BROADBAND IMPEDANCE MATCHING

OF A LOOP ANTENNA

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SUMMARY

In the frequency range of 1-30 Mc, many types of receivers and measurement devices use small loop antennas. These antennas are inefficient in this frequency range because of their low output, but they have characteristics which make them desirable for some applications. The loops are physically small, which makes them convenient for manipulation, and they have nulls in their response pattern which allow them to be used for direction finding applications.

This report develops a technique for matching the loop impedance to a receiver input over a wide band of frequencies without the need for tuning at each frequency. The technique consists of incorporating the antenna parameters as the leading elements of a filter network.

Experimental verification of the technique is given showing that the technique is useful in the frequency range of 1-30 Mc. A theoretical extension of this technique to other applications is also discussed.

CHAPTER I

INTRODUCTION

Definition of the Problem

In a great many applications, it is desirable to use an antenna that is physically small. Often, the frequency range of interest is far below the resonant frequency of the antenna. Examples of this type of operation are many, two of these being the high frequency, 1-30 Mc, loop direction finder and high frequency field intensity instruments. A typical HF loop direction finder uses a small single turn loop with a diameter of approximately 12 inches. The field intensity meter can use several antennas, among which is also the single turn 12 inch loop. For this reason, a similar antenna has been selected for this investigation. The resonant frequency of the 12 inch loop antenna is approximately 200 Mc. When used in the 1-30 Mc range, it is very inefficient. That is, the induced voltage is low, and it is very difficult to match the antenna to a receiver to obtain the maximum power transfer. This results in very low sensitivity. Since the induced voltage cannot be changed without changing the antenna design, any attempt at improvement must be directed toward obtaining a better match between the antenna and the receiver. The optimum match is considered to be that which transfers the entire induced voltage of the loop to the receiver input at all frequencies. Since this optimum is unobtainable practically, two other approaches are investigated: (1) transfer of as much of the induced voltage as possible over a limited band of frequencies, and (2) allowance of some ripples in the pass band in order to obtain a somewhat wider frequency range.

Previous Approaches to the Problem

Many types of approaches have been tried to solve the loop antenna impedance matching problem in the past. These have met with varying degrees of success. Several of these approaches are in use today, the two most popular being the capacitive antenna trimmer and the impedance transformer.

The capacitive antenna trimmer in its simpliest form consists of a variable capacitor placed in series with the antenna. In operation, the trimmer is adjusted to resonate with the antenna inductance for the best power transfer. The receiver S meter is normally used as an indicator or, if this is not available, the trimmer is tuned for maximum audio response.

The impedance transformer is simply a transformer used to more closely match the antenna impedance to the receiver input by transforming the antenna impedance level to one that is closer to the receiver input. Both conventional transformers and autotransformers are used for this purpose.

In practice, these two systems are used in the same receiver. Several transformers provide a closer match over the frequency range of interest. The transformers are usually designed to operate over a one-octave band of frequencies or one receiver band. The transformer switching is done in conjunction with the band switching, hence an extra switch is not needed. A trimmer capacitor is then placed in series with the transformer secondaries. An example of this type system using both transformers and a trimmer capacitor is shown in Figure 1.

Purpose of the Research

The purpose of the research described is to develop and evaluate a different technqiue for matching the loop antenna to the receiver input over a broad band of frequencies. The procedure consists basically of designing a filter that is flat to a given tolerance over as much of the frequency band as possible and incorporating the antenna parameters into the filter as the first, or leading, element. The desired filter response is one that will transfer the entire voltage induced in the loop to the receiver terminals over the frequency range of interest. Since this will in general not be possible, it will be shown that the desired filter response can be approximated to within a given tolerance. The frequency range of interest to this research is 1 to 30 Mc, and the filters are designed on that basis.

Review of Literature

A review of literature concerned with applications of the technique described here has yielded one report published by the Naval Research Laboratory[1] which discusses use of a loop antenna with an airborne UHF direction finder in the 200-400 Mc frequency range. In this application, the loop parameters were used as the first elements in a bandpass filter; however, the filter parameters were obtained using transmission line constants rather than the lumped constants used in the present research.

Other than this report no literature has been found dealing with the present or related technqiues.

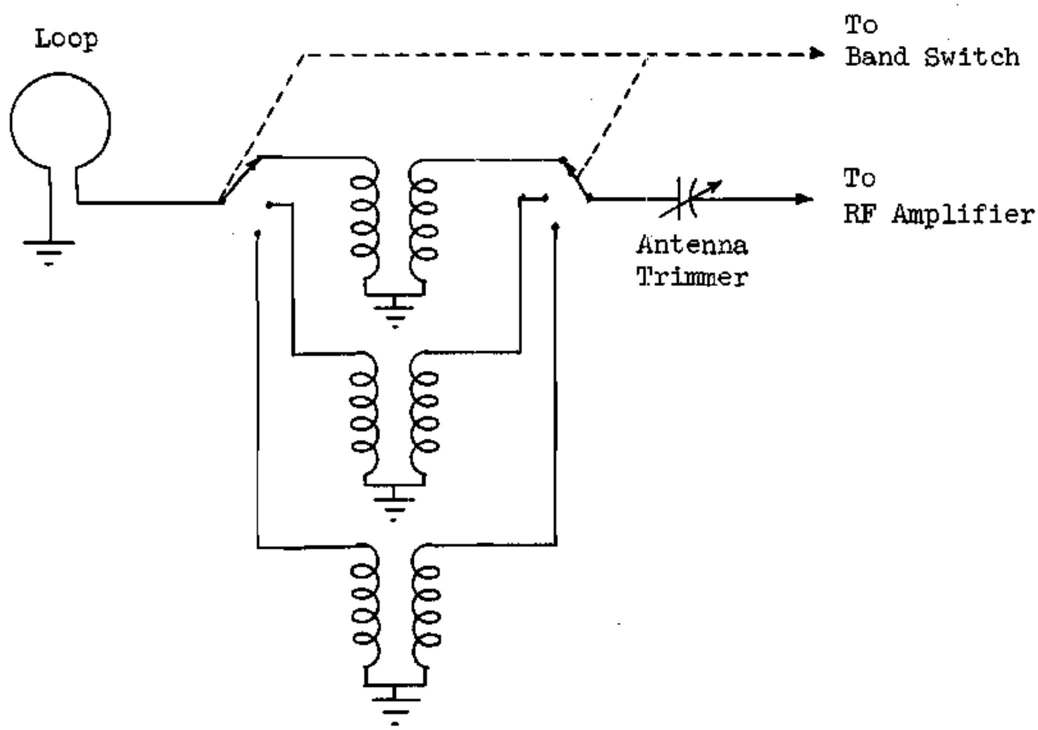


Figure 1. Typical Antenna Input Circuit for a Communications Receiver.

CHAPTER II

PROCEDURE

Antenna Parameters

Before any attempt at improving the impedance matching between the antenna and the receiver input can be made, the characteristics of both the antenna and receiver input must be known. For the purpose of this research, the receiver input will be assumed to be 50 ohms with no reactive component. This value was chosen since it appears to be the most predominant value used as a design goal. Also, the measurements become much simpler in the experimental phase of this research, since most of the instruments, cables, attenuators, etc., are designed for a 50 ohm system. This choice does not rule out using this technique with other impedances. Experimental verification of the procedure using a 50 ohm system will be valid for other impedance levels as well.

The schematic diagram shown in Figure 2 represents the loop antenna parameters with the simplifying assumption that the resistance,

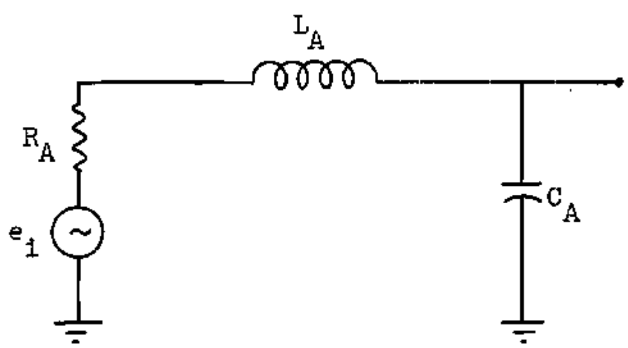


Figure 2. Loop Antenna Schematic.

inductance, and shunt capacitance can be considered as lumped constants. The induced voltage, e_i, is also considered lumped although it is actually induced over the entire loop. The three parameters, resistance, inductance, and shunt capacitance are needed, together with their behavior with frequency before the networks can be designed.

By using a grid dip meter, the antenna was found to have a self resonance at 185 Mc. Using an inductance-capacitance meter, the effective low frequency, 1 kc, inductance was found to be 1 microhenry. Calculating the shunt capacitance from these two measurments gives a value of 0.75 picofarad. This is sufficiently small not to influence the low frequency inductance measurement. The effective resistance is composed of two parts, the ohmic or copper resistance and the radiation resistance. The copper resistance is very small, being that of 3.2 feet of approximately No. 12 copper wire. Terman[2] states that the radiation resistance of a small loop antenna is

$$R_r = 3.12\,(10^4)\left(N\frac{A}{\lambda^2}\right)^2 \text{ ohms,} \tag{1}$$

where

N = number of turns on the loop,

A = area of the loop in square meters,

λ = wavelength in meters.

For a one turn loop with a 12 inch diameter, Equation (1) reduces to

$$R_r = 5.53(10^{-7})f^4 \text{ ohms} \quad (f \text{ in Mc}). \tag{2}$$

At 1 Mc this is simply $5.53(10^{-7})$ ohms and at 30 Mc it is 0.80 ohms. Impedance bridge measurements verify that the total effective resistance remains less than 1 ohm over the entire frequency range of interest.

For the purpose of this research, the antenna will be assumed to act as a 1 microhenry inductor in series with the induced voltage as shown in Figure 3. Impedance bridge measurements also verify the validity of this assumption over the frequencies of interest here. Figure 3

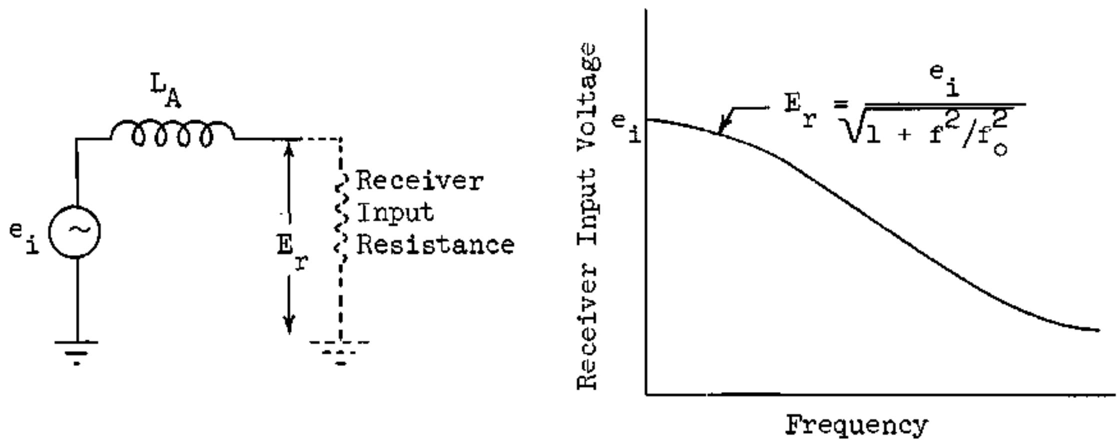


Figure 3. Simplified Loop Antenna Characteristics.

reflects this assumption and shows the simplified antenna schematic together with its transfer characteristics.

Network Design

Using the equivalent circuit shown in Figure 3, networks can be designed and experimental verifications made. When connected to the receiver, the resulting inductance-resistance network formed by the antenna and receiver input resistance is a simple lowpass filter. An improvement in the response at higher frequencies can be obtained by converting this to a simple bandpass filter as shown in Figure 4, where ω_o is the desired center frequency. It is seen that the simple bandpass filter gives an improvement in the high frequency response; however, the

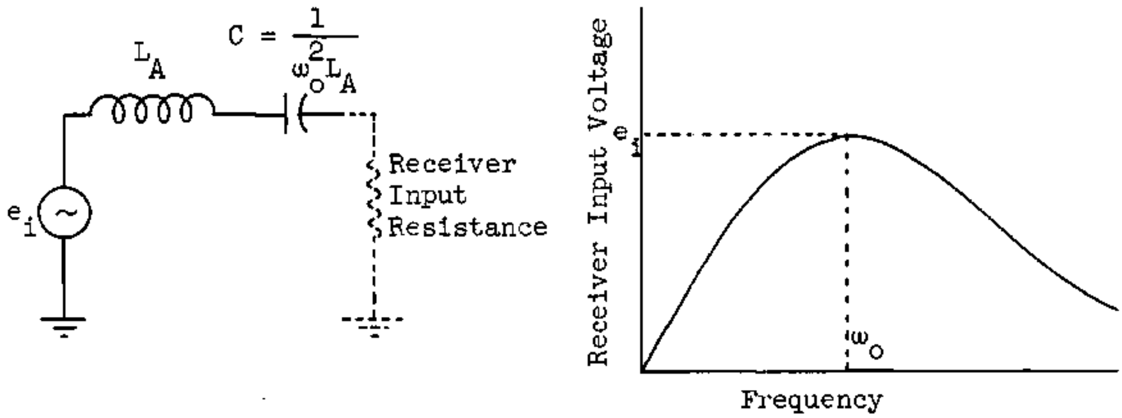


Figure 4. Simple Bandpass Filter.

improvement is only over a narrow frequency band. If this filter is to operate over a wide band of frequencies, it must be made tunable. The capacitive antenna trimmer system is identical to Figure 4 with a variable capacitor substituted for the fixed capacitor shown.

Since the antenna acts as a single series inductor, it may be used as the leading element in any type of filter whose first element is an inductor, providing the value of this leading element can be forced to be the 1 microhenry inductance of the antenna. Many types of filters exist which fulfill the above conditions. Two of these are the Butterworth and Chebychev approximations to the ideal lowpass filter response.

The Butterworth or maximally flat transmission function is given by

$$|Z_{12}(j\omega)|^2 = \frac{1}{1 + \omega^{2n}} , \tag{3}$$

where

$Z_{12}(j\omega)$ = filter impedance transfer function,

n = order of the function.

The order of the function determines the slope of the transfer function both in the passband and after cutoff. As n increases, the passband becomes "flatter" and cutoff becomes more rapid.

The Chebychev or equal-ripple approximation is based on the equation

$$|Z_{12}(j\omega)|^2 = \frac{1}{1 + \epsilon^2 T_n{}^2(\omega)} , \tag{4}$$

where

$T_n(\omega)$ = Chebychev polynomial of the first kind of order n,

ϵ = ripple tolerance.

As with the Butterworth function, the order n determines the slope of the impedance function. In the Chebychev function, n also determines the number of ripples in the passband. The height of the ripples is determined by ϵ in the relation $\frac{1}{1 + \epsilon^2}$. The height of the ripple is normally expressed in db below the maximum response.

Deriving networks from Equations (3) and (4) proves to be a laborious task; however, normalized element values have been tabulated by

Weinberg[3] making this task unnecessary. Portions of these tables are shown in the Appendix as Tables 3 through 5.

