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A MODEL STUDY OF
LATERAL PILE DEFLECTIONS

A THESIS

Presented to

the Faculty of the Graduate Division

Georgia Institute of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

By

Edwin Ellis Ware

December 1956

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A MODEL STUDY OF

LATERAL PILE DEFLECTIONS

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Date Approved by Chairman: December 15, 1956

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ABSTRACT

A MODEL STUDY OF LATERAL PILE DEFLECTIONS

(50)

Author: Edwin E. Ware

Thesis Advisor: George F. Sowers

All structural foundations supported by piles are subjected to vertical and lateral forces. The problem of determining the lateral displacements of these foundations, when acted upon by lateral forces applied above the ground, involves an analysis of the loaded piles' deflection curves. This analysis is complex because it involves the effect of both elastic and partially elastic elements, which is difficult to express mathematically.

A point of zero deflection, or point of rotation, is a characteristic of all lateral pile deflection curves. The determination of this point is essential to a solution of the deflection curve problem.

The purpose of this work is to determine experimentally the effect which the relative stiffnesses of a free-head prismatic pile and a homogeneous elastic medium have on the point of pile rotation when a lateral load is applied at the upper end of the pile. Also, an attempt

will be made to develop a simple and practical method of predicting the lateral deflection of the pile at the surface of the elastic medium.

The experimental work was done on a model basis. Flat steel stock of variable thickness and length was used for the piles in conjunction with a controlled gelatin mixture as the elastic medium. The lateral load was applied by passing weights over a pulley. Photography was used to reproduce the deflection curves.

The results of fifteen individual tests established a well defined relationship between the concerned stiffnesses and points of rotation. As the ratio of pile stiffness to elastic medium stiffness increases, the point of pile rotation moves downward. The depth to rotation approaches a limiting value of two-thirds the pile length as the stiffness ratio approaches infinity. The lateral deflection of a pile at the surface of the elastic medium may be predetermined from the lateral load and existing conditions of stiffness.

It is recommended that additional tests be made before the results of this thesis are used for any purpose which is not purely academic. Full-scale tests would be desirable, and a complete and accurate description of the existing soil conditions should be obtained.

Approved: ______________________

George F. Sowers
Thesis Advisor

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Date of Approval

CHAPTER I

INTRODUCTION

The Problem.--Every structure built above the ground surface must be capable of resisting vertical and lateral displacements. Structures built on pile foundations depend on the interaction of the piles and soil for the resistance to displacement. The phenomena resulting from applying a lateral load above the ground surface to the upper end of a vertically loaded pile embedded in soil has presented the practicing civil engineer and mathematician with a perplexing but interesting problem for many years. This problem has its beginning as well as its end connected with the shape of the deflection curve that a laterally loaded pile will exhibit. Once the deflection curve is determined, the practical problems such as the lateral deflection and the action of pile groups may be solved by consideration of structural mechanics. The problem is complex because its theoretical solution involves a three-dimensional analysis of both elastic and partially elastic elements which up to this time does not lend itself to an exact mathematical solution.

It can be shown that any pile embedded in an elastic medium will have at least one point along its length where the deflection due to a lateral load will be zero. This can be easily proven by considering the statical stability of the pile. (1)[*] Assuming that the zero point did not exist, it is seen that the entire equivalent reaction contributed by the elastic medium would be equal to the external force. This combination of forces results in an unbalanced couple which is incompatible with conditions of equilibrium. A balanced condition is created by a bending moment existing in the pile at the zero point or point of rotation. This bending moment is the result of a deflection of the pile below the point of rotation in a direction opposite to that of the external load. The deflection curve could be approximated if by some means this point of rotation could be predetermined. None of the theoretical solutions attempted heretofore agree as to the location of this point.

One of the difficulties in arriving at a valid theoretical analysis is the inability to evaluate the soil properties accurately. These constants are complex and are difficult to represent mathematically.

Historical Background.--Some of the earliest lateral load tests were made in Germany during the first quarter of this century. (2) In one of these, the investigator loaded wooden posts and filled the resulting deflection cracks with ashes. Subsequent excavation showed the absence of ashes in the lowest third of the embedded length. Here the theory of the zero point was established experimentally. Many other simple field and laboratory tests have been conducted without arriving at any concrete

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[*]Numbers in parentheses refer to the Bibliography.

conclusions. Two large-scale tests conducted in this country during construction of locks and dams on the Mississippi River have been the most important. (3)(4) The results of these tests are applicable only in like situations, and general conclusions cannot be drawn. A general study of the problem was recently made at the Georgia Institute of Technology from the results of a series of model tests. (5) Many questionable issues, however, remain for further research.

