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A Process to Obtain Robustness Metrics for Adaptive Flight Controllers

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This research effort seeks a process to draw parallels between the classical stability metrics of gain and phase margins for classical linear control systems with stability margins for adaptive controllers. The method uses a Monte Carlo simulation to yield stability threshold results for the adaptive controller based on problem-specific performance metrics. By fitting a linear controller's analytical robustness results to the adaptive stability data, the gain and phase margin for the performance-fitting linear system are considered to be the worst case equivalent gain and phase margin for the adaptive controller. This paper also discusses some experiences successfully obtaining time delay margin in a flight test setting.

Nomenclature

dB	=	decibel
$\boldsymbol{e}$	=	tracking error
K_p	=	linear system proportional gain
L_p	=	roll damping derivative, dimensionless
q	=	pitch rate, rad/s
Δ	=	control input gain variation, dimensionless real scalar
τ	=	time delay, positive real scalar in unit of seconds

I. Introduction

Adaptive control methods for aerospace applications have been well developed in control literature; however, these flight control systems are typically not implemented on manned aircraft due to the difficulty involved in validating the safety and stability of such nonlinear and time-variant systems. Verification and validation of linear, time-invariant (LTI) control laws is much easier, since the concepts of gain and phase margin quantify the controllers' ability to maintain stability in the presence of gain and phase changes. These classic stability margins stand as established standards that can be met to validate the LTI controllers[1]. Since these stability margins are only valid for linear systems, entirely new methods must be developed to validate and certify adaptive controllers[2,3,4]. One way to accomplish this goal may be to make advances in the analysis of learning algorithm stability and convergence rates[5], while another effort is to develop unique performance and stability metrics for adaptive controllers[6]. Many ideas for stability metrics for nonlinear adaptive control systems have been proposed, including gain margin and time delay margin for adaptive controllers, transient and steady state performance metrics[6], and several new stability metrics specifically suited for nonlinear systems[7]. Work has already been done to further develop, prove and verify these new metrics, including efforts to formalize gain margin in adaptive systems[8], analyze the effect of time delay on adaptive systems[9], and work to explore the effect of high gain values on time delay margin[10]. Some of these metrics can also be calculated on-line to help the controller adapt to changing conditions in what is known as metrics-based adaptive control. One example of this is the use of bounded linear stability analysis to change adaptive gains to improve closed-loop stability margins[11,12]. Metrics-based adaptive control is an important part of the NASA Intelligent Resilient Aircraft Control (IRAC) program, with the other focus being the study of robustness of adaptive controllers.[13]

The contribution of the current research effort lies in studying and measuring the robustness of adaptive controllers. Specifically, this paper proposes an offline process to obtain estimates of gain and phase margin for a

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nonlinear, adaptive controller through comparison of that adaptive controller's performance in simulation with a linear system whose stability margins are known. The method employs a Monte Carlo method to simulate an adaptive controller's response to injected gain variation and time delay in terms of a carefully chosen set of performance or stability metrics tailored to the system under scrutiny. The method then calls for finding a linear system whose analytical stability performance matches the adaptive system's performance in simulation. By matching the adaptive and linear systems in terms of their robustness to adverse gain variation and time delay, the method infers that the adaptive system's gain and phase margins may be considered at least as good as those of the matching-performance linear controller.

Sections II and III provide background information that respectively describe the simulation and stability or performance metrics utilized in this study. Section IV shows the proposed methodology applied to a simple system with an adaptive controller.

Monte Carlo analysis is normally not extensible to a flight test setting. This paper also briefly describes some recent experience measuring time delay margin (or in principle gain margin) in a flight test setting. This is described in section V.

II. Monte Carlo Simulation

Monte Carlo simulation is a powerful tool used to simulate systems with uncertain system characteristics and noise sources[14]. The method consists of many simulation iterations for a system with one or more uncertain input parameters. In each iteration, the uncertain parameters are randomly selected from a probability distribution and the system may be simulated for a chosen time interval or until it satisfies a predefined success or failure condition. By observing the aggregate results of all simulation runs, the expected behavior of the system in general can be seen without having to consider every possible parameter combination.

