

Final Report

RADIOTRACER STUDIES ON RAPID SAND FILTRATION

By T.F. Craft and Geoffrey G. Eichholz

**OWRR Project No. B-020-GA
and EES Project E-600-705, jointly**



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1970



**Engineering Experiment Station
GEORGIA INSTITUTE OF TECHNOLOGY
Atlanta, Georgia**

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ENGINEERING EXPERIMENT STATION
in cooperation with
ENVIRONMENTAL RESOURCES CENTER

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332

PREFACE

The research on which this report is based is a continuation of work performed under OWRR Project B-008-GA, which was initiated in May 1966 and completed in December 1967.

This project was set up as a matching fund project under the auspices of the Georgia Institute of Technology Water Resources Center;^{*} the federal support was designated as Project No. B-2550 (EES Project B-327) and the matching support by the State of Georgia as Project No. E-600-705.

Part of this work was comprised in the doctoral thesis of T. F. Craft which covers also some of the more theoretical aspects of this field that are not dealt with in any detail in this report. That thesis was issued as a Water Resources Center Report WRC-0469 in August 1969.

Thanks are due to Dr. H. V. Grubb and Dr. G. L. Bridger of the School of Chemical Engineering for their hospitality during all laboratory operations of this project. We are also grateful to the Atlanta Water Works (Mr. Paul Weir, director) for their assistance and cooperation; particularly to Mr. Richard R. Smith, Superintendent of the Hemphill Water Treatment Plant for his support and encouragement. The Nuclear and Biological Sciences Division of the Engineering Experiment Station provided financial and logistical support.

^{*} Since March 1970, Environmental Resources Center.

SUMMARY

Both the theoretical and practical aspects of rapid sand filtration have been investigated. A radiotracer technique made it possible to measure accumulations of suspended matter in porous filter beds. Filter coefficients were calculated and were useful in determining the mechanisms involved when suspended particles are trapped by a porous medium, such as sand or anthracite.

The experimental evidence shows that three major actions occur. These are physical hindrance, interstitial sieving, and the physico-chemical van der Waals and double layer forces. The particular mechanism by which a given particle is trapped in a porous medium is determined primarily by the size of the particle. There is a gradual transition between applicable mechanisms; more than one force may be influencing a particle, but their relative effectiveness depends on the particle size.

The effectiveness of anthracite and normally graded, reverse graded, and uniform sand was evaluated in a series of long filter runs. From the standpoint of head loss, anthracite plus sand or anthracite alone are superior. Other media in order of decreasing effectiveness are reverse-graded sand, uniform sand, and normally-graded sand. When all factors are considered, it is concluded that anthracite plus sand is the best choice.

TABLE OF CONTENTS

	Page
PREFACE	iii
SUMMARY	v
LIST OF TABLES.	ix
LIST OF FIGURES	xi
INTRODUCTION.	1
EQUIPMENT AND PROCEDURES.	5
EXPERIMENTAL RESULTS.	11
Phase I	
Determination of Filter Coefficients	11
Summary of Radiotracer Filtration Runs at Constant Flow Rate.	18
Effect of Particle Size.	24
Effect of Flow Rate.	33
Effects of Dissolved Activity.	34
Phase II	
Field Study.	38
Comparison of Filter Composition	39
CONCLUSIONS	51
REFERENCES.	55
PUBLICATIONS.	57
PERSONNEL	59

LIST OF TABLES

Table	Page
1. Summary of Observed Data for Constant Flow Rate.	12
2. Filter Coefficients (λ) at 2 gal/ft ² /min	14
3. Summary of Observed Data for 30-40 Mesh Sand	16
4. Filter Coefficients (λ) for 30-40 Mesh Sand.	17
5. Isotope Sorption by Anthrafilt	37
6. Activity of Anthrafilt Exposed to I-131.	38

LIST OF FIGURES

Figure	Page
1. Details of Lead Shield for Geiger Tube	6
2. View of Filter and Automatic Counting System	7
3. Test Filters Installed at Atlanta Water Works.	9
4. Filter Coefficients for Vermiculite Particles in 40-60 Mesh Sand	19
5. Filter Coefficients for Vermiculite Particles in 20-30 Mesh Sand	20
6. Filter Coefficients for Vermiculite Particles in 14-20 Mesh Sand	21
7. Filter Coefficients for Vermiculite Particles in Anthrafilt No. 1.	22
8. Filter Coefficients for Vermiculite Particles in 30-40 Mesh Sand at Various Flow Rates	23
9. Activity Profiles in 40-60 Mesh Sand at 2 gal/ft ² /min.	25
10. Activity Profiles in 20-30 Mesh Sand at 2 gal/ft ² /min.	26
11. Activity Profiles in 14-20 Mesh Sand at 2 gal/ft ² /min.	27
12. Activity Profiles in Anthrafilt No. 1 at 2 gal/ft ² /min	28
13. Semi-log Plot of Activity Profiles in 20-30 Mesh Sand at 2 gal/ft ² /min	29
14. Typical Count Variations at Different Sand Levels.	31
15. Filter Coefficients at 2 gal/ft ² /min for Minimum Sand Grain Diameters	32
16. Time-Activity Relationships for Effluent and Filter Bed.	35
17. Particle Size Distribution of Atlanta Water Works Filter Influent	40
18. Head Loss in Uniform Sand at Various Times	41
19. Head Loss in Normally Graded Sand at Various Times	43

LIST OF FIGURES (Concluded)

Figure		Page
20.	Head Loss in Reverse Graded Sand at Various Times.	44
21.	Head Loss in Sand Plus Anthracite at Various Times	47
22.	Head Loss in Anthracite at Various Times	49

INTRODUCTION

Filtration is an essential part of water treatment procedures that begin with a turbid water and produce as the end product a clear, sparkling effluent. Filters themselves take various forms and today may utilize one or more of a variety of media, but essentially they consist of a large container supporting a porous medium through which the water to be filtered passes, usually in the downward direction, and usually under the force of gravity alone. The traditional filter medium has been sand and it is still present in the vast majority of filters in use today.

Rapid sand filters have been in use for more than a century and their design and operation are highly developed. However, it has only been in recent years that much attention has been directed to the fundamental principles that cause suspended particles to be removed from a moving liquid during passage through a porous medium.

Virtually all filter design until quite recently has been based on empirical deduction from observations. The resulting design "recipes" have been used interchangeably with little regard for the characteristics of the water to be filtered. This seems to indicate that either the action of the filter is independent of the nature of the influent or that in many cases the filters must have been grossly oversized. Until such time as the basic mechanism is more fully understood, any improvements will likely be incremental.

There are a number of variables involved in rapid sand filtration, and they may be placed in one of three general categories: (1) hydraulic factors, (2) physical-chemical parameters of the porous bed, or (3) characteristics of the influent. Many individual factors have been studied, and the inter-

relationships of a number of these have been reported. Although a number of theories have been advanced,⁽¹⁾ it is evident that no single theory so far developed can explain the available experimental data. Without invoking scientific principles yet unknown, it seems likely that a combination of chemical and physical laws are involved, but additional information is required to ascertain the relative importance of the mechanisms so far proposed and to assist in the formulation of a definitive theory.

A major parameter of interest is the distribution of particle sizes deposited at different depths within a filter. Information on this variable appears particularly useful because there is no apparent a priori reason why all particle sizes should behave in identical fashion. If different size particles are affected by different forces, it would simplify matters to consider these influences separately. Eliassen⁽²⁾ has reported some data on deposition of different particle sizes at different depths in an experimental filter, but he was working with an iron floc which consisted for the most part of particles much larger than the minimum size that a sand filter will remove.

A technique involving radiotracers was used by Stanley⁽³⁾ in a study of some of the variables of sand filtration. The floc particles employed were passed through an orifice so that reproducible sizes could be obtained. He did not report any actual particle sizes, but he concluded that the size of the particles affects penetration. Other parameters considered included the concentration of iron, pH, ion concentration, sand particle size, and rate of flow. The present work was also intended to elucidate some of these factors. This has been done through the use of labeled particles, which were entrained in the water flow through a pilot-scale filter bed.

There are two experimental approaches to the determination of the relation between particle size and the ultimate location of deposit. One might

either use a wide range in size of suspended particulates and determine the resulting distribution at different levels, or use a narrow range of particle sizes and measure simply the gross deposit at each level. Primarily the second approach has been used in the present project, utilizing various particle size fractions up to 105 μ diameter.

A major goal of the laboratory phase of this work has been to determine the amount of material of a given size that is deposited in the porous medium as a function of bed characteristics, flow time, and position in the filter. A radiotracer technique was chosen for obtaining this information because data on particle deposition could be obtained during filtration without physically disturbing either the filter medium or the flow passing through it. This is a great advantage over the alternate procedure of removing at intervals samples of the bed and/or the flowing water. Excellent laboratory facilities for the utilization of radioisotopes were available and the authors had had previous experience with radioisotopes. In the second phase of these studies, conducted at the Atlanta Water Works, long filter runs were undertaken using actual, prefilter water. Information was obtained by measuring loss of head as a function of time at various elevations within the filter bed.

The primary purpose of this project was the investigation of the basic mechanism of filtration. The second phase was involved with the practical aspects of filtration as encountered under commercial conditions, as a way of evaluating some of the factors that arose during the work in the first phase. Special equipment required for this study was designed and constructed. The resulting data were used in the development of a theoretical interpretation to explain the action of rapid sand filters.

EQUIPMENT AND PROCEDURES

Most of the laboratory studies employed a special filter designed to be wide enough to minimize wall effects and yet small enough to handle available water flow. The laboratory-scale filter used in the radiotracer work had a bed surface area of about one square foot. It was constructed in the conventional manner with arrangements for water to flow either up or down for backwashing or filtration, respectively. A vertical plastic tube, sealed at the bottom, was positioned inside the center of the filter. The lower end of the tube was anchored above the bottom of the filter, and the upper end extended well above the highest attainable water level.

A small Geiger-Müller tube was suspended inside a lead shield which fit snugly inside the plastic tube. The lead shield had a narrow horizontal slit through which the G-M tube could detect radioactive material located in a plane at the same elevation as the slit. A drawing of this shield is shown in Fig. 1. The shield and G-M tube were suspended from a coaxial cable which could be raised and lowered to position the shield slit at desired depths in the filter. An automatic device was constructed to control the positioning of the shield and the operation of the counting and recording system. A general view of the filter with the positioning control and automatic counting system is shown in Fig. 2 and constructional details of the complete system are given elsewhere.^(4,5)

Filtration runs were performed in the following manner. The filter was backwashed and the medium compacted to a standard level. The tracer particles containing radioactivity were poured into the top of the filter while the water was flowing in the filtration mode. After allowing sufficient time for the particles to be deposited, the radioactivity at and below the surface of the filter bed was measured. This was accomplished by positioning the slit

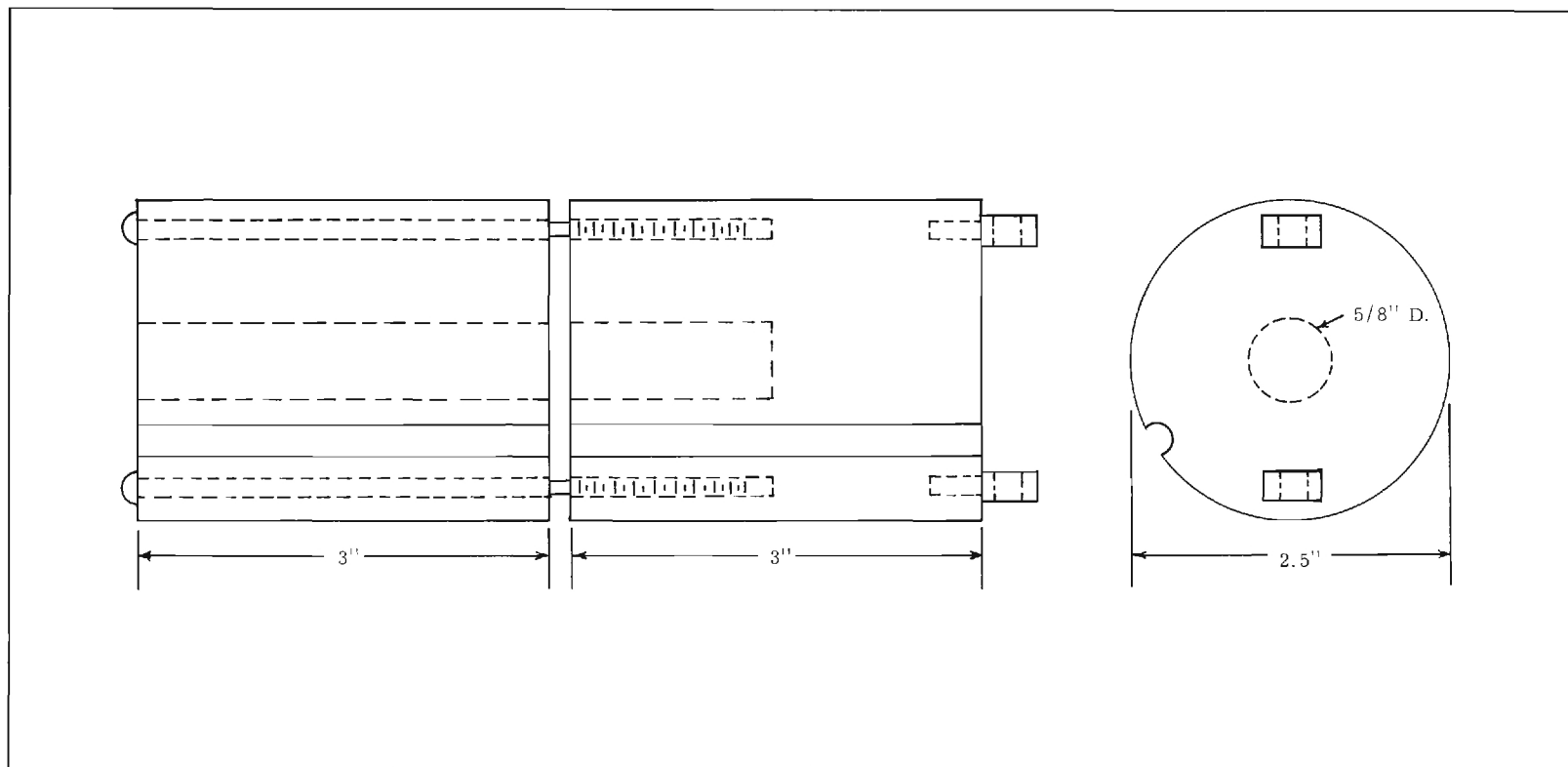


Figure 1. Details of Lead Shield for Geiger Tube

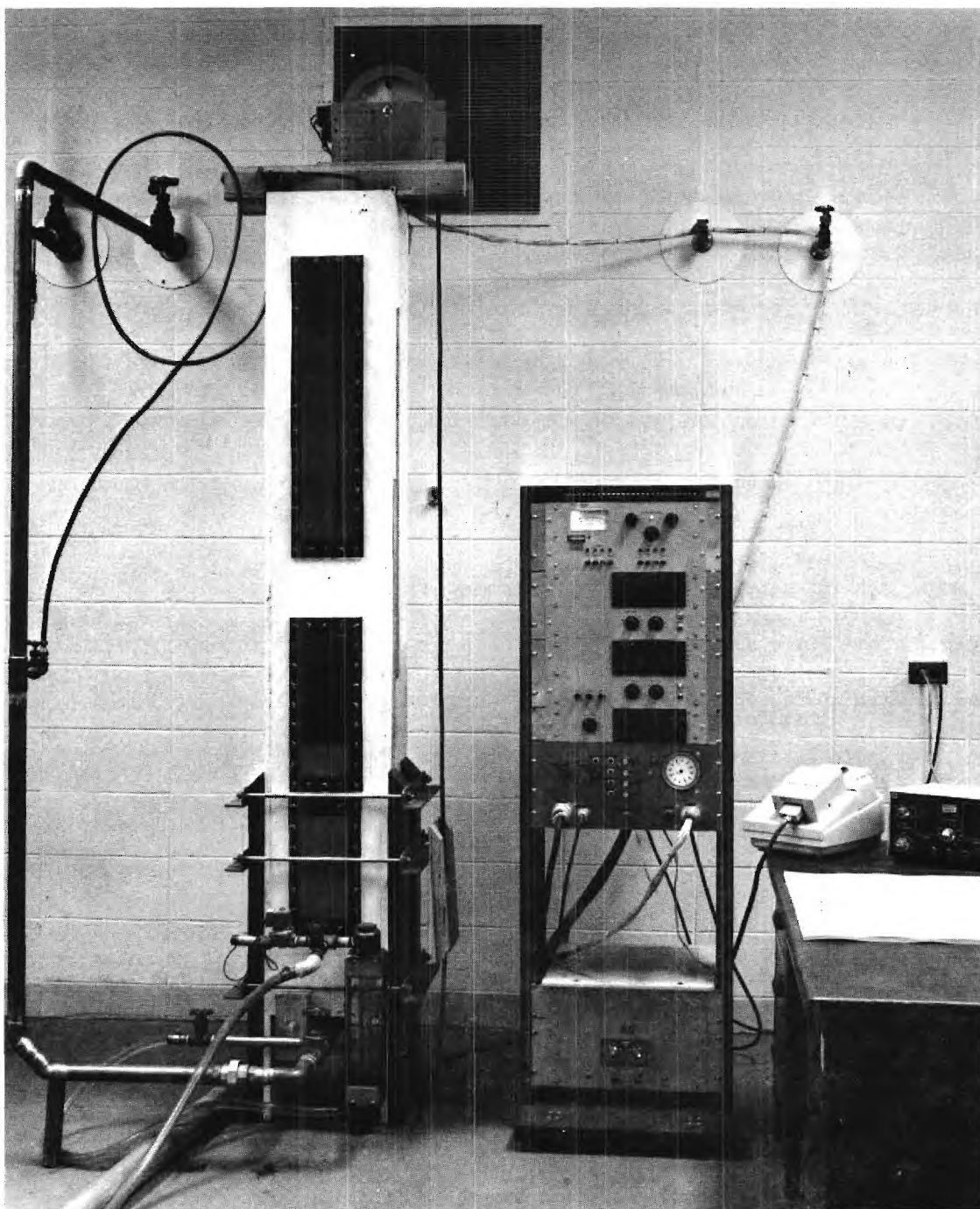


Figure 2. View of Filter and Automatic Counting System

in the lead shield at the elevation of interest and counting with the G-M tube for a definite length of time. This was repeated at all levels desired, producing a profile of activity and consequently indicating a profile of particle deposition.

