Radiosity Methods for Volume Rendering

by

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1. Introduction

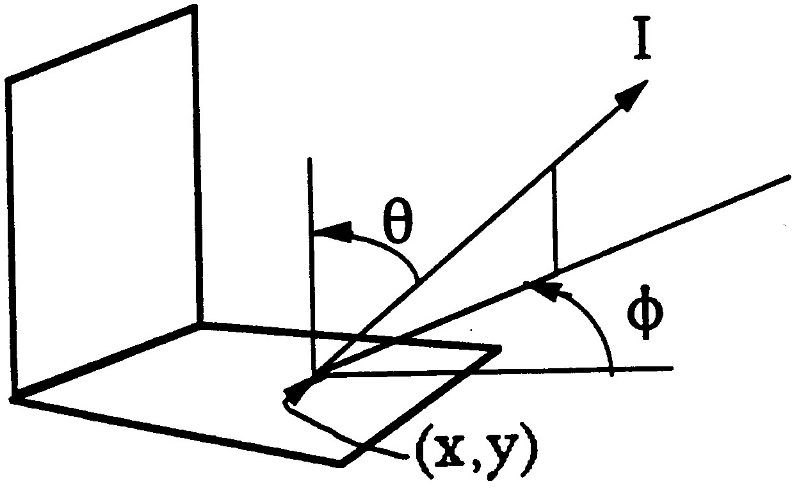
Radiosity methods are techniques for calculating radiative transfer. They were originally introduced in the field of heat transfer, and are described in many heat transfer textbooks (e.g. the undergraduate text by Incropera and Dewitt (1990), or the graduate text by Siegel and Howell (1981)). Since calculating global illumination is a radiative transfer problem, radiosity methods have been adapted to image synthesis, beginning with the work of Goral et al. (1984) and Nishita and Nakamae (1985). Over the past seven years many papers describing variations of the radiosity method for rendering surfaces have been published. In these notes, the use of these variations for rendering volumes will be considered. This material can be found in more detail in Rushmeier and Torrance (1987) and Rushmeier (1988).

2. Review of the Basic Radiosity Method for Surfaces

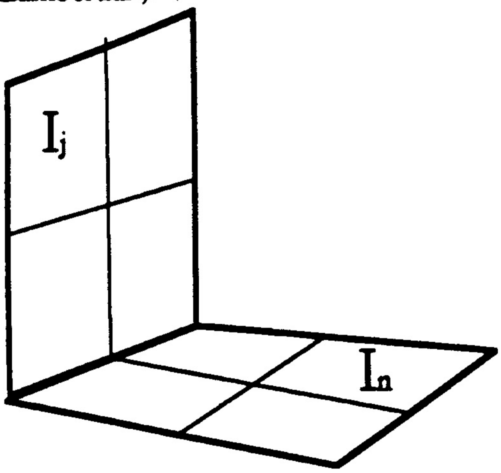
The radiance of a point on a surface is given by:

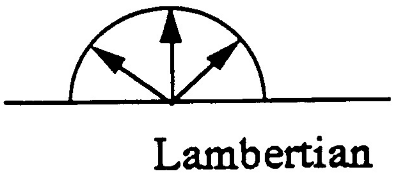
$$I(x,y,\theta,\phi) = I_e(x,y,\theta,\phi) + \int \rho_{bd}(\theta,\phi,\theta_i,\phi_i)\, I_i(x,y,\theta_i,\phi_i) \cos\theta_i \, d\omega \quad (1)$$

where (x,y) is a location on the surface, (θ,ϕ) are angles indicating direction, I_e = self emitted radiance, ρ_{bd} is the bidirectional reflectance, $d\omega$ is a differential solid angle, the subscript i indicates incident, and the integral is over the entire hemisphere of directions.



In the basic radiosity method, I is assumed to be uniform across each discrete surface, and independent of direction. The reflectance is assumed to be independent of direction (i.e. Lambertian).