In this proposed method, the uncertain parameters are gain variation and time delay. Each iteration of the simulation for the adaptive controller involves choosing the time delay and gain variation values from a probability distribution, then generating a time history of a specified length of time. For the simple example described in Section IV, a simple choice is made: A uniform distribution across the region of interest. An advantage of Monte Carlo analysis is that a more complex distribution could be utilized if more was known about the uncertainty of these parameters. In a similar application to the proposed method of this paper, a Monte Carlo method has been used previously to find probabilistic definitions of robustness for a system with parameter variation[15].

Another benefit of using the Monte Carlo method is that Monte Carlo simulation results are often already available during the development of a flight control system, since the Monte Carlo method may be utilized for performance prediction and analysis of many other aspects of an aerospace system design. The method could be added to an existing Monte Carlo simulation effort by adding performance metrics such as those described in Section III, or by analyzing previously generated simulation results.

III. Stability Tests and Performance Metrics

A test for system stability is required to judge whether each run of the adaptive controller in the Monte Carlo simulation has exhibited stable or unstable behavior at a given parameter combination. As a practical matter, the test metric is ultimately a performance metric, which will be highly problem dependent. For a launch vehicle flight control system, for example, this could include a combination of a wide array of performance checks of interest, such as: heating, desired trajectory, fuel use, aerodynamic loads, and actuator usage. By carefully choosing the performance metric(s), a determination can be made whether a given time history resulted in desirable behavior or not, then this information goes into a plot of acceptable combinations of gain variation and time delay. It is very important to properly choose the metric in terms of the validity of conclusions.

In the greater adaptive control community, there are a wide variety of metrics being developed to measure several aspects of nonlinear controller performance, dynamics, and stability. For on-line metrics actively used in control feedback, the metrics must be fast, reliable, robust, real-time, easy to understand and reproducible. For this off-line simulation based method, the metrics need to dependably give information on the system's behavior in a limited time history. A simple metric proposed in the example presented in Section IV is the discrete approximation of the L_2 norm of the error in following a reference signal[16]. The formulation of this calculation is

$$\|e\|_{L_2} = \sum_{i=1}^{n} \|e_i\|_2 dt \ , \tag{1}$$

where e_i is the tracking error between the controlled system's response and a reference input for step *i* in the time history, *dt* is the time step between simulation points, and the absolute sum norm, $||e_i||$, is given by

$$\|e_i\|_2 = \left[\sum_{i=1}^{n} e_i^2\right]^{\frac{1}{2}}, \tag{2}$$

where *n* is the length of vector e_i for each datapoint *i* in the simulation.

Another potential metric that could be utilized in the Monte Carlo simulation is the Lyapunov exponent, which can show onset of oscillations in the system and give information about the system dynamics[17]. Work has already been done to show that the Lyapunov exponent can be used to determine the threshold of time delay for instability[18].

IV. Development of Methodology for 1st Order System

The proposed method is shown for a simple, first order example problem. First the system is introduced and the control law for the linear case and adaptive case are given. Next the analytical results for instability in the linear controller in terms of gain variation Δ and time delay τ is shown. To verify the effectiveness of the Monte Carlo method and the chosen performance metric test, the linear system is simulated with the Monte Carlo method. For a given combination of randomized Δ and τ, the system's departure from a stable state is tested by measuring if the approximation for the L_2 norm of the error, given in Eq. (1), has surpassed a set value. The result of the linear Monte Carlo simulation is shown to correspond with the analytical results expected based on Routh's stability criterion and the linear system's gain and phase margins. Next the adaptive system's Monte Carlo simulation results are presented and the method of matching a linear controller to the resulting stability threshold curve is demonstrated.

A. Example System Description

The system to be simulated represents the lateral flight control of an airplane, and is represented by the first-order differential equation

$$\dot{p}(t) = L_p\left[ap(t) + \Delta\delta(t-\tau)\right] + W \tag{3}$$

where p(t) is the airplane's roll rate, δ(t) is the control input, Δ is gain variation, τ is time delay represented as a delay on the control input, L_p and *a* are both positive constants and *W* is white noise. It is important to note that since L_p and *a* are positive, the system is unstable without a control input. For the linear simulation, the control input is given by

$$\delta(t) = -K_p p(t), \tag{4}$$

while for the adaptive controller, δ(t) is calculated as

$$\delta(t) = k(t)p(t), \tag{5}$$

where k(t) is a time-varying gain which comes from solving the differential equation

$$\dot{k}(t) = -\gamma p^2(t), \tag{6}$$

where *γ* is a scalar called the learning rate.

