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AN OVERVIEW OF INPUT-OUTPUT ANALYSIS

A THESIS

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by

Mario Jorge Torre

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CHAPTER I

INTRODUCTION

Economic theory has as one of its purposes an explanation of the material aspects of our society in terms of interactions between a diversified number of variables. It establishes, for instance, the relation between the rates at which the means of production and the means of consumption increase.

Unlike physics, which has moved to entirely new premises by using the method of inductive reasoning from quantitatively observed events, the science of economics has maintained its position as a largely deductive system based on a set of premises which has not varied greatly. The problem of so called balances of national economy led many modern economists, such as Wassily W. Leontief, Leon Walras, Vilfredo Pareto, and Irving Fisher to criticism of the existing economic theory. Their efforts have become a point of departure for what is known today as input-output analysis. This new approach met with great opposition from qualified individuals who were acustomed to make analyses with the use of relatively simple data such as interest rates, gross national product, price and wage levels, and the like. This opposition was due not only to the forbidding rigor of the mathematics involved, but also to the fact that the initial results did not prove to be significantly superior to those obtained by traditional methods.

In compliance with the Constitution of the United States, which gives the Federal Government the task of taking a decennial census, ever since 1790 Federal statistical agencies have maintained the tradition of keeping a comprehensive quantitative record of the country's social and economic progress. These data, along with all the information obtained by private enterprises, were published exclusively for reference purposes and were unrelated to any particular method of analysis, causing the economic science at that time to have a concentration of theory without fact and a mounting accumulation of data without theory.

In order to link economic theory to facts, a procedure which could take a look at the actual world situations and analyze them in a detailed manner was necessary. This analysis would have to be able to study the intervening steps that take place from the time that a shift of wages occurs and its impact in the price structure is noticed. To satisfy this need the "Interindustry" or "Input-Output Analysis" was designed.

Essentially, the input-output analysis is a method of study which takes advantage of the partially stable pattern of the flows of goods among elements of the economy to bring a much more detailed statistical picture of the system.

It was not until the First World War that the previously mentioned agencies started to organize the data in order to fit a distinct picture of the economic system. The first time that a major use was made out of the input-output analysis was during the Second World War. The need for refined information was felt by the United States industrial production administrators when President Roosevelt ordered 50,000 airplanes. Of course, it was obvious to everyone that the country would have to increase its production of aluminum. It was not so obvious, however, that the building of aluminum potlines would produce a shortage of copper, which in turn would be met by borrowing silver from Fort Knox to make the bus bars that delivered electricity to the potlines. At this time government officials were very grateful to have at hand an input-output table of the United States economy for 1939 which consisted of only 42 sectors.

This input-output technique seemed promising enough so that an input-output table consisting of 200 sectors was constructed for 1947, with detailed statistical studies.

Work has now begun on the preparation of an input-output table for the United States economy of 1963. This table, which is expected to achieve great success, will contain approximately 600 sectors to deal with the increased complexity of our modern economy.

CHAPTER II

NATURE AND USES OF INPUT-OUTPUT ANALYSIS

2.1 Basic Concepts

The main function of the national economy is to supply goods and services required by individuals or groups of individuals for some specific purpose. In the modern complex economy, the task of producing these goods has been divided among a multitude of industries many of which have no direct contact with the consumer himself, but rather act as specialized groups which perform specific tasks for other processing industries which may or may not have a direct connection with the ultimate consumer. Since for the solution of many economic problems it is quite important to know the condition of these intermediate industries whose state may stagnate the potential of the economic system, we find it necessary to establish consistent connections between the demand for a finished product and the implication that it may have on a demand for products of these intermediate industries.

In order to solve the problem and obtain this information, a direct approach was utilized. This approach consisted in consulting with the industries that produced the finished product before a prospective increase in the demand would occur, and inquiring about an estimate of additional requirements imposed on their suppliers by this increased demand. The immediate supplier would then be visited and asked the same question and systematically return to the primary stages in order to estimate the total impact. Obviously, this task requires much time and expense, and in order to reduce this unnecessary amount of work the interindustry or input-output analysis has been designed.

Input-output analysis is a relatively new tool in quantitative economic analysis. Because of its basic concept it has been informally used in most of the planning for business production and purchasing. For example, the assembler who notes that a finished pump needs one impeller, two valves, several bolts, etc. and balances his future production and purchasing plans accordingly, is using the basic principle of input-output analysis.

The concept of interindustry analysis is very closely related to the direct attack of interviewing industries in our previous problem, "but it permits the capitalization of an expenditure for empirical research over a large number and variety of analytical applications."[1] It is based on an observation that the purchase of materials or services by a sector from other parts of the economy are related to the output of the individual sector. This is further simplified by the fact that in many cases the pattern of goods and services needed to continue with a determined production activity, exhibits strong elements of stability. For example the amount of iron ore (i.e. input) necessary to produce one ton of pig iron (i.e. output) would hardly change from year to year. One of the major virtues of the input-output analysis lies in the fact that it does not deal with empty numbers but rather allows one to maintain a constant workable relationship between concept and measurement.

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[1] W. Duane Evans and Martin Hollemberg, "The Interindustry Relations Study for 1947", The Review of Economics and Statistics, May 1952.

The input-output analysis, although very practical, does not present a theoretically complete picture of either the supply or the demand side of the economy. On the supply side, it does not consider the optimizing behavior of the economy since it is assumed that output is proportional to input, implying that there is only one prescribed procedure to produce a given product. On the demand side, optimizing is eliminated depending on whether the model considered is an open or closed input-output model.

2.1.1 Closed Model

The closed model of the input-output analysis was the first one to be devised but has been superseded by the newer open model.

In the closed form, all economic activities are considered interdependent, so that there are no final demands and no unproduced inputs. Households constitute an industry which, by using consumer goods in specific amounts (required inputs), is enabled to produce its output (i.e. labor), in the aggregate quantity required by other industries for their activities. Government is similarly treated as an industry, its outputs being its specific expenditures in fixed ratios to its input, and the distribution of that output being measured as the tax payments by the various sectors of the economy. Foreign trade is the industry that produces imports as its output, to meet the needs of domestic sectors, while its own outputs are the exports furnished by the latter.

In this system, Leontief[2] assumed coefficients with a fixed ratio of input to output, taking for numerators and denominators the figures

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[2] For Leontief's description of the theory underlying the closed model, see Wassily W. Leontief, The Structure of American Economy 1919-1939, Oxford University Press.

in money terms from the intersector flows and sector outputs. Because of the strict interdependence of this model, which assumed unrealistic functions as fixed patterns of consumption, investment, and government activity, it was necessary to open the model so that its variables would be independent, thus making the system more adaptable to practical applications.

2.1.2 Open Model

In this new model, the assumption of constant input to output ratio is dropped for a number of industries such as households, government, and so forth, and the input provided for these industries by another group of industries becomes part of the "bill of goods", better known as final demand. This model is the one which is presently in use and will be further developed in subsequent chapters.

2.2 Practical Uses

It is now the purpose of this section to point out some of the many practical uses of today's input-output analysis. These uses can be classified into two separate categories: nonanalytical and analytical uses.