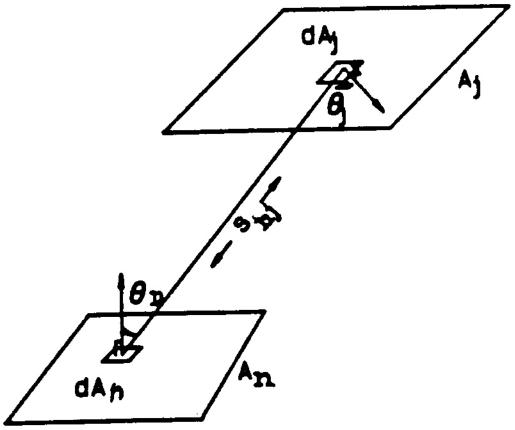


As shown in Kajiya (1986) and Rushmeier and Torrance (1990) the result of applying these assumptions to Eq. (1) is the radiosity equations:

$$I_n = I_{e,n} + \rho_n \sum I_j F_{nj} \tag{2}$$

where the subscripts n and j indicate values associated with surfaces n and j, and F_{nj} is the form factor relating n and j, given by:

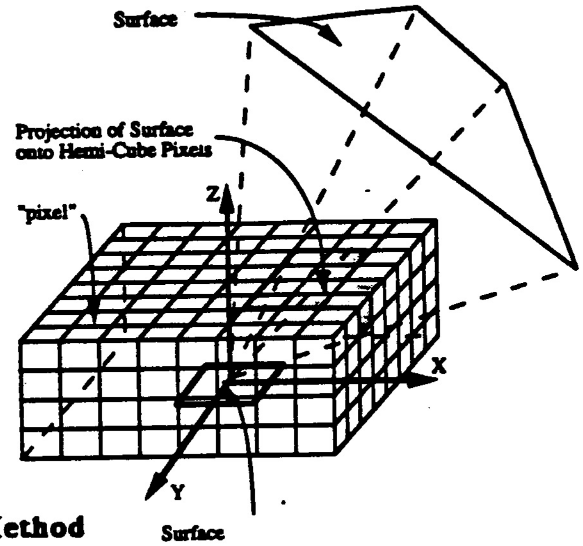
$$F_{nj} = (1/A_n) \iint \cos\theta_n \cos\theta_j \, dA_n \, dA_j / \pi s_{nj}^2 \tag{3}$$



The term radiosity is actually defined to be the energy per unit area and unit time leaving a surface. For a Lambertian surface the radiosity B is just p times the surface radiance I. Equation (2) can be written for surface radiosities simply by replacing the I's with B's.

A major portion of the calculation time in the radiosity approach is the calculation of form factors. One popular approach for this calculation introduced by Cohen and Greenberg (1985) is the

the center of a surface, and finding which surface is visible through each hemi-cube pixel. The form factor is equal to the "delta" form factors associated with each pixel through which a surface is seen.

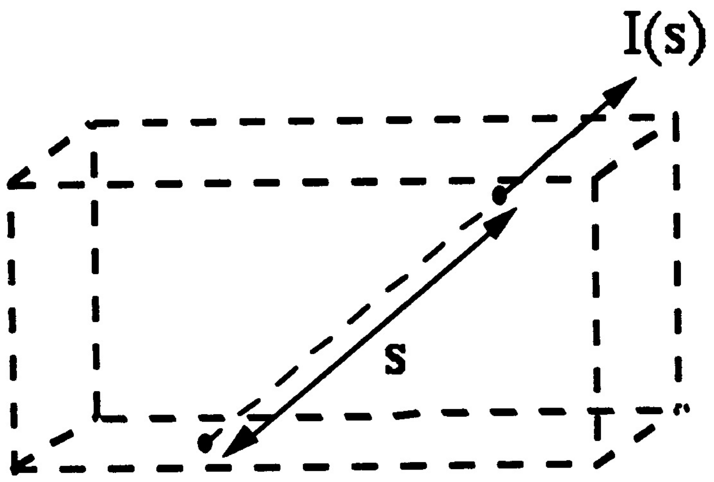


2. Radiosity for Volumes -- the Zonal Method

The equation for the radiance along a path in a general participating ambient medium, in the direction of the path can be written:

$$I(s) = I(s^{*}=0)\tau(s) + \int \kappa_t \, \tau(s-s^{*}) \, J(s^{*}) \, ds^{*} \tag{4}$$

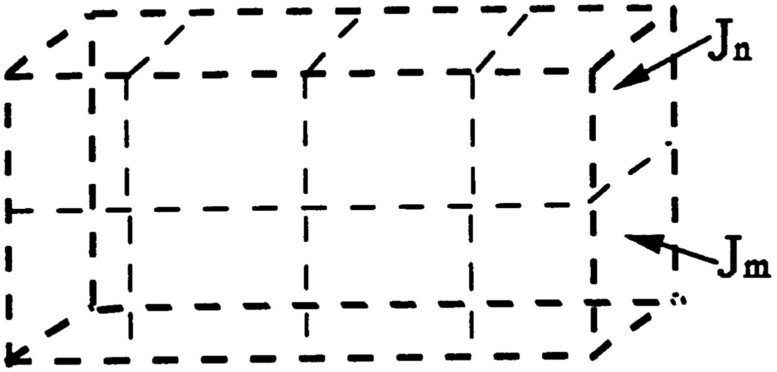
where κ_t is the coefficient of extinction for the medium, the fraction by which radiance is reduced by absorption and scattering per unit distance traveled in the medium, $\tau(s)$ is the transmittance through a distance s $J(s)$ = the source radiance at a point s, that is the radiance emitted or scattered at that point.