Next the behavior of the analytical behavior of the linear system will be calculated and shown in simulation.

B. Linear Analytical Results

In order to validate the Monte Carlo simulation method, the simple linear system is simulated and the analytical solution for the stability threshold in terms of gain variation *Δ* and time delay *τ* is compared to the simulation result. The gain and phase margins of the linear system were determined when gain K_p is chosen as 1. The system has an infinite upward gain margin, a downward gain margin of -6.94 dB and a phase margin of 63.3 degree at 1.34 rad/s.

This corresponds to an acceptable time delay limit of 0.8245 seconds at nominal gain. A second order Padé approximation[19] was added to the linear system and an analytical solution for gain variation versus time delay was determined by constructing a Routh array and solving for the parameter ranges in which Routh's stability criterion was satisfied[19]. For stability, there are two relevant criteria that must both be satisfied:

$$\Delta > \frac{a}{K_p}, \tag{7}$$

And

$$\Delta > \frac{-3 + (21 - 6L_p a\tau + (L_p a)^2 \tau^2)^{1/2}}{K_p L_p \tau}. \tag{8}$$

The utility of these inequalities are that they will be used to compare the linear system's expected stability regions with the result from Monte Carlo simulation of the system.

C. Monte Carlo Simulation Results for Linear System

Next, the Monte Carlo simulation was run on the linear controller. The simulation consisted of 5000 iterations of a time history of 100 seconds each. For each run, the value for τ was chosen randomly from the uniform distribution between 0 and 2 seconds, while Δ was chosen uniformly on the interval from -10 to 10 dB. The system attempted to track a reference of zero in the presence of white noise of standard deviation of 0.1. The stability test involved checking to see if the $\|e\|_{L2}$ value at the end of the time history had exceeded a value of 100. This threshold was chosen by trial and error after determining that a threshold of 10 was too strict and a threshold on the order of 1000 was too lax.

The resulting simulation result is shown in Figure 1.

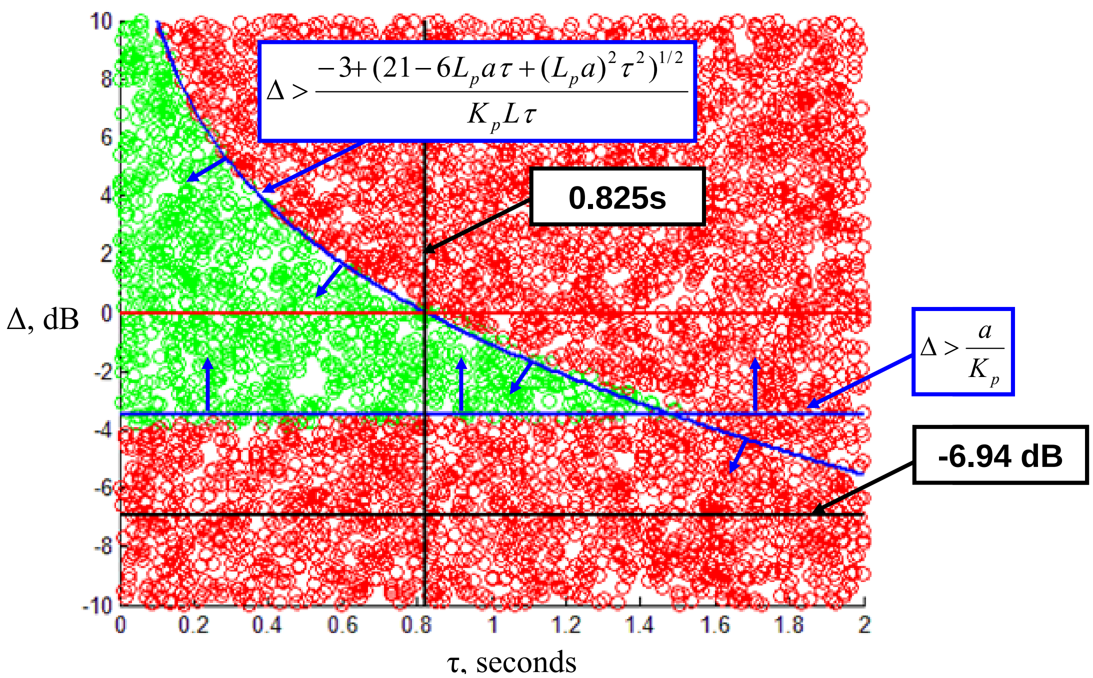

Figure 1: Linear Monte Carlo Results with analytical stability boundary

Points were plotted in green where the simulation $||e||_{L2}$ threshold was not exceeded and red where it was exceeded. The blue lines represent the analytical stability inequalities. As expected, at a Δ of 0dB, corresponding to the un-varied linear system with $Kp = 1$, the time delay threshold appears to be 0.825 seconds, as expected from the phase margin results previously computed. This result verifies the Monte Carlo simulation result as a valid method for simulating this system.

