Nontrivial Impact

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Nontrivial Impact

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For Hanna, Steve, and Jeff, whose ceaseless support never failed to encourage.

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SUMMARY

The purpose of this investigation is to identify and calculate the forces that occur on an impulsive object as it intrudes into granular media. The system analyzed is a computational model of a sinusoidally actuated spring-mass system jumping on a bed of granular material. Various types of ground reaction forces on the robot's foot are investigated and their parameters are systematically varied to compare to experimental data taken from the real-world jumping robot system. Different types and combinations of ground reaction forces are investigated since a single force type was found to be insufficient to fully explain the experimental system's dynamics. The mechanics of this setup are modeled as a set of ordinary differential equations, which are computationally solved to determine the jumping mechanics. Maximal jump heights are calculated across a wide variety jumping motions and granular media densities with different types of ground reaction force laws.The relations that are investigated include a depth-dependent spring-like force, a velocity-squared-dependent force, and an added-mass force. The results of finding a well-fitting combination of force laws across many jump types and volume fractions can be used to imply a valid comprehensive force law for impulsive motion on a granular surface. The anticipated outcome is that there exists such a comprehensive force law, but each force type's contribution will vary as a function of volume fraction. Finding optimal jumping motions using this comprehensive law may lead to better implementations of impulsive commands in fields such as robotics and biomechanics where granular material is involved.

Chapter I

INTRODUCTION

The study of the biomechanics of animal locomotion and its interaction with complex media can result in discovery of new principles that constitute the media's physics [4]. Recent advances in computational methods and power have allowed for more advanced simulations of biomechanical motion [5]. In particular, the biomechanics of locomotion upon a granular material, where fluidization [3, 7], flow [3], and packing [7] of the material may occur, is our main area of interest. In this regime, the reaction force of the material on the surface of locomotion is not trivial to calculate [3].

Research in biomechanics includes the investigation of jumping on a hard surface. A few studies examining the biomechanics of this motion have asserted that a simple spring-mass model can be sufficient to describe the complex muscular interactions involved in this motion [8, 2]. The primary paper on which this proposal is based [1], specifically studies the jumping dynamics of such a spring-mass system and concludes that a simple squatting maneuver results in an optimal jump height above the spring-mass system's resonant frequency, and a stuttering jump with a countermovement is optimal below the system's resonant frequency [1]. A similar result was found in van Werkhoven and Piazza's work, who found bouncing using countermovements in succession gave an optimal jump height [8].

To the best of our knowledge, most projects focusing on jumping in the fields of sports physiology and biomechanics are done with jumping on a hard surface [8, 2, 6]. Within the existing studies of jumping on sand, the comprehensive investigation of the physics of granular reaction forces is limited. There is a gap of study in the detailed mechanics of jumping on a granular surface, particularly on the impact of forces in granular media and optimal jump maneuvers, so we propose to combine the two areas by asking what kind of ground reaction forces exist in sand-jumping, and as a result, what optimal jumps arise?

To answer this question, we will study the reaction forces of a one-dimensional jumping robot, as used in Aguilar et. al [1], but now jumping on a bed of $\sim 1mm$ diameter poppy seeds. Experimental data for jump heights over various actuation commands and volume fractions were collected and analyzed. In parallel, a computational model for the robot was created with the purpose of finding a sufficiently reliable ground-force law. Through the computational model, we hope to achieve understanding of the correlations of volume fractions and actuation commands, thus deriving a ground-force law and finding optimal jump patterns for granular media. This investigation will result in a clearer understanding of the reaction forces of sand under a jumping-type impact. The discovery of an optimal jump pattern could be useful for applications in other fields as well, such as biomechanics, sports physiology, and robotics.

Chapter II

LITERATURE REVIEW

2.1 Jumping Robot Foot and Dense Granular Flow

Robotic mechanical systems can be used as a model for the complex interactions that occur in animal locomotion [5]. In addition, a simple computational model of the robotic system can greatly assist in the speed of that system's analysis. Understanding the physics behind both the mechanical and computational models of the locomotion process can aid in the development of more optimal robotic designs and commands, especially for machines with locomotion strategies inspired by nature. Specifically, we seek to understand the physics behind the lift-off dynamics of jumping vertically on a bed of granular media.

