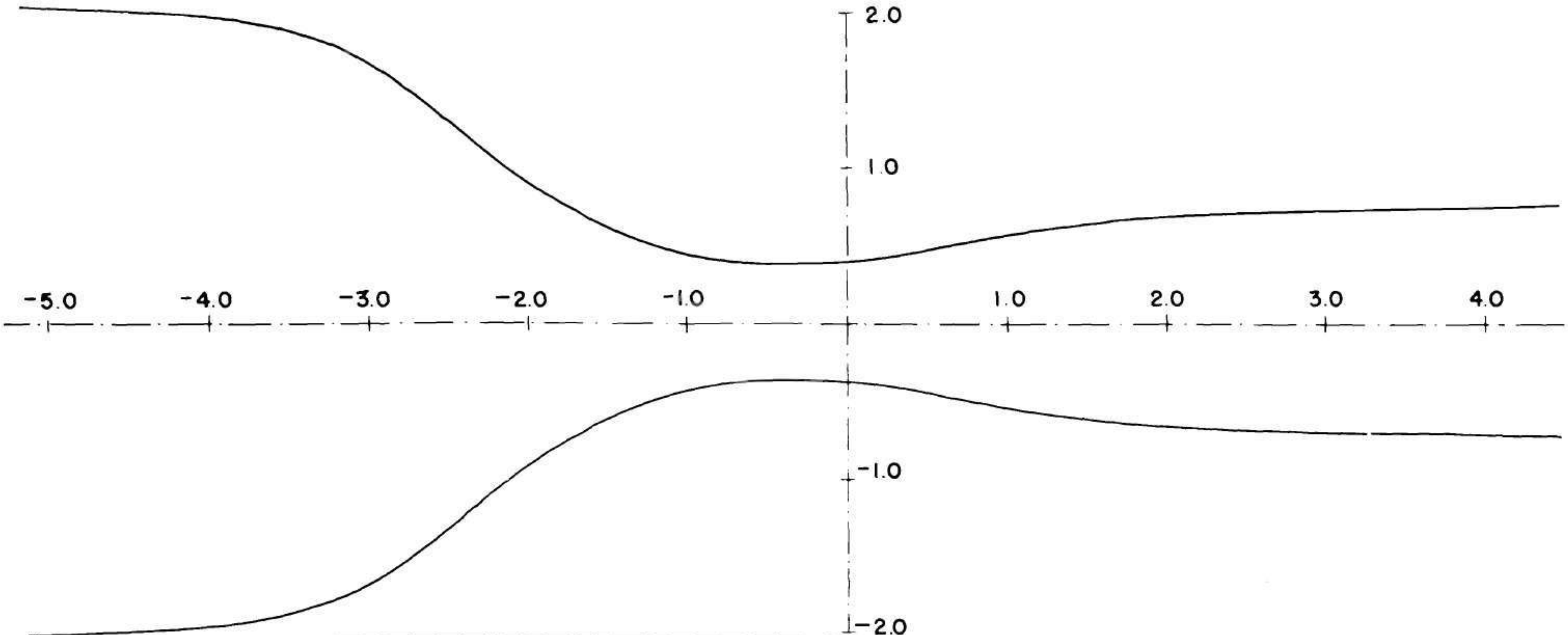


FIGURE 3. GENERAL VIEW OF WATER CHANNEL



X	±Y	X	±Y	X	±Y	±X	±Y	X	±Y	X	±Y
-5.25	2.0	-3.25	1.70	-1.75	.781	-.25	.410	+1.25	.6241	+3.0	.7426
-4.5	2.0	-3.0	1.625	-1.50	.645	0	.4138	+1.50	.6536	+3.5	.7525
-4.25	1.99	-2.75	1.53	-1.25	.540	+.25	.4426	+1.75	.6756	+4.0	.7614
-4.0	1.96	-2.5	1.35	-1.0	.470	+.50	.4814	+2.0	.6977	+4.5	.7704
-3.5	1.94	-2.25	1.15	-.75	.425	+.75	.5252	+2.25	.710		
-3.375	1.89	-2.0	.945	-.50	.410	+1.0	.5867	+2.5	.722		