2.2.1 Nonanalytical Uses

It has been pointed out that the input-output tables have a particular value independent of any analytical scheme in which they may be used. In these tables a great amount of information which has been synthesized from the Census of Manufacturers, the gross national product accounts, the foreign trade statistics of the United States, etc. has been combined with collateral information on prices, employment, productivity, and capacity, in such a way as to construct a table capable of spotlighting gaps or discrepancies in the economic system. However, the primary application of input-output analysis lies in its analytical uses.

2.2.2 Analytical Uses

Input-output analysis is considered as a relatively new approach in the economic science, and most of the people who have worked with this type of interindustry relationships claim that it provides an important new analytical tool. Making use of past transaction data, the input-output tables can yield cost ratios which indicate dollars of given inputs per dollar of given outputs; with some adjustments these ratios can be extended into the future. It is also believed that input-output tables can measure with a fair degree of accuracy changes in interindustry relations.

Let us now take a closer look at input-output analysis and consider some of its many practical uses.

2.2.3 Production Requirements Analysis

During the last few years, input-output analysis has been used to determine the additional production requirements set on the economy by an increase in the final demand of a specific sector. It has been noticed that to permit continued delivery, outside the intermediate sector, of an additional unit of output from a given sector A, additional production will be required not only in sector A but in all other sectors of the economy called on directly or indirectly to supply the needed materials or services. For example, if it is desired to increase the delivery to a dealer by one more automobile, it will be necessary to supply the automobile factory with an incremental amount of power, materials, labor, etc. The industries supplying the above automobile factory will in turn require an increase in materials, power, labor, etc. Next, the industries which supply these intermediate industries will again require more inputs, and so the chain continues. The total increase in materials requirements will be the sum of all direct and indirect increments of materials and will be the result of the production requirements analysis.

2.2.4 Factor-payment Extension of the Production Requirements Analysis

In some cases it is not necessary to know the production requirements *per se*, but rather as a function of factor payments that may be generated by an autonomous demand. In this type of analysis the main interest lies in determining what proportion of the cost incurred by a sector B for the production of one unit of its output is directly related to the cost of a sector A in producing one unit of output to satisfy an autonomous demand. For example, if the delivery of one unit of output of sector A to satisfy an autonomous demand requires 1/10 unit of the output of sector B, and if each unit of output of sector B consists of $0.50 of labor, then it may be concluded that every unit that A produces will be accompanied by $0.05 of labor in sector B.

2.2.5 Relations of Product Requirements to Employment

Up to now it is known that requirements imposed on intermediate sectors by an increase of a unit of output from any sector can be determined. It is also understood that if the demand schedule is known, then the production levels for all related industries can be calculated; these production levels are associated with factor payments. It is of great interest to realize that the implications of this relationship can be extended to other elements of the economy such as employment.

In order to make this extension, it is clear that employment and production may be related within a given sector so that it will be possible to convert statements about changes in productivity to changes in employment. The simplest use of this analysis can be made to relate the change in employment in each sector to the increase or decrease of a unit of output in a given sector. Assuming that there is no change in productivity per working hours, the desired employment information will be obtained by using an employment inverse table. Such tables would contribute to the appraisal of proposed alternative actions designed to alleviate employment and also to determine the amount and location of employment associated with current or expected patterns of exports from the domestic economy.

2.2.6 Relation of Production Requirements to other Resource Impacts

A practical application of this type of problem can be best portrayed by the following analysis. Assume that there is an adequate representation for a future processing structure. It is desired to know what conditions will a possible demand schedule impose on the rest of the economy. After this is calculated, a series of investigations will be made in order to decide if the industries that are affected by the demand schedule will have the industrial capacity to meet their respective demands. Of course, not all industries will have adequate measures for industrial capacity, but whenever the measure is available, it will be possible to compare the capacity to the expected production level. If the comparison indicates no imbalance then we can conclude that at least in this area the proposed demand schedule will not be impractical.

2.2.7 Price Equilibrium Problem

The analysis of the problem indicates that the set of price changes corresponding with a variation in the factor payment structure can be estimated. The same analogy that exists between an increase in demand for automobiles and an increase in demand for steel can exist in the transmission of changes in prices through the economy. The problem is based on the assumption that if processing and demand structure remain unchanged, then changes in price may be expected when wage levels in a designated sector are increased. By using an input-output model for this type of problem, it is assumed that there is an orderly transmission through the sectors involved of the changes in prices, each sector maintaining the same level of profit. This model can portray the necessary changes throughout the economic sectors to maintain the equilibrium of prices under the new conditions, but the actual response of the economy may be quite different.

2.2.8 Regional and Interregional Analysis

With the use of input-output analysis, rough regional inferences can be drawn from national input-output computations. If, for example, there would be a decrease in the demand for automobilies, the first repercussion would be felt by the Detroit area. Later, if this decrease in demand would not have been balanced by an increase in another type of demand, other regions which are centers of production of components necessary for the assembly of automobiles would also feel the effect of this decrease in the sales of automobiles.

A more adequate consideration of regional economic problems by the input-output method will require the availability of regional input-output data.

2.2.9 Final Demand Projections

Another important use of input-output analysis is found in the case where final demand schedules are estimated in sector detail without making reference to the gross national product. The problem that this projection is intended to answer is one of determining whether the postulated final demand is feasible within the known limits of industrial capacity and resource availability, and what levels of productivity may be required to support it.

Some other uses of input-output analysis are related to the following areas:

a - Basic Gross National Product Projections

b - Analysis of the Effect of Foreign Trade

c - Marketing Analysis

d - Industrial Mobilization Analysis

e - Forecasting Models

We may conclude that although an explanation of several common uses of input-output analysis has been presented here, there is a great number of areas in which input-output analysis may be profitably used and which have not been discussed in this synthesis.

2.3 General Assumptions of the Open Model

If a careful analysis is made of the open input-output model it will become quite obvious that this model is describing a hypothetical economy. Such an economy states that its main purpose is to produce a number of different commodities, each of which is manufactured by a different industry. Each industry uses only one process of production, and each process is one of simple addition in which a set of inputs is combined in fixed proportions to produce certain output.

In order for this model to be applicable to our economy and obtain reasonable and practical results, it will be necessary to accept a set of assumptions which will cover certain specified areas.

2.3.1 Assumptions Regarding Theory

Within this group there are three main assumptions as follows:

a - The total effect of carrying out several types of production is the sum of the separate effects.

b - The inputs purchased by each sector are only a function of the level of output of that sector.

This assumption is usually made clearer by stating that the production function is linear.

c - Each commodity is supplied by a single industry or sector of production.

Two very important corollaries can be drawn from this assumption:

1 - There is only one process used for the production of each output. This corollary eliminates the posibility of optimization since it eliminates all choices regarding the proportions in which the inputs are to be combined to produce a given output. In regard to this corollary, Paul A. Samuelson[3] states that it is not necessary to consider it due to the fact that even though production functions allow substitutions among inputs, they do not take place no matter how the final demand is changed, because the achievement

[3]Tjalling C. Koopmans, "Abstract of a Theorem Concerning Substitutionability in an Open Leontief Model", Activity Analysis of Production and Allocation , John Wiley and Sons, Inc. 1951- pgs. 142-6.

of efficiency in production always leads to a unique set of input-output ratios for each industry.

