PRODUCTION COSTING AND PLANT DISPATCHING FOR LARGE ELECTRIC UTILITY SYSTEMS

A THESIS

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Federico Angel Ramirez

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PRODUCTION COSTING AND PLANT DISPATCHING FOR
LARGE ELECTRIC UTILITY SYSTEMS

Approved:

Gerald J. Thuesen, Chairman

Leon F. McGinnis

Gunter P. Sharp

Date approved by Chairman: 8/16/78

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Dedicated to my parents

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CHAPTER I

INTRODUCTION

1.1. The Problem

Economy of production is one of the most important factors in operating a large electric generating system. The ability to efficiently produce electric energy depends in a large measure on the methods of dispatching the various generating units in the electric system to the electric demand experienced by the system.

The fundamental problem is to obtain the minimum overall operating cost at any given load or output. To attain this objective, consideration must be given to the proper allocation of the electrical load to the particular generating units of the system. Because of the fact that there are numerous generating units with different characteristics, and there exist exogenous forces associated with the availability of the generating units, the allocation of electrical load must reflect a variety of constraints.

In the power system, the generating units differ with regard to costs, fuels, maintenance costs, etc. In addition, these units have different operating purposes. The gas turbines that have low capital requirements but high generation costs are used to supply peak loads. Fossil fuel units are generally used for loads of longer duration since they have higher capital but lower operating costs. The nuclear units which require the greatest capital investment have low operating costs and are used for base (continuous) loads. The hydro units require high or low capital costs (depending on the site) and have generating costs near zero. However, there are usually constraints on the amount of energy that may be produced from these sources.

Because the electric utilities require preventive maintenance, the units are not always available throughout the year. Although there is a planned maintenance schedule, there are unexpected outages that occur. All these factors mentioned affect the total production cost of the electric generating system and they have to be incorporated in any method that dispatches the various generating units.

In order to plan the expansion of an electric generating system, it is necessary to understand how each generating unit will operate within the total system. Methods have been developed to compute production costs, but the actual algorithms utilized are extremely detailed and require a large amount of data. As a result, the computer programs used require large amounts of core memory and require hours to execute.

Because of these limitations, there is a need for a quick accurate method for determining how each unit must operate. Then based on the projected system load, the generating units can be scheduled and a reasonable estimate of the total system operating costs can be determined.

1.2. Objectives of the Study

The purpose of this study is to:

1. Develop a fast, cost effective method for determining the operating levels of each generating unit for any given level of total system demand. This method will attempt to consider the actual operating conditions experienced by an electric utility.
2. Obtain actual electric utility cost data for individual generating units and make it accessible on the computer by defining the operating cost functions for each unit.
3. Test the effectiveness of the method developed by comparing the results obtained with the more detailed method presently utilized by an electric utility.

1.3. Plan for this Study

Chapter II presents a brief literature survey about the methods developed in the load dispatching problem and determination of the operating costs. Chapter III presents the principal concepts and terms that will be used in the development of the dispatching algorithm. Chapter IV presents the development of dispatching method. Chapter V presents the comparison of the results obtained by the method developed with the more detailed method actually used for the electric utility studies. In Chapter VI the conclusions and recommendations resulting from this investigation are discussed.

CHAPTER II

LITERATURE SEARCH

During the history of power generation much effort has been applied to the problem of finding the most economical way to produce energy. As a result a number of different methods to deal with this problem have been developed.

At first, estimates of the operating costs for particular combinations of generating units and forecasted load were made on the bases that the generating capability was always available and that the load was known with certainty.

The first publication related with this problem was by F. H. Rogers (19) in 1924. In his article, for the simplest case where the input curves are all straight lines, Rogers gave a graphical proof for the following theorem:

> The load should be divided between two or more units as to obtain equal values of the first derivative of the power discharge curve of each unit.

The treatment was extended by Rogers and Moody (20) in 1925. In their paper they present a comprehensive discussion on the theory, nature, and practical application of the incremental rates. They prove mathematically that the most economical division of load is obtained when the incremental rates are equal, and suggest some methods for calculation of the units input-output curves.

Six years later, in 1931, G. R. Hahn (8), shows that load dispatching using incremental rates may be applied to any parallel operation of units. He states that in some cases certain connections must be applied to the input-output curves before the increment rates are determined.

J. E. Mulligan in his article, "The Load Division Among Generating Units for Minimum Cost," (15) expands the underlying load division by demonstrating the criteria to follow where more than two units are involved, where the curves of input plotted against output are discontinuous or inflected, and where part of the units operate at constant load as the total load changes.

In 1943, Steinberg and Smith (22) published the first book dealing with economic loading in power systems. This book is basically a summary of the research results in this area up to that year. In their book, Steinberg and Smith show the application and limitations of the incremental rate theory and they give some practical solutions to the load division problems.

After 1950, with the development of operations research techniques such as linear programming, dynamic programming and simulation, the economic dispatching problem begins to be formulated in terms of a large number of variables. One of the first articles using these techniques was the paper by L. L. Garver (5). Garver describes how the economic scheduling problem may be formulated as an integer programming model. The integer program formulation includes the discontinuous power output characteristics of the generators, the costs of starting and shutting down each unit, and the dispatching of the load by incremental costs. The objective of this approach is to minimize the cost function which is the sum of the costs of starting, stopping, producing power at minimum output and producing power above the minimum output for each time period. This objective function and the constraints are formulated in a linear form. Reasonably good results have been obtained in the application of this method, but if the number of units is too large, as in the case with the present ever increasing loads and interconnection of power systems, a large amount of memory and computer time is required to process the algorithm.

F. Anatti and D. Grohnnan, in their article, "A Method for Economic Load Dispatching in a Thermal Power System," (1), developed an iterative method for the optimum allocation of power generation for thermal units. As an initial solution, the generation schedule at equal incremental costs is determined by a very simple computer program which is also employed in the successive stage of the iterative optimization procedure. The feasible solution corresponding to the equal incremental cost schedule is subsequently obtained by the solution of power flow equations. Subsequently, the adjustments of the power injections are optimized so as to reach the minimum operating cost of the system, subject to inequality constraints imposed by equipment ratings and transmission losses. Iterative solutions of the optimality equations by the previous method are alternated with solutions of the power flow equations, until a feasible solution is reached.

In 1969, R. R. Booth (2) developed a detailed computer program to simulate the operation of the system for a period of one year. The basis of the simulation approach is to model the component elements of the power system to produce an overall representation that responds in similar manner to the actual power system in a given set of circumstances. This simulation program essentially analyses the operation of the system hour by hour of the period and considers such factors as forced and scheduled outages, randomness in the load curves, and hydro constraints. This program was written in Fortran IV and has been run as an IBM 360/40. The basic computing speed is 2-3 second per 24 hours day. The simulation of one year takes approximately 20-30 minutes. The representation includes approximately 50 generating units.

In 1971, Rees and Larson (18) used dynamic programming for purposes of economic load dispatching. In their article, Rees and Larson describe a dynamic programming successive approximation algorithm for the optimization of power generation schedules for utilities participating in coordination agreement. The power sources available for schedule can include conventional hydro units, thermal generating units and pumped storage units. The objective of this algorithm is to schedule these power sources hourly such that generation costs are minimized over the entire year. The algorithm is implemented in three separated but coupled computer programs:

1. monthly optimization over one year
2. daily optimization over one month
3. hourly optimization over 48 hours.

The monthly optimization, daily optimization and hourly optimization programs form an optimization procedure hierarchy. Each program in turn addresses power generation scheduling with a shorter time horizon but on a more detailed and refined system model. The optimal long term policy and long term optimal costs of higher-ordered program are transferred as input to the program of adjacent rank in order that the short term optimization explicitly considers long term effects of current decisions.

R. R. Booth in his article, "Power Systems Simulation Model Based on Probability Analysis," (3), developed a simulation model based on the probability distributions of load and unit availability instead of based on the chronological simulation of the system operation. In conjunction with a dynamic programming algorithm, this model helps in the solution of a variety of problems concerning the planning and operation of power systems. An advantage of this model is that it considers any type of units and many operational problems can be formulated. The accuracy is good, but it requires a large amount of computer time.

Some of the models named have found application in the electric utility industry, but they are extremely detailed and require large amount of input data and considerable time to process.

It is the purpose of this thesis to develop a method that gives good estimates of future system operating costs without lengthy and costly computer runs. In the method developed in this study, the allocation of load is accomplished, like in the other approaches, by the incremental cost method. The principal difference with respect to the methods actually used is in the procedure utilized to analyze the system demand and in the treatment of the scheduled and forced outage times. A detailed description of the approach will be presented in Chapter IV.

CHAPTER III

BACKGROUND INFORMATION

The purpose of this chapter is to present the principal concepts and terms that will be used in the development of an algorithm to obtain the total operating costs of a power system. This chapter has been divided into three parts. First, the analysis of the cost functions is briefly discussed; second, basic concepts utilized in the electric utility industry are described, and last, a description and analysis of the optimal method for load dispatching (incremental cost method) is presented.

3.1. Cost Functions

This section contains a review of some elementary theory concerning the behavior of costs. The description of the different cost functions and how they can be obtained is also presented.

3.1.1. Fixed Costs

Fixed costs are those which do not vary as output changes. They are the costs which will be incurred if the firm is to continue to operate whether its output is large or small. Total and average fixed cost curves are shown in Figure 3-1. Total fixed costs remain at OF whatever the level of output. Fixed cost per unit of output will therefore fall

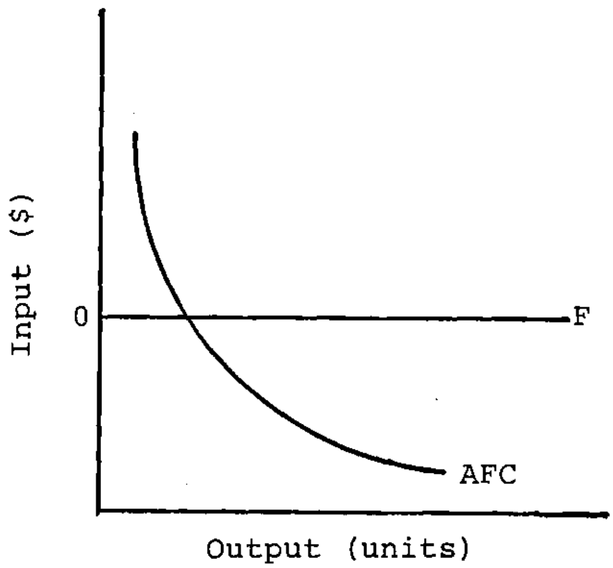


Figure 3-1. Fixed Cost Function.

steadily as output increases. This is illustrated in Figure 3-1 where the average fixed cost curve (AFC), is monotonically decreasing.

3.1.2. Variable Costs

Variable costs are those which change as output changes. The total variable cost curve has a different slope depending on the rate of change with respect to the output. Unit variable cost (UVC) is total variable cost divided by the units of output and is the slope of the total variable curve at any level of output. A typical variable cost function presented in power systems is shown in Figure 3-2a. The unit variable cost function (UVC) for this case is shown in Figure 3-2b.

3.1.3. Total Cost

The total costs are defined as the sum of the total fixed costs plus the total variable costs. The total cost curve (TC) is illustrated in Figure 3-2a. If the total cost is divided by the number of units produced in the period, the average cost is obtained (AC). Figure 3-3 represents the average cost function for the total cost curve shown in Figure 3-2a.

3.1.4. Incremental Cost or Marginal Cost

The incremental cost can be defined simply as the additional cost that will be incurred as the result of increasing the output one more unit. Conversely, it can be defined as the cost that will be saved if the output is reduced by one unit. The incremental cost is also known as the mar-

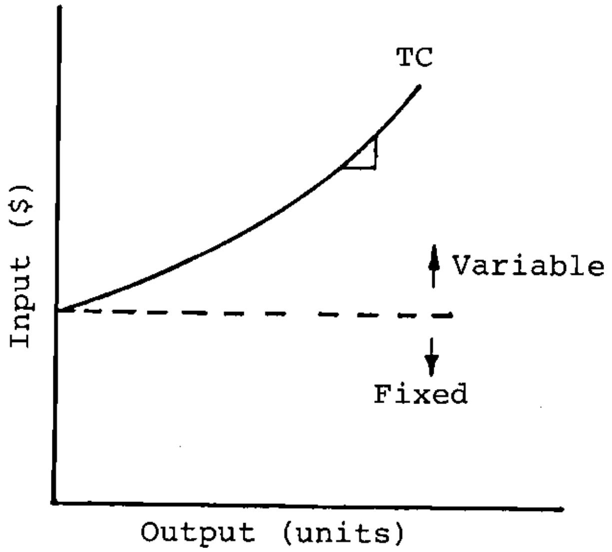


Figure 3.2a. Total Cost Function.

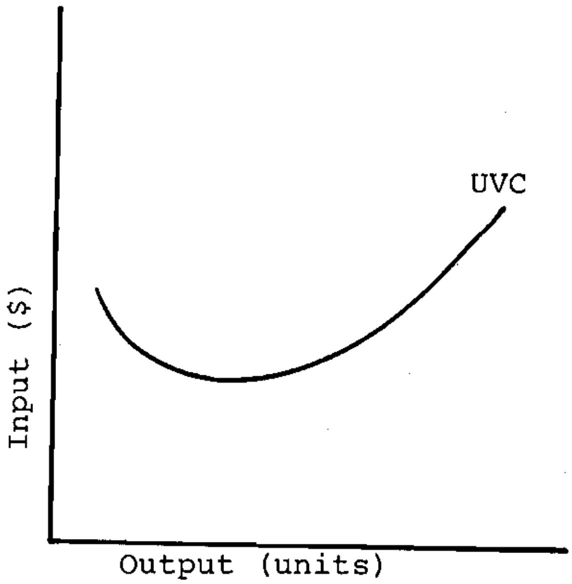


Figure 3-2b. Unit Variable Cost Function.

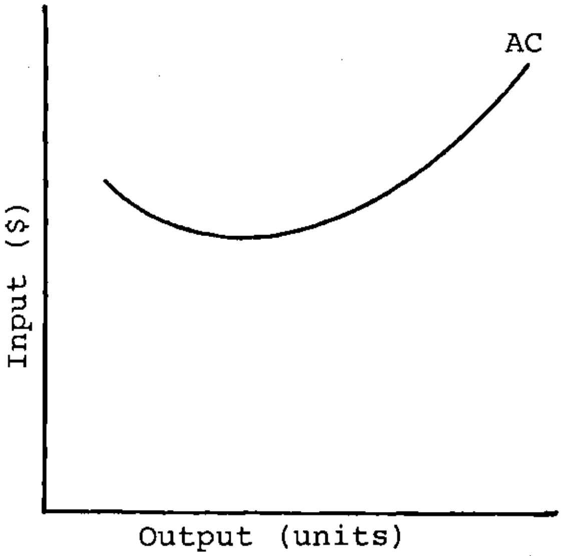


Figure 3-3. Average Cost Function.

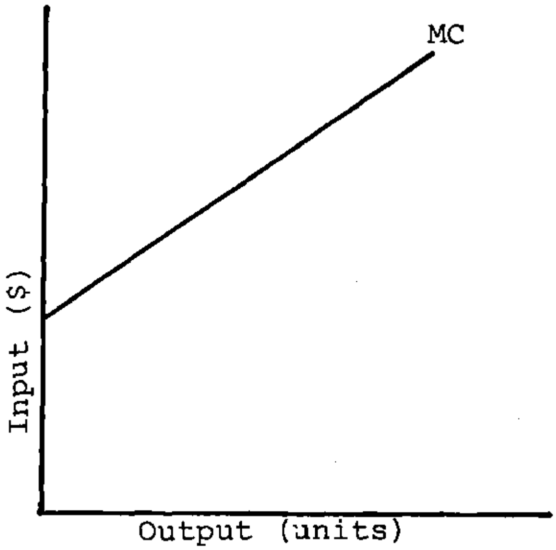


Figure 3-4. Marginal Cost Function for a Cuadratic Cost Curve.

ginal cost. It can also be defined as the change in the total variable cost for a unit change in output. It is the rate of change of input with output, and consequently, increment cost is the slope of the input-output curve for a given operating point, namely:

$$\text{Incremental Cost} = \frac{\Delta \text{ cost input}}{\Delta \text{ power output}} = \frac{d \text{ (cost input)}}{d \text{ (power output)}}$$

When the total cost function is linear or a linear approximation of some curve, the marginal cost is constant. The slope of the marginal cost function for the total cost function represented in Figure 3-2a is shown in Figure 3-4.

3.2. Demand Function

The calculation of the operating costs can be based either on two types of demand curves: the load curve or the load duration curve. The purpose of this section is to define and describe the characteristics of these curves.

3.2.1. The Load Curve

Most of the difficulties and complexities of modern power plant operation arise from an inherent variability of the load demanded by the users. Each customer requires small or large blocks of energy according to the demands of their activities. The ideal load from the standpoint of efficient equipment utilization would be one of constant magnitude and steady duration. Such an ideal load is shown in Figure 3-5a. The unit cost to produce the energy represented by the area

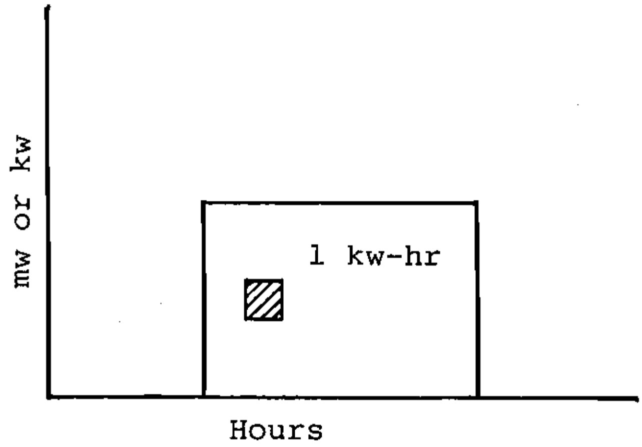


Figure 3-5a. Ideal Load Curve.

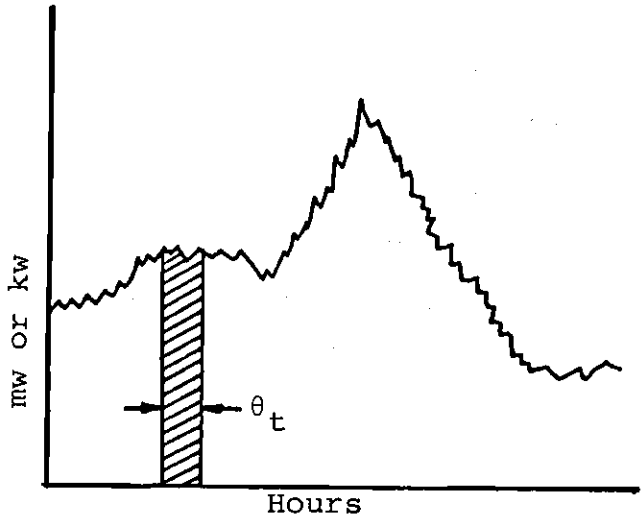


Figure 3-5b. Real Load Curve

of this load curve (one kilowatt-hour) could be from 1/2 to 3/4 of that to produce the same unit under the more frequently realized condition illustrated in Figure 3-5b. These curves which represent the hour-by-hour electric demand are commonly referred to as load curves. Daily load curves show the kilowatts demanded over a 24 hour day while annual load curves indicate the hour-by-hour kilowatts demanded at each time interval during a year. These curves also show the maximum or peak load that occurred during a particular period.