FIGURE 4
MODEL CONTOURS AND ORDINATES

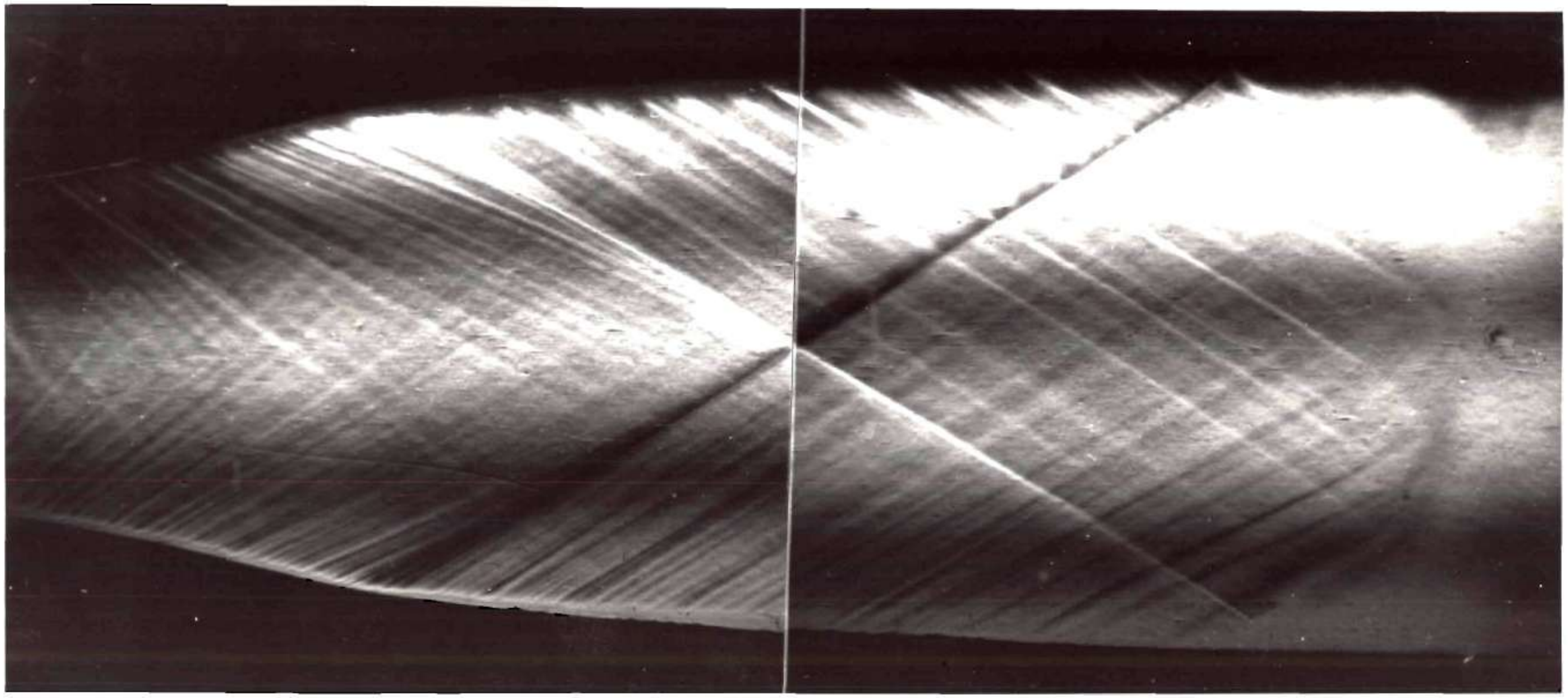


FIGURE 5. SCHLIEREN PICTURE OF FLOW IN THE WIND TUNNEL

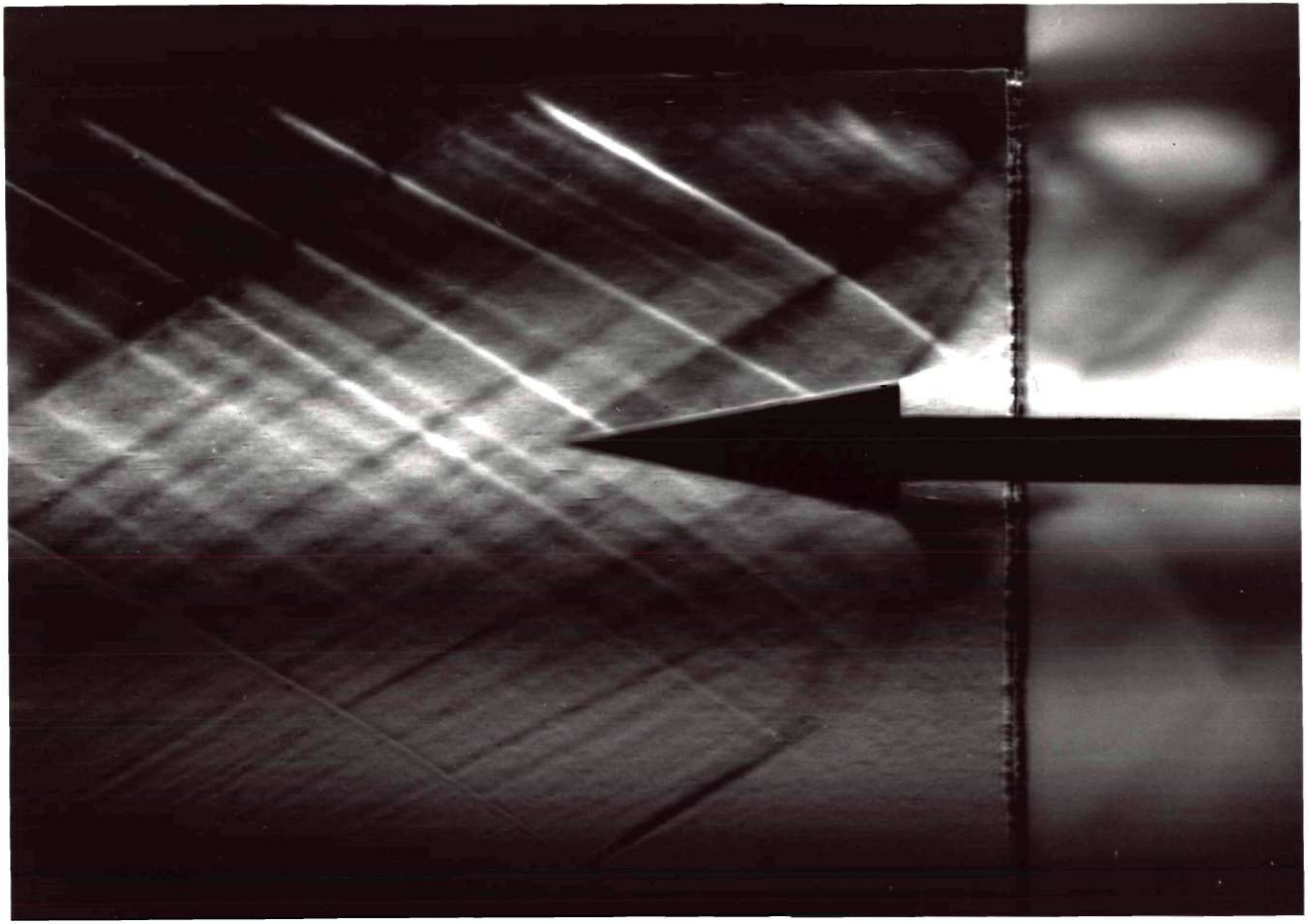


FIGURE 6. SCHLIEREN PICTURE WITH 22 DEGREE CONE

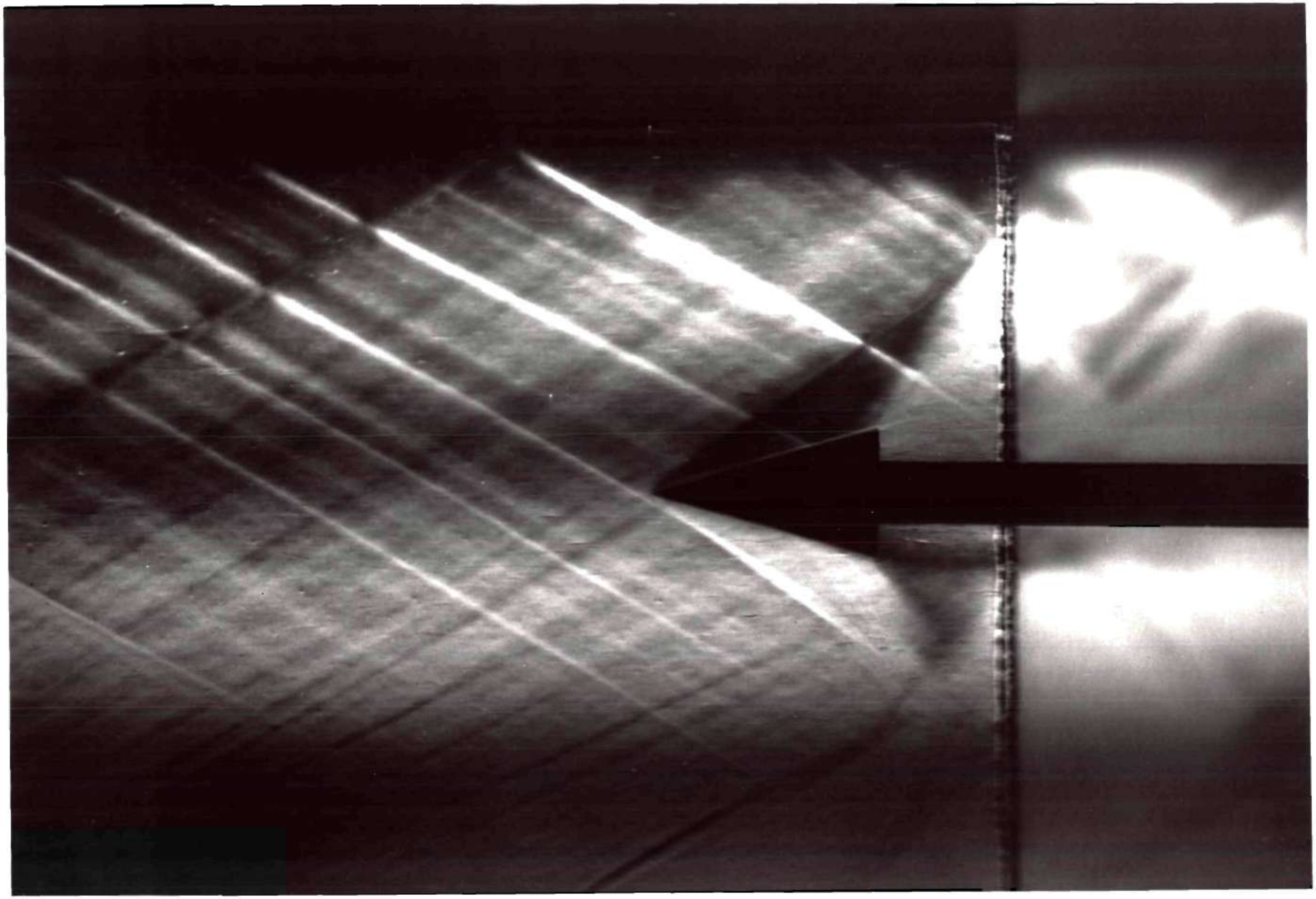