2 - Each sector has only a single primary output.

2.3.2 Assumptions Regarding the Accuracy of the Data

The statistics collected for input-output tables are appropriate for the calculation of coefficients of production.

2.3.3 Assumptions Regarding Technological Changes

The coefficients of production of the year in which the table is constructed are good approximations for the actual coefficients of the year of application.

The above assumptions are regarded as the most standard in the application of the Leontief's open input-output model. Of course, it is reasonable to expect that with certain applications, some additional assumptions for the particular cases may have to be adopted.

CHAPTER III

DATA FRAMEWORK

It is impossible to separate a discussion of input-output analysis from a consideration of the types and quality of the data on which the analysis is to be based. Some of the problems related to the selection of data, which confront the construction of an input-output table are: (a) should producer's costs or consumers prices be used; (b) how to treat secondary quantities; (c) how to handle imports of goods that are domestically produced; (d) where to charge transportation costs; (e) how to handle scrap; (f) other considerations.

It is the purpose of this chapter to present a general but comprehensive study of the procedures required to obtain the necessary data and the risks which are involved.

3.1 Concept of Transaction

In the social system, economic behavior acquires a quantitative meaning only and only when a transaction takes place; this transaction sets the limit between economic and non-economic behavior. For example, if an individual manufactures a certain article and uses it within his household, it does not in itself affect the gross national product of the country since no transaction has occurred. On the other hand, if this individual decides to sell this article, then he engages in an economic transaction and alters, at least in theory, the value of the gross national product.

If we would examine the nature of an economic transaction, it would be noticed that a typical transaction consists of four major elements. In the first place two transactors are needed. In addition, there is a flow of a commodity or service from one transactor to the other, and in exchange a counterflow of money or credit. The relationship between the money and the commodity flow is what is known as price. In Figure 1, the relationship between these four elements is clearly presented.

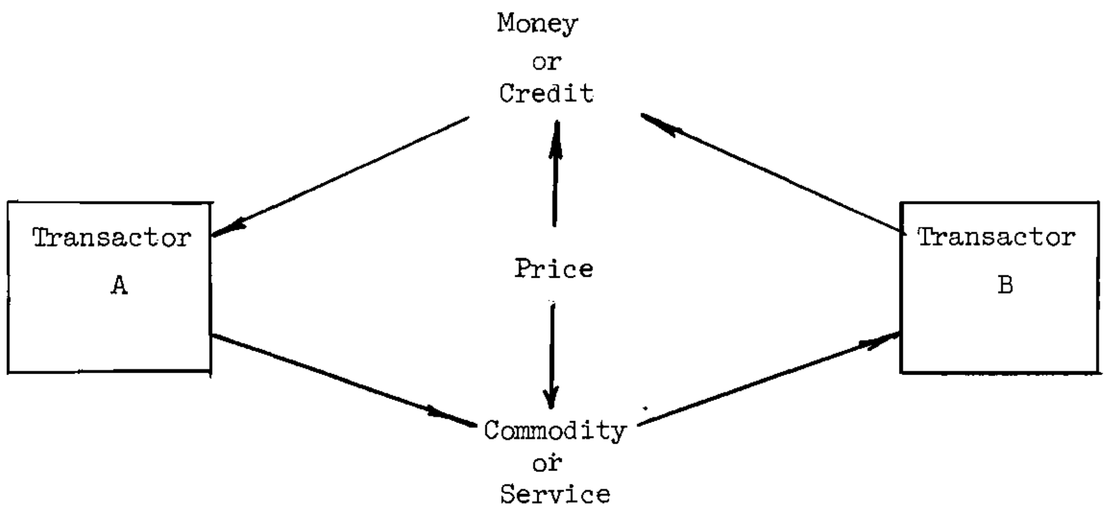

Figure 1. Basic Transaction Diagram.

3.2 Transactor Grouping

In order to condense the available data and make the task of data gathering much easier, the transactors have been grouped by two different methods. The first method groups together those transactors who, on one basis or another, are expected to exhibit similarities in functional behavior (e.g. processing or assembly). The other may attempt to group together transactors who deal with goods or services that are similar in nature. At the present time the major criterion used for grouping transactors in industrial statistics is that they should be engaged in similar functions, provided, that the products they deal with constitute a relatively homogeneous group. In forming these groups, the main objective is to maintain a simple additive measure of output which may be applied to the units making up the group.

3.3 Sector Concept

A group of transactors will be referred to as a sector of the economy. These sectors can be divided into two main categories. The first category constitutes what is known as industries which are engaged in several types of functions such as:

a - extractive

b - fabricating

c - processing

d - assembly

e - service activities.

This category of sectors does not operate independently of the rest of the system but rather in response to a given demand. In the future, this category will be referred collectively as "intermediate sectors" of the economy, but they are also known as endogenous, processing, or productive sectors.

The second category of sectors will have as their prime objective to buy goods or services as their end products. Generally, these goods will not be processed and will not be purchased with the intention of resale. In the future, this category of sectors will be known as the "autonomous sectors", the "exogenous sectors" or the "final demand sectors".

Before undertaking the study of the interrelations of the input output table, it should be noticed that the purchasing activities of an intermediate sector will be restricted in the following manner: in the first place, they will usually occur only in response to demands which take place outside the sector; second, since the sector will perform a limited range of activities, its pattern of purchases will be related to the demand.

Finally, it will be worthy to note that there will be sectors which can be identified as endogenous in some cases, and as exogenous at other occasions. Although this is quite permissible, a given sector will not appear as endogenous and exogenous at the same time.

3.4 Recording Data Transactions

Up to this point, it has been assumed that the economy is divided into two great groups composed of sectors and that for each of the intermediate sectors a measure of output can be establihsed. It is obvious that the next step consists of recording the transactions between sectors for a given period of time.

Taking a closer look at only one sector, it is noticed that such a sector can only occur in two types of transactions. In the first type, money is received due to the sale of some product or service and in the second type, money or credit is disbursed due to the acquisition of some asset. Therefore, a summary of the economic transactions of a sector should consist of both a receipts and a disbursements side. Since it may be assumed that all transactions are recorded in monetary terms, with other details as required, a monetary balance between the receipts and disbursements side of the sectors' transactions can be obtained.

In recording data transactions, two distinguishable features may be noticed; knowing how to cope with them is imperative for the proper implementation of this analysis. The first feature is associated with the fact that in some cases, the processing sectors will show some purchases that are not related to the output. These expenses will be charged to the current operating account in some cases and to the capital account in others. If we are dealing with a processing sector, it would be convenient to separate both accounts since the former is closely related to the firm's current activity levels and the second is not. Unfortunately, this cannot be done since the current statistical resources are not adequate for such an approach, and so it has been suggested as an alternative, to set up a fictious autonomous sector and combine all the charges in all intermediate sectors made to the capital accounts, which in this case would be independent of the charges made to current accounts.