The second phase of test work was primarily concerned with the effect of different filter media and different filter configurations using actual, pre-treated water. Since this work was done at the Atlanta Water Works, it was considered inadvisable to use radioactive tracers for this phase. For this reason, filter performance had to be evaluated by means of the more primitive method of measuring head losses.

The filter apparatus used for head loss studies consisted of two plastic tubes with spaced taps attached to piezometers. In these tests it was of interest to evaluate the effect of reverse-grading of a non-uniform filter medium and this necessitated a design capable of producing such a configuration of the medium. This was achieved by washing the bed in the normal manner, then placing a wire screen on top of the medium and inverting the entire filter. The filter was filled completely with water during the inversion, so a filter size small enough for manipulation by a single individual had to be chosen. A second filter that remained in fixed position was included so that direct comparison of results was possible.

The photograph (Fig. 3) shows the apparatus arranged for simultaneous flow through both filters. The filter tube on the left is the fixed tube. For backwashing, the valves at the top and bottom of the filter shown in the center of the mounting panel were closed and the tubes disconnected. The flexible leads to the piezometers were long enough to permit this tube to be inverted onto the brackets shown at the right of the panel. The tubes were reconnected, the supporting screen was removed, and backwash carried out in

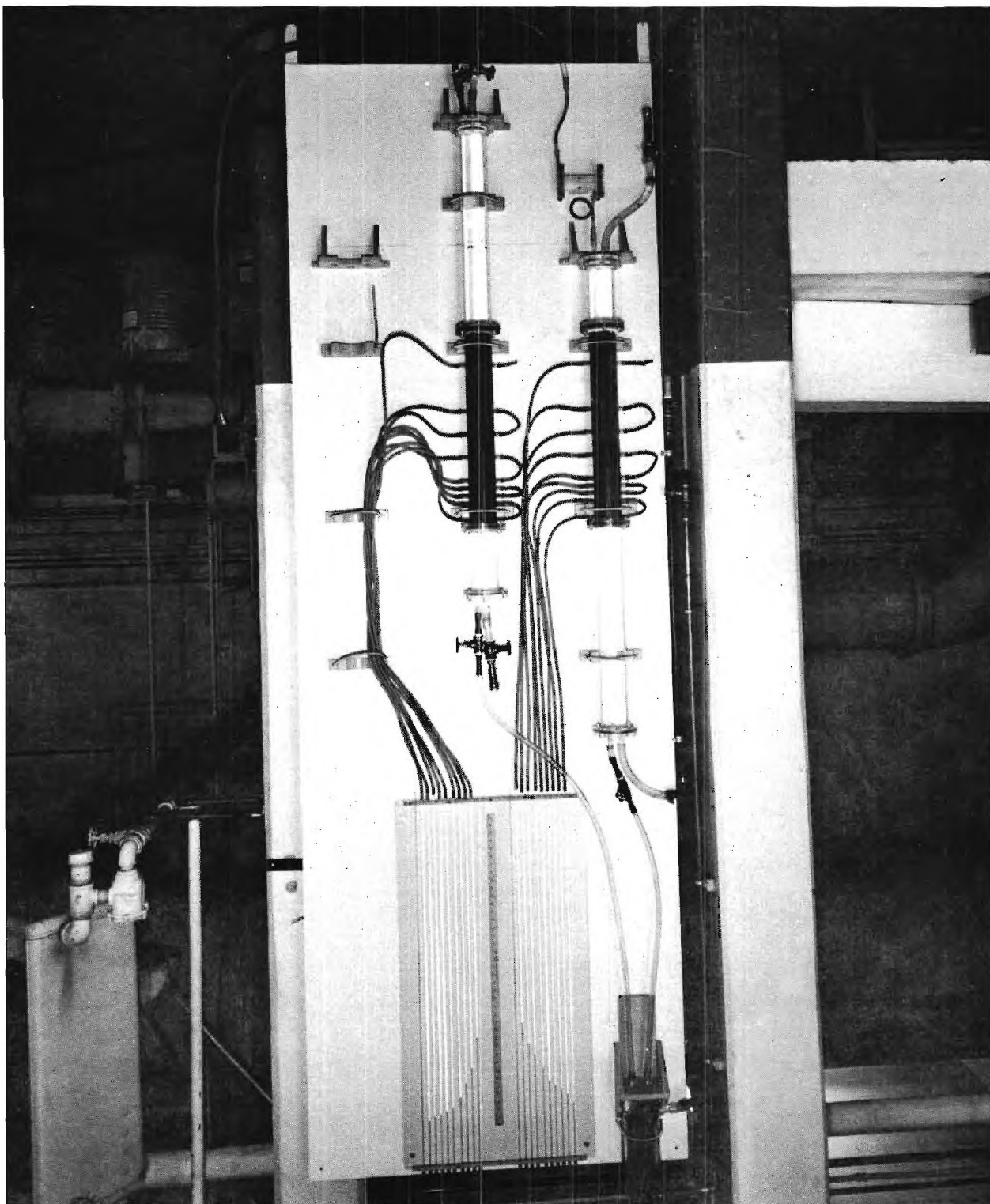


Figure 3. Test Filters Installed at Atlanta Water Works

the normal manner. After backwashing the procedure was reversed to prepare for the next run.

The materials used as filter media were sand and anthracite. A clean white silica sand furnished by the Pennsylvania Glass Sand Corp., Atlanta, Georgia was screened to obtain the desired mesh size ranges for the uniform sands. Non-uniform sand, when used in Phase 2, was filter sand obtained from the Atlanta Water Works. The ground anthracite was supplied by the Palmer Filter Equipment Company, Erie, Pennsylvania under their trade name "Anthrafilt." Details of the properties and preparations of these materials have been previously given.⁽⁶⁾

The suspended matter used in the radiotracer experiments consisted of vermiculite supplied by the Zonolite Company, Atlanta, Georgia. The preparation procedure began with an initial screening. Radioactive labeling was then done by slurrying the particles in a solution of cesium-137 chloride for an hour or more and allowing the mixture to soak overnight. The supernatant was then decanted and the particles were then calcined by heating in a furnace to a temperature of 2050°F. After crushing fused oversized particles and again screening, the sized fractions were ready for use. In this manner, particles were prepared that did not lose their radioisotope label even after long exposure to flowing water. At no time was any loss of activity from the particles ever noticed, nor could any leached out activity be detected in effluent that passed over them. The precise procedure necessary to produce these particles has been described earlier.⁽⁶⁾

EXPERIMENTAL RESULTS

Phase 1Determination of Filter Coefficients

The filter coefficient, λ , was chosen as the parameter of filter efficiency. It is defined by the equation $C = C_0 e^{-\lambda L}$, where C_0 is the concentration at any depth L within the bed. Values of λ were calculated from the relative number of counts observed at the surface and at a depth of one inch. These two points were used because in every case they revealed the highest count rates, giving numbers that were most reliable statistically. Also, the difference in λ due to different particle sizes is most pronounced when λ is calculated for the top inch of the filter bed. On the other hand, it becomes important to eliminate oversize particles as much as possible from the feed.

Typical net counts at the surface were in the range of 1000 to 2500 per 10 minutes. For convenient comparison, the results of each run were all normalized to a uniform 1200 counts at the surface level. Table 1 lists experimental results for sand and anthracite filter beds. In this summary of normalized data one can see that there is less fluctuation in the values for the one inch depth than for other depths measured. The trend of the data at other levels is in general accord with that at the one inch depth though a few values appear out of place. Table 2 summarizes the λ values calculated from the data of Table 1. A number of runs were repeated in order to ascertain reproducibility of results and to verify an occasional seemingly anomalous result. In general, such doubtful results could be traced to failure of the automatic positioning drum to lock at the desired bed elevation.

Another series of filter runs was made using 30-40 mesh sand. The objective was to determine the effect of flow rate on filter coefficient. The

Table 1. Summary of Observed Data for Constant Flow Rate*

Series I: 20-30 Mesh Sand

Bed Depth (in.)	Particle Size (μ)									
	1-3	-25	25-37	37-44	44-53	53-63	63-74	74-88	88-105	105 +
0	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200
1	1187	879	743	705	682	662	640	611	596	587
2	886	462	348	340	355	299	280	268	245	255
3	648	218	143	145	175	130	125	122	106	85
4	488	103	74	75	84	54	52	45	50	63
5	380	51	21	23	39	33	22	39	20	14
7	252	---	---	---	---	6	---	---	---	16
9	182	---	---	---	---	0	---	---	7	11

Series II: 40-60 Mesh Sand

0	1200	1200	1200	1200	1200	1200	1200	1200
1	1128	692	583	607	584	587	635	552
2	740	340	235	193	241	256	283	238
3	441	170	73	104	101	113	104	108
4	270	104	29	73	57	106	57	41
5	159	64	---	42	30	61	29	20
7	70	24	---	35	16	30	14	17
9	18	26	---	39	12	57	30	---

* Counts per 10 minutes normalized to 1200 at the bed surface.

Table 1. Summary of Observed Data for Constant Flow Rate
(Concluded)

Series III: 14-20 Mesh Sand

Bed Depth (in.)	Particle Size (μ)								
	1-3	-25	25-37	37-44	44-53	53-63	63-74	74-88	88-105
0	1200	1200	1200	1200	1200	1200	1200	1200	1200
1	1612	1260	949	966	789	737	676	656	674
2	1532	950	506	552	365	514	285	283	298
3	1318	597	244	256	165	247	140	133	125
4	1134	369	120	128	77	205	84	59	62
5	920	222	63	58	40	161	60	28	46
7	659	93	29	12	10	156	44	16	23
9	509	62	23	9	8	166	42	17	13

Series IV: Anthrafilt No. 1

0	1200	1200	1200	1200	1200	1200	1200	1200	1200
1	1451	984	877	827	774	752	733	726	732
2	1198	586	467	440	376	356	320	332	336
3	923	309	225	207	181	161	164	197	161
4	714	164	109	121	92	60	64	67	71
5	589	86	48	59	47	45	27	70	20
7	368	---	---	12	11	9	---	---	4
9	291	2	5	13	3	---	13	---	---

Table 2. Filter Coefficients (λ) at 2 gal/ft²/min

Particle Size (μ)	40-60 Mesh Sand	20-30 Mesh Sand	14-20 Mesh Sand	Anthrafilt No. 1
1-3	0.06	0.01	-0.31	-0.19
0-25	0.55	0.31	-0.05	0.20
25-37	0.72	0.48	0.23	0.31
37-44	0.68	0.53	0.30	0.37
44-53	0.72	0.56	0.42	0.44
53-63	0.71	0.60	0.49	0.47
63-74	0.64	0.63	0.57	0.49
74-88	0.78	0.68	0.60	0.50
88-105	--	0.72	0.57	0.49
> 105	--	0.71	--	--

normalized data are presented in Table 3 and the calculated filter coefficients are given in Table 4. The coefficients for the 4 gal/ft²/min were less reproducible than those obtained at lower flow rates. This may have been due in part to the presence in the water of small amounts of iron oxide-hydroxide that seemingly came from the supply main. It was manifested by a slight brownish stain that formed slowly at the sand surface and gradually penetrated into the sand. With this possible accumulation of material to alter the efficiency of the bed, timing was considered critical.

During previous runs at lower flow rates the timing was never varied much, but in the high rate runs the elapsed time between completion of backwash and introduction of tracer particles was allowed to vary no more than five minutes. Once the particulates had been deposited in the medium, the subsequent deposition of other material was of no consequence.

The statistics of the results were investigated through calculation of deviations in λ due to fluctuations in the observed number of counts at each point. The standard deviation of $e^{-\lambda}$ was calculated from the equation

$$\sigma_{e^{-\lambda}} = \frac{C_1}{C_0} \sqrt{\left(\frac{\sigma_1}{\sqrt{n_1} C_1}\right)^2 + \left(\frac{\sigma_0}{\sqrt{n_0} C_0}\right)^2}$$

where C_0 is the number of counts observed at the surface of the filter matrix, C_1 is the number of counts observed at the one inch depth in the filter matrix, σ_0 and σ_1 are the standard deviations in C_0 and C_1 ; n_0 and n_1 are the number of observations of C_0 and C_1 , respectively. The effect on λ of this statistical variability in $e^{-\lambda}$ was first added to and then subtracted from the $e^{-\lambda}$ value to yield a lower and an upper limit of $e^{-\lambda}$. The difference in λ values corresponding to these limits is, therefore, the range of two standard deviations in λ and for most filter runs fell within the range 0.02-0.05.

Table 3. Summary of Observed Data for 30-40 Mesh Sand

Bed Depth (in.)	0.5 gal/ft ² /min								
	Particle Size (μ)								
	1-3	-25	25-37	37-44	44-53	53-63	63-74	74-88	88-105
0	1200	1200	1200	1200	1200	1200	1200	1200	
1	810	601	531	510	530	530	535	525	
2	461	263	215	217	210	222	244	102	
3	216	114	104	201	93	104	99	82	
4	191	49	42	105	44	65	53	41	
5	137	36	20	0	13	30	19	28	
7	104	16	13	0	2	25	7	14	
9	50	12	4	0	3	10	9	0	
2.0 gal/ft ² /min									
0	1200	1200	1200	1200	1200	1200	1200	1200	1200
1	1239	991	827	753	706	678	627	623	636
2	851	574	442	408	330	302	294	300	284
3	679	295	214	192	144	129	130	125	125
4	530	204	78	96	75	65	56	47	42
5	339	70	38	44	25	42	40	26	5
7	256	25	0	37	0	12	9	5	0
9	60	10	0	18	0	2	0	0	0
4.0 gal/ft ² /min									
0			1200	1200	1200	1200	1200	1200	
1			1071	1008	784	701	597	589	
2			817	713	422	319	251	261	
3			589	512	222	170	129	106	
4			413	348	117	78	57	50	
5			386	222	78	39	30	10	
7			283	156	21	21	20	2	
9			245	71	0	0	7	0	

Table 4. Filter Coefficients (λ) for 30-40 Mesh Sand

Particle Size (μ)	Flow Rate (gal/ft ² /min)		
	0.5	2.0	4.0
1-3	0.375	-0.07	----
-25	0.690	0.18	----
25-37	0.815	0.37	0.11
37-44	0.825	0.47	0.17
44-53	0.820	0.53	0.42
53-63	0.815	0.58	0.54
63-74	0.805	0.65	0.69
74-88	0.825	0.66	0.71
88-105	-----	0.63	----

Many runs were duplicated and the corresponding calculated λ values usually differed by no more than 0.04, which is within the calculated expectation of variability. Uncertainties of this order of magnitude are not particularly significant in determining relationships such as those described here.

