Investigation on the Phase Characteristics of High Frequency Inertance Pulse Tube Cryocoolers above 50 K

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ABSTRACT

Phase characteristics of an inertance pulse tube cryocooler (IPTC) mainly include the mass flows, the pressure amplitudes, and the phase shifts between them. These are decisive factors for cooler performance, and are strongly affected by variations in the inertance tube design. In this work we describe theoretical analyses and experimental studies carried out on the phase characteristics of a large capacity high frequency single-stage IPTC developed in our laboratory and operated with a variety of inertance tube geometries at 80K. The theoretical analyses focused on investigating the amplitudes and phase angles at various locations of the whole system and established the phasor relationship of the cooler by combining a phasor-type analysis and a REGEN 3.2 analysis. The *COP* was calculated, and the influence on compressor efficiency was analyzed based on a force balance and Ohm's law. The experimental study stressed evaluating the phase characteristics of the cooler by making a few easy measurements of the key parameters. The measurements included the compressor piston position using LVDT (linear variable differential transformer) rods, the pressure amplitudes—in the reservoir, at outlet of compressor, and the warm end of the pulse tube — using pressure transducers, and the phase angles between them. The measured results are compared with the theoretical predictions. Both the theoretical and experimental investigations imply that the change of the inertance characteristics have a great influence on the pressure difference, cooling power, the efficiency of the cold finger, and the efficiency of the compressor. It is concluded that optimization of the inertance tube should consider both the cold finger efficiency and the compressor efficiency at the same time, in order to achieve an optimum efficiency of the overall IPTC.

INTRODUCTION

Pulse tube coolers (PTCs) have become a research focus because of their attractive cold finger design with no moving parts, minimal vibration and EMI output, and potential for long life. Highly efficient pulse tube cryocoolers operate based on the proper phase relationship between the pressure and mass flow. Inertance pulse tube cryocoolers (IPTCs) utilize the inertance tube as the phase shifter to generate the desired optimum phase shift between the

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pressure and the mass flow. An optimum phase relationship minimizes the amplitude of the mass flow through the regenerator and leads to a higher *COP* for a given acoustic power of the compressor. The optimum phase relationship can also maximize the compressor's efficiency of conversion of electric input power into acoustic power. In this paper, we focus on investigating the effect of a single-stage IPTC's phase characteristics on the efficiency of the cold finger and the compressor at the same time; this avoids the phenomenon that the cold finger does not match with the compressor. Our approach involves changing the mass flow and phase shift at the cold end of the regenerator, and then calculating the *COP* changes based on the resulting acoustic input power and electrical input power to the compressor. This combines a phasor analysis, REGEN 3.2[1,2] analysis, and force balance and Ohm's law for the compressor. The experimental study stresses evaluating the phase characteristics of the cooler by making a few easy measurements of the key parameters. These measurements are used to estimate the actual phase characteristics at the cold end of the regenerator and at the inlet of the inertance tube. Different inertance tubes were measured and the results were compared with the theoretical predictions. Both the theoretical and experimental investigations prove that achieving the highest efficiency cold finger doesn't lead to the highest efficiency of the compressor. The design of the inertance tube is the key point of the IPTC, and the optimization of the inertance tubes should consider the cooling power, the efficiency of the cold finger, and the compressor at the same time.

PHASE CHARACTERISTICS OF THE IPTC

The phase characteristics of a pulse tube cryocooler, which includes the mass flows, pressure amplitudes, and the phase angles between them, can be modeled by constructing a mass flow phasor diagram. A theoretical phasor diagram[3,4] of the IPTC is shown in Figure 1.

