Resource Modeling and Allocation in Competitive Systems

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Resource Modeling and Allocation in Competitive Systems

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SUMMARY

This thesis includes three self-contained projects:

In the first project "Bidding strategies and their impact on the auctioneer's revenue in combinatorial auctions," focusing on combinatorial auctions, we propose a simple and efficient model for evaluating the value of any bundle given limited information, design bidding strategies that efficiently select desirable bundles, and evaluate the performance of different bundling strategies under various market settings.

In the second project "Retailer shelf-space management with promotion effects," promotional investment effects are integrated with retail store assortment decisions and shelf space allocation. An optimization model for the category shelf-space allocation incorporating promotion effects is presented. Based on the proposed model, a category shelf space allocation framework with trade allowances is presented where a multi-player Retailer Stackelberg game is introduced to model the interactions between retailer and manufacturers.

In the third project "Supply-chain oriented robust parameter design," we introduce the game theoretical method, commonly used in supply-chain analysis to solve potential conflicts between manufacturers at various stages. These manufacturing chain partners collaboratively decide parameter design settings of the controllable factors to make the product less sensitive to process variations.

CHAPTER I

INTRODUCTION

This thesis includes three self-contained projects, which covers interdisciplinary areas ranging from optimization, statistical analysis, quality engineering, game theory to mechanical engineering.

The first project "Bidding Strategies and their Impact on the Auctioneer's Revenue in Combinatorial Auctions" is finished with Professor Pinar Keskinocak and Professor Wedad Elmaghraby. Focusing on combinatorial auctions, we made the following contributions:

- Propose a model for evaluating the value of any bundle given pair-wise synergies (limited information).
- Design bidding strategies that efficiently identify desirable bundles.
- Evaluate the performance of different bundling strategies under various market settings.
- Provide answers to pertinent questions, such as, how does the auctioneer's revenue change as more bidders submit bundle bids, how are revenues distributed among bidders in combinatorial auctions versus non-combinatorial auctions, and what issues should bidders consider when generating and pricing bundles under various market environments.

This is a simulation-based project and heuristics are proposed to design the bundling strategies. A journal paper has been published in Journal of Revenue and Pricing Management based on the results of this project.

The second project "Retailer Shelf-space Management with Promotion Effects" is a joint work with Professor Jye-Chyi Lu and Professor Faiz Al-Khayyal. In this research, promotional investment decisions from both the retailer and manufacturers are integrated with retail store assortment decisions and shelf space allocation. We made the following contributions:

- Present an optimization model for the category shelf-space allocation incorporating retailer's promotional investment decisions. An optimization method is provided to solve the problem.

- Introduce the impact of trade allowances from manufacturers into the retailer's shelf space allocation decision model, where a multi-player Retailer Stackelberg game is introduced to model the interactions between the retailer and manufacturers. This is the first time that trade allowances are analyzed in the context of shelf space allocation problems.

- Demonstrate potentials of the proposed method with the real data collected from a retailer. We also investigate properties of the solutions through the quantitative analysis of numerical examples.

This research includes interdisciplinary areas. Optimization methods are applied to model and solve the shelf space allocation problem. Statistical analysis methods are introduced to build the manufacturers' trade allowances response functions. Game theory is utilized to model the interactions between the retailer and manufacturers.

Both of the first two projects solve the resource allocation problems, problems of assigning available resources among competing identities. The first project focuses on analyzing the problem from the bidders' perspectives, where bidders compete for resources in the context of auctions, while the second project analyzes the problem from the resource owners' perspectives through optimization methods.

The third project "Supply-chain Oriented Robust Parameter Design" is a joint work with Professor Jye-Chyi Lu. We introduce the game theoretical method, commonly used in supply-chain analysis to solve potential conflicts between manufacturers at various process stages. These manufacturing chain partners collaboratively decide parameter design settings of the controllable factors to make the product less sensitive to process variations. This research contributes to the literature in the following aspects:

- Propose methods of solving robust parameter design problem in a single stage.
- Analyze multi-stage robust parameter design problems, where variance interactions between various stages are studied. This is the first time that the robust parameter design is analyzed cross multiple stages.
- Model the interaction between the robust parameter design in various stages with Stackelberg game.
- Provide real-life examples to demonstrate the potential of the proposed method.

This research is also a combination of knowledge from multiple areas, where the two major areas are quality engineering and mechanical engineering. Optimization models are presented to solve the robust parameter design problem. Regression analysis is applied to build the response model. Game theory is used to model the interactions between the multiple stages.

CHAPTER II

BIDDING STRATEGIES AND THEIR IMPACT ON THE AUCTIONEER'S REVENUE IN COMBINATORIAL AUCTIONS

2.1 Introduction

In markets where capacity and services are being auctioned, natural complementarities may exist across items. Two items are complements (exhibit synergies) when their combined value is larger than the sum of their independent values. Slots of capacity may be complements since there are economies of scale in the transaction costs (e.g., material handling, documentation, and tracking). Lanes in a transportation network may be complements if a group of lanes (e.g., if they are geographically close or form continuous routes) can lead to higher efficiency for a carrier. In combinatorial auctions (CA), a bidder can express his synergies among items by submitting bids on groups (or bundles) of goods, and wins either all or none of the items in a bundle. For these reasons, the use of CA in industrial settings has increased of late. For example, Sears Logistics Services (SLS) ([7]) and The Home Depot, Inc. ([4]) used CA for procuring logistics services. Sears Logistics saved over $84 million running six CA.

While the ability to submit bundle bids would appear to be a great advantage to bidders, surprisingly, in many applications of CA, most bidders do not submit bundle bids. For example, analyzing the data we received from a company which has run combinatorial auctions for transportation services, we found that in a single-round auction for 140 lanes, only 5 out of 46 bidders submitted bundles, and only 18 bundle bids were submitted compared to 2398 single-item bids. Industry observers explain this situation by the bidders' lack of understanding on how to bid in CA ([10]), which is mainly due the novelty of large CA and the complexity of identifying profitable bidding strategies. The distinguishing feature of CA is that the number of possible bundles is exponential in the number of items. In addition to the complexity of knowing their valuations over all possible bundles, the bidders face the problem of deciding *which* bundles to submit. Evaluating and submitting all possible bundles would be prohibitively time consuming both for the bidders and the auctioneer, who needs to solve the winner determination problem, which is NP-hard[1]. Therefore, it is of practical importance to develop bidding strategies that are efficient and effective, i.e., can be computed in reasonable time and result in profitable allocations for the bidders by identifying a set of bundles which best represent their preferences.

Some companies such as Logistics.com provide carriers with software tools to allow them visualize the shipper's network and the groups of lanes a carrier might consider for bidding. However, to the best of our knowledge, there are no tools available which "suggest" to bidders on which bundles to bid. In addition, research on bidding strategies has been very limited; much of the previous research focuses on multi-round CA, while single round CA are commonly used in practice. For multi-round CA, a few papers (e.g., [6], [9], [14]) consider a myopic best response bidding strategy where in each round bidders select new bundles to submit to maximize their utility given the current ask prices for bundles or items; these papers assume that the bidders know their values for all possible bundles. For single-round CA, Berhault *et al.* ([1]) propose combinatorial bidding strategies to coordinate a team of mobile robots to visit a number of given targets in partially unknown terrain. Their experimental setting emphasizes the uncertainty in information on valuations and its effect on simple bidding strategies. Song and Regan ([13]) present a bidding strategy for the procurement of freight transportation contracts. Without any information from competitors, the carriers (bidders) enumerate all feasible bundles. The cost of each bundle is determined by the empty moves of the truck. Each carrier then solves a set covering problem to select a subset of bundles with the objective of minimizing the total cost subject to the constraints that each lane is covered at least once, and then submits the selected bundles to the auctioneer.

In this chapter, we (i) propose a simple model for evaluating the value of any bundle

[1] With some special bundle structures, the winner determination problem is polynomially solvable ([11]).

given pair-wise synergies (limited information); (ii) design bidding strategies that efficiently identify desirable bundles; (iii) evaluate the performance of different bundling strategies under various market settings, and (iv) provide answers to pertinent questions, such as, how does the auctioneer's revenue change as more bidders submit bundle bids, how are revenues distributed among bidders in CA versus non-CA, and what issues should bidders consider when generating and pricing bundles under various market environments. Motivated from the transportation industry and their current practices, our focus is on single-round, first price, sealed-bid forward CA ([3]). Since there is a one to one correspondence between forward auctions and reverse auctions, our methodology and results are also applicable to reverse auctions.

Table 1: Notation

$N; n$	:	Set of individual items auctioned; size of N ($\|N\|$).
B	:	Bundles of items.
m	:	Number of bidders.
v_j^i	:	The value of item j for bidder i.
v_j^{-i}	:	The average value of item j for all bidders other than i.
$Syn^i(j,k)$	:	The pairwise synergy value between items j and k for bidder i.
V_B^i	:	The value of bundle B for bidder i.
AC_B^i	:	Average individual value of items in B + Average pairwise synergy of items in B, for bidder i.
VR_B^i	:	Value ratio of bidder i for bundle B.

2.2 Synergy Model

The key input to any bidding decision support tool (or algorithm) for a CA is the bundle values (how much a bidder values a bundle). Since it would be prohibitively time consuming for a bidder to compute all possible bundle values, it is desirable to have an *efficient* method for estimating them with *limited* input. In this chapter, we present a synergy model which takes item values and pairwise synergy values as the input and returns the bundle values for any combinations.

We know of two other papers that attempt to generate bundle bids from limited information. Using five real-world situations, including a transportation auction, as their motivation, Leyton-Brown *et al.* ([8]) assume that the bid price for a path from A to B is equal to the Euclidean distance from A to B multiplied by a random number, drawn from a uniform distribution. Addressing the bidding behavior in the FCC spectrum auctions, Gunluk *et al.* ([5]) collected data from the FCC's non-CA auctions. Using a simple synergy model (for 12 items) on this data, they approximate bundle bid prices for spectrum licenses. Their main goal is to generate hard instances of the winner determination problem.

To the best of our knowledge, this is the first time that a generic synergy model is presented to generate bundle values, as opposed to bids, for a general market environment.

A bundle value is comprised of two parts: the values of the individual items in the bundle and the "synergy" values among the items in the bundle. Given potential complementarities (or substitutability) across any two items, the minimal information one would need to compute a bundle's synergy value would be the pairwise synergies. We propose a simple model which uses as input only the individual item values and the pairwise synergies. According to this synergy model, for a singleton bid, the bundle value is the item value. For a doubleton bid, the bundle value is the sum of the two item values plus the pairwise synergy value between them. The value of a bundle with bundle size > 2 is computed as follows:

$$\begin{aligned} V_B^i &= \sum_{j=1}^{|B|} v_j^i + \text{Synergy Value of Bundle B} \\ &= \sum_{j=1}^{|B|} v_j^i + \frac{2}{(|B|-1)} \sum_{j=1}^{|B|} \sum_{k>j} Syn^i(j,k) \\ &= |B| * (\text{Average Item Value in B} + \text{Average Pairwise Synergy Value in B}) \\ &= |B| * AC_B^i \end{aligned}$$

where AC_B^i is the average unit contribution of B. We assume that $Syn^i(j,k) \geq 0$. Computing V_B^i is very efficient ($O(n^2)$); furthermore, V_B^i possesses the desirable trait that it does not have a bias for large bundles over small bundles or vice versa (it increases, on average, linearly in the bundle size). Hence, this model would be appropriate in environments, such as transportation auctions, where both small and large bundles could be valuable for the bidders ([10])[2].

2.3 Bidding Strategies

In industrial CA, bidders face the challenge of deciding on *which bundles* to bid and *how much* to bid. This task becomes especially daunting when there are hundreds or thousands of items in the auction, as in the case of transportation auctions run by major shippers. Given the limited availability and infancy of decision-support tools for bidding in CA, we found that bidders in transportation auctions commonly use some ad hoc strategies in formulating their bids: (1) Submit only singleton bids. (2) Bid on high value packages. (3) Take competition into account when generating bundles. (4) Combine a very attractive lane with less desirable lanes. (5) Put together lanes that increase the “density” in an area.

Motivated by these common practices, we propose three bundling strategies, namely, Naive Strategy, Internal-Based Strategy (INT), and Competition-Based Strategy (COMP), which correspond to strategies (1), (2) and (3), respectively. We refer to INT and COMP as wise strategies, and to the bidders using these strategies as wise bidders. Note that bidders who use the naive strategy do not submit any package bids, and are referred to as naive bidders.

In the proposed bidding strategies, we initially limit our focus to generating bundles, not on pricing, and therefore we assume that all bidders price their bundles using a fixed profit margin. That is, if the bundle value is V, then the bid price is $(1-\text{PM})V$, $0 \leq \text{PM} < 1$, where PM is referred to as the profit margin. Although this pricing method is quite simplistic, it is commonly used in practice[3] and allows us to focus and test the impact of bundling strategies as a first step. Next, we discuss the proposed bundling strategies in detail.

[2] Unfortunately, we were unable to test and validate our model since real data from combinatorial auctions is generally not publicly available. We were able to obtain some bidding data from large transportation auctions; however, the data sets have been relatively sparse, that is, (i) while there are many bidders, only a few of them submit package bids, and (ii) for those bidders who do submit package bids, they submit very few package bids and include an item in at most one package bid.

[3] In the logistics industry, 33% of third-party logistics companies (3PL) in North America used cost-plus pricing in 2000 ([12]).

2.3.1 Internal-Based Strategy (INT)

In practice, many bidders generate bids only considering their own valuations for the items, without taking competitors' valuations into account. For example, in trucking auctions, carriers usually submit package bids based on only the relative value of the shipper's lanes to their own network rather than the competitors' networks ([10]). Internal-Based Strategy (INT) mimics this practice, focusing on identifying bundles with comparably high average value per item. INT generates bundles for bidder i as follows.

For each item $j \in N$:

(1) Create a single-item bundle $B_1^j = \{j\}$. Set $n_j = 1$.

(2) Set $B = B_{n_j}^j$. Define $k = argmax_{l \in N-B}\ AC_{B\cup\{l\}}^i$.

(3) If $AC_{B\cup\{k\}}^i > AC_B^i$, then $B_{n_{j+1}}^j = B \cup \{k\}$, $n_j = n_j + 1$, and return to step (2).

This bundle creation algorithm starts from each individual item and searches for items to add to the current bundle B to increase the average unit contribution (AC) of the bundle. If such an item can be found, then we add the item which increases the AC the most, i.e., set $B = B \cup \{k\}$. We repeat this process until the bundle's AC cannot be increased further. All bundles generated (i.e., bundles $B_1^j, \ldots, B_{n_j}^j$, for $j \in N$) until the algorithm stops are considered as 'desirable' bundles. Thus, this strategy generates at least n bundles ($n_j \geq 1$ bundles for each $j \in N$) with $O(n^3)$ running time. Once these 'desirable' bundles have been identified, the bidder may submit some or all of the generated bundles, depending on whether or not the auctioneer put a limit on the maximum number of bundle bids allowed per bidder.

2.3.2 Competition-Based Strategy (COMP)

Competition-Based Strategy (COMP) focuses on identifying bundles for which a bidder has a relatively high valuation compared to his competitors. COMP is very similar to INT, except that the criteria for adding an item to a bundle is the *value ratio* of bidder i for bundle B (VR_B^i) instead of the AC_B^i, where

$$VR_B^i = \frac{V_B^i}{\sum_{j \in B} v_j^{-i}} \quad \text{and} \quad v_j^{-i} = \frac{1}{m-1} \sum_{q \neq i} v_i^q.$$

Although it would be desirable to compare a bidder's own bundle value with all of its competitors, note that in the denominator of VR_B^i we have the competitors' item values only, and not the synergy values. This is because in practice it is significantly more difficult to gain information about competitors' synergies than item values. For example, in the trucking industry, values for individual lanes (item values) are very much dependent on the cost of operating a truck, which is fairly uniform across different companies (as it depends primarily on fuel cost and driver salaries). However, the cost of operating a truck on a group of lanes put out for bid depends not only on those lanes, but also on a carrier's current network, which is usually private information to that carrier and is difficult to acquire by competitors. Therefore, it is more practical to only incorporate the competitors' item values in this model.

2.4 Simulations

We designed a series of experiments to test the performance of our proposed bundling strategies, gain insights into the value of bundle bids both for the bidders and the auctioneer, and answer the following questions: (1) How does auctioneer's revenue change as the number of bundle bids submitted increases?; (2) How different is the revenue distribution among bidders in CA compared with non-combinatorial auctions?; (3) What is the relationship between a bidder's size and the efficacy of a particular bundling strategy?; (4) Which are the critical (and non-critical) factors in determining the performance of a bundling strategy? (5) Is there a relationship between a bidders size and the optimal profit margin? (6) How good are the proposed bundling strategies compared to the ideal case where bidders are allowed to submit bundles for all possible combinations of items?

Our experiments are motivated primarily by trucking and spectrum auctions, where items auctioned off (e.g., lanes or spectrum licenses) are associated with certain geographic locations[4] and there are different types of bidders in terms of their size and valuations. To

[4]In many studies of the trucking industry, the nation was divided into seven zones: Northeast, Southeast,

capture this characteristic of real-world auctions, we designed a simulation with 4 regions and 20 items, where 5 items are associated with each region. Model 1 (Section 2.4.1) and Model 2 (Section 2.4.2) are two market environments characterized by different bidder sizes and valuations.

Given the difficulty of solving the winner determination problem, in most real-world CA, the auctioneer may set a limit on the number of bundles a bidder is allowed to submit. For example, in FCC's proposed first combinatorial auction[5], 12 package bids were allowed per bidder. To model this restriction and to test its impact on the auctioneer's and the bidders' revenues, we ran experiments by limiting the maximum number of bundle (package) bids allowed per bidder (NB) to the following values: NB={2,5,10,15,25}. If the number of generated bundles using INT or COMP exceeds NB, we assume that the bidders sort their bundles in decreasing order of AC and the VR, respectively, and submit the top NB bundles. We do not impose any restriction on the number of singleton bids and assume that all (wise) bidders submit singleton bids for all items (in addition to their bundles bids).

Unless noted otherwise, we assume that all bidders of the same type use the same bundling strategy and the same profit margin (PM). When bidders use the same PM, without loss of generality we assume that PM=0, i.e., the bidders bid their value for each submitted bid. Hence, the auctioneer's revenue is equal to the sum of the values of the winning bids and a bidder's revenue is represented by the total value of his winning bids[6].

To understand the impact of bundle bids on the auctioneer's revenue, we ran experiments by varying the number of wise bidders (submitting bundle bids) from none to all. In addition, to compare the performance of INT and COMP, we ran experiments where some bidder types use INT while others use COMP. For each experimental setting, we generated 25 random seeds[7]; the data presented below represents an average of the 25 replications.

Midwest, Southwest, Central and Northwest ([3]).

[5]FCC: http://wireless.fcc.gov/auctions/31/releases.html

[6]When PM=0, a bidder's profit will always be zero. We use the term "revenue" to reflect the total value of the bundles won by a bidder.

[7]The coefficient of variation (ratio of standard deviation over mean) of the auctioneer's revenue in a scenario is around 0.01 over 25 replications.

2.4.1 Model 1 - Different-size Bidders with Comparable Valuations

In trucking and spectrum auctions, bidders tend to be characterized by the regions in which they operate. Furthermore, the transportation industry is characterized by many small and relatively few large players, where 75% of the firms own less than six power units ([3]). To capture these properties in Model 1, we introduce 3 types of bidders: local, regional, and global. We assume that there are 12 local bidders (3 in each region), 6 regional bidders (3 in regions (R_A, R_B) and 3 in regions (R_C, R_D)) and 3 global bidders who operate in all regions (Figure 1).

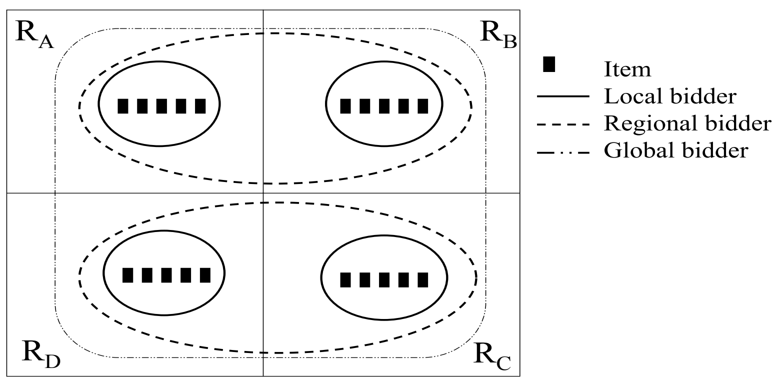

Figure 1: Model 1: Local, regional, and global bidders.

In general, given the size of their business we might expect global bidders to have a higher chance of having a positive synergy between any two items, but their synergies might be lower compared to the local bidders, who tend to maintain routes within concentrated geographical settings, and therefore have considerable opportunities for combining routes and exhausting possible synergies across customer schedules. Therefore, we use different distributions to model bidders' synergy values, but we keep the expected synergy values the same to avoid a bias against any one type of bidder. The data on trucking auctions presented in [10] (Table 4.10, pg. 45) indicates that 94% of the bundle bids are priced at 15% less than the sum of the individual values of the lanes in the bundle. Based on this observation, in our experiments we set

$$\frac{\text{average pairwise synergy value}}{\text{average item value}} = 0.18$$

[8]. The details of the experimental design are shown in Table 2.

Table 2: Experimental design for Model 1. With probability $1 - p$, the pairwise synergy value between any two items is zero; with probability p, they are drawn from the corresponding uniform distribution.

Bidder Type	Number of Bidders	Interested Regions	Item Values	Local Synergy Values	Regional Synergy Values
Local	3; 3; 3; 3	R_A; R_B; R_C; R_D	U(10, 15)	U(3,6) p=0.5	U(0,0)
Regional	3; 3	R_A R_B; R_C R_D	U(10, 15)	U(2,5) p=0.643	U(2,5) p=0.643
Global	3	R_A R_B R_C R_D	U(10, 15)	U(1,4) p=0.9	U(1,4) p=0.9

2.4.1.1 Auction Results

The bidders' total revenues (summed over each type) are summarized in Tables 3 and 5. The coefficients of variation of the bidders' revenues are listed in Tables 4 and 6. In scenario XYZ, the bidding strategies for local, regional and global bidders are X, Y and Z, respectively. For example, ICN means that local, regional and global bidders use INT, COMP, and the Naive strategy, respectively.

Figure 2 shows the auctioneer's average revenue over 25 replications as a function of the number of types of bidders using INT; for example, "1 type is wise bidders" corresponds to scenarios NNI (only global bidders are wise), NIN (only regional bidders are wise) and INN (only local bidders are wise). The vertical black lines represent the range for the auctioneer's average revenue across associated scenarios. We obtained similar results for COMP.

[8] Since we are assuming that PM=0, the bid for a bundle B is equal to its value, V_B^i. In order to calibrate our experimental setting with Plummer's observation, we set $V_B^i = 1.18 \times |B| \times$ (average item value).