FIGURE 7. SCHLIEREN PICTURE WITH 32.5 DEGREE CONE

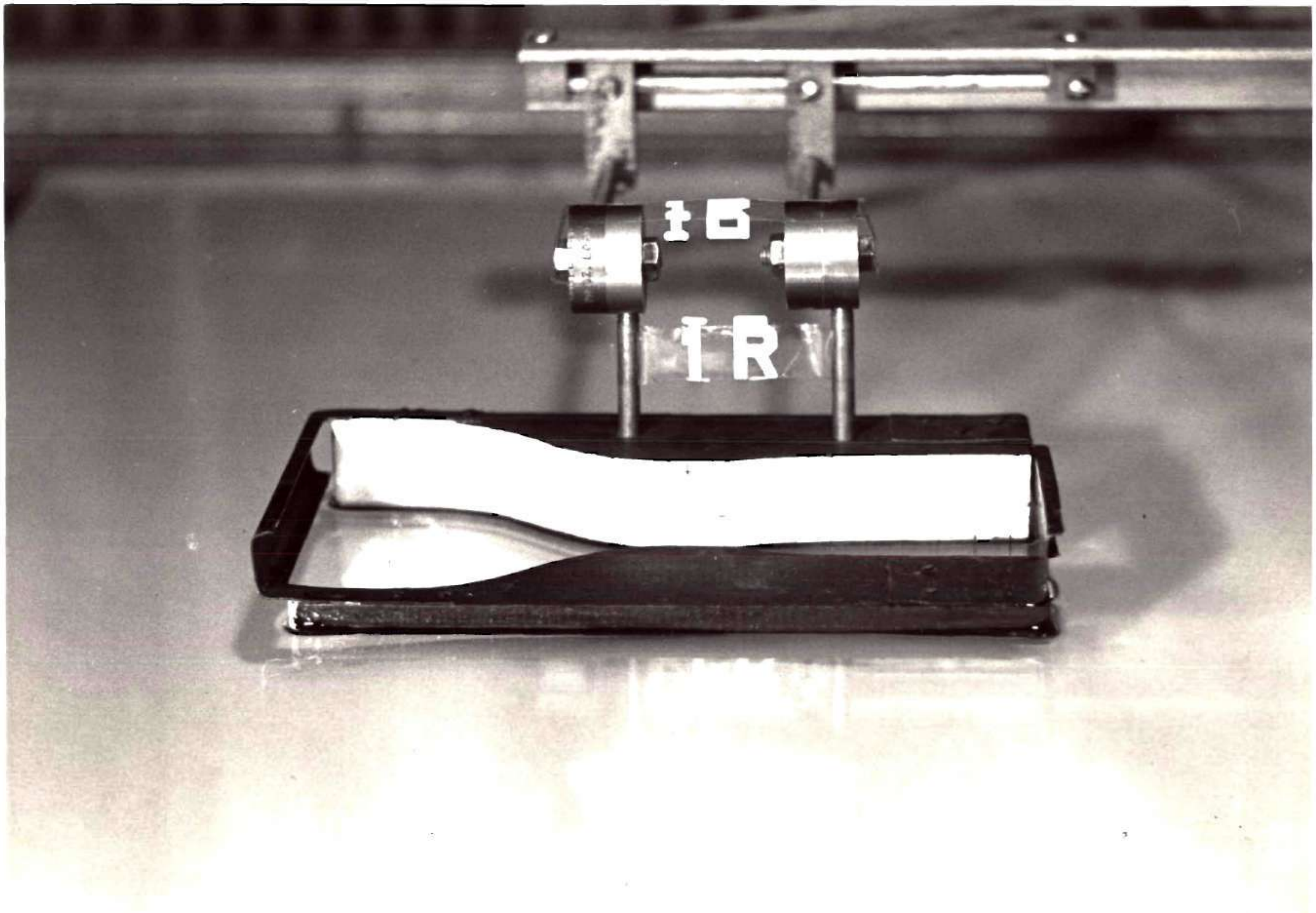


FIGURE 8. SIDE PICTURE OF WATER CHANNEL MODEL STATIC

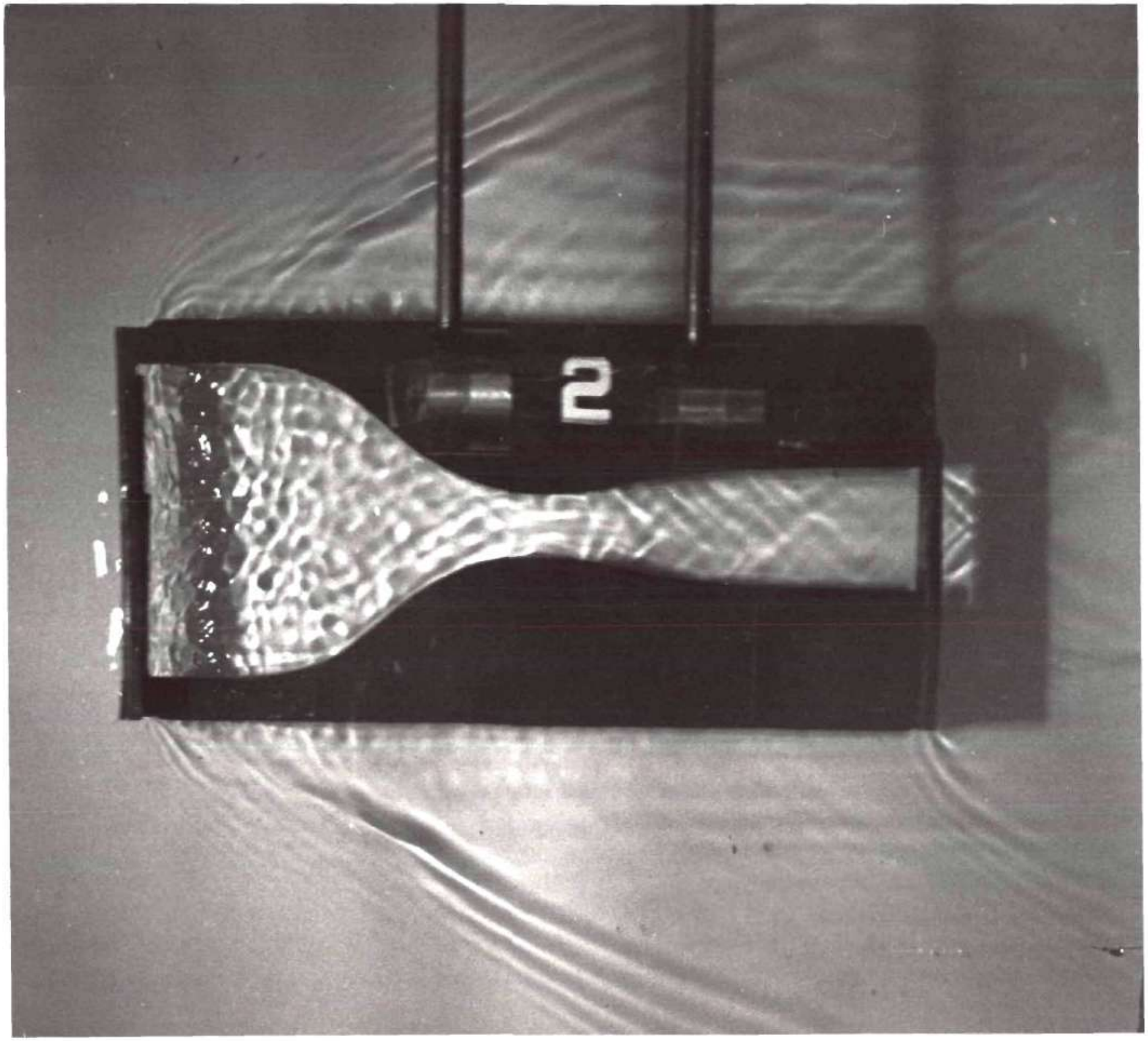


FIGURE 9. CHANNEL MODEL AT DESIGNED SPEED

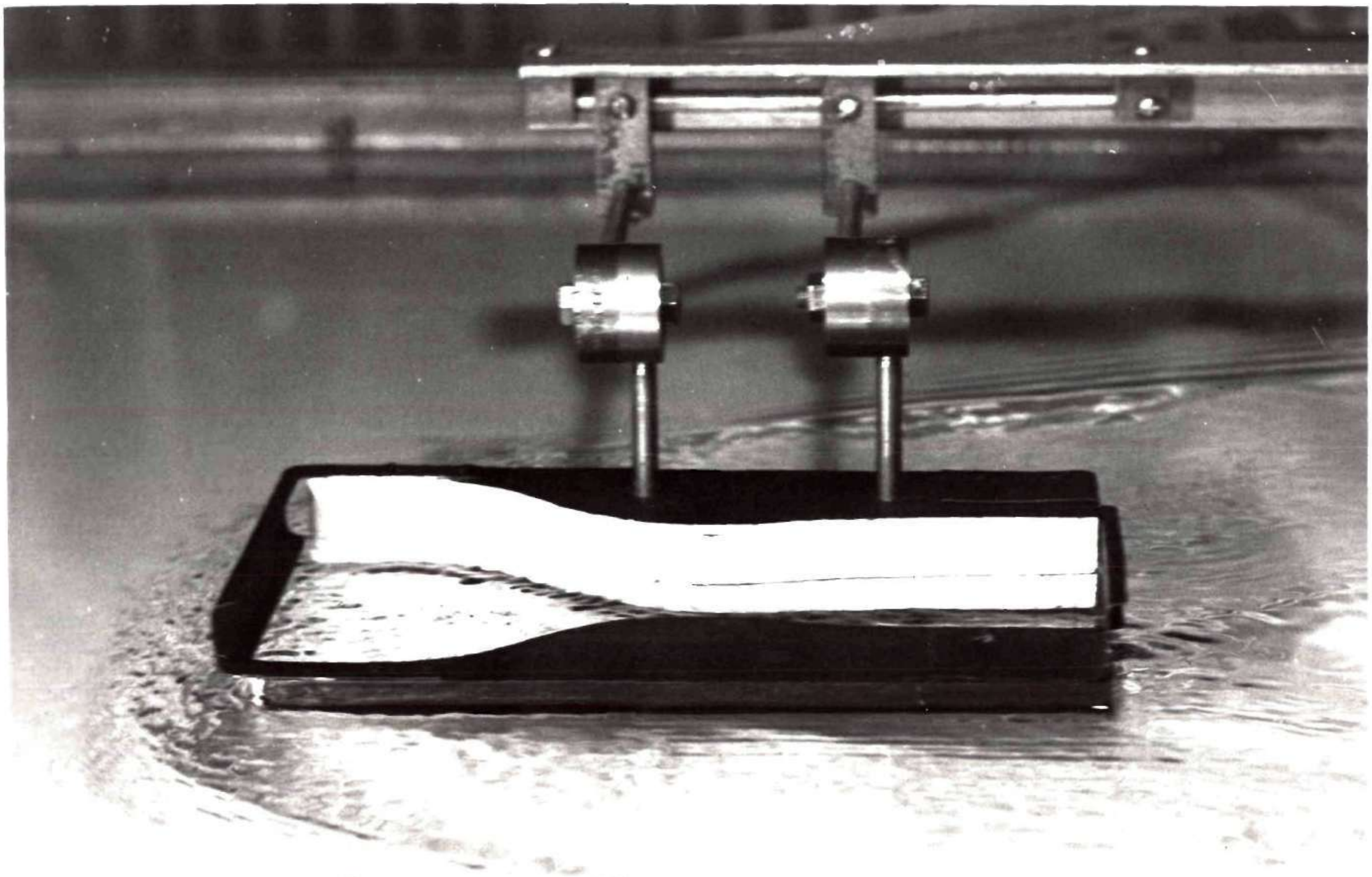