Purpose.--It is the purpose of this thesis to determine experimentally the effect which the relative stiffnesses of a free-head prismatic pile and a supporting homogeneous elastic medium have on the point of pile rotation when subjected to a lateral load applied at the upper end. Also, an attempt will be made to develop, from the experimental data, a simple and practical means of predicting the lateral pile deflection at the surface of the elastic medium.

Review of Published Theoretical Analyses.--A wide variety of theoretical approaches has been used in the past in the attempts to arrive at a workable solution to the problem of lateral pile deflections. Only a brief statement will be made concerning the approach and assumptions made by the authors of a few of these works.

Mr. E. Titze (6) presented a solution in 1932 in which he treated the problem as a two-dimensional case of plane strain. This would be analagous to considering the pile as a flat bar supported on an elastic foundation of a stiffness k. Titze assumed a k that varied with depth. This resulted in an expression for deflection that is too complex for wide application.

Messrs. A. E. Cummings (7) and Y. L. Chang (8) each presented a solution concerning piles fixed against rotation at the top. Both of these methods are simplifications of the method presented by Titze. Mr. Cummings assumed a simple expression for the soil stiffness and considered the pile fixed at a certain depth. Mr. Chang considered the soil stiffness equal to the modulus of elasticity of the soil. The differential equation

$$EI \frac{d^4y}{dx^4} = \text{load} = -E_s y \,,$$

could then be solved as long as E_s remained constant.

A method of computing pile deflections by the solution of a series of difference equations has been presented by Messrs. L. A. Palmer and J. B. Thompson. (9) This method depends on constants that must be obtained from full-scale field tests. An approximate solution may be obtained by expressing the various derivatives of the basic deflection function at any point along the pile's length in terms of adjacent points.

All of the above-mentioned methods involve the use of empirical constants, since no definite field or laboratory methods for their determination are available. No two methods are entirely in agreement as to the effect the interaction of pile and soil have on the deflection curve. It can readily be seen that the problems connected with a laterally loaded pile deserve much study.

CHAPTER II

TEST APPARATUS

An experimental study of pile deflections below the surface of the soil necessitates the utilization of some accurate system for physically measuring the induced deflection at points along the length of the pile. For economical reasons full-scale field tests are not practical. A series of model tests using a transparent, elastic material for the soil and photography as a means of measuring the deflections were used in this thesis.

Chemically pure commercial gelatin was selected as the elastic medium. Glycerine was added to prevent rapid dehydration and to increase the stiffness of the mix. It was found that a wide range in the modulus of elasticity could be obtained by varying the proportions of gelatin, glycerine and water. (10) (11) Any one original mix could be remelted and reused approximately four times. An electrical refrigerator was employed for curing the gelatin mixes. A complete summary of mixing and curing methods is given in the Appendix.

A box 12 inches long, 8 inches wide and 12 inches deep was constructed from 1/4-inch plexiglas and used as a container for the gelatin, as shown in Fig. 1. Reference grids were scribed on one side of the box to facilitate computing the scale of the photographs.