Table 3: Bidders' total revenue in Model 1

			Wise Bidders Use INT					
NB		NNN	NNI	NIN	INN	NII	INI	IIN
2	L	102.4	73.3	44.5	261.6	36.6	213.7	197.6
	R	97.3	69.7	250.1	36.4	202.3	25.4	134.8
	G	89.8	168.1	33.8	33.8	95.4	99.2	13.8
5	L	102.4	57.7	18.1	302.0	17.3	226.1	179.2
	R	97.3	55.2	299.3	19.4	212.6	12.8	157.6
	G	89.8	207.1	19.3	14.4	112.2	104.5	11.3
10	L	102.4	39.0	13.9	307.9	10.2	212.2	181.0
	R	97.3	36.5	311.6	17.6	212.4	9.1	157.9
	G	89.8	250.2	13.3	10.9	121.1	123.4	9.5
15	L	102.4	30.0	13.3	307.9	7.8	207.8	181.0
	R	97.3	32.5	312.3	17.6	216.1	7.3	157.9
	G	89.8	265.8	13.3	10.9	120.0	129.8	9.5
25	L	102.4	22.2	13.3	307.9	4.7	210.4	181.0
	R	97.3	27.1	312.3	17.6	201.8	5.5	157.9
	G	89.8	280.7	13.3	10.9	137.7	129.2	9.5
			Wise Bidders Use COMP					
NB			NNC	NCN	CNN	NCC	CNC	CCN
2	L		72.8	41.1	262.9	35.7	219.6	187.1
	R		74.8	249.8	36.4	207.1	24.3	143.0
	G		165.1	38.1	32.6	92.7	94.5	14.4
5	L		56.1	19.9	298.5	16.3	210.3	179.1
	R		55.9	297.2	21.9	224.9	12.7	155.4
	G		207.4	18.7	15.1	101.1	119.6	13.2
10	L		39.8	14.5	306.0	9.8	210.6	170.8
	R		39.8	311.9	18.2	226.1	10.3	167.8
	G		246.2	12.1	12.1	108.2	123.0	9.5
15	L		30.9	14.0	306.0	7.9	209.0	169.5
	R		30.7	313.6	18.2	219.5	7.9	169.2
	G		267.0	11.5	12.1	117.1	127.6	9.5
25	L		23.8	13.9	306.0	8.5	202.7	169.5
	R		24.7	313.6	18.2	212.7	6.1	169.2
	G		281.9	11.5	12.1	123.4	136.2	9.5

Table 4: The coefficient of variation (ratio of standard deviation over mean) of the bidders' revenues.

			Wise Bidders Use INT					
NB		NNN	NNI	NIN	INN	NII	INI	IIN
2	L	0.3	0.4	0.5	0.1	0.6	0.2	0.2
	R	0.4	0.5	0.1	0.5	0.2	0.8	0.3
	G	0.4	0.2	0.5	0.6	0.4	0.4	0.9
5	L	0.3	0.4	0.8	0.1	0.7	0.2	0.3
	R	0.4	0.4	0.1	1.0	0.3	0.9	0.3
	G	0.4	0.2	0.9	1.1	0.5	0.4	0.9
10	L	0.3	0.6	0.8	0.1	1.3	0.3	0.3
	R	0.4	0.6	0.0	1.1	0.3	1.3	0.3
	G	0.4	0.1	1.2	1.3	0.5	0.5	0.8
15	L	0.3	0.7	0.8	0.1	1.6	0.3	0.3
	R	0.4	0.5	0.0	1.1	0.3	1.4	0.3
	G	0.4	0.1	1.2	1.3	0.5	0.4	0.8
25	L	0.3	0.9	0.8	0.1	2.0	0.2	0.3
	R	0.4	0.7	0.0	1.1	0.3	1.8	0.3
	G	0.4	0.1	1.2	1.3	0.4	0.4	0.8
			Wise Bidders Use COMP					
2	L		0.4	0.6	0.1	0.4	0.2	0.3
	R		0.5	0.1	0.5	0.3	0.7	0.3
	G		0.2	0.6	0.6	0.5	0.3	0.9
5	L		0.5	1.0	0.1	0.7	0.2	0.3
	R		0.5	0.1	0.9	0.3	0.9	0.3
	G		0.2	0.8	0.9	0.6	0.4	0.8
10	L		0.6	0.9	0.1	1.3	0.3	0.3
	R		0.6	0.0	1.1	0.3	1.2	0.3
	G		0.1	0.9	1.2	0.5	0.4	0.9
15	L		0.8	0.9	0.1	1.6	0.2	0.3
	R		0.8	0.0	1.1	0.3	1.5	0.3
	G		0.1	0.9	1.2	0.5	0.4	0.9
25	L		0.8	0.9	0.1	1.5	0.2	0.3
	R		0.9	0.0	1.1	0.3	1.8	0.3
	G		0.1	0.9	1.2	0.5	0.3	0.9

Observation 1 *The auctioneer's revenue increases, at a decreasing rate, in the number of types of wise bidders and NB (Figure 2).*

The increase in the auctioneer's revenue is mainly due to the inclusion of synergies in the bundle bids. Recall that the auctioneer's revenue is directly proportional to the sum of the values of winning bids, since all bidders use the same profit margin. Due to positive

Table 5: Bidders' total revenue in Model 1 when all bidders are wise.

NB		III	CCC	CCI	IIC	CIC	ICI	ICC	CII
2	L	178.1	170.2	154.5	174.1	165.8	170.7	175.7	170.1
	R	131.2	133.7	138.6	130.1	137.1	130.4	127.2	138.5
	G	38.3	42.7	53.8	43.3	44.2	46.2	44.1	38.7
5	L	166.8	169.0	163.7	163.0	163.0	167.6	168.9	162.3
	R	117.6	114.5	124.6	114.6	114.5	120.8	114.6	122.0
	G	66.3	66.2	61.8	72.8	72.8	61.8	66.2	66.3
10	L	161.9	141.9	159.2	152.3	152.8	159.5	141.8	160.8
	R	121.0	138.7	128.1	127.6	129.2	126.4	138.8	125.7
	G	68.5	70.1	63.9	71.1	68.9	65.4	70.1	64.7
15	L	158.1	144.4	156.6	152.0	151.9	158.5	143.9	155.6
	R	116.3	134.5	127.3	117.2	121.1	121.9	131.3	122.4
	G	77.1	72.0	67.4	82.0	78.2	71.1	75.8	73.3
25	L	157.0	145.2	149.9	153.4	151.6	151.8	146.3	154.5
	R	119.2	129.5	131.4	113.5	119.5	126.0	124.0	125.3
	G	75.4	76.4	70.2	84.5	80.1	73.9	80.9	71.7

Table 6: The coefficient of variation (ratio of standard deviation over mean) of the bidders' revenues.

NB		III	CCC	CCI	IIC	CIC	ICI	ICC	CII
2	L	0.3	0.4	0.4	0.3	0.4	0.3	0.3	0.3
	R	0.3	0.3	0.4	0.3	0.3	0.3	0.3	0.3
	G	0.8	0.9	0.6	0.9	0.9	0.6	0.9	0.9
5	L	0.3	0.3	0.3	0.4	0.4	0.3	0.3	0.3
	R	0.6	0.5	0.5	0.5	0.5	0.5	0.5	0.6
	G	0.6	0.7	0.6	0.6	0.6	0.6	0.7	0.6
10	L	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.4
	R	0.5	0.5	0.5	0.4	0.4	0.5	0.5	0.5
	G	0.5	0.6	0.6	0.5	0.5	0.6	0.7	0.6
15	L	0.4	0.4	0.4	0.3	0.4	0.4	0.4	0.4
	R	0.5	0.4	0.5	0.3	0.3	0.5	0.5	0.5
	G	0.5	0.5	0.7	0.4	0.5	0.6	0.5	0.7
25	L	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
	R	0.4	0.4	0.5	0.4	0.4	0.5	0.5	0.5
	G	0.4	0.5	0.7	0.4	0.5	0.6	0.5	0.7

synergies, bundle bids lead to a higher average value per item, and in general, have a higher chance of winning than singleton bids. Hence, an increase in the bundle value due to synergies is directly passed onto the auctioneer.

From Observation 1, the auctioneer benefits from an increase in both the number of types of bidders who are "wise," i.e., submit bundle bids, and the number of bundles allowed per bidder. We also observe that the auctioneer has diminishing returns as the number of types of wise bidders or NB increases. An interesting question is which of these two parameters has a bigger impact on the revenue of the auctioneer.

Observation 2 *In general, for the auctioneer the benefit of adding a type of wise bidder is greater than the benefit of increasing NB.*

From Figure 2, we can see that having a small NB (e.g., 2) and two types of wise bidders is in general more profitable for the auctioneer than having a large NB (e.g., 25) but only one type of wise bidders. That is, as the number of types of wise bidders increases, the auctioneer can capture high revenues even with a small NB. Hence, the auctioneer might prefer to channel its resources into educating more bidders to bid wisely, rather than solving larger and possibly more complex winner determination problems (due to higher NB).

On average, naive bidders' contribution to the auctioneer's revenue decreases while wise bidders constitute a larger portion of the winning bids as NB increases (Figure 3). Furthermore, Table 4 shows that in general the coefficient of variation of revenues (i) is higher for naive bidders compared to wise bidders, and (ii) increases as NB increases for naive bidders while it remains the same for wise bidders. When bidders bid naively, the chance of winning no items is high, which results in a large coefficient of variation as well as a decrease in revenues.

When all bidders are wise, because global bidders have a larger pool of bundles from which to select, local bidders' contribution to the auctioneer's revenue decreases while global bidders constitute a larger portion of the winning bids as NB increases (Table 5). Similarly, the coefficient of variation of global bidders' revenues decreases (in general) as NB increases.

2.4.1.2 Benefits of Diversification in Submitting Bundle Bids

In the experiments, we found that due to their greedy nature, INT and COMP generate bundles with a substantial overlap for global bidders, i.e., the generated bundles contain many of the same items (Figure 4). If two bundles have one or more common items, at most one of those bundles can win, reducing the chances of winning for overlapping bundles.

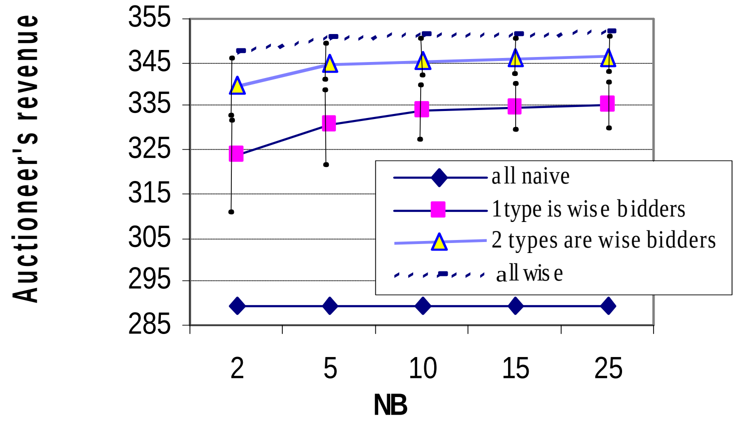

Figure 2: Auctioneer's average revenue as NB and the number of types of wise bidders (using INT) increase.

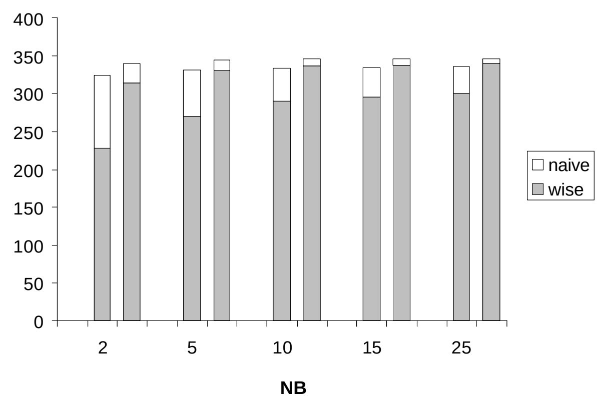

Figure 3: Wise (under INT) vs. naive bidders' contribution to the auctioneer's revenue. For each NB, the two bars correspond to "1 type is wise bidders" and "2 types are wise bidders", respectively.

This leads to the following question: Are bidders better off by increasing the diversification among the submitted bundles? In the discussion below, we focus on the effect of overlapping bundles on global bidders, for whom the overlap is most significant.

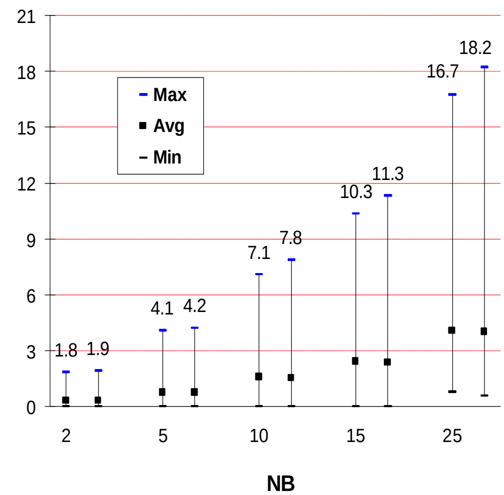

Figure 4: The average, maximum and mininum number of times an item appears in a submitted bundle for global bidders. For each NB, the two lines correspond to INT and COMP, respectively.

To measure the overlap and its effect on the efficacy of bundle bids, we introduce the parameter Restricted Overlapping Frequency (ROF) which restricts the degree of overlap across *submitted* bundles. That is, if ROF is set to x, then any one item cannot appear in more than x submitted bundles. Previously, under INT and COMP, bundles were selected for submission in decreasing order of AC and VR, respectively. We now slightly alter our bundle selection (as opposed to creation) procedure: For a particular ROF value:

1) Sort bundles in decreasing order of AC (for INT) or VR (for COMP). Set the submitted bundle set equal to the null set.

2) Select the next bundle from the top of the list. If adding this new bundle to the submitted bundle set does not violate the ROF for any item (i.e., if with the addition of this new bundle the global bidder does not include any item in more than ROF number of bundles), then add it.

3) Repeat step 2) until the size of the submitted bundle set reaches NB or the end of the bundle list is reached.

In our experiments, the number of bundles created with INT or COMP is not very large, and possess a high degree of overlap for global bidders (Figure 4), thereby limiting our ability to separate the effect of ROF from NB (the ROF bundle selection algorithm generally stops before NB bundles are selected). Therefore, we propose a Revised INT (or Revised COMP) strategy to expand the set of *created* bundles, by applying the bundling strategy INT (or COMP) on each of the following regions for global bidders: (R_A), (R_B), (R_C), (R_D), (R_A, R_B), (R_C, R_D) and (R_A, R_B, R_C, R_D). That is, a global bidder uses INT (COMP) to generate bundles using items in region R_A, then R_B, and so on. The set of generated bundles is then sorted according to AC (VR) and the ROF procedure is applied. The scenarios tested in the new experiment are presented in Table 7. Since the number of bundles satisfying ROF is not large, there is no need to test scenarios associated with the large NB; on the other hand, small NB restricts the options of ROF. Therefore, we run simulations for NB=15 and ROF={2,3,4,5,6,7,8,9,11,15}. The ROF criterion is only applied to global bidders; local and regional bidders' bundling and sorting strategies are the same as before. Figures 5 and 6 plot the auctioneer's and the bidders' total revenues by bidder type, respectively.

Table 7: Scenarios tested

Scenario	Local, Regional	Global
R1	Naive	Revised INT
R2		Revised COMP
R3	INT	Revised INT
R4	COMP	Revised COMP

Observation 3 *When only the global bidders are wise, it is best for them to submit bundles with a moderate degree of overlap.*

This is observed from scenarios R1 and R2 (Figures 5 and 6). If ROF is very small, the submitted bundles have, on average, a smaller AC (or VR). On the other hand, a large ROF leads to a high overlap among the bundles selected for submission which reduces each bundle's chances of entering into the winning set. Since a moderate degree of overlap allows

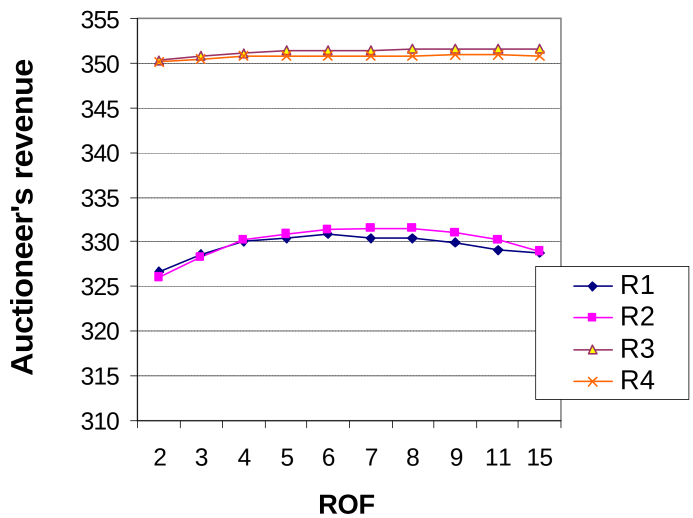

Figure 5: Auctioneer's revenue

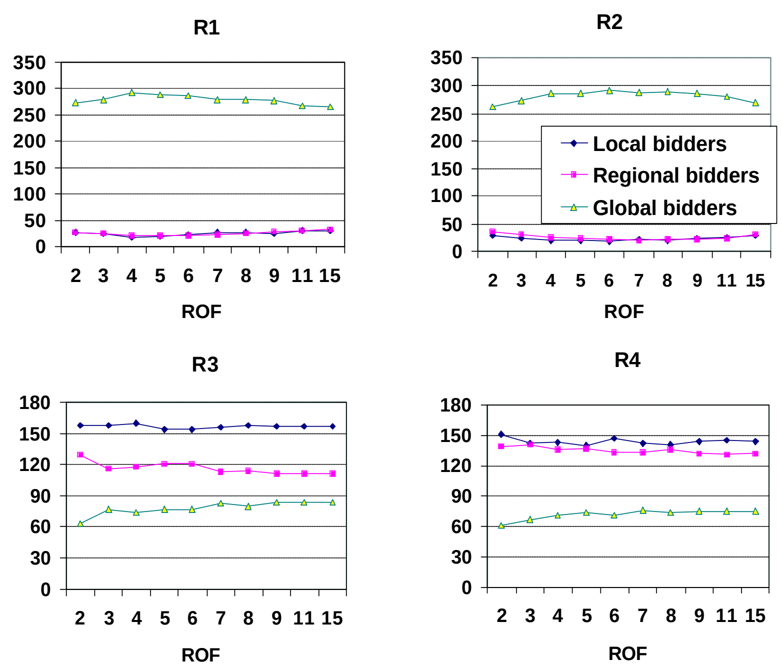

Figure 6: Bidders' revenue

more variety in the submitted bundles, it leads to higher revenues for global bidders when they are the only group of wise bidders.

Observation 4 *When all bidders are wise, ROF should not be a constraining factor in the bundle selection process for global bidders.*

This is observed from scenarios R3 and R4 (Figures 5 and 6). Note that when global bidders are the only group of wise bidders and all competitors are naive bidders, the competition is low, hence, bundles with relatively small AC (or VR) have a strong chance to win. Therefore, global bidders can win more items by spreading out their bundles over more items. However, when all bidders are wise the competition is high. Bundles with larger AC (or VR) are more competitive and therefore, have a higher chance to win. As a result, as ROF increases, AC (or VR) increases, and global bidders are better off.

In general, when bidders with a large number of positive-valued items, in comparison to their competitors, bid in CA, they should spread out their bundles over more items with a moderate degree of overlap if the competition is low; they should bid on their highest valued bundles if the competition is high.

2.4.1.3 Benefits of Taking Competition into Account

Observation 4 tells us that bidders should not be concerned with the degree of overlap when all bidders are wise. The next logical question is then, does one bundling strategy perform better for global, regional or local bidders, respectively? We compared the performance of INT versus COMP and summarized our results in Tables 8 and 9.

Table 8: The average number of items won by bundle bids when NB is 25.

	X (Local)			X (Regional)			X (Global)	
Scenario	INT	COMP	Scenario	INT	COMP	Scenario	INT	COMP
XII	8.7	8.5	IXI	6.7	7.1	IIX	4.3	5.0
XCC	8.0	7.9	CXC	6.8	7.3	CCX	4.0	4.3
XCI	8.4	8.3	CXI	7.0	7.4	CIX	4.1	4.6
XIC	8.5	8.3	IXC	6.4	7.0	ICX	4.2	4.7

Observation 5 *Global and regional bidders, in general, win more items via bundle bids with COMP; local bidders win more items via bundle bids with INT.*

Table 9: The total revenues of each type of bidders when NB is 25.

	X (Local)			X (Regional)			X (Global)	
Scenario	INT	COMP	Scenario	INT	COMP	Scenario	INT	COMP
XII	157.0	154.5	IXI	119.2	126.0	IIX	75.4	84.5
XCC	146.3	145.2	CXC	119.5	129.5	CCX	70.2	76.4
XCI	151.8	149.9	CXI	125.3	131.4	CIX	71.7	80.1
XIC	153.4	151.6	IXC	113.5	124.0	ICX	73.9	80.9

Observation 6 *Global and regional bidders, in general, earn higher revenues with COMP, while local bidders earn higher revenues with INT.*

From Observations 5 and 6, we find that INT and COMP are the preferred bundling strategies for local and global bidders, respectively. In all instances, local bidders won at least as many (up to three more) items via bundle bids using their preferred strategy (INT) than COMP. In 89% of the instances, global and regional bidders won at least as many items via bundle bids using their preferred strategy (COMP) than INT. (For global bidders, the difference in the number of items won via bundle bids from using COMP versus INT is in the range of [-3,7].) A similar comparison of a bidder type's total revenues under INT and COMP also supports Observation 6. In 93% of the instances, local bidders received equal or higher revenues using INT than COMP. (For local bidders, the difference of revenues from using INT versus COMP is in the range of [-15, 53].) In 80% of the instances, global and regional bidders received equal or higher revenues using COMP than INT. (For global and regional bidders, the difference of revenues from using COMP versus INT is in the range of [-107, 118].)

The explanation for these observations is as follows: Because global and regional bidders have more positive valued items, switching their interests from their own high valued items (INT) to the items for which their valuations are high relative to competitors (COMP), increases their winning chances of bundle bids. However, due to having a smaller number of positive valued items and higher synergies among their positive valued items, local bidders benefit more by focusing on their own high valued items.

2.4.1.4 *How to Price Bundles*

Up until now, we assumed that all bidders use the same profit margin in their bidding strategies. In practice, however, different profit margins are often used among various bidders. Caplice ([2]) observed that some small, privately held carriers seek only to earn a profit threshold, and such a carrier will lower its prices beyond those of a profit maximizing firm so long as it can earn its desired threshold. Plummer ([10]) also concluded that "smaller, profit-threshold-seeking carriers lower their discrete (singleton) bids beyond the discrete or package bids of larger more sophisticated carriers in many cases" in CA. These observations lead us to the following questions: Should small bidders always bid more aggressively than large bidders? As a bidder bids more aggressively (uses a higher PM), how does his revenue change in CA and non-CA?

To answer these questions, we designed an experiment with the scenarios listed in Table 10. This design differs from all the previous experiments in that we test the actions of an individual bidder instead of a bidder type. In each scenario, one bidder is selected as the test bidder, for whom the profit margins are set to the following values: PM = {0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07 0.08, 0.09, 0.10}. As the test bidder increases PM the remaining bidders' profit margins are fixed to 0.05. There is no restriction on NB. All bidders use INT in scenarios P1 to P3, and use the naive strategy in scenarios P4 to P6. We ran these experiments using the same instances generated in the previous experiments, i.e., the item values and the synergy values of the bidders are the same as before. Since bidders use different profit margins, a bidder's revenue is represented by the real revenue (PM)$\times V_{won}$, where V_{won} is the total value of the bidder's winning bids.