D. Adaptive System Monte Carlo Simulation Results and Comparison to Linear System

The nonlinear adaptive controller is run through the exact same tests with Δ and τ drawn from the same distributions. The initial result is shown in Figure 2.

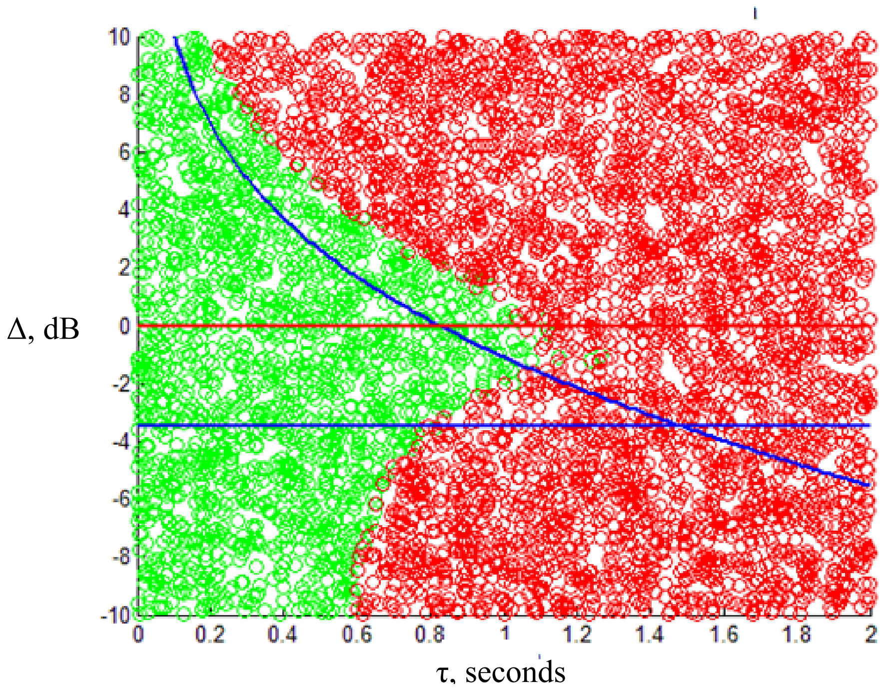

Figure 2: Adaptive Controller Monte Carlo results with overlay of linear analytical stability results with $K_p = 1$.

The blue line is the same linear analytical stability threshold results for the case when $K_p = 1$ and is shown for reference. The adaptive system can tolerate smaller values of Δ since the adaptive gain can be adjusted even if the control effectiveness is poor. Note that by varying the gain (K_p) of the linear controller, the line can be changed. By trial and error, it was found that by choosing $K_p = 0.65$, the analytical linear result lines up with the adaptive result, as shown in Figure 3.