There are studies that approach this scenario, but none were found that use a purely physical approach in modeling the system. Muramatsu et al. performed a clinical trial on humans using physiological methods [6], and found that jumping on a granular, yielding surface resulted in both a lower jump height and a higher energy expenditure. The quantitative results of this paper reaffirm the common belief that jumping from sand is overall more difficult that jumping from a hard surface. However, their methods only involved data collection and a statistical analysis of the jumpers' jump heights and energy expenditures, calculated via oxygen uptake. No analysis of the ground reaction forces and dynamics behind the jumping on a yielding surface was made. The volume fraction of the granular surface was also not varied. Aguilar et al. conducted a study which becomes the primary inspiration and source for the current project, as it used the same jumping robot [1]. He analyzed the lift-off dynamics of a simple one-dimensional robot consisting of an actuated motor and a spring-mass arrangement, which was made to jump on a hard surface. The actuator moves like an oscillator, and its frequency and phase were varied to find the optimal performance for jumping. Two optimal jump modes were found, a simple squat jump, and a peculiar stutter jump. The stutter jump included a countermovement to its initial squat that ended up making it jump higher than the simple jump. It is interesting to note that these optimal jumps did not occur at the robot's resonant frequency for its spring-mass system. This paper developed explicit equations for the dynamics of motion for the spring-mass system, and emphasized the crucial point that the system is piecewise-linear, since the equation of dynamics changes instantaneously depending on whether the robot's foot is touching the ground. This renders the equations of motion analytically difficult to solve and opens up a realm of possibilities for chaos, hysteresis, and complex dynamics. This paper provides an excellent starting point for the current project, as it defines the dynamics of motion for this system jumping on hard ground, but we now desire to introduce variable ground reaction forces on a soft surface, which may greatly influence the jump heights and motion of the system.

Other preceding studies have been done that verify points made in Aguilar et al.'s conclusions. Blickhan [2], whose spring-mass model we use, shows that a simple spring-mass system is a sufficiently similar analog to the complicated biomechanics of hopping, that the total ground reaction force is proportional to the foot's contact period, and that the equations of dynamics should be piecewise-linear, separated into contact and aerial phases. However, Blickhan's model for hopping differed in that a massless spring was used in Blickhan's system, with the spring also acting as the contact point with the ground, whereas Aguilar et al.'s model has a contact surface with mass. Also, the surface on which the jumping was performed was hard and unyielding. Another study conducted by van Wekhoven et al. showed that a countermovement is needed to produce an optimal jump height [8], which agrees with Aguilar et al.'s conclusion in regimes below the system's resonant frequency. However, a fundamental distinction must be made that Wekhoven's paper used multiple bounces and optimized for a periodic actuation frequency to induce bouncing, while Aguilar et al. used a single countermovement and systematically varied the actuated motor's actuation frequency, i.e. how quickly the motor executed its single periodic motion.

Having examined the optimal lift-off dynamics for jumping off of hard ground, it is of interest to extend the analysis and examine what occurs at different packing states in granular media. Many animals locomote in diverse terrain in the wild, and the varied mechanics of their jumping from a granular surface is interesting. To examine these mechanics, it is essential to determine a ground reaction force relation that describes how exactly the sand is pushing back on the intruding foot. Umbanhowar and Goldman introduced a ground reaction force relation that we use in modeling our robot foot near the critical packing state [7], where force is separable into additive terms that are linear in depth and quadratic in velocity. At volume fractions farther from the critical packing state, where the media does not remain constant under shear, the canonical force law for the ground reaction force is not sufficient. However, a relatively unexplored possibility is that of an added mass force, a concept used in fluid dynamics. This is an additional impact force we are investigating where the compacted media anneals the impact surface, and thus adds an additional mass to the impactor, which is affected by gravity.

De Gennes [3] provides a good elaboration via his own paper on granular material, where he introduces some of the essential concepts and terminology for the complex dynamics of granular material, such as volume fraction, critical packing, the fluid and frozen phases, and the model of macroscopic stress fields. Determining which of these models applies to the jumping robot will be essential in describing the mathematics of the added mass term of the ground reaction force. The current study will endeavor to find a sufficiently accurate force law for the spring-mass robot jumping on a granular, yielding surface of varying volume fractions. We hope to find a relation that includes depth dependent, intrusion velocity dependent, and/or added mass terms. This search will be carried out through varying parameters in a computational model of the jumping robot and periodically refining the model such that it matches the experimental output of the real-world system across a wide variety of volume fractions. If a good fit between the model and physical system is found, we can determine a more accurate representation of the underlying mechanics of jumping on granular media, and extend those mechanics to other applications.