Table 10: Scenarios tested for pricing analysis

Scenario	P1	P2	P3
Test Bidder (INT)	1 Local Bidder	1 Regional Bidder	1 Global Bidder
Scenario	P4	P5	P6
Test Bidder (Naive)	1 Local Bidder	1 Regional Bidder	1 Global Bidder

A bidder will win his maximum number of items when PM is at its smallest value; hence, in our experiment, a test bidder wins his maximum number of items when PM is 0.01. As the test bidder increases PM, his revenue increases as long as his winning bids remain unchanged. Eventually, the increase in PM typically results in the loss of some existing winning bids. When the increase in revenue from winning bids can not compensate the loss of revenue from the lost items, the bidder's revenue decreases. We define *optimal profit margin* (OPM) to be the profit margin at which the test bidder's revenue is maximized, and the *threshold profit margin* (TPM) to be the largest profit margin that yields a positive revenue. We call the ratio of OPM/TPM (always ≤ 1) the *risk measure* (RM). A larger RM implies a higher risk of using a large profit margin.

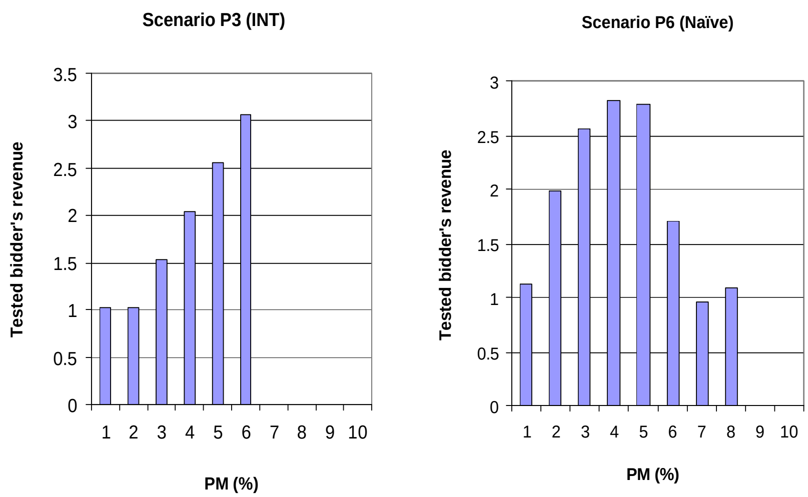

Figure 7: A global test bidder's revenue in scenarios P3 and P6 in one replication

Observation 7 *RM is larger in CA than in non-CA for all bidder types.*
Observation 8 *Local and regional bidders have a larger RM than global bidders in CA.*
Observation 9 *Local bidders do not need to bid more aggressively than regional and global bidders.*

These observations are illustrated in Table 11. As PM increases, the test bidder's winning items leave the winning set independently in non-combinatorial auctions. Therefore, even when the test bidder uses a PM which is larger than OPM, his revenue does not drop to zero immediately (Figure 7). However, when a bundle loses in CA, all the items included in that bundle are lost together.

Since a global bidder has a larger set of bundles from which to select, as he loses some existing winning bundles due to the increase in PM, he has a higher chance to win other bundles than a local bidder. For example, in the instance shown in Figure 7, at PM = 0.01 and 0.02, the test global bidder wins 2 bundles, each containing 3 items. When PM = 0.03, he loses both bundles, but wins a third bundle, of size 3, in their place. The global bidder's profit continues to increase in PM until PM = 0.07, at which point he loses all his submitted bids. In our experiments, global, regional and local bidders win alternative bids as PM increases 40%, 18%, and 0% of the time , respectively.

Table 11: Average OPM, TPM and RM in each scenario

Test bidder	**Scenario** (INT)	OPM (%)	TPM (%)	RM	**Scenario** (Naive)	OPM (%)	TPM (%)	RM
Local	**P1**	5.50	5.50	1	**P4**	4.67	5.39	0.87
Regional	**P2**	5.73	5.73	1	**P5**	6	7.04	0.85
Global	**P3**	5.07	5.60	0.90	**P6**	5.16	7.96	0.65

We observed that the local bidders' OPM (in scenario P1) equals to 5.5% on average, which is larger than the 5% profit margin used by all non-test bidders. Therefore, it is not necessarily optimal for a local bidder to bid more aggressively in CA auctions.

2.4.2 Model 2 - Same-size Bidders with Asymmetric Valuations

In model 1, observations are based on settings where bidders have different sizes, but possess similar valuations over singleton items and comparable pairwise synergies (in expectation). To understand how the results change if we reverse this relationship, we introduce Model 2, where bidders are same-sized with different valuations.

In this experiment, there are 10 global bidders in the auction. All of the bidders draw their synergy values from the same distribution; however, there are asymmetries in their item valuations: *Symmetric* bidders draw valuations for all four regions from the same distribution; *Asymmetric* bidders draw valuations for regions R_B and R_D from distributions with lower and higher means, respectively, than regions R_A and R_C (Table 12).

Table 12: Bidder types in Model 2

Bidder Type	Number of Bidders	Synergy Values	Item Values			
			R_A	R_B	R_C	R_D
Symmetric	5	U(1,5)	U(5,10)			
Asymmetric	5	U(1,5)	U(5,10)	U(3,7)	U(5,10)	U(8,12)

Table 13 reports the bidders' total revenue. Scenario XY denotes that symmetric bidders use bundling strategy X and asymmetric bidders use bundling strategy Y. Note that observations 1 and 2 still hold under Model 2.

Table 13: Bidders' total revenue in Model 2

NB	Scenario	NN	IN	NI	CN	NC	CC	II	IC	CI
2	Symmetric	90.0	177.8	79.6	130.2	55.3	96.3	140.9	124.4	118.0
	Asymmetric	107.3	57.6	145.0	94.4	180.0	153.7	106.4	130.4	130.9
5	Symmetric	90.0	208.7	71.6	153.4	45.1	106.4	151.6	128.4	130.4
	Asymmetric	107.3	38.0	161.6	80.3	200.3	156.0	106.6	134.8	131.3
10	Symmetric	90.0	234.6	58.0	174.0	26.4	114.6	146.6	125.7	125.7
	Asymmetric	107.3	20.0	182.8	66.5	227.1	151.4	118.3	140.2	139.6
15	Symmetric	90.0	238.5	50.1	187.7	19.5	109.6	145.5	123.5	122.5
	Asymmetric	107.3	17.6	194.1	55.7	236.0	157.2	120.6	143.0	143.6
25	Symmetric	90.0	241.6	42.9	198.8	18.5	108.0	148.8	122.5	128.5
	Asymmetric	107.3	15.3	204.4	47.1	237.6	159.7	118.0	145.3	138.7

Observation 10 *Symmetric bidders prefer INT and asymmetric bidders prefer COMP.*

Observation 10 follows from Table 13. For symmetric bidders, when we compare the profits between scenarios IN and CN, II and CI, and IC and CC, we find that no matter which bidding strategy the asymmetric bidders use, the best response of the symmetric bidders is INT. Similarly, by comparing the profits of the asymmetric bidders between scenarios NI and NC, II and IC, and NI and NC, we see that asymmetric bidders' profits are higher when they use COMP.[9]

These results are counter-intuitive because one would think that symmetric bidders

[9]In 82% of the instances, Symmetric bidders prefer INT. The difference of revenues from using INT versus COMP is in the range of [-10.5, 17.2]). In 80% of the instances Asymmetric bidders prefer COMP. The difference of revenues from from using COMP versus INT is in the range of [-11.5, 18.6]

would benefit from COMP by generating more bundles covering competitors' low valued regions, and asymmetric bidders would benefit from their own high-valued items using INT. To explain the results, we examined the generated bundles; in particular, we looked at the overlap among bundles generated under each strategy (Figure 8). For symmetric bidders, their bundles containing items from region R_B have a larger VR due to the low values of asymmetric bidders in this region. As a result, when they use COMP, symmetric bidders generate a large number of bundles covering items from region R_B, which leads to a high overlap among their submitted bundles. Similarly, when asymmetric bidders use INT, they generate a large number of bundles covering their high value items in region R_D, which leads to a high overlap among submitted bundles and decreases their chances of winning. It is also interesting to note that if both bidder types use their preferred bundling strategies (Scenario IC), the auctioneer's revenue is a little higher than in the other scenarios.

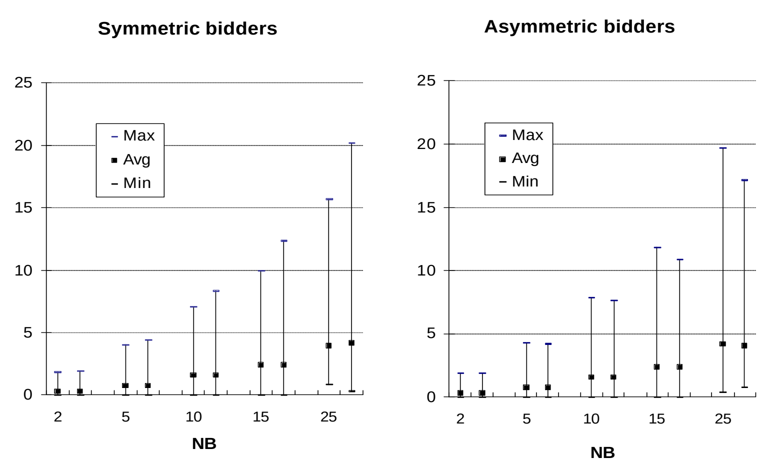

Figure 8: The average, maximum and minimum number of times an item appears in a submitted bundle for the bidder type. For each NB, the two lines corresponds to INT and COMP, respectively.

2.4.3 Comparison with Full Enumeration

The proposed bundling strategies INT and COMP create and select a subset of bundles among all possible combinations. To test their performance, we compare these bundling strategies with the "ideal case" where a bidder submits bids for all possible bundles (combinations of items). In this section, we address the following question: If bidders select bundles using INT or COMP, what is the expected revenue loss for the bidders and the auctioneer compared to the case where bidders submit bids for all possible combinations of items (full enumeration strategy)? Note that if the bidders use a fixed profit margin, the "ideal case" always leads to higher revenues for the auctioneer, but not necessarily for the bidders.

Due to the computational intractability of full enumeration of bundles in our original test instances[10], we design a smaller size experiment in this section with 10 items evenly distributed in two regions. There are 6 local bidders (3 in each region) who are only interested in items from one region and 3 global bidders who are interested in items from both regions. Similar to Model 1, both types of bidders' item values are drawn from $U(10, 15)$. Local synergy values for local bidders are drawn from $U(3, 6)$ with probability $p = 0.5$. Both local and regional synergy values for global bidders are drawn from $U(2, 5)$ with probability $p = 0.643$.

In our experiments, all bidders use the same bundling strategy (namely, INT or COMP in scenarios 1-8, and full enumeration in scenario 9) except one test bidder, who uses either the same strategy as the other bidders, or the full enumeration strategy. Bidders submit all the generated bundles in the auction, i.e., there is no bound on NB. Table 14 shows the scenarios tested in the simulation. Tables 15 and 16 present the revenues for the test bidder and the auctioneer, respectively.

In each of the 25 instances, we find that the local test bidder's revenue is the same whether he uses our proposed bundling strategies or full enumeration (Scenarios 1 versus 2 and 3 versus 4) when the remaining bidders use our proposed bundling strategies. In 92% of the instances, the global test bidder's revenue using full enumeration (Scenario 6) is within 1% of his revenue using INT (Scenario 5). Similarly, the global test bidder's revenue using full enumeration (Scenario 8) is within 0.9% of his revenue using COMP (Scenario 7). Note that the number of bundles generated with our bundling strategies is less than 20, a very small number when compared to the 1023 bundles generated under full

[10]For example, to test the scenario where each bidder submits all possible combinations in Model 1, we need to generate 2^{20} bids for each of the 21 bidders. Furthermore, for each scenario, we need to run 25 replications. The total number of bids one needs to generate for one scenario is $2^{20} \times 21 \times 25 = 550,502,400$

Table 14: Scenarios tested

Scenario	Test bidder		Remaining Bidders' Bundling Strategy
	Bidder Type	Bundling Strategy	
1	Local	INT	INT
2		Full Enumeration	
3	Local	COMP	COMP
4		Full Enumeration	
5	Global	INT	INT
6		Full Enumeration	
7	Global	COMP	COMP
8		Full Enumeration	
9	All Bidders Bid using Full Enumeration.		

Table 15: Tested bidder's revenue

Tested Bidder	Scenario				
Local	**1**	**2**	**3**	**4**	**9**
	13.5	13.5	11.3	11.3	15.6
Global	**5**	**6**	**7**	**8**	**9**
	25.8	27.1	25.4	27.7	24.1

enumeration. Hence, for a relatively small percentage of profit loss in most instances, our proposed bundling strategies offer significant computational advantages to the bidders.

When we compare the auctioneer's revenue between Scenarios 1, 3 and 9, in all instances the auctioneer's revenue loss under INT (Scenario 1) and COMP (Scenario 3) is less than 0.25% and 0.24%, respectively, compared to full enumeration (Scenario 9). Hence, an auctioneer facing bidders who strategically submit bundles, rather than submit bids on all possible bundles, gains significant computational advantages in return for a small loss in profits.

Table 16: Auctioneer's revenue

Scenario	**1**	**2**	**3**	**4**	**9**
mean	173.1	173.1	173.2	173.2	173.6
Scenario	**5**	**6**	**7**	**8**	
mean	173.1	173.2	173.2	173.3	

2.5 Conclusions

In this chapter, we proposed a simple efficient model for evaluating bundle values given pairwise synergies and developed bundling strategies to help bidders select promising and profitable bundle bids. We tested the efficiency and performance of our bundling strategies under different market environments using simulations and gained some interesting insights. Our experimental results show large benefits both for the bidders and the auctioneer from bundle bids. From the auctioneer's point of view, educating more bidders to submit a few bundle bids each is more profitable than allowing the existing few wise bidders to submit a large number of bundle bids. In addition, from the bidders' viewpoint, we provided some suggestions on the issues that should be taken into consideration when selecting and pricing bundles with respect to different market environments. We found that bidders must carefully consider the overlap in their submitted bundles as well as their competitors' comparative valuations when submitting bundles. In addition, we found that, while bidders may lose a fraction of their potential profit when they limit the number of bids submitted, a 'smart' bundling strategy such as INT or COMP, helps to minimize this loss while significantly reducing the computational complexity of the bid submission process. Our future work will focus on developing alternative bundling and pricing strategies and extending our research in to multiple round combinatorial auctions.

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CHAPTER III

RETAILER SHELF-SPACE MANAGEMENT WITH PROMOTION EFFECTS

3.1 Introduction

Shelf space is one of the most important resources in a retail store. Marketing research shows that two-thirds of all consumer purchase decisions are made in the retail store, rather than prior to store visits ([58]). Thus, the decisions of which products to stock among the large number of competing products and how much shelf space to allocate to those products is a question central to retailing. With a well-designed shelf space management system, retailers can attract customers, prevent stockouts and, more importantly, increase the financial performance of the store while reducing operation costs ([78]).

It is now a well established empirical proposition that retail promotions have a noticeable impact on sales of a product and the operations in the retail store. Blattberg and Neslin ([9]) pointed out that in many frequently purchased product categories, more than 50% of sales volume is sold on a retail promotion. Moreover, the performance of the promotion on a brand has a large impact on the demand of nonpromoted products ([36], [74]). The influence of promotions on the product demand results in a large impact on the store's operations, *i.e.*, affecting replenishment and restock operations, which in turn changes the efficiency of shelf space allocations. Due to the strong interdependencies between the shelf space allocation, promotional activities and operating costs, a growing number of retailers are now practicing "category-level shelf management"[1] (also known as "category management"), a strategy that views whole categories as individual business units and seeks to coordinate product selection, shelf space allocations, promotion, merchandising, and logistics to enhance overall

[1] In a retail store, the shelf management includes several levels, the store level includes many departments, *e.g.* food department, which then includes several product categories each, *e.g. noodle category.* In practice, a category manager can be in charge of several product categories ([71]).

category performance ([56]). Therefore, the integrated study of promotional activities with product selection, shelf space allocation and operation costs at the category level has become more critical than ever.

Although promotions have significant influence on the store operation cost and the efficiency of shelf space allocation, to the best of our knowledge, they are not formally introduced in the existing shelf space allocation models ([12], [39], [43]). This research presents a category-level shelf space allocation model for including brand promotion and cross-brand promotion effects. Our model considers promotion selection (assortment), shelf space allocation and the promotion level as the decision variables for optimizing category-level shelf space management decisions.

Furthermore, the retail promotions are often supported by trade promotions, which can take many different forms, such as off-invoice allowances, bill-back allowances, flat allowances, free goods, display allowances, and inventory financing. Manufacturers believe that a large part of the trade allowances are pocketed by retailers instead of passed through to customers ([47], [48]). For example, the Cannondale and Nielsen surveys indicate that manufacturers believe that only 51% of their trade promotion dollars are being passed through to consumers, with more than 20% of the trade promotion dollars going directly to the retailer's bottom line. Therefore, trade allowances are an important factor which significantly affects both the retailer's profits and retail promotion activities, and hence in turn affects the operation cost and shelf space management.

Most of the analytical research on trade allowances is built on simple models where the demand function is linear or implicitly defined, and retail operation costs are not included. The objectives of these research efforts are to examine the major factors which influence the "pass through"[2] ([70]) or to present possible strategies to alleviate the retail pass through problem ([48]). Simple Stackelberg games (See Appendix A for the introduction of "Stackelberg game") are usually introduced between few *i.e.*, one or two, manufacturers and retailers, and manufacturers are the leaders in these games.

[2] *Pass through* is generally defined as the percentage of trade promotions given to the consumers by retailers.

In this research, we introduce the trade promotion factor into the category shelf management framework, where nonlinear demand is represented as a signomial function [3], which provides a more realistic representation of the product demand in the retail store compared to the linear demand function. In the proposed framework, the decisions of category shelf space allocation and retail category promotion activities are affected by the trade allowances paid by manufacturers. The decisions of retail promotional efforts are determined by incorporating the cross promotional elasticity among products in the same category and also the operation costs associated with these promotional activities. Our objective is to develop a framework that optimally assigns the shelf space and promotion budgets among products in a category. Since trade allowances are usually negotiated between the retailer and the manufacturers, we introduce a one-period profit maximizing Stackelberg game to model the negotiation, where a retailer is the leader and the manufacturers are the followers. A retailer optimizes the decision of assortment, shelf space and promotion level for each product in the category by considering manufacturers' responses to a retailer's supply package – a combination of shelf space and promotion level.

This chapter is organized as follows. Section 3.2 reviews related research. Section 3.3 summarizes the expected contribution of this research. Section 3.4 presents the category-level shelf space management model. In section 3.5, we present the framework of the category-level shelf space management with trade promotions. Numerical analysis to demonstrate the potential of the proposed frame is presented in section 3.6 and section 3.7 conclude this research.

3.2 Literature Review

In this section, we review the relevant previous work in three areas: commercial and optimization models of shelf space allocation, experimental studies in retail promotion and the trade allowances models.

Commercial and Optimization Models

[3] A continuous polynomial allows real, as opposed to positive integer, powers of the variables. When all coefficients of a continuous polynomial are positive, it is called a *posynomial*. When at least one coefficient is negative, it is called a *signomial*.

Commercial software and hardware systems that apply modeling principles have gained many customers within the retailing industry due to their general simplicity and their easily implementable decisions ([79]). Today there are various PC-based systems available to retailers including Apollo (IRI) and Spaceman (Nielsen). These software products can provide the retailer with a realistic view of the shelves and are capable of allocating shelf space according to simple heuristics such as turnover, gross profit or margin, using handling and inventory costs as constraints ([28]). The drawback of all these systems results from their failure to incorporate demand effects; all ignore the existing effects of shelf space on product sales. Thus, none of the available systems can be considered seriously as an optimization tool ([28]) and the promotion effect is not included in the systems. Consequently, most retailers "use them mainly for planogram accounting purposes so as to reduce the amount of time spent on manually manipulating the shelves" ([31]).

One of the first optimization models was developed by Hansen and Heinsbroek ([39]). They use a nonlinear demand function, which incorporates individual space-elasticities but disregards cross-elasticities from similar products. Binary variables for handling assortment decisions are also included.

The model of Corstjens and Doyle ([22]) incorporates both space- and cross-elasticities. It incorporates a more detailed cost structure including procurement costs, carrying costs and out-of-stock costs, which are jointly modeled as a signomial form with respect to allocated shelf space. However, the assortment decisions are not considered.

Borin *et al.* ([12]) extend the demand function of Corstjens and Doyle ([22]) to allow simultaneous decisions about assortment selections and shelf space allocations. In addition, they explicitly consider substitution effects due to temporary or permanent unavailability of products. Yang and Chen ([78]) simplify the model of Corstjens and Doyle ([22]). The authors disregard cross-elasticities and assume that a product's profit is linear within a small number of facings, which are constrained by the product's lower and upper bound of the number of facings. However, the models of both Borin *et al.* ([12]) and Yang and Chen ([78]) have the following drawbacks. They focus on the revenue side and do not incorporate the cost side of the operation explicitly. Clearly, some of the relevant costs are not independent of the shelf space allocation. For example, the smaller the shelf space allocated to a product the greater the frequency of restocking and the higher the resulting restocking costs of this product.

In all these optimization models, promotional effects are not considered. Yang and Chen ([78]) and Irion *et al.* ([43]) suggest to introduce the marketing variables to the demand function through a general production term as an extension of the demand function of Corstjens and Doyle ([22]). However, no detailed information is provided about these marketing variables. In this research, we extend the demand function of Irion *et al.* ([43]) to incorporate both space and promotional elasticity. A more realistic shelf space allocation model is presented, which simultaneously determines the assortment selections, shelf space and promotion level. The operational costs are also clearly represented.

Retail Promotion Models

Past research on retail promotions has primarily focused on the effect of promotions on the sales of the promoted brand. However, when category sales are the objective, retailers not only evaluate sales increases for the promoted brand but also evaluate the promotion's effect on sales of competing brands. When retailer's objective is category profits, it is also necessary to take into account the switching from less profitable brands to more profitable brands and vice-versa.

Experimental methods are the most commonly used tools to analyze the effects of retail promotions. Significant amount of research is done on analyzing the promotional elasticity (e.g., [13]) and cross promotional elasticity (e.g., [61]). Furthermore, some researchers relate variability in product category sales to promotional activity in the product category. Raju ([60]) analyzes the influence of an increase in the magnitude and frequency of discounts on the category sales. Karande and Kumar ([45]) provide guidelines to retailers for planning promotions in terms of what brands to promote, and how and when to promote them. Walters ([75]) analyzes the factors which affect the product category price elasticity. Even though the close relationship between promotion effects and shelf space has been identified in these papers, no analytical model is developed to relate these two factors with category profits. In our proposed model, we build the connections through the category-level shelf space management model.

Models on Trade Promotions

There is some research concerning the empirical analysis of retail response to trade promotions. Frequently cited work in this area are Chevalier and Curhan ([17]), Curhan and Kopp ([25]) and Armstrong ([4]). Using survey data, they estimate the ratio of trade allowances pocketed by retailers or examine the factors that determine the strength of retail support for trade deals.