3.2.2. The Load Duration Curve

Another type of curve which represents the characteristics of electric demand is the load duration curve. It is constructed from the load curve by rearranging each load for each time interval θ_t to occur in descending order of magnitude (Figure 3-6). The ordinates of this curve may extend from zero to maximum demand in kilowatts or from zero to 100% maximum demand. The abscissa ranges from zero hours to the number of hours of the period (day, month, year). For example, the annual load duration curve shown in Figure 3-6 was constructed in the following manner. The number of hours during which 1000 kw, 2000 kw, 3000 kw, etc. is demanded was recorded from the daily load curves, then totaled for the year and plotted in decreasing order of magnitude. Hence, the interpretation of point A (Figure 3-6) is that at least 12 mw (12000 kw) were demanded for 5256 hours of the year, or that, through 60% (5256/8760) of the year, only 40% (12/30) of

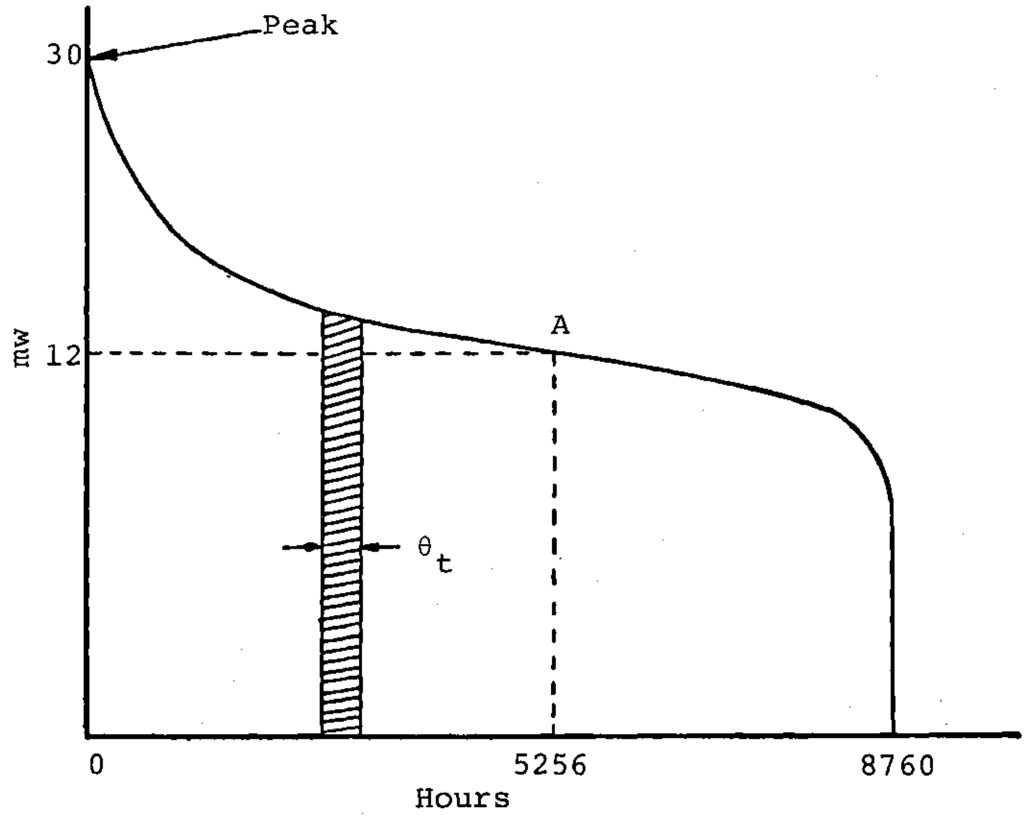


Figure 3-6. The Load Duration Curve.

maximum demand was required.

3.2.3. Computation of the Operating Costs

The calculation of the production costs can be based on the load curve or on the load duration curve. For the load curve, the operating costs for the area under this curve at each time interval θ_t are dependent on levels at which each unit in the system is operating to produce the energy represented by the interval (Figure 3-5b). Because of the high variability of power demand and the instability of the load curve, it is questionable whether this high degree of detail produces reliable estimates of the future. Due to the fact that the load duration curve represents a summary of a load curve by removing the time sequence of demand, its shape is more predictable. Thus production costs based on a load duration curve are simpler to develop and more stable from year to year.

However, a loss of information is introduced when this curve is used for calculation of operating costs, because this curve gives only the magnitude of the load and not the time at which the load occurs. For the load duration curve the operating costs are the area under this curve weighted at each time interval θ_t by the operating costs per unit energy output and the output of each generating unit operating in that interval.

3.3. Operating Characteristics

3.3.1. Input-Output Curve

For distributing load for maximum economy, it is necessary to have a knowledge of the relationship between energy input and output of the units concerned with each of the inputs and each of the outputs expressed in common units. This relationship can be expressed by the input-output curve. This curve defines the performance characteristics of the generating unit and it is the basic curve from which the incremental rates and incremental costs are derived for load dispatching purposes. An example of an input-output curve is shown in Figure 3-7. The fuel input in MMBtu/hr is plotted as a function of output in megawatts (mw). In developing these performance curves, at first consideration was given only to thermal performance. In recent years, however, it has been shown that for purposes of dispatching generating units for a particular system it is sufficient to use cost functions for the respective generating units based on the input-output curves extended by fuel prices. Usually fuel cost represents the major item of production costs. Section 4.1.2. will explain how the production cost curves can be obtained for each unit based on the input-output curve.

3.3.2. Capacity Factor

An important term used by power companies is the capacity factor. It is defined as the total energy produced by a unit during the year divided by the total energy that the

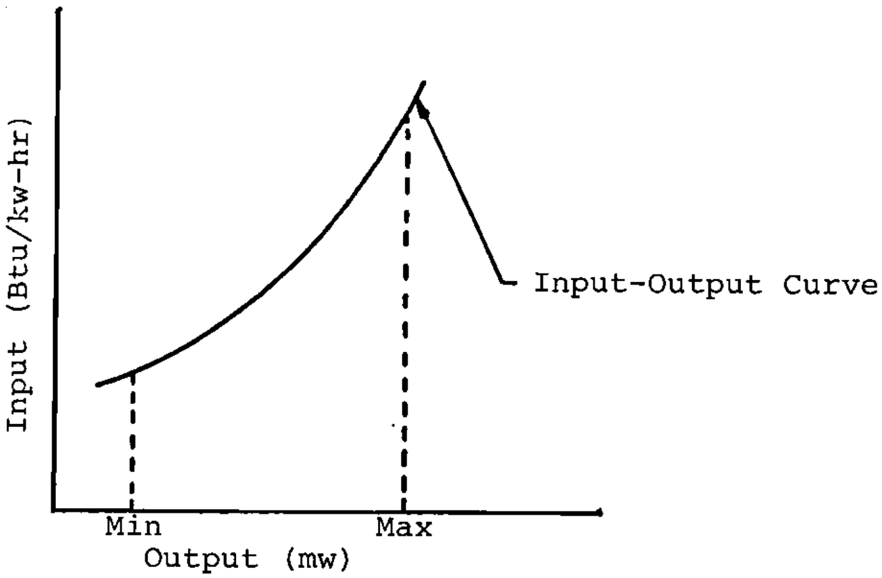


Figure 3-7. Input-Output Curve.

unit would have produced if it had operated the entire year at its maximum capacity. To illustrate, suppose there is a unit with a maximum capacity of 300 mw and it was operating at 100 mw during 4760 hours and at 250 mw during 4000 hours; then the capacity factor is:

$$\text{Capacity Factor} = \frac{\text{Energy Produced}}{\text{Potential Energy}} \tag{3-1}$$

$$= \frac{(100)(4760)+(4000)(250)}{(300)(8760)}$$

$$= 56\%$$

Thus, this factor provides information about the percentage of the unit's potential that is utilized during the year.

3.4. Optimal Method for Load Dispatching

The incremental cost method has been proven to be the optimal method for load dispatching. The subject of this section is to present the theory and application of this method to the estimation of future production costs.

3.4.1. Concept of Incremental Rate

The incremental rate of a unit at any given output is numerically equal to the slope of the input-output curve at the point corresponding to that output. Mathematically, the incremental rate is the first derivative of the input-output curve with respect to the output. It measures the rate of change of the input with respect to the output, and does not indicate the absolute value of the input. It is expressed in Btu/mw-hr.

Based on the input-output curve, the incremental rate may be derived by one of the following methods (12)(22):

1. When the input-output curve can be expressed by an algebraic equation (i.e., $y = Ax^2 + Bx + C$), the incremental rate can be determined by differentiation, since it is the first derivative of the equation of the input-output curve.
2. The incremental rate may be graphically determined by drawing a tangent to the input-output curve at the point corresponding to the output.
3. From the input-output curve, a series of output values are chosen and the corresponding input values read. The difference between successive values of the output are usually made constant and are made small enough so the characteristic shape of the incremental rate curve can be determined with reasonable accuracy. The incremental rate is merely the ratio of the input difference to the output difference, or the incremental input divided by the incremental output.

Of the three methods, the first method is the easiest to apply. The second method is not recommended because it requires plotting the input-output curve and it will generally be found that the tangent cannot be established with any reasonable degree of accuracy. Hence, the resulting values will be inconsistent and unsuitable for determining the characteristic shape of the incremental rate curve. The third method can give results closely checked by those obtained with the first one, but requires numerous computations. Therefore, the first method is the one most utilized because once the input-output curve is expressed by an equation, the incremental rate for any level of output is computed rapidly with high level of accuracy.

3.4.2. Incremental Production Costs

In section 3.3.1. it was pointed out that the input-output curve for a generating unit can be expressed by a relation between the fuel input (MMBtu/hr) and the output (mw). If this relation is expressed by a mathematical function, then

$$\text{Incremental Fuel Rate} = \frac{d\ (\text{input})}{d\ (\text{output})}. \tag{3-2}$$

The units of measure with the incremental fuel rate area MMBtu/mw-hr. The incremental fuel rate is converted to incremental fuel cost by multiplying the incremental fuel rate in MMBtu/mw-hr by the fuel cost in \$/MMBtu. Then the incremental fuel cost is expressed in \$/mw-hr.

So far, only the fuel cost has been obtained, but we are interested in the total operating cost. This cost is composed of the fuel cost plus the cost of other items such as labor, supplies and maintenance. Therefore, the incremental operating cost or incremental production cost of a given unit is made up of incremental fuel cost plus the incremental cost of these additional items. It is necessary to perform a rigorous analysis in order to be able to express the costs of these production items as a function of instantaneous output.

Arbitrary methods of determining incremental cost of labor, supplies and maintenance are used. The most common of the methods is to assume these costs are a fixed percentage of the incremental fuel costs. However, in many systems for purposes of scheduling generation, the incremental production cost is assumed to be equal to the incremental fuel cost.

3.4.3. Conditions for Maximum Efficiency (Optimum Scheduling)

Assume that there are any number of generating units in the system with the following characteristics:

1. The input-output curves are continuous.
2. The first derivatives of the input-output curves (the incremental rate curves) are continuous.
3. Convex total cost functions.
4. The value of the incremental costs always increases as the output increases.
5. Transmission losses are neglected.

It has been shown (12) that the minimum input in dollars per hour for a given total load is obtained when all generating units are operated at the same incremental production cost.

Demonstration:

Let F_n = input to unit "n" in dollars per hour

F_t = total input to the system in dollars per hour

It is desired that the total input to the system be minimized, it means,

$$F_t = \sum_n F_n = \text{Minimum} \tag{3-1}$$

under the restriction that

$$\sum_n P_n = P_R \tag{3-2}$$

where

P_R = Received load (mw)

P_n = Output of unit "n" (mw)

If it is applicated the method of Lagrange Multipliers, the equation of constraint is given by

$$\psi(P_1, P_2, P_3 \dots P_n) = \sum_n P_n - P_R = 0 \tag{3-3}$$

The minimum fuel input for a given received load is obtained when

$$\frac{\partial G}{\partial P_n} = 0 \tag{3-4}$$

where

$$G = F_t - \lambda\psi \tag{3-5}$$

and λ is the Langragian type of multiplier.

From (3-4) and (3-5)

$$\frac{\partial G}{\partial P_n} = \frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial \psi}{\partial P_n} = 0 \tag{3-6}$$

Then

$$\frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial}{\partial P_n} [\sum_n P_n - P_R] = 0 \tag{3-7}$$

$$\frac{\partial F_t}{\partial P_n} - \lambda[1-0] = 0 \tag{3-8}$$

Therefore

$$\frac{\partial F_t}{\partial P_n} = \lambda \tag{3-9}$$

But

$$\frac{\partial F_t}{\partial P_n} = \partial(\sum_n F_n)/\partial P_n = \frac{\partial F_n}{\partial P_n} = \frac{dF_n}{dP_n} \tag{3-10}$$

Then from (3-9) and (3-10)

$$\frac{dF_n}{dP_n} = \lambda \tag{3-11}$$

where $\frac{dF_n}{dP_n}$ = incremental operating cost of unit "n" in dollars per mw-hr

λ = incremental cost of received power in dollars per mw-hr

The value of λ must be chosen such that $\Sigma P_n = P_R$. Thus, the minimum input for a given combined output is obtained when the incremental production costs are the same.

The same result as in Equation (3-11) could be obtained intuitively. Assume that all the units are not operating at the same incremental cost. Therefore, some units are operating at higher incremental costs than others. It would then be possible to decrease the dollars per hour input to the system by increasing the generation in the units that are operating at lower incremental costs, and decrease the generation in units operating at higher incremental costs. In the limiting case, it will be seen that all sources should be operated at the same incremental costs.

An example of the application of this method is shown in Figure 3-8. Suppose there are two generating units in the system with the following ranges of capacity:

$$10 \leq C_1 \leq 25$$

$$5 \leq C_2 \leq 15$$

and the marginal cost functions represented in Figure 3-8. Suppose the demand of the system is 20 mw, then Unit 1 will operate at the level of 10 mw and Unit 2 also at 10 mw; now, which unit will provide the next 5 mw for the case when the system demand is 25 mw? According to the incremental cost method this load will be assigned to Unit 2 because it has less marginal cost. The next 5 megawatts will be assigned to Unit 1 and so on. In other words, after the units are already in operation, the next load increment will be picked up by the unit which will produce the next increment of energy at a minimum incremental cost.

It is important to notice that so far the optimal allocation for a given demand has been considered where all the units are always available at the time of that demand. However, in actual power systems there exist high variability

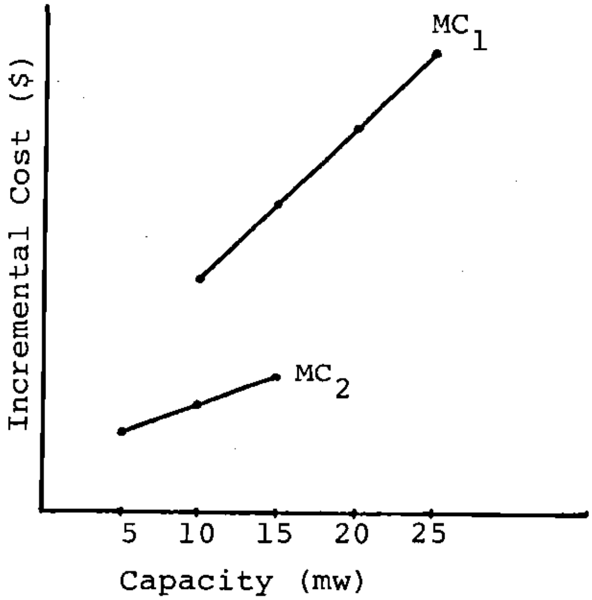


Figure 3-8. Example of the Incremental Cost Method.

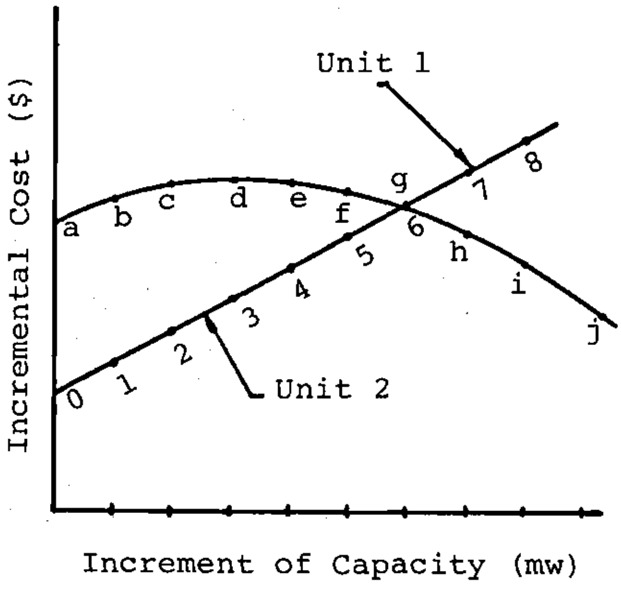


Figure 3-9. Tests for Non-Linear Marginal Costs.

in the demand and the units are not available all the time. Thus, the load dispatching is more complicated as additional factors must be considered.

In Chapter IV is the description of the algorithm developed to allocate generating units to various levels of demand. It will be shown how to use the incremental cost method when accounting for these other considerations.

3.4.4. Two Considerations with Respect to the Application of the Incremental Cost Method

The incremental rate or incremental cost method, can be applied for different type of incremental cost curves, but it is necessary to make two tests when this method is used. The following example illustrates these tests.

Suppose there are two generating units whose marginal cost functions are shown in Figure 3-9. If the marginal cost method is utilized to load these units, the capacity will be assigned to Unit 1 until point 6 is reached. If more load is required the assignment of load will continue on Unit 2 until it gets to point "g." Now the question is where to make the next increment. According to the marginal costs functions of both units, the most economical way is to assign capacity to Unit 2 (g→h). But consider these two points:

1. Would it not be better to make three increments on Unit 2 (g→h→i→j) and two decrements on Unit 1 (6→5→4)?
2. Will the total variable cost up to point "h" be minimized?

In Appendix A, it will be demonstrated that for the cost functions of the utility company studied, it is not necessary to make the two tests just examined.

CHAPTER IV

DEVELOPMENT OF THE METHOD

This chapter describes the method developed to calculate the operating costs of a large electric power system. The first part presents the type of data that was obtained from a large electric utility company in order to test the generation unit dispatching algorithm that is developed. Subsequent sections present the rationale that is the basis of the algorithm, a description of the algorithm, and a numerical example.

4.1. Characteristics of the Generating Units, Data and Why Required

4.1.1. Input-Output Curve

As seen in Section 3.3.1, in order to distribute the load for maximum economy, it is first necessary to have a knowledge of the relationship between input and output for an electric generating unit. This relationship is expressed by an input-output curve. A frequently used method for expressing this relationship is the quadratic function shown in Equation 4-1.

$$Y = Ax^2 + Bx + C \qquad (4\text{-}1)$$

where Y = power (the input in MMBtu/hr)

x = operating capacity (the output in mw)

A = coefficient (MMBtu/mw^2-hr)

B = coefficient (MMBtu/mw-hr)

C = coefficient (MMBtu/hr)

For each producing unit in the system, the coefficients of the input-output curve have been obtained, so the power of each unit at different levels of capacity can be computed directly.

4.1.2. Operating Cost Functions

There are several components of the operating cost of an electric generating unit. These components include the operating and maintenance cost, fuel handling cost and the fuel cost which is the principal component. These costs are usually expressed in dollars per million-btus.

$$\mathrm{CFD}(\$/\mathrm{MMBtu}) = \mathrm{VOM}(\$/\mathrm{MMBtu}) + \mathrm{FHC}(\$/\mathrm{MMBtu}) + \mathrm{FUC}(\$/\mathrm{MMBtu}) \tag{4-2}$$

where CFD = total operating cost

VOM = operating and maintenance cost

FHC = fuel handling cost

FUC = fuel cost

In addition to these variable costs are the fixed costs (independent of the MMBtu/hr), expressed in \$/week. These fixed costs are indicated by FIX.

By having the operating cost in \$/MMBtu and the input-output curve for each unit, the total cost, average cost and marginal cost can be computed directly.

If the operating cost is in \$/MMBtu and the power of the unit (Y) is in MMBtu/hr, then the total operating cost per hour of operation can be computed by multiplying the input-output curve by the total operating cost (CFD) of the unit. Therefore, the total operating cost curve will be:

$$TC = A \cdot CFD\ x^2 + B \cdot CFD\ x + C \cdot CFD$$

$$(\$/hr) = (MMBtu/mw^2\text{-}hr)(\$/MMBtu)\ (mw^2) + (MMBtu/mw\text{-}hr)(\$/MMBtu)(mw) + (MMBtu/hr) \qquad (4\text{-}3)$$

If the operating cost curve is divided by the amount of units produced (in this case mw), the average cost per megawatt per hour of operation is found by

$$AC = TC/x = A \cdot CFD \cdot x + B \cdot CFD + C/x$$

$$(\$/mw\text{-}hr) = (MMBtu/mw^2\text{-}hr)(\$/MMBtu)(mw) + (\$/MMBtu)(MMBtu/mw\text{-}hr) + (MMBtu/mw\text{-}hr)(\$/MMBtu) \qquad (4\text{-}4)$$

This figure represents the average cost of produced energy.

Next, the marginal cost or incremental cost function must be calculated. The marginal cost is defined as the first derivative of the input with respect to the output. This cost is also expressed in \$/mw-hr.

$$MC = \frac{dTC}{dx} = 2 \cdot A \cdot CFD \cdot x + B \cdot CFD$$

$$(\$/mw\text{-}hr) = (MMBtu/mw^2\text{-}hr)\ (\$/MMBtu)(mw) + (MMBtu/mw\text{-}hr)(\$/MMBtu) \qquad (4\text{-}5)$$

Notice that the marginal cost function is a straight line, and this characteristic will simplify the use of an incremental cost method when finding the most economical operating levels for the generating units in the system.

For purposes of this study, the generating units of the system will be divided into two types: Type I, which include all the nuclear and fossil steam units, and Type II units which are the combustion turbines (CT). Each type unit has its capacity range expressed by minimum and maximum capacities, and it is operated all the year (1). Since Type II units generally have high operating costs, they usually operate when the energy demanded cannot be met by Type I units. These units are either turned off or operate at their maximum capacity.

Using these relationships, the cost function for any of the units of the system can be constructed. These curves are illustrated in Figures 4-1a and 4-1b.

4.1.3. Scheduled Maintenance

An important variable to consider for the load dispatching problem is the maintenance time. Each unit needs a certain amount of time during the year for maintenance. The time and the period when the maintenance is scheduled is usually the periods of low demand, in the spring and fall months. For the algorithm developed in this study, the maintenance time in indicated by TMAIN.

(1) Except for maintenance and forced outages.