Chapter III

METHODS & PROCEDURES

We use the MATLAB library of Simulink to computationally model the actuated spring-mass system as a system of ordinary differential equations. We define coupled equations of position for the actuated rod, the robot foot, and the level of granular material in the bed. The three main independent variables that enter into the model are the motor's positional command, the volume fraction of the granular material, and the ground reaction force law that dictates what force the granular material will exert on the intruding foot. By systematically varying these three independent variables and comparing the resultant simulated jump heights to experimental data from the same jumping command and volume fractions, we hope to find relations that will give an understanding of the ground reaction force across a variety of volume fractions and jumping commands.

3.1 Volume Fractions

Volume fraction is a term used to define the packing density of granular media. It is defined as the volume of the material divided by the occupied volume. At the critical packing state (CPS) of the granular media, Goldman and Umbanhowar suggest that the ground reaction force decouples into additive terms which are linear with respect to depth and quadratic with respect to velocity when using a spherical intruder. A more fundamental definition of CPS is that it is the point at which the granular media transitions from consolidating (in loose packed) to dilating (in close packed) upon undergoing shear from an intruder. We determine that CPS was approximately 0.612 ± 0.001 by dropping a ball into the media at different volume fractions, comparing the volume of the media before and after the ball drop, and defined CPS as the volume fraction in which there was zero change in volume (volume change transitioned from being negative at low volume fraction to positive at high volume fraction). We systematically vary our volume fractions in the simulation from 0.02

below CPS to 0.02 above CPS, giving a range of tested volume fractions of 0.592-0.632. This range gives a regime of both comparatively low (0.592) and high (0.632) volume fractions from CPS.

3.2 Jumping Commands

We test five types of jumping commands as inputs for the actuation motor. These varying actuation commands will generate different accelerations on the actuated rod, which will result in different kinds of jumps. The jumping commands we test are:

3.2.1 Single Jump

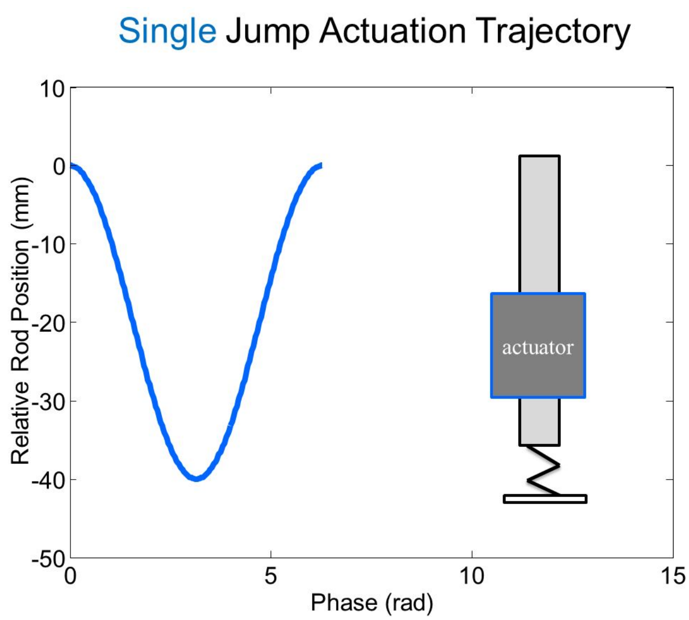

Figure 1: The actuation trajectory for a single jump.

A single cycle of actuation that consists of one period of a sinusoidal wave. The amplitude of the motion is 40 mm, in the frame of reference of the actuator relative to the thrust rod. The phase offset of the wave is $3\pi/2$.

3.2.2 Preload Jump

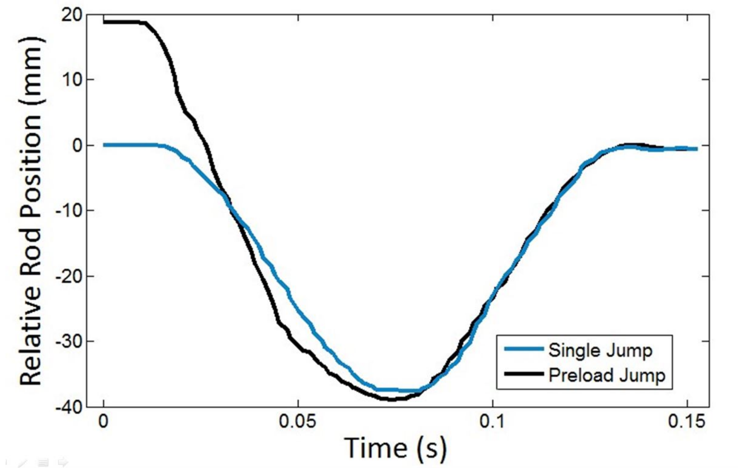

Figure 2: Comparative rod trajectories for the single jump vs. the preload jump.