Another research stream analytically investigates various strategic issues relating to trade promotions. Kim and Staelin ([47]) offer a framework that helps explain why manufacturers offer trade promotions despite poor pass throughs. A Stackelberg game is presented between two manufacturers and two retailers where manufacturers are leaders. Tyagi ([70]) investigates the factors affecting the extent of retail pass through. Specifically, with a simple Stackelberg game between one manufacturer and one retailer where the manufacturer is the leader, the paper analyzes relations between retail pass through decision and the curvature, *i.e.*, linear, concave or convex, of consumer demand functions. No explicitly represented demand function is presented in the analysis. Kumar *et al.* ([48]) analyzes the factors that affect the retail pass through as well as the strategy of alleviating the problem. A two-period game among a manufacturer, a retailer and customers are presented. The effects of depth and frequency of trade promotions, customers' knowledge of the trade promotions on retailers' pass through decision are analyzed. Furthermore, they show that by complementing trade promotions with advertising that informs customers about ongoing promotions, the manufacturer can enhance retail pass through.

All of these papers study the problem using analytical models, which are highly simplified, *i.e.*, they use linear price functions and ignore the operation costs. The objectives are to find out managerial suggestions based on the analytical results. However, due to the strong interdependencies between trade promotions, retail promotions, retail operational costs and retailer's profits, our objective is to suggest a realistic category management framework incorporating the impact of trade promotions on retail promotions.

3.3 Contributions in this Chapter

In this chapter, we present the following work:

1. Model the promotional effects on product demand in a product category.
2. Formulate category-level shelf space allocation problem, which simultaneously determines the assortment decision, space allocation and promotion level for each product.
3. Provide optimization methods to solve the category-level shelf space allocation problem.
4. Propose a framework to model the impact of trade promotions on the category-level shelf space allocation based on a Stackelberg game between a retailer and manufacturers.
5. Provide numerical examples to demonstrate the potential of the proposed models and methods.

3.4 Shelf Space Allocation Model

Space allocated to a brand and the promotion activities in the product category are both important factors that affect a customer's brand selection. In this section, following Irion, *et al.* ([43]), we reexamine the shelf-space model developed therein and imbed it in a more general category-level shelf space management model that simultaneously determines the assortment decision, shelf space allocation and promotion level for each product.

3.4.1 Assumptions

(i) The objective of the retailer is to maximize product category profit.

(ii) Consistent with prior research, the direct space-elasticity for product i satisfies $0 \leq \beta_i \leq 1$, the cross-elasticity between product i and product j satisfies $-1 \leq \delta_{ij} \leq 0$, and the scale factor α_i for product i is generally taken to be positive.[4]

(iii) All shelved products are owned by the retailer.

[4] Note that our model assigns shelf space to products within a product category. Since such products are usually very similar in nature, we would expect them to have substitution properties amongst each other ($\delta_{ij} \leq 0$). Nevertheless, it is straightforward to extend our model and the linearization technique to the more general case where $-1 \leq \delta_{ij} \leq 1$.

(iv) Products are restocked individually. As soon as the number of units on the shelves is zero, the product is fully restocked. This assumption allows inventory holding costs to be calculated easily and also makes possible a disregard of substitution effects due to temporary stockouts.

(v) There is no backroom space to store additional inventory. This assumption allows that only inventory holding costs of product-units stored on the shelves are considered.

Table 17: Notation

n_i	:	number of *facings* allocated to product i.
z_i	:	indicator variable is 1 if product i is selected for shelving, and 0 otherwise.
x_i	:	level of promotional expense for product i.
N	:	number of products in category.
F_i	:	shelf space of one facing for product i (inches).
S	:	total amount of available shelf space within the product category (inches).
U_i	:	upper bound on the number of facings allocated to product i (inches).
L_i	:	lower bound on the number of facings allocated to product i (inches).
G_i	:	number of units of product i that can be stored in one facing.
P_i	:	selling price of product i ($).
W_i	:	wholesale price of product i ($).
C_i	:	unit production cost of product i ($).
CR_i	:	cost each time product i is restocked ($).
CF_i	:	fixed cost to include product i in the assortment ($).
CP_i	:	unit replenishment cost for product i ($)[5].
β_i	:	*space elasticity* of product i.
α_i	:	*scale factor* for product i.
δij	:	*cross space elasticity* between products i and j.
I	:	current investment/interest rate (%).
Ω	:	product category profit ($).
μ_i	:	*promotion elasticity* of product i.
ν_{ij}	:	*cross promotion elasticity* between products i and j.
A_i	:	scale factor for the promotional expense for product i.
B	:	budget constraint on the promotion cost.
X_{Li}	:	lower bound on the promotion level for product i.
X_{Ui}	:	upper bound on the promotion level for product i.

3.4.2 Demand Formulation

The major approach that is currently used for specifying demand functions in shelf space management models follows the nonlinear structure proposed by Corstjens and Doyle ([22]).

The demand structure incorporates both the individual space elasticity and the cross elasticities between products within the same store, which is:

$$d_i = \alpha_i (s_i)^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (s_j)^{\delta_{ij}} \tag{1}$$

where, s_i denotes the shelf space allocated to product i. The parameter α_i is a scale factor for product i demand, β_i is the product i *space elasticity*, and δ_{ij} is the *cross space elasticity* between products i and j. For details about this model, see ([22]).

Yang and Chen ([78]) and Irion *et al.* ([43]) extended 1 to include marketing variables. In their models, marketing variables, "*possibly* include price, advertisement, promotion, store characteristics, and other marketing mix variables" are suggested to be introduced by a *general* production term. However, no discussion is provided concerning issues, such as promotion elasticity and cross promotion elasticity. Since both theoretical and empirical results lead to nonlinear models for capturing market responses for promotional activities ([19], [77]), following the models of Yang and Chen ([78]) and Irion *et al.* ([43]), we explicitly introduce the promotional factors as:

$$d_i = \alpha_i (F_i n_i)^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (F_j n_j)^{\delta_{ij}} (x_i^{\mu_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} x_j^{\nu_{ij}}) \tag{2}$$

where $F_i n_i$ denotes the shelf space allocated to product i[6]. The parameter x_i ($x_i \geq 1$) is introduced to denote the promotion level, which has been called the promotion intensity[7]. When product i is promoted, $x_i > 1$. On the other hand, $x_i = 1$ when there is no promotion of product i. The demand in ([43]) is obtained from 2 by setting $x_i = 1$ for all i.

We introduce μ_i and ν_{ij} to represent *promotion elasticity* and *cross promotion elasticity*, respectively. In practice, the parameters α_i, β_i, δ_{ij}, μ_i and ν_{ij} can be determined via

[6] A *facing* is a segment of shelf space with dimensions width, height and depth when viewed by a customer. The sizes (or widths) of facings can vary with products, so that each facing dedicated to product i would have width F_i. Moreover, for the purposes of our model, different products *cannot* share the same facing. If there is enough height and depth space available, products can be stacked and lined up many rows deep. The total number of products that fit on a facing is G_i, which allows for stacking multi-rows deep.

[7] For example, various level of the price reduction can be used in promotions. When "display" is used as a promotion measure, major display, secondary display and no display are used to classify the intensity. When promoting through advertisement, investments associated with advertisement varies ([17]). In this research, promotion level represents the intensity of the comprehensive promotion efforts from various of promotion measures.

regression analysis using cross-sectional data ([22]). For given cross-sectional data, the magnitude of the scale factor α_i depends on the size of the time interval considered, while the elasticities β_i, δ_{ij}, μ_i and ν_{ij} can be assumed to be independent of the time interval. The demand d_i defined by (2) is for an arbitrary interval, and all products must have the same sized interval.

The demand models have the following properties concerning the promotional effects:

1) *Diminishing returns*: In practice, sales increase substantially when an item is promoted even though the depth of the promotion is very small ([37]), and a subsequent increase in the depth of promotion leads to additional incremental sales but at a much slower rate ([29]). The proposed demand function reflects these two facts by restricting $\mu_i \in [0, 1]$.

2) *Substitution effects*: Gupta ([38]) estimates that 85% of a brand's promotion elasticity is due to brand switching while the rest is due to the changes in the quantity normally purchased or in the frequency of purchase, *i.e.*, purchase incidence. The parameter *cross promotion elasticity* is introduced to capture the brand switching effect. When product i's substitute product j is promoted, with $\nu_{ij} \in [-1, 0]$, the increase in the promotion level, x_j, reduces the demand of product i with a diminishing rate. Furthermore, Walters ([73]) found only a weak relationship between retail promotion and sales of complementary products. Therefore, we assume that the promotion of a product has no effect on its complementary products, which is $\nu_{ij} = 0$ for the complementary products i and j.

3) *Asymmetry*: the cross promotional effects are asymmetric, *i.e.*, promoting higher quality brands impacts weaker brands disproportionately ([6], [7], [44], [54]). Thus, ν_{ij} does not necessarily equal to ν_{ji}.

Finally, we need to point out that in the proposed demand function, the promotion level x_i represents the aggregate promotional effects on the demand, *i.e.*, price discounts, coupons, display and feature advertisements. This is because of the following reasons:

1) Category managers usually draft a yearly/quarterly category promotions and management plan ([71]); the aggregate promotion effects on each product and cross products

in the same category are their major concerns. Therefore, analyzing the aggregate promotion effects has realistic applications.

2) Furthermore, researchers observed that the temporary nature of promotional price reduction results in a higher sales spike due to consumers' forward purchases, *promotional elasticities* are greater than *price elasticities* ([8], [49], [54]). Therefore, it is more reasonable to introduce these price discount promotional effects together with the other types of promotional effects through the promotional term x_i. For details about promotional expenses, see section 3.4.3.

3.4.3 Category Shelf-Space Management Model

With P_i as the product i selling price and W_i as the product's wholesale price, the unit profit is $P_i - W_i$ and the *total gross margin* for product i is

$$a_i = (P_i - W_i)d_i \tag{3}$$

where W_i denotes the wholesale price.

Turning to in-store costs for shelf space allocation, in addition to the *fixed cost* CF_i, we adopt the structure of Irion *et al.* ([43]) for the *variable costs*

$$b_i = CP_i d_i + \left(\frac{W_i I G_i}{2}\right) n_i + \left(\frac{CR_i}{G_i}\right) \frac{d_i}{n_i}. \tag{4}$$

With a unit replenishment cost of CP_i, the first term gives the total replenishment cost for product i, which includes costs, such as insurance of products, deterioration, and processing costs of sending items back to the supplier (in case they are broken or not needed any more). With G_i as the number of units of product i that can be stored in a single facing, the second term gives the discounted inventory holding cost for product i. As demand is deterministic and product i is restocked (instantaneously) to its maximum level of $G_i n_i$ only when the shelves are depleted (by Assumption(iv)), the *average* shelf-inventory level is $G_i n_i/2$, and this is multiplied by the unit cost W_i and the discount rate I to get the discounted inventory holding cost for product i. Since the shelves for product i are replenished $d_i/(G_i n_i)$ times and each restock operation costs CR_i, the last term is the restocking cost component, which includes order processing expense, transportation expense, and loading and unloading cost.

Finally, costs associated with the promotion activities are considered. The total expense of promotions on product i is

$$e_i = A_i(x_i - 1). \tag{5}$$

where A_i is a scale factor, $x_i = e_i/A_i + 1$ is a scaled promotion level, which is introduced as a decision variable in the category-level shelf space allocation problem. Because the demand function (2) yields zero demand for a given product if the promotional expense of any other product in the category is zero, we set $x_i = 1$ for a non-promoted product. Therefore, x_i is larger or equal to 1. The total promotion expenditure would be $\sum_{i=1}^{N} e_i$ and it cannot exceed a given promotion budget B.

The objective of the retailer is to maximize the category profit, which equals to the total gross margin minus the total operation costs and promotional expenses. Though retail promotion is sometimes supported by manufacturers through trade allowances, we deduct the promotion cost from the retailer's profit in the model for two reasons. First, there is often no direct link between the magnitude of the allowance and the actions taken by a retailer ([47]). In category management, the retailer usually has an annual budget plan for the promotion activities ([71]). The major concerns for a retailer are to maximize the profit in a category by using the promotional budget efficiently. Second, a retail store stocks a large number of private store brands, and promotions on these private brands constitute a large expenditure for the retailer which should be deducted from the retailer's profit[8].

There are a number of constraints in a retailing environment that have to be included in the model formulation. Similar to Corstjens and Doyle ([22]), our model includes space capacity and control constraints for the allocation. The space capacity constraint ensures that any shelf space allocation must not exceed total available shelf space (Constraint (7)). Space control constraints impose lower and upper bounds for the number of facings allocated to each product (Constraints (9)). Furthermore, we also introduce a promotion budget constraint (Constraint (10)) and promotion control constraints (Constraint (12)), where

[8]For example, it is reported that store brands accounted for 40% of retail sales in Europe and 20% US in 2002 ([40]). Messinger and Narasimhan ([53]) also reported that store brands, which typically offer higher retail margins, accounted for 13% of super market sales in the year ending June 30, 1991.

the budget constraint ensures that the total expense on the promotions of all products in the store must not exceed the preset values, and the promotion control constraint imposes lower and upper bounds for the promotion level. The bounds for the promotion level are closely related to the trade allowances received from the manufacturers. In our model, the magnitude of allowances from a manufacturer changes the lower/upper bound of promotion level for the corresponding product. Also, the retailer's promotional budget is affected by the magnitude of manufacturer allowances.

Since there exists high competition among manufacturers for the scarce shelf space in the retail store, it is impossible to stock the products from all manufacturers. Thus, as in Irion *et al.* [43], we introduce assortment decision variables

$$z_i = \begin{cases} 1 & \text{if product } i \text{ is included in the assortment} \\ 0 & \text{otherwise.} \end{cases}$$

In contrast to the unit profit $a_i - b_i - CF_i$ in [43], here we have the unit profit for product i as $a_i - b_i - CF_i - e_i$, which yields the store profit function

$$\begin{aligned} \Omega_0 &= \sum_{i=1}^{N} [a_i - b_i - CF_i - e_i] z_i \\ &= \sum_{i=1}^{N} \left[\left(P_i - W_i - CP_i - \left(\frac{CR_i}{G_i} \right) \frac{1}{n_i} \right) d_i - \left(\frac{W_i IG_i}{2} \right) n_i - CF_i - A_i(x_i - 1) \right] z_i \end{aligned}$$

Using (2) to rewrite the objective, we obtain the following category shelf space allocation model, which we call **Problem** $P0$:

Find (n_i, z_i, x_i), for $i = 1, 2, \ldots, N$, that maximize

$$\begin{aligned} \Omega_0 = \sum_{i=1}^{N} \Bigg[\alpha_i F_i^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} F_j^{\delta_{ij}} \Bigg(& (P_i - W_i - CP_i) z_i n_i^{\beta_i} x_i^{\mu_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (n_j^{\delta_{ij}} x_j^{\nu_{ij}}) \\ & - \left(\frac{CR_i}{G_i} \right) z_i n_i^{\beta_i - 1} x_i^{\mu_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (n_j^{\delta_{ij}} x_j^{\nu_{ij}}) \Bigg) - \left(\frac{W_i IG_i}{2} \right) z_i n_i - CF_i z_i - A_i(x_i - 1) z_i \Bigg] \end{aligned} \tag{6}$$

subject to

$$\sum_{i=1}^{N} F_i n_i z_i \leq S \tag{7}$$

$$(n_i - 1)(z_i - 1) \geq 0 \qquad i = 1, \ldots, N \tag{8}$$

$$L_i \leq n_i \leq U_i \qquad i = 1, \ldots, N \tag{9}$$

$$\sum_{i=1}^{N} A_i (x_i - 1) \leq B \tag{10}$$

$$(x_i - 1)(z_i - 1) \geq 0 \qquad i = 1, \ldots, N \tag{11}$$

$$X_{Li} \leq x_i - 1 \leq X_{Ui} \qquad i = 1, \ldots, N \tag{12}$$

$$n_i \in \aleph^+ \qquad i = 1, \ldots, N \tag{13}$$

$$x_i \geq 1 \qquad i = 1, \ldots, N \tag{14}$$

$$z_i \in \{0, 1\} \qquad i = 1, \ldots, N \tag{15}$$

where $\aleph^+$ is the set of *positive* integers and the objective is a signomial function, which makes our model NP-Hard.

Similar to the reason of setting $x_i \geq 1$, we restrict n_i to be a positive integer (instead of a nonnegative integer). As pointed out in Irion *et al.* [43], the following rule must be enforced to ensure positive n_i: if $z_i = 0$ (product i is not in the assortment), then $n_i = 1$. This is achieved by nonlinear Constraints (8). Similarly, for our model, the implication if $z_i = 0$ then $x_i = 1$ is ensured by nonlinear Constraints (11).

The above model falls into the class of Mixed Integer Nonlinear Programming (MINLP) problems, which has recently experienced much research activity ([14]). MINLP problems are very hard to solve since they encompass both the combinatorial nature of Mixed Integer Programs (MIP) and the difficulties of solving Nonlinear Programs (NLP). Indeed, our model is further complicated by being a *nonconvex* NLP, which could have several local optima in the continuous case. Using a similar method proposed by Iron *et al.* ([43]), this category shelf space allocation model is reformulated using a piecewise linearization technique (see Appendix B). A linear MIP is derived, whose feasible region, when projected onto the decision space of the shelf space model, is identical to that of the nonconvex shelf space allocation model, and whose optimal objective value is an upper bound on the optimal objective value of the nonconvex model. The reformulated linear MIP model can be solved with CPLEX.

3.5 Shelf Space Allocation with Manufacturer Trade Allowances

Due to the limited shelf space in retail stores and growing competition among manufacturers, manufacturers have pursued growth in mature markets by using trade promotions to increase their market share ([52]). Retailers have become gatekeepers controlling the extent of a manufacturer's influence on consumers, which has allowed retailers to *demand* increased levels of trade promotion for limited shelf access and display feature. It is a well-accepted fact that over the last 20 years, there has been a rise in the magnitude and frequency of trade allowances. An annual survey indicates that grocery manufacturers have increased their allocations to trade promotions from 39% in 1976 to 47% in 1993 ([53]).

When retailers get trade allowances from manufacturers, there is no obligation/contract to guarantee that retailers use those allowances on the promotion of their products. Industry sources estimate that up to 35% of a supermarket chain's profit and up to 75% of a wholesaler's income are derived from retaining trade promotions ([52]). Therefore, trade allowances not only affect shelf space allocation and promotion level, but also significantly influence retailers' profits.

To capture this realistic practice, we introduce the trade allowance into the shelf-space allocation model proposed in Section 3.4.3. A game theoretical method is applied to represent the interaction between retailer and manufacturers. Following the notation in Choi ([18]), the interaction between firms can create one of the following three scenarios due to the variation in bargaining power:

1) *Manufacturer Stackelberg*: The manufacturers have more bargaining power than the retailer and thus are the Stackelberg leader.
2) *Retailer Stackelberg*: The retailer has more bargaining power than the manufacturers and thus is the Stackelberg leader.
3) *Vertical Nash*: Every firm in the system has equal bargaining power.

In modeling the problem, the level of bargaining power possessed by each firm (as compared to the other firms) can be translated into whether the firm is a leader or a follower. In the game-theoretical approach, the firm with more bargaining power can have the first-mover advantage (Stackelberg leader). The firm with less power would then have to respond to the leader's decisions.

As mentioned before, retailers demand trade allowances from manufacturers due to the scarce space and high competition. Therefore, a one-period profit maximizing Retailer Stackelberg game is proposed in this chapter. The retailer is the leader who suggests to manufacturer[9] i a "supply package," $(\mathbf{n_i}, \mathbf{x_i})$, which is defined as several combinations (n_{ik}, x_{ik}), $k = 1, \ldots, K$, of shelf space n_{ik} and promotion level x_{ik} in a period of time, *e.g.*, a quarter. Here, i is the index for manufacturers and k denotes the kth element in $(\mathbf{n_i}, \mathbf{x_i})$[10]. Manufactures are followers who decide trade allowances, $\mathbf{t_i}$, as a response to $(\mathbf{n_i}, \mathbf{x_i})$, where $\mathbf{t_i}$ is a vector with components t_{ik} for each (n_{ik}, x_{ik}). The retailer then finds optimal solutions for the assortment decision, shelf space allocation and promotion level for each product by taking manufacturers' responses into account.

3.5.1 Objective Functions for Retailer and Manufacturers

Before presenting the game, we first model the retailer's shelf space allocation problem with the introduction of trade allowances. Since trade allowances are paid by manufacturers and may contribute to a retailer's profit, they are added into the retailer's profit function (6). Furthermore, the manufacturer will pay a trade allowance with respect to a specific (n_{ik}, x_{ik}); with the regression analysis, we can build functions of trade allowances which depend on (n_i, x_i) as shown in Section 3.5.2. Therefore, the retailer's objective function is:

$$max \quad \Omega_1 = \sum_{i=1}^{N} [a_i - b_i - CF_i - e_i + \breve{t}_i(n_i, x_i)] z_i, \qquad (16)$$

where $\breve{t}_i(n_i, x_i)$ denotes trade allowance paid on product i which will soon become clear (see formula (19)). The retailer maximizes the profit Ω_1 subject to all constraints (7) through

[9] To keep the description simple, we assume that each manufacturer supplies only one product to the retailer.

[10] For example, a supply package for manufacturer 1 includes two elements ($K = 2$): $(\mathbf{n_1}, \mathbf{x_1}) = \{(5, 3), (2, 4)\}$, where $n_{11} = 5$, $n_{12} = 2$, $x_{11} = 3$ and $x_{12} = 4$.

(15) of “Problem P0.” We call the new problem **Problem** $P1$.

For each given (n_{ik}, x_{ik}), manufacturer i's objective is to maximize the profit by selecting an optimal trade allowance, which equals to the total gross margin minus the trade allowance paid to the retailer:

$$max \quad \Pi_{ik} = (W_i - C_i)d_{ik} - \widetilde{t}_{ik}, \qquad k = 1, 2, ..., K, \tag{17}$$

where W_i and C_i are the wholesale price and unit production cost, respectively, which are constants in the problem. d_{ik} is the demand for product i when entertaining a combination of (n_{ik}, x_{ik}); $\widetilde{t}_{ik}$ is the corresponding trade allowance and is also the decision variable of manufacturer i's problem. Based on the manufacturer's *historical data*, for a given n_{ik} and x_{ik}, a manufacturer's demand d_{ik} can be represented as a function of $\widetilde{t}_{ik}$, *i.e.*, $d_{ik} = h_i(\widetilde{t}_{ik})$. Using this relation in the objective (17), manufacturer's optimal solution t_{ik} is found with respect to (n_{ik}, x_{ik}). Then, $\mathbf{t}_i$ is the vector consisting of trade allowances with respect to all combinations of (n_{ik}, x_{ik}) for $k = 1, \ldots, K$, provided in the retailer's supply package[11].

Here, the relations between t_{ik} and d_{ik} are built with the following suggested steps:

1) *Relations between product demand and* (n_{ik}, x_{ik}): Although for a given (n_{ik}, x_{ik}), the demand for product i (see Equation (2)) will not only depend on (n_{ik}, x_{ik}) but also $(\mathbf{n_j}, \mathbf{x_j})$ from other manufacturers ($j \neq i$), manufacturer i usually has a “believed demand” $\tilde{d}_{ik}$ when a combination (n_{ik}, x_{ik}) is entertained. Intuitively, a manufacturer can use the “first principle idea” to guess the demand (called the “believed demand”), where the first principle idea gives a “rough approximation of the demand function.” Therefore, without knowing the values of a competitor's n_j and x_j, this manufacturer will take a few educated guesses of n_j and x_j and apply the “well-known” demand function (2) with the parameters estimated in an educated manner. Here, one can assume that the demand function is given to a manufacturer if it is a part of a more expensive contract. With these few guesses and the resulting demands, the manufacturer i will take the “average” and use it as the “believed demand” $\tilde{d}_{ik}$.