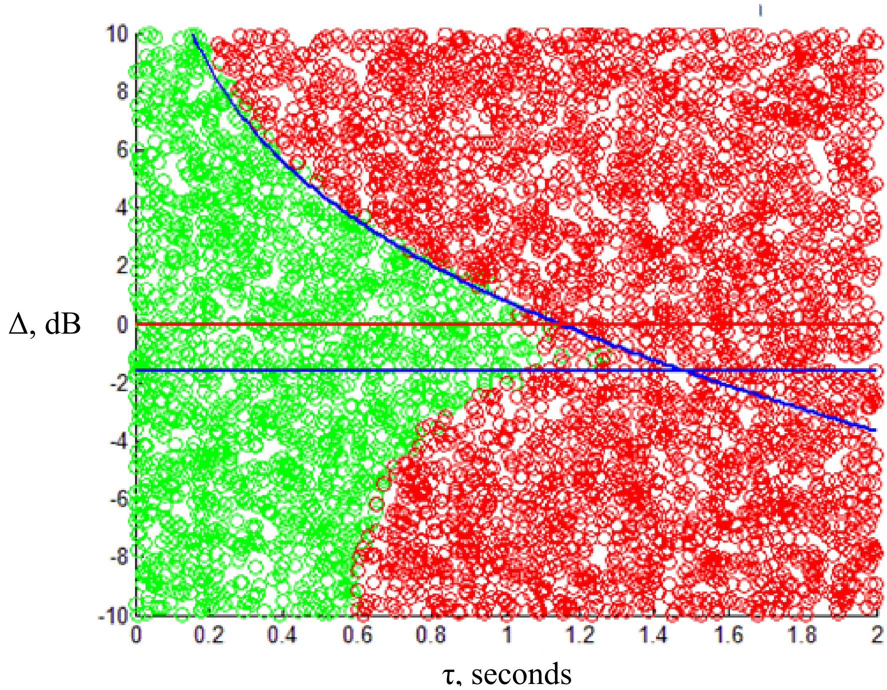

Figure 3: Adaptive Controller Monte Carlo results with overlay of linear analytical stability results with K_p = 0.65.

Note that the top level constraint is a very good match for the simulated data, though the adaptive controller cannot handle as much time delay as the linear system in the area of -2 to 0 dB of gain variation Δ. However, at very low delta and very high time delays, the linear system would also be exhibiting very poor performance and it is likely that in practice, other performance tests would be violated in this region. This result leads to the proposal that the adaptive controller's equivalent gain and phase margin must be the same level as the fit linear controller's or better under these conditions. Note that the corresponding linear system with K_p= 0.65 has an infinite upward gain margin, a downward gain margin of -3.19 dB and a phase margin of 46.2 degrees, corresponding to a time delay of 1.15 seconds. This adaptive controller is expected to have approximately the same gain and phase margin under these conditions and additionally is expected to handle smaller gain variations as well.

V. Flight Test Estimation of Time Delay Margin

Normally, it not possible in a flight test setting to produce the number of trials necessary to consider a result a true Monte Carlo analysis where probability distributions can be predicted. That said, it may still be possible to inject time delay and gain alterations in a flight test setting. One can then evaluate performance metrics to determine boundaries of allowed gain and time delay alternation and therefore margins. Clearly this is only possible or practical when there is no risk of harm to exceeding limits or otherwise failing a performance test – such as is the case for an aircraft without relevant structural limits and a backup flight control method. It also important to point out that the test provides a margin beyond and unknown time delay already present in the system. That is, it is how much delay can be added beyond what is already present in the real system under the current conditions.

Such a test was recently conducted to evaluate the proposed Adaptive Loop Recovery (ALR) method to increase the time delay margin of an adaptive controller[20]. This was possible because the aircraft utilized, a small foam airplane model called the Multiplex Twinstar, is sufficiently strong to enable carefree stability testing at low speeds. The vehicle is depicted in Figure 4. The backup flight control method is for the airplane to be flown by a human pilot.

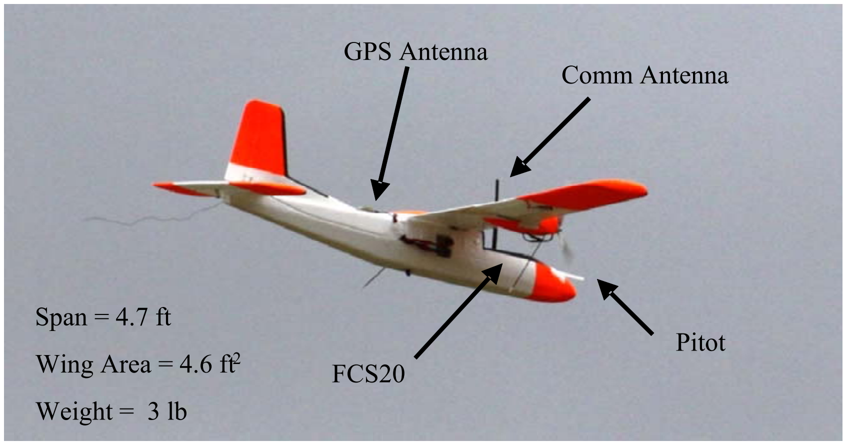

Figure 4. Small foam airplane utilized for in-flight estimation of time delay margin; primary avionics are the FCS20[21] digital autopilot.