The second feature of the recording of data transactions will suggest itself when a transaction between two transactors occurs with an intermediary acting as a wholesaler or distributor. This relationship is illustrated by Figure 2.

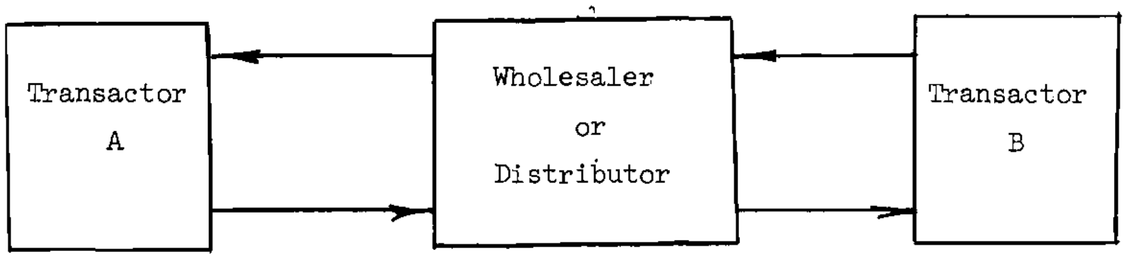

Figure 2. Transaction Diagram Based on the Presence of a Distributor

Since there is no technological necessity for this intermediary to exist, it is possible to regard the transaction as having taken place directly between the two sectors, with the producing sector engaging the services of a distributive sector to carry out the transfer. In this case, the cost of distribution will be charged to the producer and the transaction will be expressed in terms of "purchaser's price". This situation is represented in Figure 3.

The other alternative to this feature consists in having the cost of distribution charged to the purchaser. In this case, the money transferred to the producer would be less than in the previous case and the transaction would be recorded in terms of "producer's price". This situation is represented in Figure 4.

Currently, both forms of pricing can be found in economic statistics since both methods will accomplish the purpose of recording economic behavior of the producer and the user. However, once a particular method is selected, such method should be used throughout the analysis.

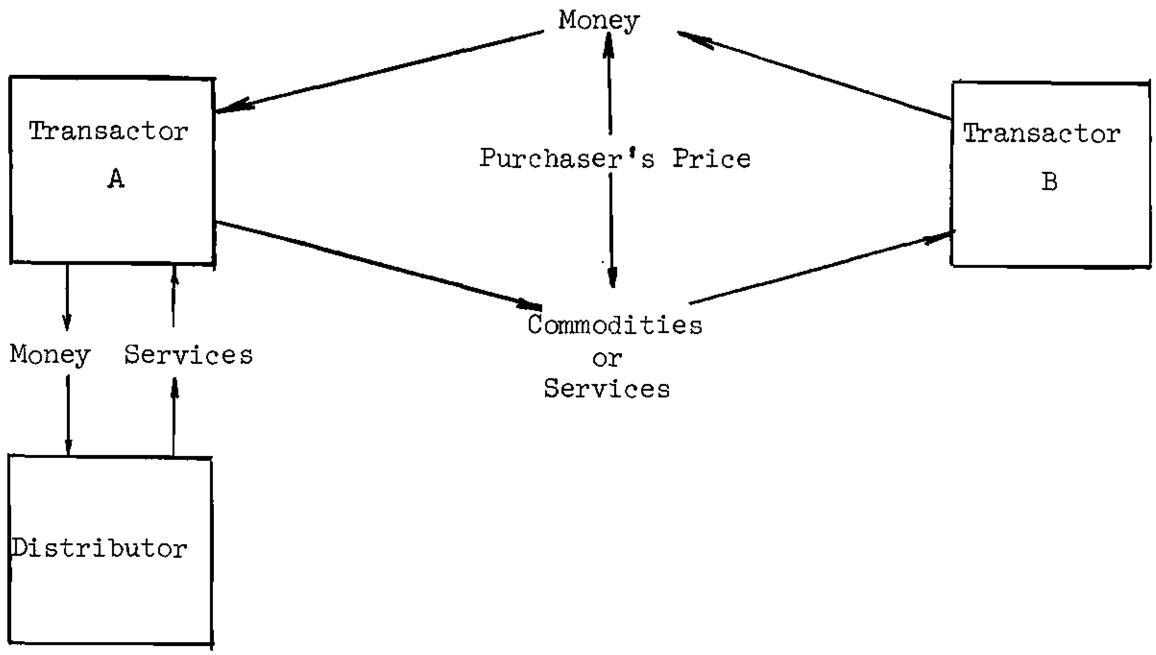

Figure 3. Transaction Diagram Based on Purchaser's Price

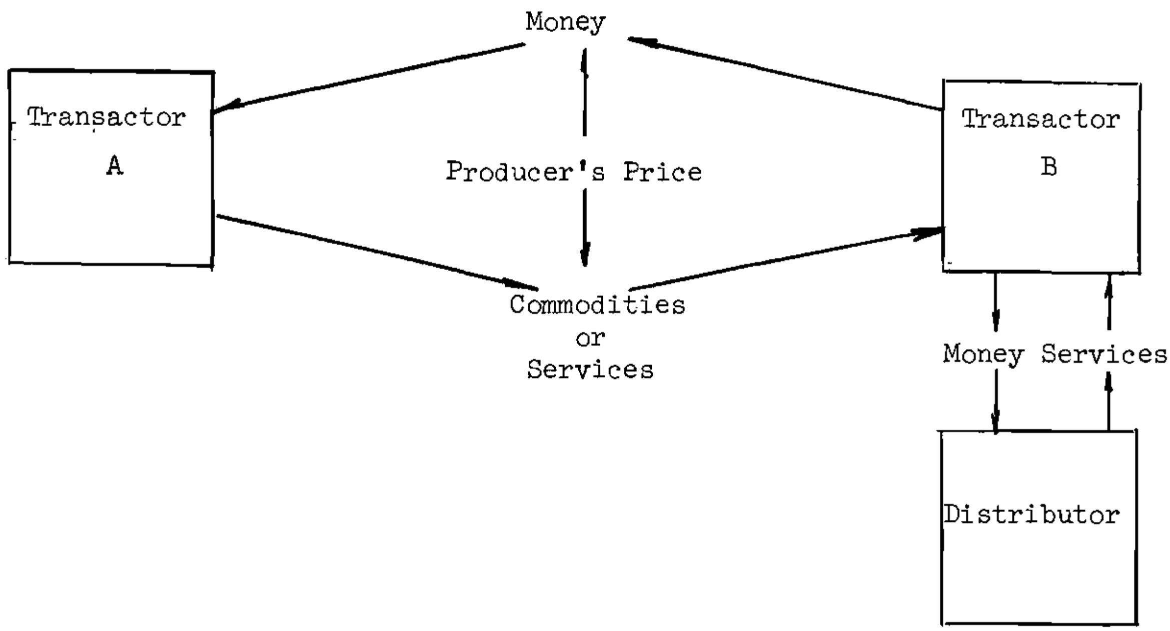

Figure 4. Transaction Diagram Based on Producers' Price

3.5 Basic Problems of Data Gathering

Upon initiating a request for information and basic data to use in an input-output analysis, it will become quite clear that some important questions will arise. Some of them are related to the following areas: accuracy of individual entries, dependence on indirect estimating, application of formal statistical methods and unification of the data.