A single cycle of actuation that consists of one period of a sinusoidal wave, starting from a negative relative position, pushing into the granular material and loading the spring before executing a jump. Figure 2 shows the resultant rod trajectories of a single jump versus a preload jump. As seen in the figure, the preload jump begins its trajectory offset from the single jump as it has loaded the spring as it pushed into the media.

3.2.3 Stutter Jump

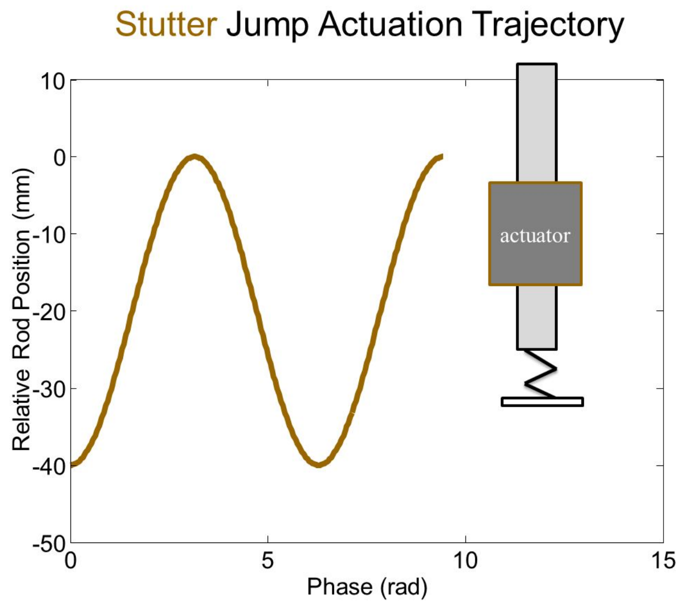

Figure 3: The actuation trajectory for a stutter jump.

Generates 1.5 cycles of sinusoidal actuation starting from positive amplitude relative to thrust rod, phase offset by $\pi/2$; essentially executes a countermovement into the granular material before executing a jump with a preliminary hop.

3.2.4 Delayed Stutter Jump

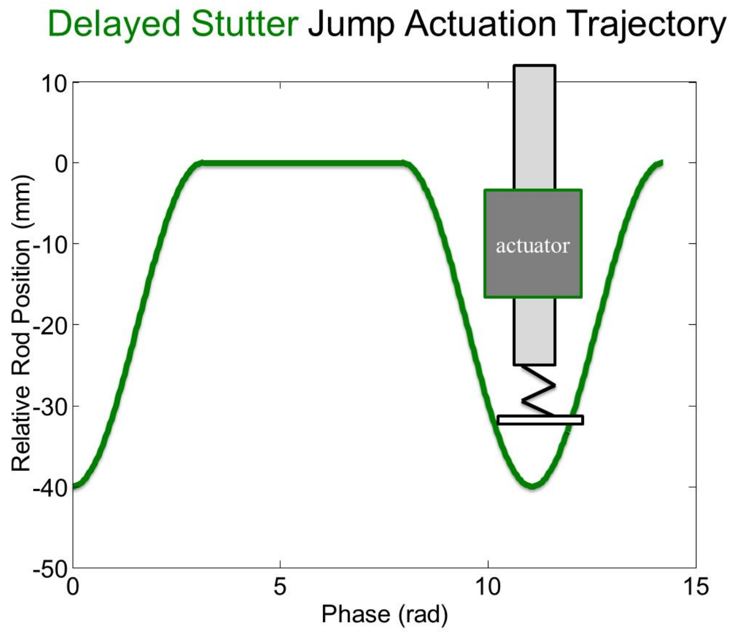

Figure 4: The actuation trajectory for a delayed stutter jump.

Executes a countermovement like the stutter jump, but then waits short while before executing a single jump.

3.2.5 Delayed Stutter Preload Jump

Executes a countermovement, and then waits a few periods before executing a preload jump.