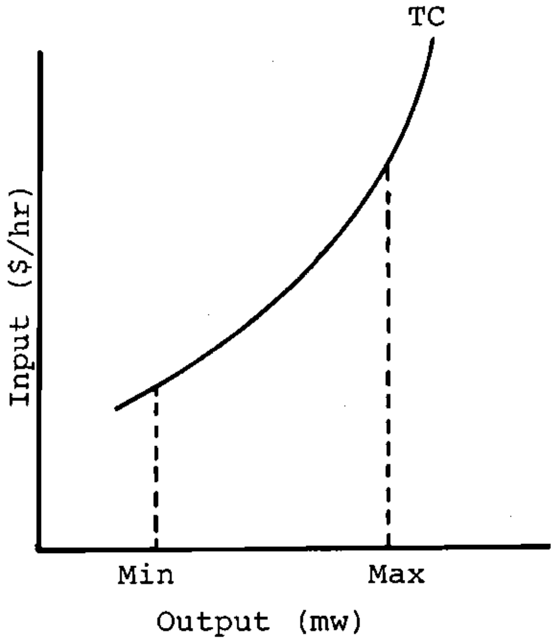


Figure 4-1a. Generating Units Total Cost Function.

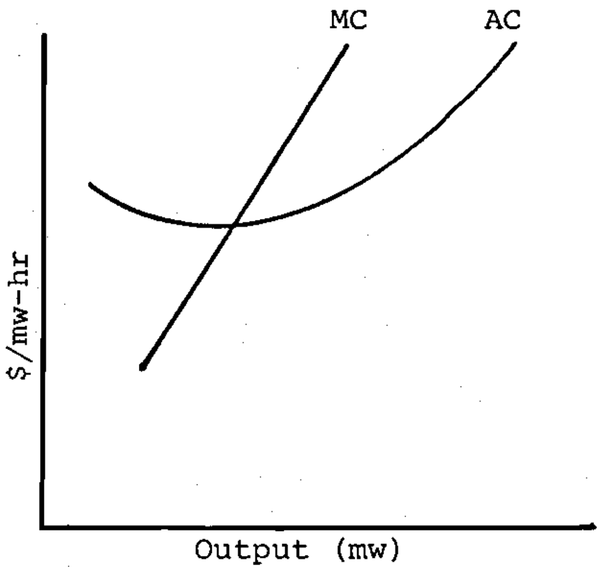


Figure 4-1b. Generating Units Average and Marginal Cost Functions.

TMAIN = number of hours out of the system due to maintenance.

4.1.4. Forced Outages

Another important variable that affects the dispatching of generating units is the forced outage time. This is the time that the unit is out of the system due to unanticipated failures. The problem in computing this time is that it is a random variable, so it has to be calculated by probabilistic methods. The forced outage rate (TFOR) is defined as:

TFOR = number of hours out of the system due to forced outages.

It is important to mention that in developing the algorithm, in this study it has been assumed that the combustion turbines (Type II units) are never withdrawn from the system because of maintenance or forced outages. Since these units only operate when the energy demand cannot be met by Type I units, they are out of the system most of the year. In this study, it has been supposed that the maintenance is performed during this time.

4.1.5. The Load Duration Curve

The total operating cost of the operating units in a power system can be obtained from two types of demand curves: the load curve and the load duration curve. As stated in Section 3.2.3, the advantage of the load curve is that the magnitude of the load and the time it occurs is described, but because of the high variability of demand and the instability of this curve from year to year, it is not desirable as the basis for the efficient calculation of production cost. On the other hand, the advantage in using the load duration curve in computing the operating costs is that it can be represented by a simple function and it is generally stable from year to year. However, a loss of information is introduced because this curve gives only the magnitude of the load and the portion of the time that it occurred.

It is desired that the method developed in this study be used to compute the operating costs for future years. Since the load duration curve has more stability and can be represented by simpler functions, this curve was chosen for use in computing the system operating costs.

The load duration curve data has been obtained by collecting about 90 points (hour, megawatt) that describe the shape of an annual load duration curve. Section 4.2.8 shows how these points will be used to obtain more points that describe more precisely this load duration curve. Table 1 contains the data obtained so far and their identification codes.

4.2. Generating Unit Dispatching

This section describes the actual steps required with the use of this generating unit scheduling scheme. The steps include an analysis on the load duration curve and the procedure utilized to compute the generating units' energy costs. The flow diagram of the algorithm and a numerical example are also presented in this section.

Table 1. Identification Code of the Information Obtained.

Code	Description	Units	Units Description
A	Heat rate coefficient	$MBtu/mw^2\ hr$	$\frac{\text{Millions of Btu}}{\text{Megawatt}^2\text{-hour}}$
B	Heat rate coefficient	MBtu/mw-hr	$\frac{\text{Millions of Btu}}{\text{Megawatt} \cdot \text{Hour}}$
C	Heat rate coefficient	MBtu/hr	$\frac{\text{Millions of Btu}}{\text{Hours}}$
KAP	Operating capacity	mw	Megawatts
Y	Heat rate	MMBtu/hr	Million of Btu/hour
FUC	Fuel cost	$/MMBtu	Dollars/million of Btu
FHC	Fuel handling cost	$/MMBtu	Dollars/million of Btu
VOM	Op. and Maint. cost	$/MMbtu	Dollars/million of Btu
CFD	Total operating cost	$/MMBtu	Dollars/million of Btu
TC	Total cost	$/hr	Dollars/hour
AC	Average cost of energy	$/mw-hr	Dollars/hour
MC	Marginal cost	$/mw-hr	Dollars/hour
TMAIN	Maintenance time	hrs	hours
TFOR	Forced outage time	hrs	hours
(RH, PL)	Load duration curve points	(hr, mw)	(Hours, megawatt)
FIX	Fixed cost	$/week	Dollars/week

4.2.1. Analysis of the Load Duration Curve

To begin development of the algorithm, an analysis of the load duration curve is presented. Figure 4-2 shows the classical load duration curve over a one year period of operation. The total area under this curve represents the total annual energy (mw-hr) demanded over the year. It is desired that this energy be produced at minimum cost.

This curve can be divided in a number of rectangles as shown in Figure 4-3a. If

L_j $j = 1 \ldots n$ = The different levels of loads (mw) demanded during the year.

T_j $j = 1 \ldots n$ = The respective time in a year (hours) that each load was demanded.

Then, the product of $L_j \cdot T_j$ (the area under the rectangle) represents a portion of the total energy demanded. This portion will be identified by e_j. Hence,

$$e_j = L_j \cdot T_j \tag{4-6}$$

Then, the total energy demanded (TOTENE) during the year will be the sum of the e_j's.

$$\text{TOTENE} = \sum_{j=1}^{n} e_j = \sum_{j=1}^{n} T_j \cdot L_j \tag{4-7}$$

Notice that T_j is an interval of time and therefore the sum of all these intervals must be equal to the total number of hours in a year (8760).

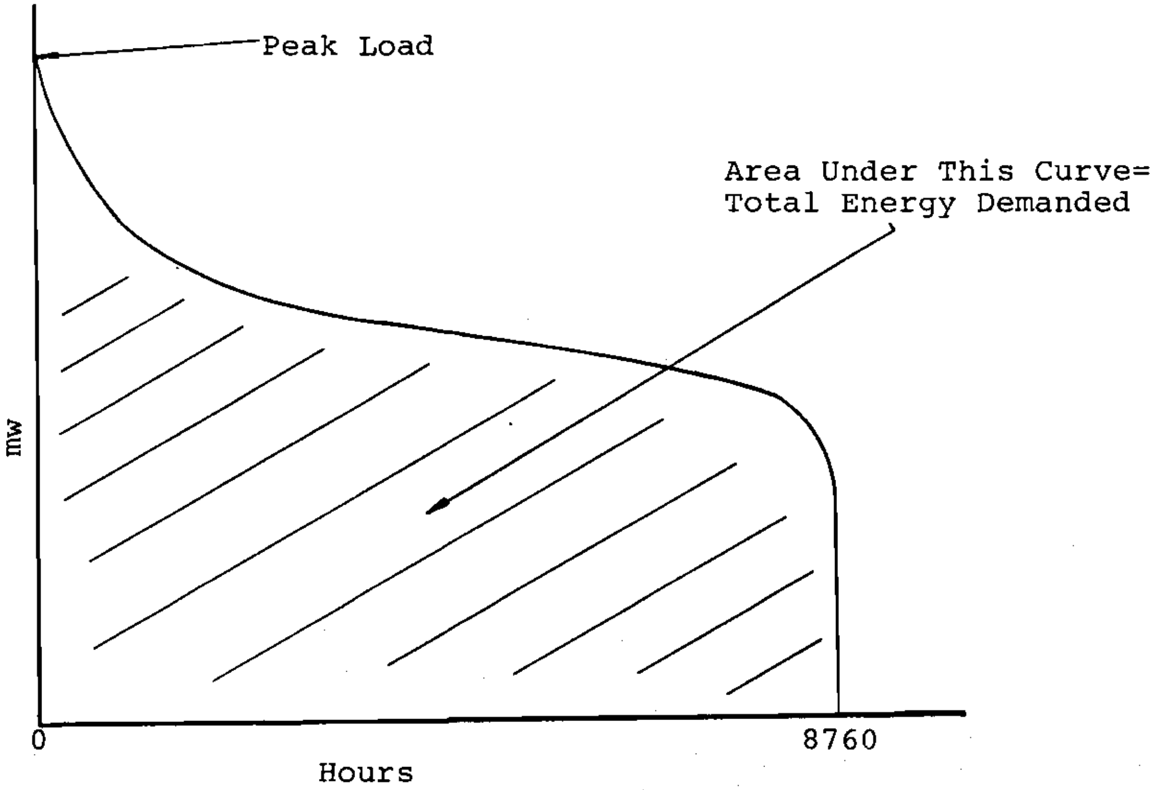


Figure 4-2. Load Duration Curve.

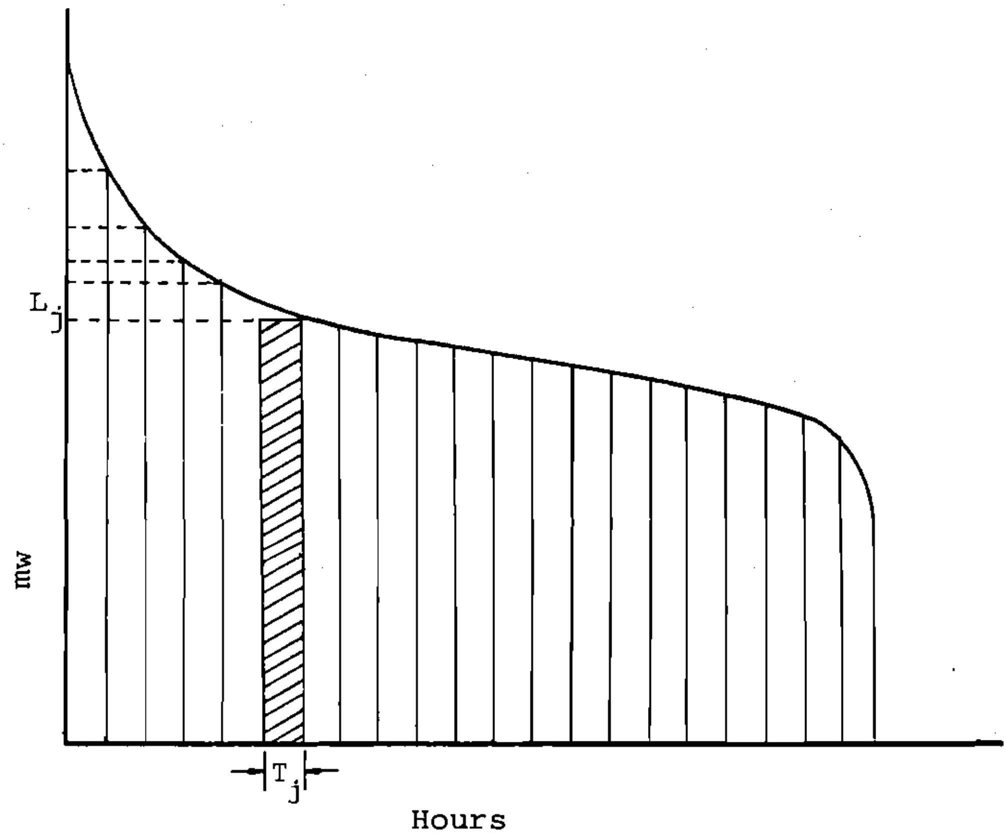


Figure 4-3a. Division of the Load Duration Curve.

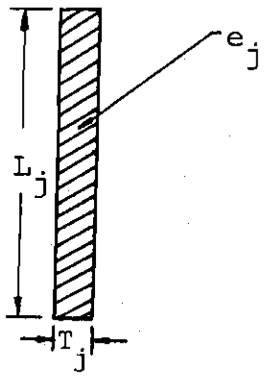


Figure 4-3b. A "Rectangle" of Energy.

$$\sum_{j=1}^{n} T_j = 8760 \tag{4-8}$$

As seen in Figure 4-3a and 4-3b, because of the form in which the load duration curve was divided, each load L_j is demanded T_j hours. Therefore, with this information, it is possible to compute the probability of the demand of each load L_j. This probability may be found by dividing the number of hours that each load is demanded by the total number of hours in a year.

$$PL_j = \frac{T_j}{8760} \text{ and } \sum_{j=1}^{n} PL_j = 1 \tag{4-9}$$

where PL_j = the probability that load "j" (L_j) is demanded during the year.

4.2.2. Determination of the Time Operated by the Unit

In a power system, the units are not available all year because of maintenance time and forced outages. If the load curve is used, it is possible to relate the unavailability of the unit to the demand at the exact time the unit is unavailable. This cannot be done when the load duration curve is used since information about the chronological time is lost. However, the total number of hours that the generating units will not be operating during the year due to maintenance and forced outages can be obtained. With this information, it is possible to calculate the probability that any unit will be unavailable. This probability is computed by dividing the total number of hours the unit will be unavailable by the number of hours of the year. Let's call

$HRSOUT_i$ = the number of hours that unit "i" will not be operating in the year.

then,

$$HRSOUT_i = TMAIN_i + TFOR_i \tag{4-10}$$

where $TMAIN_i$ = maintenance time for unit "i" during the year

$TFOR_i$ = forced outage time for unit "i" during the year.

Therefore, the probability that unit "i" will be out is:

$$PO_i = \frac{HRSOUT_i}{8760} \tag{4-11}$$

and the probability that unit "i" will be operating is:

$$PI_i = 1 - PO_i = 1 - \frac{HRSOUT_i}{8760} \tag{4-12}$$

With these values determined, the expected number of hours that each unit will be operating during each T_j can be computed. If PL_j is the probability of occurrence of L_j and PI_i the probability that unit "i" will be operating when the demand is L_j, then

$$JOINT_{ij} = (PI_i)(PL_j) \tag{4-13}$$

where $JOINT_{ij}$ = probability that unit "i" is operating when the demand is L_j.

Now, by definition of expected value (10), the expected number of hours that unit "i" will be operating during the year is equal to the probability that unit "i" operates by the number of hours of the year.

$$EHRS_i = PI_i \cdot 8760 \qquad (4\text{-}14)$$

Because of the fact that the load duration curve is being analyzed by intervals (T_j), it is necessary to obtain the expected number of hours that unit "i" operates during T_j (i.e., during the time that L_j is demanded). This is obtained simply by multiplying the $EHRS_i$ by the probability of occurrence of L_j.

$$TIMEIN_{ij} = (EHRS_i)(PL_j) \qquad (4\text{-}15)$$

$$= (JOINT_{ij})(8760) \qquad (4\text{-}16)$$

where $TIMEIN_{ij}$ = expected number of hours that unit "i" will be operating when the demand is L_j.

Notice that if we have "n" demands (loads) during the year

$$\sum_{j=1}^{n} TIMEIN_{ij} = 8760 - HRSOUT_i \qquad (4\text{-}17)$$

4.2.3. Determination of the Energy Produced

The next step is to obtain the energy produced by the unit over the year (ENPRO). It was shown in Section 4.2.1 that the energy demanded when the load is L_j (e_j) is equal to $T_j \cdot L_j$. Therefore, the energy produced by unit "i" when demand is L_j will be the product of the unit time operated and the unit operating capacity during L_j.

$$ENPRO_{ij} = TIMEIN_{ij} \cdot KAP_{ij} \tag{4-18}$$

where KAP_{ij} is the level of capacity that unit "i" is operating when demand L_j occurs.

The energy produced by unit "i" during the entire year is obtained by simply adding the energy produced by the unit at each L_j.

$$ENPRO_i = \sum_{j=1}^{n} ENPRO_{ij} \tag{4-19}$$

Therefore, if there are "m" units in the system, for each load the energy produced when this load occurs is the sum of the energies produced by each unit when demand is L_j.

$$\sum_{i=1}^{m} ENPRO_{ij} = \sum_{i=1}^{m} TIMEIN_{ij} \cdot KAP_{ij} \tag{4-20}$$

<u>This sum must be equal to the energy demanded e_j.</u> Hence, to meet the requirement of energy demanded during the year (the area under the load duration curve) it is necessary that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} ENPRO_{ij} = \sum_{j=1}^{n} e_j = TOTENE \tag{4-21}$$

where TOTENE is equal to the total energy demanded in the year.

4.2.4. Capacity Factor

The capacity factor is defined as the total energy produced by a unit during the year divided by the total energy that the unit would have produced if had it been operated at maximum capacity the entire year. From (4-19), it is possible to calculate the capacity factor for unit "i."

$$CAPFAC_i = \frac{ENPRO_i}{KAP_{max_i} \; 8760} \tag{4-22}$$

$$CAPFAC_i = \frac{\sum_{j=1}^{n} ENPRO_{ij}}{KAP_{max_i} \cdot 8760} \tag{4-23}$$

where KAP_{max_i} represents the maximum capacity that unit "i" can operate. This factor provides information about the percentage of the unit's potential that is used during the year.

4.2.5. Determination of the Optimal Operating Capacity

To illustrate the procedure utilized to obtain the level of capacity at which each generating unit must operate to minimize the operating cost, the following example is presented.

Suppose there are three units in a power system, Figure 4-4 illustrates one of the divisions of the load duration curve of this system. Assume that the units do not have restrictions in capacity, they have cost function like Figure 4-1, and they operate all the interval T_j. The objective is to schedule these units in the most economical way so the energy e_j is met.

For the assumptions previously described, the incremental cost method as explained in Section 3.4.3 is applied, and the units are dispatched in the most economical way to meet the energy demand (e_j). In other words, increments of capacity will be added to the units, so that each increment picked up will be produced at the minimum incremental cost. This assignment will continue until the total megawatts produced are equal to the load L_j. Suppose that the levels of capacity for each unit are KAP_{1j}, KAP_{2j}, and KAP_{3j} then,

$$KAP_{1j} + KAP_{2j} + KAP_{3j} = L_j. \tag{4-24}$$

Also, the energy produced has to be equal to the energy demand. Therefore,

$$\begin{aligned} &(KAP_{1j})(TIMEIN_{1j}) + (KAP_{2j})(TIMEIN_{2j}) \\ &+ (KAP_{3j})(TIMEIN_{3j}) = L_jT_j = e_j \end{aligned} \tag{4-25}$$

but due to the assumption that the units operate all the time,

$$TIMEIN_{1j} = TIMEIN_{2j} = TIMEIN_{3j} = T_j \tag{4-26}$$

Consequently, if the generating units are operating all the time, the load dispatching can be based only on the capacities. In other words, we only have to be concerned that Equation (4-24) holds.

Now assume that the units are out of the system a certain number of hours during the year. From (4-16) $TIMEIN_{ij}$

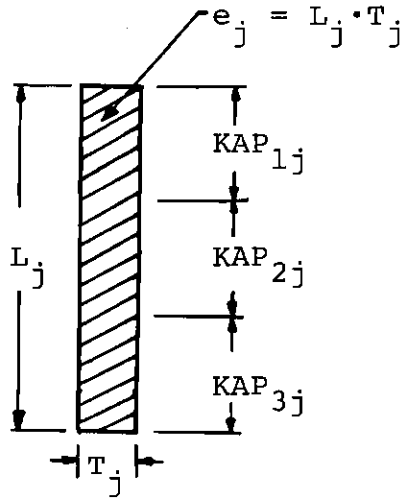


Figure 4-4. A "Rectangle" of Energy (3 Generating Units).