3.5.1 Accuracy of Individual Entries

Unfortunately, the question of numerical accuracy cannot be answered as easily as it was posed. In order to know how inaccurate are the figures presented, it would be necessary to have the exact true values, but if these values could be obtained the problem of accuracy would cease to exist.

Since the problem of accuracy does exist and is very real, the first question which may be desired to answer is how closely does each figure really represent its true value. Is the amount of electricity used in the aluminum industry correctly stated? How closely does the dollar value indicating the tonnage of paint used by the automobile industry correspond to its actual magnitude? In order to answer all of these questions it will be necessary to define every term in each sector, and to know exactly what it is supposed to represent.

In an input-output table, the less aggregative[4] the manner in which the measurements are obtained, the more accurage the values will be and the firmer will be our terminological foothold. For this reason, as the

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[4] "Criteria for Aggregation in Input-Output Analysis", Review of Economics and Statistics, August 1958, v. 48, pg. 250.

number of rows and columns increases in an input-output table, the more precise the analysis will be.

3.5.2 Dependence on Indirect Estimating Procedures

Actually, none of the figures entered in the input-output tables are obtained by direct observation but rather by indirect estimating procedures.

Compilation of such a large number of organized statistical information involves the use of many special techniques which have been proven to be correct by repeated applications. These techniques are not the ones found in general textbooks but rather the ones used informally by individuals who have been working in this field for a long time and have familiarity with these particular sectors of the economy.

3.5.3 Application of Formal Statistical Methods

It has been often asked why the data used in input-output analysis are not presented with their corresponding standard deviations, coefficients of variation and other measures of statistical reliability. The reason is that in its present stage this type of economic analysis cannot make efficient use of these powerful statistical tools.

Quantitative phenomena can be subdivided into three groups. The first group is exemplified by models of classical mechanics and are formulated in terms of two, three, four or even eight variables. The second group is known as "mass phenomena" and involves very large number of variables (thousands, millions, etc.), in which statistical tools are commonly used.

The third group is of the intermediate type and consists from ten to one hundred variables. This number of variables is too large for exact observation but too limited for dependable statistical analysis. It is in this bothersome intermediary group that the field of quantitative economics is found.

Based on this reasoning it may as well be stated that the economic system can be satisfactorily defined and explained in terms of a conceptual scheme involving a large and statistically irreducible number of operationally distinct but mutually dependent variables.

Relying on the previous statement, the researcher must formulate a unified conceptual framework and then proceed with the systematic collection and organization of the necessary information. The researcher should start with a simple and direct collection of data and facts based on some overall theoretical design. After the principal parts of the analytical structure have been built, mathematical statistics may then become useful.

3.5.4 Theoretical Unification of the Data

Input-output analysis with its footing on the classical general equilibrium theory is very much oriented towards a general system approach. This approach emphasizes that the explanation of what happens in any one part or aspect of the economy is systematically connected with the explanation of what happens in each of its other sectors. For example, the study of a price system is based on the same set of factual data as the analysis of the physical quantities.

This does not mean, of course, that simplified special-purpose formulations are not used in appropriate circumstances. The more they proliferate, the more important it becomes to concentrate an ever-increasing part of our exploratory efforts on the development of an integrated, and still not overaggregative analysis of the economic system as a whole.

CHAPTER IV

ANALYTICAL FRAMEWORK

Having obtained a general conception of the scope of input-output analysis and of some of its objectives, we are ready to deal with the analytical framework of the system.

Since virtually any input-output analysis will involve one segment or another of the national economy, it may be reasonable to refer to such an analysis as a model or a structure analogue of some aspect of the complete national economic system.

Before going into the mathematical theory behind the input-output analysis it may be necessary to point out that in this study only the Leontief open model will be discussed.

4.1 Input-Output Matrix

In order to fully comprehend the significance of the inter-industry analysis, it is necessary to obtain thorough understanding of the input-output matrix.

The input-output matrix was originally intended to deal with the flows of commodities and services (rather than money) through the economic system, and this remains its main function. In this context, the flows may be in physical terms, but the heterogeneity of measurement resulting from aggregation of elementary transactors in almost every sector makes it fairly inevitable that all flows will be expressed in money terms, even though they represent physical quantities. The distribution of output from one sector to another is equivalent to the sales by the supplying sector and to the costs or purchases of the inputs by the using sectors.

The standard input-output matrix is an interlocking mechanism in which each row represents a particular sector's output in the base period. The cell at which this row intersects with any column contains the amount (in some specified unit, usually money) of the row sector's output received by the column sector. For example, looking at Table I, which is based on the Italian input-output data, under the row sector idenfified as "Basic Industry", we can see that of the total output of basic industry (150 units), 25 are used up by agriculture, 45 by establishments in the basic industry sector itself and 40 by producers of finished goods. Alternatively, each column sector's cell entries represent the amount of the row sector output received by that column sector as inputs, from the successive intersecting row sectors. For example, looking at Table I, the role of the basic industry as a purchaser of inputs is column B. In this case, the total purchased from all industries is 60 units, 15 being from the services' sector and 45 from the basic industry sector itself.

In a standard matrix, it is customary to place the endogenous (i.e. producing) sectors first: reading from the top, all endogenous sector rows are placed before any of the primary rows; and reading from left to right, all endogenous sector columns are placed before any exogenous (i.e. final demand) sector columns. The next to the last row shows the distribution of primary inputs among column sectors. The second group of columns from the left side shows the set of inputs

Table 1. Input-Output Matrix Based on Italian Data

Producing Sectors	Using Sectors S	A	B	F	Total Intermediate Use	Final Use	Total Use
S = Service	20	25	15	80	140	60	200
A = Agriculture	0	25	0	120	145	105	250
B = Basic Industry	0	25	45	40	110	40	150
F = Finished Goods	0	0	0	80	80	320	400
Total Purchase	20	75	60	320	475		
Primary Inputs	180	175	90	80		525	
Total Outputs	200	250	150	400			1000

flowing to the final demand sectors from the successive supplying sector rows.