The procedure for using these tables consists of selecting the type of filter desired, and determining the ratio of input to output resistance. This resistance ratio is shown in the tables as r. The element values are read from the line of the table corresponding to the desired number of elements, n. These values are those of a lowpass filter terminated in a 1 ohm resistor and with a cutoff frequency of 1 radian per second. The impedance and frequency normalizations are removed by converting the impedance level from 1 ohm to the desired level, and transforming the cutoff frequency to the desired value. As shown by the sample calculations in the Appendix, the frequency is transformed by an amount that depends on the value of the inductance of the first element. The larger this element, the higher the cutoff frequency will be. A brief inspection of the tables shows that for Butterworth filters, as the number of elements increases, the value of the leading inductor decreases for equal source and load resistances ($r = 1$) and increases slowly for zero source resistance ($r = 0$). For the Chebychev filters with both $r = 0$ and $r = 1$, the value of the leading element increases slowly with an increasing number of filter elements but increases rapidly with increasing ϵ. Since the amount of frequency transformation available is directly proportional to the value of the leading element of the prototype filter, it follows that the use of large values of ϵ will result in large bandwidths of the final lowpass filters. Additional modest increases in bandwidth can be obtained by increasing the number of filter elements. The degree of impedance normalization also affects the cutoff frequency of the final lowpass filter. Since the value of the leading inductance in the filter is directly proportional to the impedance level, a high impedance will produce a large bandwidth. For example, doubling the impedance level will increase the bandwidth by a factor of two.

In the present application, frequencies below 1 Mc are of no interest; therefore, the lowpass filters designed using the tables can be converted to bandpass filters and the upper cutoff frequency thus raised. The bandpass transformation is performed by a change of independent variable in Equations (3) and (4). The frequency variable ω is replaced by $\bar{\omega}$, where

$$\bar{\omega} = \frac{\omega^2 - \omega_o^2}{\omega} . \tag{5}$$

This transformation is accomplished in the physical network by series resonating each inductor and parallel resonating each capacitor at the geometric mean frequency given by

$$\omega_o^2 = \omega_1 \omega_2 , \tag{6}$$

where

ω_1 = desired lower cutoff frequency,

ω_2 = desired upper cutoff frequency.

The bandwidth of the bandpass filter, $\omega_2 - \omega_1$, thus obtained is equal to the cutoff frequency, ω_c of the lowpass filter.

Although the equivalent source impedance of the loop is very small and is neglected in this research, this fact does not limit the filters to only those with zero source impedance. An artificial source impedance of any value can be inserted in series with the antenna and filters designed using this impedance. In this research, a 50 ohm source impedance is considered in addition to zero source impedance. The reasons for this choice become apparent in the next chapter. All of the filters are designed in the same manner; however, different tables are used for different values of r, where r is defined as the ratio R_s/R_L. Tables 3 through 5 give element values for r = 0 and r = 1. Weinberg lists element values for several values of r, including zero and one.

CHAPTER III

INSTRUMENTATION AND EQUIPMENT

General Considerations

For ease of measurement, the experimental portion of this research is divided into two phases. Phase I is a closed system of measurements, that is, the signal is injected directly into the network without being radiated. Phase II is an open or radiated system of measurements. Here the signal is radiated by a standard antenna and received by the loop under consideration. Phase II is much closer to the final use of the antenna; however, the measurements of Phase I are more accurate and prove useful in determining the correlation of the measured results with the theoretical values.

In the closed system, the filters are designed with equal input and output impedances of 50 ohms for compatability with test equipment. The output impedance corresponds to the receiver input while the filter input impedance is an artificial resistance placed in series with the antenna to allow a signal to be injected into the network. If the induced voltage in the loop is zero, the loop will act as a single 1 microhenry inductor. The equivalent circuit of the signal generator appears as a voltage source in series with a 50 ohm resistor. Therefore, with the signal generator connected to the antenna, the circuit is identical to the initial portion of Figures 29 and 30. The networks are then designed from Weinberg's data using the tables for equal source and load impedances.

For the open system, filters designed for zero source impedance and a 50 ohm output impedance are measured in addition to those previously discussed with 50 ohm source and output impedances. The 50 ohm source impedance consists of a 50 ohm resistor placed in series with the antenna, with the remainder of the filter designed as described above for the closed system measurements. For the zero source impedance filters, the 50 ohm resistor is removed and the networks designed using the r = 0 or zero source impedance tables. In the open system, only bandpass filters are evaluated because the induced voltage in the loop is below the receiver sensitivity for frequencies below 1 Mc. All bandpass filters have center frequencies of

$$f_o = \sqrt{(1 \times 10^6)(30 \times 10^6)} = 5.48 \text{ Mc.}$$

The closed system technique is used initially to evaluate the filter design in terms of the theoretical response. In this phase, the insertion loss of the loop and additional components and the shape of the passband are measured and compared to theoretical values. In Phase II, the open system is used and the receiving efficiency is measured in addition to evaluating the passband shape. Since the closed system measurements are more accurate and more easily conducted, they are used as the basic approach. These tests verify the assumption that the antenna acts as a 1 microhenry inductor. The radiated measurements are used to check the matching technique under actual conditions when operating with a receiver and are also used to evaluate networks that do not lend themselves to the closed system, such as the networks with zero source impedance.

Phase I - Closed System Measurements

The closed system measurements were made using the two test systems shown in Figures 5 and 6. Figure 5 shows a system using a sweep generator and an oscilloscope as the signal source and indicator, respectively. The sweep generator used has a 50 ohm output impedance. A two foot length of 50 ohm coaxial cable terminated with a 10 db pad connects the generator to the circuit input. This combination appears to the loop as a 50 ohm resistance and serves as the source impedance. Another 10 db pad is placed at the output of the matching network followed by a diode detector with a 50 ohm input impedance. Again, this appears to the network as a 50 ohm resistance and serves as the load impedance. The dc output of the detector is displayed on an oscilloscope. The system is calibrated by removing the loop and matching network from the system and noting the position of the oscilloscope trace. Successive attenuation is then added and the trace position recorded at each step. The horizontal or frequency axis is calibrated by displaying the marker from the sweep generator and recording its frequency at each 0.5 cm on the oscilloscope graticule. With the system thus calibrated, various types of networks are placed in the system and photographs taken of the oscilloscope presentations. Values are then scaled from the photographs and plotted with more appropriate scales.

The measurement system shown in Figure 6 rather than the system of Figure 5 is used to evaluate the bandpass networks. A conventional

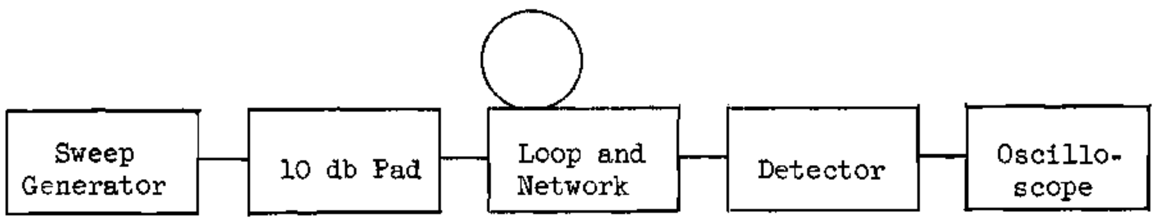


Figure 5. Closed System Measurements-Sweep Generator Method.

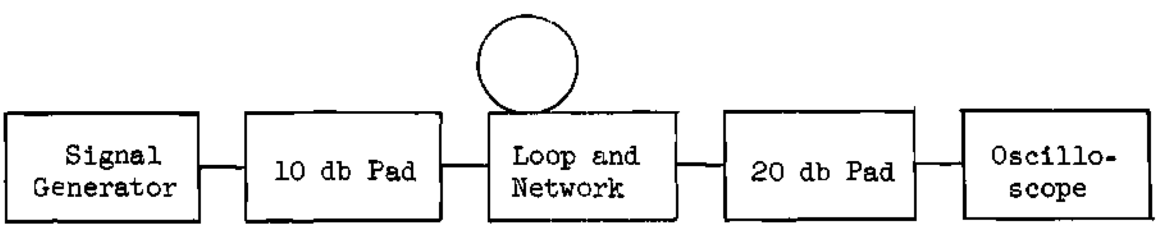


Figure 6. Closed System Measurements-Signal Generator Method.

signal generator is used because of the difficulty in obtaining accurate readings at low frequencies with the sweep generator, which gives poor information concerning the lower cutoff frequency of the bandpass filters. The conventional signal generator can be tuned to any frequency desired and the output displayed. In this system, the signal generator is connected to the network input as before. The output, however, is connected to the oscilloscope through a 20 db pad without a detector. The input impedance of the 20 db pad is sufficiently close to 50 ohms when terminated in the high impedance of the oscilloscope for the purpose of these measurements. The system is calibrated by removing the network and measuring the voltage with the oscilloscope. The network responses are then measured and the output voltages converted to db below the input by a simple digital computer program, and the values plotted.

Phase II - Open System Measurements

Open system measurements are made using the system shown in Figure 7. The method consists of establishing a standard field and then measuring the network output when the antenna is placed in this known field. The standard field is established using the system developed by the National Bureau of Standards[4] for calibrating field intensity meters up to 30 Mc. The transmitting antenna is a balanced unshielded loop whose circumference is less than $\lambda/8$ at the highest frequency in order to have an essentially constant phase throughout the loop. This loop is fed from a signal generator and balancing transformer to prevent radiation from the coaxial line which would be present in an unbalanced system. The loop under test is placed coaxially with the transmitting loop

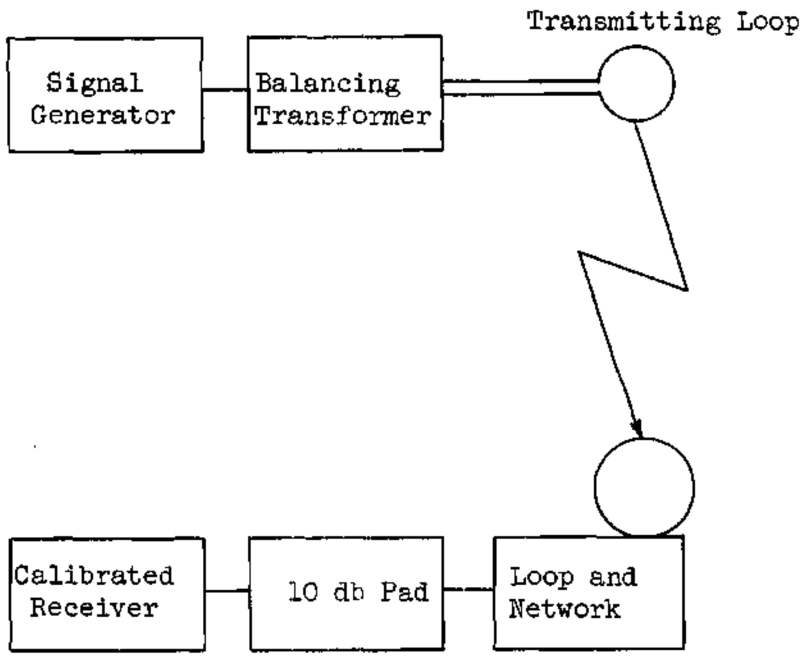


Figure 7. Open System Measurements.

at a known distance away from the loop. Although the measurements are made in the near field in terms of the induction field, H, they are related to the equivalent electric component, E, which would exist in the far field by E = ZH, where Z is the impedance of free space, 120π. The National Bureau of Standards gives the equivalent E field as,

$$|E| = \frac{60\pi\, r_1^2\, I}{(d^2 + r_1^2 + r_2^2)^{3/2}} \sqrt{1 + \frac{(2\pi d)^2}{\lambda}}\ , \tag{7}$$

where

E = equivalent free-space electric field intensity in volts per meter,

r_1 = radius of transmitting loop in meters,

r_2 = radius of test loop in meters,

d = axial spacing between loops in meters,

I = transmitting loop current in amperes,

λ = free-space wavelength in meters.

The loop and network under test are connected through a cable approximately eight feet long to a receiver with a 10 db pad and a 50 ohm input. This receiver is equipped with a meter that is calibrated in terms of input voltage. At each frequency the receiver is calibrated using the signal generator and the gain control on the receiver to give a constant meter reading for a standard input voltage. The signal generator is connected to the transmitting loop and the signal generator output adjusted to give a preset value of current through the loop. This value is 67 ma as determined by the voltage drop across a 1.2 ohm resistor placed in series with the transmitting loop. The test loop is then connected to the receiver and the output measured in terms of db below the calibration voltage.

CHAPTER IV

DISCUSSION OF RESULTS

The results of the experimental phases agree very closely with the theoretical results. Curves showing the response characteristics of various types of networks are shown in the Appendix. The experimental data are summarized in Tables 1 and 2 together with the calculated theoretical values. It is seen that very close agreement between the two does exist. The individual networks are discussed in more detail below. Figure 11 through Figure 19 describe closed system measurements while Figure 20 through Figure 28 describe open system measurements.

The most desirable filter type for this application is one which has a flat response curve over the frequency range of interest. This at once points to the Butterworth approximation to the ideal lowpass response. A three element Butterworth lowpass filter was designed using the procedure described earlier. Figures 11 and 12 show the results of the measurements made using this network and its transformed bandpass equivalent. As the figures indicate, this filter gives a flat response over the passband; however, the bandwidth is not wide enough for the present applications.

By allowing some ripple in the passband, a Chebychev filter can be designed that will give a much wider passband. Figures 13 and 14 show a three element Chebychev filter designed for a 1 db ripple. This network gives approximately twice the bandwidth obtained with the Butterworth filter. The 3 db Chebychev filter of Figures 15 and 16 gives approximate-

Table 1. Closed System Measurement Summary

Fig.	Type of Filter	Theoretical Ripple	Theoretical f_1	Theoretical f_2	Theoretical BW	Measured Ripple	Measured f_1	Measured f_2	Measured BW
8	3 Element Butterworth LP	0	-	7.95	-	0	-	7.75	-
9	3 Element Butterworth BP	0	2.81	10.76	7.95	0	3.15	11.4	8.25
10	3 Element Chebychev LP	1	-	16.10	-	1.0	-	17.7	-
11	3 Element Chebychev BP	1	1.68	17.78	16.10	1.3	1.57	18.3	16.73
12	3 Element Chebychev LP	3	-	26.60	-	2.8	-	24.1	-
13	3 Element Chebychev BP	3	1.10	27.70	26.60	3.5	1.10	26.1	25.0
14	5 Element Butterworth BP	0	3.59	8.51	4.92	.6	.6	8.35	4.75
15	5 Element Chebychev BP	1	1.60	18.6	17.0	.8	.8	17.9	16.31
16	5 Element Chebychev BP	3	1.05	28.75	27.7	2.5	2.5	27.0	25.92

ly three times the bandwidth of the Butterworth network.

The next logical extension of this theory is to evaluate filters using more than three elements. As seen in Figure 17, the five element Butterworth does not give as wide a bandwidth as the three element Butterworth. This agrees with the theory of Chapter II where it was noted that this will hold true for any number of elements with the Butterworth function and with $r = 1$. As is shown in Chapter II, this is not true with the Chebychev function. As the number of elements is increased, the bandwidth increases. Figures 18 and 19 show five element Chebychev functions with 1 db and 3 db ripples, respectively.

The measurements made using the open system are shown in Figures 20 through 28. All of these curves have been normalized with respect to the induced voltage. When these networks are placed in an electromagnetic field with a constant field strength, the induced voltage will not remain constant but will increase at a rate of 6 db per octave, i.e., the output voltage will double each time the frequency doubles. This is shown in the Appendix, where Equation (18) indicates that the expression for the induced voltage in a loop contains a frequency term. With all other factors constant, this gives a 6 db per octave increase in induced voltage. The open system curves are plotted in db below the theoretical induced voltage calculated from the Appendix, Equation (20) and the value of the standard field. This allows easier comparison with the closed system measurements.