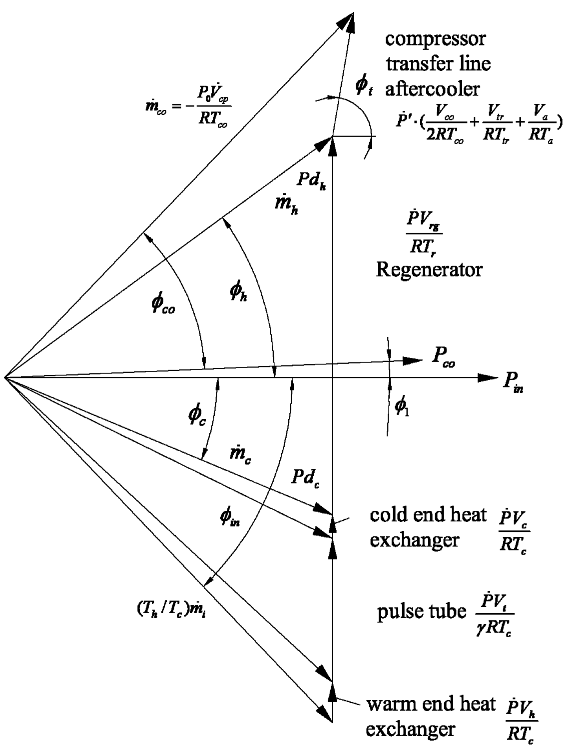

Figure 1. Phase characteristics diagram of the IPTC

From a phasor diagram, one can visualize the important flow magnitudes and phase angles, leading or lagging, that influence overall cryocooler performance. The phase characteristics at the cold end of the regenerator determine the cooling power, and the phase characteristics at the face of the compressor piston are related to the required acoustic power of the compressor. The impedance of the inertance tube is determined by its geometries. Varying the inertance tube geometry leads to a change of the phase characteristics at the cold end of the regenerator, which results in different *COP* values. Thus, the design of the inertance tube is the most important part of the cooler. The theoretical phase characteristics at various locations can be calculated as follows: $\dot{m}_h$, Pd_h and φ_h at the warm end of the regenerator can be calculated exactly by the software REGEN 3.2 with a given mass flow $\dot{m}_c$, pressure amplitude Pd_c and the phase angle φ_c at the cold end of the regenerator. Other segments can be constructed by the equations written in Figure 1. The phasor representing mass changes in volume for the transfer line, dead volume of the compressor, and the aftercooler, can be constructed with φ_t equal to the empirical value of 80 degrees; the pressure amplitude P' is approximately equal to $1.06Pd_h$. The phasor representing mass changes in volume for the cold end heat exchangers, warm end heat exchangers, and the pulse tube, can be constructed using the pressure amplitude Pd_c. All of the segments use the pressure at the warm end of the pulse tube as reference. φ_1, which is the angle between the pressure at the warm end of the pulse tube and the outlet of the compressor, is about 2 degrees. Now using trigonometry, we can calculate the mass flow $\dot{m}_{co}$, phase angle φ_{co}, the volume amplitude $\dot{V}_{cp}$ of the compressor, and the phase characteristics of the inertance tube; then the theoretical phase characteristics of the key elements are all constructed.

EFFECT OF PHASE CHARACTERISTICS ON THE COMPRESSOR EFFICIENCY

The efficiency of the compressor η_c written in this paper presents the transduction efficiency of the electrical input power to acoustic power of the compressor. The related equations of the compressor are[4]:

$$\Delta V_1 = (R_{elec} + iX_{elec})I_1 - \frac{BLU_1}{A} \tag{1}$$

$$\Delta p_1 = \frac{BLI_1}{A} + \frac{R_{mech} + i(\omega M - K_s/\omega)}{A^2}U_1 \tag{2}$$

$$-\frac{\Delta p_1}{\Delta U_1} = Z_{acoust} = R_{acoust} + iX_{acoust} \tag{3}$$

where U_1 is the complex volume flow rate, Δp_1 is the pressure difference on the front and back sides of the piston, Z_{acoust} is the acoustic impedance of the cold finger, R_{elec} is the electric resistance, X_{elec} is the electric reactance, B is the magnetic field, L is the length of the wire, A is the area of the piston, R_{mech} is the mechanical resistance, M is the moving mass , K_s is the spring constant, and ΔV_1, I_1 are the complex voltage and current.

Δp_1 and U_1 are approximately equal to the value P' and $\dot{V}_{cp}$ calculated from Figure 1 using the method written above; thus R_{acoust} and X_{acoust} can be obtained. The first two equations express Ohm's law (including electromotive force) and Newton's law, respectively. From the three equations, the efficiency η_c can be expressed as follows:

$$\eta_c = \frac{W_{pv}}{W_{elec}} = \frac{R_{acoust}}{R_{acoust} + \frac{R_{mech}}{A^2} + \frac{A^2 \cdot R_{elec}}{(BL)^2} \cdot [(R_{acoust} + \frac{R_{mech}}{A^2})^2 + (X_{acoust} + \frac{\omega M - K_s/\omega}{A^2})^2]} \tag{4}$$

Then, for a given compressor, the efficiency η_c responding to different acoustic impedance of the cold finger, can be solved. As shown in the equation, the phase characteristics of the cold finger influence the efficiency of the compressor significantly. Obviously $X_{acoust} + \frac{\omega M - K_s/\omega}{A^2} = 0$ leads to a higher efficiency, as the system operates at resonance, but different R_{acoust} will result in different efficiencies of resonance, thus it is necessary to investigate the effect of phase characteristics on the efficiency of the compressor. It also can be seen that larger BL and smaller R_{elec}, R_{mech} result in a higher efficiency.