In order to clarify the functional relationship of the input-output matrix (see Table 2), it is useful to divide the matrix into four different quadrants. The first quadrant (i.e. upper right section) includes all cells showing individual flows from the endogenous rows to the exogenous columns; that is, it contains the final use of the produced commodities and services (Y_i) broken down by major types of use (I_i, C_i, G_i, and E_i). The mathematical relationship within this quadrant is represented by the following formula:

$$Y_i = I_i + C_i + G_i + E_i \qquad (1)$$

The second quadrant (i.e. upper left section) comprises the main part of the interindustry accounts and is usually called the endogenous quadrant since all entries are flows from endogenous sectors to endogenous sectors. Each entry X_{ij} indicates the amount of commodity i used by sector j and measured in constant units. The total intermediate use of any commodity is represented by W_i:

$$W_i = X_{i1} + X_{i2} + \text{-----} + X_{ij} + \text{---} + X_{in} \qquad (2)$$

and the total purchases from other sectors by a particular industry is given by U_j:

$$U_j = X_{1j} + X_{2j} + \text{----} + X_{ij} + \text{----}\ X_{nj} \qquad (3)$$

The third quadrant (i.e. lower left section) contains inputs which are "primary" in the sense of not being produced within the system. In a

Table 2. Symbolic Input-Output Matrix

		Purchasing Sectors									
		Intermediate Use		Final Use						Supply	
Sector of the Economy		Sector 1 ... j ... n	Total Intermediate Use	Investment	Consumption	Government	Exports	Total Final Use	Total Use = Total Supply	Imports	Production
	1	$X_{11} \ldots X_{1j} \ldots X_{1n}$	W_1	I_1	C_1	G_1	E_1	Y_1	Z_1	M_1	X_1
	2	. .									
	.	.Quadrant II.		Quadrant I							
	.	. .									
Producing	i	X_{i1} X_{in}	W_i	I_i	C_i	S_i	E_i	Y_i	Z_i	M_i	X_i
Sector	.	. .									
	.	. .									
	.	. .									
	n	$X_{ni} \ldots X_{nj} \ldots X_{nn}$	W_n	I_n	C_n	G_n	E_n	Y_n	Z_n	M_n	X_n
Total Produced inputs		U_1 U_j U_n									
		Quadrant III		Quadrant IV							
Primary inputs (value added)		V_1 V_j V_n		V_1	V_C	V_G	V_E		V	V	
Total Production		X_1 X_j X_n		I	C	G	E		Z	M	X

static model the use of existing capital stock is a primary input, as is the use of the other primary factors such as labor and land. The total payment for primary inputs by each sector, therefore, corresponds approximately to the value added in production. Value added is defined as the difference between the value of output and the cost of inputs produced outside the given establishement. The term V_j will represent the total use of primary inputs by a given sector:

$$V_j = Z_j - \sum_{i=1}^{n} X_{ij}. \qquad (4)$$

The fourth quadrant (i.e. lower right section) contains the direct input of primary factors to final use, which, in Table 2 consists of investment, consuption, government and exports. These transactions do not enter into most interindustry models, but they should be recorded to make the totals consistent.

The last two columns of Table 2 break down the total supply of each commodity between imports and domestic production. These imports may be treated as a deduction from final use or may be added to the primary inputs. If each commodity is only produced by one sector and there are no joint products, the total supply of commodity i is equal to the production in sector i plus imports to sector:

$$Z_i = M_i + X_i \qquad (5)$$

The structure of the input-output table can be expressed by the following symbols:

Z_i = Total supply of commodity i.

X_i = Total production of commodity i.

M_i = Imports of commodity i.

X_{ij} = Amounts of commodity i used in sector j.

Y_i = Final demand for commodity i.

W_i = Total intermediate use of commodity i.

U_j = Total use by sector j of inputs purchased from other industries.

V_j = Total use of primary units in sector j.

These symbols may be combined to express the two balance equations for the input-output matrix. The first balance equation applies to rows in Table 2 and states that for each commodity total supply is equal to total demand, which in turn, is composed of intermediate demand plus final demand:

$$\underbrace{Z_i = M_i + X_i}_{\text{Supply}} = \underbrace{\sum_{j=1}^{n} X_{ij} + Y_i}_{\text{Demand}} = W_i + Y_i \qquad (6)$$

The second balance equation applies to columns and states that the total production in each sector is equal to the value of the inputs purchased from other sectors plus the value added in that sector:

$$X_j = \sum_{i=1}^{n} X_{ij} + V_j = U_j + V_j \qquad (7)$$

Within the input-output matrix there are totals and relationships which have analytic and economic meaning. As expressed in the first balance equation, the row and column totals are the same for each endogenous sector. Also, the value of its output equals the value of its sum of inputs. For the exogenous and primary input sectors the situation is not the same. Here, the equality of any particular row total with any column total is not logically necessary or factually probable, even where the titles of a row and a column sector specify a common set of transactors. The third quadrant contains the primary inputs used by endogenous sectors. Now, if all endogenous sectors are producing units and if the exogenous column sectors include only goods purchased by transactors conventionally identified as final buyers (i.e. government, exports, consumption, etc.) then the sum of inputs to the exogenous sectors is the gross national product plus imports, which, in the matrix, consists of all entries in the first and fourth quadrant. On the other hand, the sum of primary inputs distributed to all sectors consists of all entries in the third and fourth quadrant. It is evident that the two totals, i.e. final good purchased and primary input supplies, are equal. If the value of imports is subtracted from the sum of the exogenous sectors' total inputs, the result is exactly the gross national product. As an additional parallel, the surplus or deficit in the balance of trade is shown by subtracting the total of the imports row from the total of the exports column in the exogenous group.

From these relationships, it is easy to demonstrate that

$$\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} + \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} M_i \qquad (8)$$

and adding across all columns we have:

$$\sum_{j=1}^{n} X_j = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} + \sum_{j=1}^{n} V_j \qquad (9)$$

and since,

$$\sum_{i=1}^{n} X_i^{\cdot} = \sum_{j=1}^{n} X_j^{\cdot} \tag{10}$$

we may combine equations (8), (9), and (10), to obtain the expression for the basic national account:

$$\sum_{i=1}^{n} Y_i^{\cdot} - \sum_{i=1}^{n} M_i^{\cdot} = \sum_{j=1}^{n} V_j \tag{11}$$

This arithmetical analysis underlies the view that an input-output matrix is an expanded version of the standard social accounts, with the difference that it includes intermediary sector flows.

4.2 The Basic Model

With the understanding that the main purpose of the input-output model is to explain the magnitude of the interindustry flows in terms of the levels of production in each sector, we are now ready to present equations for the system representing an open model of goods (or money) flows.

It is essential to distinguish between an input-output system of goods flows in terms of a table or a set of balance equations, and the important matrix of coefficients that may be derived from the above table and used for prediction or analysis. The flow of goods table is a historical picture which essentially involves a set of definitions expressed in terms of equations but which portrays no projective relevance or empirical assumptions. However, with the assumption that: (a) constant returns to scale exist; (b) no process produces more than one product, and (c) a given product is only supplied by one sector, Leontief has brought this system to the point of transition from a mere interrelated table to a predictive application. The crossover involves the hypothesis that the term "a_{ij}" is not simply a chance average input of i per unit of output of j, but a relatively stable ratio resulting from a characteristic and continuing technology in the production of the j^{th} good output.

In order to carry out a conceptual representation of the open system model, we will let Z_1, Z_2, Z_i, ... Z_n stand for the totals of producible outputs, and Z_o stand for the total of a primary nonproduced good such as labor; let Y_1, Y_2, ... , Y_n stand for the total final consumption of each of the produced goods; let X_{ij} stand for the amount of input of the i^{th} good used in the production of the j^{th} good output. Therefore, X_{oj} represents the labor used for the production of the output of good j.

