NATIONAL SCIENCE FOUNDATION
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FINAL PROJECT REPORT
NSF FORM 98A

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PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology Atlanta, Georgia 30332	Theoretical Physics	PHY 7921541
	4. Award Period From 1/15/80 To 6/30/83	5. Cumulative Award Amount $43,678

6. Project Title

Fluctuation Phenomena in Thermal Physics

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

During the period of this grant, 10 papers were published and 3 Ph.D. theses were produced. Some of the work concerned hydrodynamic fluctuations away from equilibrium and light scattering measurements for such processes. Other work dealt with coupled rotational and translational diffusion of molecules in liquids. The phenomenon of long time tails was studied and an analysis of computer simulations, theory, and experiments related to this phenomenon was published. Mathematical methods for the study of multiplicative stochastic processes were developed. These processes arise in the study of laser noise problems for which a new theory has been developed. The attached report contains greater detail about each of these topics.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (√)	TO BE FURNISHED SEPARATELY TO PROGRAM: Approx. Date
a. Abstracts of Theses		X 3			
b. Publication Citations				√ 10	Oct. 1983
c. Data on Scientific Collaborators					
d. Information on Inventions					
e. Technical Description of Project and Results		X			
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Ronald F. Fox		Oct. 18, 1983

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Annual Report for NSF Grant PHY-7921541 by Ronald F. Fox

"Fluctuation Phenomena in Thermal Physics"

This report covers the third and final year of activity under this grant.

In December, 1982, a graduate student, Barry J. Cown, concluded hig Ph.D. dissertation: "Stochastic Near-Field Theory and Techniques for Wideband Electromagnetic Emitters of In-band and Out-of-band Frequencies", under my supervision. Dr. Cown is an applied physicist working on radar problems at the Engineering Experiment Station here at Georgia Tech. This work represents a very applied utilization of my expertise in stochastic processes.

Another graduate student, Byron Burel, who will earn his Ph.D. in June, 1983, has concluded his studies of light scattering from a fluid in a thermal gradient. These studies were designed to test a theory for hydrodynamic fluctuations in a fluid far from thermal equilibrium. The fluid is driven to a steady state by imposition of a thermal gradient. The thermal gradient induces spatial dependence in all fluid properties, including the dissipative parameters such as viscosity and heat conductivity. Cohen, Dorfman, and Kirkpatrick have used kinetic theory to show that a major enhancement of Rayleigh scattering is to be expected under appropriate conditions. This is to be contrasted with the earlier work which showed the modification of the Brillouin side bands under the same conditions, but to a much less detectable level. I reviewed the Brillouin situation in the paper: "Testing Theories of Nonequilibrium Processes with Light-Scattering Techniques", J. Phys. Chem. 86, 2812 (1982). Recently, Ronis and Procaccia confirmed the work of Cohen, Dorfman and Kirkpatrick for the Rayleigh line. Both groups, however, have chosen a situation in which the spatial dependence of the transport coefficients can be ignored. Burel and I have attacked the much harder problem where this dependence is not ignorable.

The problem of hydrodynamic fluctuations away from equilibrium in the non-linear regime continues to provide debate among theorists. My paper: "Stress-Strain Fluctuations in Non-linear Hydrodynamics", Physica 112A, 505 (1982) discusses the approach of Bedeaux and Mazur which I have criticized.

Having decided that hydrodynamics may not be the best context in which to test non-equilibrium fluctuation theory, work commenced on dye laser fluctuations. This context simultaneously involves non-equilibrium fluctuations, multiplicative stochastic processes, and non-Markovian processes. In the last regard, a paper: "Correlation Time Expansion for Non-Markovian, Gaussian, Stochastic Processes", has been accepted for publication by Physics Letters. Collaboration with Raj Roy, an experimentalist who recently joined our School of Physics provides added impetus to this work on laser noise.

Two papers appeared from Ph.D. work of Ulrich Steiger, who graduated two years ago. They were: "Coupled Translation and Rotational Diffusion in Liquids", J. Math. Phys. 23, 296 (1982), and "Boson Operator Representation of Brownian Motion", J. Math Phys. 23, 1678 (1982). This work provides a remarkable representation of diffusion processes in quantum field theory language. The objective of these studies is a better understanding of diffusion controlled reactions in liquid solvents.