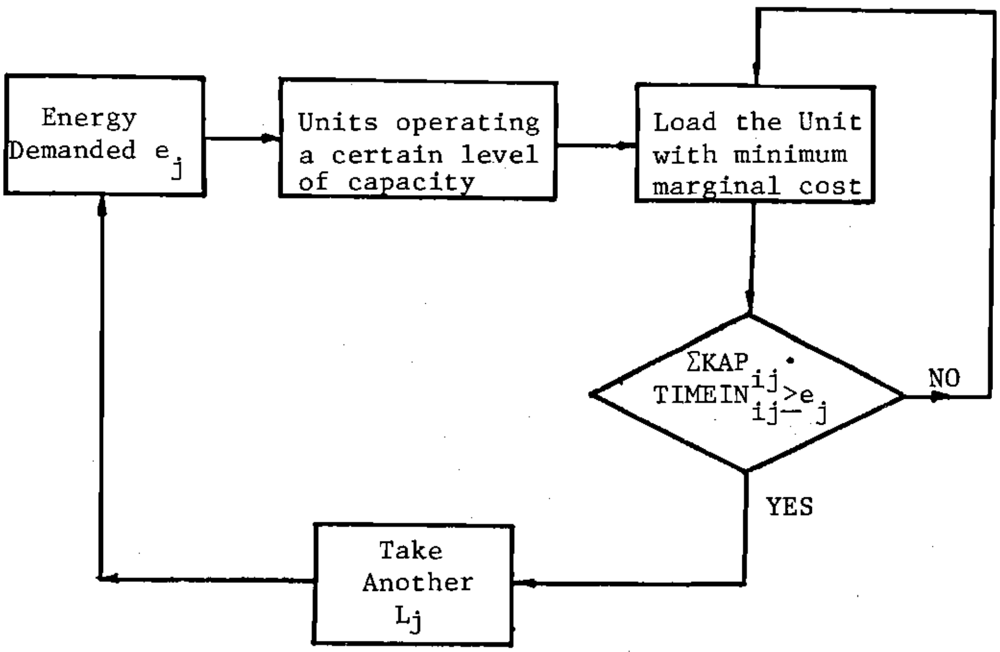


Figure 4-5. Flow Diagram of Units Dispatching.

can be calculated and the total energy produced during T_j may be computed by Equation (4-25). But $TIMEIN_{1j}$, $TIMEIN_{2j}$, $TIMEIN_{3j}$ are less than T_j so the energy produced will be less than the energy demanded e_j. The $TIMEIN_{ij}$ cannot be altered because it depends on the unit maintenance time and forced outage time, so the only way to meet the energy demanded is to increase the operating capacities (KAP_{ij}) of the units until the energy produced is equal to the demanded. In other words, use the incremental cost method to load the units, but stop when the sum of the products of $KAP_{ij} \cdot TIMEIN_{ij}$ is equal to e_j. This procedure is shown in Figure 4-5. Notice that for this case the <u>total megawatts produced</u> ($\sum_i KAP_{ij}$) are greater than the <u>megawatts of load L_j demanded</u> for that interval T_j. This means

$$\sum_i KAP_{ij} > L_j \tag{4-27}$$

but the <u>energy produced</u> ($\sum_j KAP_{ij} \cdot TIMEIN_{ij}$) is equal to the <u>energy demanded</u> ($L_j \cdot T_j$). This approach leads to operating units at higher capacities than would be found under actual conditions.

4.2.6. Computation of the Total Energy Costs (Operating Costs)

Once the KAP_{ij}, the CFD_i (operating cost) and the input-output relationship Y_i have been computed for each generating unit "i," it is only necessary to use Equation (4-3) to calculate the operating cost per hour of operation for unit "i" when the demand is L_j.

$$TC_{ij} = A_i\ CFD_i \cdot KAP_{ij}^{\ 2} + B_i\ CFD_i KAP_{ij} + C_i \tag{4-28}$$

The energy cost for unit "i" for demand L_j is calculated by multiplying TC_{ij} by the number of hours operated by unit "i" when demand L_j occurs. This variable will be identified by EC_{ij}.

$$EC_{ij} = (TC_{ij})(TIMEIN_{ij}) \tag{4-29}$$

$$EC_{ij} = (A_i CFD_i KAP_{ij}^{\ 2} + B_i CFD_i KAP_{ij} + C_i)(TIMEIN_{ij}) \tag{4-30}$$

From Equation (4-29), it is possible to obtain the total energy cost when the demand is e_j (TEC_j).

$$TEC_j = \sum_{i=1}^{m} EC_{ij} \tag{4-31}$$

The calculation of the total energy cost of unit "i" in one year of operation ($ENCOST_i$) is obtained by adding the EC_{ij} for L_j j=1...n.

$$ENCOST_i = \sum_{j=1}^{n} EC_{ij} \tag{4-32}$$

Consequently, if the load duration curve is divided in "n" intervals of time T_j j=1...n and there are "m" units in the system (i=1...m), the total energy cost of the system for one year of operation (TOTCOS) will be equal to the sum of the unit's total energy costs. Therefore,

$$TOTCOS = \sum_{i=1}^{m} ENCOST_i \qquad (4\text{-}33)$$

Now, if the fixed cost of unit "i" (FIX_i) is added to the unit total energy cost ($ENCOST_i$), the overall operating cost for unit "i" is obtained

$$GRANC_i = ENCOST_i + FIX_i \qquad (4\text{-}34)$$

Hence the total overall cost for the system will be

$$GRANCOST = \sum_{i=1}^{m} GRANC_i \qquad (4\text{-}35)$$

4.2.7. Operation of Combustion Turbines

The procedure developed to dispatch the generating units and the equations derived to obtain the energy produced and the operating costs can be applied either for Type I or Type II units. As stated in Section 4.1.2, units Type II operate at a fixed level of capacity (maximum). Also, it will be assumed that they are not out of the system because of maintenance or forced outages. Due to this last assumption, the time operated for each of these units at each interval (T_j) will be the number of hours of the interval T_j. In other words, for Type II units:

$$TIMEIN_{ij} = T_j \qquad (4\text{-}36)$$

Now, because of the fact that Type II units produce power at a fixed operating level (maximum), they have constant marginal cost. For instance, assume that in a power system the unit No. 11 (Type II) has the following characteristics:

Input-output curve:	$Y = .17287x^2 + 3.539x + 153.73$
Levels of operation:	Min = 0 mw (turned-off)
	Max = 24 mw (turned-on)
Operating Cost:	2.359 \$/MMBtu.

The marginal cost for this unit is:

$$Y = 2 \cdot A_{11} \cdot CFD_{11} x_{11} + B_{11} \cdot CFD_{11}$$

$$= (2)(.17287)(24)(2.359) + (3.539)(2.359)$$

$$= 27.9 \text{ \$/mw-hr}$$

Notice that the marginal cost was obtained only for maximum capacity (24 mw). To avoid overestimation of produced energy when the incremental cost method is applied, the marginal cost for these units will be calculated at full capacity, but the load assigned will be only that required to meet the demand.

4.2.8. Approximation on the Load Duration Curve

It has been shown how to calculate the energy cost for each one of the energy "rectangles," but in order to obtain the energy e_j under these "rectangles," it is necessary to know the value of the demand L_j.

It was pointed out in Section 4.1.5 that 90 data points are used to describe an annual load duration curve. It will be shown how the intermediate points based on this data were obtained. Figure 4-6 illustrates a load duration curve. Assume that the points (PH, PLR) shown on this curve are two of those 90 data points to be supplied. It is a reasonable approximation in most parts of the curve to assume that the distance between two next points is a straight line. Let's take the two next points indicated by the circle in Figure 4-6. With these two points, it is possible to construct a triangle as shown in Figure 4-7 and calculate any point between them. To illustrate this procedure, suppose it is desired to compute which is the value of L_q for the $HOUR_q$ (Figure 4-7). By simple trigonometry,

$$\frac{PL_{j+1} - PLR_j}{PH_{j+1} - PH_j} = \frac{PLR_{j+1} - L_q}{PH_{j+1} - HOUR_q} \tag{4-37}$$

therefore,

$$L_q = PLR_{j+1} - \frac{(PLR_{j+1} - PLR_j)(PH_{j+1} - HOUR_q)}{PH_{j+1} - PH_j} \tag{4-38}$$

where the $HOUR_q$ is obtained by substracting the interval T_j from the last $HOUR_q$ calculated (in this case the last PH_j is the last data point computed, but it can be a new point calculated). Therefore, by Equation (4-38) different values of the load demand L_j can be obtained and it will depend on the interval size T_j chosen to analyze the load duration curve.

It is important to notice here that if the load duration curve is divided in rectangles like shown in Figure 4-3a, the

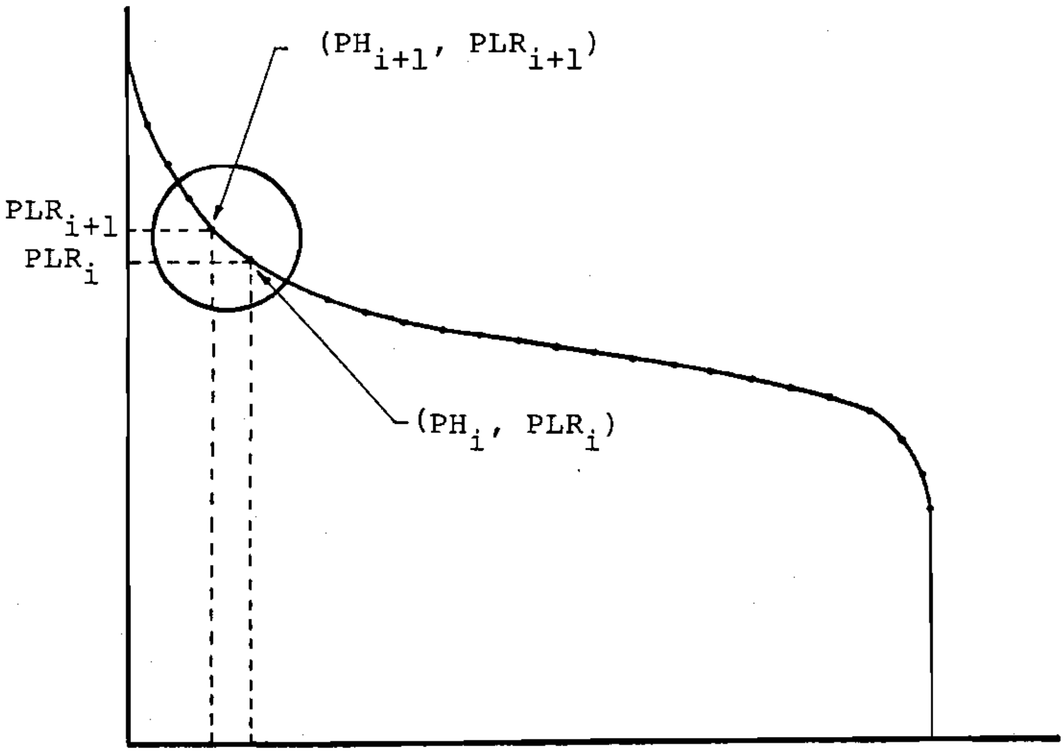


Figure 4-6. Approximation of the Load Duration Curve.

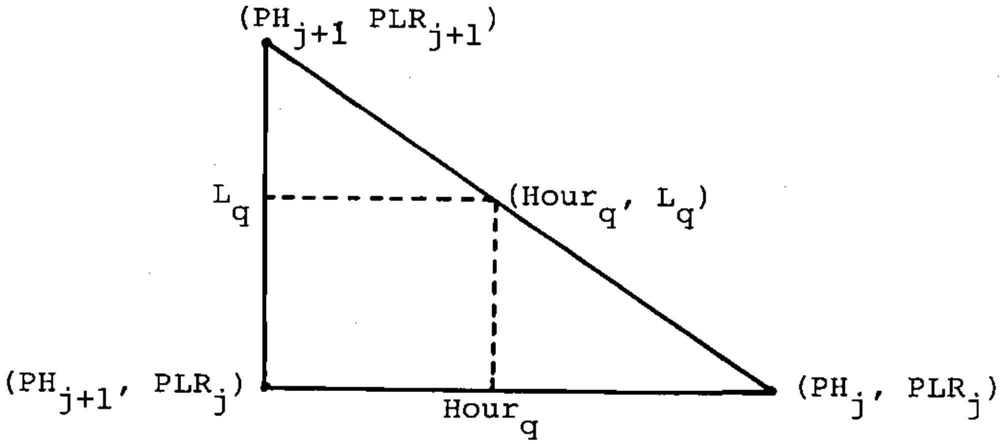


Figure 4-7. Computation of the Load Demand L_j.

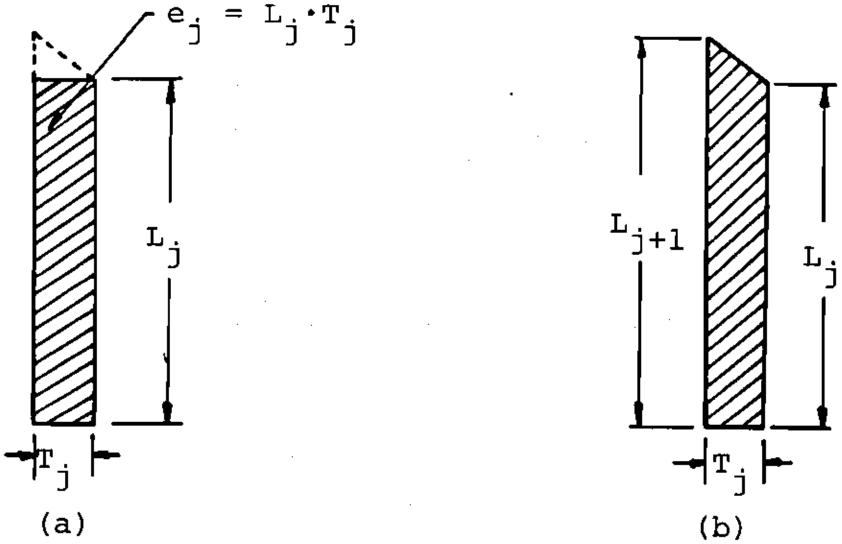


Figure 4-8. Approximation of e_j.

total energy demanded is underestimated. It would be more exact if instead of a "rectangle" a figure like that shown in Figure 4-8b is used. The area under this trapezoid will be:

$$e_j = T_j \cdot L_j + T_j \cdot ((L_{j+1} - L_j)/2) \qquad (4\text{-}39)$$

$$e_j = T_j \cdot ((L_j + L_{j+1})/2) \qquad (4\text{-}40)$$

4.3. Steps and Flow Diagram of the Method Developed

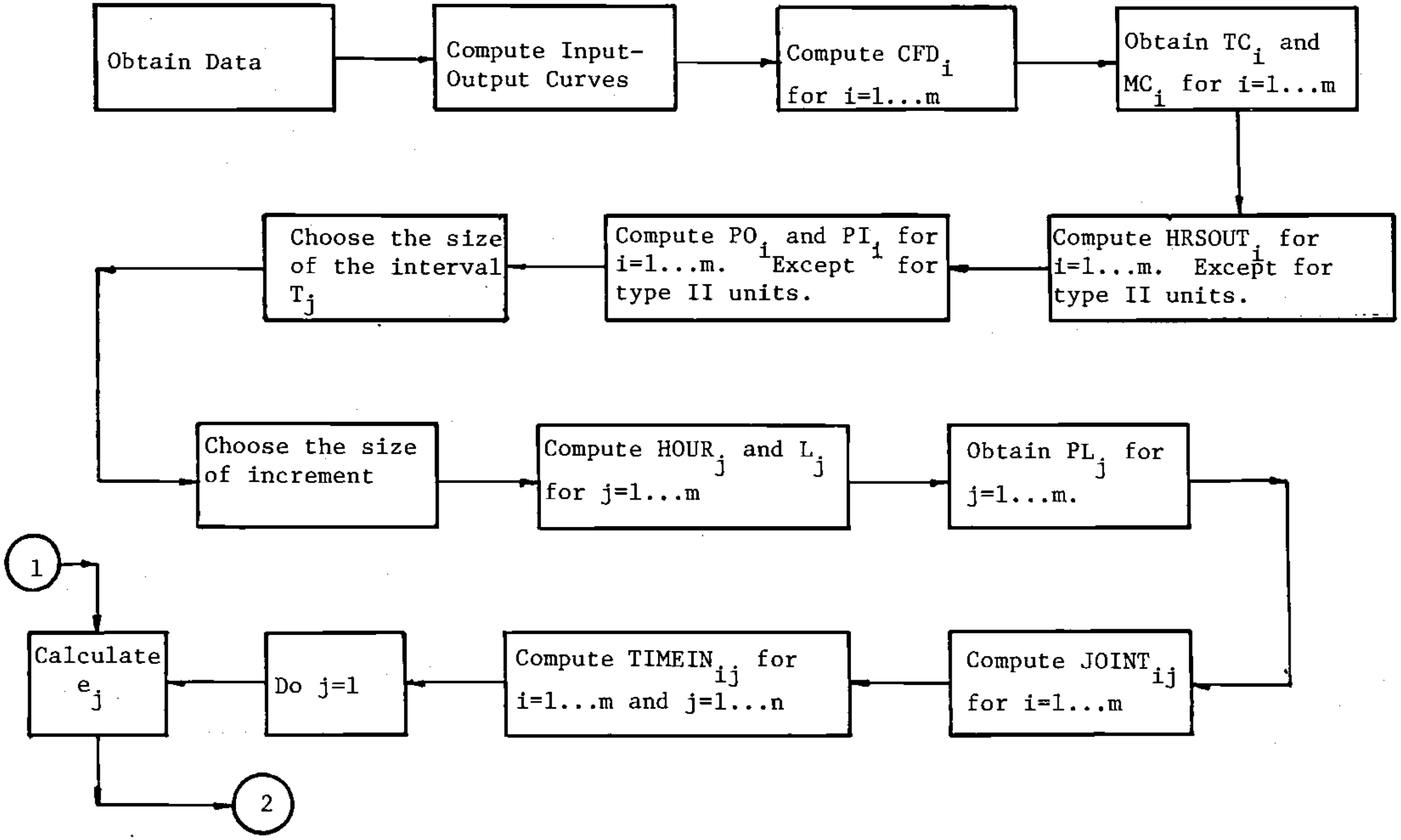
Up to this point the equations necessary to obtain the operating costs have been presented. The purpose of this section is to explain the procedure for applying these equations. The first part enumerates the steps to follow and the second part presents the flow diagram of the algorithm.

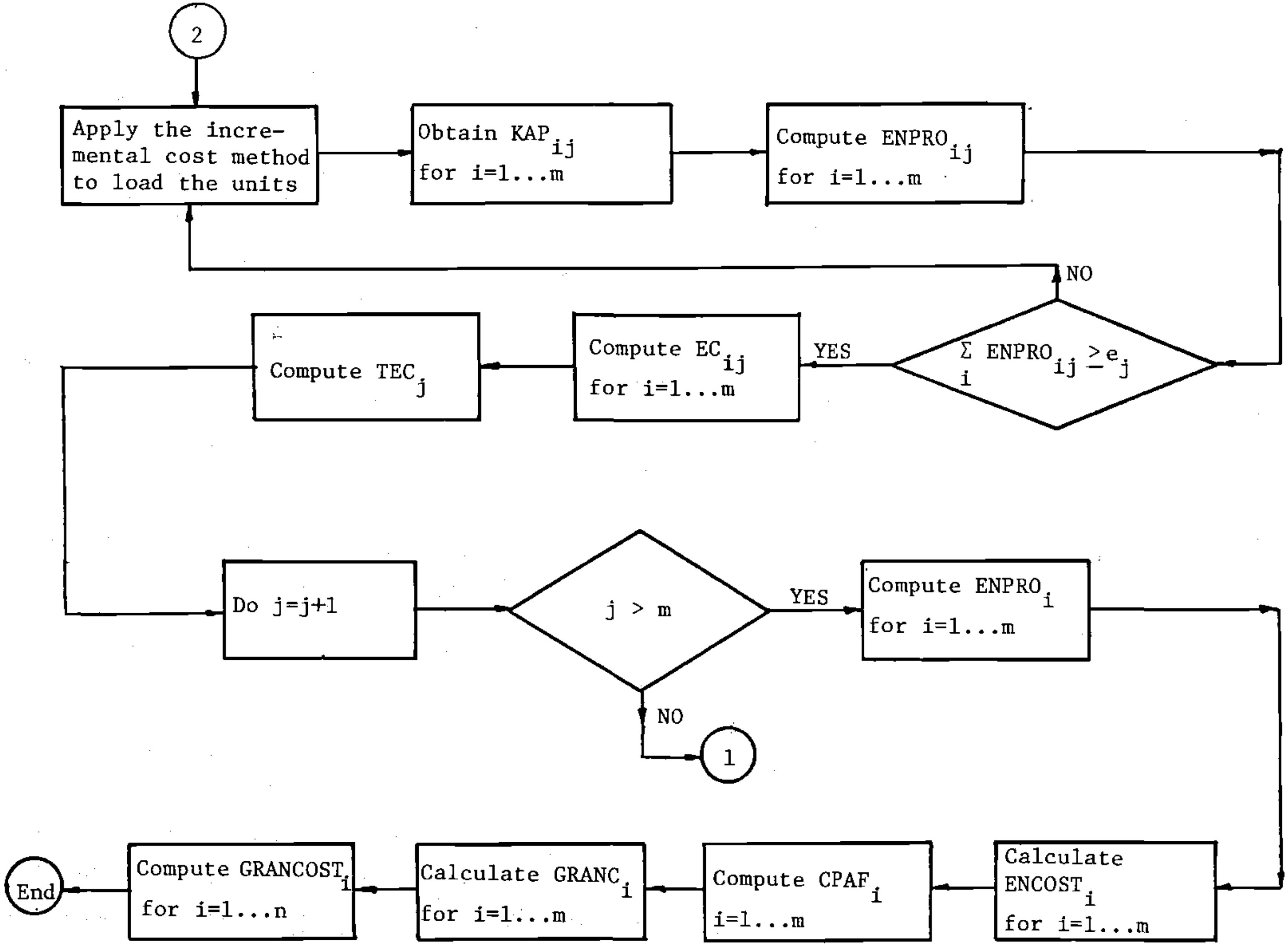
4.3.1. Steps

1. Obtain the data (Eq. 4-1).
2. Compute the input-output curves (Eq. 4-1) for i=1...m.
3. Calculate CFD_i (Eq. 4-2) for i=1...m.
4. Obtain TC_i and MC_i function (Eq. 4-3 and Eq. 4-5, respectively) for i=1...m.
5. Calculate $HRSOUT_i$ (Eq. 4-10) for i=1...m except for Type II units.
6. Compute PO_i and PI_i (Eq. 4-11) and (Eq. 4-12) for i=1...m, except for Type II units.
7. Choose T_j. Remeber the size of T_j determines the number of intervals "n" that divides the load duration curve.

