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**GEORGIA INSTITUTE OF TECHNOLOGY**
OFFICE OF CONTRACT ADMINISTRATION
**SPONSORED PROJECT INITIATION**

Date: October 15, 1976

Project Title: Nonlinear Systems and Multidimensional Digital Signal Processing

Project No: E-21-601

Co- Project Directors Dr. Edward W. Kamen and Dr. Aubrey M. Bush

Sponsor: U. S. Army Research Office, Research Triangle Park, NC 27706

Agreement Period: From 10/15/76 Until 1/14/78

Type Agreement: Grant No. DAAG29-76-G-0346

Amount: $58,887 ARO
6,077 GIT (E-21-348)
$64,964

Reports Required: Progress Report
Technical (when justified)
Final Report

cc Sam Renneker on RRR

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Contractual Matters
(thru OCA)

Chief, Procurement Office
U. S. Army Research Office
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Research Triangle Park
North Carolina 27706

Defense Priority Rating: None

Assigned to: Electrical Engineering (School/Laboratory)

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# SPONSORED PROJECT TERMINATION

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Date: 2/17/78

Project Title: Nonlinear Systems and Multidimensional Digital Signal Processing

Project No: E-21-601

Project Director: Dr. E.W. Kamen and Dr. A.M. Bush

Sponsor: U.S. Army Research Office; Research Triangle Park, NC 27709

Effective Termination Date: 1/14/78

Clearance of Accounting Charges: 1/14/78

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- X Final Fiscal Report and Closing Documents
- X Final Report of Inventions
- X Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ______________________

NOTE: SEE E-21-B00

Assigned to: Electrical Engineering (School/Laboratory)

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Other ______________________

CA–4 (3/76)

E-21-601

PROGRESS REPORT

1. ARO PROPOSAL NUMBER: 14340 EL

2. PERIOD COVERED BY REPORT: October 15, 1976 through June 30, 1977

3. TITLE OF PROPOSAL: Nonlinear Systems and Multidimensional Digital Signal Processing

4. CONTRACT OR GRANT NUMBER: DAAG29-76-G-0346

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: E. W. Kamen and A. M. Bush

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:
   E. W. Kamen, "Decomposition of two-dimensional transfer functions into one-dimensional components," submitted to IEEE Trans. on Acoustics, Speech, and Signal Processing, March 1977, to be revised.
   E. W. Kamen and R. M. Mersereau, "Exact and approximate decomposability of two-dimensional transfer functions," paper to be given to 20th Midwest Symposium on Circuits and Systems, Lubbock, Texas, August 1977.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
   Mr. Paul Steiner, graduate student

   Dr. C. Y. Chong, Assistant Professor

14340EL

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## BRIEF OUTLINE OF RESEARCH FINDINGS

A new concept of decomposability has been developed for two-dimensional digital filters. A system is said to be decomposable into one-dimensional components if it has a transfer function of the form

$$H(z_1,z_2) = \frac{A(z_1,z_2)}{B_{10}(z_1)B_{01}(z_2)B_{11}(z_1z_2) \;\ldots\; B_{ij}(z_1^iz_2^j) \;\ldots\; B_{MN}(z_1^Mz_2^N)}$$

where $A(z_1,z_2)$ and $B_{ij}(z_1^iz_2^j)$ are polynomials in the complex variables $z_1$ and $z_2$. Such decompositions are of interest because the denominator factors can be treated as one-dimensional functions of the variable $z_{ij} = z_1^iz_2^j$. Note that the class of separable two-dimensional digital filters is a special case. Techniques for determining the existence of low-order decomposable approximations have been developed, and it has been shown that in general, good decomposable approximations require rather high order system functions.

The study of two-dimensional signal processing system functions of the separable and the decomposable class have begun on an experimental basis by examination of the magnitude response function in perspective plots generated on the digital signal processing facility for a variety of system parameters. These plots should prove useful in developing some deeper insights into the types of system magnitude functions one may hope to represent efficiently with those structures.

A search for efficient approximation procedures utilizing decomposable sections as basic structures continues and will be implemented as developed.

A second approach to the representation of two-dimensional digital filters is based upon polynomials and formal power series in one variable with coefficients in an infinite dimensional vector space (e.g. a Hilbert space). A major new observation that has already been obtained in this extremely general framework is that a 2-D digital filter defined over the field of real numbers can be treated as a 1-D digital filter defined over an infinite-dimensional vector space. This observation is of central importance, for it makes it possible to apply the rich theory of 1-D techniques in an infinite-dimensional vector space to the study of 2-D digital filters. Results have been obtained on the construction of such representations in an exact or least-squares sense.

K-21-601

PROGRESS REPORT

1. ARO PROPOSAL NUMBER: 14340 EL

2. PERIOD COVERED BY REPORT: July 1, 1977 through December 31, 1977

3. TITLE OF PROPOSAL: Nonlinear Systems and Multidimensional Digital Signal Processing

4. CONTRACT OR GRANT NUMBER: DAAG29-76-G-0346

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHORS OF REPORT: E. W. Kamen and A. M. Bush

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

E. W. Kamen, "Minimal-order decomposition of two-dimensional transfer functions," to be submitted to _IEEE Transactions on Circuits and Systems_.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT:

E. W. Kamen, co-principal investigator
A. M. Bush, co-principal investigator
C. Y. Chong, assistant professor
P. Steiner, graduate student

# BRIEF OUTLINE OF RESEARCH FINDINGS

The work has centered on the study of systems that can be represented by two-dimensional transfer functions. Such systems include one-dimensional externally-bilinear systems and two-dimensional linear digital filters. New results have been obtained on the representation and realization of these systems. In particular, a canonical realization having minimal order has been constructed for externally-bilinear systems. This appears to be the first solution to the minimal-realization problem in the bilinear case. Furthermore, the canonical form carries over to a class of two-dimensional digital filters, yielding highly efficient designs.

Results have also been obtained on the problem of constructing approximate realizations of two-dimensional sequences. Approximation procedures have been developed that employ one-dimensional techniques (such as the Burrus-Parks method) row-by-row, column-by-column, and diagonal-by-diagonal. The performance of these procedures is now under study.

Work has also begun on the application of the theory to the problem of modelling one-dimensional and two-dimensional nonstationary random processes. The idea is to utilize linear and bilinear operations in the computation of autocorrelation functions, which hopefully will lead to fast algorithms for the computation of model parameters. Part of this work is based on the recent observation (made by T. P. Barnwell) that windowing operations can be implemented using bilinear processors.

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FINAL REPORT

Contract No.: DAAG29-76-G-0346

Title: "Nonlinear Systems and Multidimensional Digital Signal Processing"

Principal Investigator: E. W. Kamen and A. M. Bush

The research has centered on the study of, and the relationship between, two-dimensional linear digital filters and one-dimensional externally-bilinear systems. Many new results have been obtained in the areas of representation and realization theory. In particular, a canonical realization having minimal order has been constructed for externally-bilinear systems. This is the first, and so far the only, solution to the minimal-realization problem in the bilinear case. Furthermore, the canonical form carries over to a class of two-dimensional digital filters, yielding highly efficient designs. Additional results include algebraic criteria for exact realizability and procedures for the construction of approximate realizations.