My program of study of correlation functions using cumulant methods lead to a direct confrontation of the question of long-time tails. In June, 1982, I attended a conference on Nonlinear Phenomena in Fluids at Boulder, Colorado and gave a talk: "The Long Time Tail Conumdrum in Nonequilibrium Statistical Mechanics", to appear in Physica A. I addressed the computer simulation, theoretical, and experimental bases for long time tails in diffusion processes. I enunciated a variety of qualms regarding the firmness of each type of evidence. A much more detailed account of my objections: "Long Time Tails and Diffusion", will soon appear in Phys. Rev. A. Three incidental papers were also produced during this period of research. "The Ideal Gas and the Second Law of Thermodynamics", Am. J. Phys. 50, 804 (1982), presented my thoughts after my encounter with I. Prigogine in 1981. "Elementary Analysis of Electric Dipole Transitions Induced by Semi-Classical $\vec{A}\cdot\vec{p}$ and $\vec{E}\cdot\vec{r}$ Perturbations", written with Bill Harter, is submitted to Am. J. Phys. It clears up a long standing, published confusion about $\vec{A}\cdot\vec{p}$ versus $\vec{E}\cdot\vec{r}$. The resolution involves a gauge transformation, originally devised by Maria Mayer in 1931! This paper is relevant in multiphoton processes in molecules. The third paper is a remark about a paper recently published in Phys. Rev. Letters. My paper, with H. A. Gersch: "Critique of 'Quantum Statistics for Distinguishable Particles'", shows that the paper critiqued does not achieve its claimed objective, the establishment of quantum statistics using an argument based on distinguishable particles, but instead contains a blunder.

Overall the year has proved productive. Research activity has shifted towards quantum noise problems.

"Analysis of Coupled Translational and Rotational Diffusion Using Operator Calculus," by Ulrich R. Steiger, Ph.D. 1981.

SUMMARY

The equations for coupled translational and rotational diffusion of asymmetric molecules immersed in a fluid are obtained. The method used begins with the Kramers-Liouville equation and leads to the generalized Smoluchowski equation for diffusion in the presence of potentials. Both external potentials and intermolecular potentials are considered. The contraction of the description from the Kramers-Liouville equation to the Smoluchowski equation is achieved by using a combination of operator calculus and cumulants. Explicit solutions of these equations are obtained in the two-dimensional case. The formalism also allows the calculation of corrections to the generalized Smoluchowski equation. Smoluchowski's result is precisely the second cumulant, in the cumulant expansion.

The next non-vanishing term, the fourth cumulant, leads to diffusion equations with position dependent diffusion coefficients. The higher order cumulants lead to evolution equations for the reduced probability density which contain partial derivatives of order m with $m \geq 3$. Explicit expressions are given up to the sixth order in the cumulant expansion for translation diffusion. From a practical point of view, this formalism is very useful because partial differential equations can be solved numerically by using a finite element calculation.

The contraction of the Liouville-Kramers description into the Smoluchowski description is achieved by using a creation-destruction operator representation. In an appropriately defined inner product space, these operators possess the operator algebra of boson operators in quantum field theory. The discovery of this representation for the Liouville-Kramers description including rotational effects greatly facilitated obtaining the Smoluchowski contraction

"Stochastic Near-Field Theory and Techniques for Wideband Electromagnetic Emitters at In-Band and Out-of-Band Frequencies," by Barry J. Cown, Ph.D. 1982.

SUMMARY

Antenna systems play an important role in the defense of both the civilian and military populations against incoming threat missiles. Typical antenna installations aboard surface ships or at Army field sites may have various radar, navigation, and communication antenna systems located within a relatively small area. The scattering and coupling of electromagnetic energy among the various antennas can cause severe degradations in the ability of the antenna systems to perform their designated functions.

While all of the antenna systems are important, the directive microwave radar antenna systems are crucial. These systems have the tasks of detecting and tracking incoming missiles and, in some installations, they also guide the defensive surface-to-air missiles to the incoming threat missiles. Reliable detection and accurate tracking of a target can be hindered, or even prevented altogether, by the coupling of unwanted signals among antennas operating in the same frequency band (in-band coupling) or different frequency band (in-band to out-of-band coupling or out-of-band to out-of-band coupling). Further, the electromagnetic energy scattered by the antennas, the support structures, and other scattering objects can result in reduced detection range, tracking errors, and elevated sidelobe radiation. The increased sidelobe radiation can render the installation more susceptible to enemy anti-radiation missiles (ARM) which depend on high average sidelobe radiation levels for detection of and guidance to their target.