8. Choose the size of the increment to use in the incremental cost load dispatching method (Section 5.3.1 presents some recommendations to choose the increment size).
9. Compute $HOUR_j$ and L_j using Equation 4-38 for j=1...n.
10. Obtain PL_j (Eq. 4-9) for j=1...n.
11. Compute $JOINT_{ij}$ (Eq. 4-13) and $TIMEIN_{ij}$ (Eq. 4-16) for i=1...m and j=1...n.
12. Do j=1, in other words, start with last "rectangle." Compute e_j (Eq. 4-40).
13. Apply the incremental cost method for load dispatching as explained in Section 4.2.5. Remember the special treatment mentioned in Section 4.2.7 for Type II units.
14. Compute $ENPRO_{ij}$ for i=1...m (Eq. 4-18).
15. If $\sum_{i=1}^{n} ENPRO_{ij}$ is greater or equal than e_j, go to step 16, if it is not go to 13.
16. Compute EC_{ij} (Eq. 4-30) for i=1...m and TEC_j (Eq. 4-31). Go to step 17.
17. Do j=j+1 and compute e_{j+1} (Eq. 4-40).
18. If j > m go to 19, if it is not, go to 13.
19. Calculate $ENPRO_i$ (Eq. 4-19), $ENCOST_i$ (Eq. 4-32) and $CAPF_i$ (Eq. 4-23) for i=1...m.
20. Compute $GRANC_i$ (Eq. 4-34) and GRANCOST (Eq. 4-35).
21. End.

4.3.2. <u>Flow Diagram</u>.




2
Apply the incremental cost method to load the units
Obtain KAP_ij for i=1...m
Compute ENPRO_ij for i=1...m
NO
Σ_i ENPRO_ij ≥ e_j
YES
Compute EC_ij for i=1...m
Compute TEC_j
Do j=j+1
j > m
YES
NO
1
Compute ENPRO_i for i=1...m
Calculate ENCOST_i for i=1...m
Compute CPAF_i i=1...m
Calculate GRANC_i for i=1...m
Compute GRANCOST_i for i=1...n
End

4.4. An Example of Economical Dispatching

To assist in the description of the dispatching algorithm a numerical example is presented. The operating costs and capacity factor for each of the system's generating units will be computed by the method developed and compared with those obtained if they have been calculated based on the load curve.

4.4.1. Statement of the Problem

Suppose a power system with 4 Type I units with characteristics shown in Table 2 and projected system's load curve shown in Figure 4-9. Assume these units will not be out of the system because of forced outages. It is desired to obtain the generating units operating cost and their capacity factors for the year projected.

4.4.2. The Load Duration Curve

Figure 4-9 shows the projected system's load curve for the year "x." In this curve, the demand load is plotted against the time sequence. For instance, a load of 150 mw will be demanded from hour 0 to hour 500, a load of 160 mw will be required from hour 501 to hour 1000, and so on. The next step is to calculate the load duration curve. It is computed from the load curve by plotting each load demanded versus the percent of time or number of hours that the load demand is greater than or equal to that amount. For example, for the load curve shown in Figure 4-9, the greatest load demand is 210 mw. This load is demanded 125 hours, therefore, the time percent of the year that the demand is greater or

Table 2. Data for the Example.

Unit Number	Input-Output Coefficients			Capacities (mw)		Maint. Time (hrs)	Forced Out. Time (hrs)	Oper. Cost (CFD) $/MMBtu
	A	B	C	Min	Max			
1	.02364	9.039	57.79	25	70	1000	0	1.12
2	.09476	5.650	104.25	15	40	1000	0	1.12
3	.02590	8.860	62.77	30	35	1000	0	1.12
4	.02277	7.608	128.55	30	100	1500	0	1.12

Table 3. Data to Plot the Load Duration Curve.

No.		L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
1	Megawatts	210	200	195	190	185	180	175	170	160	150
2	Hours Dem	125	375	500	625	875	1000	1250	1250	1250	1510
3	% Time	1.4	4.2	5.7	7.1	9.9	11.4	14.2	14.2	14.2	14.2
4	% Time $\geq L_j$	1.4	5.6	11.3	18.4	28.3	39.7	53.9	68.1	82.3	100.0

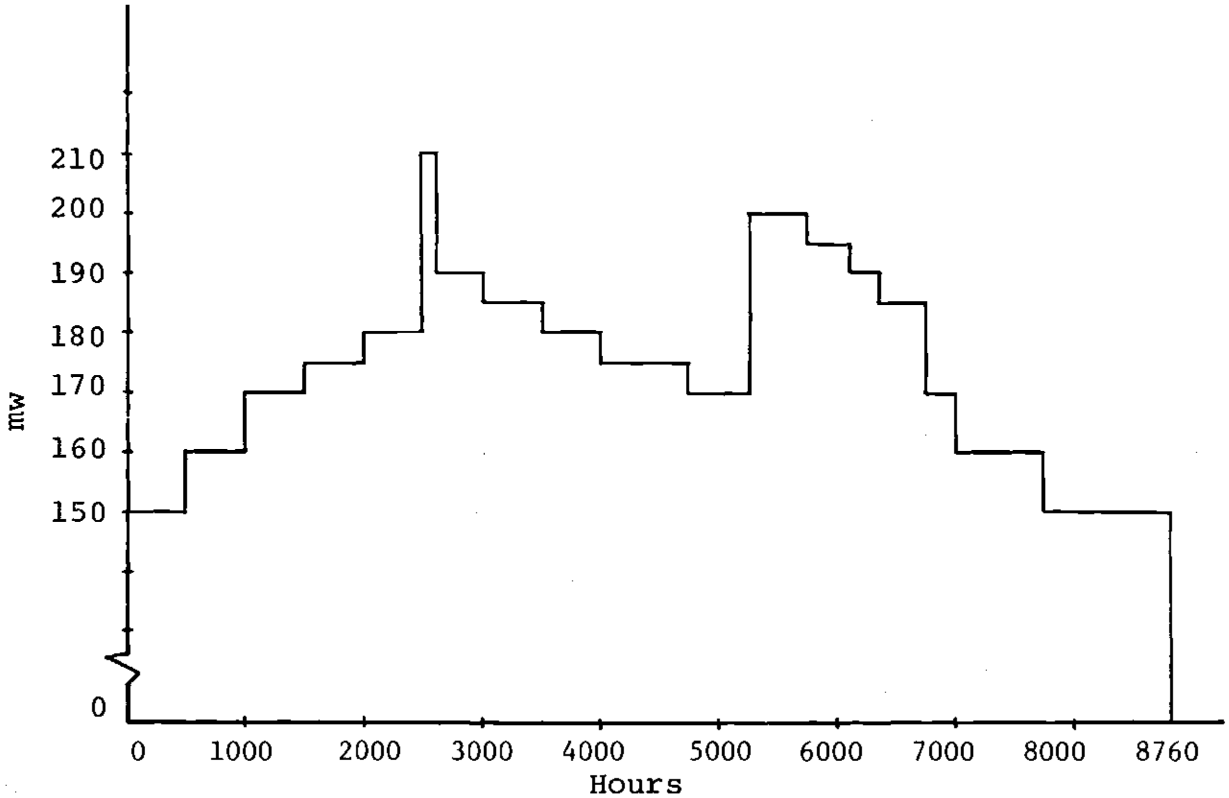


Figure 4-9. Load Curve for the Example

equal than 210 mw was 1.42% (125/8760). Table 3 shows the time percent that each load is demanded. In row 4 of this table is illustrated the percent time that the demand is greater or equal than the load shown in row 1. The load duration curve is obtained by plotting row 4 vs 1. Figure 4-10 shows the load duration curve for this example.

4.4.3. Computation of Probabilities of Operation and Load Demand Probability

The next step is to compute the probability that each unit will be operating during the projected year. Table 4 shows the number of hours each unit will not be available (TMAIN and TFOR). From Equation (4-12) can be calculated the probability of operation for each unit, (PI_i).

$$PI_i = 1 - HRSOUT_i/8760$$
$$= 1 - PO_i$$

where $HRSOUT_i = TMAIN_i + TFOR_i$.

Table 4 illustrates the PI_i for each unit on the system.

Table 4. Probabilities of Operation.

Unit	$TMAIN_i$	$TFOR_i$	$HRSOUT_i$	PI_i
1	1000	0	1000	.885
2	1000	0	1000	.885
3	1000	0	1000	.885
4	1500	0	1500	.828

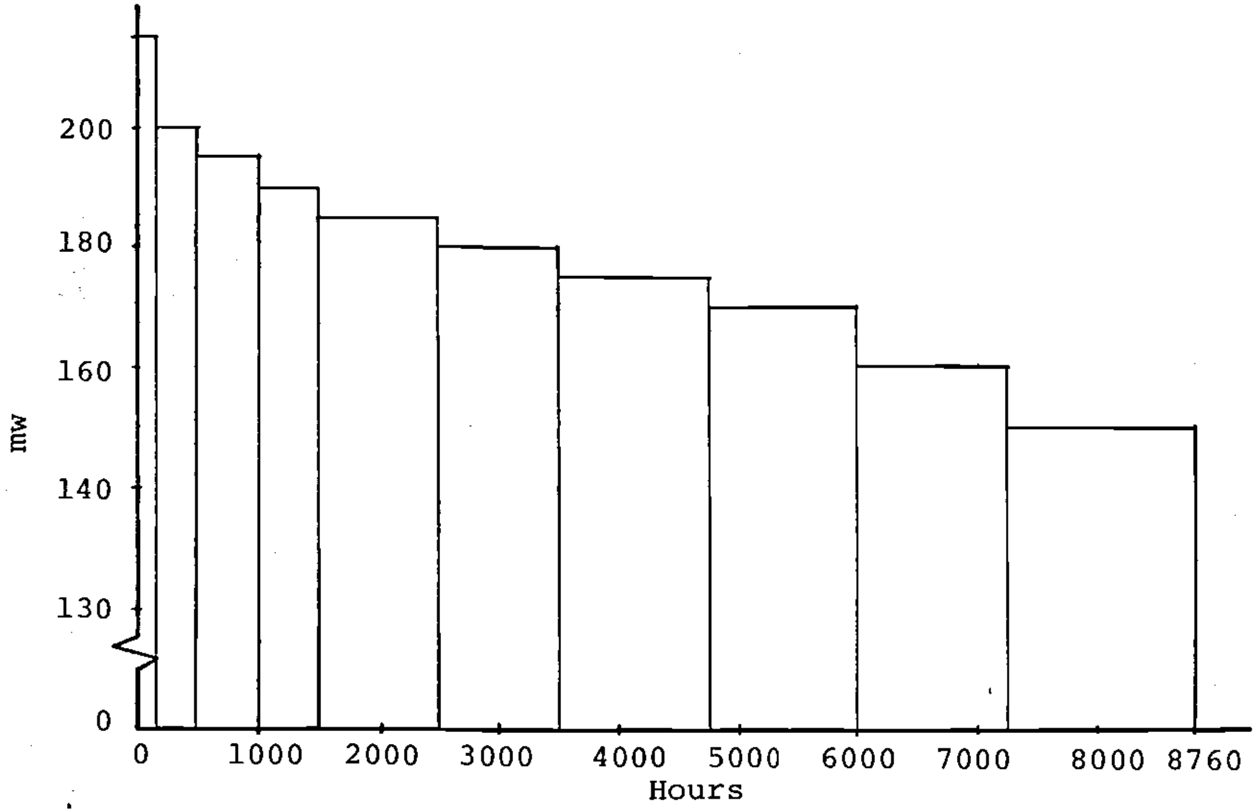


Figure 4-10. Load Duration Curve for the Example.

Once the PI_i's are computed, the next step is to obtain the probability of each load demand. From the load duration curve, it is possible to obtain the number of hours that each load is demanded. Therefore, the probability that load "j" is demanded during the year (PL_j) can be computed. This can be done by Equation (4-9).

$$PL_j = \frac{T_j}{8760}$$

Table 5 shows the PL_j for each load.

4.4.4. Expected Number of Hours Operated During Each Interval T_j

The next step is to obtain the probability that unit "i" will be operating when the demand is L_j. It is calculated by Equation (4-13).

$$JOINT_{ij} = (PI_i)(PL_j)$$

For instance, the probability that unit 1 will be operating when the demand is L_1 (210 mw) is

$$JOINT_{11} = (PI_1)(PL_1) = (.8858)(.014) = 0.012.$$

By $JOINT_{ij}$ and Equation (4-16), the expected number of hours that unit "i" will be operating when the demand is L_j can be calculated.

$$TIMEIN_{ij} = (JOINT_{ij})(8760)$$

For example, the expected number of hours that unit 1 will be

Table 5. Probabilities of the Loads.

No.		L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
1	Megawatts	210	200	195	190	185	180	175	170	160	150
2	Hours (T_j)	125	375	500	675	875	1000	1250	1250	1250	150
3	PL_j	.014	.042	.057	.071	.099	.114	.142	.142	.142	.172

operating when the demand is L_1 is

$$TIMEIN_{11} = (JOINT_{11})(8760) = (.012)(8760) = 105 \text{ hours}$$

The $JOINT_{ij}$ and $TIMEIN_{ij}$ for i=1...4 and L_j j=1...10 are shown in Table 6 and Table 7, respectively.

4.4.5. Economic Dispatching

In order to dispatch the units in the most economical way, it is necessary to know their marginal cost functions. This function can be obtained from Equation (4-5).

Unit 1: $MC_1 = .05295\ x_1 + 10.123$

Unit 2: $MC_2 = 0.2122\ x_2 + 6.328$

Unit 3: $MC_3 = .0580\ x_3 + 9.923$

Unit 4: $MC_4 = .0510\ x_4 + 7.916$

The next step is to obtain the level of capacity that each unit must operate to meet the demand e_j. The step can be done using Equation (4-24) and by the procedure shown in Figure 4-5. For example, the energy demanded corresponding to the load of 150 mw (L_{10}) is Equation (4-6) [(1)]

$$e_{10} = T_j \cdot L_j$$

$$e_{10} = T_{10} \cdot L_{10}$$

$$e_{10} = (1500)(150) = 225000 \text{ mw-hr}$$

It is necessary to dispatch the units in such a way that the energy produced is equal to the energy demanded e_{10}. There-

(1) It can be used Equation (4-40) for more accuracy.

Table 6. Values of $JOINT_{ij}$.

i \ j	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
1	.012	.037	.051	.063	.088	.101	.126	.126	.126	.152
2	.012	.037	.051	.063	.088	.101	.126	.126	.126	.152
3	.012	.037	.051	.063	.088	.101	.126	.126	.126	.152
4	.012	.035	.047	.059	.082	.095	.118	.118	.118	.143

Table 7. Values of $TIMEIN_{ij}$.

i \ j	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
1	105.12	324.12	324.12	446.76	551.8	884.76	886.1	1103.76	1103.76	1331.5
2	105.12	324.12	324.12	324.12	551.8	884.76	886.1	1103.76	1103.76	1331.5
3	105.12	324.12	324.12	324.12	551.8	884.76	886.1	1103.76	1103.76	1331.5
4	105.12	306.20	306.60	324.12	551.8	884.76	886.1	1103.76	1103.76	1331.5

fore the operating level for each unit must be:

$$\begin{aligned} \text{Unit } 1 &= 35 \text{ mw} \\ \text{Unit } 2 &= 27 \text{ mw} \\ \text{Unit } 3 &= 35 \text{ mw} \\ \text{Unit } 4 &= \underline{80 \text{ mw}} \\ & 177 \text{ mw} \end{aligned}$$

because for these levels

$$\sum_{i=1}^{4} KAP_{i10} \cdot TIME_{i10} \geq e_{10}$$

in other words,

$$(34)(1331.5) + (25)(1331.5) + (35)(1331.5) + (80)(1252.68) = 225375.4$$

and

$$225375.4 \text{ mw-hr} \geq e_{10}$$

Notice that the energy produced by each unit $ENPRO_{i10}$ (Equation 4-18) is

$$\begin{aligned} &\text{Unit 1:} \quad (34)(1331.5) = 45271.00 \text{ mw-hr} \\ &\text{Unit 2:} \quad (25)(1331.5) = 33287.00 \text{ mw-hr} \\ &\text{Unit 3:} \quad (35)(1331.5) = 46602.50 \text{ mw-hr} \\ &\text{Unit 4:} \quad (80)(1257.68) = 100214.40 \text{ mw-hr} \end{aligned}$$

The same procedure is applied for the remaining e_j.

4.4.6. Calculation of Energy Costs and Capacity Factors

The unit energy cost for the energy demand e_j is computed by Equation (4-30)

$$EC_{ij} = (A_i CFD_i \cdot KAP_{ij}^{\ 2} + B_i CFD_i \cdot KAP_{ij} + C_i)(TIMEIN_{ij})$$

therefore, for e_{10}:

$$EC_{11} = ((.0264)(34)^2 + 10.123(34) + 64.72)(1331.5)$$
$$EC_{11} = \$585243.65$$

$$EC_{12} = ((.1061)(25)^2 + 6.328(24) + 116.76)(1331.5)$$
$$EC_{12} = \$454430.25$$

$$EC_{13} = ((.02901)(35)^2 + 9.923(35) + 70.30)(1331.5)$$
$$EC_{13} = \$603368.16$$

$$EC_{14} = ((.0255)(80)^2 + 7.916(80) + 143.97)(1252.68)$$
$$EC_{14} = \$1,178,125.7$$

Following the same procedure the EC_{ij} for all i, j is calculated.

Now, using Equations (4-32) and (4-33), the total energy cost for each unit ($ENCOST_i$), and the system total energy cost for one year of operation (TOTCOS) can be computed. Similarly, by Equation (4-23), the capacity factor is calculated. After applying these equations, the results are as follows:

Unit	$ENPRO_i$ (mw-hr)	$ENCOST_i$ ($)	$CAPFAC_i$
1	340,802.19	4,352,648.63	56%
2	217,492.60	2,927,610.28	62%
3	337,187.95	4,320,150.45	70%
4	641,067.95	7,574,900.97	70%
	1,536,550.69	19,175,310.33	

When the time and period of occurrence of the forced outages are known, load dispatching based on the load curve is optimal. The results obtained using the load curve in Figure 4-9 are:

Unit	$ENPRO_i$ (mw-hr)	$ENCOST_i$ ($)	$CAPFACI_i$
1	345,145.00	4,436,303.75	50%
2	215,480.00	3,001,414.40	61%
3	337,415.00	4,339,469.70	70%
4	606,210.00	7,143,853.70	69%

Next, the differences in percent of the results obtained by the method developed with respect to those obtained by using the load curve are presented.

Unit	$ENPRO_i$	$ENCOST_i$	$CAPFAC_i$
1	-1%	-2%	0%
2	+1%	-2%	1.6%
3	-0.06%	-0.04%	0%
4	+6%	+6%	1.4%

Difference in total cost: 1.34%
Difference in energy produced: 2.14%

For example, the difference between the results obtained by the two methods in computing the $ENPRO_1$, $ENCOST_1$, and $CAPFAC_1$ are -1%, -2%, and 0%, respectively.

For this particular problem, the differences between the two approaches are not significant. In the next chapter, the method developed in this study will be tested by using data for a large electric utility company with approximately 120 generating units of different types. The results developed are then compared with the results provided by a production costing routine actually used by a large electric utility. The purpose is to test the dispatching algorithm developed with a more accurate method presently in use. The two characteristics that are of greatest interest are the speed and the accuracy with which the two methods determine total system production costs.

CHAPTER V

COMPARISON OF THE METHOD DEVELOPED WITH THE METHOD CURRENTLY UTILIZED

The purpose of this chapter is to test the algorithm developed in this study with existing methods. For this purpose, the accuracy and computer time required by the developed method are compared with results obtained from a large scale system presently utilized by a large electric utility.

5.1. General Information

A large power system containing approximately 120 thermal units has been used to test the algorithm developed. For purposes of the test, the units have been identified by numbers. From Unit 1 to Unit 88 are included all Type I units. Type II units (combustion turbines) are identified with numbers between 89 and 123.

A computer program of the algorithm developed has been written in Fortran IV. This program attempts to compute all the variables mentioned in Chapter IV. The runs of this program have been performed on a CDC Cyber 74 at the Georgia Institute of Technology.