J is given by the following equation:

$$J(s) = (1-\Omega)\, I_e(s) + \Omega \int \Phi(\theta,\phi,\theta_i,\phi_i)\, I_i\, d\omega \qquad (5)$$

where Ω is the scattering albedo of the medium (the ratio of the scattering coefficient to extinction coefficient), and Φ is the scattering phase function, the fraction of scattered energy incident from a direction which is scattered into a second direction.



Completely analogously to the radiosity method for surface, the radiosity method for volumes is formed by assume J is uniform over discrete volumes and does not depend on direction. Analogous to the Lambertian assumption for surface reflectance, scattering is assumed to be isotropic, that is the phase function is the constant $1/4\pi$. An equation for the source radiances is then given by:

$$4\kappa_t J_k = 4\kappa_a J_{e,k} + \Omega_k/V_k\,(\Sigma I_j\, \underline{SjVk} + \Sigma J_m\, \underline{VmVk}) \qquad (6)$$

and the equation for surface radiances is given by:

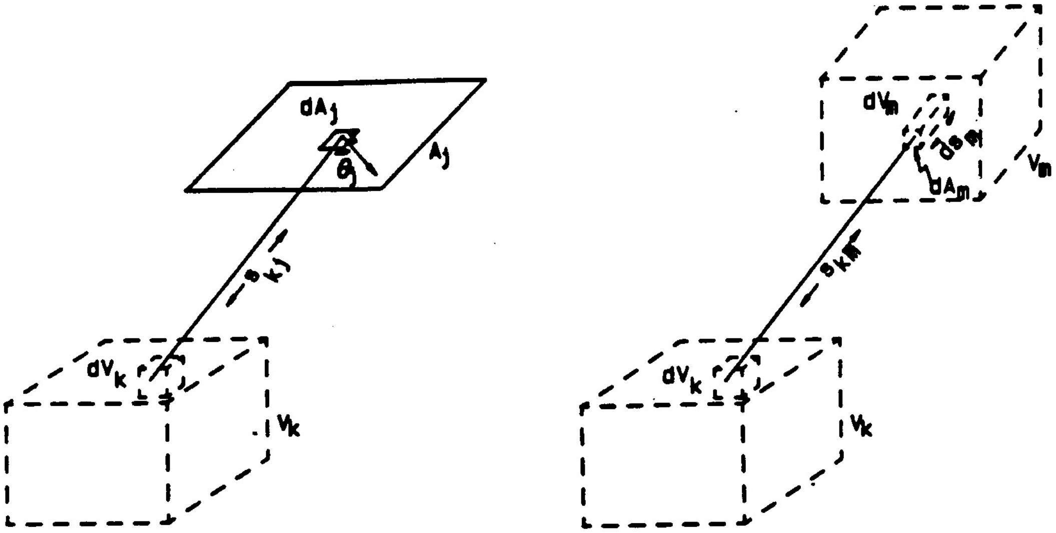
$$I_n = I_{e,n} + \rho_n/A_n\,(\Sigma I_j\, \underline{SjSn} + \Sigma I_k\, \underline{VkSn}) \qquad (7)$$

The form factors appearing in Eqs. (6) and (7) are given by:

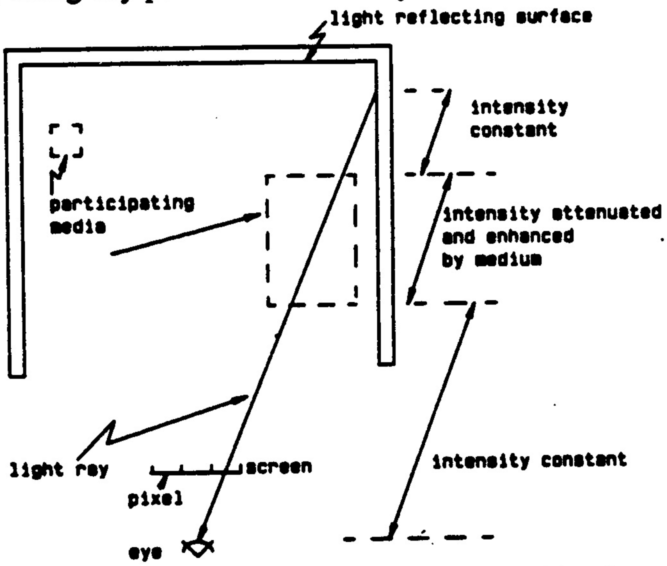
$$\underline{SjSn} = \iint \tau(s_{jn}) \cos\theta_j \cos\theta_n\, dA_j\, dA_n/\pi s_{nj}^2 \qquad (8)$$

$$\underline{VkSj} = \underline{SjVk} = \iint \tau(s_{jk})\kappa_t\, dV_k \cos\theta_j\, dA_j/\pi s_{kj}^2 \qquad (9)$$

$$\underline{VkVm} = \iint \tau(s_{km})\, k_{t,m} k_{t,k}\, dV_k dV_m/\pi s_{km}^2 \qquad (10)$$



Once all the values of I's and J's are known, I(s) along any path can be found by a straightforward integration of Eq. (4).

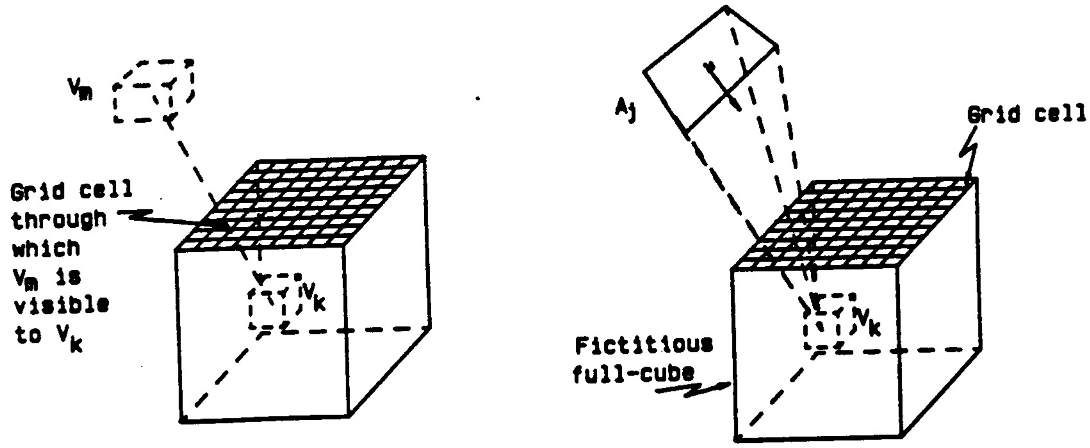


The radiosity of a volume is defined to be the energy/time and area emitted and or scattered by the volume, and it is equal to πJ. So, analogous to the radiosity method for surfaces, Eq.(6) can be written in terms of volume radiosities by substituting B's for J's.

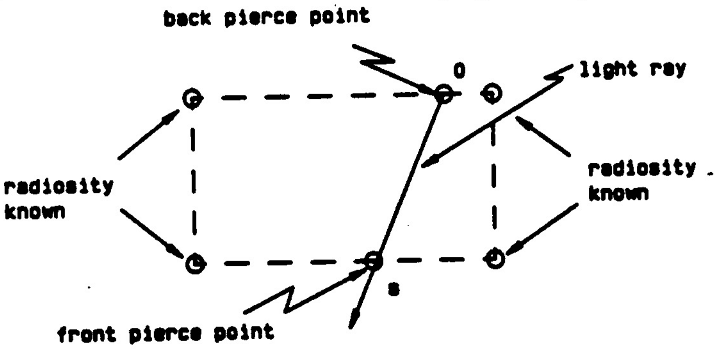
The radiosity method for volumes was originally developed for heat transfer analysis by Hoyt Hottle (Hottel and Sarofim, 1967), and is generally referred to in the heat transfer literature as the zonal method.

An important feature of the zonal method is that the method does not solve for the total radiance leaving each volume (which would include an extremely complicated distribution of radiances arriving from other surfaces and volumes in the environment). This is a strength of the method versus others used in heat transfer (i.e. six flux or discrete ordinate) because the discretization of the volume has minimal impact on distorting radiative paths through the volume. It is a weakness in the sense that walkthroughs of an environment can't be rendered simply by displaying polygons with workstation hardware. Rendering requires performing a line integral at each pixel.