Summary of Radiotracer Filtration Runs at Constant Flow Rate

The calculated values of the filter coefficient were plotted against the arithmetic mean of the particle diameters, and the results are shown in Figures 4-8. The theoretical implications of these curves have been previously discussed in detail⁽⁶⁾ but briefly the interpretation is as follows. Examining Figure 5, as a typical example, the curve may be divided into three sections, each due to a different predominating mode of capture of suspended particles by the porous medium.

That portion of the curve for particles with diameters larger than about 90 μ is essentially flat and is due to retention of the previously suspended particles on the surface. This occurs where the suspended particles are larger than the openings in the sand. When smaller suspended particles are involved, here in the range of about 60 μ to 30 μ , penetration of the porous bed becomes progressively easier, and the filter efficiency decreases linearly. In this range of particle sizes, the major capture mechanism is considered to be mechanical interstitial sieving.

For particles of diameter less than 30 μ the relationship is no longer linear. While particle capture continues to occur, it is at a much lower efficiency. It is believed that these small particles are influenced largely by van der Waals and double-layer forces and not by purely mechanical forces of interference. These physico-chemical forces are relatively less effective, and filtration depending on them is less efficient than that where other forces are involved.

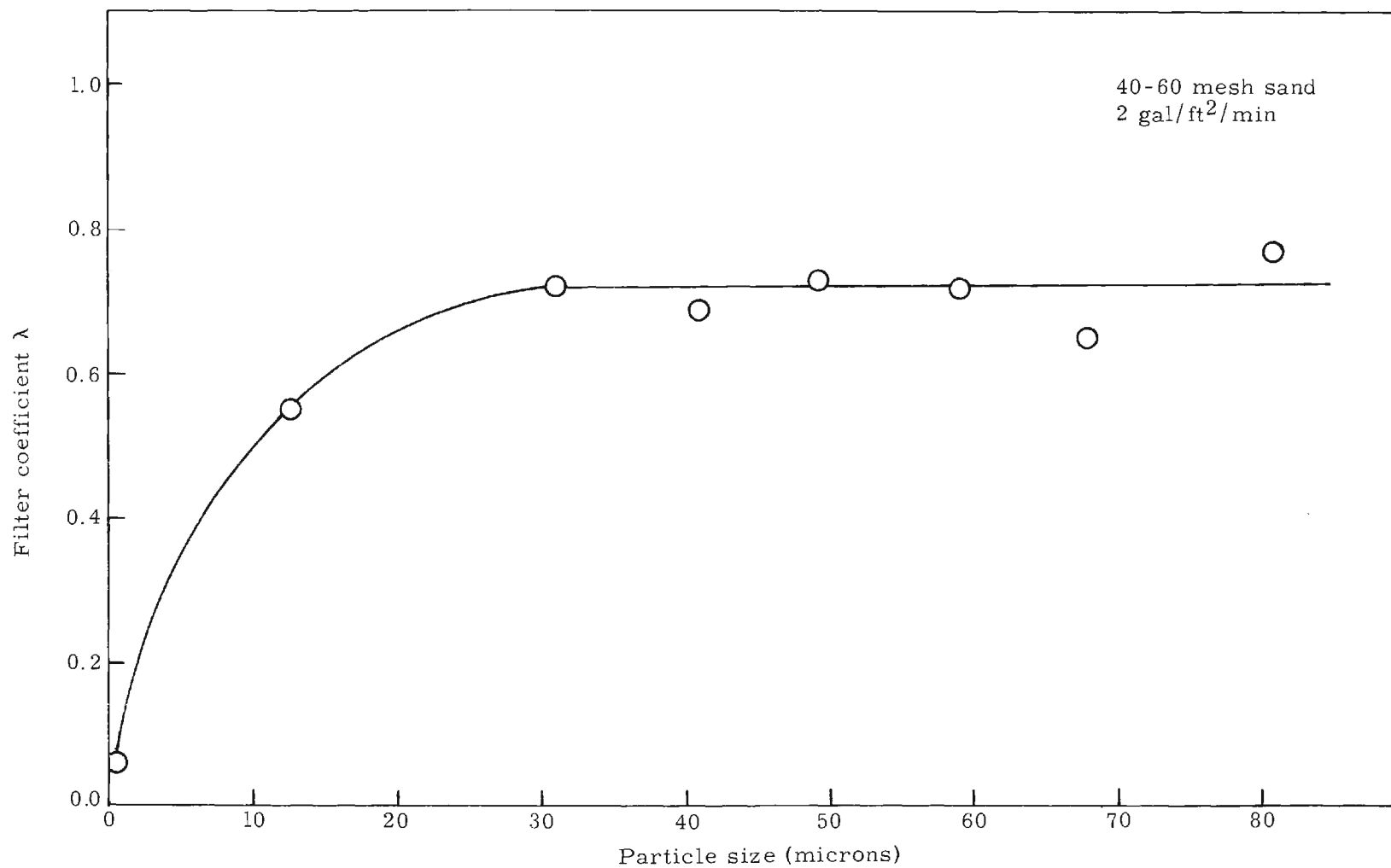


Figure 4. Filter Coefficients for Vermiculite Particles in 40-60 Mesh Sand

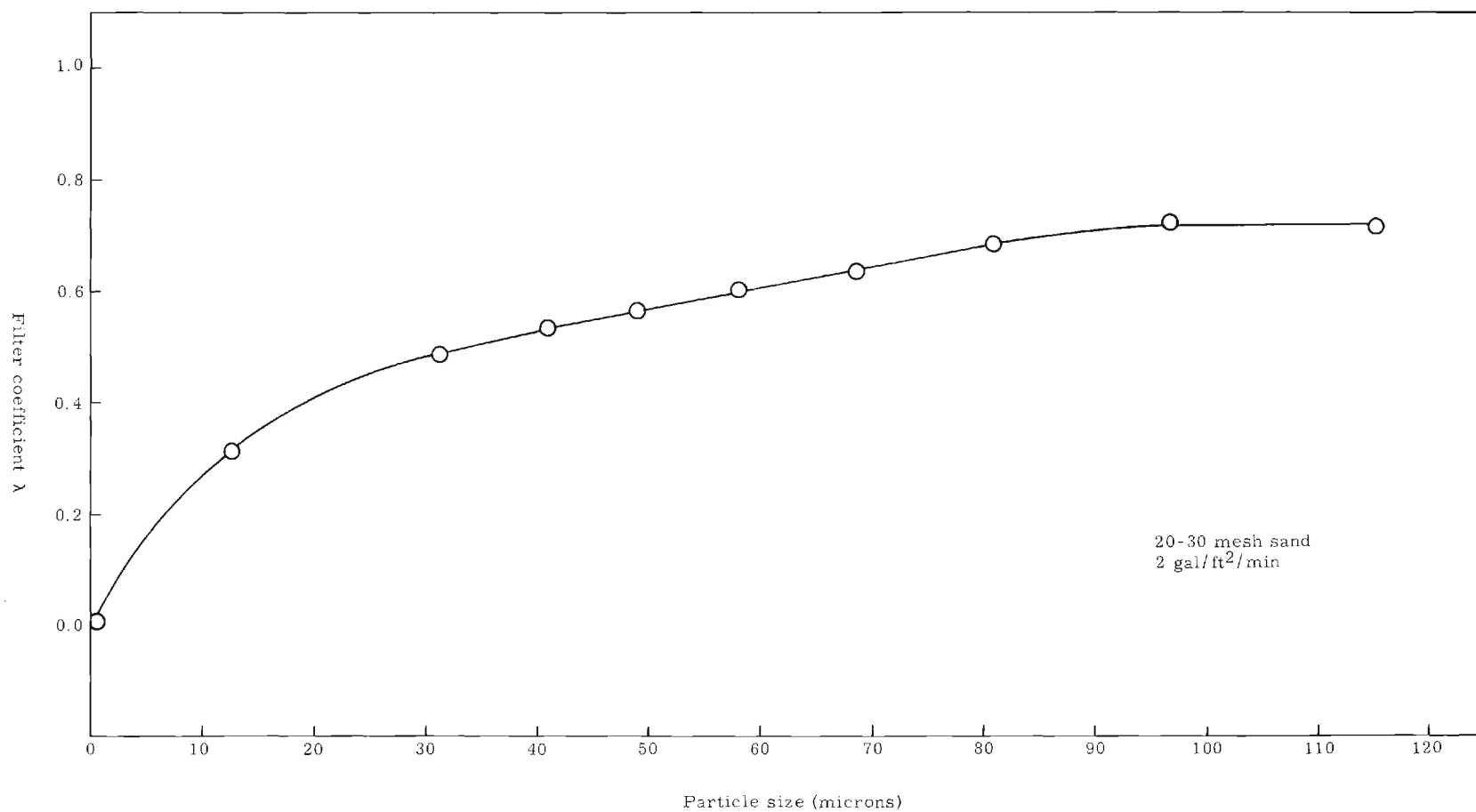


Figure 5. Filter Coefficients for Vermiculite Particles in 20-30 Mesh Sand

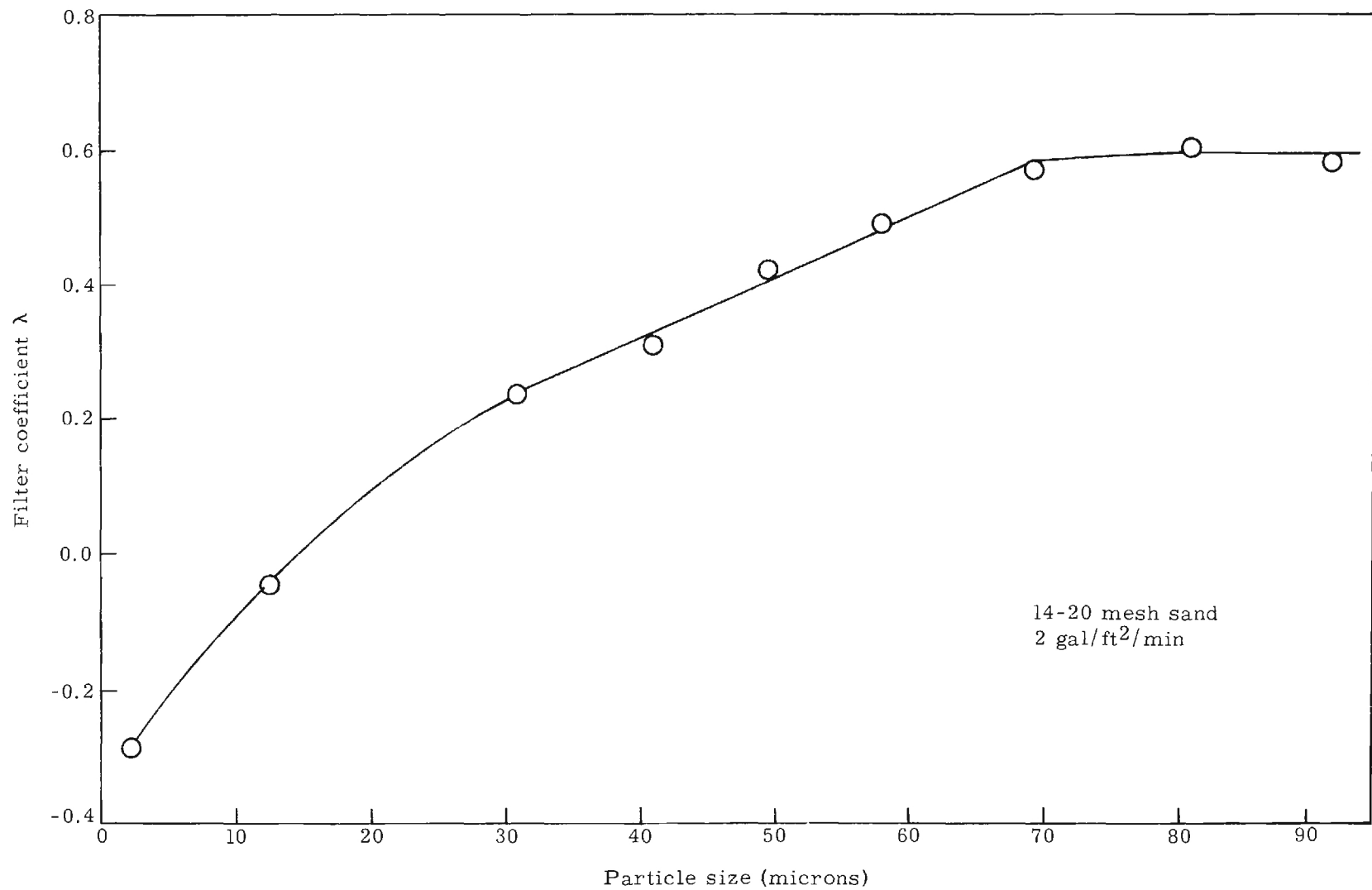


Figure 6. Filter Coefficients for Vermiculite Particles in 14-20 Mesh Sand

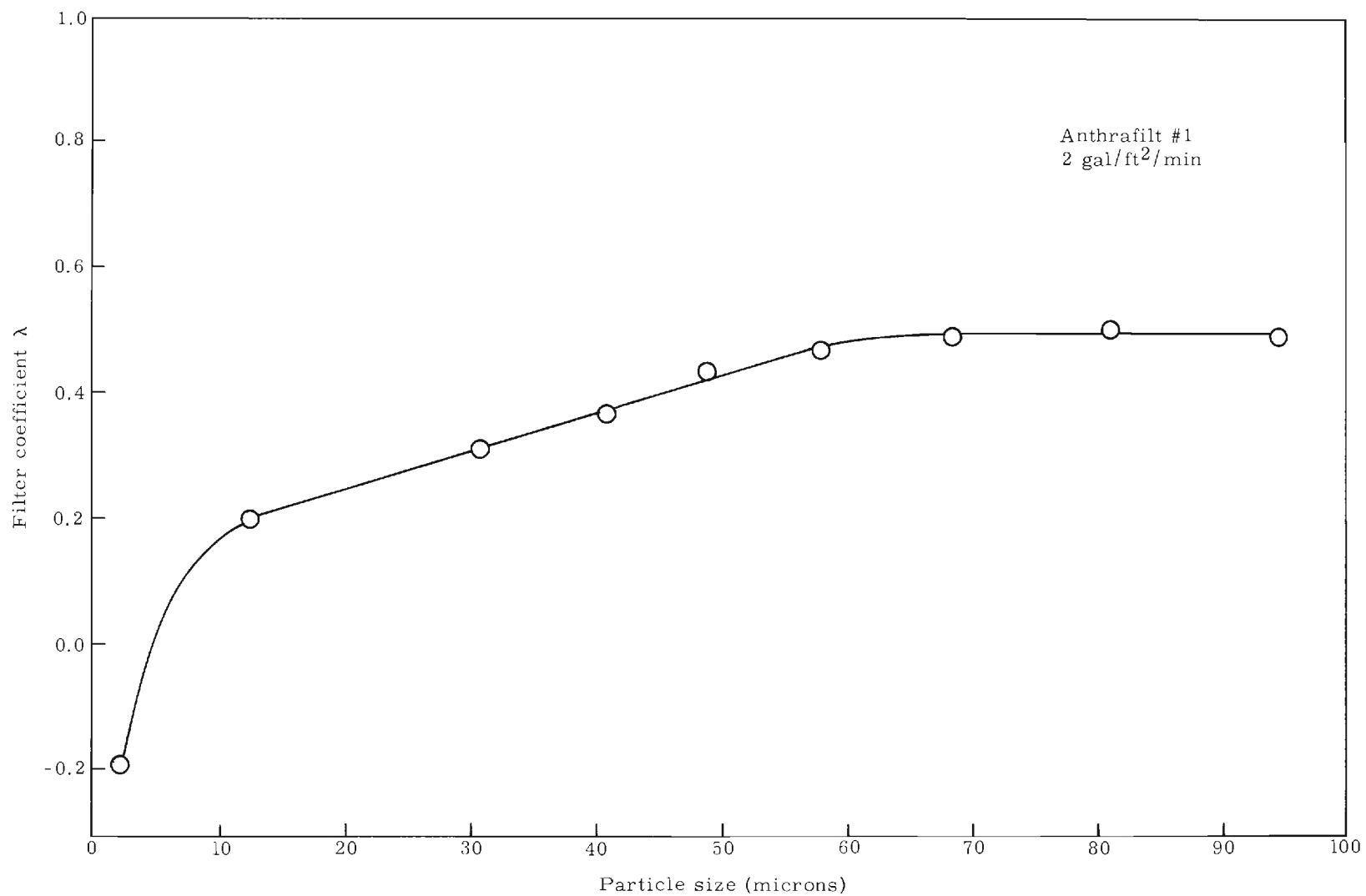


Figure 7. Filter Coefficients for Vermiculite Particles in Anthrafilt No. 1

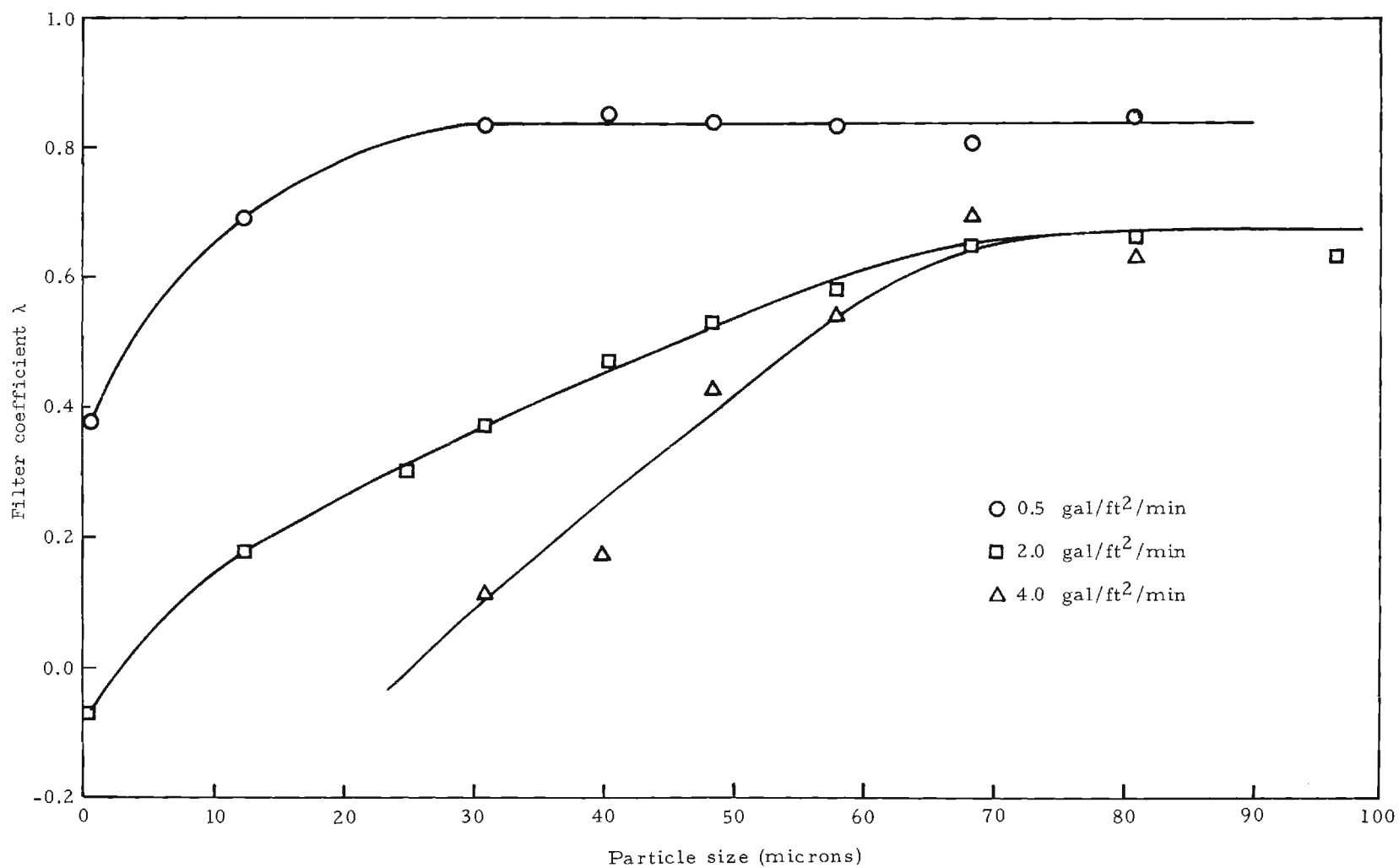