2) *Relations between* z_i *and* $\widetilde{t}_{ik}$: When a very low trade allowance is paid to the retailer,

[11] We will not consider the case where manufacturers provide untruthful information in this research.

manufacturer i may lose the chance to get shelf space, which is $z_i = 0$ in "Problem P1." Therefore, a different trade allowance $\widetilde{t}_{ik}$ leads to a different probability of having nonzero z_i. We denoted this probability by $p_{i,z_i=1}$. Based on *historical data*, $p_{i,z_i=1}$ can be found as an explicit function of $\widetilde{t}_{ik}$ with logistic regression analysis ([55]), *i.e.*, $p_{i,z_i=1} = f_i(\widetilde{t}_{ik})$ for some f_i. Thus, d_{ik} is obtained as:

$$d_{ik} = p_{i,z_i=1}\tilde{d}_{ik} = f_i(\widetilde{t}_{ik})\tilde{d}_{ik}, \tag{18}$$

which shows the explicit representation of d_{ik} as a function of $\tilde{t}_{ik}$.

3.5.2 One-period Retailer Stackelberg Game

The one-period Retailer Stackelberg game between a retailer and manufacturers is as follows:

1) The retailer will design a supply package $(\mathbf{n_i}, \mathbf{x_i})$ for each product i.

2) A manufacturer submits to the retailer a vector $\mathbf{t_i}$ in response to $(\mathbf{n_i}, \mathbf{x_i})$.

3) Based on the information of $\mathbf{t_i}$ and $(\mathbf{n_i}, \mathbf{x_i})$, the retailer finds out manufacturers' trade allowance response functions by using regression analysis ([55]). We denote the response function by $\breve{t}_i(n_i, x_i) = g_i(\mathbf{n_i}, \mathbf{x_i}, \mathbf{t_i})$. Note that for a specific combination of n_i^* and x_i^*, the response function $g_i(\mathbf{n_i}, \mathbf{x_i}, \mathbf{t_i})$ gives the "prediction" $\breve{t}_i(n_i^*, x_i^*)$. The combination (n_i^*, x_i^*) does not have to be in the retailer's supply package $(\mathbf{n_i}, \mathbf{x_i})$ which only has a limited number of "discrete" combinations. By bringing these response functions into the objective function of "Problem P1," the retailer's problem becomes:

$$Max \quad \Omega_1 = \sum_{i=1}^{N}[a_i - b_i - CF_i - e_i + g_i(\mathbf{n_i}, \mathbf{x_i}, \mathbf{t_i})]z_i, \tag{19}$$

subject to all constraints (7) through (15), which is:

Find (n_i, z_i, x_i), for $i = 1, 2, \ldots, N$, that maximize

$$\begin{aligned}\Omega_1 = \sum_{i=1}^{N}\Bigg[\alpha_i F_i^{\beta_i}\prod_{\substack{j=1\\ j\neq i}}^{N} F_j^{\delta_{ij}}\Bigg((P_i - W_i - CP_i)z_i n_i^{\beta_i} x_i^{\mu_i}\prod_{\substack{j=1\\ j\neq i}}^{N}(n_j^{\delta_{ij}} x_j^{\nu_{ij}})\\ -\left(\frac{CR_i}{G_i}\right) z_i n_i^{\beta_i - 1} x_i^{\mu_i}\prod_{\substack{j=1\\ j\neq i}}^{N}(n_j^{\delta_{ij}} x_j^{\nu_{ij}})\Bigg) - \left(\frac{W_i IG_i}{2}\right) z_i n_i - CF_i z_i\\ -A_i(x_i - 1)z_i + g_i(\mathbf{n_i}, \mathbf{x_i}, \mathbf{t_i})\Bigg]\end{aligned} \tag{20}$$

subject to constraints (7) to (15). When using linear regression, the regression function g_i is linear. Solving this problem, the retailer gets the optimal solutions (n_i^*, z_i^*, x_i^*), for $i = 1, 2, \ldots, N$, which incorporate manufacturers' responses of trade allowances.

This shelf-space allocation model with Stackelberg game has the following properties. For a manufacturer,

1) larger trade allowance responses lead to a higher chance to be assigned a positive shelf space in the retail store;

2) larger trade allowance response does not necessarily lead to a larger shelf space or promotion level.

Both of these two properties reflect industry practice. The first property is consistent with the purpose of trade allowances, *i.e.*, for competition of scarce shelf space and supporting the retail merchandising activities. The second property is consistent with the present market situations, which are the "number-one" concern among manufacturers, as indicated by the 1998 Trade Promotion Best Practices survey by Cannondale Associates ([42]).

Remark: This model can be modified to model the situations where the retailer promises a shelf space or a promotion level with respect to a trade allowance as well. To guarantee the levels of shelf space or promotional efforts with respect to a trade allowance, the lower bound of shelf space L_i and promotion level X_{Li} in constraints (9) and (12) can be represented as a function of $g_i(\mathbf{n_i}, \mathbf{x_i}, \mathbf{t_i})$, *i.e.*, $X_{Li} = k_i g_i(\mathbf{n_i}, \mathbf{x_i}, \mathbf{t_i})/A_i$. However, to keep the problem simple and easy to understand, this modification will not be included in this chapter of thesis work.

3.6 Numerical Examples and Quantitative Analysis

As mentioned before, the proposed framework has two major improvements over the existing shelf space management model. First, it considers manufacturers' trade allowance as a response to the resource assignment. Second, it models the interactions between the space allocation, promotion activities and trade allowance charged from manufacturers. In this section, the proposed framework is analyzed from both the retailer's and manufacturers' perspectives in order to demonstrate the potentials and properties associated with these two improvements.

For retailers, one of the major indices for evaluating a space allocation method is the profit improvement. Therefore, experiment 1 is designed to show the benefits of the proposed methods. Specifically, it aims to answer the following questions:

- Question 1: Will taking the manufacturers' trade allowance response into account make the retailer better off?
- Question 2: How much profit can a retailer gain if the proposed methods are utilized to replace his existing space assignment?

Simulations are designed to answer these two questions in section 3.6.2. Section 3.6.2.1 illustrates the problem solving procedure of the proposed framework. In section 3.6.2.2, solutions are compared with the case where the response function is not included and also compared with a retailer's existing shelf-space allocation solutions.

Since a manufacturer's trade allowances have an impact on his resource competition in the proposed framework, it is interesting to see what the impact is regarding to his chance for winning the shelf space and promotion efforts from a retailer. Thus, in section 3.6.3, experiment 2 is designed to answer the question:

- Question 3: As the magnitude of the trade allowance increases, how do the assigned shelf space, retailer's promotion efforts and manufacturer's product demand change?

Simulations are run with real data collected from a retail store, which is also presented in the related paper ([43]). We will first introduce the data set in section 3.6.1.

3.6.1 Data Set and Parameters

"Bulbs category" is selected in this study due to the inclusion of a large variety in product demands. The dimension of shelf space for the bulbs category is (Height, Depth, Length) = (40, 37, 166). Product information collected during a non-promoted period are listed in Table 18. The current space allocation in the category is: $\Lambda = (n_1, n_2, n_3, n_4, n_5, n_6) =$ (6, 6, 4, 6, 6, 6).

Table 18: Product information in bulbs category

Product ID	Product Dimension Height	Depth	Length	Retail Price [$]	Wholesale Price [$]	Units Sold
1	7	3	5	1.37	0.92	651
2	7	3	5	1.37	0.92	627
3	7	3	4	2.17	1.45	130
4	7	3	5	1.37	0.92	451
5	7	3	5	0.96	0.64	1268
6	7	3	5	1.37	0.92	845

With the information from the store management, parameters related to operation costs and space restrictions are estimated and assumed to be equal for all products in a category:

Restocking cost CR_i is \$5;

Replenishment cost $CP_i = 0.01 * W_i$;

Fixed cost to include product i in the assortment CF_i is \$25;

Investment/interested rate $I = 1\%$ per month.

In addition, to increase product variety, the retailer restricts the maximum space assigned to a product to be no larger than $\frac{1}{4}$ of the total space. There is no restriction on the lower bound of space assigned to a product.

We estimate space elasticity, promotion elasticity and cross-elasticities based on past research since cross-sectional data needed to estimate them via regression is not available, In the past research (e.g., [24], [22]), space elasticity typically ranges between 0.06 and 0.25, whereas cross space elasticities are assumed to take on values between -0.01 and -0.05. Thus, space and cross-elasticities are assigned randomly to the investigated products within these ranges. Note that the product sales data shown in Table 18 does not include promotional effects. With the sales data as demand and the estimated space and cross space elasticities, the scaling factor α_i can be calculated using equation (2), where x_i and x_j are 1 for all i and j. Promotions have a larger impact on demand than space allocation, *i.e.*, promotion elasticity on advertisement can be around 0.3 ([64]) and the impacts of promotional price cuts are much higher ([19]). Furthermore, the promotion and cross promotion elasticity in this research represent the comprehensive promotional effects from multiple promotion activities, such as price cut, advertisement and so on, therefore, promotion elasticities are assigned values between 0.4 and 0.43. Compared to promotion elasticities, cross promotion elasticities are very small ([64]), however they are higher than cross space elasticities. They are assigned randomly within the range of -0.06 to -0.1 to the investigated products.

Parameters related to promotional budget are estimated. Retailers are assumed to invest no more than 30% of the total gross margins obtained from the previous planning cycle for the category's promotion activities, which is $B = \$500$, and no more than 85% of the gross margin obtained from a product for its related promotions. In empirical analysis, promotion intensity is usually classified as three levels: "Strong," "Moderate," or "None" ([17]). Here, the maximum promotion level is set as 5 for all products. The scale factor for the promotional expense, A_i, is then represented by the division of product promotion budget by 5, which are 49.8, 48, 15.9, 34.5, 69, 64.6 with respect to products 1 to 6.

3.6.2 Experiment 1: Profit Improvements for the Retailer

Focusing on answering the first two questions, we identify the following four cases.

- Case 1: We implement the proposed framework to solve the retail shelf space allocation problem, Problem $P1$.

- Case 2: We solve Problem $P0$, the optimization problem where the response functions of manufacturers' trade allowances are not introduced into the model.

To see the potential profit improvements over the retailer's existing space allocation, the other two cases are identified as:

- Case 3: We solve Problem $P1$ by setting n_i for $i = 1, \ldots, 6$ with the current assignment Λ in the retail store.

- Case 4: We solve Problem $P0$ by setting n_i for $i = 1, \ldots, 6$ with the current assignment Λ in the retail store.

In the remainder of this section, the problem solving procedure is introduced followed by the solution comparisons.

3.6.2.1 The Proposed Framework - Case 1

Stage 1: Retailer Defines Supply Package

The retailer suggests to manufacturer i a supply package $(\mathbf{n_i}, \mathbf{x_i})$, $i = 1, \ldots, 6$ in order to acquire the manufacturers' response functions. Since the maximum number of facings assigned to a product is 10 for product 3 and 8 for the remaining products[12], $\mathbf{n_3}$ includes all integers in the range of [1, 10] and $\mathbf{n_i}$ includes all integers from [1, 8] for all $i \neq 3$. We assign $\mathbf{x_i}$ values (1, 3, 5), which corresponds to "None", "Moderate" and "Strong" promotion intensity. Therefore, the supply package $(\mathbf{n_i}, \mathbf{x_i})$ includes the full enumeration between values from $\mathbf{n_i}$ and $\mathbf{x_i}$.

Stage 2: Manufacturers Make the Best Response

For each given (n_{ik}, x_{ik}), retailer i solves the optimization problem (17). As suggested in section 3.5.1, to build the relations between product demand and $(n_{ik}\ x_{ik})$, manufacturer i needs to find out the "believed demand" $\tilde{d}_{ik}$. When historical data is available, this is easily obtained with a regression analysis. However, historical data might not be available. In such a situation, the retailer needs to provide demand function (2) to the manufacturers. With the demand function (2), $\tilde{d}_{ik}$ is computed in the following way.

- Treat all the other products $j \neq i$ as a single product J, and identify the possible accumulated space and promotion level for these products. For example, when (n_{ik}, x_{ik}) is (5, 1), the total available space for the remaining products is 141. Then there are 3 possible combinations of (s_{Jk}, x_{Jk}) for the remaining products: (141, 1), (141, 3) and (141, 5). The reason for treating all remaining products as a single product here is because when estimating the demand for a manufacturer, it is more interesting to see the aggregated effects from the rest of the products, instead of the detailed information from each individual product. Furthermore, it is computationally intractable to enumerate all of the possible combinations among all of the remaining individual products.

[12] The maximum number of facings assigned to a product equals to the maximum length allowed for a product divided by the product length, which is $(\frac{1}{4} * 166)/$ product length.

- For each combination, compute manufacturer i's demand with demand function (2), where (n_{ik}, x_{ik}) and (s_{Jk}, x_{Jk}) are used as the input, and the average value of cross-elasticities between i and $j \in J$ is applied to represent the cross-elasticity between i and J. The expectation of these demands then represents $\tilde{d}_{ik}$. For example, when $(n_{ik}, x_{ik}) = (5,\ 1)$, demands for manufacturer i are 799, 732 and 703 with respect to the combinations of (s_{Jk}, x_{Jk}) of (141, 1), (141, 3) and (141, 5). We obtain $\tilde{d}_{ik}$ as $\frac{1}{3}(799 + 732 + 703) = 745$.

Note that demand $\tilde{d}_{ik}$ is obtained only when manufacturer i wins the assignment (n_{ik}, x_{ik}) by paying a high enough trade allowance, t_{ik}. Zero demand is incurred otherwise. Therefore, a logistic regression is applied to build the relations between the binary response of losing versus winning: $Y_{ik} = \{0, 1\}$ and the trade allowance t_{ik} paid by i with historical data. The mean response $E[Y_{ik}]$ denotes the probability that $Y_{ik} = 1$, which is $p_{i,z_i=1}$ in step 2) presented in section 3.5.1.

Due to the lack of historical data, the following mean response function is used where the major relations between Y_{ik} and t_{ik} representing the real practice are expressed:

$$E[Y_{ik}] = p_{i,z_i=1} = \begin{cases} b_{ik} & 0 \leq t_{ik} < t_{ika}, \\ a_{ik} ln(\lambda_{ik} t_{ik}) + b_{ik} & t_{ika} \leq t_{ik} < t_{ikb}, \\ 1 & t_{ik} \geq t_{ikb}. \end{cases}$$

Here a_{ik} ($a_{ik} > 0$) and λ_{ik} ($\lambda_{ik} > 0$) are scale factors, and b_{ik} ($b_{ik} \geq 0$) is a constant. Since the probability $p_{i,z_i=1}$ is in the range of [0, 1], by solving equation

$$a_{ik} ln(\lambda_{ik} t_{ik}) + b_{ik} = 0 \text{ and } a_{ik} ln(\lambda_{ik} t_{ik}) + b_{ik} = 1,$$

it is found that $t_{ika} = \frac{1}{\lambda_{ik}} e^{\frac{-b_{ik}}{a_{ik}}}$ and $t_{ikb} = \frac{1}{\lambda_{ik}} e^{\frac{1-b_{ik}}{a_{ik}}}$, respectively. Intuitively, when t_{ik} is small, a small increase in t_{ik} has no impact on the retailer's resource allocation. Hence, $p_{i,z_i=1}$ is a constant, b_{ik}, for $0 \leq t_{ik} < t_{ika}$. As t_{ik} increases, the probability, $p_{i,z_i=1}$, increases, but at a diminishing rate until it reaches 1. Thus, $p_{i,z_i=1}$ is represented by a non-linear function with diminishing return in the interval of $[t_{ika}, t_{ikb})$. As $p_{i,z_i=1}$ reaches the upper bound, 1, an increase in t_{ik} has no effect on $p_{i,z_i=1}$.

Given the believed demand $\tilde{d}_{ik}$ of realizing (n_{ik}, x_{ik}) and the probability function $E[Y_{ik}]$ of winning this demand by paying a trade allowance t_{ik}, the relation between the expected demand d_{ik} and t_{ik} is obtained via formula (18) as follows:

$$d_{ik} = p_{i,z_i=1}\tilde{d}_{ik}. \tag{21}$$

Substituting d_{ik} into the manufacturer's profit maximizing problem (17), the optimal solutions are represented as:

$$t^*_{ik} = \begin{cases} \frac{a_{ik}}{\lambda_{ik}}(W_i - C_i)\tilde{d}_{ik} & 0 \leq t^*_{ik} < t_{ikb}, \\ t_{ikb} & t^*_{ik} \geq t_{ikb}, \end{cases}$$

Due to the high level of competition over the scarce space, t_{ikb} can be assumed to be very large. Thus, for each given (n_{ik}, x_{ik}), an optimal t^*_{ik} is easily found as $\frac{a_{ik}}{\lambda_{ik}}(W_i - C_i)\tilde{d}_{ik}$ and $\mathbf{t_i}$ represents the set of t^*_{ik} with $k = 1, \ldots, K$.

Let ρ_{ik} denote $\frac{a_{ik}}{\lambda_{ik}}(W_i - C_i)$, where ρ_{ik} can be counted as the trade allowance that manufacturer i would like to pay for a unit of the demand. To simplify the computation, ρ_{ik} is assumed to be the same for all k. In practice, it is found that the promotional allowances for products vary. The size of the promotional allowance is usually greater than 20% of the product cost. Furthermore, there is a tendency for manufacturers to offer smaller promotional allowances for products with large market shares within their merchandise categories ([17]). Therefore, ρ_{1k} through ρ_{6k} are assigned the values 0.248, 0.258, 0.58, 0.332, 0.128 and 0.212, which equates to 25%, 26%, 58%, 32%, 13% and 21% of the product unit cost.

Stage 3: Retailer Determines the Resource Allocations

After collecting the information $\mathbf{t_i}$ from all of the manufacturers, the retailer determines the response functions through a regression analysis on $(\mathbf{n_i}, \mathbf{x_i})$ and $\mathbf{t_i}$. We employ the method of least squares, where the estimators are unbiased and have minimum variance among all unbiased linear estimators ([55]). The resulting response functions turned out to be the linear forms:

$$\check{t}_1(n_1, x_1) = 53.41 + 7.95n_1 + 57.88x_1, \tag{22}$$

$$\check{t}_2(n_2, x_2) = 60.54 + 9.43n_2 + 65.17x_2, \tag{23}$$

$$\check{t}_3(n_3, x_3) = 39.85 + 5.02n_3 + 40.19x_3 \tag{24}$$

$$\check{t}_4(n_4, x_4) = 52.81 + 7.86n_4 + 57.32x_4 \tag{25}$$

$$\check{t}_5(n_5, x_5) = 60.54 + 9.86n_5 + 64.51x_5 \tag{26}$$

$$\check{t}_6(n_6, x_6) = 86.97 + 9.69n_6 + 74.27x_6 \tag{27}$$

With these functions, the retailer can solve Problem 1.

3.6.2.2 **Solutions Analysis**

The optimal solutions for problem Case 1 through Case 4 are solved with GAMS. The optimal solutions are listed in Table 19.

Table 19: Solutions comparison

		Product ID						
Case		1	2	3	4	5	6	Profit
1	z_i	1	1	1	1	1	1	2940.916
	n_i	8	7	1	1	8	8	
	x_i	1	1	1	1	4.501	5	
	t_i	174.89	191.72	85.06	117.99	429.808	535.84	
	promotion cost	0	0	0	0	241.6	258.4	
2	z_i	1	1	1	1	1	1	2653.765
	n_i	8	7	1	1	8	8	
	x_i	1	1	1	1	1	2.59	
	t_i	174.89	191.72	85.06	117.99	203.93	356.878	
	promotion cost	0	0	0	0	0	102.739	
3	z_i	1	1	1	1	1	1	2764.867
	n_i	6	6	4	6	6	6	
	x_i	5	1	1	1	1	5	
	t_i	390.51	182.29	100.12	157.29	184.21	516.46	
	promotion cost	199.2	0	0	0	0	258.4	
4	z_i	1	1	1	1	1	1	2440.988
	n_i	6	6	4	6	6	6	
	x_i	1	1	1	1	1	1	
	t_i	158.99	182.29	100.12	157.29	184.21	219.38	
	promotion cost	0	0	0	0	0	0	

The proposed method, Case 1, shows a significant advantage over the remaining cases. When comparing the proposed method with Case 2, where the manufacturers' response functions are not included, 12% profit improvement is observed. Similarly, 13% profit improvement is obtained in Case 3 over Case 4 due to the consideration of the manufacturers' response in Case 3 but not in Case 4. These facts demonstrate the importance of the introduction of the manufacturers' response function in the model, which clearly answers Question 1. Furthermore, our method has a 20.5% profit improvement over Case 3, where the existing assignment Λ is utilized and trade allowance responses from the manufacturers are not considered, which indicates the significant benefits from implementing the Retailer Stackelberg game framework.

To further understand the practical feasibility of the proposed solutions from Case 1, we compare the space and promotion levels among these 6 products. Products 3 and 4 are low-demand products, and therefore, are assigned few facings without promotions. High-demand products, products 5 and 6, receive the largest space with strong promotion activities. The remaining two products with moderate demand structures are given large space without promotions.

It is also found that the retailer requests different magnitudes of trade allowances from different manufacturers for various purposes. The results show that no promotion effort is allocated to products 1 through 4. Trade allowances therefore are charged as "rental" of the shelf space, which is commonly charged in industry ([11]). As we can see, the "rental" per unit space is different across various products. In addition, trade allowances charged from products 4 and 5 are both for the "rental" of the shelf space and for supporting promotions. However, a larger portion of trade allowances from product 6 is pocketed by the retailer than from product 5. These results indicate the unfairness from the competition perspective. However, they are feasible and represent real industrial practice. As mentioned before, since the trade allowances are usually negotiated between retailers and manufacturers, no public information concerning the magnitudes and specifications is available, therefore, retailers usually charge different "rental" even for the same space from different manufacturers, and various pass through is often applied to various products in practice ([11], [17]).

In summary, the proposed framework results in a significant profit increase for the retailer. Furthermore, it suggests practically feasible solutions which systematically assign space and promote efforts with the consideration of both the manufacturers' response and interactions among products.

3.6.3 Experiment 2: Impact of Trade Allowance for Manufacturers

In practice, manufacturers tend to pay higher trade allowances to increase the shelf space and promotion intensity in retail stores in the hopes of increasing the market share ([34]). Therefore, if the retailer implements the proposed method for resource allocations, it is critical to understand the impact of trade allowances on resource allocations from the manufacturers' perspective. Experiment 2 is designed to test the model for this purpose.

Factor η_i ($\eta_i > 0$) is introduced to denote the magnitude of trade allowances from manufacturer i. Instead of submitting $\mathbf{t_i}$, manufacturer i submits $\eta_i \mathbf{t_i}$, and the remaining manufacturers (all $j \neq i$) submit $\mathbf{t_j}$, which amounts to taking $\eta_j = 1$. We solve the shelf space allocation problem with the proposed method for each value of η_i in the set {0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2}. The changes of product demand, trade allowances charged by the retailer and the retailer's profit are then observed as a function of η_i. This is run on each of the 6 products. Solutions are shown in Figures 9 through 11.