FIGURE 10. CHANNEL MODEL AT DESIGNED SPEED

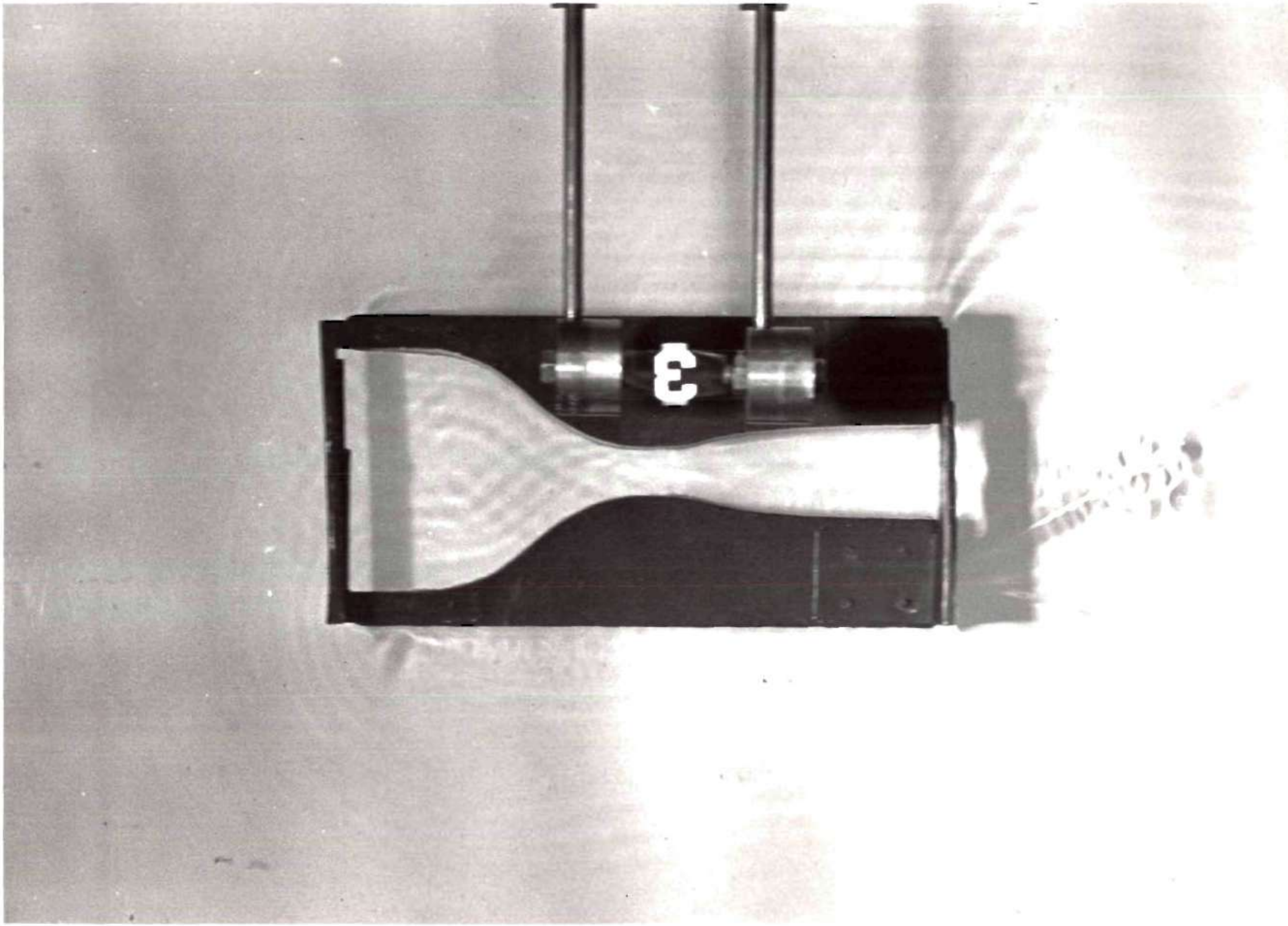


FIGURE 11. CHANNEL MODEL BELOW DESIGNED SPEED

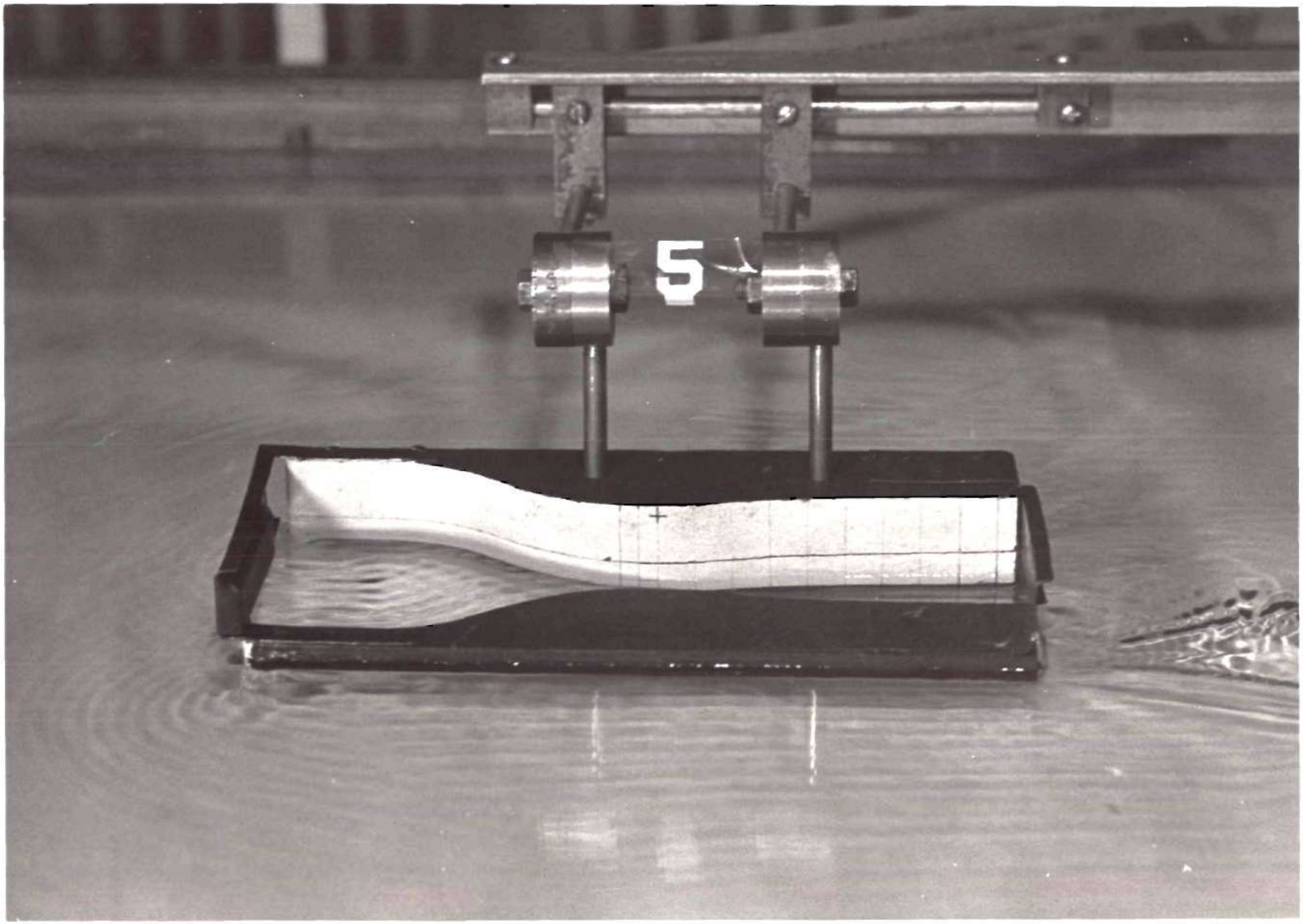


FIGURE 12. CHANNEL MODEL BELOW DESIGNED SPEED

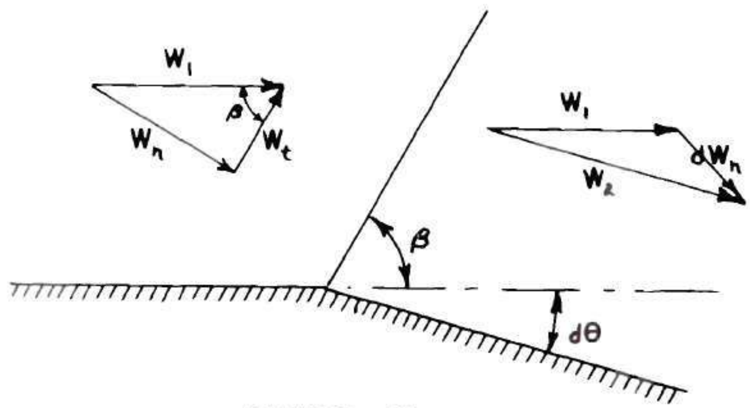