The objective of the test is to make comparisons of the operating costs, capacity factors and computer execution time of the method developed with the method actually used by the electric utility.

5.2. Steps Followed to Test the Method

The test has been divided in three parts:

1. In the first part are the comparisons of the operating results and computer execution times obtained by the method developed. Comparisons are made for different megawatts increments (marginal cost method) and different intervals hours (T_j). This is done for one year of operation.
2. In Part II are shown the values of the unit's operating costs and unit's capacity factors obtained by both methods. The differences between these results are presented and analyzed.
3. In the last part, the total operating cost, memory core requirements, and computer execution time of the two methods are compared. This comparison is made for four projected years.

5.3. The Test

5.3.1. Part I: Results Obtained by the Method Developed

In this part of the test is presented the results obtained by the method developed in this study using different megawatts increments and different interval times. Table 8 shows the total operating cost utilizing intervals (T_j) on the load duration curve of 10, 30, 60, and 120 hours

Table 8. Total Operating Costs and Execution Time for Different Time Intervals and an 5mw Increment (19x1).

Interval (Hours)	Operating Cost (Thousands of Dollars)	Exec. Time (Seconds)
10	1,107,931.00	34.69
30	1,096,908.00	26.67
60	1,094,908.00	24.60
120	1,093,144.00	23.99

Table 9. Total Operating Cost and Execution Time for Different Increments (mw) and a 60 Hours Interval.

Increment (mw)	Operating Cost (Thousands of Dollars)	Exec. Time (Seconds)
1	1,094,347.00	108
5	1,094,908.00	24.6
10	1,094,327.00	14.09
100	1,114,547.00	5.34

for an increment (in the application of the marginal cost method) of 5 mw. The right column of this table shows the computer execution time for these intervals. Notice that for increments larger than 10 hours the differences in execution time are not significant. In other words, the time to process the algorithm does not depend on the interval, T_j. Table 9 shows the total operating cost and computer execution time obtained for different increments and a 60 hours interval. Notice how the variation in the total operating cost for the different increments are insignificant, even though the differences in execution time are quite large.

From these results can be concluded that the computer execution time for the method developed depends predominately on the increment size utilized to dispatch the units by the incremental cost method. The increment size must be chosen depending on the unit's capacity ranges and on the slopes of the marginal cost functions. The larger are the unit's capacity ranges, the larger can be the capacity increments. The smaller the slope of the marginal cost function of the unit, the smaller the change in marginal cost for a given increment. Thus, the smaller are these slopes, the larger can be the capacity increments without adversely affecting the results.

With respect to the interval time size T_j (Table 8 and Figure 5-3 and 5-4), it can be concluded that the differences in total operating cost and execution time do not vary significantly as the interval varies. The smaller the interval,

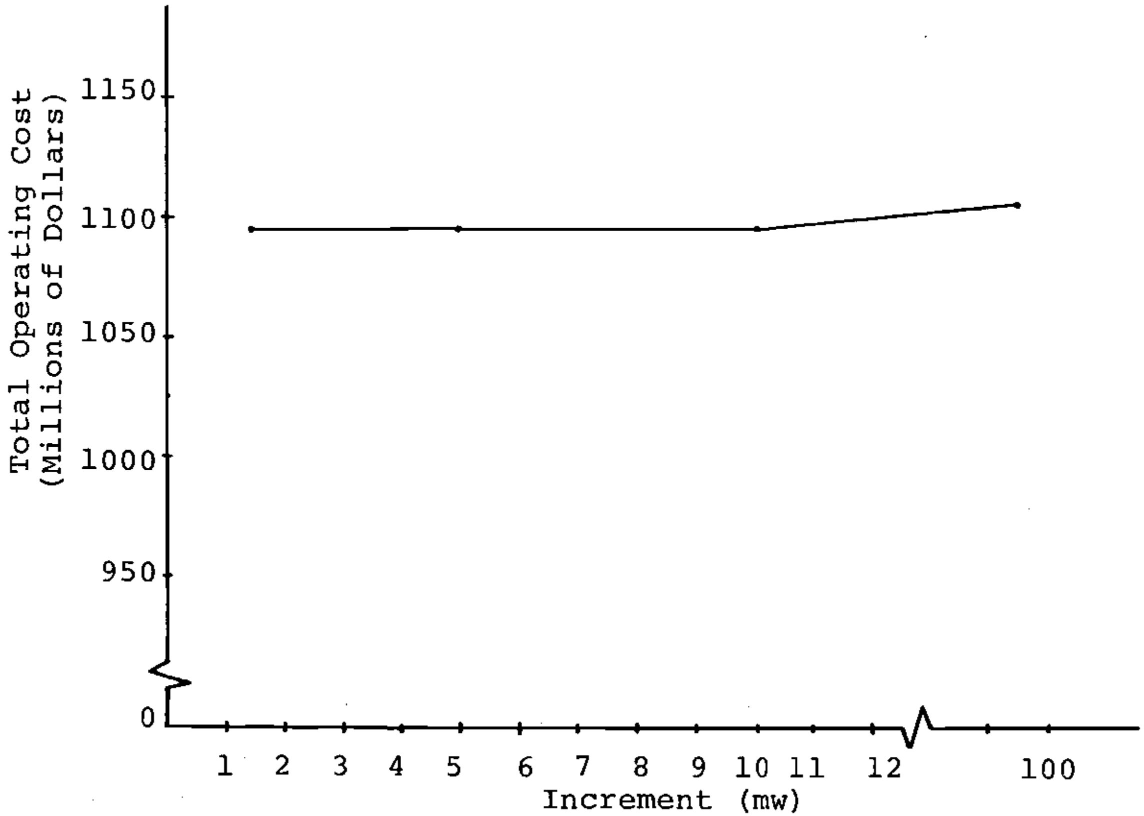


Figure 5-1. Comparison of the Total Operating Costs for 19x1 Using Different Capacity Increments and 60 Hours Interval.

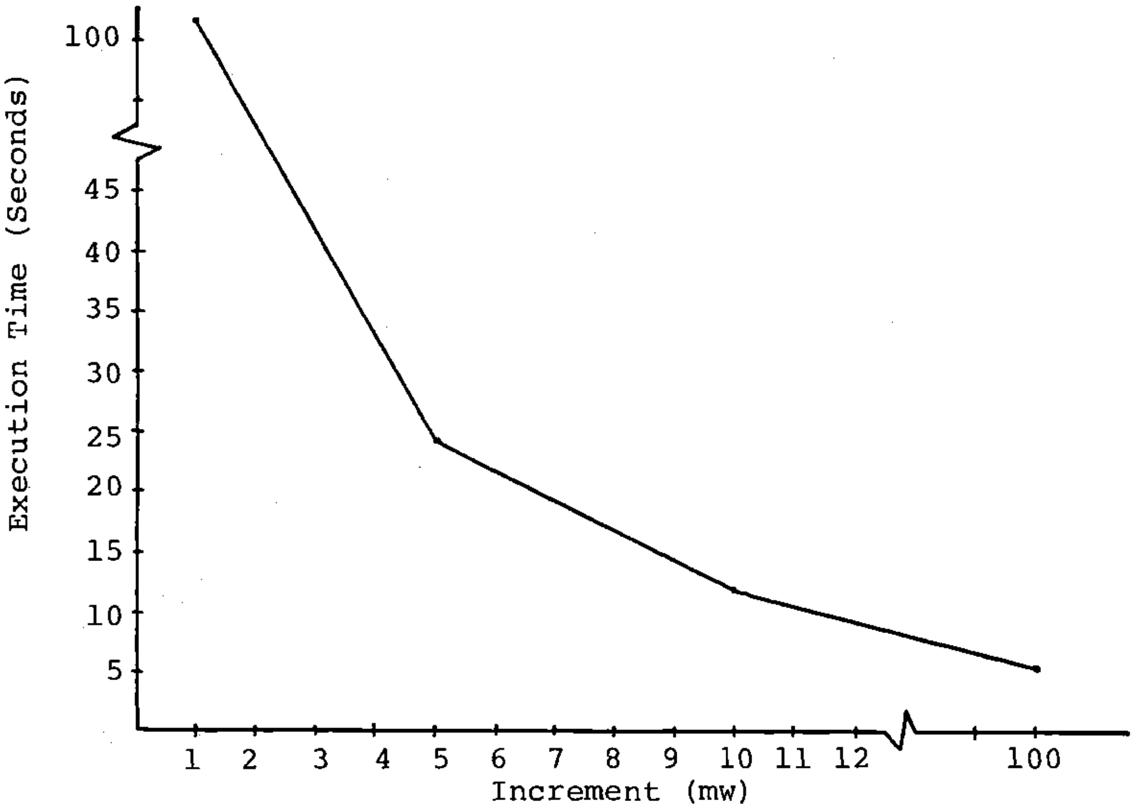


Figure 5-2. Comparison of the Total Execution Time for 19x1 Using Different Increments (mw) and 60 Hours Interval.

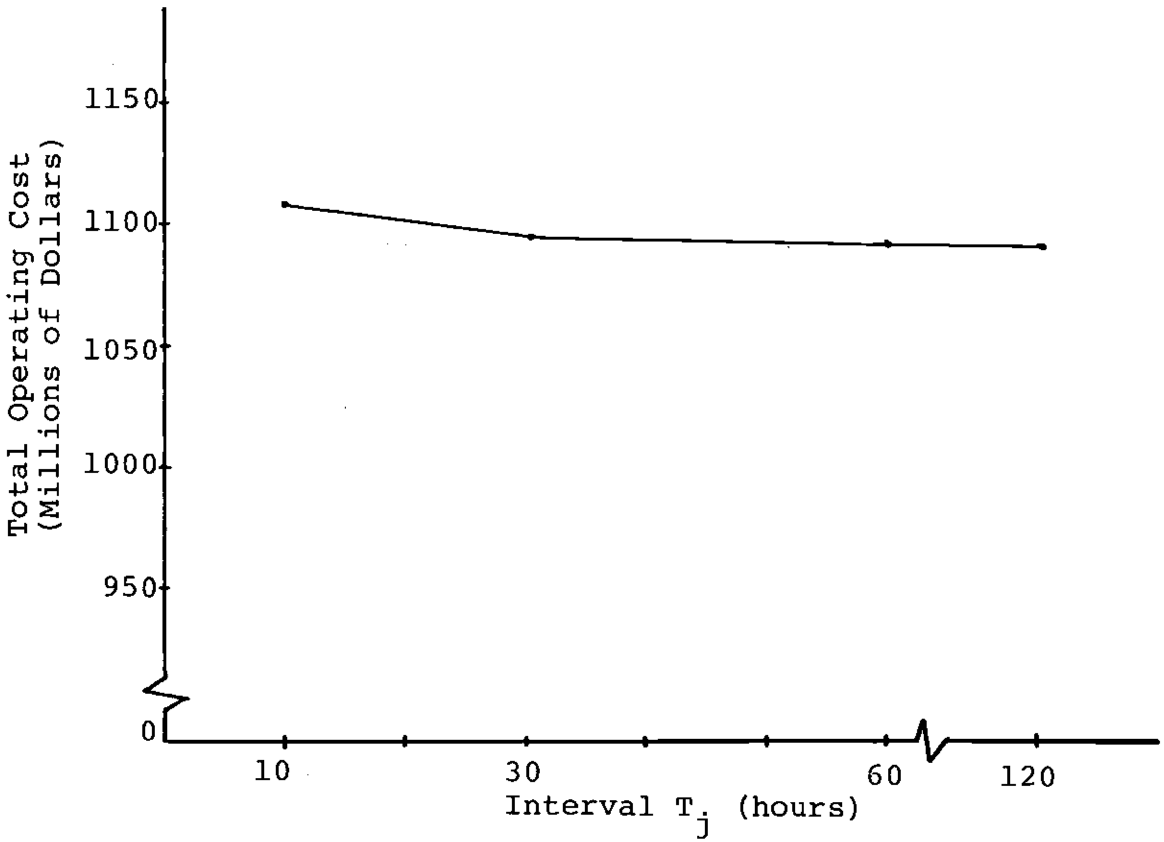


Figure 5-3. Comparison of the Total Operating Costs for 19x1 Using Different Time Intervals and a 5 mw Capacity Increment.

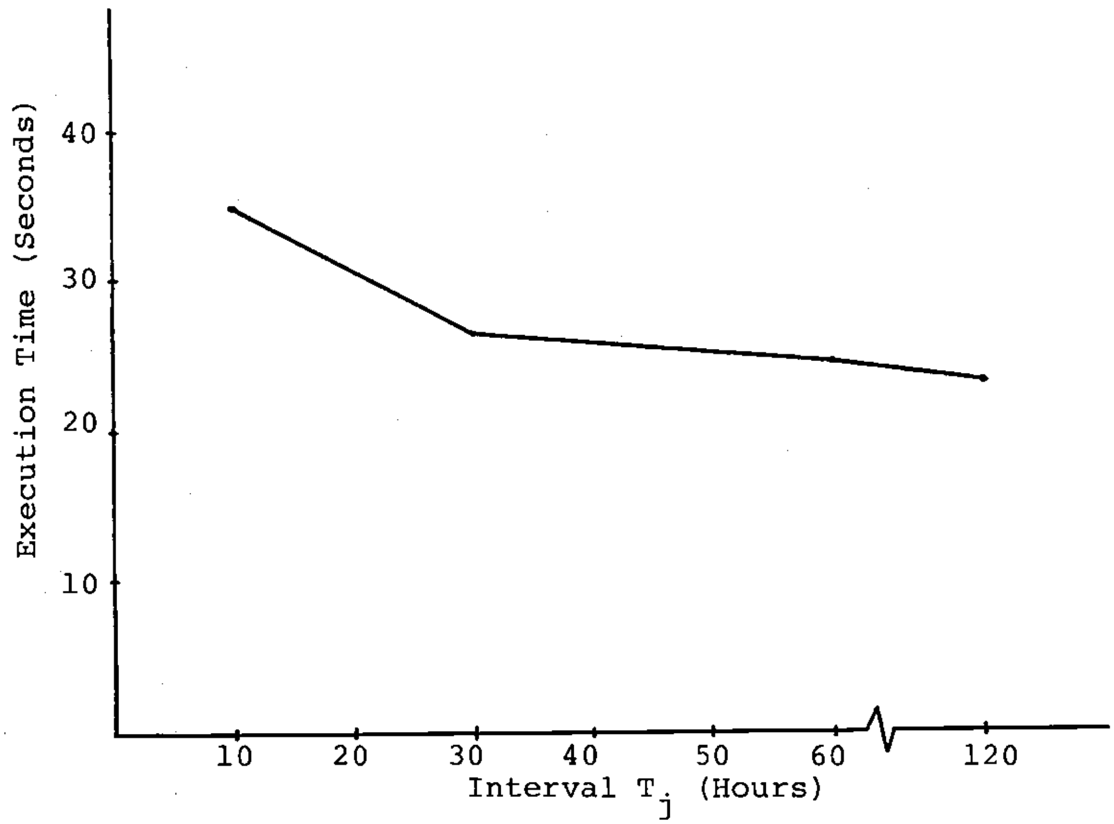


Figure 5-4. Comparison of the Total Execution Time for 19x1 Using Different Time Intervals and a 5mw Capacity Increment.

the more precisely the energy demand is computed, so the operating costs are more accurate.

5.3.2. Part II: Results Obtained by the Two Methods - Analysis by Unit

In this part are compared the operating results obtained for each unit by the two methods. These results were obtained from the two approaches for one year of operation.

Appendix B shows these results. First, observe the operating cost percent differences. For Type I generating units of high capacity the percent differences in operating costs between the two methods are in the range of ±5%. Since these units usually have low marginal costs, they are operated all the year(1) at their maximum capacity. Consequently, their operating capacity at each interval of time does not depend on the scheduled units at that interval of time. This is not the case for low capacity Type I units and for the combustion turbines (Type II units). Because of the fact these generating units have high marginal costs, they do not operate at full capacity during the year, so their level of operation at each interval (T_j) depends on the scheduled units at that interval. This variation in operating capacity causes that the differences in operating costs for small units to be significant (20%-60%) between the two methods (see Appendix B).

Figure 5-5 shows the frequency distribution of the

(1) Except during outages.

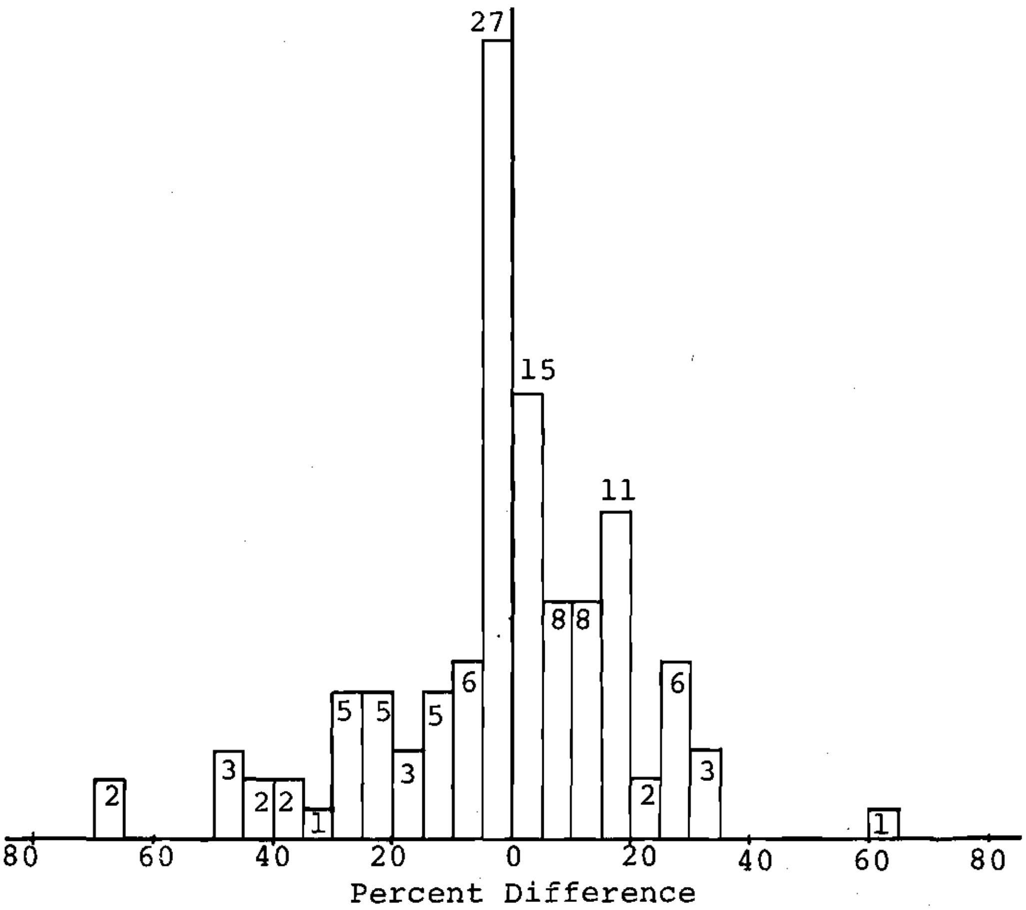


Figure 5.5. Frequency Distribution of the Units' Operating Costs Percent Differences for Type I and Type II Units for 19x1 Obtained Between the Two Methods. (Inc. = 5 mw, T_j = 60 hrs.).

percent differences between the operating costs computed by both methods. Notice how this curve is skewed to the left and that most of the percent differences fall in a range of ±10%. If we separate the percent differences due to Type I units (Figure 5-6) from the percent differences due to Type II units (Figure 5-7), it is observed that most of the negative percent differences are associated with the combustion turbines and most of the positive percent differences with Type I units. The explanation of this is related to the assumption made in the method developed that the units maintenance can be performed at any time of the year.[2] In other words, it is known that Type II units usually operate during the periods of high demand and sometimes operate during the periods of low demand when Type I units are in maintenance. If this assumption is stated and the algorithm developed is used to compute the units operating costs, the units Type II will be introduced into the system last because they have high operating costs. Consequently, this method does not consider that these units produce energy during the periods of low demand so their energy produced is underestimated. On the other hand, the energy not allocated by this method to Type II units is allocated to Type I units. This is the reason that for most of the Type I units an overestimation in the energy produced occurs.

Another factor that causes the differences in the

(2) The assumption is implied when $PI_i = 1 - HRSOUT_i/8760$ is obtained.