The data of Figure 20 is from the same Chebychev filter as that of Figure 11. This curve is used to check the accuracy of the open field measurement system. The expected accuracy of this system is ± 2.0 db.

Factors affecting the accuracy are, (1) signal generator calibration, (2) receiver calibration, (3) calibration of pads, (4) losses in cables, and (5) accuracy of establishing the standard field. The curves in general show agreement with the theoretical response to the expected accuracy. The open system curves indicate that the induced voltage in the loop is approximately 1 db lower than the calculated value. This inaccuracy could have been caused by the equipment used to measure the current in the transmit loop or inaccuracy in the dimension of either loop or spacing between loops. The curves also tend to be lower at the higher frequencies. This indicates that at the higher frequencies, reflections from the walls of the room may be present. Also as the frequency increases, one of the basic assumptions of a loop antenna used with a direction finder, i.e., that the phase around the loop is constant, becomes progressively less valid. A combination of these two effects may be causing the higher frequency response to be lower.

It should be noted that the network response of Figure 20 is 6 db below the calculated value. This is because of the 50 ohm source impedance that was added to the network. One half of the induced voltage is developed across this resistor and one half across the load resistor. This fact makes the network with zero source impedance more desirable if sufficient bandwidth can be obtained.

Results of the open system measurements are summarized in Table 2. Although these measurements do not agree as well with the calculated values as in the closed system, they do in general fall within the expected accuracies. All of these measurements point out two important features of the networks with zero source impedance, (1) the bandwidth

Table 2. Open System Measurement Summary

		Theoretical				Measured			
Fig.	Type of Filter	Ripple	f_1	f_2	BW	Ripple	f_1	f_2	BW
18	3 Element Butterworth	0	2.10	14.06	11.94	1.0	2.20	12.0	9.80
19	3 Element Chebychev	1	2.10	14.10	12.00	2.5	1.90	14.3	13.40
20	3 Element Chebychev	3	1.70	17.90	16.20	3.3	1.80	16.8	15.00
21	5 Element Butterworth	0	2.08	14.38	12.30	2.0	2.30	12.5	10.20
22	5 Element Butterworth	1	2.00	15.25	13.25	1.5	2.05	13.7	11.65
23	5 Element Butterworth	3	1.60	18.70	17.10	3.2	1.87	19.0	17.13

of these networks is lower than with comparable type networks with equal source and load impedances, and (2) increasing the number of elements yields only a minor increase in bandwidth. This was expected from the theory of Chapter II.

Figure 27 shows the loop antenna with no filter, for comparison with other measurements. Figure 28 shows the antenna with a capacitive trimmer added. Although this improved the antenna response, it must be readjusted at each frequency. This is both inconvenient and if the antenna is used in a measurement device it will add inaccuracies in the measurements because of the resetability of the trimmer.

CHAPTER V

THEORETICAL EXTENSION OF MATCHING TECHNIQUE

Although the networks for this research were designed using the Weinberg tables, they need not be in practice. Other design procedures which yield the proper value for the first element can be used. In the case of the bandpass networks, other techniques can be used which would yield equivalent networks to the ones used here and with fewer elements. The present design method is not presented as the most efficient but rather as one simple, convenient method of achieving the desired results.

If an impedance level other than 50 ohms is desired, it is easily obtained when the network is designed. The impedance level of the normalized network is simply transformed to a different level. When a wider bandwidth is needed, a higher impedance level is desired. As shown in the Appendix, with an impedance level of 200 ohms rather than 50 ohms, four times the bandwidth can be achieved. If this matching technique is to be used with a receiver that is in the design stages, the choice of a higher impedance level should be considered.

When the load impedance is not a pure resistance or when the antenna equivalent circuit contains more than one reactive element, the matching technique described here is still valid; however, the Weinberg tables can be used only in special cases which will be discussed later.

Consider the matching situation shown in Figure 8. The antenna consists of a source resistance and more than one reactive element. The configuration shown is only an example. Neither the antenna nor the

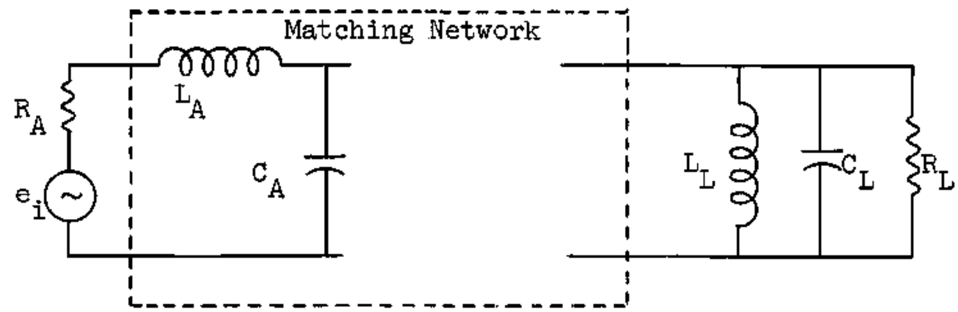


Figure 8. Generalized Matching Parameters.

load need be limited to these particular parameters. Fielder[5] outlines a method of matching a source to a load where both the source and load are arbitrary but known impedances. By a simple redefinition of the matching network, Figure 8 becomes the type network described by Fielder. Figure 9 illustrates this network. Because of the additional constraints placed on the network by the source and load impedances, less control

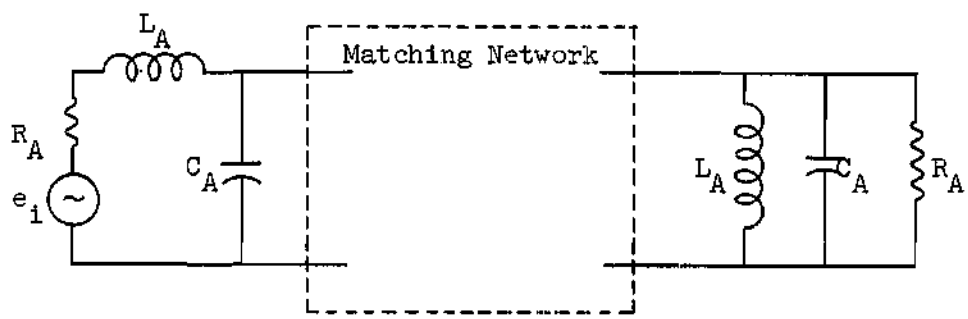


Figure 9. Matching Network Described by Fielder.

over the bandwidth and matching network parameters is possible than when the Weinberg tables can be used; however, a matching network is assured for the more complex source and load.

If the source can be reduced to a single reactive element such as with the 12 inch unshielded loop, more control can be obtained when matching to the arbitrary load shown in Figure 9. The method of Fielder would still be used, but with fewer restrictions placed on the matching network. This type analysis would be used to match the loop to a receiver input when the input is other than a pure resistance.

When the antenna equivalent circuit consists of more than one reactive element, but the load is a resistance, the method described by Fano[6] is applicable. This method allows an arbitrary source to be matched to a resistive load. Figure 10 illustrates this situation.

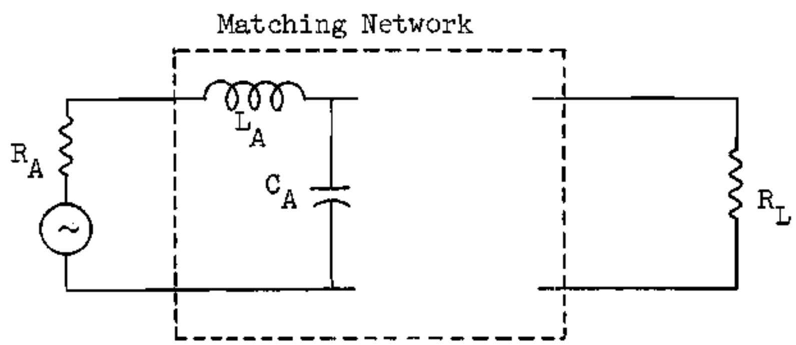


Figure 10. Matching Network Described by Fano.

The methods of Fano and Fielder provide a matching technqiue for the general cases described above. As mentioned earlier, the Weinberg tables may be used in certain specialized cases. The circuit of Figure 10 can be matched using the Weinberg procedure providing the antenna shunt capacitance, C_A, is no larger than the first shunt capacitor in the Weinberg network. The first shunt capacitor would then consist of C_A in parallel with an added capacitor of proper size to produce the needed value. This would be the case when using a 12 inch shielded loop. The capacitance of the shielded loop is too large to be neglected, being approximately 10 picofarads. This is lower than the normal values obtained from the Weinberg tables, hence, the tables could be used. An antenna such as the shielded loop, with two reactive elements cannot yield a bandpass network using the Weinberg analysis since a resonating capacitor cannot be added in series with the antenna inductance. If the cutoff frequency of the lowpass design is high enough, it can be used, however.

A shunt capacitor across the load resistor may be handled in the same manner described above subject to the same restriction, that the shunt capacitance be less than that calculated from Weinberg. If the load is a parallel combination of resistance, inductance and capacitance, the use of the Weinberg tables is even more restricted. For this situation, the antenna must act as a single element to allow a bandpass filter to be used. The restriction placed on the shunt capacitance above is again imposed and, further, the shunt inductance of the load must be high enough to allow the final shunt capacitance to be resonated when the filter is transformed to a bandpass network. These restrictions are severe enough to, in general, rule out the use of the Weinberg procedure when the load contains resistance, capacitance and inductance.

The emphasis in this research has been on matching of small loop antennas. The methods are also applicable to other antennas whose parameters can be determined and used as the first elements of a suitably designed filter.

The possibility also exists of using this technique for other more specialized purposes. For example, the cutoff frequency may be chosen such that an interfering signal out of the receiver band would be attenuated. This could reduce the harmful effects of some spurious responses and intermodulation products of the receiver. More specialized response shaping could also be done such as adding notches in the passband or adding stopbands.

CHAPTER VI

CONCLUSIONS

The object of this research was to develop a procedure for matching a small loop antenna to a receiver input over a broad band of frequencies. The method of attack is presented in Chapter II, and a summary and comparison of the calculated and experimental results are given in Chapter IV. A theoretical extension of these results is discussed in Chapter V.

The good correlation between experimental and calculated data indicate that the methods described herein are applicable for matching a small loop antenna to a receiver input over a broad band of frequencies without the need for tuning at each frequency.

The salient features of the developed technique are:

(1) The results of the experimental evaluation agree well with the theoretical analysis.

(2) The design procedures are comparatively simple because of the readily available Weinberg tables.

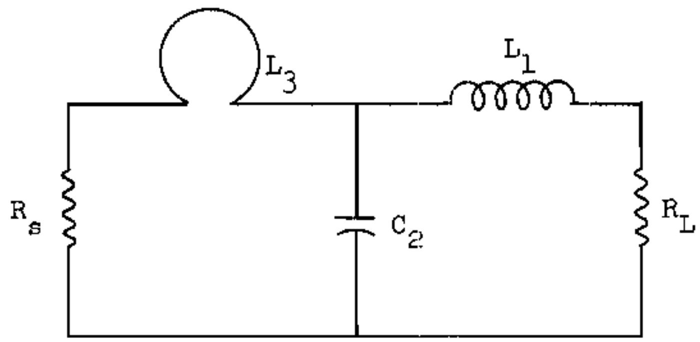
(3) Realizable elements are used, i.e., the networks do not depend on ideal components for their operation.

These features as well as the general results make this technique worthy of consideration for matching problems of the type discussed.

APPENDIX

SAMPLE NETWORK CALCULATIONS

The basic network is shown below. Assume it is required to design a three element Chebychev function with a 1 db ripple, and equal source and load impedances.



From the Weinberg tables the normalized values of the "Lowpass Prototype" are:

$R_s = R_L = 1$ ohm	$L_3 = 2.0236$ henry
$L_1 = 2.0236$ henry	$\omega_o = 1$ radian
$C_2 = 0.9941$ farad	

Transforming R_L to 50 ohms we multiply all R's and L's by 50 and divide C's by 50, ω_c is unchanged.

$R_s = R_L = 50$ ohm	$L_3 = 101.2$ henry
$L_1 = 101.2$ henry	$\omega_c = 1$ radian
$C_2 = 0.0198$ farad	

Now removing the frequency normalization we divide all L's and C's by 101.2×10^6 to force L_3 to 1 μh. Multiply ω_c by 101.2×10^6.

The final values are now:

$R_s = R_L = 50$ ohms $\qquad L_3 = 1 \times 10^{-6}$ henry

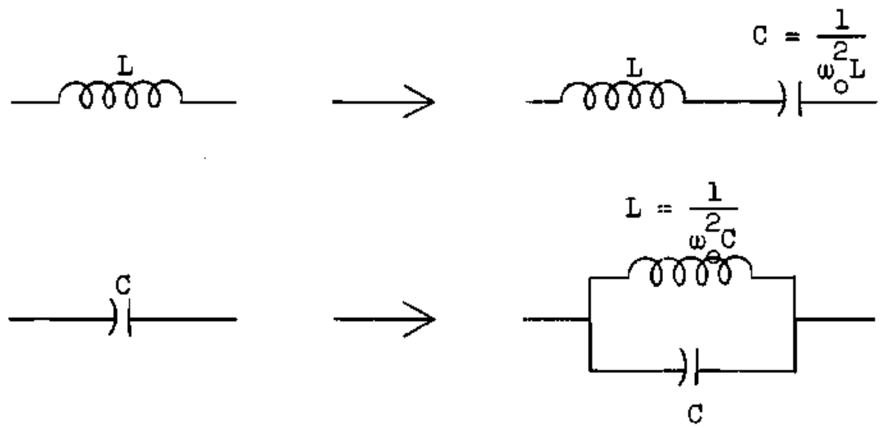
$L_1 = 1 \times 10^{-6}$ henry $\qquad \omega_c = 101.2 \times 10^{6}$ radian

$C_2 = 196 \times 10^{-12}$ farad $\qquad f_c = 16.1$ Mc

This yields the desired lowpass filter.

To obtain a bandpass filter from the above lowpass design the transformation indicated graphically below is made.

Lowpass $\qquad\qquad$ Bandpass



The center frequency is now ω_o and the bandwidth is equal to that of the lowpass prototype.