3.2.6 Jumps Focused On

The two jumping commands which we focus most on are the preload jump and the delayed stutter preload jump. This is because the spring-like coefficient 'k' for the ground reaction force was initially calculated through a quadratic fit of volume fraction vs. k in the preload, fitting to k values that produced simulated jump heights that best matched experiment. Based on these fits, the relationship between k and volume fraction could be approximated by a quadratic curve, which provides a good baseline on which to further develop the ground reaction force, since it is known that for the preload jump, the quadratically fit 'k' function alone should give jump heights similar to experiment across most volume fractions. In addition, we focus on the delayed preload stutter jump because it was found experimentally that a delayed preload stutter could produce higher jump heights in loose-packed media than any other jump. It is of interest to understand the mechanism for this improved jump height.

3.3 Ground Reaction Force

$$F_{GRF} = kz + \alpha \|v\|^2 + \beta f(z, \frac{dv}{dt}) \tag{1}$$

We establish a ground reaction force equation (Equation 1) that contains three additive terms: a spring-like depth-dependent force, a velocity-squared dependent force, and an added mass term. The added-mass term has not been thoroughly investigated yet, but is proposed to be some function of intrusion depth and the time-derivative of velocity. The depth-dependence is due to the hypothesis that, as the foot intrudes further into the granular media, more material is displaced, thus more added mass is applied to the foot as it intrudes deeper. The acceleration dependence is proposed to be due to the Newtonian form of the force law, $F = m_a ddeda$, which is acceleration dependent. The coefficients 'k', 'α', and 'β' are the three parameters that will be systematically varied across different jump types and volume fractions in order to find possible regions where certain values of 'k', 'α', and 'β' produce accurate jump heights for multiple jump types and/or volume fractions.

Chapter IV

RESEARCH RESULTS

4.1 Simulink Simulations of Granular Jumping

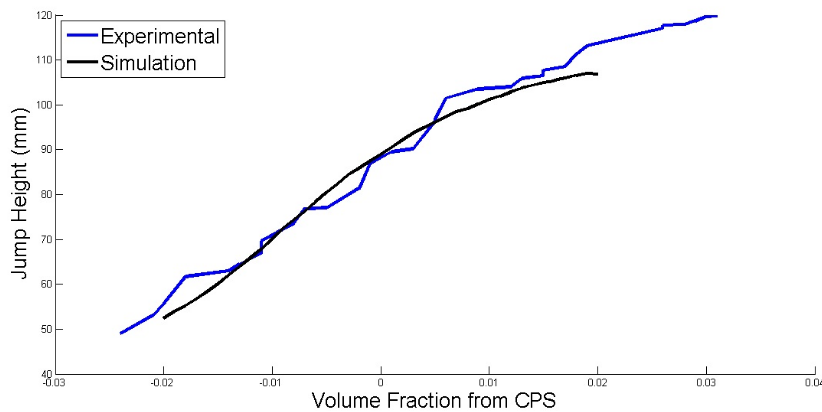

Figure 5: Jump Height vs. Volume Fraction for the Preload Jump, kz model.

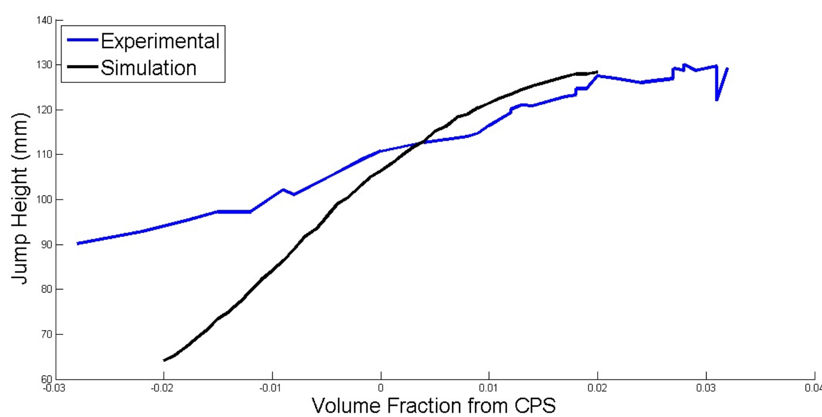

Figure 6: Jump Height vs. Volume Fraction for the Delay Stutter Preload Jump, kz model.

As seen in Fig. 5, since the value of 'k' is quadratically fit from the experimental jump heights of the preload-type jump, when put back into the simulation the simulated jump heights fall reasonably close to the experimental jump heights, as expected. However, Fig.