FIGURE 13

EXPANSION WAVE AND NOMENCLATURE

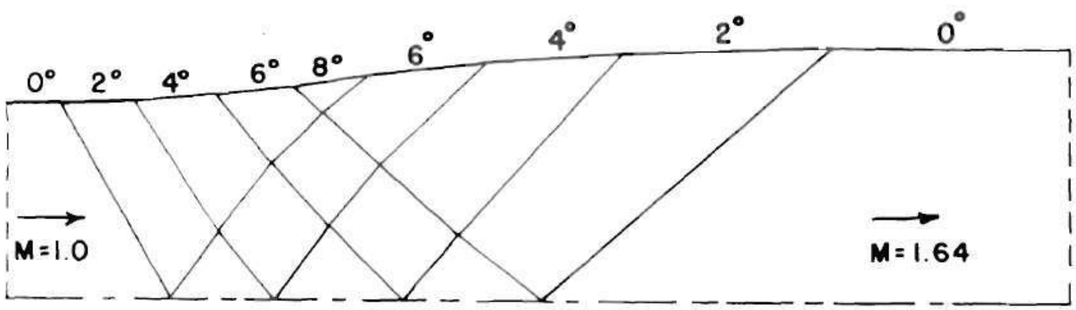


FIGURE 14

NOZZLE DESIGN CONSTRUCTION

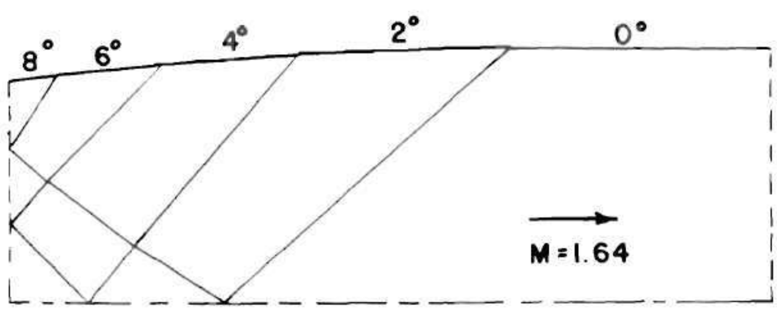


FIGURE 15

APPROXIMATE METHOD NOZZLE DESIGN CONSTRUCTION

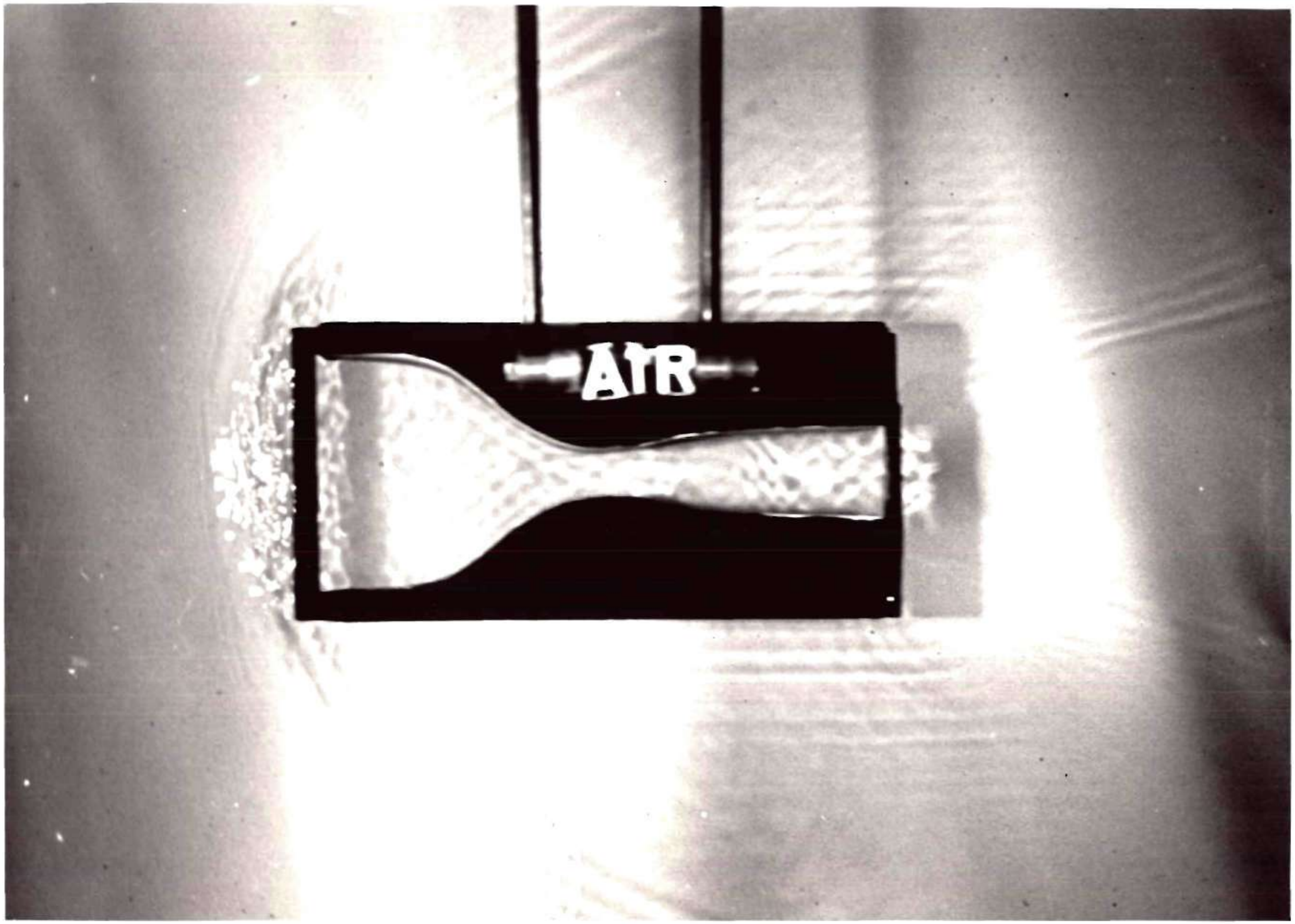