The form factors for the zonal method can be calculated using variations of the hemi-cube method. The surface to surface factors are modelled essentially as before, with the exception that the delta form factors have to be attenuated by the value of transmittance along the path between the surfaces. Volume to surface factors are calculated using a full cube rather than a hemi-cube, with delta form factors which vary from the surface to surface factors by a cos θ factor.



The rendering of a scene can be performed by any path integration method. In Rushmeier and Torrance (1987) the method used was to use z-buffer projections, rather than a ray tracing.



Fill a z-buffer and an item buffer with all the surfaces visible for a particular view;
Fill a buffer *accum_rad* with the radiances of all visible surfaces;
For each volume in the environment, from back to front {
    Project the bounding surfaces on the back of the volume into a buffer;
    Project the bounding surfaces on the front of the volume into a buffer;
    For each pixel covered by the volume which is at least partially in front of the closest
        opaque surface{
        Interpolate κ_t and I for the back surface;

Interpolate κ_t and I for the front surface;
Attenuate *accum_rad* by multiplying by a transmittance calculated using
interpolated κ_t;
Increase *accum_rad* by adding in the integral of source radiance for this
volume;}}

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4. Applying Extensions of the Radiosity Method to Volumes

Many extensions and improvements to the basic radiosity method have been developed over the years. Most of these modifications can be grouped into form factor variations, solution speed ups, discretization methods and generalization of assumptions. We will look at each type of modification briefly, and examine its applicability to volumes.

4.1 Form Factor Variations

In heat transfer, many variations of the form factor calculation were developed prior to the application of radiosity to image synthesis and the development of the hemi-cube method These variations have in general been adapted in some form to image synthesis.

Contour Integration In contour integration the double area integral for the form factor is converted to a double line integral. This method was developed by Sparrow (see Sparrow and Cess (1970)) and was used by Goral et al. in their radiosity method. The line integral from a point to a plane polygonal has a closed form solution. This solution was used by Nishita and Nakamae (1985). Contour integration has not been applied to volume form factors. Its application to the factors SjSn is practical if the transmittance between the surfaces is assumed to be uniform.

Ray Tracing A general method for finding form factors is to divide surfaces up into very small surfaces, and approximate form factors by assuming that the cosine of the angles to the surface normals and the distances between surfaces are constant. Rays are traced to test for occlusion. In graphics this idea has been used by many researchers, including, Wallace et al. (1989). Ray tracing can equally well be applied to finding volume form factors. The major disadvantage is that occlusions between every possible pairing of surfaces, surfaces and volumes and pairs of volumes need to be considered. There are several remedies for this problem. For example, the radius of volumes considered for each volume may be limited, as in Borel and Gerstl (1991).

Hemi-cube Variations A large number of hemi-cube variations have been suggested. One idea is to replace the hemi-cube with a plane to reduce the number of projections needed for each surface (Sillion and Puech, 1989). The justification for the plane is the size of $\cos\theta$ for glancing angles. Since the $\cos\theta$ term doesn't appear in the volume factor integrals, the plane is not suitable for calculating the zonal method form factors.

4.2 Solution Speedups

The most important method for speeding up calculations is progressive refinement radiosity (Cohen et al. 1988). In this method, instead of finding all the form factors and then solving simultaneous equations, factors are found "on the fly" as emitted or reflected energy is "shot out" surface by surface. Progressive refinement could be applied to volume radiosity. The advantage of being able to watch the solution evolve however would be lost since the volume solution can't be displayed by simply using hardware to display polygons. A compromise might be to display the surfaces without the medium during the solution, just to give an idea of what the environment looks like, and then add in the medium in a postprocess.