EFFECT OF PHASE CHARACTERISTICS ON COP VALUES

The theoretical COP values based on the acoustic power generated in the compressor and the electrical input power of the compressor are respectively defined as follows:

$$COP_{pv} = \frac{\dot{Q}_c}{\dot{W}_{pv}} \tag{5}$$

$$COP_{elec} = \frac{\dot{Q}_c}{\dot{W}_{elec}} \tag{6}$$

where $\dot{Q}_c$ is the cooling power, $\dot{W}_{pv}$ is the acoustic power of the compressor, and $\dot{W}_{elec}$ is the electrical input power of the compressor.

From the theoretical analysis above, the phase characteristics at the cold end of the regenerator are the core of the cooler's phase characteristics. The inertance tube is used to generate the desired phase characteristics at the cold end of the regenerator. Figures 2, 4, and 6 show the change of the COP values with different pressure ratios, mass flows and phase angles at the cold end of the regenerator. When considering the COP_{pv} and COP_{elec} separately, it is found that the optimum mass flows are not consistent, and the difference between the optimum phase angles is 5 degrees with the same $\dot{m}_c$ and Pr_c. For example, in Figure 2, the $\dot{m}_c$ of 0.0033kg/s has the maximum COP_{pv}, but a medium COP_{elec}. The $\dot{m}_c$ of 0.003 kg/s has the maximum COP_{elec} because of a higher efficiency of the compressor. The phase angle of -30 degrees has the maximum COP_{pv} and the phase angle of -25 degrees has the maximum COP_{elec} at the $\dot{m}_c$ of 0.003kg/s. The data in Figures 4 and 6 also have the same regulations with that of Figure 2. Figures 3, 5, and 7, respectively, illustrate the required impedance Z_{in} of the inertance tube corresponding to the $\dot{m}_c$, φ_c and Pr_c shown in Figures 2, 4, and 6. The impedance of the inertance tube is defined as:

$$Z_{in} = \frac{Pd_{in}}{U_{in}} \tag{7}$$

where Pd_{in} is the pressure amplitude, and U_{in} is the volume flow rate at the inlet of the inertance tube. The horizontal axis and vertical axis represent the real part and the imaginary part of Z_{in}. The larger mass flow requires a smaller impedance amplitude. The imaginary part of the impedance increases significantly, and the real part increases slowly with the decrease of $\dot{m}_c$ at a certain φ_c and Pr_c.

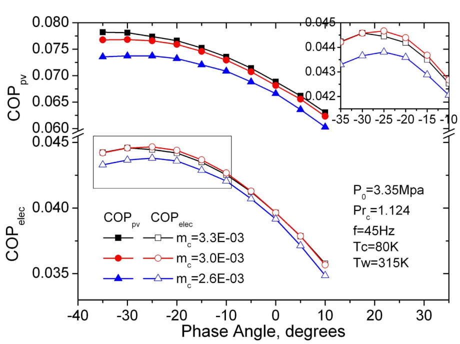

Figure 2. $\dot{m}_c$ and φ_c at the cold end of the regenerator versus COP_{elec} and COP_{pv} (Pr_c=1.124)

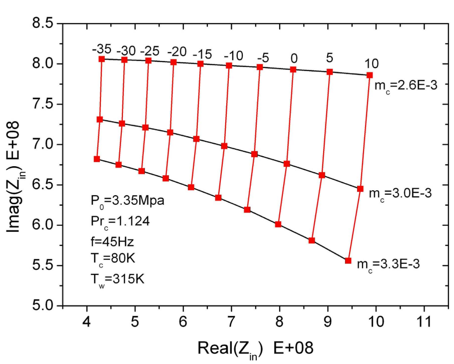

Figure 3. Required impedance of inertance tube corresponding to $\dot{m}_c$ and φ_c in Figure 2

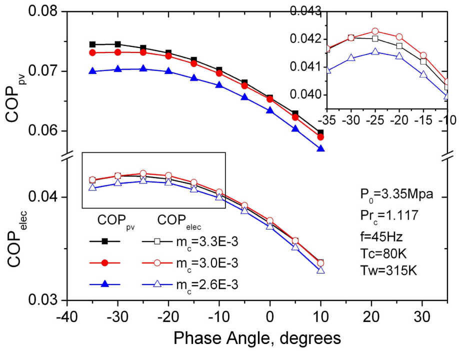

Figure 4. $\dot{m}_c$ and φ_c at the cold end of the regenerator versus the COP_{elec} and COP_{pv} (Pr_c=1.117)