Figure 8. Filter Coefficients for Vermiculite Particles in 30-40 Mesh Sand at Various Flow Rates

Effect of Particle Size

Figures 9, 10, 11, and 12, consisting of linear plots of total counts, C , versus position in the filter bed for representative particle sizes represent the actual pattern of deposition of suspended matter within the filter and illustrate the effects of particle size on activity profiles. In each case, the greatest penetration of the bed was by the particles of smallest size and penetration decreased with progressively larger particles. In Figure 9, the profile produced by all particle sizes larger than $25\ \mu$ is approximated by the curve for $53\text{-}63\ \mu$ particles. Plots of the other profiles are omitted for the sake of clarity. The same situation prevails in Figures 10 and 12 where the $25\text{-}37\ \mu$ curve represents all larger particle sizes. There were greater differences between the profiles produced by different particle size ranges in the 14-20 mesh sand illustrated in Figure 11. Those omitted from this figure were in the approximate region of the $44\text{-}53\ \mu$ and $63\text{-}74\ \mu$ curves.

When comparison between figures is made, it is found that the profile for a given particle size range is not constant but is subject to variation. This difference is due to differences in the filter bed grain size. These findings were expected, as there is no apparent reason that porous media with different characteristics should yield identical profiles even if the particles involved are the same in each case; however, in view of the similarities of the mechanisms, curves of comparable character would be expected.

If particle deposition is exponential with depth as is generally assumed, the profile curves should plot as straight lines on a semi-log grid. Accordingly, semi-log plots were prepared for a number of different particle size ranges in each medium; Figure 13 is typical. In most cases, these curves are essentially linear for depths of one to five inches. The lack of linearity in the first inch of depth, particularly for the smallest particles, is