FIGURE 16. CHANNEL MODEL AFTER MODIFICATION

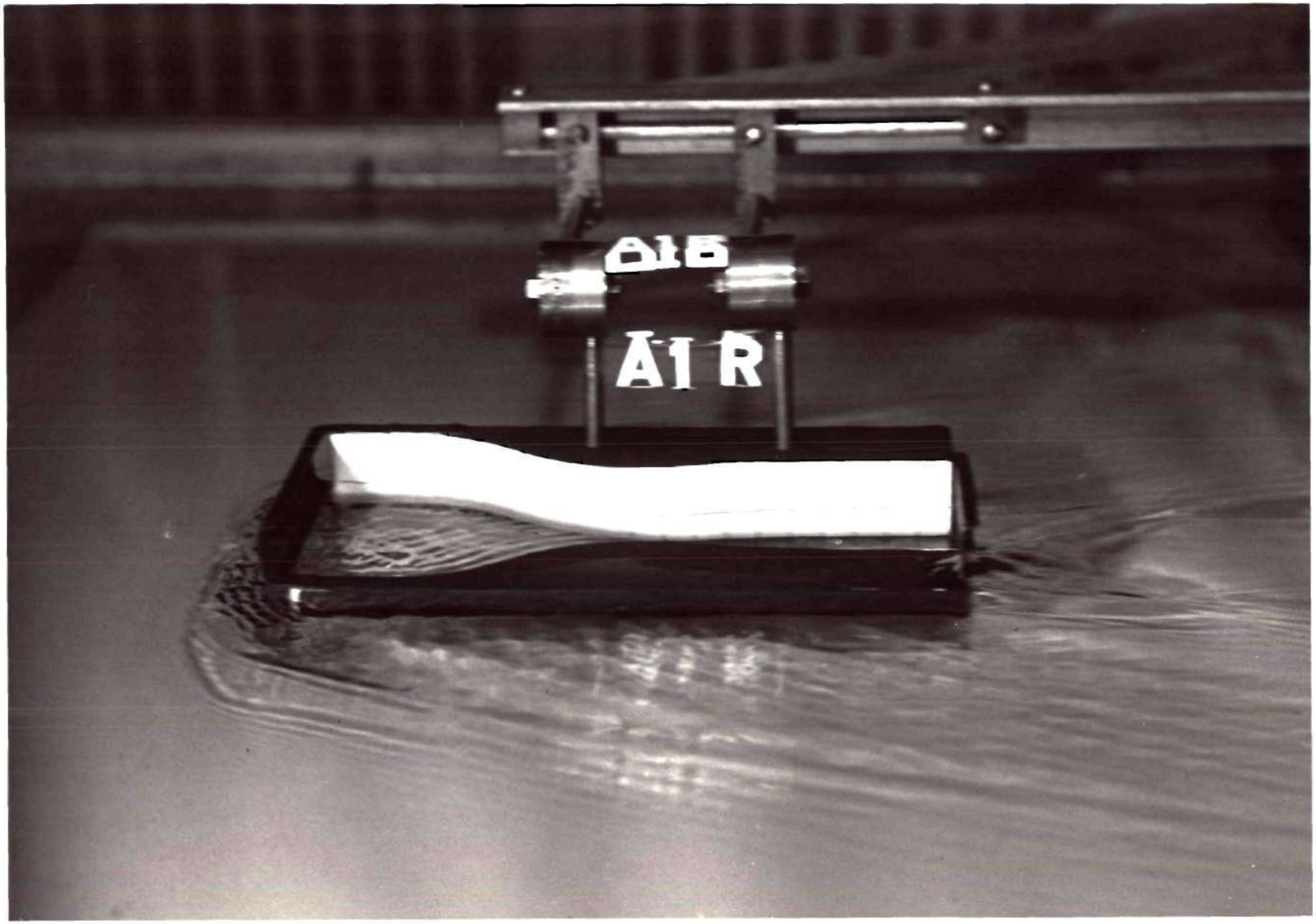


FIGURE 17. CHANNEL MODEL AFTER MODIFICATION

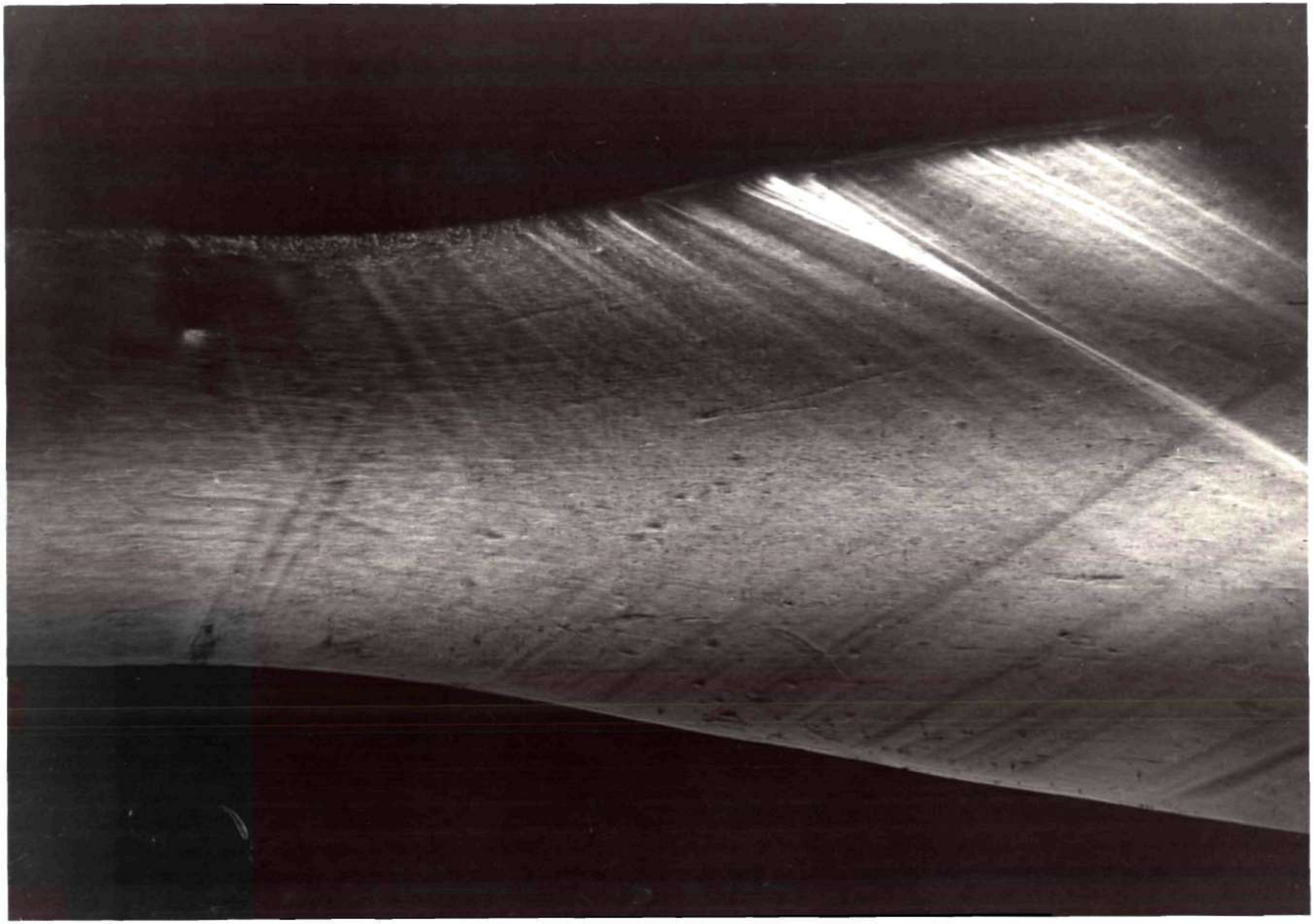


FIGURE 18. SCHLIEREN OF WIND TUNNEL EXPANSION AFTER MODIFICATION

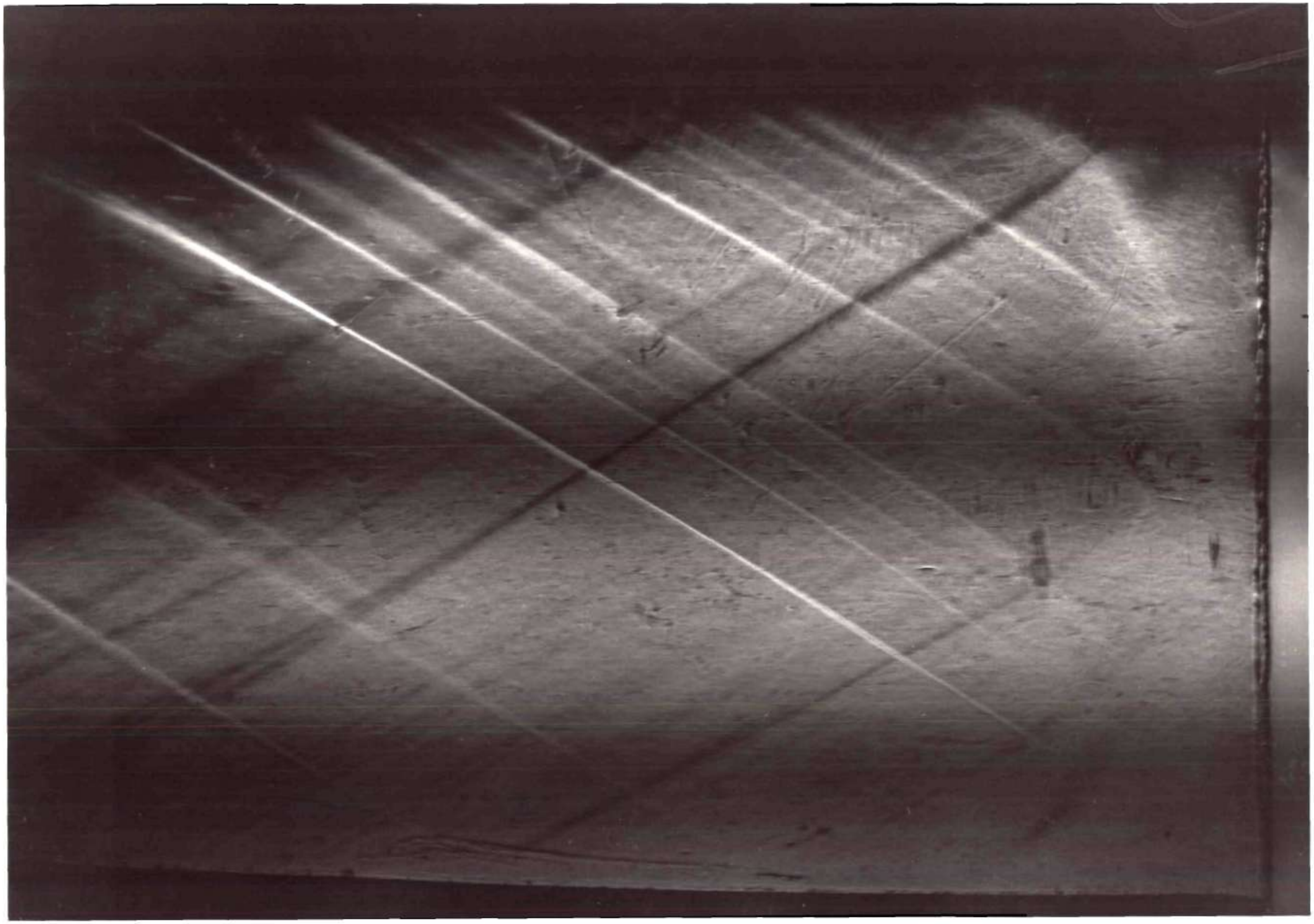