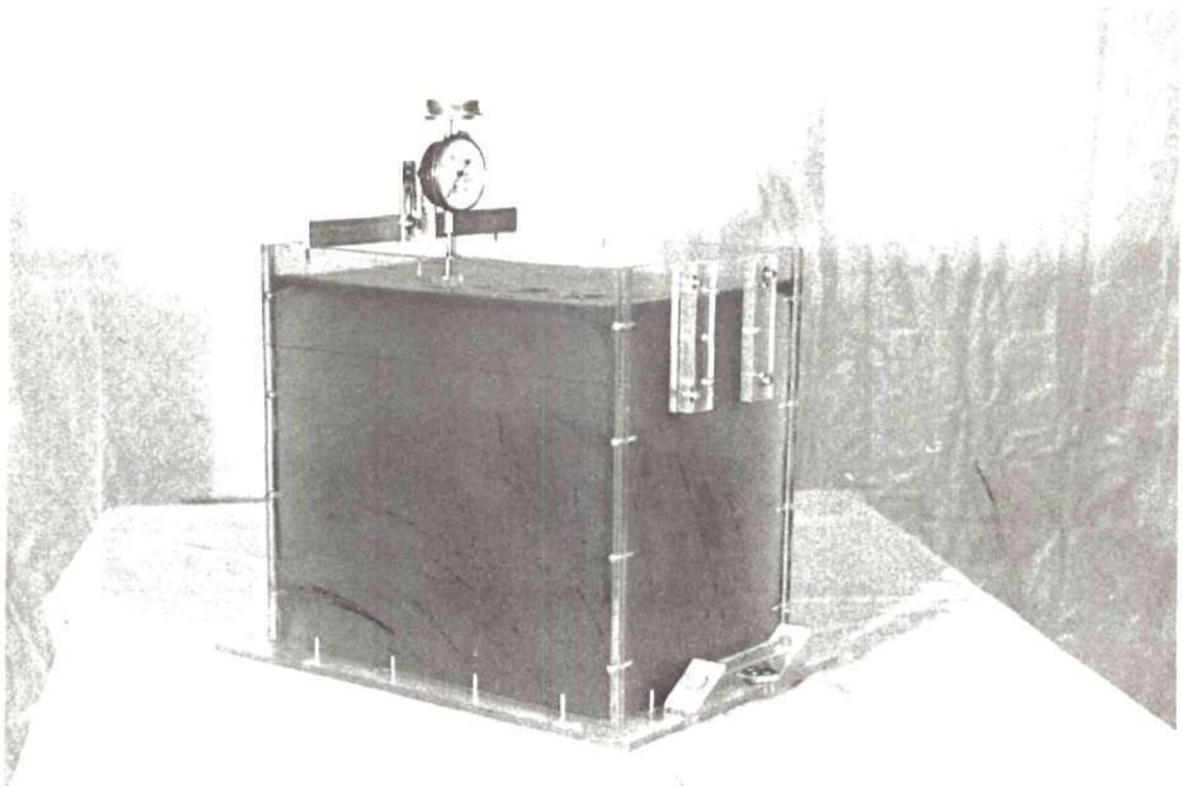


Figure 1. Test Box and Plate Load Test Apparatus.

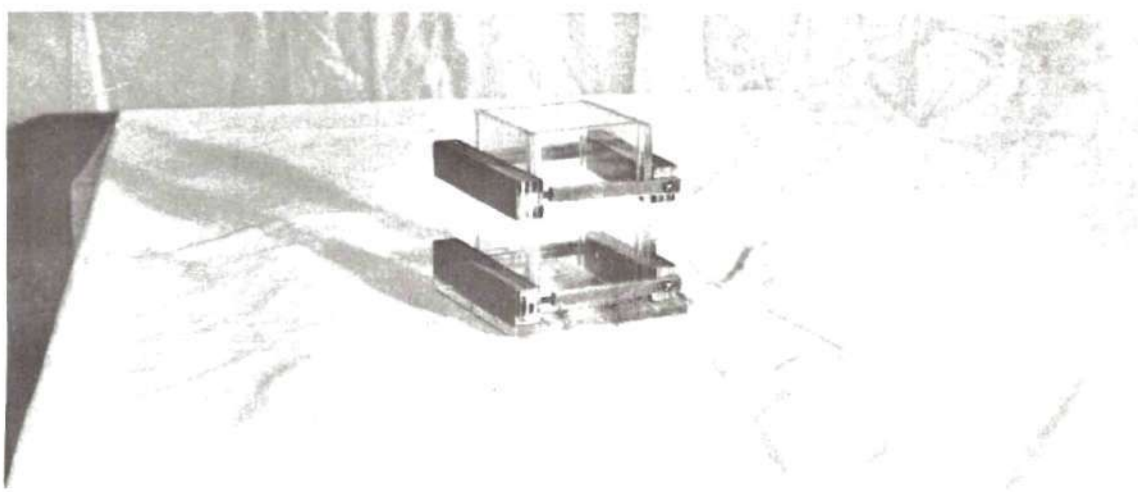


Figure 2. Collapsible Mold for Modulus of Elasticity Specimen.

The model piles were made from flat, ground steel stock of a constant 1/2-inch width with five thicknesses in 1/64-inch increments, beginning with 1/64 inch and ending with 5/64 inch. Black lacquer was sprayed on the steel piles to prevent rust and to help in producing sharp and clear photographs.

A pulley set in ball bearings, adjustable both laterally and vertically, was used to convert a vertical gravity load into a lateral load. Fig. 3 shows this pulley along with the small loading cable and adjustable pile clamp.

A plate load test and an unconfined compression test were used to measure the gelatin modulus of elasticity. A setup of the equipment for a plate test is illustrated by Fig. 1. The plates were machined from 1/4-inch plexiglas and were used in conjunction with a micrometer dial with its inner spring removed. Two holes drilled in the top of the test box insured that the plates would all be located in the same position for the various tests made. A collapsible, plexiglas mold, 3 inches square and 5 inches deep, served as a mold for forming cubes used in the unconfined compression test. Fig. 2 shows this mold and the clamps used to keep it intact.