The foregoing considerations make it imperative that the radar antenna systems function in a reliable, predictable, and electromagnetically compatible manner (compatible implying the absence of "too much" coupling) if they are to be electromagnetically effective against threat missiles. Hence, electromagnetic compatibility (EMC) and electromagnetic effectiveness (EME) are issues of vital concern. Of course, the analysis and design (or re-design) of real-world antenna installations to achieve optimum EMC and EME is a very complex problem area in applied physics, even if one were to consider electromagnetic coupling and scattering at only a single in-band frequency. However, the modern trend is toward operation over a wide range of frequencies, i.e., toward wideband continuous wave (cw) or pulsed systems. In addition, the ability to predict and control the effects of radiation at

out-of-band frequencies has also become very important as the sensitivity and performance requirements of modern radar systems have increased.

An accurate electromagnetic coupling and scattering analysis of cosited antenna systems for a given operating in-band or out-of-band frequency requires, as the first step, a knowledge of the nominal, or "clear-site", antenna electric field pattern for each antenna that would be obtained in the absence of other antennas or scattering obstacles. The most accurate patterns are obtained via the planar near-field measurement technique whereby the antenna's electric field pattern is computed numerically from electric field data obtained on a plane located within the near-field, or "Fresnel zone", of the antenna. However, this technique was developed for measuring very stable, single mode antennas at a single operating frequency. Consequently, the application of the near-field technique to measure wideband in-band and out-of-band antennas should be investigated to identify and, if possible, to overcome special problems that can arise. For example, it is known from experiments and theoretical considerations that out-of-band antenna patterns can exhibit erratic, seemingly random behavior. Thus, it is anticipated that stochastic theory and analysis will be needed in order to characterize out-of-band antenna patterns properly and efficiently.

The second step in the analysis of coupling and scattering at a single frequency requires the use of the clear-site antenna patterns to compute coupling and scattering based on either the Plane Wave Spectrum (PWS) analysis technique or other established electromagnetic analysis techniques. Thus, the extension to wideband in-band and out-of-band frequencies needs to be studied. Again, a stochastic technique is indicated for analyzing in-band to out-of-band coupling, out-of-band to out-of-band coupling, or out-of-band scattering.

Advances in the state-of-the-art of wideband antenna analysis can be achieved through a basic theoretical and numerical research effort. Accordingly, theoretical and numerical analyses were performed to study the application of near-field theory and techniques to characterize the radiation and coupling characteristics of wideband, in-band and out-of-band pulsed or cw radiating systems. Specifically, stochastic theory and equations were developed for characterizing the radiation patterns of wideband cw or pulsed antennas over both in-band and out-of-band frequency intervals from measured data collected via near-field measurement techniques. The results are applicable to either phased array or reflector antennas. Three analytical techniques for analyzing the in-band and out-of-band coupling between pairs of cosited antennas were studied. The three techniques are (1) the Plane Wave Spectrum (PWS), (2) the Spherical Wave Spectrum (SWS), and (3) Geometrical Theory of Diffraction (GTD). The existing theory and equations that are applicable to selected common waveguide components under normal in-band operation were extended to describe wideband out-of-band responses. Also, the theory and equations were formulated for computing the higher-order mode coefficients at the aperture of a waveguide radiating element from a knowledge of the measured far-field electric field of the radiating element when surrounded by a large conducting "ground" plane. Equations for describing the effects of near-field obstacles located in the antennas' forward half-plane on the performance of a wideband cw or pulsed antenna were derived via the Plane Wave Spectrum (PWS) analysis technique. The resultant statistical average pattern versus frequency is expressed explicitly in terms of the antenna system mode excitation statistical parameters.

"Fluctuations in a Nonequilibrium Steady State: Light Scattering from a Thermal Gradient," by Byron L. Burel, Ph.D. 1983.

SUMMARY

Near equilibrium fluctuation theory is extended into the nonequilibrium regime through the use of the postulate of local equilibrium, and the resulting nonequilibrium fluctuation theory is applied to a hydrodynamic system in which a steady state thermal gradient is present. This approach to nonequilibrium fluctuation theory is applied in an experimentally testable context to light scattering from water. In particular, the effects of the temperature dependence of the hydrodynamic transport coefficients upon the intensity of Rayleigh scattering are investigated.

This calculation is accomplished by using Fourier series to reduce the hydrodynamic equations to matrix form. The resulting matrix equation is solved for the mass density fluctuations, and the mass density autocorrelation function is then computed. The connection between the mass density autocorrelation function and the structure function describing the scattered light intensity is established, and the Rayleigh intensity is computed.

The result of this calculation indicates that the presence of a thermal gradient induces an enhancement of the Rayleigh intensity which may be a substantial fraction of the corresponding equilibrium intensity.