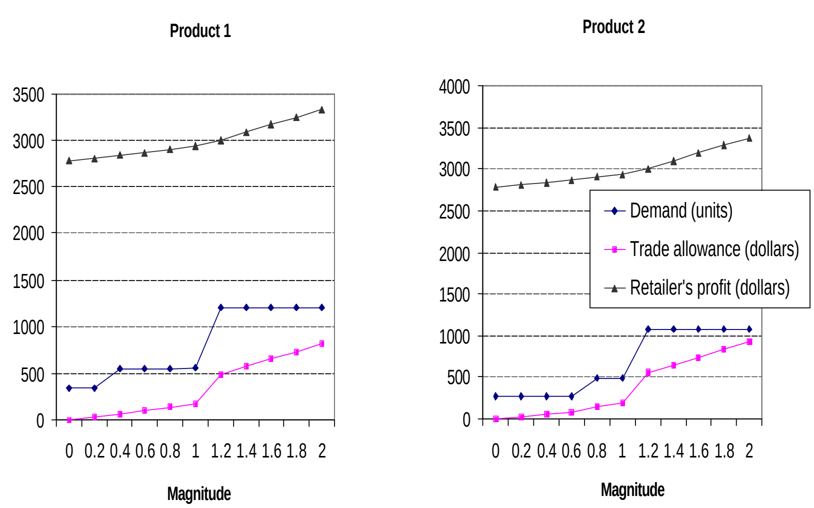

Figure 9: Solutions for product 1 and 2

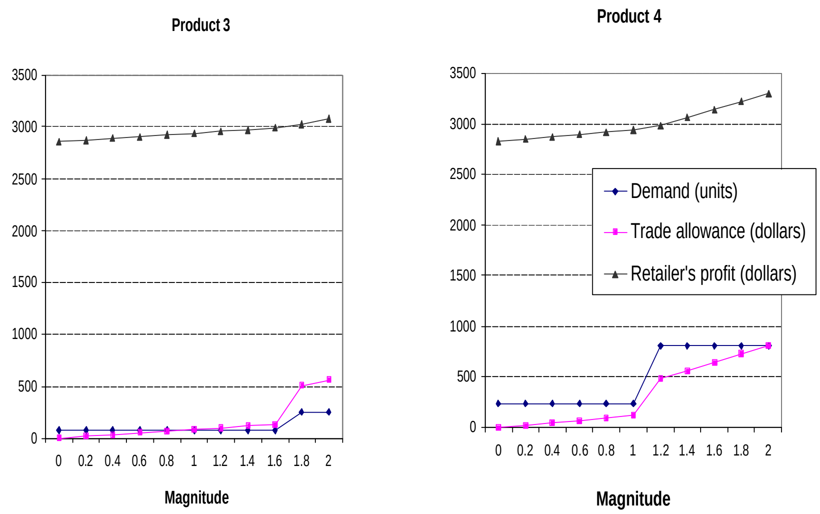

Figure 10: Solutions for product 3 and 4

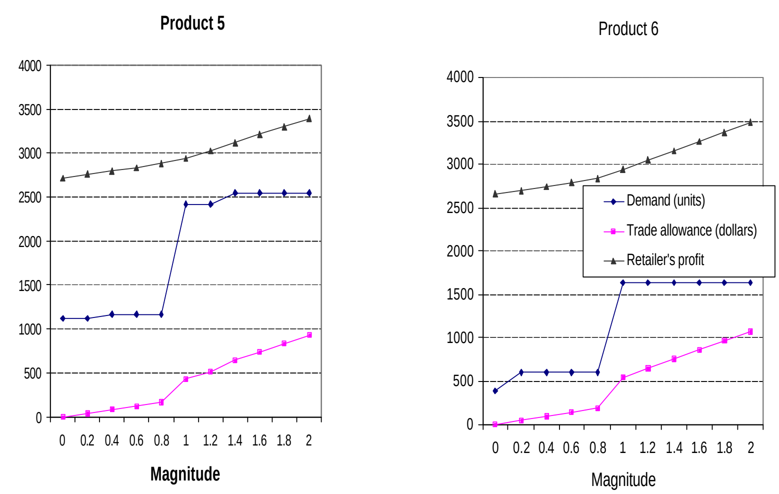

Figure 11: Solutions for product 5 and 6

The solutions show the following trends:

- First, with an increase in η_i, product demand d_i is non-decreasing. When η_i increases, the product demand does not increase continuously, but rises up until it reaches a threshold point. This pattern repeats until a product is assigned with the maximum allowed space and highest possible promotion intensity. Therefore, with the proposed framework, manufacturers can expect the trend of increasing demand as a result of an increase in the magnitude of trade allowances. However, it is hard to forecast the change in n_i and x_i associated with an increase of η_i. For example, the demand increase for product 1 at $\eta_1 = 0.4$ is due to the increase of n_1, while the demand increase at $\eta_1 = 1.2$ is from the increase of t_1[13].

- On the other hand, the increase in η_i can also lead to a larger retailer's pocket rate, *i.e.* the percentage of trade allowance pocketed by the retailer. For example, product 1's demands are fixed values when η_i is in the intervals [0.4, 1] and [1.2, ∞). The increased trade allowances from manufacturer 1 do not help to obtain a larger shelf space or to pass through to customers via promotions, instead, it contributes to a retailer's profits in this case.

Although it is a monotonic nondecreasing function between trade allowances and demands, the increase in trade allowances *generally* increases demands via enlarging the shelf space and promotion activities. Therefore, this is a rational resource allocation framework from the manufacturers' perspectives.

When more than one manufacturer tends to pay larger trade allowances, solutions are affected by the relative magnitudes among these trade allowances, which is not helpful for evaluating the proposed model due to the complexity, hence, this case will not be discussed in this section.

Finally, to further investigate properties of the proposed framework, it is also interesting to examine model sensitivity to input data errors. For example, elasticities estimations and manufacturers' response functions are obtained through regression analysis, and the

[13] The jump points in the demand figure are either from the increase of n_i, x_i or both.

manufacturers' estimation of product demands with respect to a trade allowance is also obtained via regression analysis. All these regressions can lead to errors. Due to time constraints, we will examine the robustness of the proposed model to these errors in the future.

3.7 Conclusion

In this project, promotion effect is introduced to the decision of product selection and shelf space allocation in the management of category-level shelf space, and is modeled as an MINLP. A piecewise linearization method is proposed to solve the MINLP problem. Furthermore, based on the proposed category shelf space allocation model, a category shelf space allocation framework with trade allowances is presented, where a multi-player Retailer Stackelberg game is introduced to model the interactions between retailer and manufacturers. With this framework, a retailer maximizes the profit by taking the manufacturers' trade allowances response into account, which provides a realistic approach of simultaneously determining both the promotion level and pass through of trade allowances. Numerical examples demonstrate significant potentials of the proposed framework.

3.8 References

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Appendix A: Stackelberg Game

Stackelberg ([65]) proposed a dynamic model with 2 players in which a dominant (or leader) player moves first and a subordinate (or follower) player moves second. It is assume that players' payoff functions are common knowledge. The timing of the game is as follows: 1) player-one (leader) chooses a decision q_1; 2) player-two (follower) observes q_1 and then chooses a decision q_2; 3) the payoff of player i is then given by the profit function $\pi_i(q_i, q_j)$.

This game sometimes has multiple Nash equilibriums and backwards-induction ([19]) is the most commonly used method solving for one of the Nash equilibriums-Stackelberg equilibrium. To solve for the backwards-induction outcome, let Q_1 and Q_2 denote the feasible action set for q_1 and q_2, respectively. When player-two knows the player-one's move at the second stage of the game, he or she will face the following problem, given the action q_1 previously chosen by player-one:

$$max_{q_2 \in Q_2} \quad \pi_2(q_1, q_2), \tag{28}$$

Assume that for each q_1 in Q_1, player-two's optimization problem has a unique solution, denoted by $R_2(q_1)$. This is player-two's reaction (or best response) to player-one's action. Since player-one can solve player-two's problem as well as player-two can (due to the assumption of "players' payoff functions are common knowledge"), player-one should anticipate player-two's reaction to each action q_1 that player-one might take, so player-one's problem at the first stage becomes:

$$max_{q_1 \in Q_1} \quad \pi_1(q_1, R_2(q_1)). \tag{29}$$

Assume that this optimization problem for player-one also has a unique solution, denoted by q_1^*. We will call $(q_1^*, R_2(q_1^*))$ the backward-induction outcome of this game, which is the Stackelberg equilibrium where no player wants to deviate from this solution.

It is straightforward to extend what follows to allow for more than one following players.

Appendix B: Model Reformulation and Piecewise Linearization

Following the treatment in [43], this appendix details the technique for solving the MINLP. Focusing first on the constraints of the product category model, observe that the bilinear term $z_i n_i$ in constraints (7) and (8) and the bilinear term $x_i z_i$ in constraints (11) are nonlinear. In order to linearize these constraints we use the technique of Al-Khayyal and Falk ([3]) and Al-Khayyal ([2]), who propose a reformulation technique for finding global solutions of bilinear programming problems (see also Sherali and Adams ([63]) who extended this technique). The technique involves the use of the convex and concave envelope of a bilinear function over a rectangular region. Each bilinear term (in our case $z_i n_i$ and $z_i x_i$) is replaced by a new variable and four additional linear constraints are imposed on each of these variables. It is crucial to note that the nonlinear constraints (7), (8) and (11) are replaced, using the technique in Al-Khayyal and Falk ([3]) and Al-Khayyal ([2]), by an *equivalent* system of *linear* inequalities.

Turning our attention to the objective function (54), we define the following two intermediate variables that facilitate the description of our linearization scheme

$$u_i = n_i^{\beta_i} x_i^{\mu_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (n_j^{\delta_{ij}} x_j^{\nu_{ij}}) \tag{30}$$

$$v_i = n_i^{\beta_i - 1} x_i^{\mu_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} (n_j^{\delta_{ij}} x_j^{\nu_{ij}}). \tag{31}$$

Our objective function can now be written more concisely as

$$\Omega_0 = \sum_{i=1}^{N} \left[\alpha_i F_i^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} F_j^{\delta_{ij}} \left((P_i - C_i - CP_i) z_i u_i - \left(\frac{CR_i}{G_i} \right) z_i v_i \right) \right. \\ \left. - \left(\frac{C_i IG_i}{2} \right) z_i n_i - A_i x_i z_i - (CF_i - A_i) z_i \right] \tag{32}$$

which exhibits a linear component z_i and bilinear components $z_i u_i$, $z_i v_i$, $z_i n_i$ and $x_i z_i$. The bilinear components can be reformulated into equivalent linear forms subject to additional side constraints using the reformulation technique cited above; however, that would still leave nonlinear monomial constraints (30) and (31). To circumvent this difficulty, we replace u_i with e^{m_i} and v_i with $e^{m'_i}$ in (32) to obtain the equivalent objective function.

$$\Omega_0 = \sum_{i=1}^{N} \Bigg[\alpha_i F_i^{\beta_i} \prod_{\substack{j=1 \\ j \neq i}}^{N} F_j^{\delta_{ij}} \left((P_i - C_i - CP_i) z_i e^{m_i} - \left(\frac{CR_i}{G_i} \right) z_i e^{m_i'} \right)$$
$$- \left(\frac{W_i IG_i}{2} \right) z_i n_i - A_i x_i z_i - (CF_i - A_i) z_i \Bigg] \tag{33}$$

with side constraints for all $i = 1, \ldots, N$

$$m_i = \beta_i ln(n_i) + \mu_i ln(x_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[\delta_{ij} ln(n_j) + \nu_{ij} ln(x_j) \right] \tag{34}$$

$$m_i' = (\beta_i - 1) ln(n_i) + \mu_i ln(x_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[\delta_{ij} ln(n_j) + \nu_{ij} ln(x_j) \right]. \tag{35}$$

These constraints are still nonlinear, but each individual function $ln(n_i)$ and $ln(x_i)$ can be approximated by a piecewise linear function over their interval $[L_i, U_i]$ and $[X_L, X_U]$. Since the functions $ln(n_i)$ and $ln(x_i)$ are concave, their piecewise linear representations are greatly simplified by using separable programming theory (see [66]).

For reasons that will soon become clear, we need to derive lower and upper bounds, denoted as A_i and B_i, on m_i given by (34). Similarly, we compute lower bounds A_i' and upper bounds B_i' on m_i' given by (35). For this discussion, recall that $0 \leq \beta_i \leq 1$ and $0 \leq \mu_i \leq 1$, whereas $\delta_{ij} \leq 0$ and $\nu_{ij} \leq 0$. Therefore, m_i is at its upper bound B_i if n_i is at its maximum value U_i and n_j is at its minimum value L_j. A similar but opposite argument can be made for finding the lower bound A_i of m_i. Thus, we have

$$A_i = \beta_i ln(L_i) + \mu_i ln(X_{Li}) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[\delta_{ij} ln(U_j) + \nu_{ij} ln(X_{Uj}) \right] \tag{36}$$

$$B_i = \beta_i ln(U_i) + \mu_i ln(X_{Ui}) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[\delta_{ij} ln(L_j) + \nu_{ij} ln(X_{Lj}) \right] \tag{37}$$

and

$$A_i \leq m_i \leq B_i. \tag{38}$$

Since $\beta_i \leq 1$, the bounds A_i' and B_i' for m_i' are

$$A_i' = \beta_i ln(U_i) + \mu_i ln(X_{Ui}) + \sum_{\substack{j=1 \\ j \neq i}}^{N} [\delta_{ij} ln(U_j) + \nu_{ij} ln(X_{Uj})] \tag{39}$$

$$B_i' = (\beta_i - 1) ln(L_i) + \mu_i ln(X_{Li}) + \sum_{\substack{j=1 \\ j \neq i}}^{N} [\delta_{ij} ln(L_j) + \nu_{ij} ln(X_{Li})] \tag{40}$$

and

$$A_i' \leq m_i' \leq B_i'. \tag{41}$$

Although the two new sets of constraints (34) and (35) can be linearized using the foregoing separable programming arguments, the objective function (33) is still nonlinear. We deal with the exponential terms by judiciously approximating them over the bounds on their arguments. In particular, we want our approximating objective to *overestimate* (33) so that the optimal objective value of the estimating problem provides an upper bound on the true optimal object value. Specifically, we approximate e^{m_i} by a *convex* piecewise linear *over*estimating function, and $e^{m_i'}$ is approximated by a *convex* piecewise linear *under*estimating function. Notice that we want *both* lower and upper approximations of a convex function to be convex. Our choice of which approximation (lower or upper) to choose is based on the objective coefficient of the exponential term in (33). Since we must have $P_i - C_i - CP_i \geq 0$ (otherwise, $z_i = 0$ would always be optimal), the coefficient of e^{m_i} is nonnegative, so we overestimate it. On the other hand, the coefficient of $e^{m_i'}$ is nonpositive, so we need an underestimate of $e^{m_i'}$ in order to have an overestimate of its negation.

A convex piecewise linear overestimating function of e^{m_i} created by choosing one grid points $E_i \in (A_i, B_i)$. The two linear functions are given by

$$\begin{aligned} \Delta_i &= e^{A_i} + \left(\frac{e^{E_i} - e^{A_i}}{E_i - A_i} \right) (m_i - A_i) \\ \Phi_i &= e^{E_i} + \left(\frac{e^{B_i} - e^{E_i}}{B_i - E_i} \right) (m_i - E_i). \end{aligned}$$

In general, E_i can be taken anywhere in the open interval (A_i, B_i), but we use

$$E_i = ln \left(\frac{e^{B_i} - e^{A_i}}{B_i - A_i} \right)$$

which maximizes the absolute difference between e^{m_i} and the line segment connecting e^{A_i} and e^{B_i}. The piecewise linear function $\max\{\Delta_i, \Phi_i\}$ overestimates e^{m_i} over the interval $[A_i, B_i]$ with Δ_i defined on the subinterval $[A_i, m_i]$ and Φ_i defined on the subinterval $[m_i, B_i]$. Hence, we have, for all $i = 1, \ldots, N$,

$$\begin{aligned} e^{m_i} &\leq \max\{\Delta_i, \Phi_i\} \\ &\equiv y_i \Delta_i + (1 - y_i)\Phi_i \\ &= s_i \end{aligned}$$

for all y_i satisfying

$$y_i(E_i - m_i) \geq 0 \qquad (42)$$

$$(1 - y_i)(E_i - m_i) \leq 0 \qquad (43)$$

$$m_i \in [A_i, B_i] \qquad (44)$$

$$y_i \in \{0, 1\}. \qquad (45)$$

Therefore, replacing e^{m_i} by s_i in our objective (33) and incorporating the constraints (42)–(45) yields a linear mixed integer reformulation of the piecewise linear overestimating function of the exponential term. The remainder of the paper is restricted to the case $K = 1$, since the nominal improvement in the approximate solutions of several test problems did not justify the significant increase in the additional computation times for $K \geq 2$.

With bounds on s_i easily computed from (36) and (37), and after replacing all e_{m_i} by s_i in the objective function (33), we linearize all occurrences of the bilinear terms $z_i s_i$ in (33) and $y_i m_i$ in the constraints (42) and (43) using convex and concave envelopes (as detailed in Appendix C). This linearization produces equivalent linearly constrained linear reformulations of all bilinear terms.

Turning to the other exponential term in (33), recall that we need to construct an underestimating function of $e^{m'_i}$ over $[A'_i, B'_i]$ since its coefficient in (33) is nonpositive. We will take these underestimating linear segments to be defined by tangent lines of $e^{m'_i}$. In the spirit of the foregoing, we initially restrict our attention to the case of two segments defined by tangent lines at the interval end points; namely, lines tangent to the graph at $(A'_i, e^{A'_i})$ and $(B'_i, e^{B'_i})$. These are given by

$$
\begin{aligned}
\Theta_i &= e^{A'_i} + e^{A'_i}(m'_i - A'_i) \\
\Psi_i &= e^{B'_i} + e^{B'_i}(m'_i - B'_i).
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
-e^{m'_i} &\leq -\max\{\Theta_i, \Psi_i\} \\
&= \min\{-\Theta_i, -\Psi_i\} \\
&= g_i.
\end{aligned}
$$

If we replace $e^{m'_i}$ in (33) by g_i, we must add the constraint $g_i = \min\{-\Theta_i, -\Psi_i\}$. Since our objective is to maximize Ω_0 and the coefficient of g_i is nonnegative (*i.e.*, $(CR_i/G_i)\, z_i \geq 0$), then, by separability, the constraint $g_i = \min\{-\Theta_i, -\Psi_i\}$ is satisfied by maximizing $(CR_i/G_i)\, z_i g_i$ subject to the constraints

$$
\begin{aligned}
g_i &\leq -\Theta_i \\
g_i &\leq -\Psi_i.
\end{aligned}
$$

To complete the overestimating linearization of (33), the bilinear terms $z_i g_i$ are linearized via an equivalent linearly constrained reformulation using the convex and concave envelope technique of Appendix C, as the bounds for g_i are easily computed from (39) and (40). If more than two underestimating linear segments of $e^{m'_i}$ are desired, we would need to introduce not only additional linear constraints for each subinterval but also new binary variables to ensure that the proper linear function holds in its subinterval domain.

Thus, we have derived a linear MIP whose feasible region, when projected onto the decision space of the shelf space model, is identical to that of the nonconvex shelf space allocation model, and whose optimal objective value is an upper bound on the optimal objective value of the nonconvex model.

Appendix C: Convex and Concave Envelope of a Bilinear Function ([43])

This appendix is taken directly from [43] with permission, and is included to support the development in Appendix B. For a bilinear function xy, where $(x, y) \in \Re^2$, Al-Khayyal and Falk ([3]) prove that the *convex envelope*[14] over the rectangular domain $[x^L, x^U] \times [y^L, y^U]$, where L (U) denotes to a known lower (upper) bound on the variable, is given by:

$$max\{x^L y + y^L x - x^L y^L, x^U y + y^U x - x^U y^U\}$$

The term xy can be replaced everywhere in a model by introducing a new variable g wherever xy appears and imposing the two linear constraints:

$$g \geq x^L y + y^L x - x^L y^L \tag{46}$$

$$g \geq x^U y + y^U x - x^U y^U \tag{47}$$

Analogously, the em concave envelope[15] of xy, where $(x, y) \in \Re^2$, over the rectangular domain $[x^L, x^U] \times [y^L, y^U]$ is given by (see [2]):

$$min\{x^U y + y^L x - x^U y^L, x^L y + y^U x - x^L y^U\}.$$

When replacing xy by g everywhere in the model, g will better approximate xy if the following additional constraints implied by the concave envelope are added:

$$g \leq x^U y + y^L x - x^U y^L \tag{48}$$

$$g \leq x^L y + y^U x - x^L y^U \tag{49}$$

It is easy to show that if either x or y is at one of its bounds, then constraints (46) through (49) guarantee that $g = xy$.

Appendix D: Logistic Regression ([55])

In a variety of regression applications, the response variable of interest has only two possible qualitative outcomes, and therefore can be represented by a binary indicator variable takeing on values of 0 and 1. In such case, the response of the regression model is binary.

[14]The convex envelope of a function over a convex domain is the highest convex underestimating function over the domain.

[15]The concave envelope of a function over a convex domain is the lowest concave overestimating function over the domain.

Consider the simple regression model:

$$Y_i = \theta(X_i) + \varepsilon_i \qquad Y_i = 0, 1 \tag{50}$$

Where the outcome Y_i is binary on the value of either 0 or 1. The expected response $E\{Y_i\}$ has a special meaning in this case. Since $E\{Y_i\} = 0$ we have:

$$E\{Y_i\} = \theta(X_i) \tag{51}$$

Consider Y_i to be a Bernoulli random variable for which we can state the probability distribution as follows:

1) $Probability\ (Y_i = 1) = \pi_i$;

2) $Probability\ (Y_i = 0) = 1 - \pi_i$.

Thus, π_i is the probability that $Y_i = 1$, and 1-π_i is the probability that $Y_i = 0$. By the definition of expected value of a random variable, we obtain:

$$E\{Y_i\} = 1 * \pi_i + 0 * (1 - \pi_i) = \pi_i. \tag{52}$$

We thus find:

$$E\{Y_i\} = \theta(X_i) = \pi_i. \tag{53}$$

The mean response $E\{Y_i\}$ as given by the response function is therefore simply the probability that $Y_i = 1$ when the level of the predictor variable is X_i. This interpretation of the mean response applies whether the response function is a simple linear one or a complex multiple regression one. The mean response, when the outcome variable is a 0, 1 indicator variable, always represents the probability that $Y_i = 1$ for the given levels of the predictor variable.

CHAPTER IV

SUPPLY-CHAIN ORIENTED ROBUST PARAMETER DESIGN

4.1 Introduction

Robust parameter design is a methodology of choosing controllable factors to make a manufacturing system less sensitive to noise variations, which is normally defined for the single stage operation. When a system receives operations at several stages, the noises inherent from the upstream operations could have a great impact on the quality of products in the downstream operations ([13], [8]). For example, as shown in Section 4.4, when a part is machined with a rough milling process followed by a finish milling process, the variance of the surface roughness on the part generated in the rough milling process will affect the finish milling process ([23]).

Different efficiency or costs are usually incurred if various values of controllable factors in robust parameter design are selected. When operations are owned by different parties, the objectives of each operation in the supply chain are not aligned, which can lead to a high cost over the entire supply chain and large variances in the end output. Therefore, it is challenging to coordinate supply-chain partners with different objectives for manufacturing products meeting robust parameter design goals. Due to the growing trend of manufacturing outsourcing all over the world, such as the use of third-party manufacturers in the computer and automobile manufacturing industries, the study of robust design problems across several partners in the supply chain has become more critical than ever.