6 shows that for a different jump type, namely, the delayed stutter preload jump, the simulation produces a jump height up to 20mm below the experimental jump heights for lower volume fractions, while remaining relatively close to experimental jump heights at higher volume fractions. The x-axis in both figures is defined as the volume fraction ϕ minus the critical packing state volume fraction ϕ_c, which was described previously to be equal to 0.612 ± 0.001.

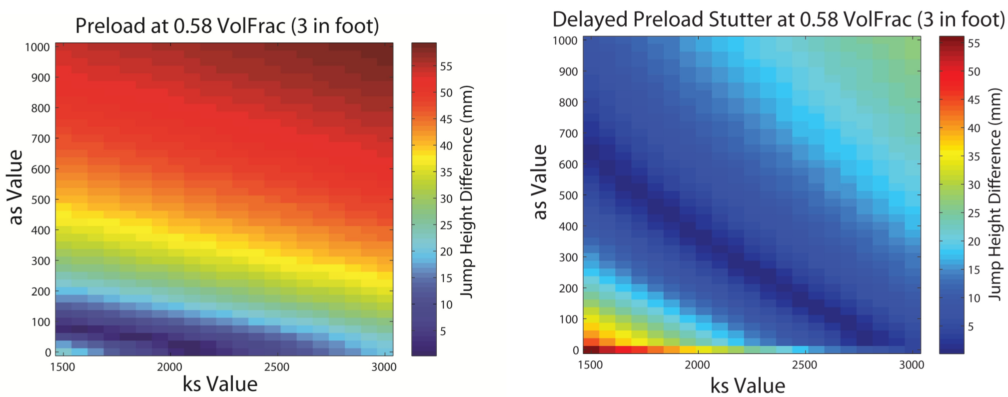

Figure 7: k vs. α sweeps for Preload Jump and Delayed Preload Stutter Jump at low volume fraction.

We then expand our model into including both the spring-like depth-dependent term and the velocity-squared dependent term, while still excluding the added-mass term. We systematically sweep through a range of numerical values of ‘k’ and ‘α’ for different jump types at one low volume fraction. The colormaps in Fig. 7 represent the absolute jump height difference between simulation and experiment. Two quasi-linear curves emerge across the values of ‘k’ and ‘α’ that follow regions of least difference (in blue).