For the same type network with a 200 ohm impedance level the values become:

$R_s = R_L = 200$ ohms $\qquad L_3 = 1 \times 10^{-6}$ henry

$L_1 = 1 \times 10^{-6}$ henry $\qquad \omega_c = 404.7 \times 10^{6}$ radian

$C_2 = 12.3 \times 10^{-12}$ farad $\qquad f_c = 64.4$ Mc

EFFECTIVE HEIGHT CALCULATIONS

In finding the effective height of an antenna, a plane wave with components H_x and E_y is assumed. The flux, Φ, linking the loop is

$$\Phi = BA, \quad \text{where } A = \text{Area of Loop}, \tag{8}$$

but

$$B = \mu\vec{H}, \tag{9}$$

therefore

$$\Phi = \mu\vec{H}A. \tag{10}$$

For a sinusoidal time varying H field, with a component in the x direction only,

$$\vec{H} = H_x\cos\omega t. \tag{11}$$

Then,

$$\Phi = \mu AH_x\cos\omega t. \tag{12}$$

The induced voltage in the loop is

$$e_i = -N\frac{d\Phi}{dt},$$

$$= -N\mu AH_x\frac{d}{dt}(\cos\omega t),$$

$$= \omega N\mu AH_x\sin\omega t. \tag{13}$$

The peak of the induced voltage is

$$e_{ip} = \omega N\mu AH. \tag{14}$$

For a loop in air $\mu = \mu_o = 4\pi \times 10^{-7}$,

also

$$E_y = \eta H_x = 120\pi H_x, \tag{15}$$

therefore,

$$H_x = \frac{E_y}{120\pi}, \tag{16}$$

and

$$e_{ip} = (2\pi f)N(4\pi)(10^{-7})A\left(\frac{E_y}{120\pi}\right),$$

$$= \frac{\pi NfA(10^{-7})E_y}{15}. \tag{17}$$

For a 1 turn loop (N = 1)

$$e_{ip} = \frac{\pi f A(10^{-7})E_y}{15} , \tag{18}$$

but the definition of effective height is

$$h_e = \frac{e_{ip}}{E_y} = \frac{\pi f A(10^{-7})}{15} \quad \text{(meters)}. \tag{19}$$

For 12" diameter loop A = .0735 meters^2,

$$h_e = .0154(10^{-7})f$$

$$= .00154f \text{ (meters)} \quad \text{(f in Mc)} \tag{20}$$

CLOSED SYSTEM MEASUREMENT RESULTS

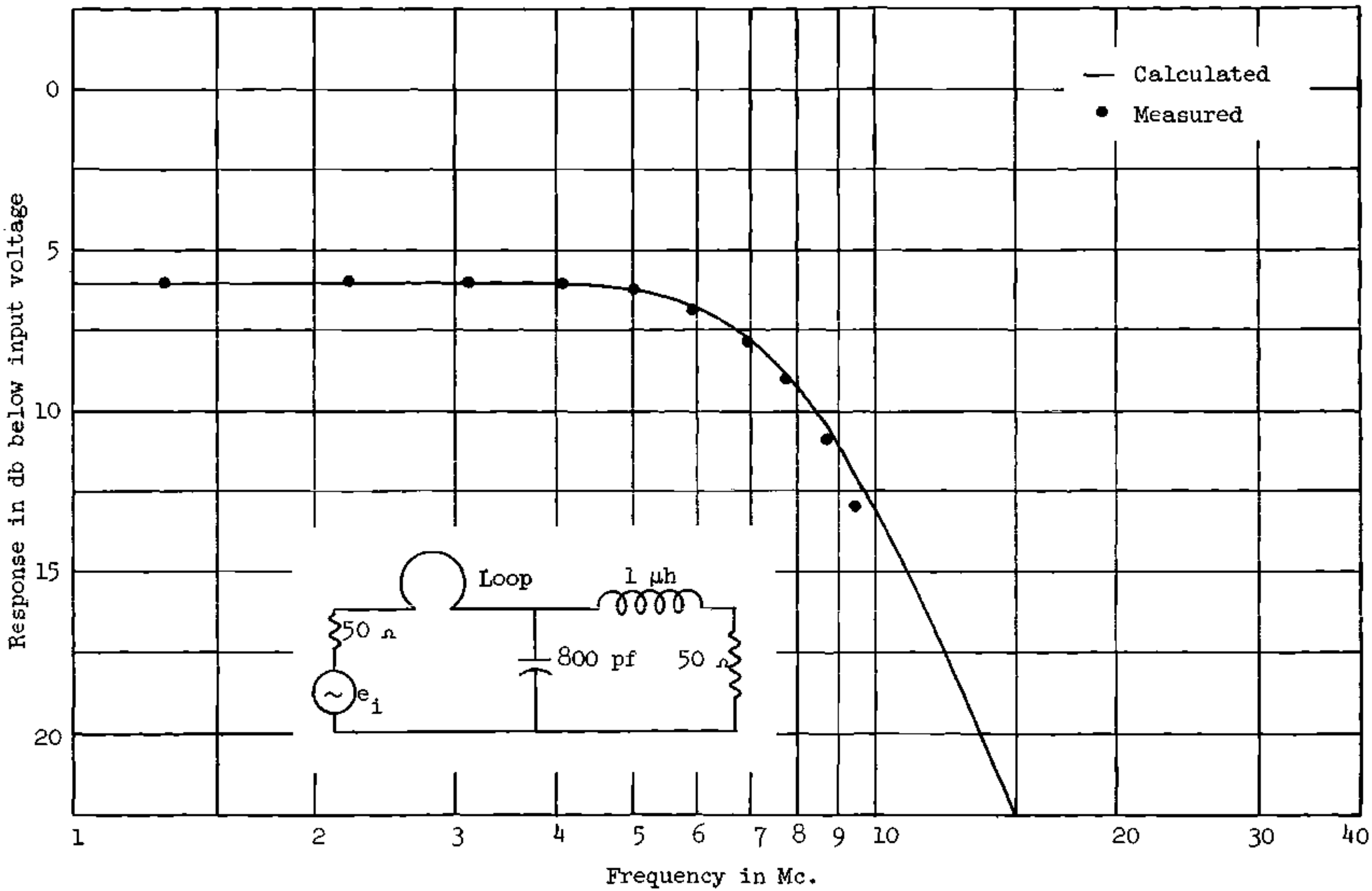


Figure 11. Response of a Three Element Butterworth Lowpass Filter (R_s = 50 ohms).

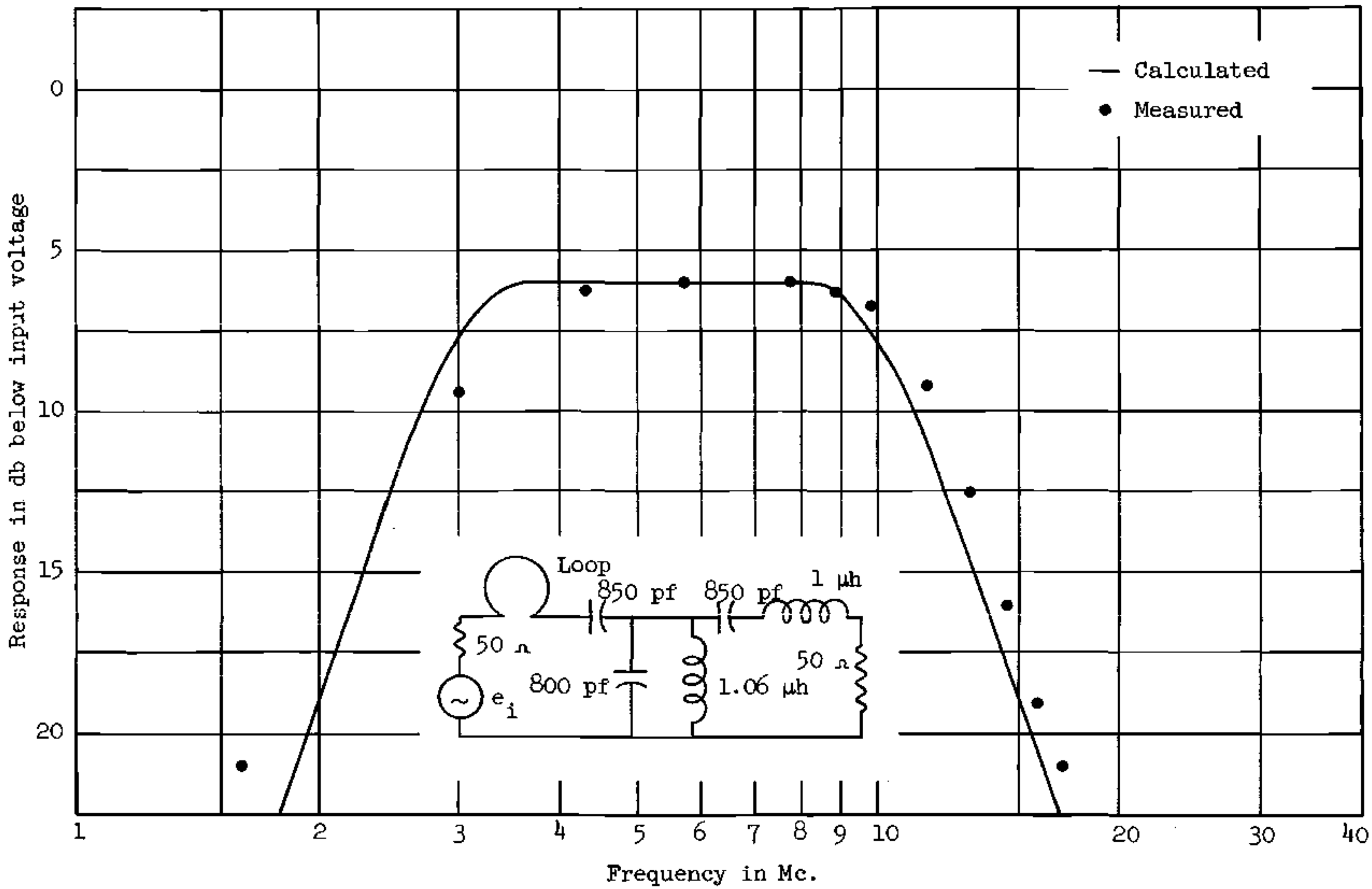


Figure 12. Response of a Three Element Butterworth Bandpass Filter (R_s = 50 ohms).

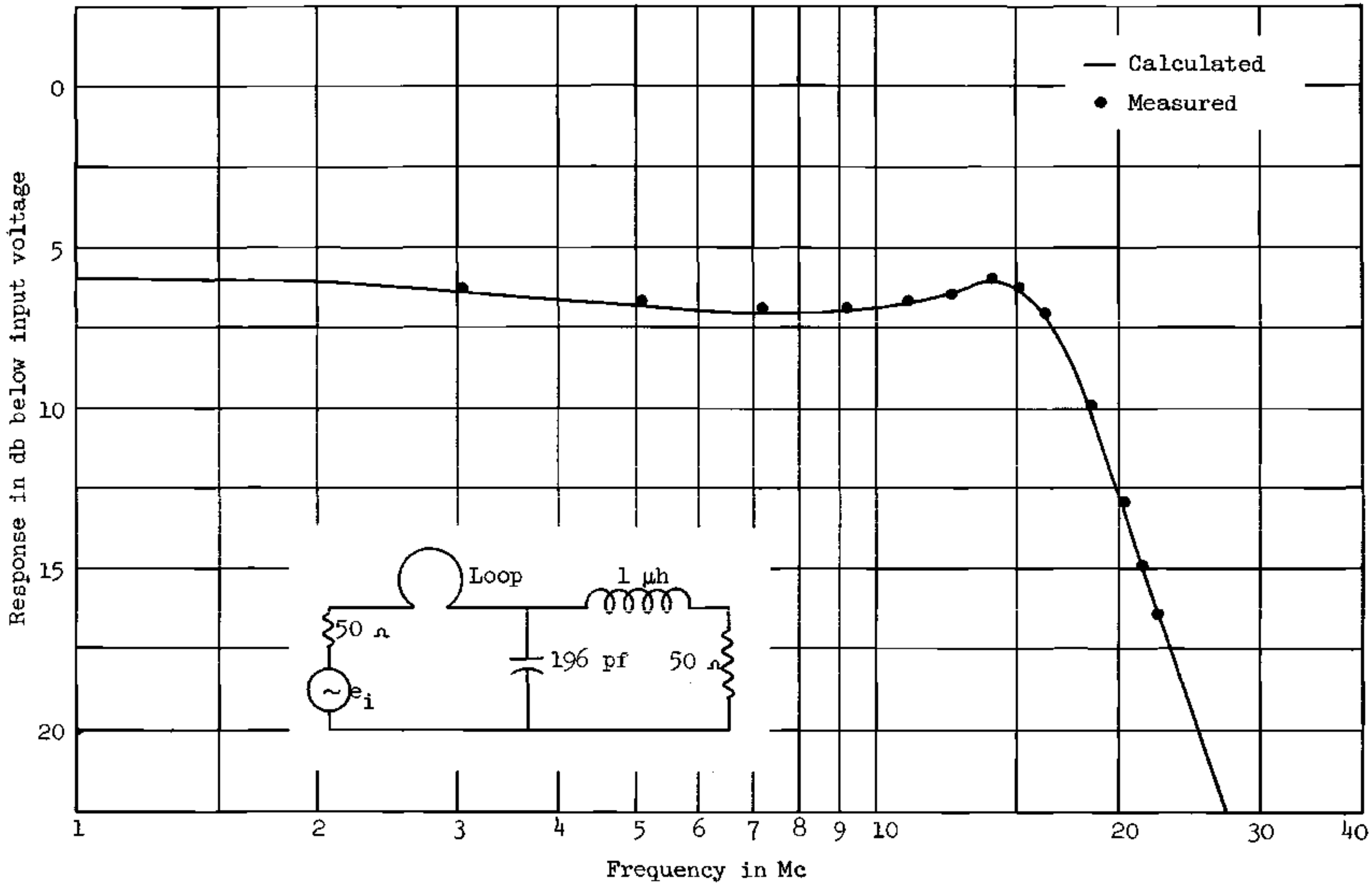


Figure 13. Response of a Three Element Chebychev Lowpass Filter with a 1 db Ripple (R_s = 50 ohms).

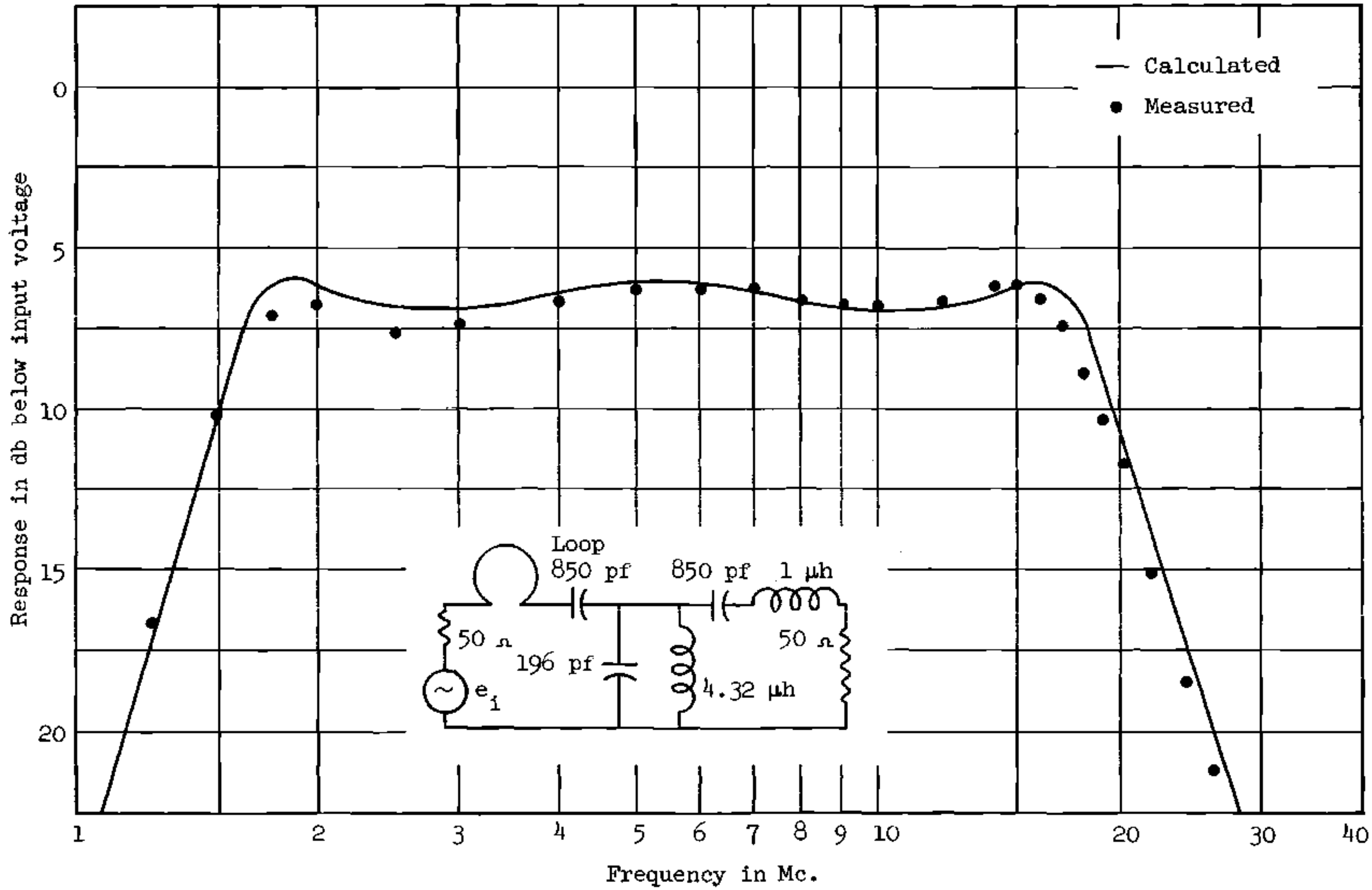


Figure 14. Response of a Three Element Chebychev Bandpass Filter with a 1 db Ripple (R_s = 50 ohms).