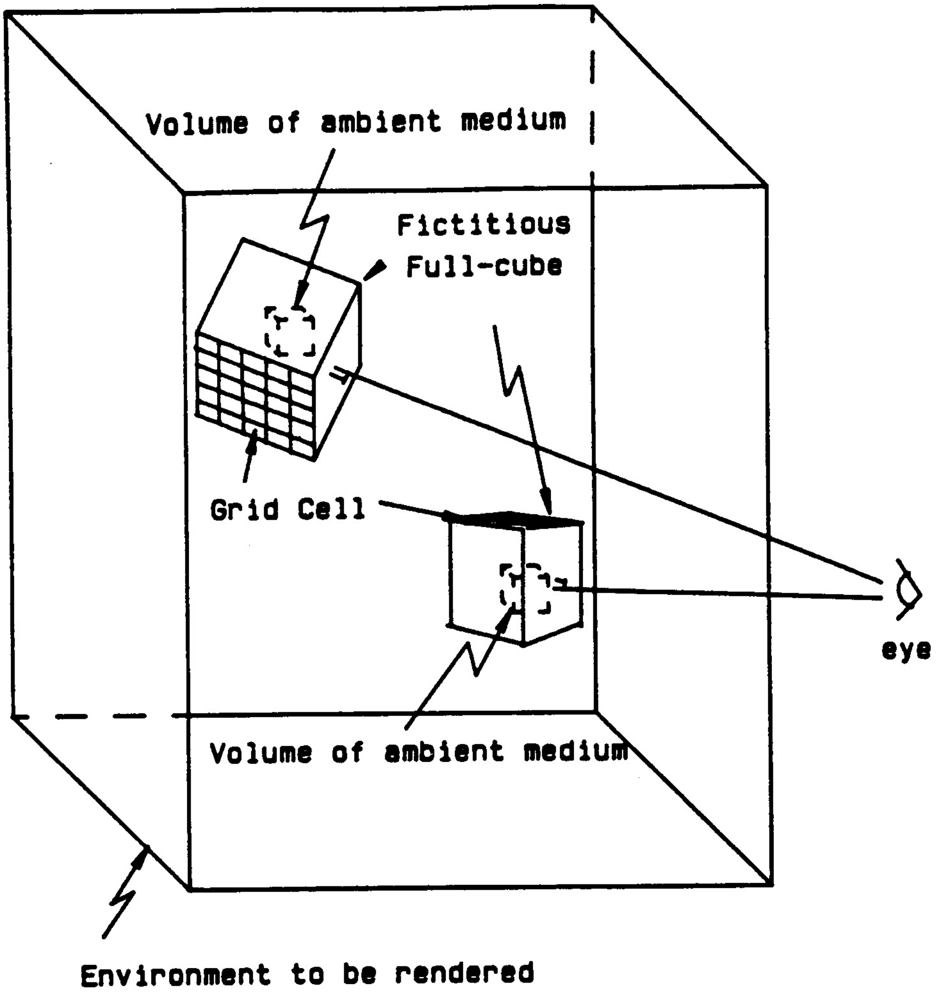
Another important speedup for animated sequences is the use of incremental radiosity presented by Chen (1990) . No work has been done in the area of animation of volume radiosity solutions.

4.3 Discretization Methods

A disadvantage of the radiosity method is the need to mesh all surfaces, with the quality of the mesh affecting the quality of the final image. Generally there are two phases of meshing -- the initial mesh, and the adaptive refinement of the mesh during the solution. A couple of methods for generating good initial meshes for surfaces have been developed (Campbell and Fussel 1990, Baum et al. 1991) but have not been applied to volumes. Adaptive refinement of the mesh for surfaces was presented by Cohen et al. 1985. In this case a complete solution is found for an initial mesh. The mesh is refined, and the radiances for the new smaller surfaces are calculated using the radiances of the larger surfaces in the initial mesh. This method has been applied to volumes to capture effects such as the sharp edges on a shaft of light shining through a medium (Rushmeier, 1988). In an extreme case, the initial solution can approximate the medium as a single discrete volume, with a more detailed surface discretization. The volume can then be rediscretized, and the new volume radiances calculated from the surface radiances found in the initial solution.