4.1.1 Experimental Results and Comparisons

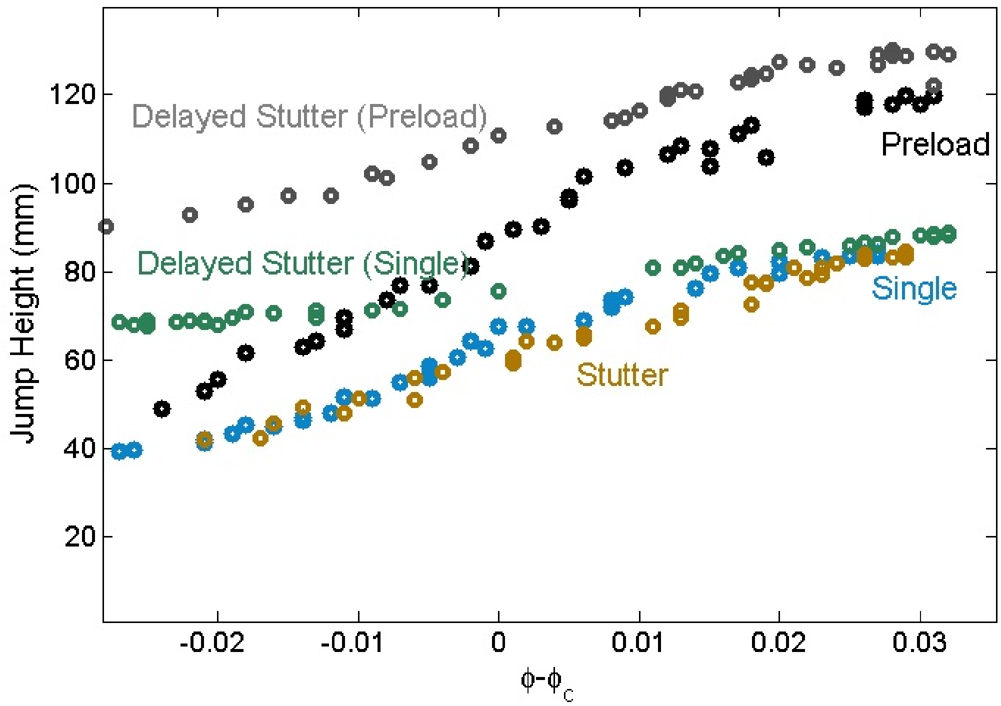

Figure 8: Experimental Jump Heights obtained for Various Forcing Commands

Many experimental trials with the jumping robot foot resulted in Fig. 8, which gives experimental jump heights for each tested volume fraction for the five tested forcing commands. These experimental data points are instrumental in assessing the correctness of the proposed model, as they provide a real world baseline to which we measure our simulated output. It was from these values that the simulated output is differenced from in order to create differential colormaps as those seen in Fig. 7.

4.2 Added Mass Impulse

We next investigated how the added mass term influenced the simluated jump heights once added to the simulation. We used a conically growing "snowplough" mechanism that anneals the impacted grains onto the intruder into a jammed region, modeled after Waitukaitis and Jaeger's model of added mass under impact-activated solidification [9]. This added mass term results in a modification to the foot's equation of motion for external forces, as momentum conservation requires:

$$[m_f + m_a(t)]a_f = F_{ext} - \frac{dm_a}{dt}v_f \tag{2}$$

Where 'f' subscripts indicate the foot and 'a' subscripts indicate the added mass. We varied the added mass coefficient 'κ' to attempt to better fit the simulated jump heights to the experimental results. Unfortunately, no clear correlation between volume fractions and best κ value has yet been found that is consistent across many jump types. Some interesting correlations emerge, however, when examining the impulse that the added mass delivers to the foot during the jump cycle.

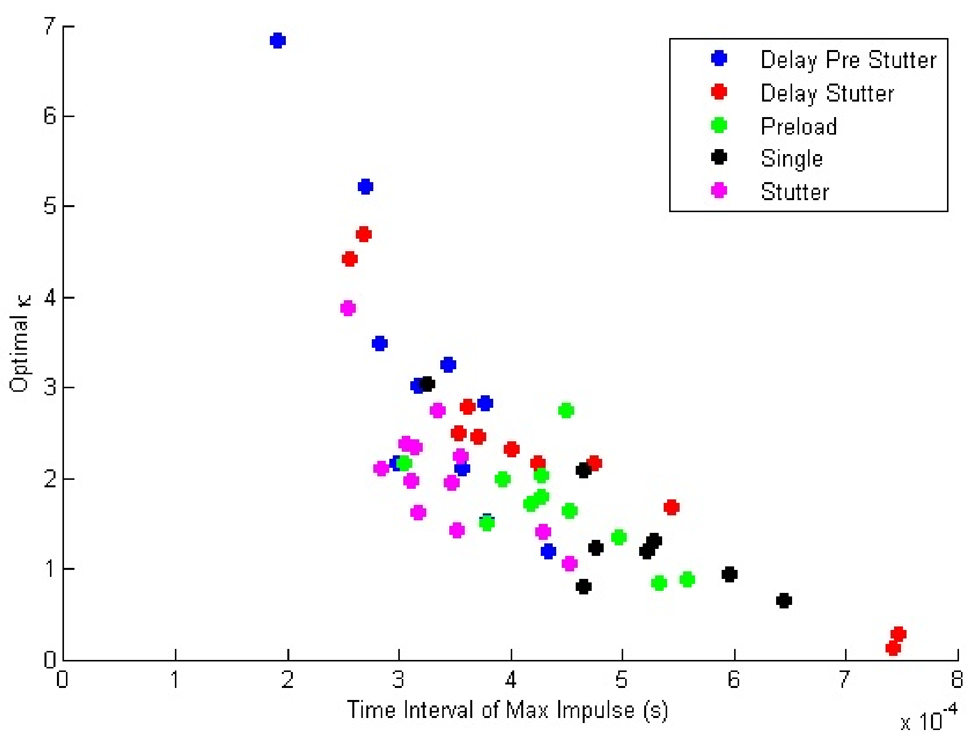

Figure 9: Time Interval of Greatest Added Mass Impulse in Relation to κ's Magnitude

After matching the κ value for each jump type and volume fraction that gave best agreement to experimental values, we examined the length of the temporal interval where the maximal added mass impulse occurred, which was usually at first intrusion. As seen in Fig. 9, there emerged an overall negative correlation between κ's magnitude and the duration of the impulse it generates.