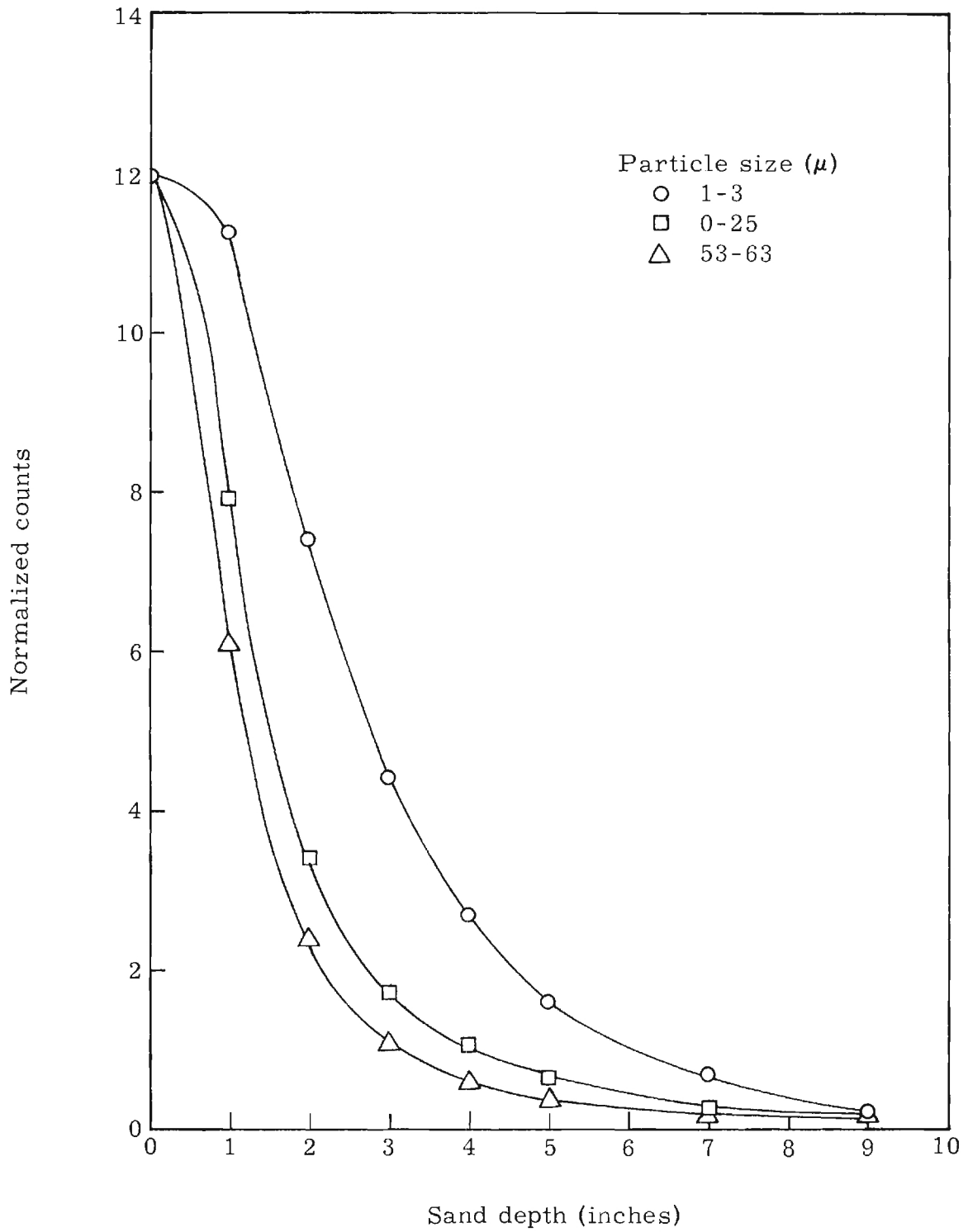


Figure 9. Activity Profiles in 40-60 Mesh Sand at 2 gal/ft²/min

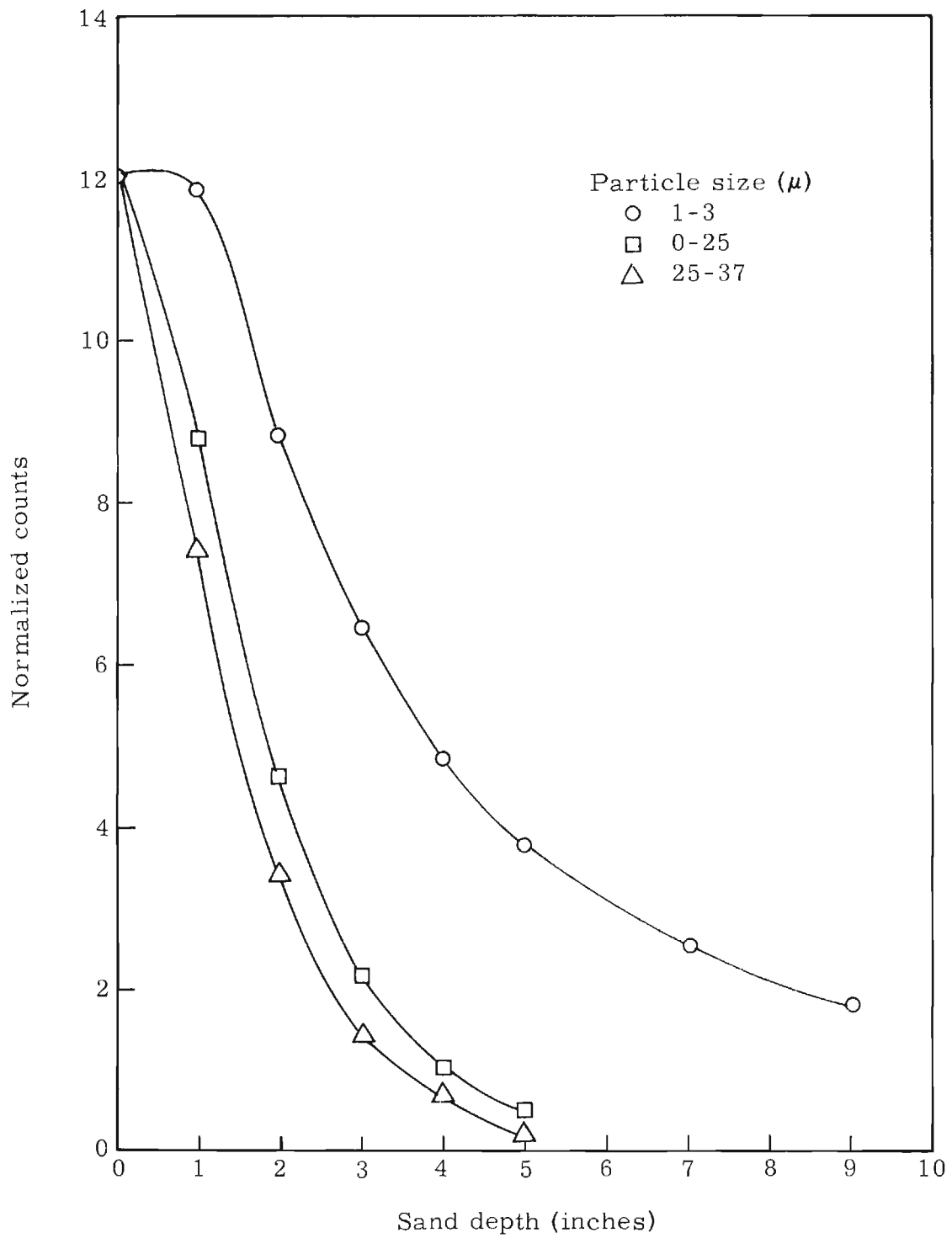


Figure 10. Activity Profiles in 20-30 Mesh Sand at 2 gal/ft²/min

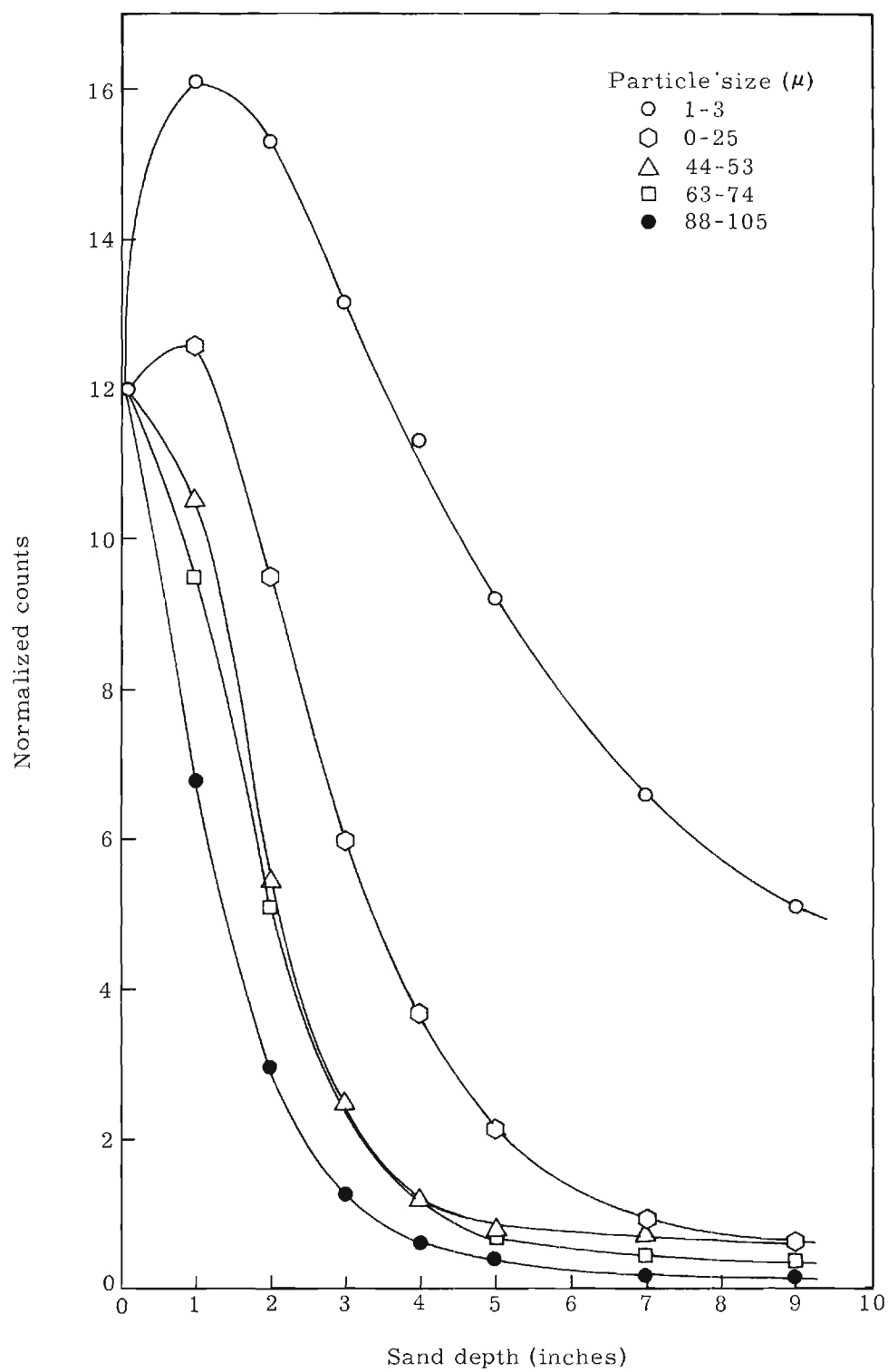


Figure 11. Activity Profiles in 14-20 Mesh Sand at 2 gal/ft²/min

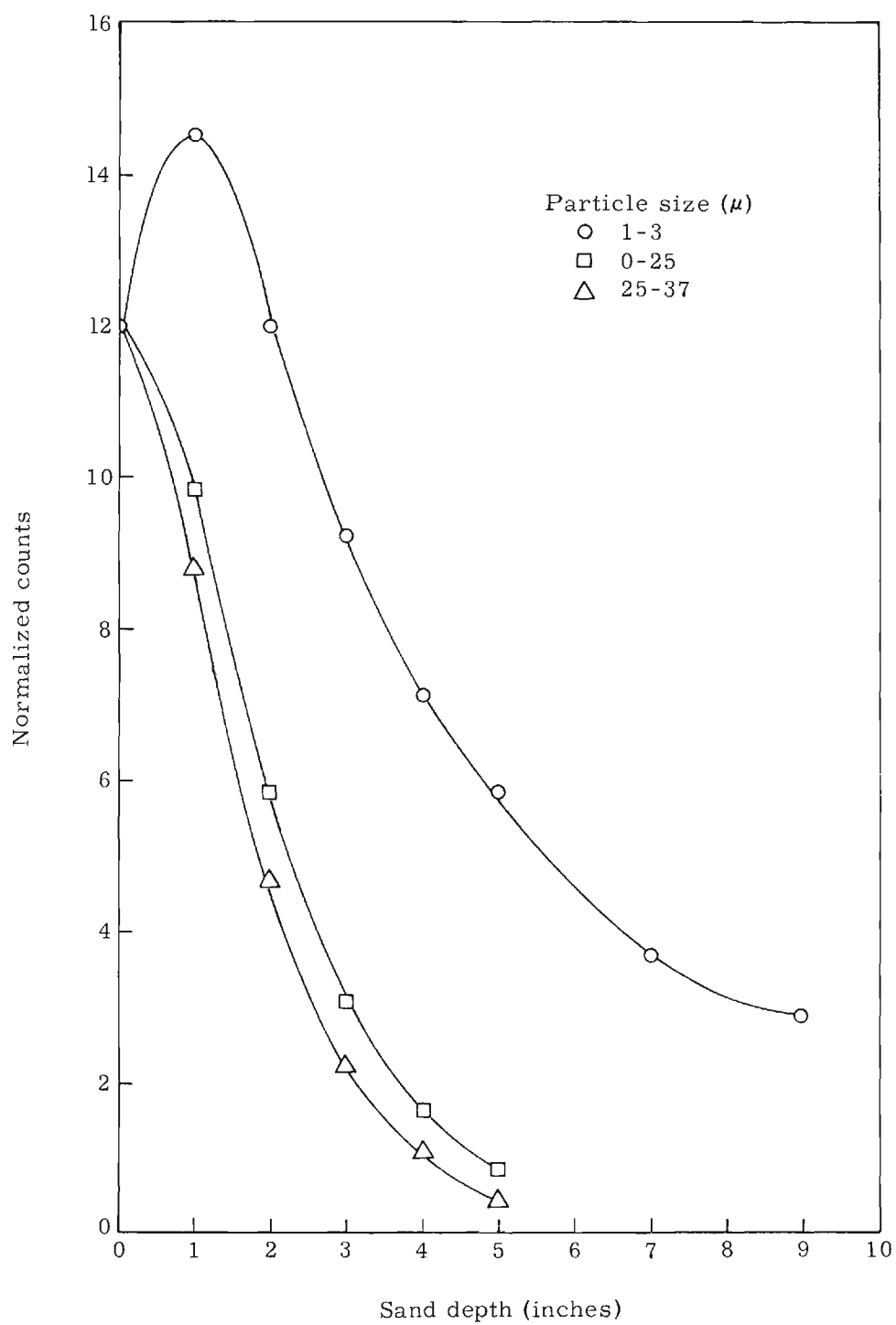


Figure 12. Activity Profiles in Anthrafil No. 1 at 2 gal/ft²/min

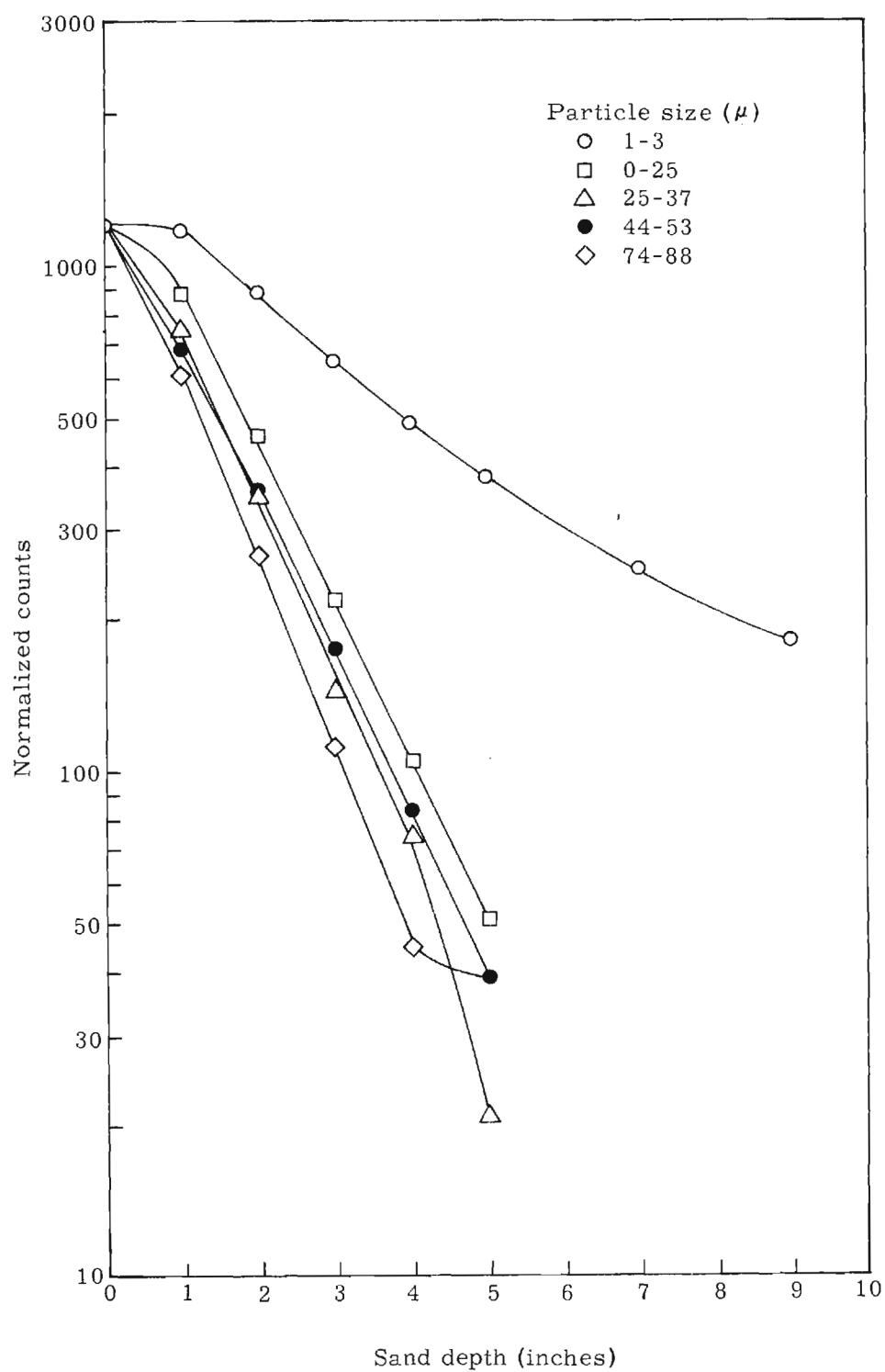


Figure 13. Semi-log Plot of Activity Profiles in 20-30 Mesh Sand at 2 gal/ft²/min

attributed to geometrical considerations involved where there is much penetration. Below the five inch depth the actual count rate was quite low and was about the same order as the background. Little significance can therefore be attached to the values obtained at the lower levels of the filter. The expectation of an exponential pattern of removal, at least over a specific range, is therefore confirmed.

The time relationship of observed counts was examined in plots such as Figure 14, where the total number of counts at various levels is shown as a function of time. It is seen that a steady level is obtained within minutes of starting the test and there is no subsequent displacement of the particles as flow continues. The random fluctuations around the steady level shown by these values are well within the range of normal statistical variation.

The exact relationship of bed grain size and filter coefficient has been the subject of discussion in the literature. The data obtained in this study were consequently plotted in a manner so that this relationship could be examined. In Figure 15, it may be seen that this relationship depends to a considerable degree on the particle sizes involved. The only straight lines observed are for the 63-74 μ and 74-88 μ particles, and the latter is questionable due to an exceptionally high value (0.78) for λ in the 250 μ (40-60 mesh) sand. Referring to Figure 4, it seems more likely that this value should be about 0.72, which would produce the dotted line included in Figure 15. The filter coefficient as measured in this study therefore appears as a linear function of sand grain diameter only for 63-74 μ particles. A severe limitation is imposed on the interpretation of these plots due to the availability of only three points for each curve, but it is clear that the size of the suspended particles is a factor.