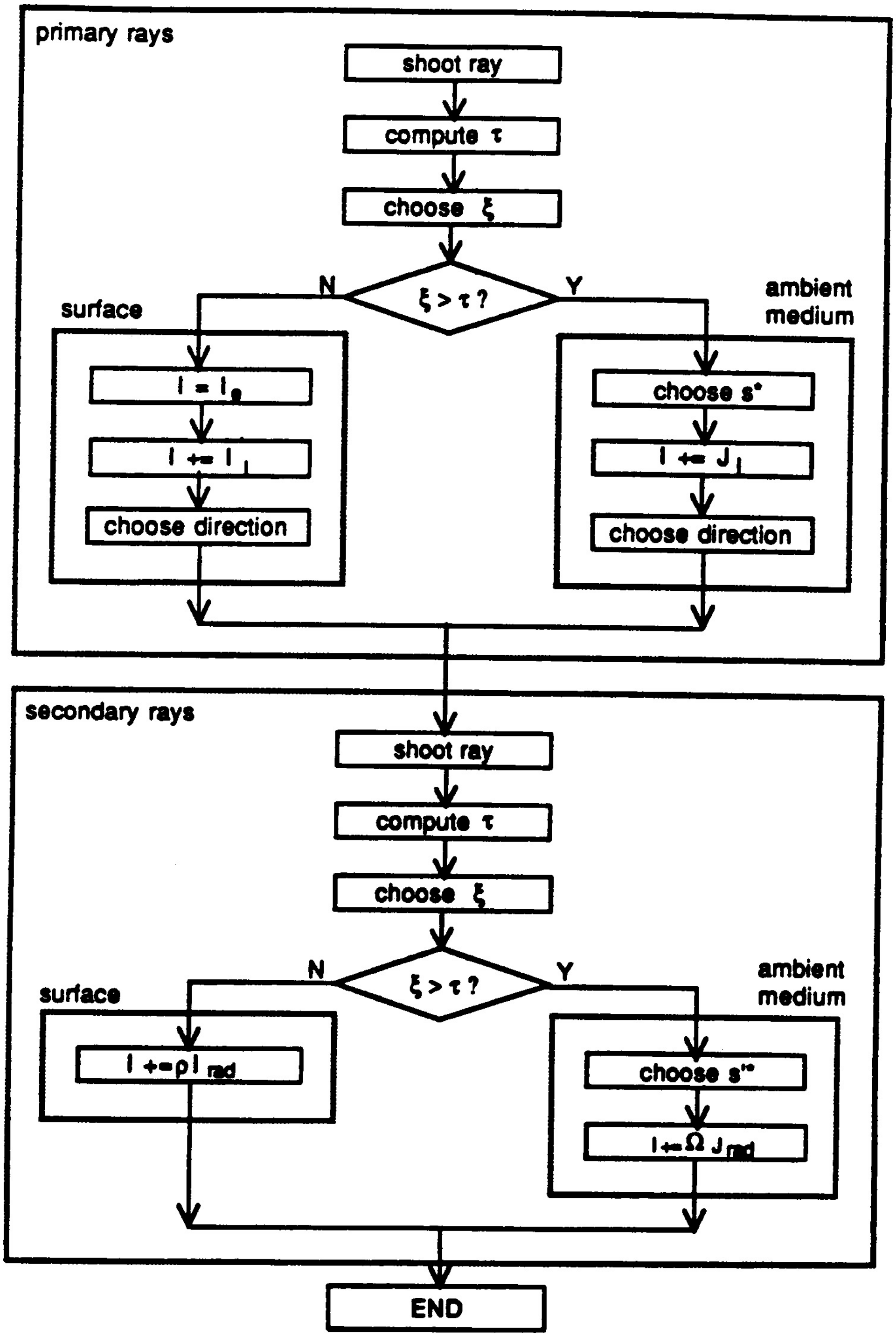
4.4 Generalization of Assumptions

Many methods have been developed to alter the basic radiosity method given in Eq. (2) so that it gives a result closer to the more accurate Eq. (1) in which directional variations are allowed. There are fundamentally two approaches. One is to discretize directions as well as surfaces. This was initially presented by Immel et al. (1986). A simple application of this type of approach to generalize the zonal method is to allow directionality in the last scatter to the user by gathering in light at each volume using a full cube aimed at the user. The delta-form factors on the full cube are weighted to account for the scattering phase function.



The alternative approach are the multi-pass methods in which ray tracing is used to "add in" directional effects to a radiosity solutions. Such multi-pass methods for surfaces have been presented by Wallace et al. (1987), Sillion and Peuch (1989), Shirley (1990), and others. In a sense, the basic application of the zonal method is a two pass method because the major directional portion of the radiance of each volume -- i.e. the light that passes through undeflected -- is added in by evaluating a path integral at the time of rendering. Beyond this, however, many other versions of a multipass method can be formulated to take into account the various interactions of directionally reflecting surfaces and directionally scattering volumes of media.

One multi-pass method is a combined zonal/ Monte Carlo method similar to the one described by Chen et al. (1991) for surfaces. The rendering is essentially a Monte Carlo solution, with the zonal solution only used for higher order reflection/scattering. The outline of the method is shown in the following flow chart :


primary rays
shoot ray
compute τ
choose ξ
ξ > τ ?
N
Y
surface
ambient medium
I = I_e
I += I_i
choose direction
choose s*
I += J_i
choose direction
secondary rays
shoot ray
compute τ
choose ξ
ξ > τ ?
N
Y
surface
ambient medium
I += ρ I rad
choose s'*
I += Ω J rad
END

5. References

Baum et al. (1991) "Making Radiosity Practical..." to appear in 1991 Proceedings of SIGGRAPH.