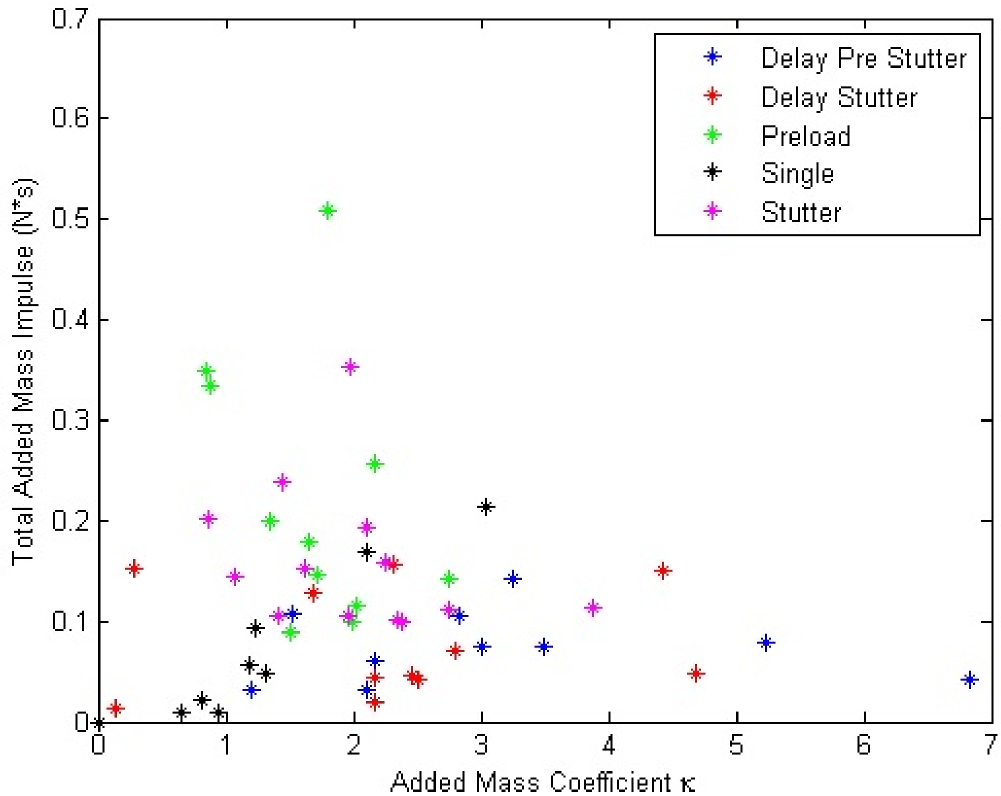

Figure 10: κ's Magnitude in Relation to the Total Added Mass Impulse

We also examined the total of added mass impulses for the entire jump cycle, and compared it against the corresponding κ value. There seems to be no strong correlation across all jump types between these two, implying that a larger κ value will not neccessarily lead to a greater total impulse on the foot due to added mass.

Chapter V

DISCUSSION & CONCLUSIONS

As seen in Fig. 6, the simple 'kz' model was not sufficient to explain the mechanics of the ground reaction force for different jump types. Some additional force, such as the velocity-squared dependent term or the added mass term may be acting on the foot such that more jump height is gained at lower volume fractions. Our systematic parameter sweeps across 'k' and 'α' in Fig. 7 also reveal that, when the colormaps are overlain, the lines of least difference for the two jump types do not intersect at any point. Namely, there is no unique combination of 'k' and 'α' for the two differing jump types at the same low volume fraction that produces accurate jumps for both jump types. Further investigations will examine the introduction of the proposed added-mass term, with attempts to create three-dimensional parameter sweeps across 'k', 'α', and 'β' to try to find unique values of the three parameters that give accurate jump heights across multiple jump types and volume fractions. If unique values for these parameters are found, we can examine their relation to each other and their dependence of jump type and/or volume fraction, and perhaps create a more comprehensive understanding of the impulsive mechanics on granular media.

5.1 Future Work

Recent developments in this project have given rise to new insights into what kind of forces affect different jump types. It has been found that added mass creates a negligible effect for all jumps except for the single stutter jump. The velocity-squared dependent term in the GRF equation contributes sufficient correction for most jumps.We found via intrusion experiments that the stiffness constant of the springlike ground reaction force actually changes with penetration depth. Finally, a new method of simulation to experiment matching was also implemented involving trajectory comparisons.

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