A baseline neural network adaptive controller was developed for aircraft flight control[21]. The structure of this controller is a Model Reference Adaptive Controller (MRAC) whose adaptive element is a neural network. The ALR modification was also made, which could be turned on and off. The modification itself is a new term added to neural network training. That is, it is a change to the differential equation describing how the gains change.

These two controllers (one with ALR, one without) were tested in flight test to find their time delay margin in the pitch axis. This was done by injecting time delay in the elevator command until there was unacceptable performance. In both cases, this was manifested by a limit cycle of unacceptable magnitude or frequency. It was found that the time delay margin without ALR was 0.10 seconds. With ALR the time delay margin was 0.14 seconds.

To further explore the behavior of this modification of the gain adaptation, a time delay in between (0.12 seconds) was selected, and the ALR method was turned on and off. The results are shown in Figure 5. In this time history, ALR is initially engaged and there are no oscillations. When ALR is turned off at 2189.0 seconds, the baseline adaptive controller experiences divergent oscillations. When ALR is once again engaged at 2194.8 seconds, the aircraft is able to recover much of the performance even in the presence of the time delay.

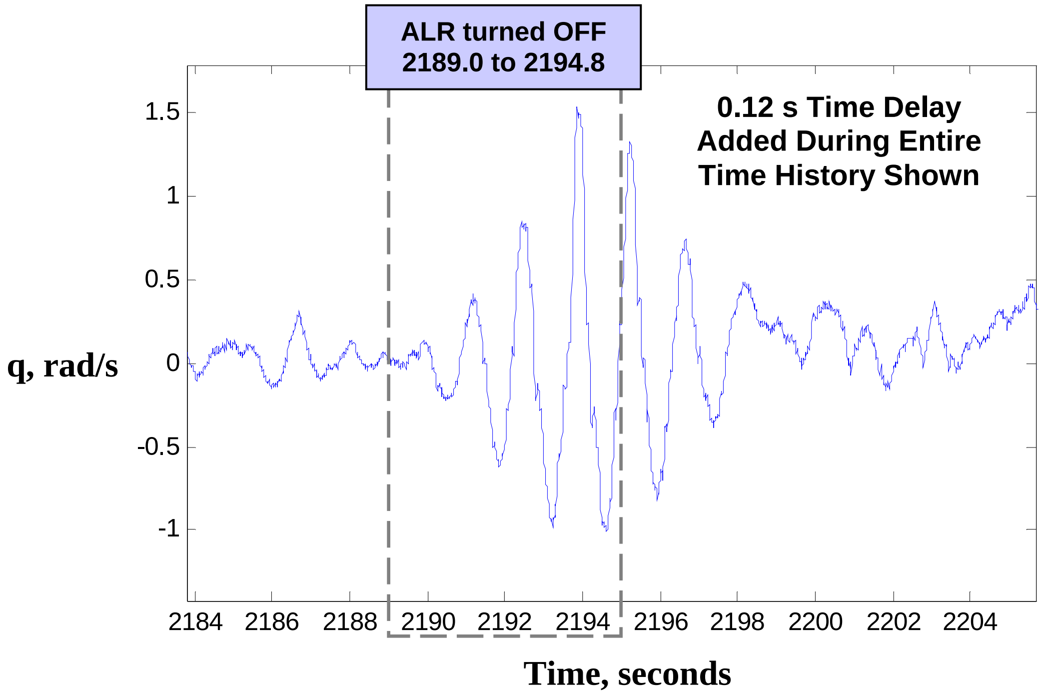

Figure 5. Flight test data showing time response (pitch rate) with injected time delay in pitch control.

This test is notable for two reasons: (1) that a repeatable time delay margin test was possible in flight test in this case; (2) the results were reasonable and consistent with the theory. In principle, the same method could be utilized for upward or downward gain margin determination. The same caveats about structural and other limitations would apply.

VI. Conclusions and Future Work

In this paper, a method for simulating adaptive controllers is presented with the potential for fitting a linear system to the adaptive simulation results to suggest worst case gain and phase margins for the adaptive controller. This initial work can be developed further through the simulation of higher order systems, and by using new performance metrics to detect stability thresholds, onset of oscillation, and control input usage. Flight test results were also presented for in-flight estimation of margins and have great potential for real-time stability margin estimation.

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