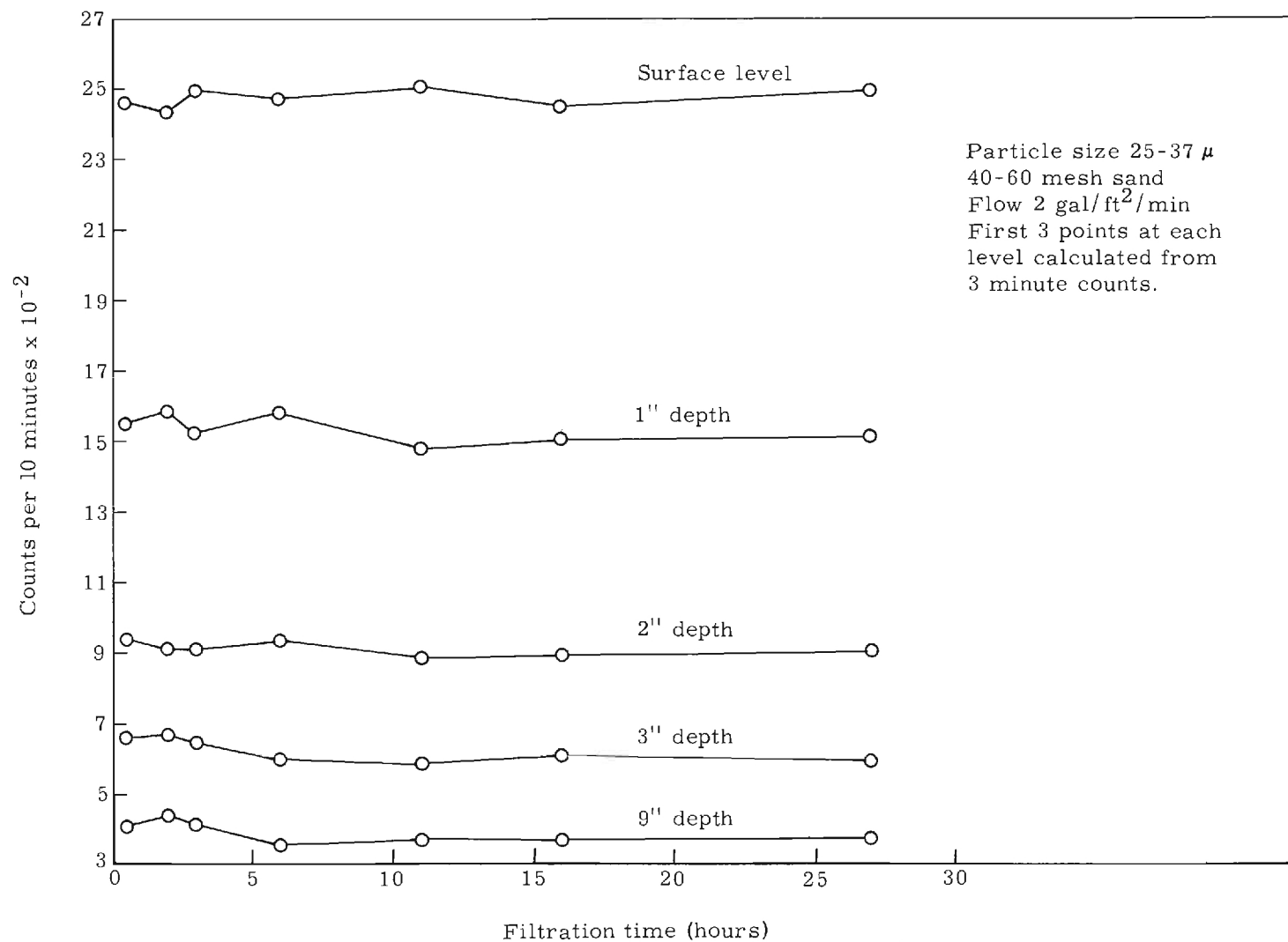


Figure 14. Typical Count Variations at Different Sand Levels

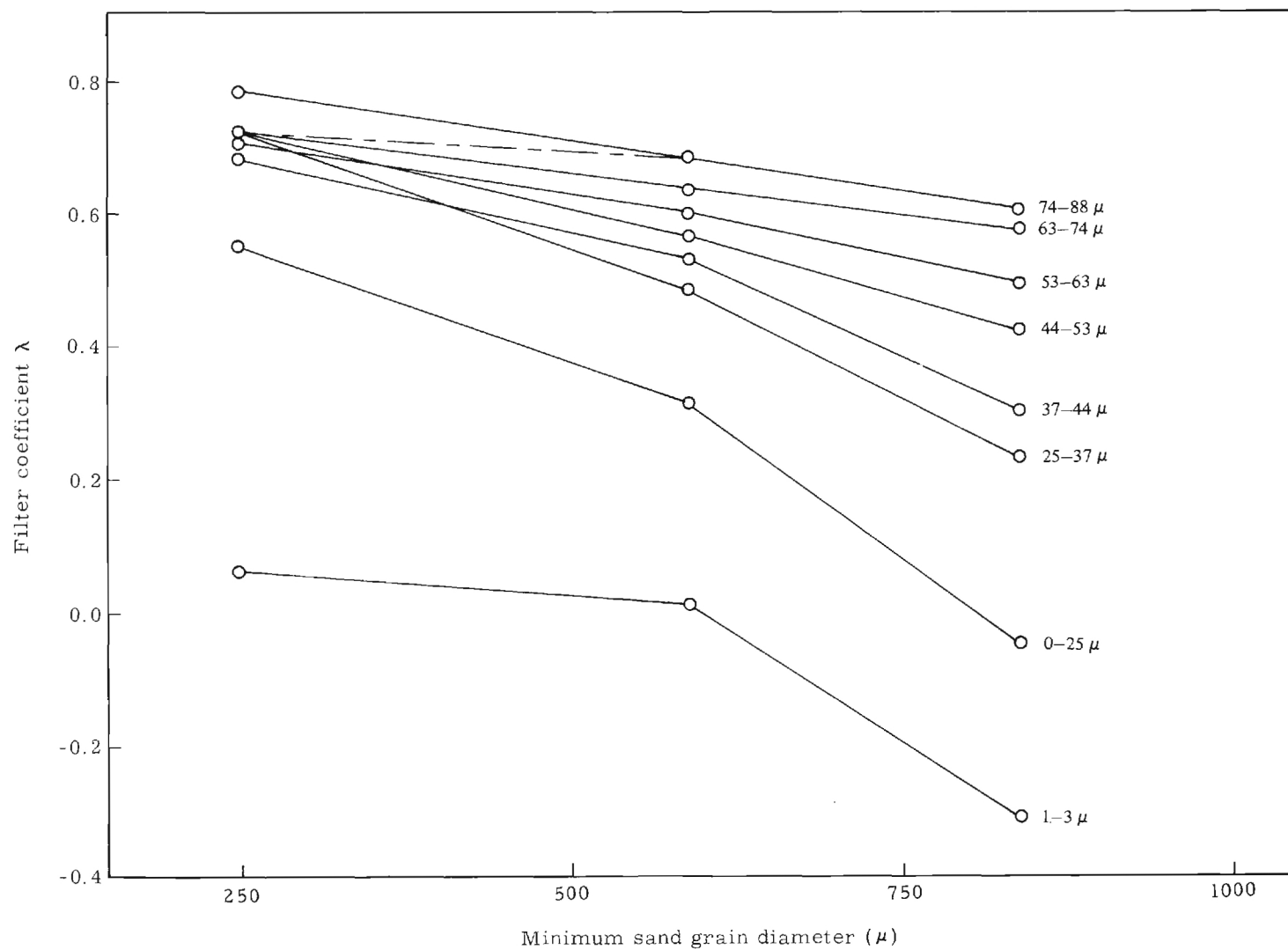


Figure 15. Filter Coefficients at 2 gal/ft²/min for Minimum Sand Grain Diameters

Effect of Flow Rate

An extended series of experiments were performed to determine the relationship of filter coefficient to flow rate, other factors being held constant. The medium was 30-40 mesh sand and flow rates of 0.5, 2.0, and 4.0 gal/ft²/min were selected. Deposition of radiotracer particles in a number of different size fractions was observed at each of the flow rates.

Figure 8 presents in graphic form the filter coefficients determined. Sand of this size should theoretically pass spheres of 65 μ diameter, and a constant coefficient value for all particles larger than 65 μ is to be expected. This supposition is confirmed in that at flow rates of 2 and 4 gal/ft²/min the coefficients are constant in this region. But the coefficients are constant at the 0.5 gal/ft²/min flow rate down to particles of only 30 μ in diameter.

The magnitude of the coefficients indicates that most of the deposited tracer has come to rest on the sand surface, a situation probably due to gravitational sedimentation of the particles. It is concluded that there is insufficient hydrodynamic force at the low flow rate to cause penetration of the bed by any except the smallest particles. The limit of gravitational action would be at zero flow rate, in which case the filter coefficient would be independent of particle size as all particles would simply settle on or very close to the surface of the porous medium.

The shape of the curve for a flow rate of 2 gal/ft²/min is very similar to that for the same rate in 20-30 mesh sand which is shown in Figure 5. Three different slopes can be identified signifying the same general considerations.

For a flow rate of 4 gal/ft²/min the curve exhibits some of the characteristics of the 2 gal/ft²/min curve, but the two are not identical. Above a certain particle size there appears to be no significant penetration of the bed regardless of flow rate. This is certainly logical if the particles are

indeed larger than the openings. For particles small enough to enter the interstices of the medium, the hydrodynamic force of the flowing water serves to force the suspended particles deeper into the bed. This may be due to the change in the shape of the velocity parabola which would tend to increase the flow rate in the forward direction, allowing less time for lateral dispersion into parts of the flow stream where capture can occur.

A second complete run was made at 4 gal/ft²/min and the shape of the curve obtained was in agreement with the initial results. There was, however, a slight displacement of the actual values, but this was probably due to a slight difference in the compaction used in the two sets of experiments. No precise mathematical expression relating velocity, particle size, and filter coefficient can be derived from these data, but it is apparent from Figure 8 that for particles small enough to penetrate the filter medium, the filter coefficient varies inversely with flow rate.

Effects of Dissolved Activity

To compare the action of the filter medium on particulates and dissolved materials, the effects on the filter effluent and the filter medium due to passage of dissolved activity through a filter were investigated. A sudden influx of dissolved radioactivity was caused by pouring a solution of cesium-137 chloride into the influent of the experimental filter, which contained 20-30 mesh sand, in order to simulate the sudden arrival of a slug of radioactivity such as might occur following an upstream accident. The resulting activity was monitored by two Geiger counters, one placed inside the filter bed and one immersed in the filter effluent line. Readings were recorded continuously.

Figure 16 shows the results after the background was subtracted and the data were normalized to a constant area under the curves. Curve A illustrates

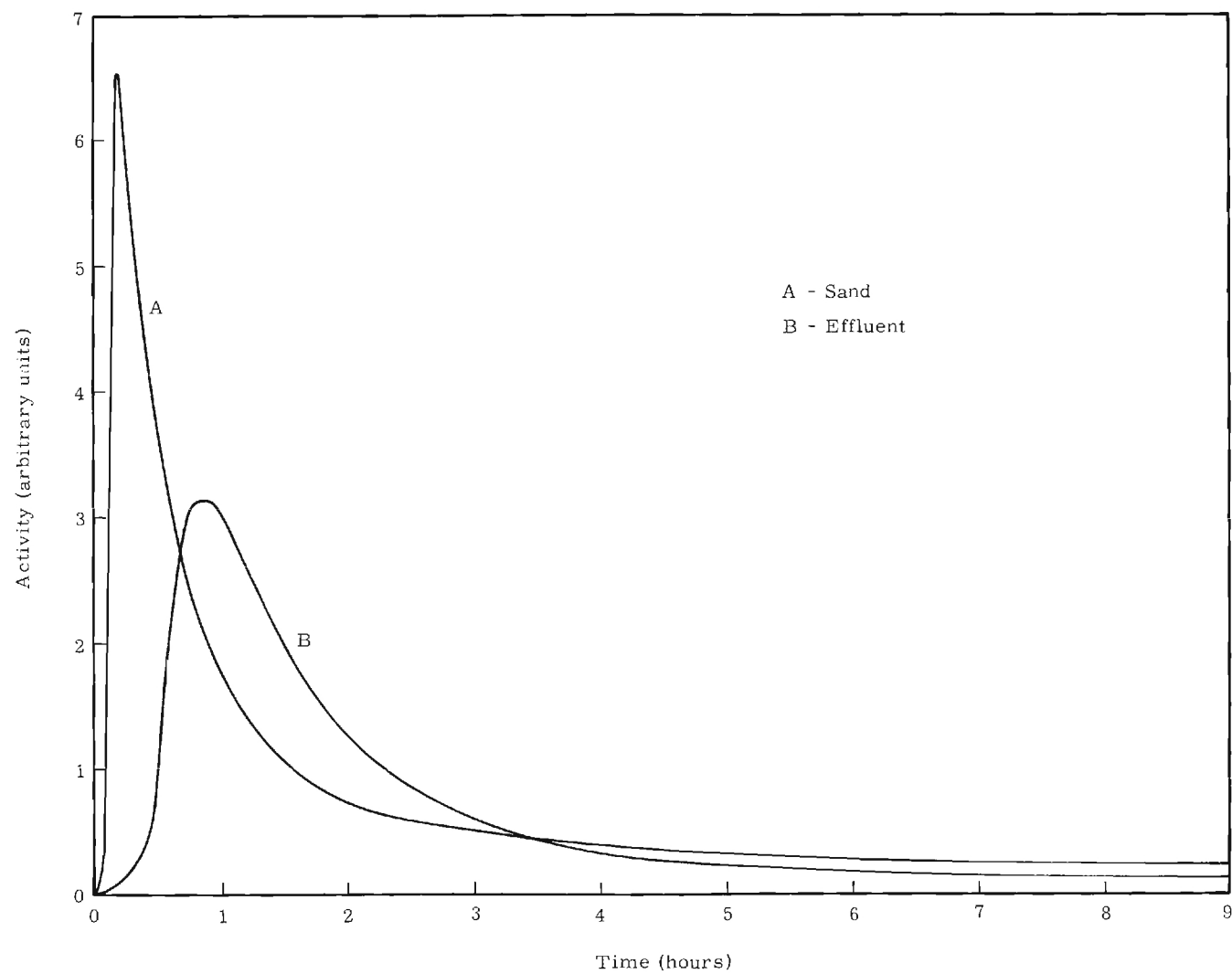


Figure 16. Time-Activity Relationships for Effluent and Filter Bed

the rise and fall of the activity level within the sand bed and curve B gives similar information on the filter effluent. From the initial background level denoted by zero on the activity scale, the activity level in the sand (curve A) is seen to rise very abruptly as the radioisotope begins to enter the sand. A sharp maximum is reached in less than 30 minutes, and thereafter the curve decreases rapidly for a time and then at a more gradual rate. The activity becomes almost constant after six hours.

Activity of the effluent (curve B) follows a somewhat similar pattern, but at a time later than that of the sand. The delay in arrival of activity at the filter exit is due to 1) the time required for liquid to pass through the filter and 2) the capacity of the filter to sorb and retain activity. Sorption by the sand is proportional to concentration of dissolved matter in the liquid, so that a portion of any entering activity will be retained so long as the concentration is increasing and the storage capacity of the sand is not exceeded. If the effluent concentration remains constant, an equilibrium is established and no further changes take place.

The effect of dissolved activity on different bed materials was observed in another test where samples of the filter media were exposed to dissolved activity resulting from the presence of uncalcined labeled particles.

A 1.5 gram sample of 88-105 μ vermiculite was stirred for two hours with a Cs-137 solution, and another sample was treated in the same manner with an I-131 solution. In each case, the particulate matter was then washed repeatedly by stirring with tap water and decanting.

The particles were stirred for 30 minutes with 10 grams of washed Anthra-filt No. 1 in about 500 ml of water. The mixture was then allowed to stand approximately 70 hours. A sample of the supernatant was removed, the Anthra-filt washed free of visible vermiculite particles, and five ml portions of

both the supernatant and the Anthrafil^t were counted in a well-type scintillation crystal.

Under the conditions of this experiment, the activities of the wash waters were very similar and the final count rates of the supernatants were essentially identical. Under equilibrium conditions, however, the Anthrafil^t had sorbed almost 17 times more Cs-137 activity than I-131 activity indicating a strong affinity for cations. Actual values are shown in Table 5.

Table 5. Isotope Sorption by Anthrafil^t
(gross counts per minute)

Sample	Cs-137	I-131
Final wash water	565	157
Supernatant	6,050	5,946
Anthrafil ^t	267,427	16,412

In another experiment, the affinity of the Anthrafil^t for the I-131 was quantitatively determined. A 40 gram sample of washed Anthrafil^t No. 1 was put into a beaker containing about 500 ml of water. Iodine-131 was added, the mixture stirred for an hour and allowed to stand overnight. The supernatant was decanted and the activity of a sample of the Anthrafil^t was determined with the well-crystal scintillation counter. The Anthrafil^t was then washed by vigorous stirring with tap water, allowing it to stand several minutes, and decanting the supernatant. The activity of the Anthrafil^t was measured several times, and the results are shown in Table 6.

Table 6. Activity of Anthrafilt Exposed to I-131
(gross counts per minute)

Before Addition of I-131	135
After Addition of I-131	3,508
After 4 Wash Cycles	1,830
After 7 Wash Cycles	1,189
After 10 Wash Cycles	606

Phase II

Field Study

The radiotracer experiments performed in the laboratory provided valuable information on the theoretical aspects of rapid sand filtration, but it was recognized from the beginning that carefully controlled laboratory conditions are not necessarily identical with actual field conditions. A major difference lay in the nature of the suspended particles: light, fluffy, floc particles in the actual filtration process and denser vermiculite particles in the laboratory. It was therefore decided to experiment with water that had been treated in a full-scale operation and was ready for filtration.

Through the splendid cooperation of the Atlanta Water Works, small-scale filters were installed at the Hemphill Water Treatment Plant. These were located so that water could be piped directly to them from the main line feeding water to the plant filters. Before arriving at the filters this water (known to the cognoscenti as "coag water") has passed from the Chattahoochee river into a reservoir with approximately 10 days detention period. Chemical treatment involves liquid alum, plus lime and carbon as needed. After flocculation

and sedimentation, the turbidity of the water is about 1 ppm. A microscopical examination of this water showed that the majority of particles present were smaller than $4\ \mu$ in diameter, although there was present an occasional agglomerate in the range of 25-40 μ diameter. Figure 17 is a histogram of the observed particle sizes.

Several filter runs were made to evaluate the effectiveness of closely graded sand, anthracite, the regular filter sand being used by the water works, and a combination of sand plus anthracite. The effect of reverse grading of the water works sand was also determined.

A filtration run was made using a portion of the 20-30 mesh sand previously used in the radiotracer study. Head losses throughout the filter were recorded at the beginning of the run and at later times; the results are plotted in Figure 18. Except for the first inch of bed depth, head loss is initially very close to linear throughout the bed. This indicates a very uniform porosity, which means a very uniform size of sand grain.

During the 30 hours of filtration, changes in head loss occurred only in the top three inches; at lower depths the head loss per inch of depth remained constant. This means that only the top three inches were involved in the removal of particulates after 30 hours. It is believed that progressively lower layers are involved as filtration proceeds, but at 46 hours head loss had increased beyond the capacity of the piezometers, and no readings were possible.