The literature in the supply-chain oriented robust design field is limited. Shang *et al.* ([17]) introduce the Taguchi's method for supply chain design. They select values for control factors in a supply chain design which includes manufacturers' capacity, delayed differentiation, information sharing between retailers and manufacturers, retailer's (S, s) policy, replenishment lead time and supplier reliability to dampen the noise factors, such as the variance in demand and inventory holding cost. Different from their work, the robust design in this chapter is applied in each individual stage where machining control factors are selected to dampen noises factors which are the sources of variances in the manufacturing system. Cooperation and information sharing are introduced between the robust parameter design across various stages in the supply chain.

When analyzing the supply chain across different parties/channels, the channel conflict, the potential objection of one channel to the actions taken by another is the critical problems to be addressed. Game theory is the most commonly used tool to analyze the interactions among various parties in the supply chain analysis. For example, games are formulated between manufacturers and retailers to model conflict and coordination in various distribution systems (e.g. [18], [14], [9]). The major purposes of these studies are analyzing the channel structure, such as, which channel to use for a manufacturer to distribute products and how to participate in a given distribution system for a retailer. Different from these research, we introduce a game to model the interaction among various parties in a multi-stage manufacturing process. The objective is to analyze the conflicts between these parties from the quality engineering perspective, which is the first time that game theoretical method is introduced in this area. Since the leader and follower relation where one player dominates the others is common in practice ([2]), Stackelberg game ([19]) is applied to model the interaction among various parties in the multi-stage process in this chapter.

In multi-disciplinary product design, game theory has been introduced to solve the conflicts between different disciplines ([10], [6], [7], [16], [20]). Most publications have considered optimal product design issues in a deterministic environment. Thus, the variations induced by noise factors were not considered in these works. Chen [2] integrates the "robust optimization" concept relevant to robust parameter design into game theory for providing flexibility in solving conflicts between product designers. Specifically, instead of sending an optimum design value to the downstream designer, an upstream designer delivers a range of design values which are near to the optimum but have less sharp changes to the product performance, i.e., select the "design window" for producing more consistent performance. However, in Chen's research the above robust optimization concept is only used by the upstream designer and the system is deterministic. Thus, there exists no variance accumulation study in the robust parameter design for multi-stage processes (see [8] for an example of modeling accumulated errors for the assembly process). In this research, variance accumulation across the various stages of the manufacturing process is studied.

This research focuses on variations presented in manufacturing processes. "Negotiations" between the supply-chain partners is modeled as a Stackelberg game, where there are leaders and followers in deciding a set of equilibrium conditions for establishing excellent robust processes. To the best of our knowledge, this is the first time that supply chain analysis has been introduced in multi-stage robust parameter design. In Section 4.3.1, we introduce the experimentation and modeling for a two-stage process. In Section 4.3.2, the Stackelberg game model is presented. Section 4.4 uses an example to illustrate the proposed methodology. The concluding remarks are given in Section 4.5.

4.2 Contributions in this Chapter

In this chapter, we make the following contributions:

1. Propose methods of solving robust parameter design problem in a single stage.
2. Analyze multi-stage robust parameter design problems, where variance interactions between various stages are studied. This is the first time that the robust parameter design is analyzed across multiple stages.
3. Model the interaction between the robust parameter design in various stages as a Stackelberg game.
4. Provide real-life examples to demonstrate the potential of the proposed method.

4.3 Models

We start with a two-stage process (Figure 12). In stage i, inputs are the control factors X_i and noise factors N_i. Response Y_i is the output from stage i. The output Y_1 is also input of stage 2 and thus, serves as a "connection" between the two stages. The target value of response Y_i is denoted by T_i for stage i. Here, T_i has a different definition for different problems: in the nominal-the-best problem[1], it defines the mean value of the response function; in the smaller-the-better and larger-the-better problems, it is zero and infinite, respectively.

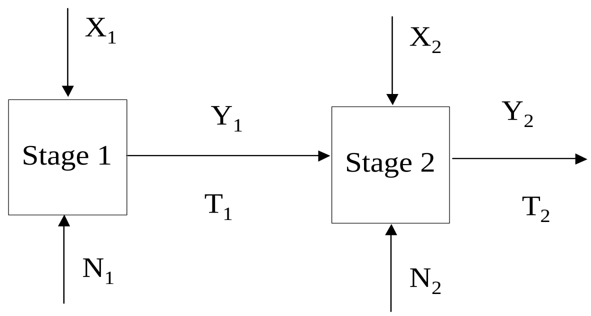

Figure 12: The two-stage process

4.3.1 Experimentation and Modeling

Before introducing the model of the two-stage game between the partners, we first model the relationships between the input and output in each stage based on data collected from experimental design methods, e.g., cross-array experiments for both noise and control factors are usually used to collect data in robust parameter design. Since modeling, coordination and optimization tasks for robust parameter design have to be explored before looking into optimal experimental design, we will not discuss the experimental design issues in this chapter and leave it as a future work. Assuming that data are available, two commonly used data modeling approaches are: Location and Dispersion Modeling and Response Modeling ([22]).

In the Location and Dispersion Modeling approach, both mean (location) and variance (dispersion) of the response are modeled as functions of control factors independently. In the Response Modeling approach, the response is modeled as a function of both the control and noise factors. Based on the fitted response model, the variance of the response is computed with respect to the variation among the noise factors and control factors with

[1]In the nominal-the-best problem, the measured response always has specific target value. In the smaller-the-better problem, the measured responses never have a negative value, and their targeted response is ideally zero. In the larger-the-better problem, the measured response, while never having negative values, are better as their value gets larger.

selected settings representing process variations, which is called transmitted variance model.

Based on the models from experiments, the robust parameter design for a single stage i can be established in the following two methods. Here, we denote the expectation and variance of Y_i by $E(Y_i)$ and $Var(Y_i)$, respectively.

Method 1:

We formulate the robust design problem as an optimization problem to select the settings of the control factors X_i, which are the decision variables in the problem. Usually, the objective of a robust design is to minimize the quality loss. However, costs associated with the different levels of a control variable can be very distinct, *i.e.*, smaller cutting speed reduces the thermal error in a machining process, but may result in a longer machining time. Therefore, the cost associated with parameter design should also be considered. Thus, in our model, the objective function is to minimize the expected quality loss, $E(L(Y_i, T_i))$, and the cost, $C(X_i)$, associated with the parameter design. The problem is formulated as:

$$\begin{aligned} Find \quad & X_i, \\ min \quad & f(E(L(Y_i, T_i)), C(X_i)), && (54) \\ s.t. \quad & T_{imin} \leq E(Y_i) \leq T_{imax}, && (55) \\ & X_{iL} \leq X_i \leq X_{iU}. && (56) \end{aligned}$$

This model can be solved using optimization methods. Constraint (55) and (56) define the feasible region of the response and control variables, respectively, where T_{imin} and T_{imax} denote the lower bound and upper bound of the response value. X_{iL} and X_{iU} are the lower bound and upper bound of the feasible value of X_i. In practice, the control variables are assumed to be completely controlled, therefore, we assume there is no variance associated with control variable X_i.

Note that in terms of problem types, constraint (55) has various expressions. Let ΔT_i denote the tolerance of the response value, we have $T_{imin} = T_i - \Delta T_i$ and $T_{imax} = T_i + \Delta T_i$ in the nominal-the-best problem. In the smaller-the-better problem, $T_i = 0$. Therefore, the lower bound T_{imin} is zero. Constraint (55) becomes:

$$0 \leq E(Y_i) \leq T_{imax}, \tag{57}$$

where $T_{imax} = \Delta T_i$. Similarly, in the larger-the-better problem, $T_i = \infty$, Constraint (55) becomes:

$$T_{imin} \leq E(Y_i), \tag{58}$$

where $T_{imin} = \Delta T_i$.

Method 2:

When some control factors and noise interact in their joint effect on the response function, the variation in the response can be reduced by changing the settings of these control factors. The remaining control factors with no interaction with noise can be used to adjust the mean value. Thus, two-step procedure ([22]) which adjusts the mean and the variance in two sequential steps is popular in robust parameter design.

Following the two-step procedure, we develop the following two-step procedure for the nominal-the-best case:

i) Select the levels of the control factors X_i to minimize $Var(Y_i)$. The control factors which have interaction with noise factors in the formulation of $Var(Y_i)$ are selected to dampen the variance of noise factors.

ii) Select the level of a control factor that does not have interaction with noise factors to minimize the difference between the mean value of the response function, i.e., $E(Y_i)$ and the target value T_i. If adjusting one control factor is not sufficient to bring the objective to the target value, it may require two or more control factors for the adjustment in this step.

When implementing the two-step procedure, we need to take the costs into consideration as well, such as the cost associated with the robust design.

For the case of smaller-the-better (larger-the-better), decreasing (increasing) the mean value is considered to be a more difficult task, it should be done in the first step. Therefore, it is recommended that the order of the two-step procedure is reversed. The two-step procedure is then modified as follows: 1) Select the levels of control variable to minimize (maximize) the response value, i.e., $E(Y_i)$ in the first step. 2) Select the levels of control factors to minimize the variance, $Var(Y_i)$. Recall that ΔT_i serves as the upper bound (lower bound) in these two cases. Thus, $E(Y_i)$ should be moved into the feasible region defined by the bound ΔT_i in the first step.

Finally, we need to point out that solutions found with Method 1 are no worse than Method 2 because Method 1 optimizes both mean and variance simultaneously.

4.3.2 Leader/follower Game Model

In Section 4.3.1, two methods are proposed to solve the robust design problem in one stage. Game theory can be applied to both methods. To keep the presentation brief, we focus on Method 1 here. Note that when multiple stages are correlated as shown in Figure 12, the process design in one stage can have significant impact on the performance of the other stage.

There are various ways to model the cooperations among product/process designers. Although full cooperation between all partners is an ideal way of addressing the needs of different partners, it is rare in practice due to the issues of organizational challenges ([2]). Sequential approach is another popular method in multi-disciplinary product-design processes when one designer dominates the other or in a design process that involves a sequential execution of interrelated processes. However, with this approach the initial decisions are made without any formal consideration of the later decisions of the follower.

In this section, we model the two-stage process as a two-player Stackelberg game, where one player is the leader (usually with higher negotiation power) makes decisions first. The other player, the follower will then take his/her "best response" ([19]) to the given action of the leader. In this game, it is assumed that players' payoff functions are common knowledge (can be expected from common sense). The timing of the game is as follows: 1) player 1 (leader) chooses a decision q_1; 2) player 2 (follower) observes q_1 and then chooses a decision q_2; 3) the payoff of player i is then given by the profit function $\pi_i(q_i, q_j)$.

This game sometimes has multiple Nash equilibriums and backwards-induction [19] is the most commonly used method solving for one of the Nash equilibriums-Stackelberg equilibrium. To solve for the backwards-induction outcome, let Q_1 and Q_2 denote the feasible action set for q_1 and q_2, respectively. When player 2 knows the player 1's move at the second stage of the game, he or she will face the following problem, given the action q_1 previously chosen by player 1:

$$max_{q_2 \in Q_2} \quad \pi_2(q_1, q_2). \tag{59}$$

Assume that for each q_1 in Q_1, player 2's optimization problem has a unique solution, denoted by $R_2(q_1)$. This is player 2's reaction (or best response) to player 1's action. Since player 1 can solve player 2's problem as well as player 2 can (due to the assumption of "players' payoff functions are common knowledge"), player 1 should anticipate player 2's reaction to each action q_1 that player 1 might take, so player 1's problem at the first stage becomes:

$$max_{q_1 \in Q_1} \quad \pi_1(q_1, R_2(q_1)). \tag{60}$$

Assume that this optimization problem for player 1 also has a unique solution, denoted by q_1^*. We will call $(q_1^*, R_2(q_1^*))$ the backward-induction outcome of this game, which is the Stackelberg equilibrium where no player wants to deviate from this solution. It is straightforward to extend what follows to allow for more than one following players.

Stage 1 and 2 in this chapter are referred as player 1 and 2, respectively in the following discussion. In terms of the leadership, two cases are identified.

Case 1- Player 1 is the leader:

This game fits for the situations where the upstream stage takes the overall process quality into consideration when making decisions. Therefore, the objective function of player 1 is a function of both Y_1 and Y_2, which is denoted by $\varphi_1(Y_1, Y_2)$. Without loss of generality, we assume that player 1's objective is to minimize the total quality loss resulting from the end output, Y_2, and also the total costs associated with the parameter design in both of these stages. The objective of player 2 is to minimize the quality loss of the final product plus the cost associated with parameter design in stage 2. We denote the objective function of player 2 by $\varphi_2(Y_2)$. The timing of the game is as follows: player 1 selects X_1 and the corresponding Y_1 is sent to player 2 as an input information. Player 2 then makes the best response Y_2 respect to Y_1 by selecting X_2.

The backward induction solutions for the game are solved in the following steps:

1. Player 1 solves player 2's problem to obtain the response function $R(Y_1)$: for a given Y_1, player 2's optimization problem is to optimize the objective $\varphi_2(Y_2)$ by selecting X_2 with the methods presented in section 4.3.1. Since it is assumed that player 2's optimization problem, the payoff function, is common knowledge. Thus, by selecting a set of values for Y_1, denoted by $\mathbf{Y_1}$ and solving player 2's optimization problem with respect to each component of Y_1 in $\mathbf{Y_1}$, player 1 gets a set of solutions $\mathbf{Y_2}$ corresponding to $\mathbf{Y_1}$. Player 2's response surface model between Y_1 and Y_2 is then obtained with regression analysis as follows:

$$\tilde{Y_2} = \tilde{f}(Y_1). \tag{61}$$

2. Given player 2's best response function $\tilde{Y_2} = \tilde{f}(Y_1)$, player 1 selects X_1^* to optimize the objective: $\varphi_1(Y_1, Y_2)$=$\varphi_1(Y_1, \tilde{Y_2}) = \varphi_1(Y_1, \tilde{f}(Y_1))$. X_1^* and Y_1^* are obtained.

3. Substituting Y_1^* into the player 2's optimization problem where Y_2 is a function of Y_1 (see Figure 4.4.1), X_2^* and Y_2^* are obtained.

Case 2- Player 2 is the leader:

This case is applicable to the situations where player 2 has more power, i.e, player 2 sets the target value for player 1. In the nominal-the-best problem, this target value is T_1. However, in the smaller-the-better and the larger-the-better problem, the target values are T_{1max} and T_{1min}, respectively. For the ease of description, we denote the target values for all these three problems by T_1.

The objective of player i is to minimize his own quality loss and the cost associated with the parameter design in stage i. We denote the objective function for player i by $\varphi_i(Y_i)$. The timing of the game is as follows: player 2 sets a target T_1 for Y_1, player 1 then make a best response to the target value. Therefore, Y_1 is a function of the target value in this case. The decision variable for player 1 is X_1. And both X_2 and T_1 are decision variables for player 2 in this case.

The backward induction solutions for the game are solved in the following steps:

1. Player 2 solves player 1's problem to obtain the response function $R(T_1)$: for a given T_1, player 1 selects X_1 to optimize his objective. Since it is assumed that player 1's

optimization problem, the payoff function, is common knowledge. Player 2 can select a set of values for T_1, denote as $\mathbf{T_1}$ and solve player 1's optimization problem for each T_1 in $\mathbf{T_1}$. A set of solutions of $\mathbf{Y_1}$ are obtained with respect to $\mathbf{T_1}$. Player 1's best response function is then obtained with the regression analysis:

$$\tilde{Y}_1 = \tilde{r}(T_1). \tag{62}$$

2. Given player 1's best response function $\tilde{Y}_1 = \tilde{r}(T_1)$, player 2 optimizes the objective $\varphi_2(Y_2)$. Note that Y_2 is a function of Y_1, we denote the relation between Y_1 and Y_2 by $Y_2 = g(X_2, N_2, Y_1)$. Thus, the objective function of player 2 can be represented as: $\varphi_2(Y_2) = \varphi_2(g(X_2, N_2, Y_1)) = \varphi_2(g(X_2, N_2, \tilde{Y}_1)) = \varphi_2(g(X_2, N_2, \tilde{r}(T_1)))$. X_2^* and T_1^* are obtained.

3. Substituting T_1^* into the player 1's optimization problem, we find X_1^*.

Two-stage games can involve multiple players. The leadership of the game depends on the power of the players. For example, when parts machined on one stage, stage A, are sent to different downstream processes, i.e., B_1 through B_n for the further machining, stage A can function as a leader who selects the design parameter to minimize the total losses in final outputs by taking the best responses from followers. On the other hand, when B_1 through B_n are the customers who provide specifications for products, the manufacturer A becomes a follower who selects design parameters to be robust to the variety in the specifications.

In addition, this model can be extended to more than two stages, and multiple responses can also be introduced with small modifications on the model proposed above.

4.4 An Example

Milling is a fundamental machining operation for generating machined surfaces by removing a predetermined amount of material progressively from the workpiece ([12]). It is widely used in a variety of manufacturing industries including the aerospace and automotive sectors, where quality is an important factor in the production of slots, pockets, precision molds and dies. A good-quality milled surface significantly improves fatigue strength, corrosion resistance, or creep life. Therefore, robust designs are usually implemented to improve the surface quality ([5]). A suitable level of control variables, such as spindle speed, feed rate, depth of cut and geometries of the cutting tool[2] (see Figure 13 and 14) are selected to dampen the noise factors caused by the tool wear, occurrence of chatter or vibrations of the machine tool.

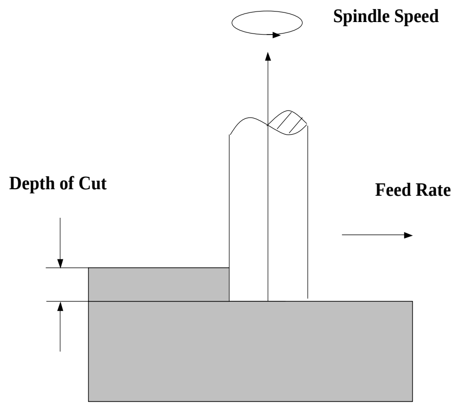

Figure 13: Cutting geometry

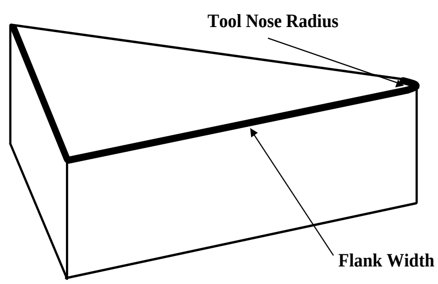

Figure 14: The geometries of the cutting tool tips

In the United States, non-precision components with rough cutting are seldom manufactured, but are often outsourced to low-cost, offshore suppliers ([15]). Finish cutting is then performed on these parts to meet the high precision requirement. In this section, we illustrate the proposed approach by using an example of the surface roughness control for a

[2]Spindle speed is the peripheral linear speed resulting from the rotation of the cutter. Feed rate is the speed or rate at which the workpiece moves past the cutter. Depth of cut is the depth of the material to be removed in one operation.

two-stage end-milling process. A manufacturer purchases aluminum parts from a supplier who finishes the rough surface milling process. A finish surface milling process is then implemented on the this part by the manufacturer. The surface roughness requirement for the final part is 1.65μm.

The manufacturer is the leader of the game, who gives the target value, the maximum mean of roughness T_{1max}, to the follower, the supplier. The supplier responds with $\tilde{Y_1}(T_{1max})$ and the price of the part $\tilde{P_1}(T_{1max})$. The manufacturer then takes the best response functions of the supplier into his own design problem to optimize his objective.

4.4.1 Performance Measure

The average surface roughness R_a, which is the most widely used surface finish parameter in industry, was selected for this study. This parameter, also known as the arithmetic mean roughness value, is the arithmetic average of the absolute value of the heights of roughness irregularities from the mean value measured within a sampling length (see Figure 15), which is:

$$R_a = \frac{1}{D}\int_0^D |y(x)|dx, \tag{63}$$

where D denotes the sampling length and $y(x)$ denotes the ordinate of the profile curve at point x. In general, a smaller R_a is desired and the response never have a negative value. Therefore, controlling the roughness can be regarded as a smaller-the-better problem.

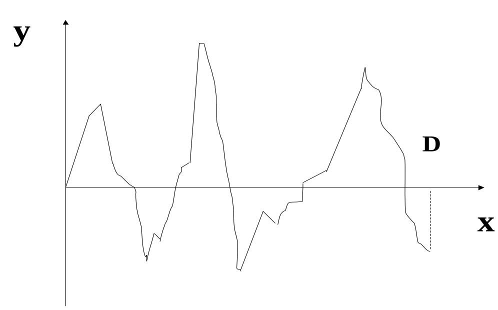

Figure 15: Profile of surface roughness

4.4.2 Method Implementation

In literatures of mechanical engineering, location effects are usually modeled as a function of control factors through robust parameter design. However, dispersion effects are rarely formulated. Furthermore, to the best of our knowledge, multistage robust parameter design has not been studied, therefore, it is difficulty to find an example which includes two-consecutive-stage robust parameter design from both literature and practice. To overcome these difficulties, an example is designed by the modification of two single-stage robust parameter design models([4][11]).

1) Supplier's Response Function

The major control parameters for the milling machining are listed in Table 20.

Table 20: Control factors

Control Factor	Level x_{1i}^-	Level x_{1i}^+
x_{11}: Spindle Speed (m/min)	31.42	235.6
x_{12}: Feed Rate (mm/edge)	0.03	0.2
x_{13}: Depth of Cut (mm)	0.4	6
x_{14}: Flank Width (mm)	0.01	0.3
x_{15}: Tool Nose Radius (mm)	0.1	1.2

Noise factors, such as occurrence of chatter or vibrations of the machine tool and thermo-errors are hard to control in the machining process, we build models for measures of location and dispersion separately in terms of the control factor main effects and interactions ([22]). The location effects of the surface roughness is proposed with the experiment implemented by Fuh ([4]). With the data shown in Table 21, variance function is estimated with Harvey's Method ([1]), where residuals from the location effects model are used to estimate dispersion effects.