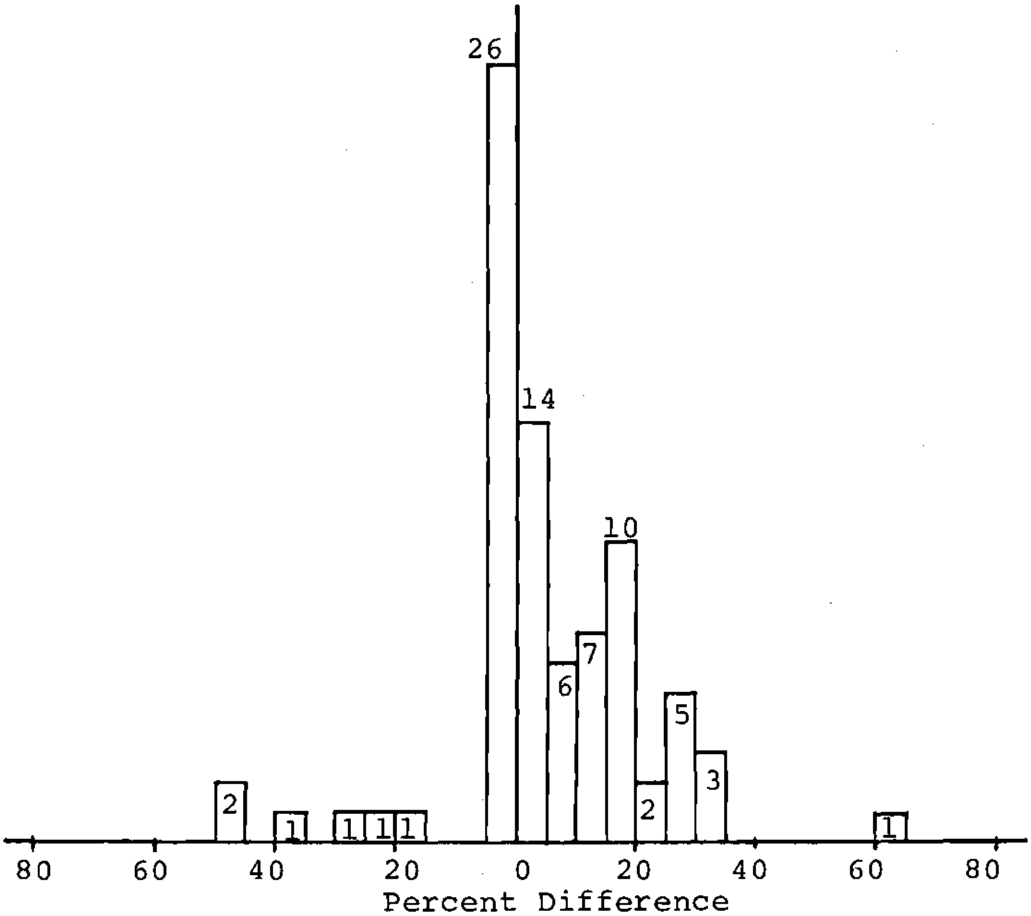


Figure 5-6. Frequency Distribution of Units' Operating Costs Percent Differences for Type I Units Obtained for 19x1 by the Two Methods (Inc.= 5 mw, T_j = 60 hrs.).

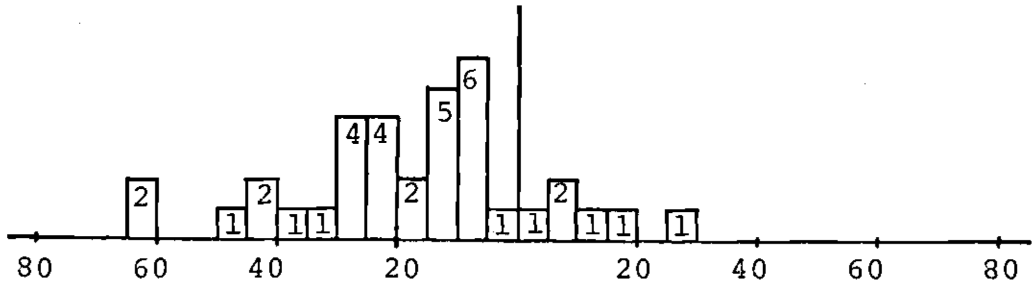


Figure 5-7. Frequency Distribution of the Units' Operating Costs Percent Differences for Type II Units Obtained for 19x1 Between the Two Methods (Inc.= 5 mw, T_j = 60 hrs.).

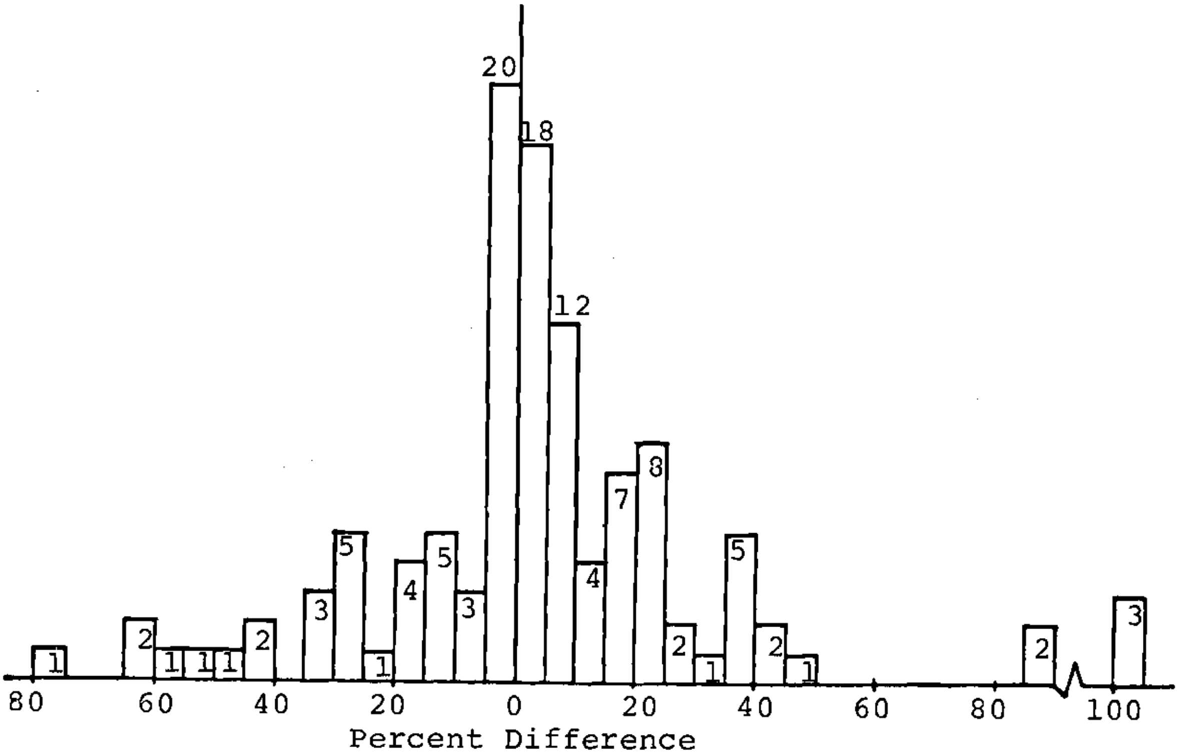


Figure 5-8. Frequency Distribution of the Units' Capacity Factors Percent Differences for Type I Units and Type II Units Obtained for 19x1 Between the Two Methods (Inc.= 5 mw, T_j = 60 hrs.).

operating costs is that it has been assumed that the time out of the units is equal to the sum of the forced outages time and the maintenance time. In the real system, the forced outage time is a random variable and the maintenance time can be scheduled.

Figure 5-8 shows the frequency distribution for the unit's capacity factors percent differences. Similarly, these percent differences have been separated for Type I and Type II units (Figure 5-9 and Figure 5-10). The capacity factors' differences are due to the same factors affecting the units' operating costs.

5.3.3. Part III: Comparison of the Total Operating Cost, Memory Core Requirements, and Execution Time for Four Projected Years

A computer run for four years of operation has been made for each method. An increment of 5 mw and an interval of 60 hours have been used for the method developed.

Table 10 and Figure 5-11 illustrate the total operating cost computed by both methods for each year. For any of the years, the difference between the total operating cost obtained by the two methods ranges from -1.27% to 1.07%. The reason for this small difference is that the high capacity generating units (low operating costs) are those which produce the bulk of system's energy output. As stated in the last section, the operating costs for these generating units are calculated with high levels of accuracy by the method developed in this study. For some generating units (i.e., combustion turbines)

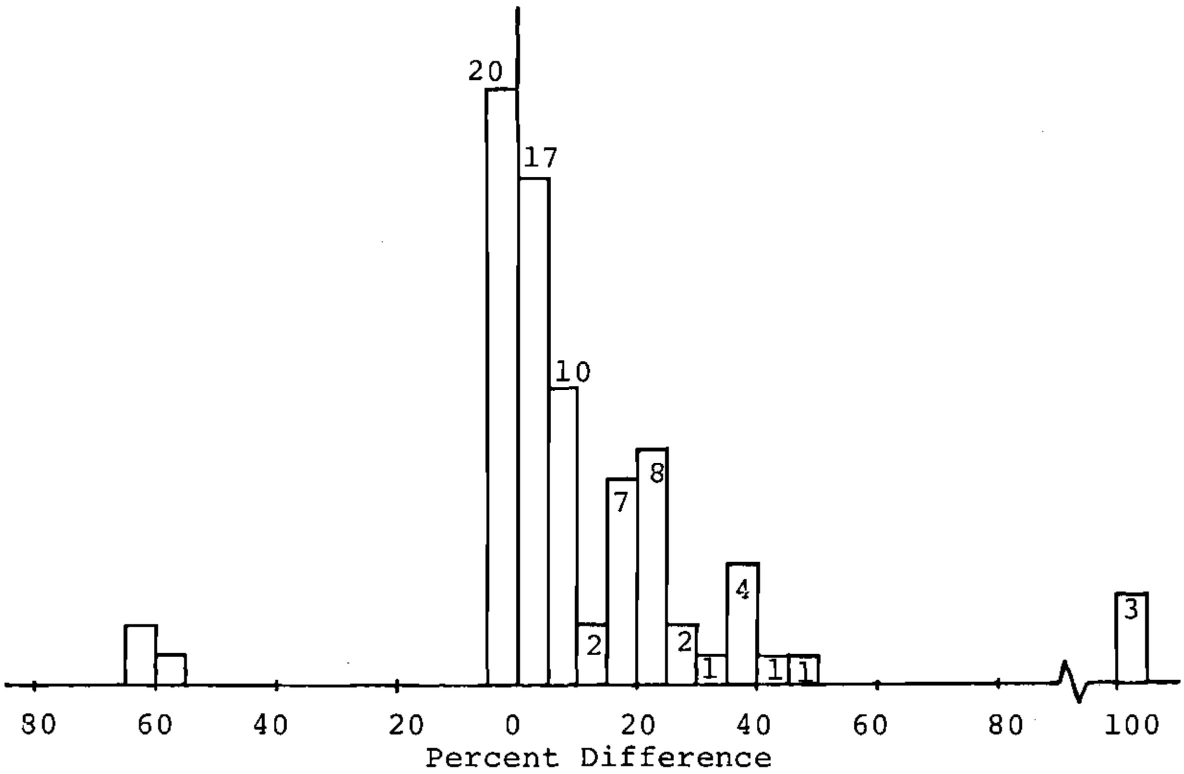


Figure 5-9. Frequency Distribution of the Units' Capacity Factors Percent Differences for Type I Units Obtained for 19x1 Between the Two Methods (Inc.= 5 mw, T_j = 60 hrs.).

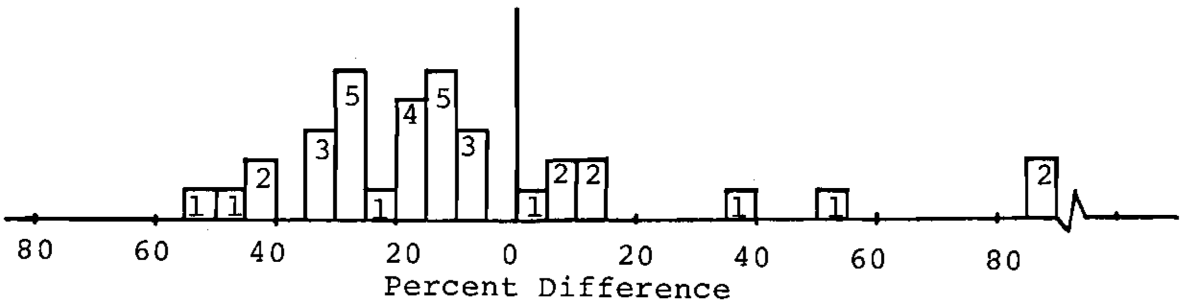


Figure 5-10. Frequency Distribution of the Units' Capacity Factors Percent Differences for Type II Units Obtained for 19x1 Between the Two Methods (Inc.= 5 mw, T_j = 60 hrs.).

Table 10. Comparison of the Projected Total Operating Cost, Core Memory and Computer Execution Time Obtained by the Two Methods for 4 Years of Operation (Inc. = 5 mw, T_j = 60 hrs.).

Year	Total Operating Cost Actual Method ($1000)	Total Operating Cost Method Developed ($1000)	Difference (%)
19x1	$1,108,345.00	$1,094,392.00	-1.27%
19x2	$1,334,559.00	$1,348,952.00	+1.07%
19x3	$1,555,126.00	$1,562,285.00	+0.46%
19x4	$1,773,039.00	$1,764,737.00	-0.46%
Execution Time	45 minutes	2 minutes	
Core Memory	500 K bytes	150 K bytes	

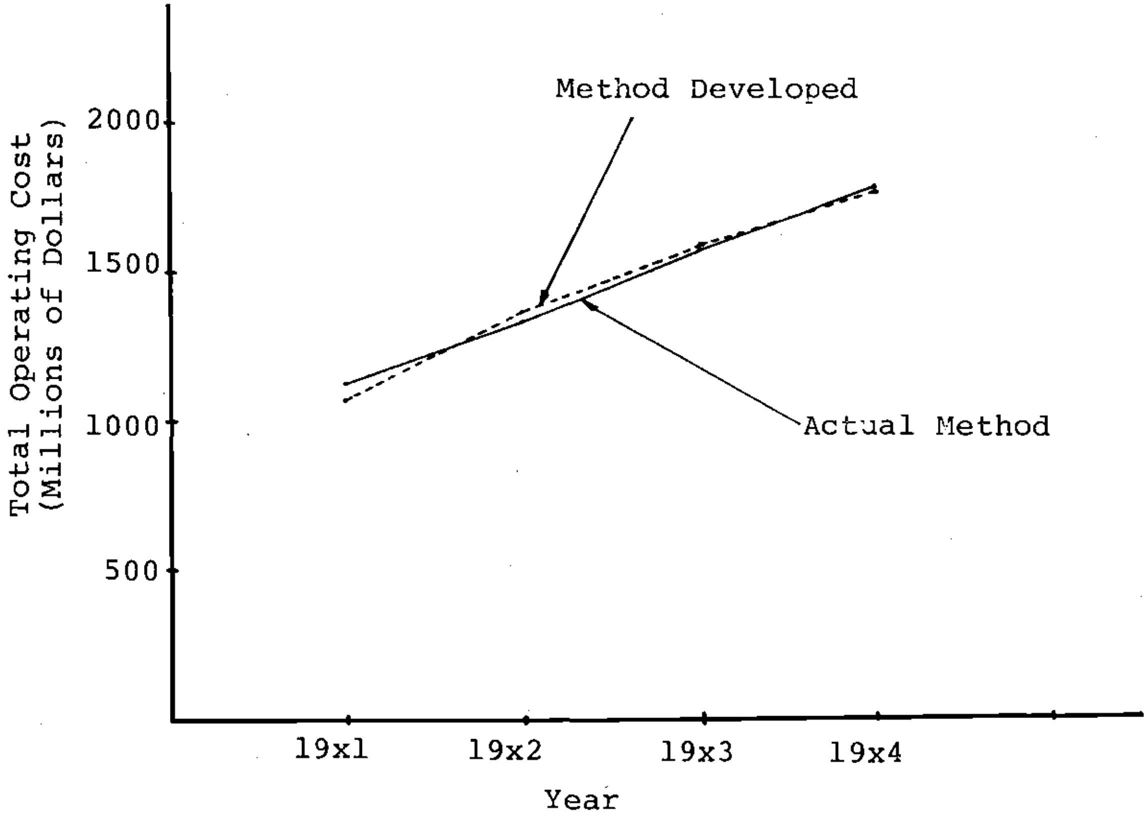


Figure 5-11. Comparisons of the Projected Total Operating Cost Obtained by the Two Methods for 4 Years of Operation (T_j = 60 hrs., Inc. = 5 mw).

the percent differences in operating costs are large, but in terms of dollars most of these differences are small if they are compared with the total operating cost of the system (see Appendix B).

Table 10 shows the execution time for each of the two approaches. The basic computer speed of the method developed when running on the CDC Cyber 74 is 2 minutes for 4 years of operation. For the actual method, a run for 4 years of operation on a AMBOL V6 takes approximately 45 minutes. These execution times cannot be directly compared since the programs have been run in different computer systems. However, according to the results, it can be concluded that the method developed does not require lengthy and costly computer runs.

Table 10 also illustrates the memory requirements by both methods. The core memory utilized by the actual method and the method developed are 500k bytes and 150 bytes, respectively.

It has been shown that the method developed in this study does not require a large amount of input data and considerable computer time to process. That the difference in the system's total operating cost computed by both methods is minor, and that for units of higher capacity the operating costs obtained by both approaches are very similar.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1. Conclusions

The accuracy and computer speed of the model developed in this study has been tested by comparison with the results obtained by a more detailed method actually utilized by a large electric utility.

The following conclusions have been drawn from this test:

1. The computer speed of the method developed depends primarily on the increment size (mw) utilized in the application of the marginal cost method. The size of this increment depends on the slopes of the marginal cost functions and on the unit's capacity ranges.
2. The accuracy of this method is determined principally by the following factors:
   a. Increment size. The smaller the increment size used in the application of the marginal cost method, the more accurate the method, although a larger amount of computer time is required.
   b. Interval time size. It has been observed that for interval time sizes between 10 hours and 120 hours, for a given increment size, the variation in the operating cost is not significant. Also changes in

these intervals have little effect on the computer time required to execute the dispatching technique.

c. Type and diversity of generating units operating in the system.

From the results obtained in the test, it has been shown that the total operating cost obtained by this method is practically the same as that obtained by the more detailed approach. As stated, the reason is that most of the energy is produced by units of high capacities (low operating costs) which are operated at maximum capacity for most of the year. Since for typical electric utilities a big portion of the energy demanded is produced by these types of units, the total operating cost of the system will be calculated by this method with high degree of accuracy.

3. For units of small capacity, this method does not compute the operating cost with high precision. The reason for these results is the assumption made about maintenance and forced outage times.
4. The method developed in this study does not require a large amount of computer time to process.
5. Much less data is required by the algorithm developed. To apply this method only the following information is necessary: input-output curves, total operating cost (CFD_i), fixed costs (FIX_i), total time that the unit will be unavailable during the year ($TMAIN_i + TFOR_i$) and the load duration curve.

6. Hydro generating units can be handled by modifying the load duration curve. The energy produced by the hydro units can be removed from the load duration curve and analyzed separately.

6.2. Recommendations for Further Research

It would be interesting to investigate if there exists another approach for expressing the probability that the generating unit will be unavailable to the system. This method would require that more weight be given to this probability when there are periods of low demand. This consideration would help to reduce the error in the operating cost estimation for the small generating units.

APPENDIX A

DEMONSTRATIONS FOR TEST 1 AND TEST 2

4.1. Demonstration for Test 1

Suppose there are "n" units in the system with linear marginal cost functions with slopes ≥ 0. Figure A-1 shows these curves. Assume the incremental cost method has been used to load the units and we are on $F_{10}(d)$. According to this method, the next increment will be on unit 10. Therefore, the next point reached will be $F_{10}(d+1) = F_{10}(e)$.

Now because of the incremental cost method, if we are on $F_{10}(d)$ the marginal cost for the level of operation for all the units has to be less than or equal to $F_{10}(d)$ (except for the units with higher marginal costs and these units are at their minimum capacity). Therefore,

$$\begin{aligned} &F_{10}(d) \geq F_8(e) \\ &F_{10}(d) \geq F_7(d) \\ &F_{10}(d) \geq F_6(d) \\ &\vdots \\ &F_{10}(d) \geq F_4(g) \\ &\vdots \\ &F_{10}(d) \geq F_1(l) \end{aligned} \tag{a}$$

If the increment is made on unit 10 then

$$\begin{aligned} &F_{10}(d+1) \leq F_8(e+1) \text{ if } F_8(e+1) \text{ exists} \\ &F_{10}(d+1) \leq F_7(d+1) \text{ if } F_7(d+1) \text{ exists} \\ &F_{10}(d+1) \leq F_6(g+1) \text{ if } F_6(e+1) \text{ exists} \\ &\vdots \\ &F_{10}(d+1) \leq F_4(j+1) \text{ if } F_4(j+1) \text{ exists} \\ &F_{10}(d+1) \leq F_1(l+1) \text{ if } F_1(l+1) \text{ exists} \end{aligned} \tag{b}$$

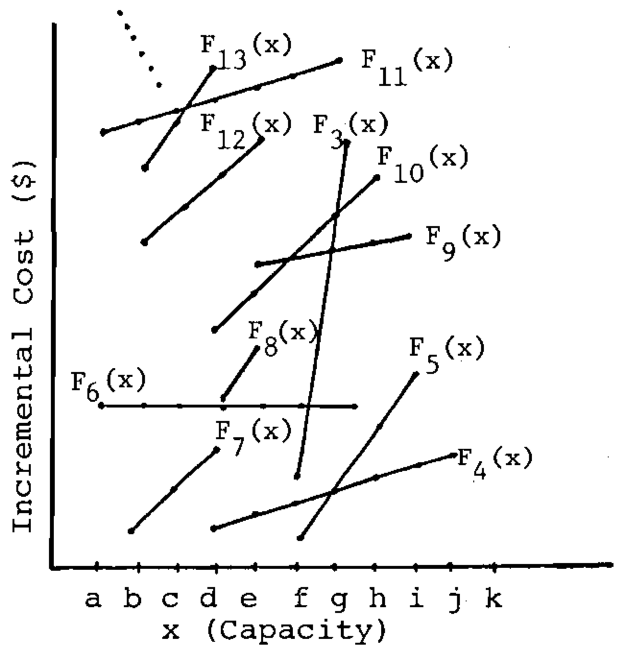


Figure A-1. Units with Linear Marginal Cost Functions and Positive Slopes (Test 1).