Comparison of Filter Composition

Sand removed from a filter at the Hemphill Plant was used⁽⁴⁾ in the comparison of normal versus reverse-graded sand. A portion of this sand was thoroughly mixed by stirring and pouring from one container to another. Alternate scoopfulls of the homogenized sand were then used to fill each of the small filters. As a result of this technique, the particle size distributions

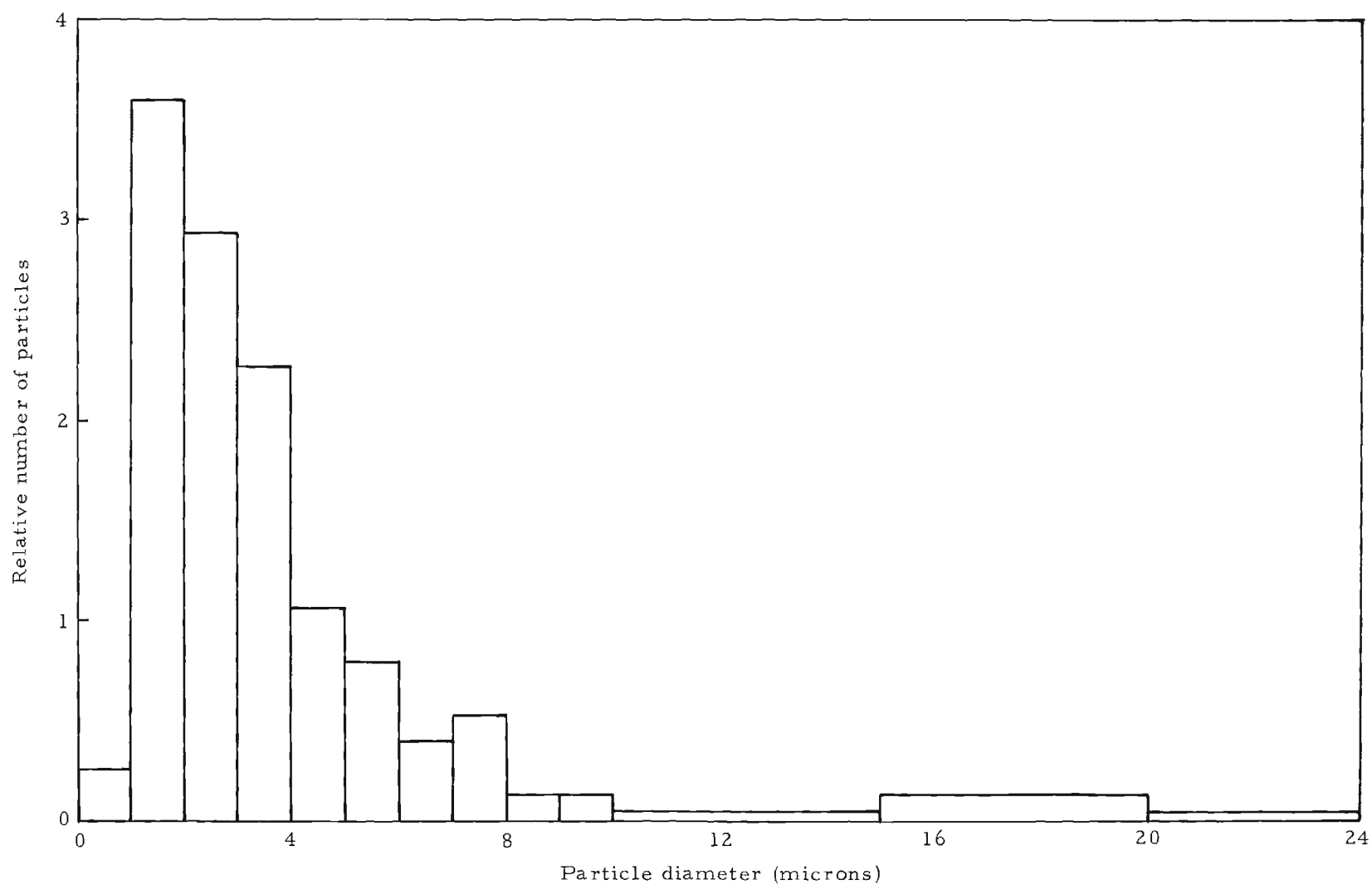


Figure 17. Particle Size Distribution of Atlanta Water Works Filter Influent

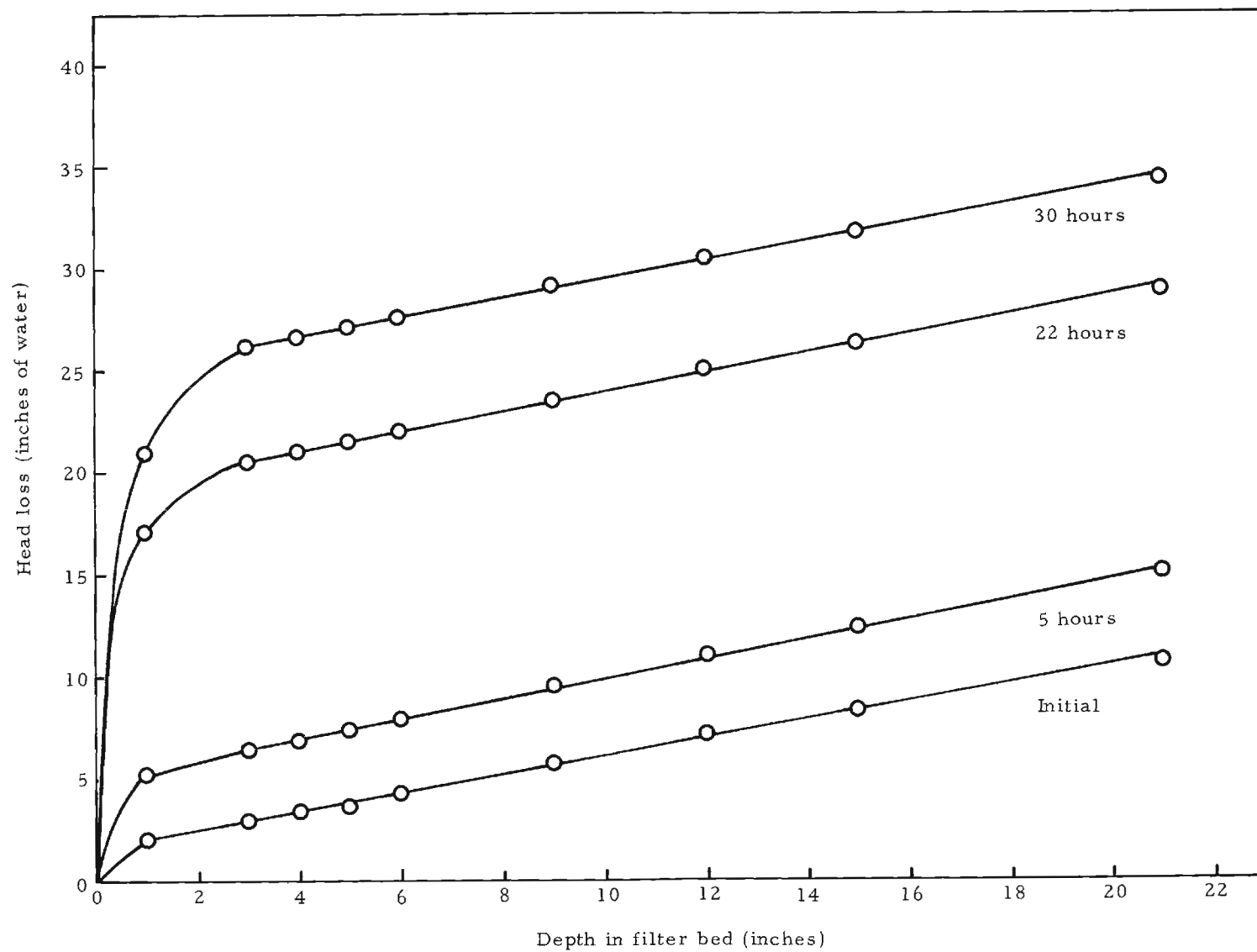


Figure 18. Head Loss in Uniform Sand at Various Times

in the pilot filters were considered identical.

The normally-graded sand was backwashed in the ordinary manner; i.e., water entered the filter at the bottom and exited at the top. Upon completion of backwashing the water was turned off and the sand was compacted by tapping gently on the filter body until the sand surface subsided to a standard level which coincided with the plane of separation at a flanged joint. After this initial compaction, no change in sand surface elevation was ever observed during filtration.

Reverse grading was accomplished in the second filter by first backwashing and compacting the sand in the same manner as in the first filter. Following this, the water level in the filter was lowered until it coincided with the sand surface. The upper portion of the filter was then removed by unbolting the flanged joint, and a 75 mesh screen was placed on top of the sand. The edges of this screen extended into the flange and it was secured by an auxiliary gasket. The upper portion of the filter was then replaced and bolted into position. Water was then admitted from the bottom until the filter was completely filled. Valves at the top and bottom were then closed and the lines to them detached. At this stage the filter medium was held between two screens, and the entire filter could be inverted without disturbing the medium. A set of brackets was provided so that the sand columns in both filters were at the same elevation. Feed and drain lines were attached, and the filter was ready for operation.

Filtration was started at a rate of $2 \text{ gal/ft}^2/\text{min}$. As soon as the piezometers reached equilibrium (a matter of a couple of minutes), the elevation of water in each of them was recorded. Successive readings were made at intervals, and the results plotted. Figures 19 and 20 are for the run which was begun on 11-10-69. Here it is seen that the initial total head loss is greater

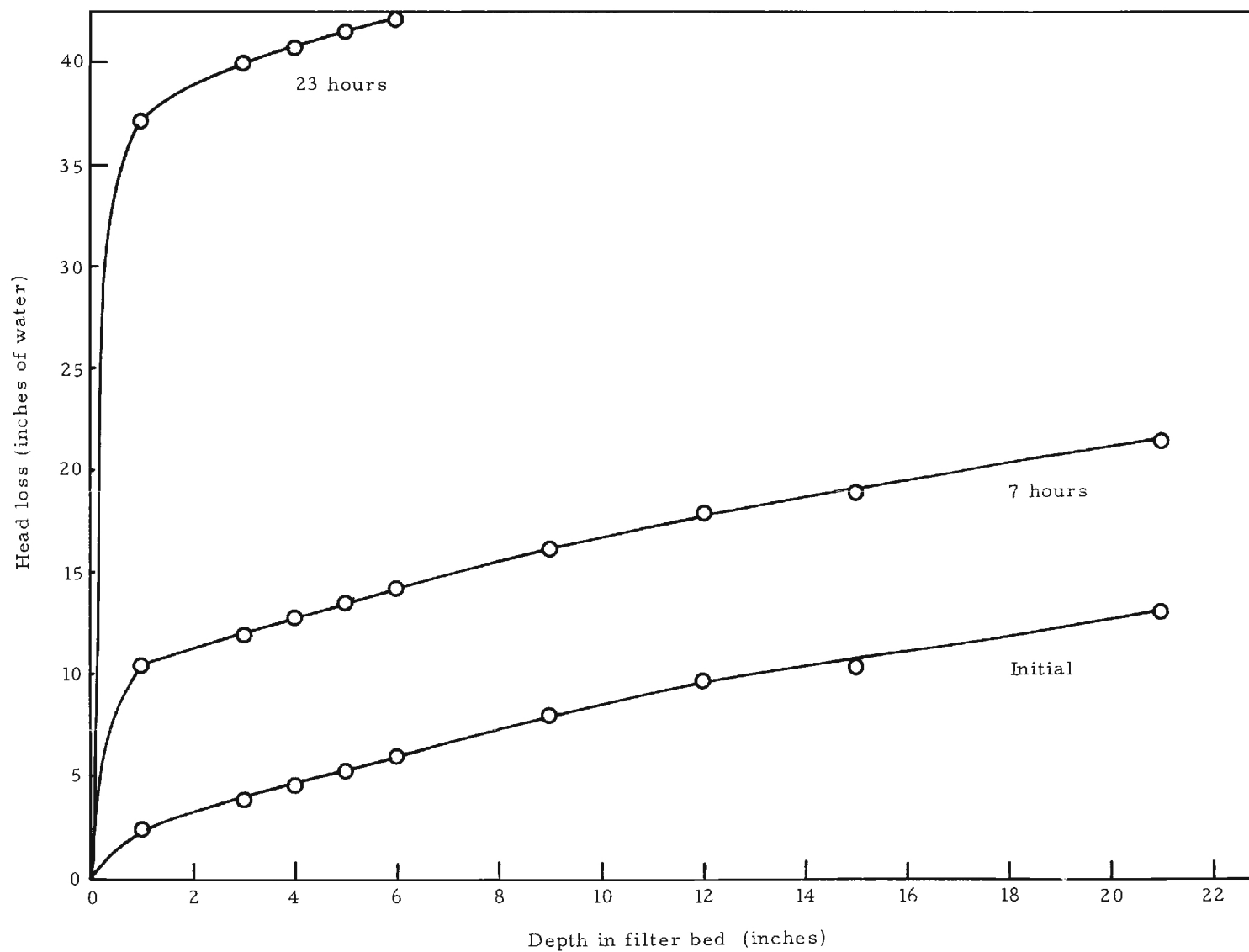


Figure 19. Head Loss in Normally Graded Sand at Various Times

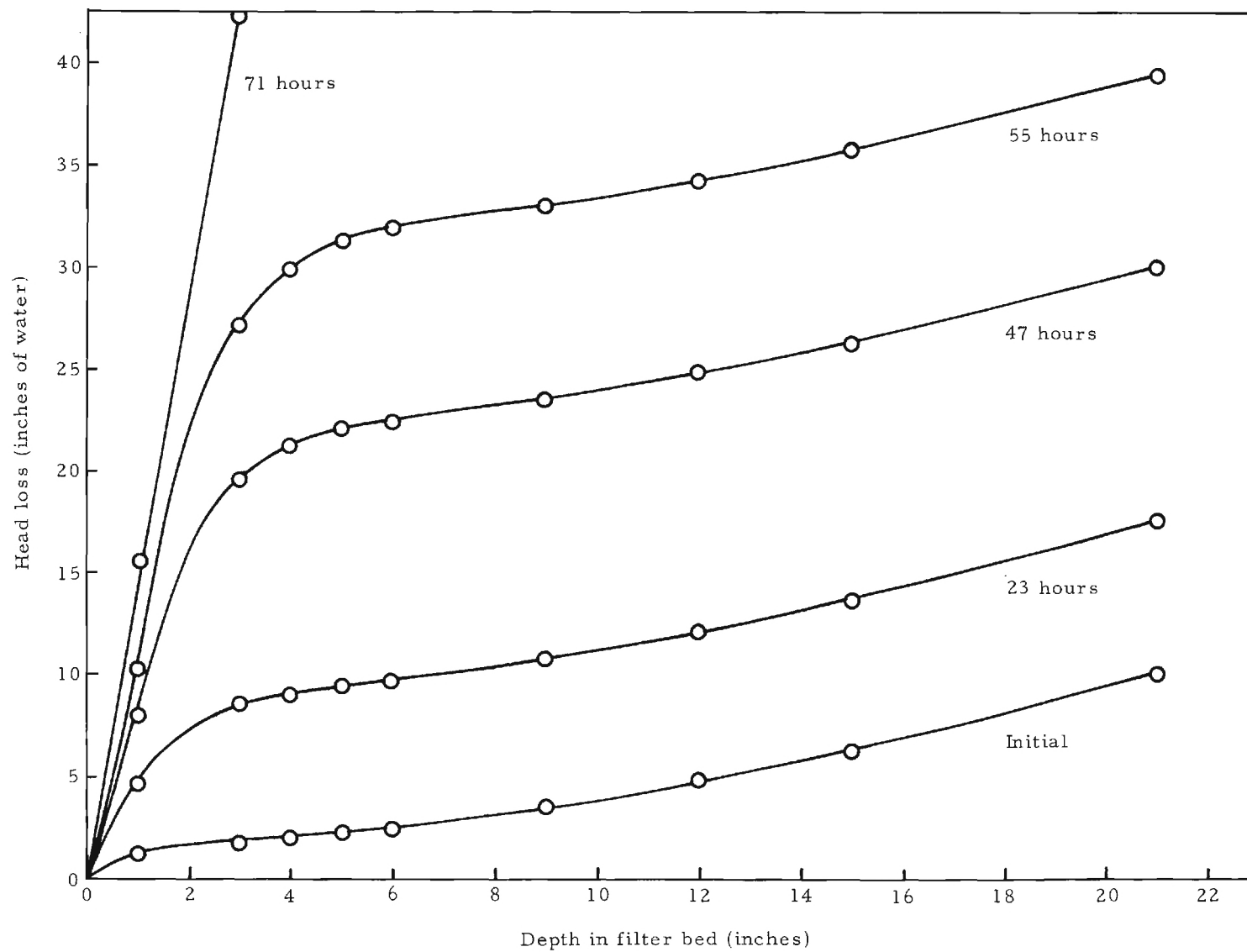


Figure 20. Head Loss in Reverse Graded Sand at Various Times

for normal-graded sand (12 inches) than for the reverse-graded (9 inches). As filtration proceeds, the difference between the two increases. After five hours, for instance, the "normal" loss has risen to 17 inches, while the "reverse" loss has increased only to $10\frac{1}{2}$ inches.