The first relation which will be considered is that the total output of any good, Z_i, is the sum of final consumption Y_i and intermediary inputs X_{ij}, so that:

$$Z_i = X_{ij} + X_{i2} + \text{---} X_{in} + Y_i \tag{12}$$

Making use of the assumptions that no process produces more than one product, and that there exist constant returns to scale, it is possible to relate the total output of a good to its inputs so that:

$$Z_j = F\ (X_{oj}, X_{ij}, X_{nj}) \tag{13}$$

and assuming that each input X_{ij} is required in fixed proportions to the output Z_j, we obtain the technical coefficient which is derived by dividing each interindustry purchase by the total output of the sector, such that:

$$a_{ij} = \frac{X_{ij}}{Z_j} \qquad (14)$$

This coefficient represents the ratio of the input unit required per output unit. Combining equations (12) and (14) we obtain for each variable:

$$Z_i = a_{11}Z_1 + a_{12}Z_2 + Y_1$$

$$= \sum_{j=1}^{n} a_{ij}\, Z_j + Y_1 \quad \text{for } i = 1,2,\text{--}n \qquad (15)$$

It is now possible to formulate a procedure which will enable us to determine the gross outputs which would meet specified final consumption requirements. In order to clarify this presentation we will assume that we are dealing with a very simple economy which consists of only two producer industries and one primary industry. Such economy may be represented by Table 3.

Table 3. Simplified Input-Output Matrix

	Inputs to Industry I	Inputs to Industry II	Final Demand	Total Output of Industry
Industry I	X_{11}	X_{12}	Y_1	Z_1
Industry II	X_{21}	X_{22}	Y_2	Z_2
Labor	X_{01}	X_{02}		Z_o

Dividing each term X_{ij} by the total output Z_j, a set of technical coefficients for this economy is portrayed in Table 4.

Table 4. Technical Coefficients

	Input to Industry I	Input to Industry II	Final Demand	Total Output of Industry
Industry I	a_{11}	a_{12}	Y_1	Z_1
Industry II	a_{21}	a_{22}	Y_2	Z_2
Labor	a_{01}	a_{02}		Z_o

Starting with the final demands Y_1 and Y_2 it will be possible to arrive at the total production required with the aid of the following procedure: to the final demand Y_1 of the commodity produced by industry I, we add the "first round" input requirements of commodity I, which is:

$$a_{11} Y_1 + a_{12} Y_2 \tag{17}$$

We then proceed to the "second round" of inputs of commodity I into each of the four first round inputs. For commodity I we would have:

$$a_{11} (a_{11} Y_1 + a_{12} Y_2) + a_{12} (a_{21} Y_1 + a_{22} Y_2) \tag{18}$$

and for commodity II:

$$a_{21} (a_{11} Y_1 + a_{12} Y_2) + a_{22} (a_{21} Y_1 + a_{22} Y_2) \tag{19}$$

Next, we may go to the "third round" inputs and so on, until we arrive at the k^{th} round input. However, it is evident from the form of the first and second round terms established previously, that the general solution will be of the form:

$$Z_1 = Y_1 + a_{11}Y_1 + a_{12}Y_2 + ---- + a_{11}(a_{11}Y_1 + a_{12}Y_2)$$

$$+ a_{12} (a_{21}Y_1 + a_{22}Y_2) \qquad (20)$$

When algebraically manipulated equation (20) will acquire the following form:

$$Z_1 = Y_1 + a_{11}Y_1 + a_{11}^2 Y_1 + a_{12}a_{21}Y_1 + ----$$

$$+ a_{12}Y_2 + a_{11}a_{12}Y_2 + a_{12}a_{22}Y_2 + ----$$

$$Z_1 = (1 + a_{11} + a_{11}^2 + a_{12}a_{21} + ----) Y_1 +$$

$$(a_{12} + a_{11}a_{12} + a_{12}a_{22} + ----) Y_2 \qquad (21)$$

Letting A_{11} substitute for the term $(1 + a_{11} + a_{11}^2 + a_{12}a_{21} + ----)$ and A_{12} for the term $(a_{12} + a_{11}a_{12} + a_{12}a_{22} + ----)$ in equation (21), we arrive at:

$$Z_1 = A_{11}Y_1 + A_{12}Y_2 \qquad (22)$$

and similarly for commodity II:

$$Z_2 = A_{21}Y_1 + A_{22}Y_2 \qquad (23)$$

With the aid of equations (22) and (23) it is clearly seen that the gross outputs are linear functions of the final demands. However, it is desirable to find an easier way to evaluate the A's. To do so we construct Y_1 in two steps: (1) the final consumption itself Y_1 and

(2) the supplementary demand. The gross output necessary to support this supplementary demand will be $A_{11}(a_{11}Y_1 + a_{12}Y_2) + A_{12}(a_{21}Y_1 + a_{22}Y_2)$, which when added to final consumption Y_1, gives:

$$Z_1 = Y_1 + A_{11}(a_{11}Y_1 + a_{12}Y_2) + A_{12}(a_{21}Y_1 + a_{22}Y_2) \quad (24)$$

$$= (1 + A_{11}a_{11} + A_{12}a_{21})Y_1 + (A_{11}a_{12} + A_{12}a_{22})Y_2 \quad (24a)$$

Similarly:

$$Z_2 = Y_2 + A_{21}(a_{11}Y_2 + a_{12}Y_2) + A_{22}(a_{21}Y_1 + a_{22}Y_2) \quad (25)$$

$$= (A_{21}a_{11} + A_{22}a_{21})Y_1 + (1 + A_{21}a_{12} + A_{22}a_{22})Y_2 \quad (25a)$$

Having obtained equations (22), (23), (24a), and (25a) we now have two ways of calculating the values of Z_1 and Z_2. Since both of these methods give the same result, it is possible to compare corresponding coefficients:

$$A_{11} = 1 + a_{11}A_{11} + a_{21}A_{12}$$

$$A_{12} = \quad a_{12}A_{11} + a_{22}A_{12} \quad (26)$$

or

$$(1 - a_{11})A_{11} - a_{21}A_{12} = 1$$

$$- a_{12}A_{11} - (1 - a_{22})A_{12} = 0 \quad (27)$$

By substitution, equations (26) and (27) can be solved to obtain the values of A_{11} and A_{12}:

$$A_{11} = \frac{1 - a_{22}}{(1 - a_{11})(1 - a_{22}) - a_{12}a_{21}} \tag{28}$$

$$A_{12} = \frac{a_{12}}{(1 - a_{11})(1 - a_{22}) - a_{12}a_{21}} \tag{29}$$

For Z_2, we obtain:

$$A_{21} = \frac{a_{21}}{(1 - a_{11})(1 - a_{22}) - a_{12}a_{21}}$$

$$A_{22} = \frac{1 - a_{11}}{(1 - a_{11})(1 - a_{22}) - a_{12}a_{21}} \tag{30}$$

Substituting the values obtained for A_{11}, A_{12}, A_{21}, and A_{22} in equations (22) and (23), we arrive at our final solution for the computation of the total output based on the final demand:

$$Z_1 = \frac{1 - a_{22}}{(1-a_{11})(1-a_{22}) -a_{12}a_{21}} Y_1 + \frac{a_{12}}{(1-a_{11})(1-a_{22}) -a_{12}a_{21}} Y_2 \tag{32}$$

$$Z_2 = \frac{a_{21}}{(1-a_{11})(1-a_{22}) -a_{12}a_{21}} Y_1 + \frac{1 - a_{11}}{(1-a_{11})(1-a_{22}) -a_{12}a_{21}} Y_2 \tag{33}$$

In order to appreciate the results obtained with the previously derived formulas a numerical example will be presented. This example will assign specific values to the simplified economic system shown in Table 4. These values are given in Table 5.