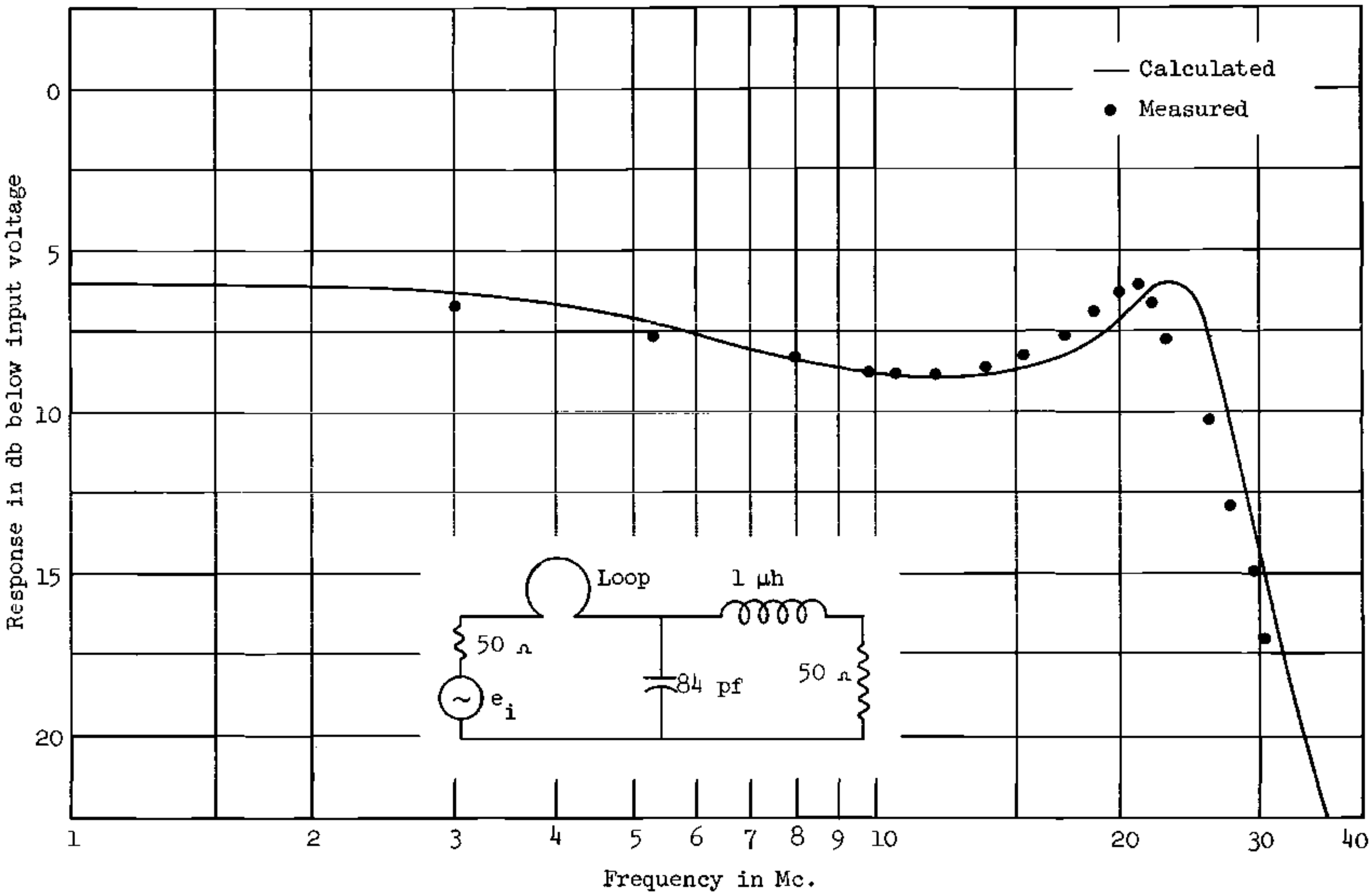


Figure 15. Response of a Three Element Chebychev Lowpass Filter with a 3 db Ripple (R_s = 50 ohms).

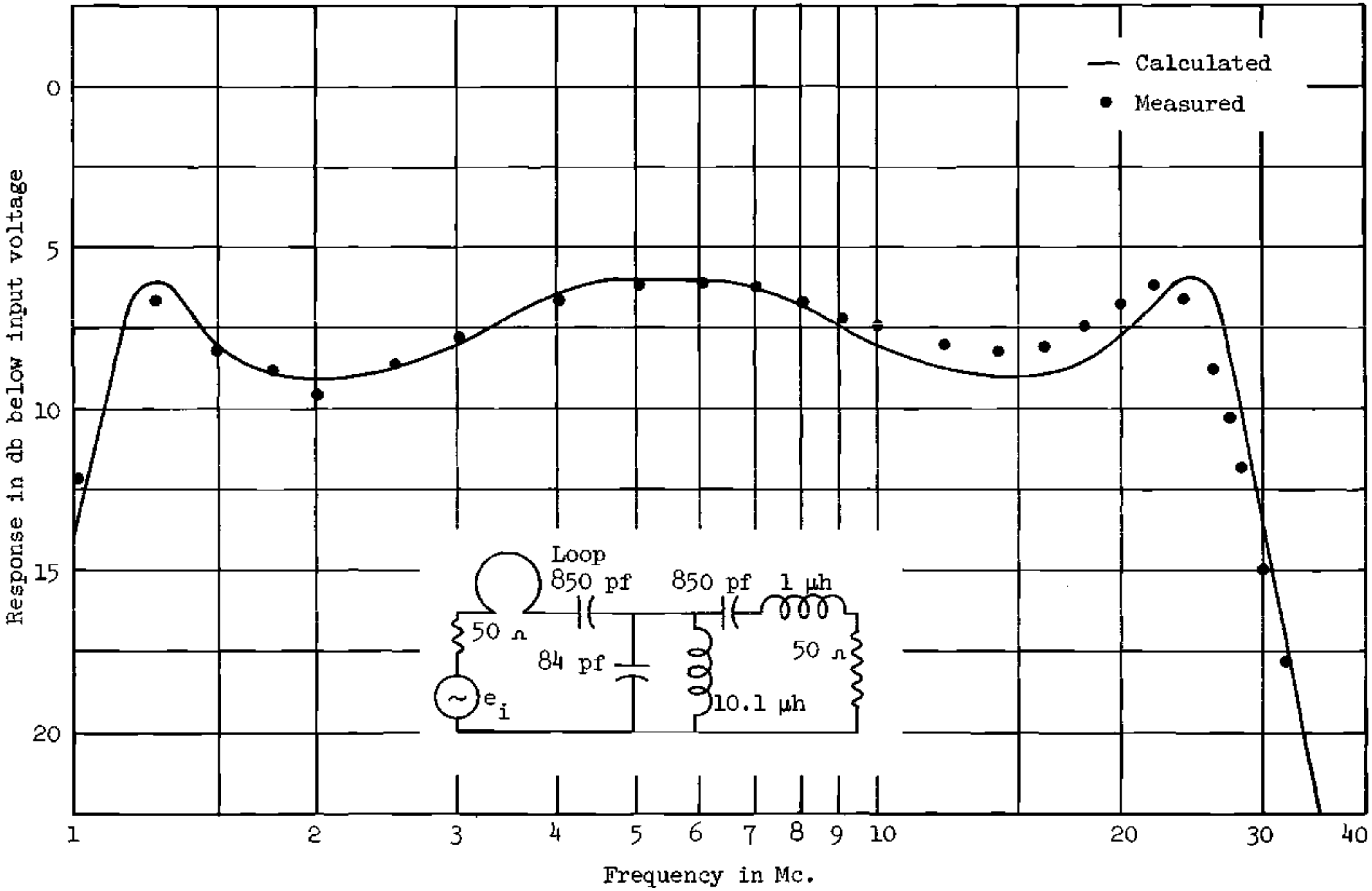


Figure 16. Response of a Three Element Chebychev Bandpass Filter with a 3 db Ripple (R_s = 50 ohms).

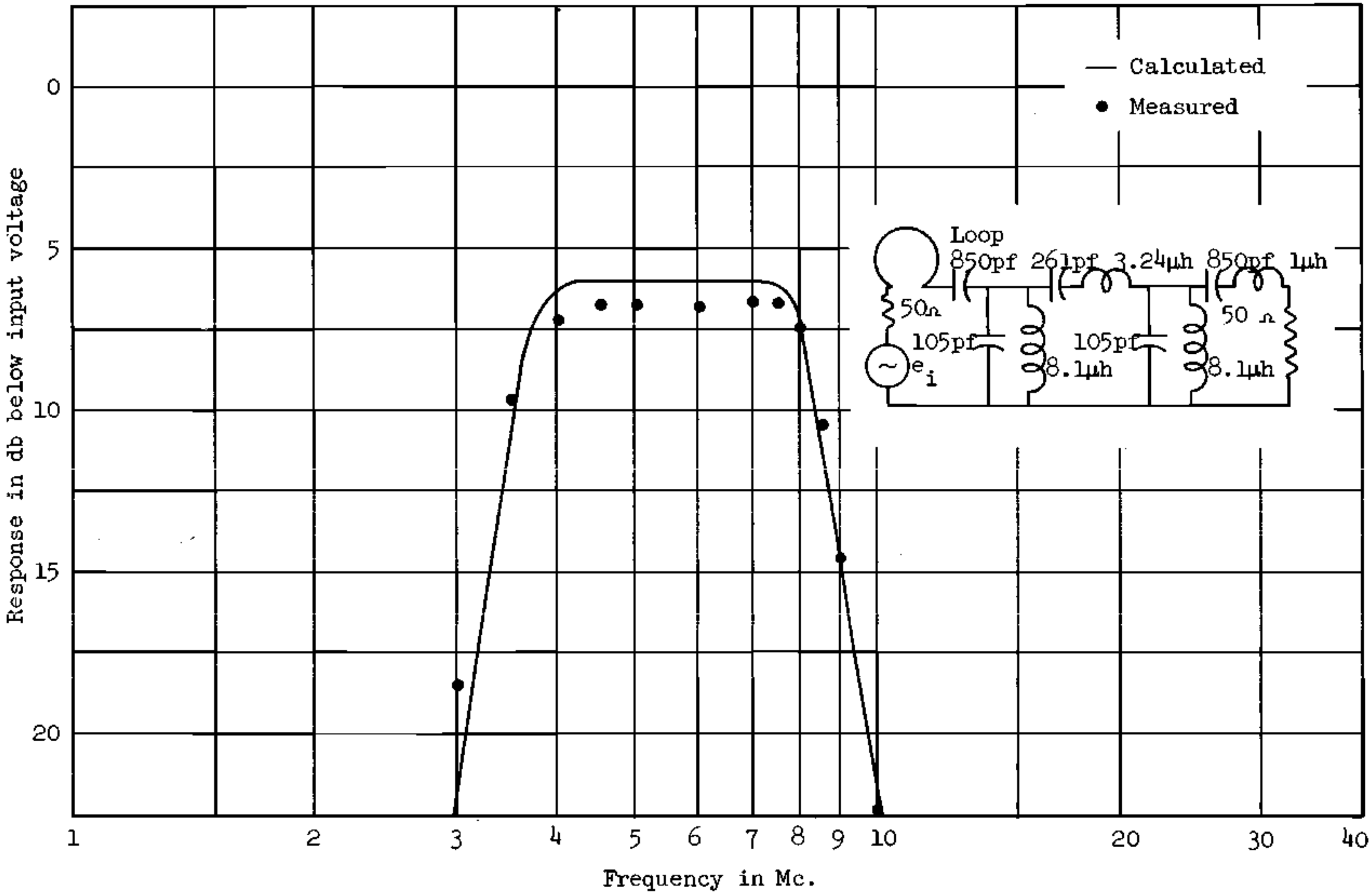


Figure 17. Response of a Five Element Butterworth Bandpass Filter (R_s = 50 ohms).