Borel, C.C. and Gerstl, S.A.W., (1991) "Simulation of Partially Obscured Scenes Using the Radiosity Method," SPIE Aerospace Sensing Symposium, Conference on Characterization, Propagation, and Simulation of Sources and Backgrounds, paper 1486-24, Orlando, FL.

Campbell, A.T. and Fussell, D.S. (1990)"Adaptive Mesh Generation for Global Diffuse Illumination," *Computer Graphics*, 24(3), pp. 155-164.

Chen, S.E. (1990) "Incremental Radiosity: An Extension of Progressive Radiosity to an Interactive Image Synthesis System," *Computer Graphics*, 24(3), pp. 135-144.

Chen et al. (1991) "A Progressive Multi-Pass Method ...." to appear in 1991 Proceedings of SIGGRAPH.

Cohen, M. F. and Greenberg, D. P. (1985) "The Hemi-Cube: A Radiosity Solution for Complex Environments," *Computer Graphics* 19(3) pp 31-40.

Cohen, M.F., Greenberg, D.P., Immel, D. and Brock, P. (1986) "An Efficient Radiosity Approach for Realistic Image Synthesis," *IEEE Computer Graphics and Applications*, 6(2), pp. 26-35.

Cohen, M.F., Chen, S.E., Wallace, J.R. and Greenberg, D.P. (1988) "A Progressive Refinement Approach to Fast Radiosity Image Generation," *Computer Graphics*, 22(4), pp. pp.75-84.

Goral, C.M., Torrance, K.E., Greenberg, D.P., and Battaile, B. (1984) "Modeling the Interaction of Light Between Diffuse Surfaces," *Computer Graphics* 18(4), pp 213-222.

Hottel, H. C. and Sarofim, A.F. (1967) *Radiative Transfer*, McGraw-Hill, NewYork, NewYork.

Immel, D., Cohen, M.F. and Greenberg, D.P. (1986) "A Radiosity Method for Non-Diffuse Environments," *Computer Graphics*, 20(4), pp. 133-142.

Incropera, F. P. and DeWitt, D. P. (1990) *Fundamentals of Heat and Mass Transfer*, 3rd Ed., John Wiley and Sons, New York, New York.

Kajiya, J.T. (1986) "The Rendering Equation" *Computer Graphics*, 20(4), pp. 143-150.

Nishita, T. and Nakamae, E. (1985) "Continuous Tone Representation of Three-Dimensional Objects Taking Account of Shadows and Interreflection," *Computer Graphics*, 19(3), pp. 23-30.

Rushmeier, H. and Torrance, K.E,. (1987) "The Zonal Method for Calculating Light Intensities in the Presence of a Participating Medium," *Computer Graphics* 21(4), pp.293-202.

Rushmeier, H. (1988) *Realistic Image Synthesis for Scenes with Radiatively Participating Media.* PhD thesis, the Sibley School of Mechanical and Aerospace Engineering, Cornell University.

Rushmeier, H.E,. and Torrance, K.E., (1990) "Extending the Radiosity Method to Include Specularly Reflecting and Translucent Materials," *ACM Transactions on Graphics*, 9(1), pp. 1-27.

Siegel, R. and Howell, J. (1981) *Thermal Radiation Heat Transfer*, 2nd Ed., Hemisphere, Washington, DC.

Shirley, P. (1990) "A Ray Tracing Algorithm for Global Illumination," *Graphics Interface '90.*

Sillion, F. and Peuch, C., (1998) "A General Two Pass Method Integrating Specular and Diffuse Reflection," *Computer Graphics* 23(3) p. 335-344.

Sparrow, E. M. and Cess, R.D. (1970) *Radiation Heat Transfer,* Brooks-Cole, Belmont, CA.

Wallace, J.R., Cohen, M.F. and Greenberg, D.P. (1987) "A Two-Pass Solution to the Rendering Equation: a Synthesis of Ray Tracing and Radiosity Methods," *Computer Graphics*, 21(4),pp. 311-320.

Wallace, J.R. , Elmquist, K.A. and Haines, E.A. (1989) "A Ray Tracing Algorithm for Progressive Radiosity," *Computer Graphics* 23(3), pp.335-344.