Table 5. Simplified Input-Output Matrix with Numerical Values.

	Inputs to Agriculture	Inputs to Manufacturing	Final Demand	Total
Agriculture	25	175	50	250
Manufacturing	40	20	60	120
Labor	10	40	0	50

Table 6 shows a set of technical coefficients based on data given in Table 5.

Table 6. Technical Coefficients for Simplified Input-Output Matrix.

	Inputs to Agriculture	Inputs to Manufacturing	Final Demand	Total
Agriculture	0.10	1.46	50	250
Manufacturing	0.16	0.17	60	120
Labor	0.04	0.33		50

From equations (28), (29), (30), and (31), the values for A_{11}, A_{12}, A_{21}, and A_{22} are obtained as follows:

$$A_{11} = \frac{1 - 0.17}{(1-.1)(1-.17) - (1.46)(0.16)} = \frac{0.830}{0.513} = 1.61$$

$$A_{12} = \frac{1.46}{(1-.1)(1-.17) - (1.46)(0.16)} = \frac{1.46}{0.513} = 2.84$$

$$A_{21} = \frac{0.16}{(1-.1)(1-.17) - (1.46)(0.16)} = \frac{0.160}{0.513} = 0.31$$

$$A_{22} = \frac{1 - .1}{(1-.1)(1-.17) - (1.46)(0.16)} = \frac{0.9}{0.513} = 1.75$$

Substituting these values in equations (22) and (23), we obtain the total output necessary to satisfy the specified demand :

$$Z_1 = (1.61)(50) + 2.84\ (60) = 250.9$$

$$Z_2 = (0.31)(50) + 1.75\ (60) = 120.5$$

The values obtained in this fashion are very close to the ones already available in Table 6, thus proving the usefulness of this procedure in solving this type of input-output problems. However, it is apparent that as the size of the input-output matrix increases, the expressions for the A_{ij} factors become more and more complex.

4.3 Price Relationship

The Leontief input-output model consists of n identities, each specifying that the value of the output equals that of the cost of inputs plus profits.

Let p_i be the price of the product of industry i, p_j the unit cost of product j used by industry i, w the wage rate, and q_i the profits per unit of output in industry i. Let $a_{n+1,i}$ be the labor input coefficient of industry i. With this set of variables the unit cost of product i will be denoted by:

$$\sum_{j=1}^{n} a_{ij}\ p_j + wa_{n+1,i} \qquad (34)$$

and the price for one unit of cost i will be:

$$p_i = \left(\sum_{j=1}^{n} a_{ij} \, p_j + w a_{n+1,i} \right) + q_i \qquad (35)$$

If we consider a long-run competitive equilibrium, profits vanish in all industries and the previous equation reduces to:

$$p_i = \sum_{j=1}^{n} a_{ij} \, p_j + w a_{n+1,i} \qquad (36)$$

CHAPTER IV

CONCLUSIONS

5.1 Interpretation of the Results

The end result of the input-output computations considered in this analysis is a set of production figures which represent the levels of activity required from the different sectors of the economy under the assumed conditions.

The computations are usually carried out by making use of coefficients of production which in some cases will provide the answer to the problem initially posed. For other problems, however, the coefficients of production will determine the production levels which in turn may only be an intermediate step in solving for the problem of interest. Real interest, for example, may lie in the employment implications of the conditions initially assumed, in which case several estimates of productivity may be combined with production figures to yield industry employment estimates.

In general, it is quite acceptable to state that although the general methodology of input-output analysis remains similar throughout different types of studies, the significance of the results varies according to the statement of the problem.

5.2 Evaluation of the Worth of Input-Output Analysis

One of the important uses that an analytical tool can have is the ability to make good predictions of important facts, and this is one of the criteria on which input-output analysis may be judged. This judgment may be viewed from a theoretical and from an empirical point of view.

5.2.1 Theoretical Evaluation

From the point of view of accepted economic theory, input-output analysis is condemned. The reason is that the accepted theory states that the proportions in which inputs are combined in production depend upon the relative prices of the inputs; however, input-output analysis has violated this law by assuming that input proportions are fixed technologically. Nevertheless, the defenders of input-output analysis declare that this assumption does not nullify its validity since even though substitution among inputs takes place, it is inconsequential enough in many occasions. Another defense of the input-output analysis is that the analyst may adjust his technical coefficients to take into account any important substitution that may take place.

5.2.2 Empirical Evaluation

The usual terminology which has been adopted is perhaps misleading in stating that the input-output model determines industry outputs and primary inputs required for a given final demand. The certainty of these requirements holds only so far as the formal analysis is concerned: given a specified set of final demands, the model implies precise output solution values. How close the latter comes to the actual industry outputs associated with such a bill of goods is a matter for factual test, and depends on the empirical validity and sufficiency of the theoretical assumptions with regard to the real economic processes.

The value of the input-output table showing the flow of goods and services from one industry to another is almost negligible from an analytical point of view. Such a table, which may be constructed by having a classification scheme and a great quantity of data makes no assumptions about substitution of resources or constant returns to scale and does not consider production functions. The input-output table is only a contribution to the factual knowledge of the economic system. However, the input-output analysis has a much different and significantly more meaningful value as determined in our previous discussion.

Although input-output analysis is of great practical value, there are numerous shortcomings which are inherent to such an analysis. Some of these faults are noticeable when it is realized that a simple Leontief model would never be able to explain why in periods of prosperity the food prices rise relative to other prices.

5.3 Recent Developments in Input-Output Analysis

The philosophy of input-output analysis has been incorporated in one of the most ambitions research projects concerned with the structure of the American economy. This research study, known as "The Brookings Quarterly Econometric Model of the United States" classifies the economy into 45 sectors and then quantifies the flows between the sectors by means of econometric equations. These equations include wage rate equations, production functions, hours worked equations, price equations, and final demand regressions. In other words, the Brookings model capitalizes on the empirical strengths of input-output analysis and minimizes its theoretical weaknesses by introducing time-dependent functional relationships between model variables. Due to its ingenious concepts, the Brookings model can be used to give quantitative answers to such problems as forecasting, simulation of the effects of policy actions, and simulation of business cycle theories.

In final conclusion, it may be stated that the introduction of econometric methods into the input-output analysis eliminates most of its theoritical flaws. With the addition of econometrics, the range of applications of input-output models covers not only the study of systems structure but also their behavioral aspects.

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