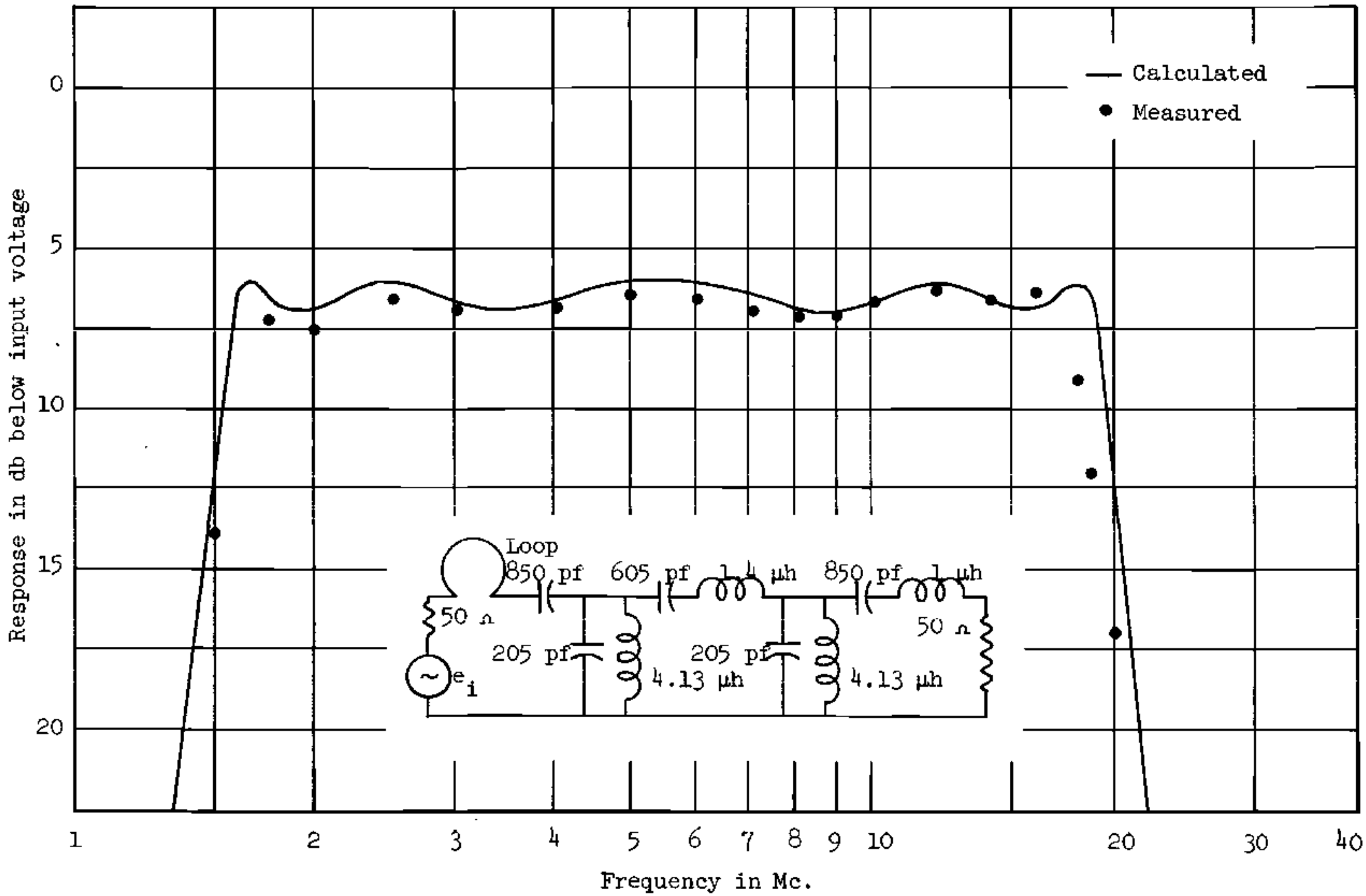


Figure 18. Response of a Five Element Chebychev Bandpass Filter with a 1 db Ripple (R_s = 50 ohms).

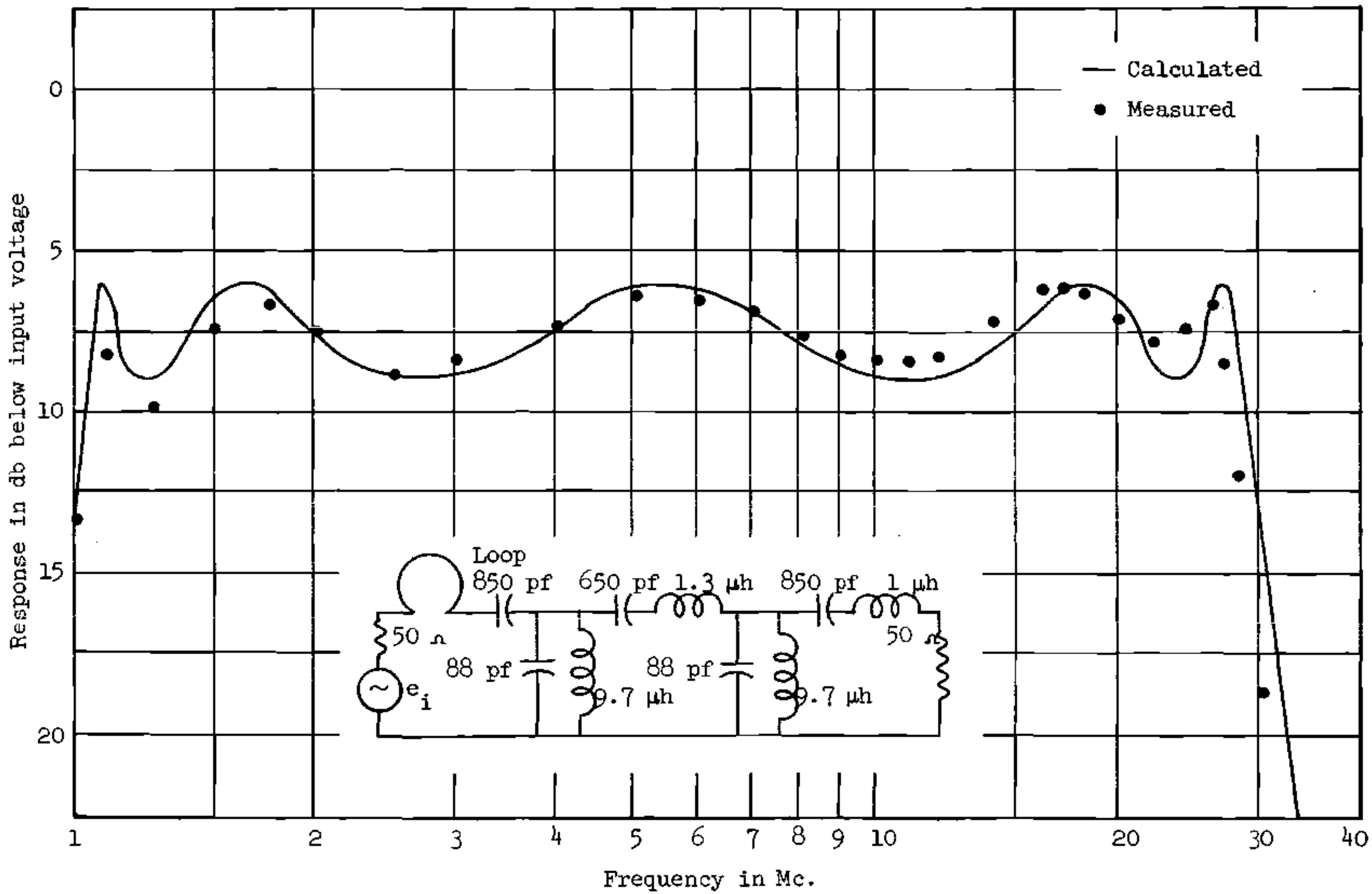


Figure 19. Response of a Five Element Chebychev Bandpass Filter with a 3 db Ripple (R_s = 50 ohms).

OPEN SYSTEM MEASUREMENT RESULTS

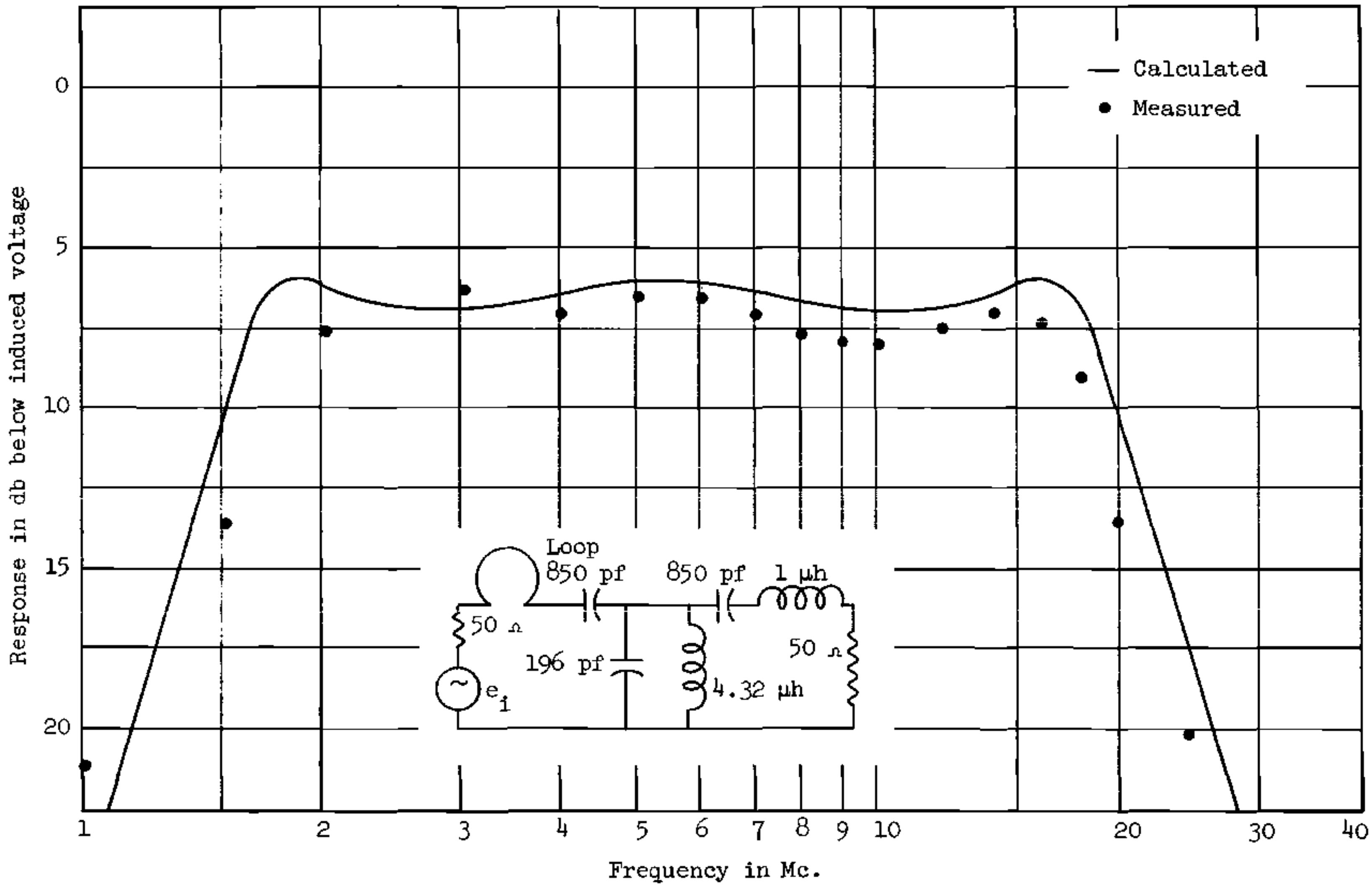


Figure 20. Response of a Three Element Chebychev Bandpass Filter with a 1 db Ripple (R_s = 50 ohms).

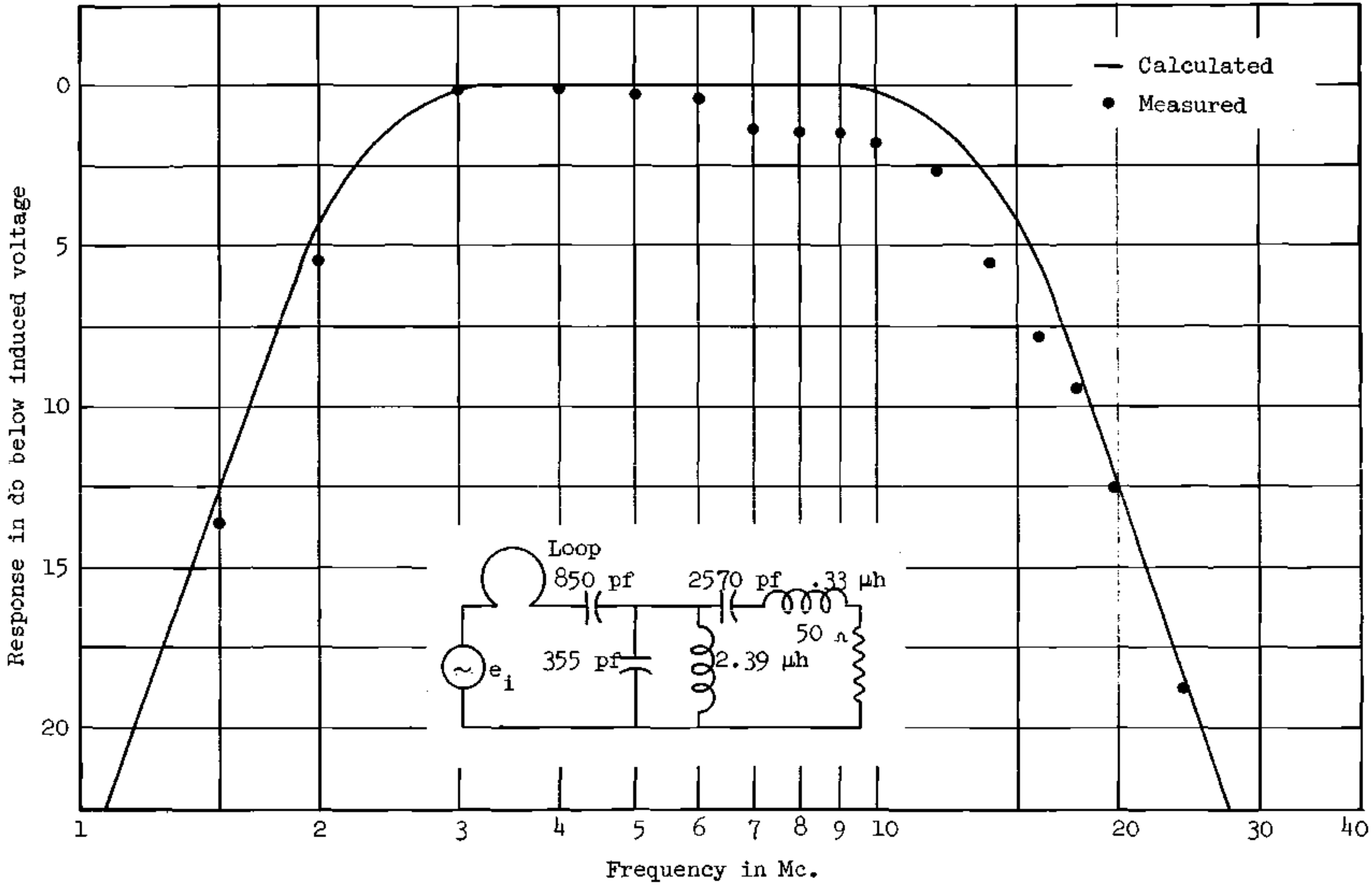


Figure 21. Response of a Three Element Butterworth Bandpass Filter (R_s = 0 ohms).