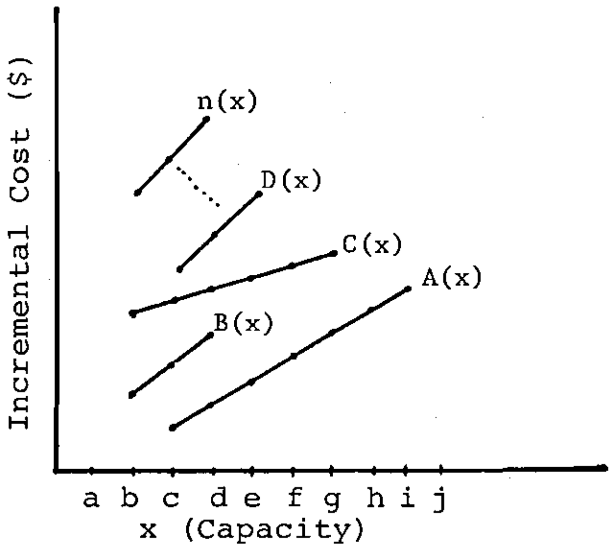


Figure A-2. Units with Linear Marginal Cost Functions and Positive Slopes (Test 2).

Since the units have positive slopes

$$F_i(x+1) \geq F_i(x-1) \text{ for } i=1\ldots n \tag{c}$$

To undertake the next increment there are two options:

1. $F_{10}(d)$ to $F_{10}(d+1)$
2. Using the incremental cost method, add "m" increments after $F_{10}(d)$ and m-1 decrements in any of the other units.

The marginal cost for option 1 is:

$$MC_1 = F_{10}(d+1)$$

and for option 2:

$$MC_2 = F_{10}(d+1) + F_{10}(d+2) + F_9(e+1) + F_9(e+2) + \ldots - [F_8(e) + F_5(i) + F_6(g) + \ldots]$$

$$MC_2 = F_{10}(d+1) + [F_{10}(d+2) + F_9(e+1) + F_9(e+2) + \ldots - F_8(e) - F_5(i) - F_6(g) - \ldots]$$

Because (a), (b), and (c) the amount $[F_{10}(d+2) + F_9(e+1) + \ldots - F_8(e) - F_5(c)\ldots]$ is greater than or equal to zero, consequently,

$$MC_2 = F_{10}(d+1) + (\text{cost} \geq 0)$$

Therefore, option 1 is more economic, so test 1 is not necessary for this type of marginal cost functions.

4.2. Demonstration for Test 2:

Assume there are "n" units in the system with linear marginal cost functions with slopes ≥ 0. These curves are shown in Figure A-2. The total variable cost before adding capacity to the units is:

$$TVC = TVA + TV_B + TV_C + TV_D + \ldots + TV_n$$

where TV_i is the total variable cost for unit "i" for i = A, B, C,...n.

Now, according to the incremental cost method, the first incremental "Δ" will be on unit A, and the variable cost added is going to be A(d)·Δ, so

$$TVC = TV_A + (A(d)\cdot\Delta) + TV_B + TV_C + TV_D + \ldots TV_n$$

At this point, the total variable cost is being minimized since it was the minimum incremental cost added and any other increment made later will have a greater marginal cost since the units have increasing or constant marginal cost functions.

Now suppose we want to make the next increment. It will be done to A(e). One more time, the TVC is automatically being minimized up to this second increment because the two smallest incremental costs have been added.

Consequently, if the unit marginal cost functions are straight lines with slopes ≥ 0 and the incremental cost method is used to dispatch the units, the total variable cost is minimized. Therefore it is not necessary to perform test 2.

APPENDIX B

COMPARISON OF THE RESULTS OBTAINED BY THE TWO METHODS: ANALYSIS BY UNIT

Unit No.	Max. Capacity (mw)	Total Operating Cost Method Developed ($1000)	Total Operating Cost Actual Method ($1000)	Difference ($1000)	Difference (%)
1	133	6,715.42	6,898.00	-182.58	-2.65
2	133	6,687.78	6,851.00	-163.22	-2.38
3	243	13,791.74	14,020.00	-228.26	-1.63
4	356	20,520.53	20,634.00	-113.47	-0.55
5	700	39,067.78	38,832.00	235.78	0.61
6	28	4,531.77	5,501.00	-969.23	-17.62
7	47	5,039.65	8,405.00	-3,365.55	-40.04
8	68	4,287.95	4,038.00	199.45	4.89
9	68	4,296.99	4,090.00	206.99	5.06
10	56	5,489.22	3,398.00	2,091.22	61.54
11	107	5,906.77	5,630.00	276.77	4.92
12	108	6,037.99	5,703.00	334.99	5.87
13	168	13,217.07	13,145.00	72.07	0.55
14	178	13,890.40	14,052.00	-161.60	-1.15
15	699	42,541.37	42,619.00	-77.63	-0.18
16	259	17,583.34	17,899.00	-315.66	-1.76
17	260	18,302.98	18,639.00	-336.02	-1.80
18	255	12,943.54	16,239.00	-3,295.46	-20.29
19	255	16,402.26	16,696.00	-293.74	-1.76
20	258	16,478.28	16,713.00	-234.72	-1.40
21	266	17,264.64	17,328.00	-63.36	-0.37
22	878	47,583.66	47,541.00	42.66	0.09
23	42	2,626.87	2,286.00	340.87	14.91
24	42	2,763.16	2,446.00	317.16	12.97
25	45	2,481.15	2,439.00	402.15	16.49
26	45	2,859.67	2,454.00	405.67	16.53
27	60	7,796.78	6,770.00	1,026.78	15.17
28	62	7,882.18	6,783.00	1,099.18	16.20
29	63	7,500.30	6,333.00	1,167.30	18.43
30	64	6,845.47	5,667.00	1,178.47	20.80

Unit No.	Max. Capacity (mw)	Capacity Factor Method Developed	Capacity Factor Actual Method	Difference (Units)	Difference (%)
1	133	43.13	44.00	.87	-1.97
2	133	42.95	44.00	-1.05	-2.39
3	243	50.82	52.00	-1.18	-2.26
4	356	52.29	52.00	0.29	0.55
5	700	51.65	51.00	0.65	1.27
6	28	53.02	87.00	-33.98	-39.06
7	47	36.70	87.00	-50.30	-57.81
8	68	50.41	48.00	2.41	5.03
9	68	50.54	48.00	2.54	5.29
10	56	53.82	51.00	2.82	5.52
11	107	48.23	45.00	3.23	7.17
12	108	48.87	45.00	3.87	8.61
13	168	83.37	83.00	0.37	0.44
14	178	82.56	83.00	-0.44	-0.53
15	699	65.67	66.00	-0.33	-0.50
16	259	63.10	64.00	-0.90	-1.41
17	260	65.57	67.00	-1.43	-2 14
18	255	43.10	61.00	-17.90	-29.34
19	255	61.80	63.00	-1.20	-1.91
20	258	61.34	62.00	-0.66	-1.06
21	266	62.38	62.00	0.38	0.61
22	878	50.75	51.00	-0.25	-0.49
23	42	31.80	26.00	5.80	22.30
24	42	35.07	30.00	5.07	16.89
25	45	33.97	28.00	5.97	21.32
26	45	34.24	28.00	6.24	22.28
27	60	34.28	32.00	2.28	7.14
28	62	33.45	31.00	2.45	7.91
29	63	39.56	36.00	3.56	9.90
30	64	34.56	31.00	3.56	11.48

Unit No.	Max. Capacity (mw)	Total Operating Cost Method Developed ($1000)	Total Operating Cost Actual Method ($1000)	Difference ($1000)	Difference (%)
31	651	38,408.01	38,924.00	-515.90	-1.33
32	655	41,778.58	42,185.00	-406.42	-0.96
33	881	55,685.18	56,216.00	-530.82	-0.94
34	881	53,464.98	53,453.00	11.98	0.02
35	221	15,738.82	15,841.00	-102.18	-0.65
36	300	19,975.85	20,457.00	-481.15	-2.35
37	481	28,619.24	28,715.00	-95.76	-0.33
38	466	28,582.10	28,237.00	345.10	1.22
39	104	5,260.62	4,995.00	265.62	5.32
40	105	5,227.56	4,936.00	391.56	5.91
41	90	4,955.92	4,829.00	126.92	2.63
42	465	23,736.16	23,451.00	285.16	1.22
43	245	19,659.33	19,871.00	-211.67	-1.07
44	245	19,659.33	19,779.00	-119.67	-0.61
45	50	4,890.29	4,602.00	288.29	6.26
46	81	7,093.83	6,615.00	478.83	7.24
47	23	1,557.54	1,367.00	190.54	13.94
48	24	1,567.28	1,380.00	187.28	13.57
49	161	8,861.04	8,252.00	609.04	7.38
50	103	4,477.00	3,647.00	830.00	22.76
51	103	5,040.03	4,303.00	737.03	17.13
52	103	5,063.08	4,311.00	752.08	17.45
53	135	6,327.61	5,512.00	815.61	14.80
54	136	6,458.14	5,543.00	915.14	16.51
55	355	15,999.30	14,052.00	1,947.30	13.86
56	355	17,016.74	14,773.00	2,243.71	15.19
57	870	47,249.10	48,355.00	-1,105.90	-2.29
58	730	11,751.40	11,844.00	-92.60	-0.78
59	21	1,628.75	2,957.00	-1,328.25	-44.92
60	22	1,631.34	3,096.00	-1,464.66	-47.31
61	32	3,156.62	4,276.00	-1,119.38	-26.18

Unit No.	Max. Capacity (mw)	Capacity Factor Method Developed	Capacity Factor Actual Method	Difference (Units)	Difference (%)
31	651	56.13	57.00	-0.87	-1.53
32	655	60.08	60.00	-0.08	-0.13
33	881	58.63	59.00	-0.37	-0.63
34	881	56.07	56.00	0.07	0.12
35	221	61.49	62.00	-0.51	-0.83
36	300	58.65	60.00	-1.35	-2.24
37	481	51.02	51.00	0.02	0.03
38	466	53.16	52.00	1.16	2.23
39	104	35.45	33.00	2.45	7.42
40	105	34.79	32.00	2.79	8.73
41	90	39.10	38.00	1.10	2.90
42	465	39.07	38.00	1.07	2.82
43	245	74.82	75.00	-0.18	-0.24
44	245	74.82	75.00	-0.18	-0.24
45	50	37.15	33.00	4.15	12.57
46	81	38.11	33.00	5.11	15.47
47	23	35.78	30.00	5.78	19.25
48	24	34.30	29.00	5.30	18.26
49	161	37.04	34.00	3.04	8.94
50	103	27.03	21.00	6.03	28.71
51	103	31.96	26.00	5.96	22.91
52	103	32.12	26.00	6.12	23.54
53	135	33.38	28.00	5.38	19.22
54	136	33.87	28.00	5.87	20.97
55	355	32.85	28.00	4.85	17.31
56	355	35.18	30.00	5.18	17.28
57	870	54.08	55.00	-0.92	-1.68
58	730	74.48	74.00	0.48	0.65
59	21	33.93	87.00	-53.07	-61.00
60	22	32.39	87.00	-54.61	-62.77
61	32	49.55	87.00	-37.45	-43.05

Unit No.	Max. Capacity (mw)	Total Operating Cost Method Developed ($1000)	Total Operating Cost Actual Method ($1000)	Difference ($1000)	Difference (%)
62	83	5,619.19	5,562.00	57.19	1.03
63	84	5,619.18	5,569.00	50.18	0.90
64	324	16,129.88	16,238.00	-108.12	-0.67
65	487	23,322.77	22,704.00	618.77	2.73
66	48	2,900.33	2,520.00	380.33	15.09
67	48	2,909.10	2,526.00	383.10	15.17
68	159	10,027.03	9,891.00	136.03	1.38
69	186	10,769.93	11,253.00	-483.07	-4.29
70	24	1,435.66	1,088.00	347.66	31.95
71	24	1,434.54	1,086.00	348.54	32.09
72	25	1,452.00	1,101.00	351.00	31.88
73	44	2,788.87	2,205.00	583.87	26.48
74	45	2,792.05	2,210.00	582.05	26.34
75	80	4,671.47	3,675.00	996.47	27.11
76	80	4,661.29	3,658.00	1,003.29	27.43
77	116	6,378.55	5,041.00	1,337.55	26.53
78	75	16,809.12	17,321.00	-511.88	-2.96
79	504	30,906.13	30,950.00	-43.87	-0.14
80	807	5,998.51	6,236.00	-237.49	-3.81
81	-	--	--	--	--
82	-	--	--	--	--
83	-	--	--	--	--
84	-	--	--	--	--
85	-	--	--	--	--
86	-	--	--	--	--
87	-	--	--	--	--
88	-	--	--	--	--
89	15	726.81	651.00	75.81	11.65
90	15	584.72	651.00	-66.28	-10.18
91	24	1,100.81	852.00	248.81	29.20
92	25	1,080.67	907.00	173.67	19.15

Unit No.	Max. Capacity (mw)	Capacity Factor Method Developed	Capacity Factor Actual Method	Difference (Units)	Difference (%)
62	83	62.26	62.00	0.26	0.41
63	84	61.48	61.00	0.48	0.79
64	324	44.94	45.00	-0.06	-0.14
65	487	42.37	41.00	1.37	3.35
66	48	36.32	30.00	6.32	21.07
67	48	36.45	30.00	6.45	21.48
68	159	58.25	57.00	1.25	2.19
69	186	49.88	52.00	-2.12	-4.08
70	24	14.76	7.00	7.76	110.92
71	24	14.75	7.00	7.75	110.73
72	25	14.29	7.00	7.29	104.12
73	44	25.71	19.00	6.71	35.31
74	45	25.12	19.00	6.12	32.19
75	80	23.23	17.00	6.23	36.63
76	80	23.18	17.00	6.18	36.35
77	116	21.27	15.00	7.27	41.78
78	75	65.91	68.00	-2.09	-3.08
79	504	60.20	60.00	0.20	0.33
80	807	18.14	72.00	-53.86	-74.80
81	-	--	--	--	--
82	-	--	--	--	--
83	-	--	--	--	--
84	-	--	--	--	--
85	-	--	--	--	--
86	-	--	--	--	--
87	-	--	--	--	--
88	-	--	--	--	--
89	15	99.32	88.00	11.32	12.86
90	15	99.32	88.00	11.32	12.86
91	24	14.98	10.00	4.98	49.83
92	25	13.97	10.00	3.97	39.73

Unit No.	Max. Capacity (mw)	Total Operating Cost Method Developed ($1000	Total Operating Cost Actual Method ($1000)	Difference ($1000)	Difference (%)
93	19	1,000.66	591.00	409.66	69.32
94	19	1,000.66	590.00	410.66	69.60
95	19	575.01	784.00	-208.99	-26.66
96	15	468.38	443.00	25.38	5.73
97	15	468.38	431.00	37.38	8.67
98	34	1,470.51	1,588.00	-117.49	-7.40
99	34	1,470.51	1,544.00	-73.49	-4.76
100	38	1,256.91	1,618.00	-361.09	-22.32
101	34	1,557.50	1,661.00	-103.50	-6.23
102	35	1,416.19	1,554.00	-137.81	-8.87
103	43	2,096.26	2,053.00	16.26	0.79
104	44	1,953.72	2,194.00	-240.28	-10.95
105	44	1,978.05	2,146.00	-167.95	-7.83
106	53	1,906.89	2,805.00	-898.11	-32.02
107	53	2,002.26	2,701.00	-698.74	-25.87
108	54	1,677.86	3,279.00	-1,601.14	-48.83
109	54	1,731.08	3,151.00	-1,419.92	-45.00
110	54	1,739.07	3,042.00	-1,248.93	-41.06
111	54	1,841.96	2,947.00	-1,105.04	-37.50
112	37	1,231.88	1,468.00	-236.12	-16.08
113	37	1,231.88	1,432.00	-200.12	-13.97
114	38	1,142.02	1,568.00	-425.98	-27.17
115	54	1,619.23	1,962.00	-342.77	-17.47
116	55	1,396.13	1,899.00	-502.87	-26.48
117	55	1,396.13	1,837.00	-440.87	-24.00
118	55	1,483.73	1,774.00	-290.27	-16.36
119	55	1,524.80	1,682.00	-157.20	-9.35
120	55	1,524.80	1,631.00	-106.20	-6.51
121	37	1,474.64	1,691.00	-216.36	-12.79
122	36	1,216.78	1,542.00	-325.22	-21.09
123	37	1,175.54	1,384.00	-208.46	-15.06

Unit No.	Max. Capacity (mw)	Capacity Factor Method Developed	Capacity Factor Actual Method	Difference (Units)	Difference (%)
93	19	12.98	7.00	5.98	85.39
94	19	12.98	7.00	5.98	85.39
95	19	8.65	12.00	-3.35	-27.90
96	15	7.53	7.00	0.53	7.63
97	15	7.53	7.00	0.53	7.63
98	34	11.99	13.00	-1.01	-7.80
99	34	11.99	13.00	-1.01	-7.80
100	38	9.19	12.00	-2.81	-23.40
101	34	12.29	13.00	-0.71	-5.47
102	35	10.27	12.00	-1.73	-14.38
103	43	14.34	14.00	0.34	2.40
104	44	13.08	15.00	-1.92	-12.83
105	44	13.31	15.00	-1.69	-11.27
106	53	10.66	16.00	-5.34	-33.36
107	53	11.24	16.00	-4.76	-29.73
108	54	9.07	19.00	-9.93	-52.27
109	54	9.26	18.00	-8.74	-48.56
110	54	9.77	17.00	-7.23	-42.55
111	54	9.89	17.00	-7.11	-41.80
112	37	8.89	11.00	-2.11	-19.22
113	37	8.89	11.00	-2.11	-19.22
114	38	7.93	11.00	-3.07	-27.90
115	54	7.48	10.00	-2.52	25.16
116	55	6.16	9.00	-2.84	-31.51
117	55	6.16	9.00	-2.84	-31.51
118	55	6.60	8.00	-1.40	-17.50
119	55	6.85	8.00	-1.15	-14.38
120	55	6.85	8.00	-1.15	-14.38
121	37	11.85	14.00	-2.15	-15.38
122	36	9.13	12.00	-2.87	-23.90
123	37	8.24	1.00	7.24	723.77

APPENDIX C

GLOSSARY OF ABBREVIATIONS

Code	Description
A	Coefficient of the heat rate curve.
AC	Average cost of energy
B	Coefficient of the heat rate curve
C	Coefficient of the heat rate curve
$CAPFAC_i$	Capacity factor of the unit "i"
CFD_i	Total operating ocst of the unit "i"
E_j	Energy demanded when the load demand is L_j
EC_{ij}	Energy cost for unit "i" when demand is L_j
$ENRS_i$	Expected number of hours that unit "i" will be operated during the year
$ENCOST_i$	Total energy cost for unit "i" during one year of operationg
$ENPRO_i$	Total energy produced by unit "i" during the entire year
$ENPRO_{ij}$	Energy produced by unit "i" when demand is L_j
FIX_i	Fixed costs for unit "i"
FHC_i	Fuel handling cost for unit "i"
FUC_i	Fuel cost for unit "i"
$GRANC_i$	Overall operating cost for unit "i"
GRANCOST	Total overall cost for the system
$HRSOUT_i$	The number of hours that unit "i" will not be operating in the year
$JOINT_{ij}$	Probability that unit "i" is operating when the demand is L_j
KAP_{ij}	Level of capacity that unit "i" is operating when demand L_j occurs

Code	Description
L_j	Load "j" demand
MC	Marginal cost
PI_i	Probability that unit "i" will be operating
PL_j	Probability that L_j is demanded during the year
PO_i	Probability that unit "i" will be out of the system
(RH, PL)	Load duration curve points
T_j	Number of hours taht load L_j is demanded
$TFOR_i$	Number of hours that unit "i" will be out of the system due to forced outages
$TMAIN_i$	Number of hours that unit "i" will be out of the system due to maintenance
TC	Total cost
TC_{ij}	Total operating cost per hour of operation for unit "i" when the demand is L_j
TEC_j	Total energy cost when the energy demand is e_j
$TIMEIN_{ij}$	Expected number of hours that unit "i" will be operating when the demand is L_j
TOTCOS	Total energy cost of the system for one year
TOTENE	Total energy demanded during the year
VOM_i	Operating and maintenance cost for unit "i"
Y_i	Input-output curve for unit "i"

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