FIGURE 19. SCHLIEREN OF AFT PORTION OF WIND TUNNEL AFTER MODIFICATION

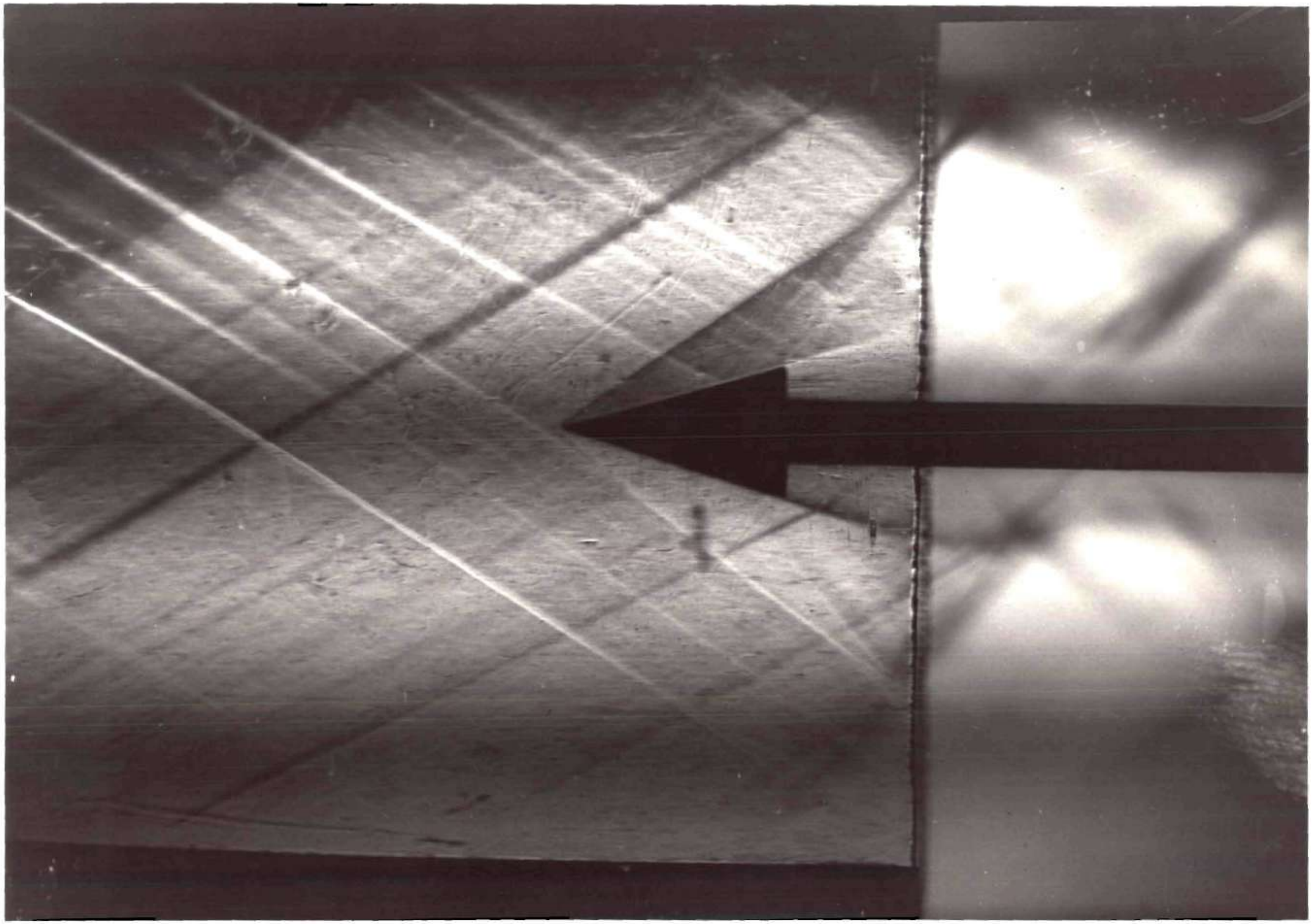


FIGURE 20. SCHLIEREN PICTURE WITH CONE AFTER MODIFICATION

Figure 21

Variation Of Mach Number With Area

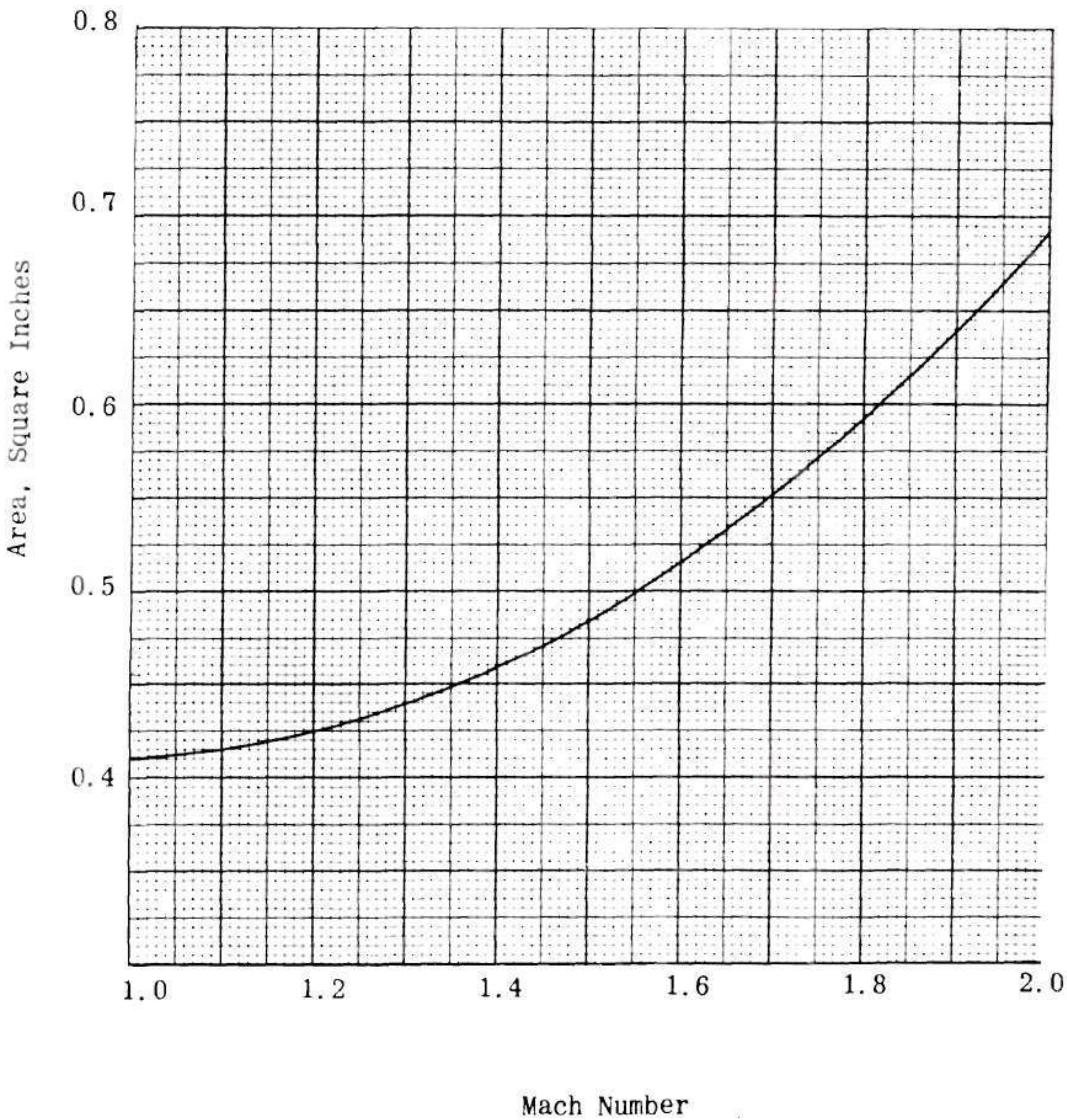