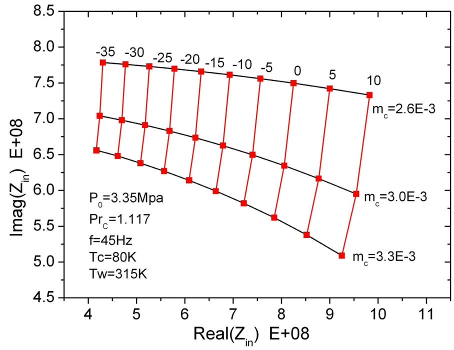

Figure 5. Required impedance of inertance tube corresponding to $\dot{m}_c$ and φ_c in Figure 4

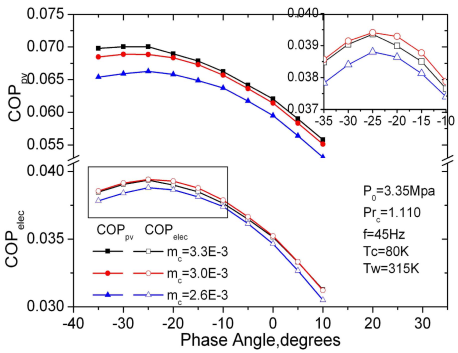

Figure 6. $\dot{m}_c$ and φ_c at the cold end of the regenerator versus the COP_{elec} and COP_{pv} (Pr_c=1.110)

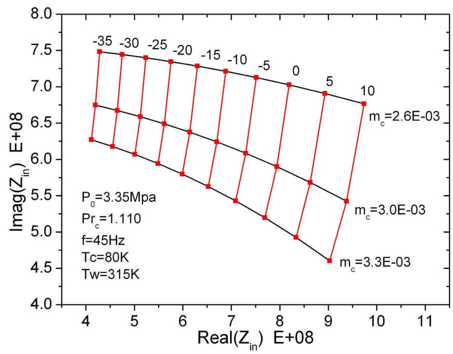

Figure 7. Required impedance of inertance tube corresponding to $\dot{m}_c$ and φ_c in Figure 6

EXPERIMENTAL MEASUREMENTS OF THE PHASE CHARATERISTICS

Cryocoolers often perform somewhat differently than expected, so it is quite useful to evaluate some key phase characteristics. Some that are very helpful are phase shifts and mass flows in the regenerator and the inertance tube. However, direct measurements of the mass flow in the regenerator and inertance tube are more difficult and may introduce additional volume to the system that can influence system performance. An easy method is adopted to measure the compressor piston position using LVDT (linear variable differential transformer) rods, the pressure amplitudes in the reservoir, at the outlet of the compressor, and the warm end of the pulse tube using pressure transducers, and the phase angles between them.

Figure 8 shows a schematic diagram of the experimental system. The measured values can be used to indirectly estimate the phase shifts and mass flows at the regenerator and the inertance tube. $\dot{m}_{co}$, φ_{co} , φ_1 in Figure 1, acoustic power of the compressor, and η_c can be obtained directly through the measurements. The pressure amplitude Pd_{in}, measured at the warm end of the pulse tube, is approximately equal to that at the cold end of the regenerator. The pressure amplitude measured at the outlet of the compressor is equal to $\dot{P}'$ in Figure 1. The pressure amplitude at the reservoir is used to calculate the mass flow at the reservoir. Then we use the phasor diagram to calculate the values of $\dot{m}_h$, $\dot{m}_c$, φ_c , Z_{in} and φ_{in} shown in Figure 1.