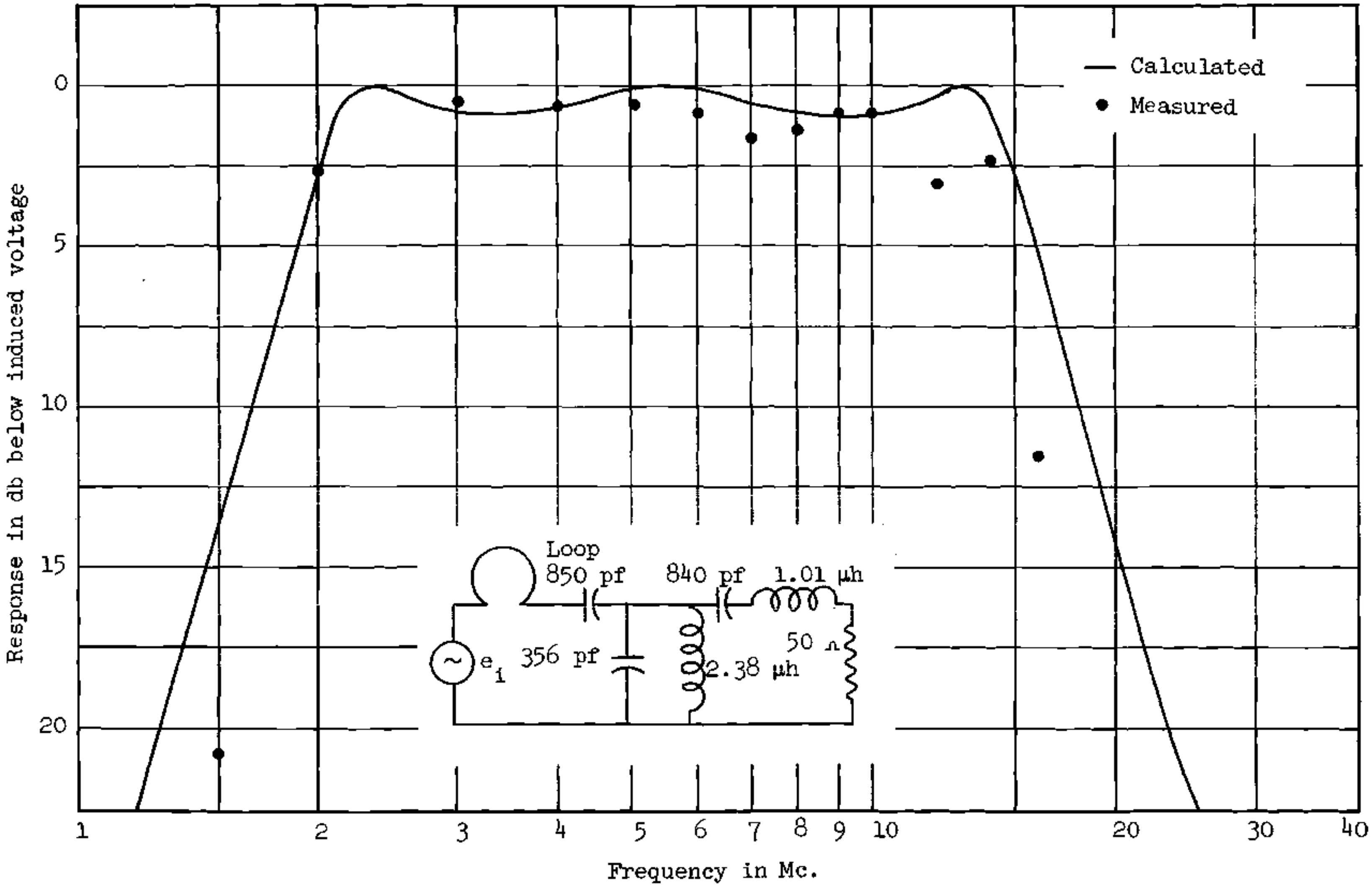


Figure 22. Response of a Three Element Chebychev Bandpass Filter with a 1 db Ripple (R_s = 0 ohms).

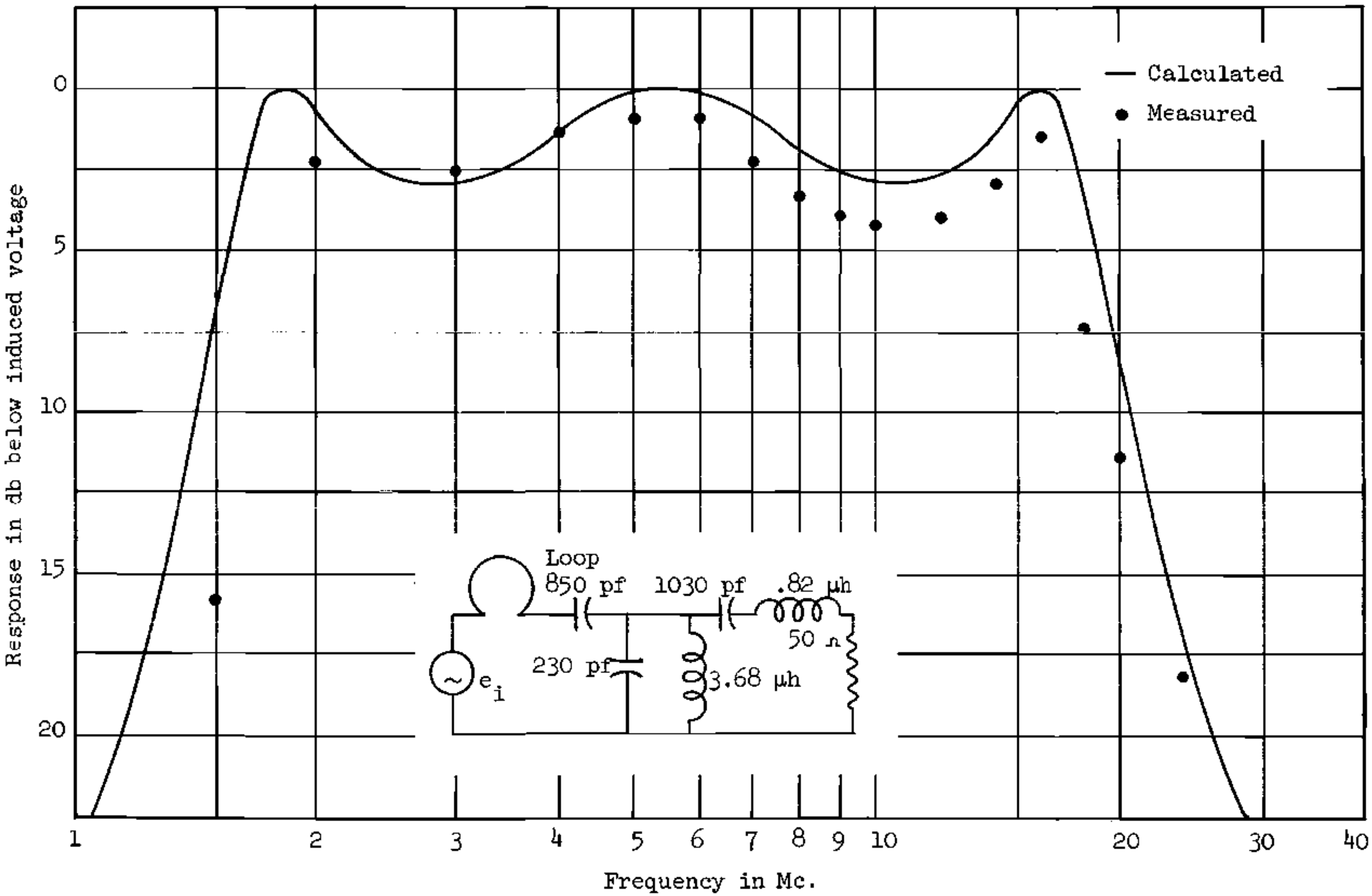


Figure 23. Response of a Three Element Chebychev Bandpass Filter with a 3 db Ripple (R_s = 0 ohms).

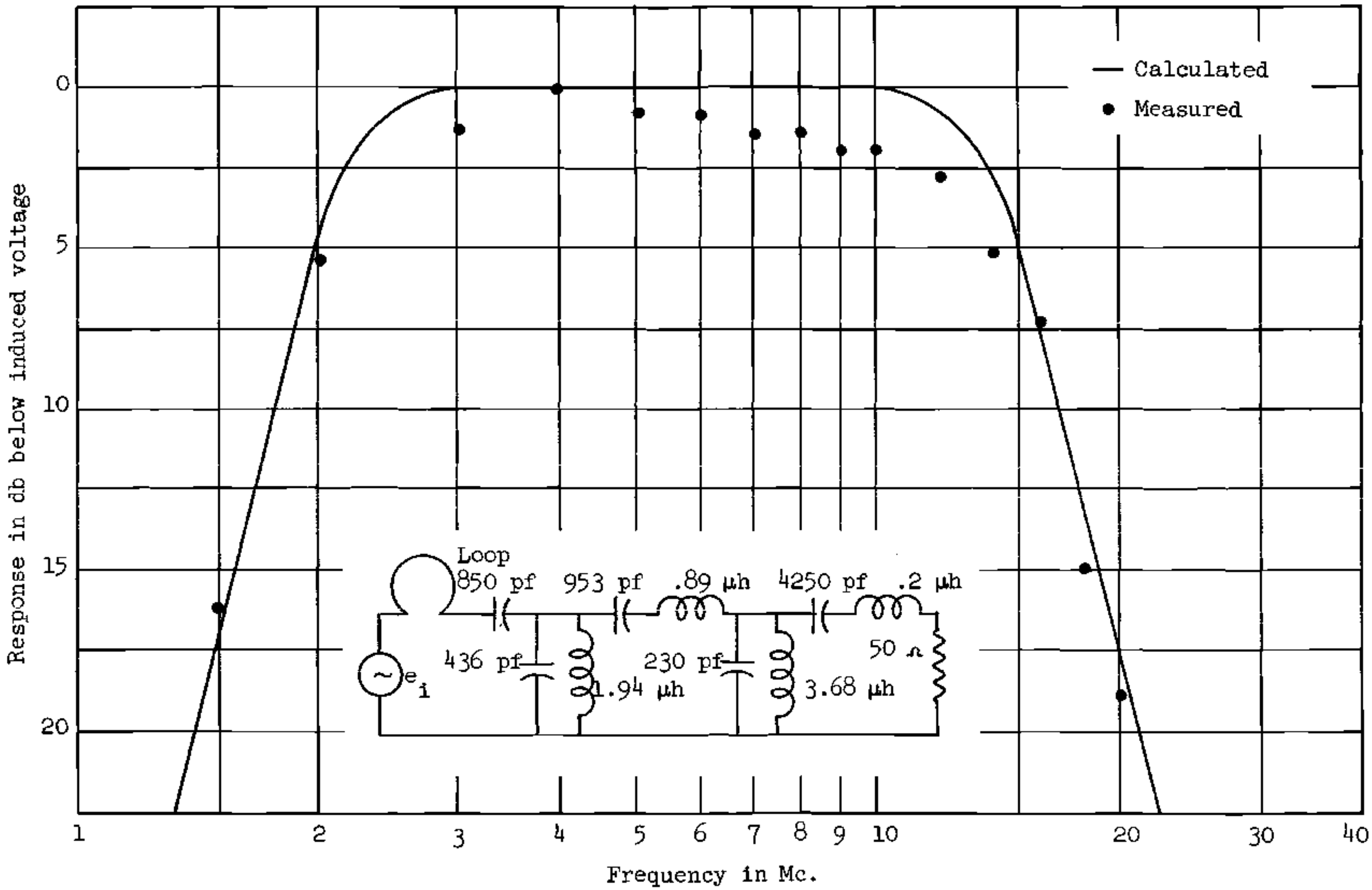


Figure 24. Response of a Five Element Butterworth Bandpass Filter (R_s = 0 ohms).

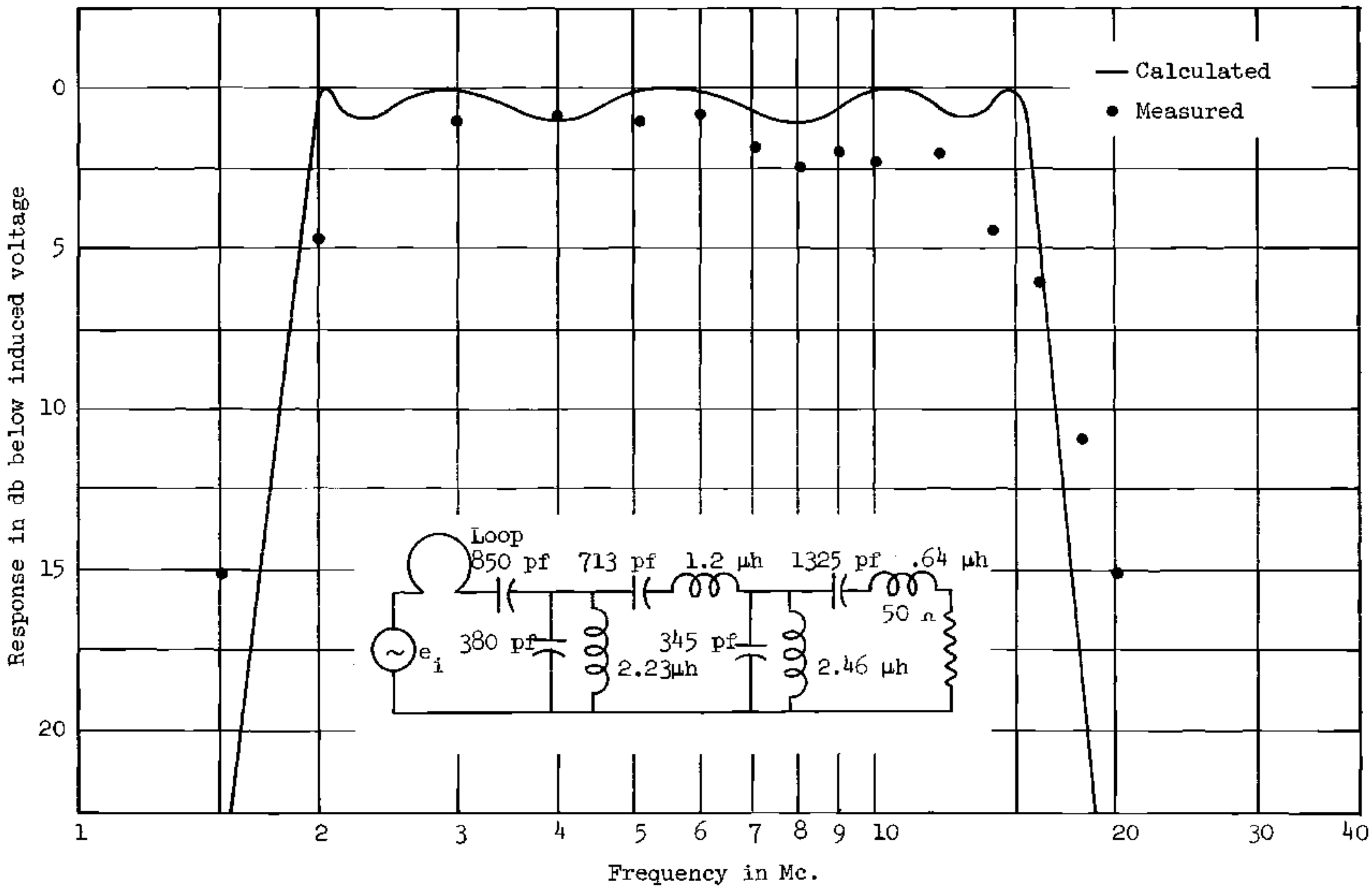


Figure 25. Response of a Five Element Chebychev Bandpass Filter with a 1 db Ripple (R_s = 0 ohms).