Table 21: Experimental design matrix and surface roughness

	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	y_1	$\hat{y}_1$	$r_1 = y_1 - \hat{y}_1$	$r_1^2 = (y_1 - \hat{y}_1)^2$
1	62.8	0.05	1	0.05	0.2	15.12	19.42	-4.30	18.47
2	62.8	0.15	4	0.05	0.2	35.98	38.58	-2.60	6.77
3	157.8	0.05	4	0.05	0.2	8.1	17.23	-9.13	83.44
4	157.8	0.15	1	0.05	0.2	43.67	45.51	-1.84	3.37
5	62.8	0.05	4	0.05	0.8	4.75	1.70	3.05	9.30
6	62.8	0.15	1	0.05	0.8	9.45	3.92	5.53	30.56
7	157.08	0.05	1	0.05	0.8	3.2	-3.17	6.37	40.55
8	157.08	0.15	4	0.05	0.8	12.15	4.18	7.97	63.58
9	62.8	0.05	4	0.2	0.2	8.13	11.21	-3.08	9.50
10	62.8	0.15	1	0.2	0.2	29.42	30.02	-0.60	0.36
11	157.08	0.05	1	0.2	0.2	4.17	10.11	-5.94	35.24
12	157.08	0.15	4	0.2	0.2	44.5	48.83	-4.33	18.76
13	62.8	0.05	1	0.2	0.8	3.4	2.15	1.25	1.57
14	62.8	0.15	4	0.2	0.8	8.23	7.05	1.18	1.40
15	157.08	0.05	4	0.2	0.8	5.83	4.27	1.56	2.44
16	157.08	0.15	1	0.2	0.8	8.45	4.36	4.09	16.77
17	125.66	0.1	2.5	0.01	0.4	8.73	15.23	-6.50	42.27
18	125.66	0.1	2.5	0.3	0.4	12.06	13.04	-0.98	0.96
19	125.66	0.1	2.5	0.1	0.1	72.33	39.99	32.34	1046.15
20	125.66	0.1	2.5	0.1	1.2	5.76	19.52	-13.76	189.23
21	31.24	0.1	2.5	0.1	0.4	9.26	9.47	-0.21	0.04
22	235.6	0.1	2.5	0.1	0.4	8.16	13.17	-5.01	25.11
23	125.66	0.03	2.5	0.1	0.4	2.95	-1.34	4.29	18.39
24	125.66	0.2	2.5	0.1	0.4	24.2	31.30	-7.10	50.40
25	125.66	0.1	0.4	0.1	0.4	5.06	12.63	-7.57	57.36
26	125.66	0.1	6	0.1	0.4	16.56	17.91	-1.35	1.83
27	125.66	0.1	2.5	0.1	0.4	9.73	13.25	-3.52	12.42

The location and dispersion effects are modeled as follows:

$$
\begin{aligned}
\hat{y}_1 &= 33.576 - 0.02x_{11} + 255.004x_{12} - 4.164x_{13} - 152.068x_{14} - 88.675x_{15} - 0.0002x_{11}^2 \\
&\quad -164.787x_{12}^2 + 0.185x_{13}^2 + 72.10x_{14}^2 + 88.121x_{15}^2 + 0.777x_{11}x_{12} + 0.013x_{11}x_{13} \\
&\quad +0.301x_{11}x_{14} - 0.122x_{11}x_{15} + 7.458x_{12}x_{13} + 92.518x_{12}x_{14} - 376.620x_{12}x_{15} \\
&\quad +15.935x_{13}x_{14} - 0.125x_{13}x_{15} + 88.139x_{14}x_{15}, \\
ln(r_1^2) &= 5.13 + 0.0579x_{11} - 82.7x_{12} + 0.35x_{13} - 11.8x_{14} - 10.3x_{15} - 0.000152x_{11}^2 \\
&\quad +211x_{12}^2 - 0.052x_{13}^2 + 1.4x_{14}^2 + 10.4x_{15}^2 + 0.061x_{11}x_{12} - 0.00118x_{11}x_{13} \\
&\quad +0.046x_{11}x_{14} - 0.0208x_{11}x_{15} + 3.12x_{12}x_{13} - 2x_{12}x_{14} + 45.6x_{12}x_{15} \\
&\quad +0.35x_{13}x_{14} - 0.849x_{13}x_{15} - 11x_{14}x_{15}.
\end{aligned}
$$

Where $\hat{y}_1$ denote the sample mean (μm) and r_1^2 is an estimate of the variance of roughness.

For a given restriction of roughness from the manufacturer, T_{1max}, the supplier's problem is modeled with Method 1 presented in section 4.3.1 as follows, we call it "Problem $M1$":

$$
\begin{aligned}
Find \quad & x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, \\
min \quad & c_1(E(Y_1)^2 + Var(Y_1)) + \frac{c_2}{x_{12}x_{13}}, \\
s.t. \quad & 0 \le E(Y_1) \le T_{1max}, \\
& x_{1i} \in [x_{1i}^-, x_{1i}^+], \quad i = 1, \ldots, 5.
\end{aligned}
$$

Where $c_1(E(Y_1)^2 + Var(Y_1))$, denoted as $E(L(Y_1))$ represents the cost associated with the quality loss[3], e.g., rework and customer dissatisfaction. Here $c_1 = 0.005$ is estimated. In addition, since the feed rate and cut depth directly affect the process efficiency, larger feed rate and cut depth lead to shorter machining time ([21]) which results in lower labor and equipment costs in general, thus, $EC_1 = c_2/(x_{12}x_{13})$ denotes the efficiency cost, where $c_2 = 50$ is estimated by the machining cost. When solving the problem, the sample mean $\hat{y}_1$ and sample variance r_1^2 are used to represent $E(Y_1)$ and $Var(Y_1)$, respectively.

Note that the quality loss and the efficient cost are both highly dependent on $E(Y_1)$,

[3]For the smaller-the-better problem, the expected quality loss $E(L(Y)) = cE(Y^2) = cVar(Y) + cE(Y)^2$ ([22]).

which is then affected by T_{1max}. When $E(Y_1)$ decreases, the expected quality loss decreases. However, a higher machining cost is incurred. Therefore, a tradeoff point of $E(Y_1)$ exists which balances the quality loss cost and machining cost in condition of satisfying the restricted target value T_{1max} from the manufacturer.

Remember that the supplier's problem is assumed to be common knowledge, thus, the manufacturer can solve Problem M1 for any target value T_{1max}. To find out the supplier's response function, the manufacturer selects a set, $\mathbf{T_{imax}}$, and solves Problem M1 for each $T_{imax} \in \mathbf{T_{imax}}$. Solutions ($\mathbf{E(Y_1)}$, $\mathbf{Var(Y_1)}$, $\mathbf{EC_1}$) with respect to $\mathbf{T_{imax}}$ are obtained. The resulting relationships between variables $E(Y_1)$, $Var(Y_1)$, EC_1 and T_{1max} are then fitted via a regression analysis as follows:

$$\begin{aligned} \tilde{E(Y_1)} &= 1.87 + 0.87T_{1max}, \\ \tilde{Var(Y_1)} &= 1.5 + 0.696T_{1max}, \\ \tilde{EC_1} &= 57 - 0.458T_{1max}. \end{aligned}$$

2) Decision of Manufacturer

The parameters of a milling machine used for the finish cut are listed in Table 22.

Table 22: Control factors

Control Factor	Level x_{2i}^-	x_{2i}^+
x_{21}: Spindle Speed (ipm)	750	1500
x_{22}: Feed Rate (rpm)	6	24
x_{23}: Depth of Cut (in)	0.01	0.05

The location and dispersion effects of surface roughness are built based on the experiments proposed in [11]. Similar to the supplier's problem, the dispersion effects are estimated with the data (Table 23) with Harvey's method. Since the surface roughness of the raw material affects the surface quality of the end products ([23]), which has not been captured in the existing model, we assume that the mean value of Y_2 linearly increases on the increase of Y_1. As a result, the transmitted variance of Y_1 is introduced into the dispersion model of Y_2.

Table 23: Experimental Design Matrix and Surface Roughness

	x_{21}	x_{22}	x_{23}	y_2	$r_2 = y_2 - \hat{y}_2$	$r_2^2 = (y_2 - \hat{y}_2)^2$		x_{21}	x_{22}	x_{23}	y_2	$r_2 = y_2 - \hat{y}_2$	$r_2^2 = (y_2 - \hat{y}_2)^2$
1	1000	18	0.01	3.51	0.02	0.001	31	1000	6	0.05	1.57	-0.35	0.122
2	1500	9	0.03	1.85	-0.06	0.003	32	1500	21	0.05	2.87	0.45	0.201
3	1250	6	0.01	1.27	-0.27	0.075	33	1250	12	0.01	2.57	0.28	0.076
4	750	24	0.03	4.32	-0.09	0.008	34	1000	12	0.01	3.30	0.73	0.533
5	1250	21	0.05	2.67	-0.11	0.012	35	1250	18	0.05	2.41	-0.20	0.041
6	750	21	0.05	3.81	0.32	0.104	36	750	9	0.03	2.51	0.26	0.066
7	1250	21	0.01	3.18	-0.23	0.054	37	1500	24	0.03	2.62	-0.22	0.049
8	1000	21	0.03	3.68	0.15	0.022	38	750	6	0.03	1.60	-0.23	0.052
9	1250	9	0.01	2.01	0.09	0.008	39	750	21	0.03	4.14	0.16	0.027
10	1500	15	0.01	2.69	0.39	0.155	40	1500	24	0.01	3.02	-0.15	0.023
11	1000	6	0.03	1.98	0.19	0.035	41	1500	24	0.05	2.77	0.27	0.072
12	750	18	0.05	3.07	-0.09	0.008	42	1250	9	0.03	2.06	0.03	0.001
13	1000	6	0.01	1.47	-0.19	0.036	43	750	6	0.01	1.65	-0.13	0.017
14	1000	12	0.05	2.34	-0.07	0.005	44	1000	21	0.01	3.78	-0.15	0.023
15	1000	9	0.05	2.59	0.43	0.181	45	1250	18	0.01	2.92	-0.11	0.013
16	1250	24	0.01	3.94	0.16	0.024	46	750	12	0.03	2.59	-0.10	0.009
17	750	9	0.05	2.41	0.22	0.047	47	1250	6	0.05	1.80	-0.17	0.029
18	1250	18	0.03	2.34	-0.49	0.239	48	1250	15	0.03	2.44	-0.12	0.015
19	1500	12	0.01	2.24	0.23	0.052	49	1250	9	0.05	2.34	0.20	0.041
20	1000	15	0.05	2.67	0.02	0.000	50	1250	6	0.03	1.60	-0.16	0.025
21	1250	24	0.03	2.77	-0.59	0.349	51	1500	18	0.01	3.02	0.43	0.187
22	750	18	0.01	4.70	0.77	0.596	52	750	15	0.05	2.64	-0.20	0.040
23	1500	21	0.01	3.00	0.12	0.013	53	750	12	0.05	2.39	-0.13	0.017
24	750	15	0.03	3.10	-0.02	0.000	54	1500	6	0.01	0.94	-0.48	0.234
25	1000	24	0.03	3.89	0.00	0.000	55	1250	21	0.03	2.54	-0.55	0.305
26	1000	15	0.03	2.74	-0.09	0.009	56	1000	24	0.01	4.14	-0.25	0.062
27	750	6	0.05	1.83	-0.04	0.002	57	1000	15	0.01	2.57	-0.46	0.213
28	1500	9	0.01	0.86	-0.85	0.725	58	1250	12	0.05	2.16	-0.14	0.019
29	750	9	0.01	2.77	0.45	0.202	59	1500	15	0.05	2.51	0.25	0.063
30	1000	12	0.03	2.13	-0.36	0.127	60	1500	18	0.05	2.64	0.30	0.090

The resulting models are represented as follows:

$$\begin{aligned}
\hat{y}_2 &= 0.5828 + 0.2778x_{22} - 0.000109x_{21}x_{22} + 0.01725x_{21}x_{23} \\
&\quad -1.772x_{22}x_{23} + 0.001E(Y_1), && (64)\\
ln(r_2^2) &= -1.648 - 191.95x_{23} + 2160x_{23}^2 + 0.041x_{21}x_{23} + 0.000001Var(Y_1). && (65)
\end{aligned}$$

The quality loss for the manufacturer, $E(L_2)$, is from the rework cost and the possible loss generated for end users. It is estimated that the quality loss resulting from the variance is at the rate of c_3 =5. For the same reasons as mentioned before, the feed rate and cut depth determine the machining efficiency which is represented by the cost $EC_2 = c_4/(x_{22}x_{23})$, where $c_4 = 3.5$ is estimated with the machining cost. In addition, the manufacturer makes a payment of $P(T_{1max})$ to the supplier for the procurement of parts, where the price $P(T_{1max})$ is proportional to the cost $\tilde{EC_1}$. Without loss of generality, we assume that $P(T_{1max}) = c_5\tilde{EC_1}$, where $c_5 = 1.1$. Thus, the optimization problem for the manufacturer, called "Problem $M2$" is formulated as:

$$\begin{aligned}
&Find && x_{21}, x_{22}, x_{23}, T_{1max}\\
&min && c_3(E(Y_2)^2 + Var(Y_2)) + \frac{c_4}{x_{22}x_{23}} + c_5\tilde{EC_1}\\
&s.t. && 0 \le E(Y_2) \le T_{2max}\\
& && x_{2i} \in [x_{2i}^-, x_{2i}^+]; \quad i = 1, 2, 3.
\end{aligned}$$

Sample mean $\hat{y}_2$ (μm) and sample variance r_2^2 are used to represent $E(Y_2)$ and $Var(Y_2)$ when solving Problem $M2$. Substituting $E(Y_1)$ and $Var(Y_1)$ in (64) and (65) with the best response functions of $E(\tilde{Y_1})$ and $Var\tilde{(}Y_1)$, the solutions for the manufacturer can be obtained. We solve the optimization problem with GAMS, which returns solutions for the manufacturer as: ($objective$, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) = (48.32, 235.6, 0.18, 6, 0.12, 0.77). Solutions for the supplier can be then found by substituting T_{1max} into the supplier's optimization problem, problem $M1$. The solutions for the supplier are obtained: ($objective$, x_{21}, x_{22}, x_{23}, T_{1max}) = (96.51, 1500, 6, 0.02, 21.66).

4.4.3 Comparisons with Other Scenarios

Scenario 1: full cooperation

When full cooperation is possible, values of the control factors for the two stage process can be determined in a single optimization problem. If the objective is to minimize the manufacturer's costs associated with the parameter design and the quality loss, the optimization model is formulated as follows:

$$
\begin{aligned}
Find \quad & x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{22}, x_{23} \\
min \quad & c_3(E(Y_2)^2 + Var(Y_2)) + \frac{c_4}{x_{22}x_{23}} + \frac{c_5 c_2}{x_{12}x_{13}}, \\
& E(Y_2) \leq T_2, \\
& E(Y_1) = 33.576 - 0.02x_{11} + 255.004x_{12} - 4.164x_{13} - 152.068x_{14} - 88.675x_{15} \\
& -0.0002x_{11}^2 - 164.787x_{12}^2 + 0.185x_{13}^2 + 72.10x_{14}^2 + 88.121x_{15}^2 + 0.777x_{11}x_{12} \\
& +0.013x_{11}x_{13} + 0.301x_{11}x_{14} - 0.122x_{11}x_{15} + 7.458x_{12}x_{13} + 92.518x_{12}x_{14} \\
& -376.620x_{12}x_{15} + 15.935x_{13}x_{14} - 0.125x_{13}x_{15} + 88.139x_{14}x_{15}, \\
& ln(Var(Y_1)) = 5.13 + 0.0579x_{11} - 82.7x_{12} + 0.35x_{13} - 11.8x_{14} - 10.3x_{15} \\
& -0.000152x_{11}^2 + 211x_{12}^2 - 0.052x_{13}^2 + 1.4x_{14}^2 + 10.4x_{15}^2 + 0.061x_{11}x_{12} \\
& -0.00118x_{11}x_{13} + 0.046x_{11}x_{14} - 0.0208x_{11}x_{15} + 3.12x_{12}x_{13} - 2x_{12}x_{14} \\
& +45.6x_{12}x_{15} + 0.35x_{13}x_{14} - 0.849x_{13}x_{15} - 11x_{14}x_{15}, \\
& E(Y_2) = 0.5828 + 0.2778x_{22} - 0.000109x_{21}x_{22} + 0.01725x_{21}x_{23} - 1.772x_{22}x_{23} \\
& +0.001E(Y_1), \\
& ln(Var(Y_2)) = -1.648 - 191.95x_{23} + 2160x_{23}^2 + 0.041x_{21}x_{23} + 0.000001Var(Y_1), \\
& x_{ij} \in [x_{ij}^-, x_{ij}^+], \quad \forall i, j.
\end{aligned}
$$

Solving this problem with GAMS, the solutions are obtained as follows: the supplier's solutions are ($objective$, x_{11}^*, x_{12}^*, x_{13}^*, x_{14}^*, x_{15}^*, $E(Y_1)^*$, $Var(Y_1)^*$) = (49.16, 235.6, 0.18, 6, 0.1, 0.77, 17.96,14.4399) and the manufacturer's solutions are ($objective$, x_{21}^*, x_{22}^*, x_{23}^*, Y_2^*, $Var(Y_2)^*$) = (96.51, 1500, 6, 0.02, 1.65, 0.0351).

Scenario 2: manufacturers set different T_{imax} to suppliers

In practice, with the sequential approach the initial decisions are made without any formal consideration of the later disciplines. Therefore, in our example, without implementing the Stackelberg game, the manufacturer may select a target value for the roughness which is different from our suggested value. To see the possible loss from these decisions, we tested another two cases where $T_{1max} = (7, 27)$. These cases represent scenarios where tighter and looser specifications are provided by the manufacturer. Furthermore, we also tested the case where no restriction of T_{1max} is set on the output Y_1, which equates to set $T_{1max} = \infty$. Solutions for these cases together with solutions from Scenario 1 and Stackelberg game are compared in Table 24 and 25.

Table 24: Comparison with other solutions - rough milling

T_{1max}	*objective*	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$E(Y_1)$	$Var(Y_1)$	$E(L_1)$	EC_1
7	56.27	235.6	0.15	6	0.03	0.79	7	5.6	0.27	55.99
17.96	49.16	235.6	0.18	6	0.1	0.77	17.96	14.44	1.68	47.48
21.66*	48.32	235.6	0.18	6	0.12	0.77	21.66	17.33	2.43	45.88
27	48.63	200	0.19	6	0.18	0.76	27	21.6	3.75	44.88
∞	47.78	235.6	0.19	6	0.16	0.76	28.44	22.75	4.16	43.62

Table 25: Comparison with other solutions - finish milling

T_{1max}	*objective*	x_{21}	x_{22}	x_{23}	$E(Y_2)$	$Var(Y_2)$	$E(L_2)$	EC_2	$P(T_{1max})$
7	101.02	1500	6	0.02	1.65	0.0302	13.76	25.67	61.59
17.96	96.51	1500	6	0.02	1.65	0.0351	13.79	30.49	52.23
21.66*	96.83	1500	6	0.02	1.65	0.0373	13.8	32.56	50.47
27	99.27	1500	6	0.02	1.65	0.0412	13.82	36.09	49.37
∞	98.98	1500	6	0.02	1.65	0.0424	13.82	37.17	47.98

* Solutions from Stackelberg game

As we can see, the manufacturer has the lowest cost (objective value) when the full cooperation is applied. The manufacturer's cost with Stackelberg game is very close to the value from full cooperation, but is much lower than the remaining cases. Thus, the manufacturer benefits from the proposed Stackelberg game. Furthermore, we can also observed that as T_{1max} increases, the supplier's quality loss increases, which results in a higher quality loss for the manufacturer. On the other hand, an increase in T_{1max} results in a lower efficient cost for the supplier, hence, a lower price is charged from the manufacturer. This clearly demonstrates the importance of selecting the right T_{1max} in the system.

Scenario 3: Weighted Sum of the Objective

In scenario 1, the objective of the manufacturer is to minimize costs associated his own operation. However, in practice, the manufacturer and the supplier can belong to the same company, therefore, the supplier's cost function is also a concern for the manufacturer. In both the Stackelberg game and the full cooperation scenario, an combined objective for the manufacturer can be proposed to solve the problem.

In Stackelberg game, the optimization problem for the supplier stays the same, and the manufacturer's optimization problem is modified as:

$$
\begin{aligned}
Find \quad & x_{21}, x_{22}, x_{23}, T_{1max} \\
min \quad & w_1\{c_1(E(Y_1)^2 + Var(Y_1)) + \tilde{EC_1}\} + w_2\{c_3(E(Y_2)^2 + Var(Y_2)) + \frac{c_4}{x_{22}x_{23}} + 9\tilde{EC_1}\} \\
s.t. \quad & 0 \leq \bar{y}_2 \leq T_{2max} \\
& x_{2i} \in [x_{2i}^-, x_{2i}^+]; \quad i = 1, 2, 3.
\end{aligned}
$$

Where: w_1 and w_2 are weights set by the manufacturer which reflect the relative importance of the objective of the supplier and the manufacturer. Note that the best response functions of the variance and the efficient costs are used to represent the supplier's objective function in this case. Table 26 and 27 show solutions when different weights are selected in Problem $M2$.

Table 26: Other solutions for different weights - rough milling

w_1	w_2	T_{1max}	$objective$	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$E(Y_1)$	$Var(Y_1)$	$E(L_1)$	EC_1
0.1	0.9	23.42	48.06	235.6	0.18	6	0.13	0.76	23.42	18.74	2.84	45.23
0.5	0.5	27.58	48.57	200	0.19	6	0.18	0.76	27.58	22.06	3.91	44.66
0.7	0.3	27.58	48.57	200	0.19	6	0.18	0.76	27.58	22.06	3.91	44.66

Table 27: Other solutions for different weights - finish milling

w_1	w_2	T_{1max}	$objective$	x_{21}	x_{22}	x_{23}	$E(Y_2)$	$Var(Y_2)$	$E(L_2)$	EC_2	$P(T_{1max})$
0.1	0.9	23.42	97.2	1500	6	0.02	1.65	0.0384	13.8	33.64	49.75
0.5	0.5	27.58	99.46	1500	6	0.02	1.65	0.0416	13.82	36.52	49.13
0.7	0.3	27.58	99.46	1500	6	0.02	1.65	0.0416	13.82	36.52	49.13

Similarly, the weighted sum of the objective can also applied to the full cooperation scenario. In this case, given the constraints are the same as the ones in scenario 1, the objective function is modified as follows:

$$min \;\; w_1\{c_1(E(Y_1)^2 + Var(Y_1)) + \frac{c_2}{x_{12}x_{13}}\} + w_2\{c_3(E(Y_2)^2 + Var(Y_2)) + \frac{c_4}{x_{22}x_{23}} + \frac{c_5c_2}{x_{12}x_{13}}\}$$

Solutions with respect to different weights are listed in Table 28 and 29.

Table 28: Other solutions for full optimization with different weights - rough milling

w_1	w_2	*objective*	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	$E(Y_1)$	$Var(Y_1)$	$E(L_1)$	EC_1
0.1	0.9	48.99	235.6	0.18	6	0.1	0.77	18.57	14.88	1.8	47.19
0.5	0.5	48.32	235.6	0.18	6	0.12	0.77	21.63	17.29	2.43	45.9
0.9	0.1	47.82	235.6	0.19	6	0.15	0.76	26.53	21.33	3.63	44.19

Table 29: Other solutions for full optimization with different weights - finish milling

w_1	w_2	*objective*	x_{21}	x_{22}	x_{23}	$E(Y_2)$	$Var(Y_2)$	$E(L_2)$	EC_2	$P(T_{1max})$
0.1	0.9	96.52	1500	6	0.02	1.65	0.0354	13.79	30.82	51.91
0.5	0.5	96.82	1500	6	0.02	1.65	0.0373	13.8	32.54	50.49
0.9	0.1	98.17	1500	6	0.02	1.65	0.0408	13.82	35.75	48.61

Results from both Stackelberg game and full cooperation show that by changing the relative weights between w_1 and w_2, solutions are changed. Larger w_i helps to decrease the corresponding player's objective values. Therefore, in terms of the relative importance between the supplier's and manufacturer's objective functions, various values of w_1 and w_2 are applied.

4.5 Conclusion

Considering the changes in American company's manufacturing environment, supply-chain oriented robust parameter design is critical for companies to produce quality products and improve their competitiveness. This project formulates problems involved in this new research area and presents our solution strategies. Several examples successfully demonstrate the potential of the proposed methods.

4.6 References

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CHAPTER V

FUTURE WORK BEYOND THESIS

There are several interesting topics that I would like to continue in the future. Due to the combinatorial property, combinatorial auctions provide a large opportunity for collusion among bidders. However, to the best of my knowledge, few research is done on this topic. Therefore, collusion in combinatorial auction is an interesting topic for my future work.

For the shelf space allocation project, to further investigate properties of the proposed framework, it is interesting to examine model sensitivity to input data errors.

For the supply chain oriented robust design project, in the current work, we only consider single response in each stage. When multiple responses are considered, which is a very common situation in practice, the problem becomes more complicated. It is a challenging topic for the further study. In addition, in the current work, the experimental design in the supply-chain environments is not discussed, which is also an interesting topic for the future work.