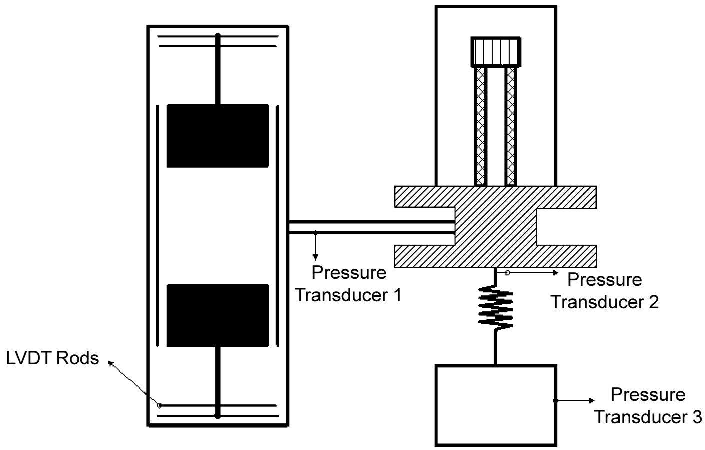

Figure 8 Schematic diagram of the experimental system

Table 1 shows the typical experimental results including the cooling power, COP_{elec}, COP_{pv}, η_c, and Pd_{in}, and also the indirect results $\dot{m}_c$ and φ_c. The cooler was operated under ambient air cooling with a reject temperature of 315 K. The theoretical cooling power is calculated by REGEN 3.2 using the values of $\dot{m}_c$,φ_c and Pd_{in}. The calculated values of η_c is quite close to the measured results. The theoretical results of cooling power are about 1.15 times the actual results. The scale factor 1.15 is acceptable, because some losses are not taken into account in REGEN3.2. The indirect method to obtain $\dot{m}_c$ and φ_c of the regenerator has acceptable accuracy, and also can be used conveniently to indirectly estimate the phase characteristics of other segments such as the inertance tube. As shown in Table 1, inertance tube 2 has the largest phase shift and the smallest impedance amplitude. The phase shift and impedance amplitude of inertance tube 3 are exactly opposite. The cold finger with inertance tube 1 has the maximum COP_{elec} and COP_{pv}. The cold finger using inertance tube 2 has higher COP_{pv}, and lower COP_{elec} compared with inertance tube 3. Experimental investigations also prove that the change of the inertance characteristics have a great influence on the cooling power, COP_{pv} and the efficiency of the compressor, and the optimization of the inertance tube should consider both the cold finger efficiency and the compressor efficiency at the same time, in order to achieve an optimum efficiency for the overall IPTC.

Table 1. Comparisons of the experimental results and theoretical results

$T_w = 315K$	Cooling Power (W@80K)	COP_{elec}	η_c	COP_{pv}	Pd_{in} (Mpa)	$\dot{m}_c$ (kg/s)	ϕ_c (°)	Cooling Power calculated by REGEN 3.2 (W@80K)	Theoretical η_c
Inertance tube 1	3.2	3.57%	59.06%	6.04%	0.35	0.0026957	-14.77	3.71	58.56%
Inertance tube 2	3.745	3.36%	58.68%	5.72%	0.35	0.0031563	-19.51	4.46	56.96%
Inertance tube 3	3.18	3.40%	59.60%	5.70%	0.35	0.0026398	-11.13	3.64	58.70%
Inertance tube 1	5.26	4.01%	58.95%	6.80%	0.39	0.0032417	-11.14	5.84	58.09%
Inertance tube 2	5.968	3.77%	58.66%	6.43%	0.39	0.0037124	-14.72	6.76	56.82%

CONCLUSIONS

Investigation of the phase characteristics of a pulse tube cryocooler is an instructive and valuable technique to evaluate the design of the cooler, and can focus optimizations in the right direction. In this paper, both theoretical and experimental investigations on the phase characteristics are presented. Both the theoretical and experimental investigations certify that phase characteristics of the IPTC influence both the performance of the cold finger and the efficiency of the compressor, and for a given refrigerator, the geometries of the inertance tubes determine the phase characteristics of the cooler. Thus the optimization of the inertance tube should consider both the cold finger efficiency and the compressor efficiency at the same time in order to achieve an optimum efficiency of the overall IPTC. The investigation in this paper is based on the same cooler. Further investigation will involve the phase characteristics of different coolers working at different cold end temperatures.

REFERANCES

1. Gary, J., Radebaugh, R., "An Improved Model for the Calculation of Regenerator Performance (REGEN3.1)," *Proc. Fourth Interagency Meeting on Cryocoolers*, David Taylor Research Center Technical Report DTRC91/003, (1991), pp.165-176.
2. Gary, J., Gallagher, A., Radebaugh, R., Marquardt, E., *Regen 3.2: User Manual*, NIST, Boulder, CO (2001).
3. Radebaugh, R., "Thermodynamics of Regenerative Refrigerators," *Generation of Low Temperature and It's Application,* Tech. Center, Kamakura, Japan, (2003), pp. 1-20.
4. Bradley, P.E., Radebaugh, R., Lewis, M., Bailey, R., "Comparison of Measurements and Models for a Pulse Tube Refrigerator to Cool Cryo-Surgical Probes," *Cryocoolers 13*, Springer Science +Business Media, New York (2005), pp. 671-679.
5. Swift, G., *Thermoacoustics: A unifying perspective for some engines and refrigerators*, Acoustical Society of America, New York (2002), pp.223-227.