From the standpoint of length of filter runs, it can be seen that the 7 hour head loss of the normal filter is not reached until $30\frac{1}{2}$ hours' operation by the reverse-graded filter. It is not possible to read accurately any losses above 40 inches, and the normal-graded filter exceeded this limit at $22\frac{3}{4}$ hours. This limit had not been reached by the reverse-graded filter when filtration was halted at 54 hours. It is to be noted, however, that the rate of head loss was not constant but increased as filtration proceeded.

It is to be noted also that in the normal-graded filter the loss of head is concentrated in the first inch of sand, whereas in the reverse-graded situation the loss is spread through a greater depth. The actual depth depends on the length of the filter run but is perhaps as much as 6 inches after a run of 54 hours.

From this it is seen that (with reverse-graded sand) a greater proportion of the filter is actively accumulating solids; head loss remains lower because there is no single layer of highly clogged pores to throttle the liquid flow.

The results of this experiment are in accord with those of Oeben, Haines, and Ives⁽⁷⁾ who worked with ferric hydroxide floc. They produced their reverse-graded condition by syphoning or spooning backwashed sand from one filter to another. It is believed that the inversion technique produces more uniformity in gradation of the size of the bed particles, but differences between the methods of achieving reverse grading are apparently of no consequence.

The comparison of 20-30 mesh sand with the Water Works sand graded in its normal fashion is instructive. Figure 18 shows a total head loss at 22 hours

of almost 29 inches in the uniform 20-30 mesh sand. Figure 19 for normal sand at 23 hours reveals a head loss far too large to measure but estimated at about 48 inches. The normally graded sand with the smallest particles on top tends to constrict the filter at the top and to concentrate more of the deposited material and, consequently, more of the head loss at the top of the filter bed. Total head loss is lessened in the uniform bed where the deposited load is spread over a greater depth, and there is less constriction in any given layer.

From the practical standpoint of water filtration, it is obvious that reverse grading of sand would be desirable if it could be accomplished easily. However, there is no obvious method for changing either the gravitational or hydraulic forces which produce particle size classification. The methods used by investigators for small-scale situations are not suitable for commercial applications.

Nevertheless, the information is valuable in that it emphasizes the advantage of having a coarse medium at the top of the bed and finer material at the bottom. This can easily be accomplished by the use of two materials of different density. Sand and Anthrafil are a common combination, and head losses for such a "half and half" filter bed are shown in Figure 21. When the bed is clean (initial condition) it may be seen that head loss per unit depth is slightly greater for the sand (depth greater than 12 inches) than for the Anthrafil (depth less than 10 inches). There is no sharp inflection in the curve as the difference is not markedly different and there is a transition zone between the 10 and 12 inch depths where there is intermingling of the two media.

The sand plus Anthrafil bed is clearly a more efficient arrangement than reverse graded sand alone and is much superior to normally graded sand. Head losses at the six inch bed depth at 23 hours are 42 inches in normally-graded

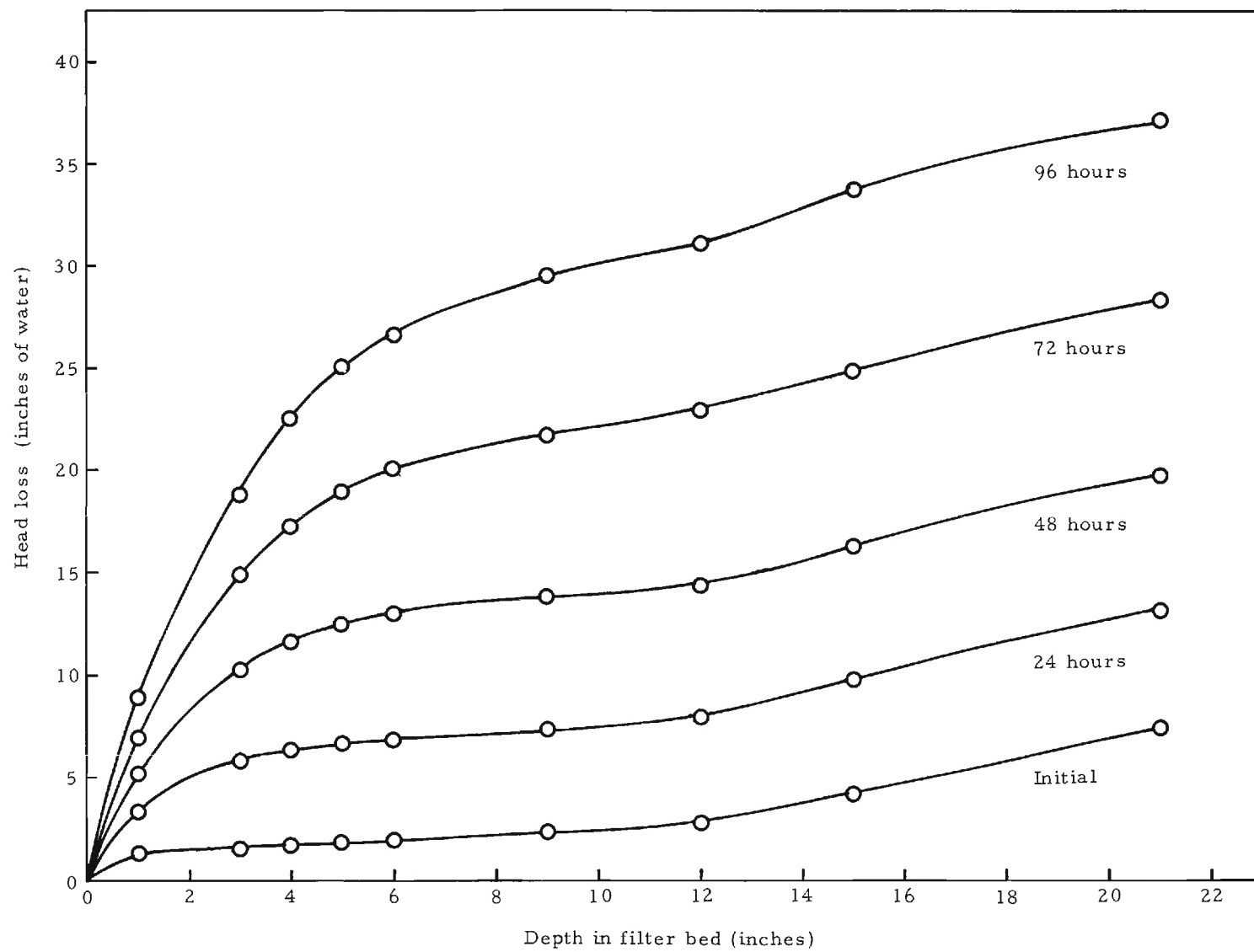


Figure 21. Head Loss in Sand Plus Anthracite at Various Times

sand, $9\frac{1}{2}$ inches in reverse-graded sand, but less than 7 inches in the mixed medium.

To evaluate the situation more fully, a bed consisting of Anthrafilt alone was prepared and tested; Figure 22 summarizes the result. While these data show Anthrafilt to be superior to either normally- or reverse-graded sand alone, the mixed media bed was the best investigated. This is because the loss of head is more linear with depth, indicating that particle deposition is taking place throughout a greater fraction of the bed depth. It is believed that this gives an advantage from the standpoint of the hydraulics involved. If there is a layer where pores are clogged to a high degree, flow through such a layer must be at a higher rate than through subsequent layers where there is more void space. In the limiting case, turbulence could result, but even before the onset of turbulence there could be a deviation in the flow regime away from the strictly laminar condition.

It is to be noted that the total head loss across the entire depth of the bed is somewhat different for Anthrafilt and Anthrafilt-sand. Due to the lower porosity of the sand layer, the initial head loss is higher for the combination but increases at a lower rate; there is no significant difference between the two beds after 96 hours of filtration.

All of the data presented above are based on flow rates of 2 gal/ft²/min. The influent consisted in all cases of the coagulated water that was entering the filters of the 1923 plant of the Hemphill Water Treatment Plant. This flow is monitored continually by personnel of the plant laboratory, and at no time during the experimental runs did the turbidity exceed 1 ppm. The turbidity of the effluent from the experimental filters was typically in the range of 0.01 to 0.05 ppm, and only a few values as high as 0.1 were ever observed. All these turbidities are highly acceptable with respect to the requirements

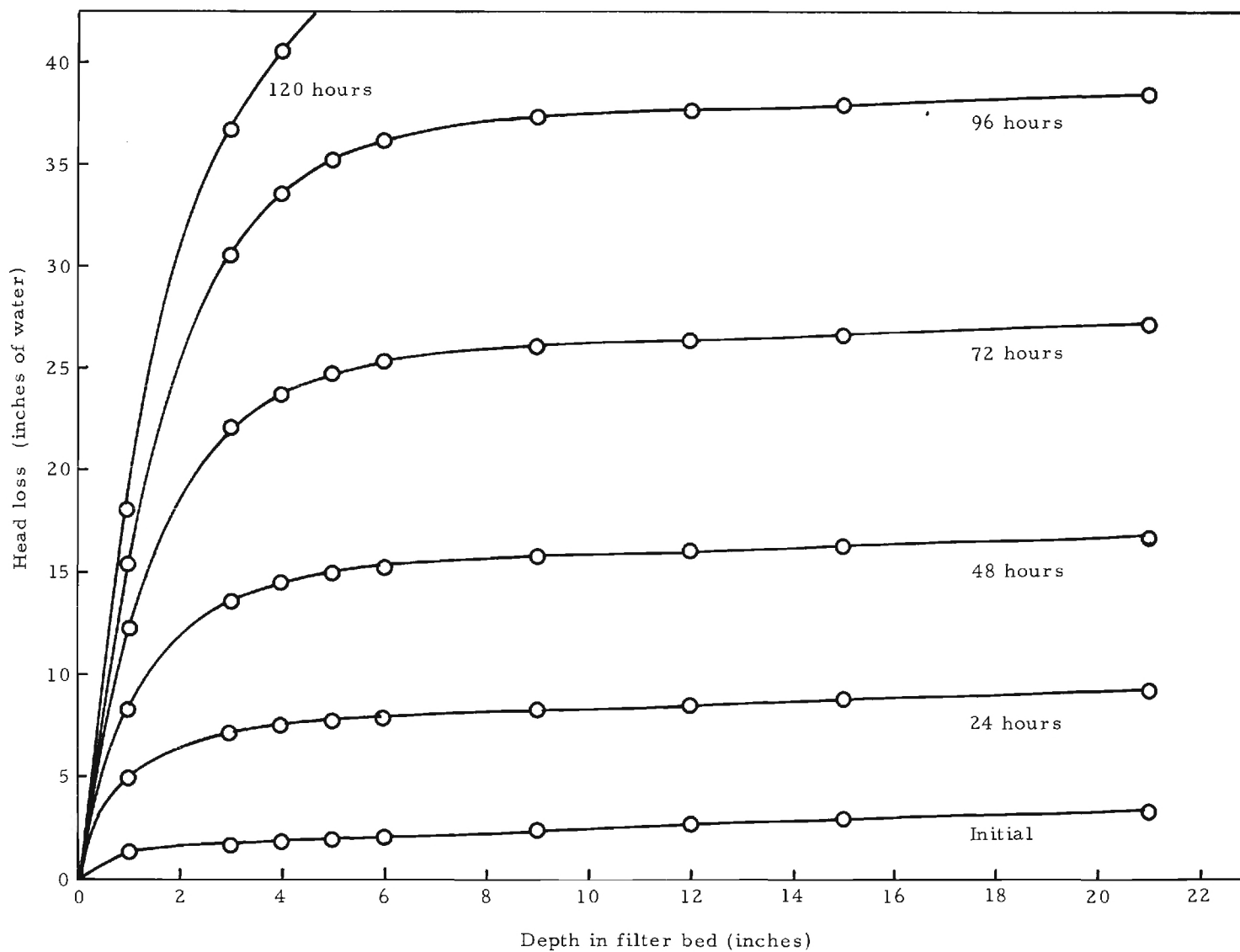


Figure 22. Head Loss in Anthracite at Various Times

of the Drinking Water Standards of 1962, which call for a turbidity not to exceed 10 ppm.

While rate of head loss is an important factor, there are other matters that should be considered in the choice of a filter medium. Traditionally sand filters have been known for their reliability in producing acceptable effluents even when the quality of the influent is poor. This is indeed important, for even in the best regulated water treatment plants equipment failure, sudden fluctuation in raw water quality, or human error can occur with the result that poor quality water reaches the filter. In such a circumstance it is desirable to have a medium capable of withstanding a shock load of high turbidity, and sand is superior to Anthrafilt in this respect.

Based on the studies of sand and Anthrafilt reported here, it is concluded that the most desirable situation is a combination of sand and Anthrafilt. Second is Anthrafilt alone, followed by reverse-graded sand, uniform sand, and then normally-graded sand.

CONCLUSIONS

The theoretical aspects of filtration through a porous medium were emphasized in the initial phase of this study. Through use of the radiotracer technique it was possible to determine the concentration of suspended particles which were deposited at various elevations within the filter. It was clearly shown that deposition occurs in an exponentially decreasing manner. This observation is in agreement with the usual assumption, first noted by Iwasaki,⁽⁸⁾ that filtration may be regarded as a first order reaction.

From a measurement of the concentration of deposited tracer material on the surface and at a given depth in the filter bed, the filter coefficient was calculated. This coefficient has a definite physical significance as a measure of filter operation; it is directly proportional to the percentage of suspended matter removed by each layer of the filter. Filter coefficients were determined for suspended particles of different sizes when they were filtered by uniform sand and anthracite beds. Plots of filter coefficient against size of suspended particle were most useful in interpreting the mechanisms involved in the filtration process. It was shown⁽⁶⁾ that the shape of the curves such as that of Figure 5 is due to the successive predominance of three different mechanisms. The particular mechanism by which a given particle is trapped in a porous medium is determined primarily by the size of the particle.

Physical hindrance traps on the surface those particles which are too large to pass through the interstices of the porous material. Particles very small in relation to the size of the interstices are governed mainly by physico-chemical forces, namely the van der Waals force and the double layer forces. Particles with sizes between these two extremes are trapped through the process of interstitial sieving, which was initially detailed by Hall.⁽⁹⁾

There is a gradual transition between applicable mechanisms; electrical and mechanical forces both may be influencing a given particle, but their relative effectiveness depends on the particle size. The transition is also accentuated experimentally because neither the suspended particles nor the bed media were absolutely uniform in size or shape.

The experimental evidence shows clearly that the mechanical mechanisms are more efficient and should be utilized whenever possible. This implies use of a finer-grained filter medium; however, the head loss would be greater and filter runs shorter. Aside from these very practical considerations, there are logical reasons that suggest that media with lower filter coefficients are actually more effective when all factors are considered.

The ultimate efficiency would involve a filter coefficient of 1, all suspended matter being trapped in the first layer. In this case the top layer would present a zone where interstitial spaces were lessened due to deposited matter and flow would be hindered. Actually it would be preferable to have a somewhat less efficient medium so that appreciable deposition can take place in the lower layers of the medium. Clogging of the medium is then delayed, and filter runs are thereby lengthened.

These conclusions are supported by the experiments of the second phase of this study. Any combination or configuration that spreads the accumulating solids over a greater depth is advantageous from the standpoint of length of filter run. This spreading of the deposit is enhanced by larger grained media and uniformly sized media.

The results obtained here show that reverse-graded sand is superior to normally-graded sand. Grading occurs during backwash, and it is the combination of gravitational and hydraulic forces that produces a bed with the smallest grains at the top and the largest grains at the bottom. It would be advantage-

ous to reverse this order, but no engineering solution to this problem is presently available.

Based on the experimental runs utilizing commercially prepared pre-filter water, a combination of ground anthracite plus sand was found to be the best filter medium tested. Use of the coarser anthracite means less head loss per unit time and therefore potentially longer filter runs. Anthracite alone would be acceptable under ordinary operating conditions, because it lowers turbidity as effectively as sand, but it is considered to be less reliable when there is a sudden worsening of the quality of the filter influent. Such changes in filter influent may be rare, but because of the fallibility of man and machine they do occur.

Cost is an omni-present factor in design, and medium selection may well be influenced by it. Geographical location may have a considerable influence on relative costs due to freight charges involved, but in the final analysis, the designer himself must make a decision concerning the cost-benefit ratio. However, in the absence of compelling influences, it is felt that the optimum filter medium of those tested is the combination of anthracite and sand.

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