Figure 22

Variation Of Pressure Ratio With Mach Machine

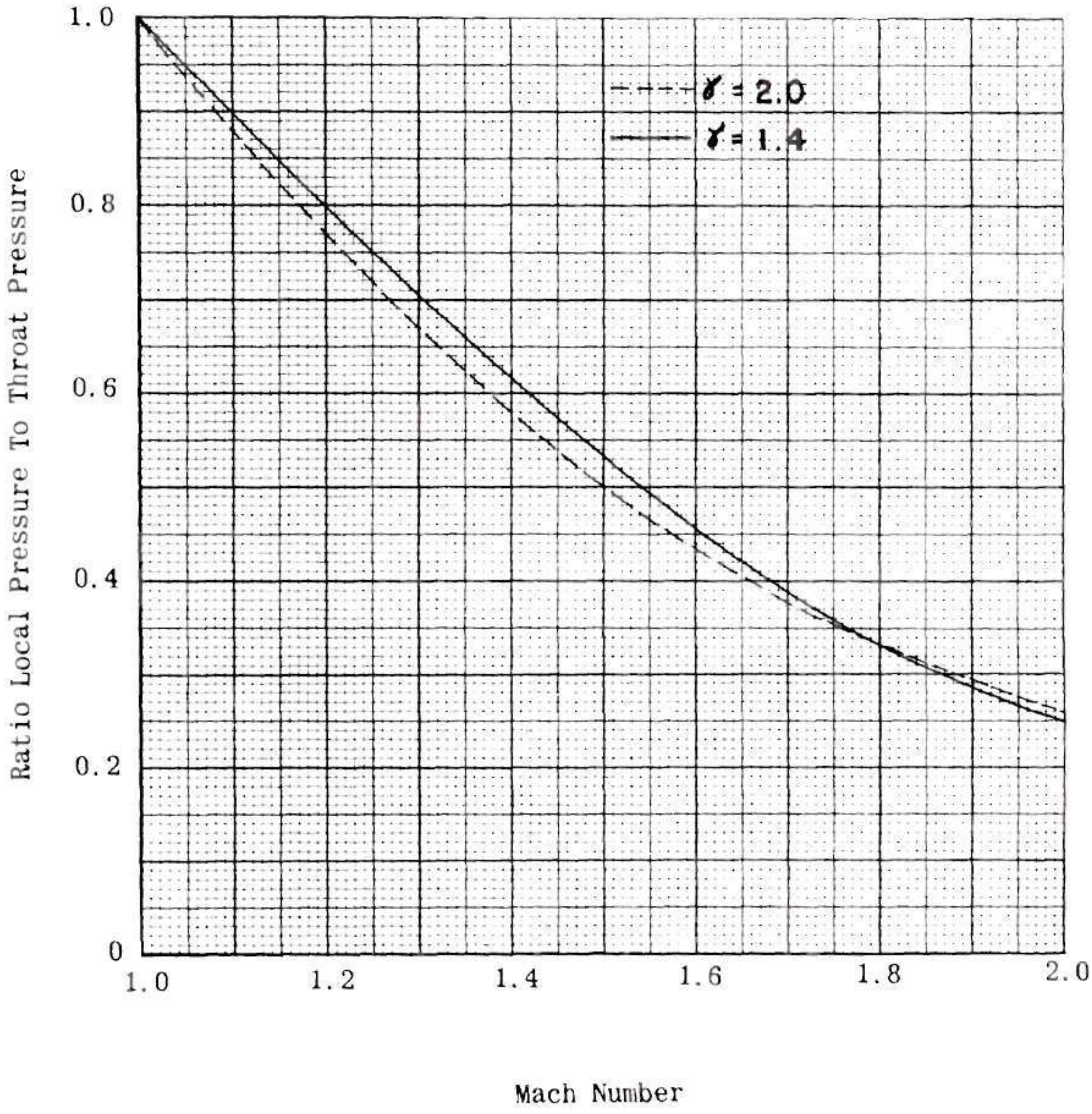


Figure 23

Variation Of Pressure Ratio In The Models

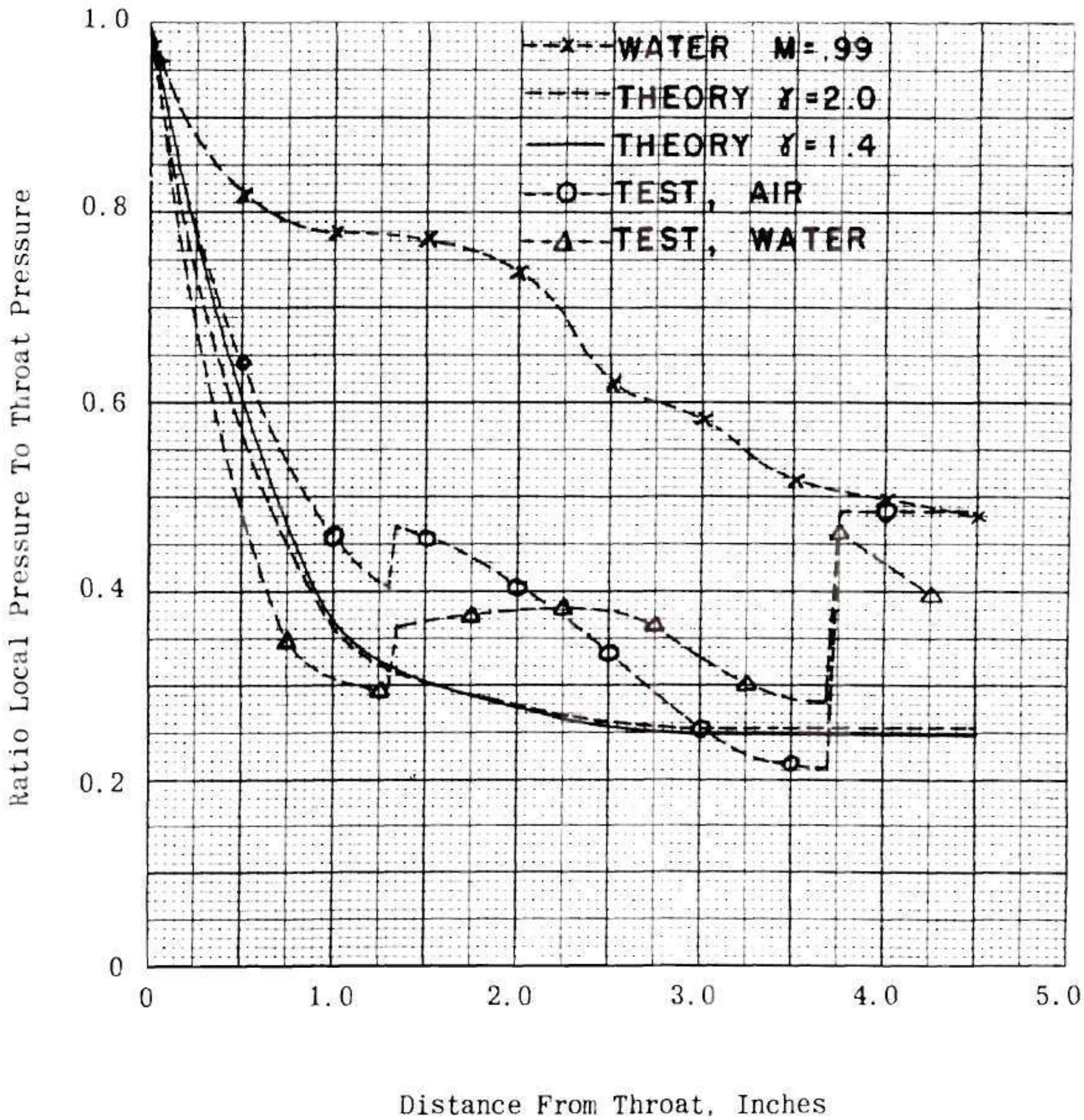


Figure 24

Variation Of Pressure Ratio In The Models After Modification