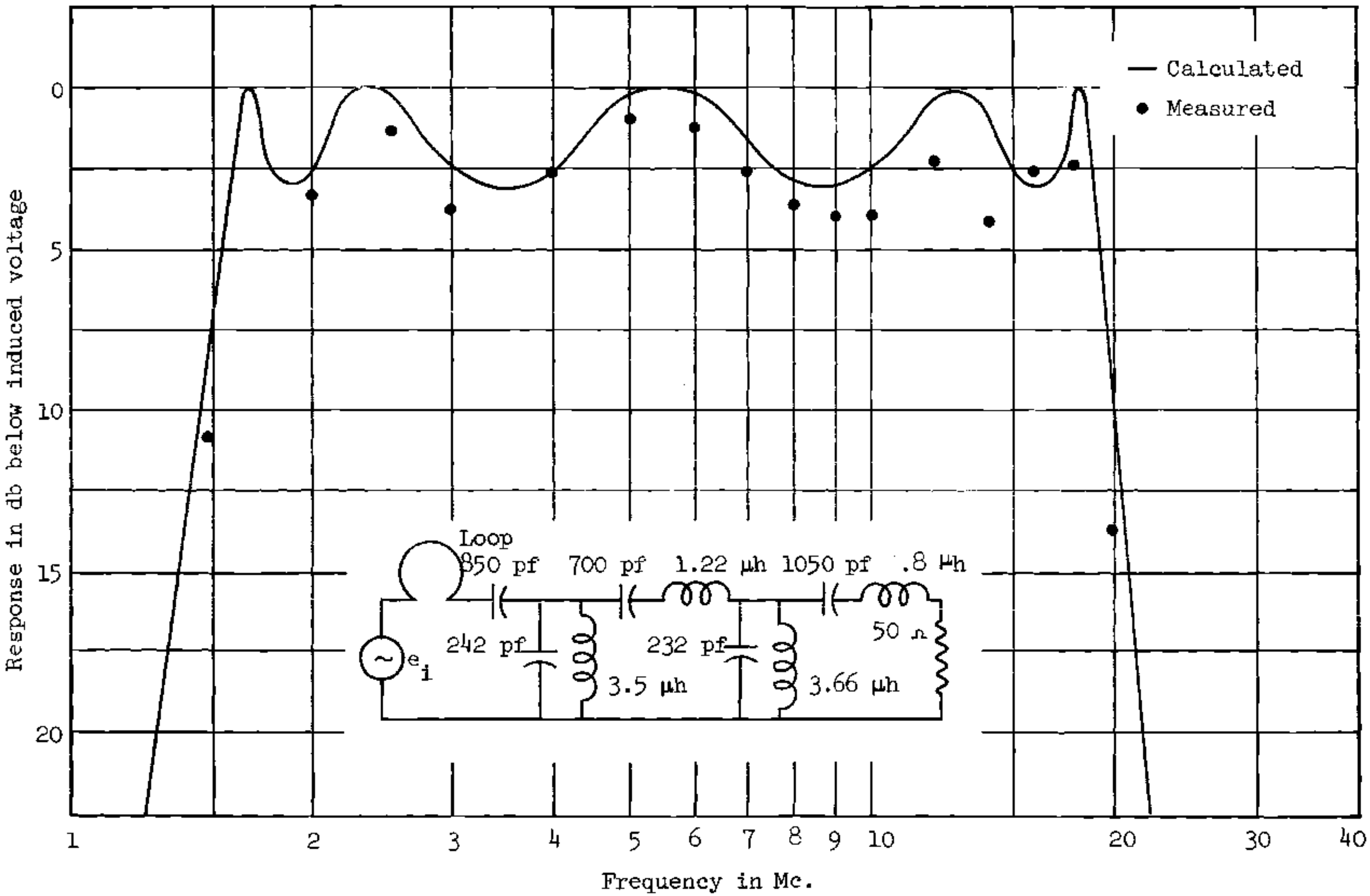


Figure 26. Response of a Five Element Chebychev Bandpass Filter with a 3 db Ripple (R_s = 0 ohms).

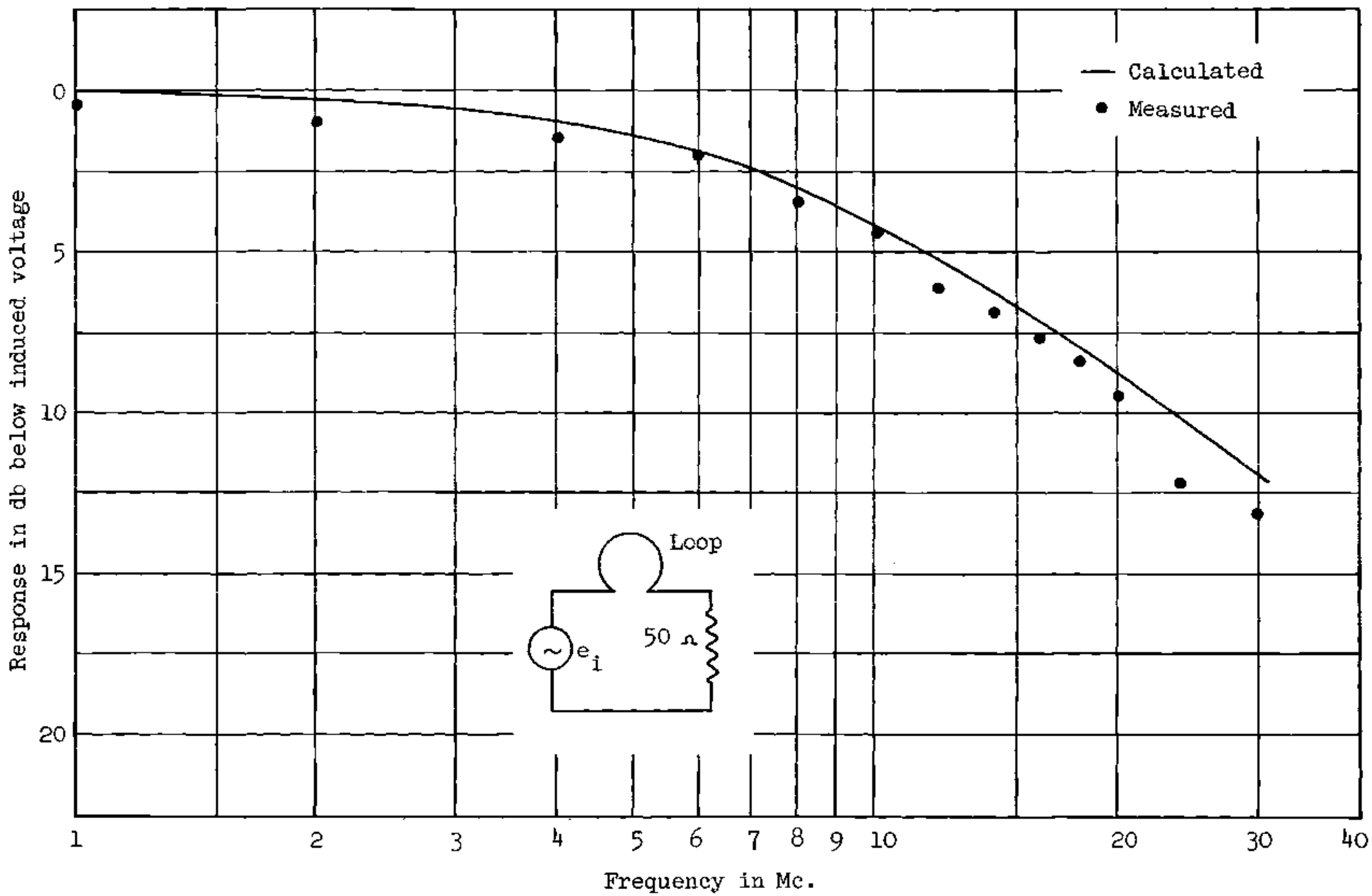


Figure 27. Response of a 12 inch Loop Antenna

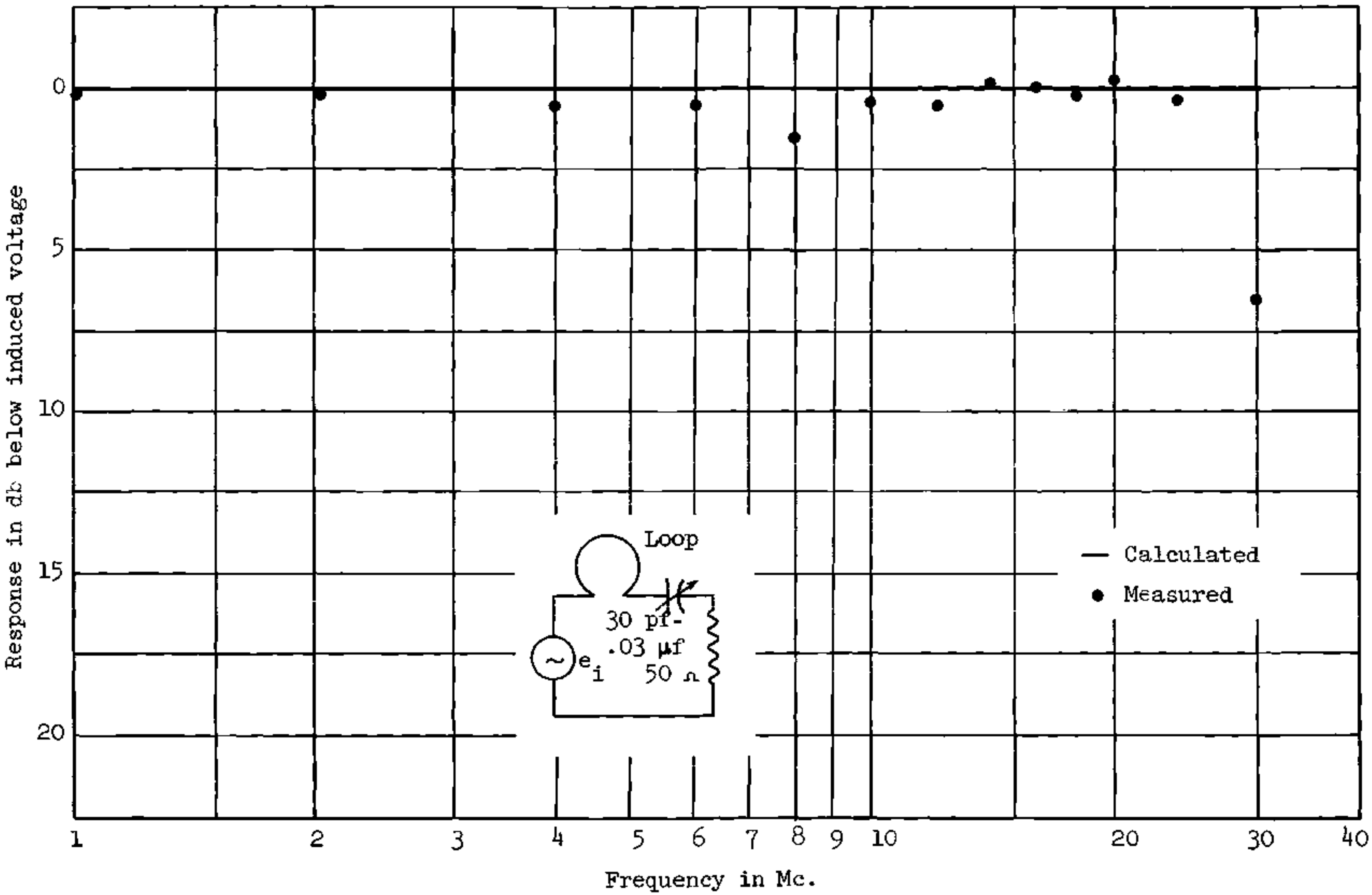


Figure 28. Response of a 12 inch Loop Antenna with a Capacitive Trimmer.

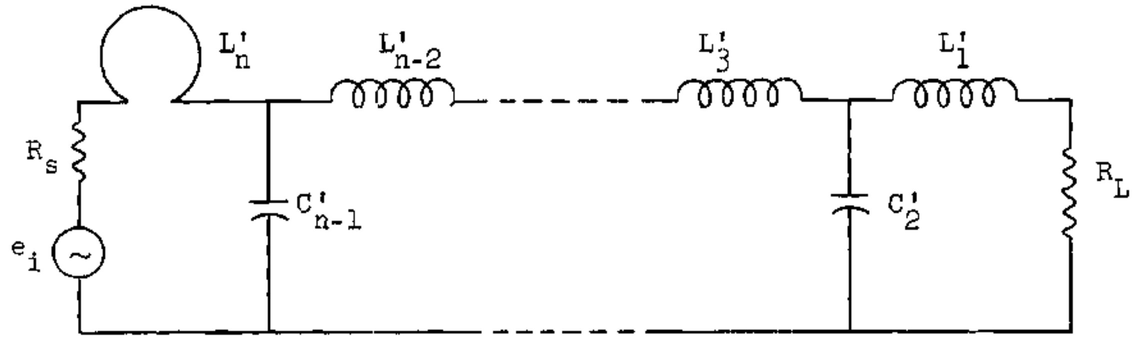


Figure 29. General Form of Lowpass Network for n odd.

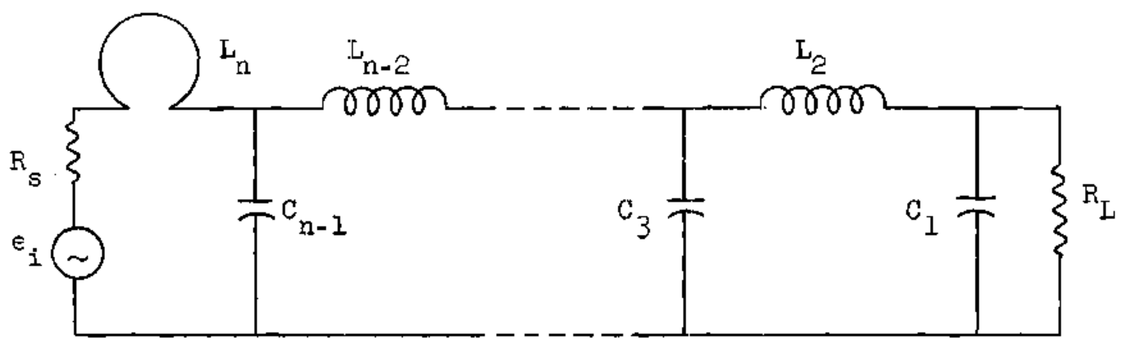


Figure 30. General Form of Lowpass Network for n even.

Table 3. Element Values for a Normalized Butterworth Filter*

n	C_1L_1'	L_2C_2'	C_3L_3'	L_4C_4'	C_5L_5'
		(a) r = 0			
1	1.000				
2	0.707	1.414			
3	0.500	1.333	1.500		
4	0.383	1.082	1.577	1.531	
5	0.309	0.894	1.382	1.694	1.545
		(b) r = 1			
1	2.000				
2	1.414	1.414			
3	1.000	2.000	1.000		
4	0.765	1.848	1.848	0.765	
5	0.618	1.618	2.000	1.618	0.618

Table 4. Element Values for a Normalized Chebychev Filter with a 1 db Ripple ($\epsilon = 0.5088$)**

n	C_1L_1'	L_2C_2'	C_3L_3'	L_4C_4'	C_5L_5'
		(a) r = 0			
1	0.509				
2	0.911	0.996			
3	1.012	1.333	1.509		
4	1.050	1.413	1.909	1.282	
5	1.067	1.444	1.994	1.591	1.665
		(b) r = 1 (see note)			
1	1.018				
3	2.024	0.994	2.024		
5	2.135	1.091	3.001	1.091	2.135

Note: n even is not physically realizable for this value of r.

*This table is based on data found in Weinberg, Network Analysis, p. 604-605.

**This table is based on data found in Weinberg, Network Analysis, p. 612-613.

Table 5. Element Values for a Normalized Chebychev Filter with a 3 db Ripple (ϵ = .9976)*

n	C_1L_1'	L_2C_2'	C_3L_3'	L_4C_4'	C_5L_5'
			(a) r = 0		
1	0.998				
2	1.551	0.911			
3	1.674	1.740	2.030		
4	1.720	1.229	2.527	1.058	
5	1.741	1.250	2.623	1.302	2.149
			(b) r = 1 (see note)		
1	1.995				
3	3.349	0.712	3.349		
5	3.481	0.762	4.538	0.762	3.481

Note: n even is not physically realizable for this value of r.

*This table is based on data found in Weinberg, <u>Network Analysis</u>, p. 616-617.

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