All the photography was done with an Argus 35-mm camera. A photo flood light behind a tracing-paper screen was the diffused light source. The camera was mounted approximately 4-1/2 feet from the pile in an adjustable clamp that remained stationary throughout the series of tests.

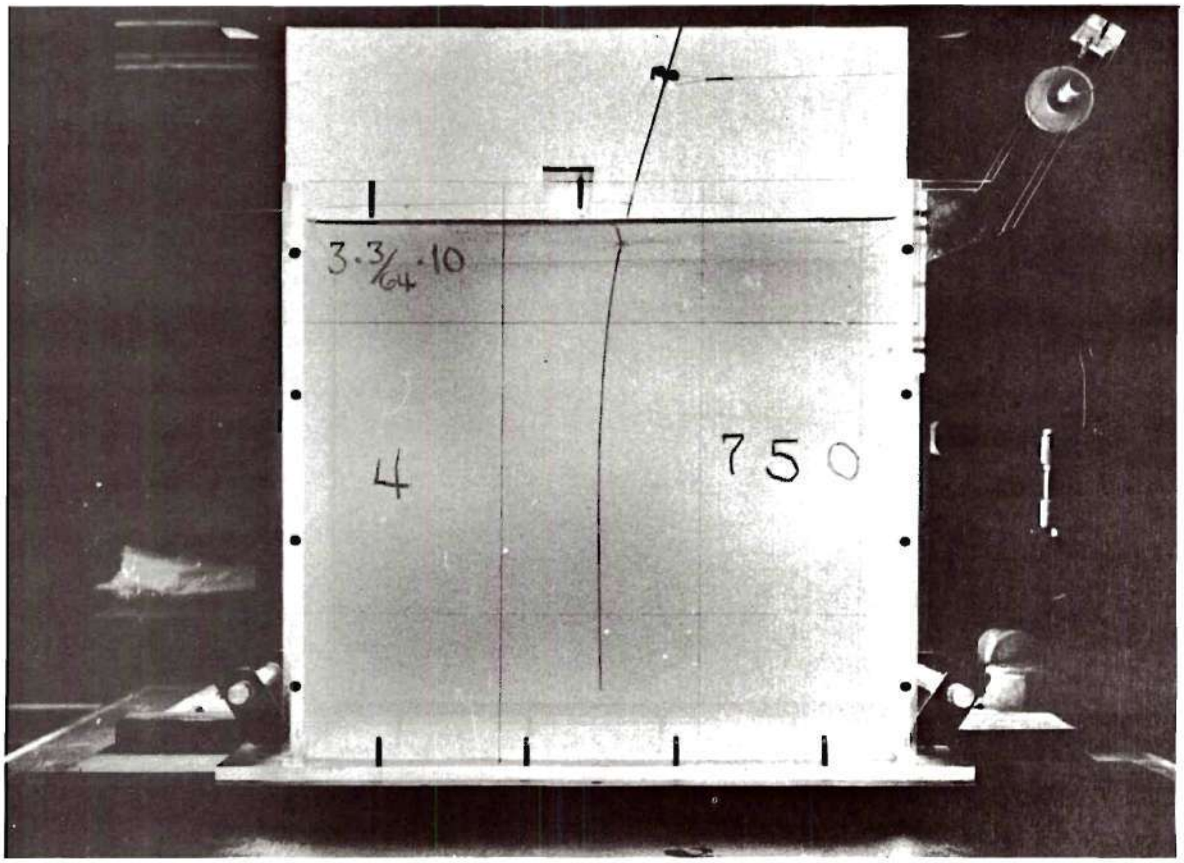


Figure 3. Test Photograph Illustrating Load System, Identification System, Grid Lines, Positive Measurement of Deflection, Parallax and Pile Having Two Points of Rotation.

CHAPTER III

PROCEDURE

Preparation of Gelatin and Pile.--A gelatin mix of an approximate desired stiffness was first selected and prepared, as outlined in the Appendix. The liquid gelatin was poured into the test box to a depth of eleven inches. The selected pile was next placed in position by using a self-centering clamp. It was made plumb by aligning it with a square placed on the base of the test box. The entire setup was allowed to cure overnight, as is detailed in the Appendix.

Determination of Gelatin Modulus of Elasticity, E_s.--It was found that the E_s of the gelatin changed by an average of 5-1/2 per cent during the lateral load tests. This change was due to heat from the photo light. For this reason a plate load test was made before and after each lateral load test, the average value being used for the entire test. The theoretical expression for the E_s from a plate test is based on the assumption that the elastic medium is of infinite extent. Since the test box was small, the average results of using three different round plates was assumed to be sufficient to compensate for small testing errors as well as for the size of the box. The computation of E_s from the results of the plate load test is detailed in the Appendix.

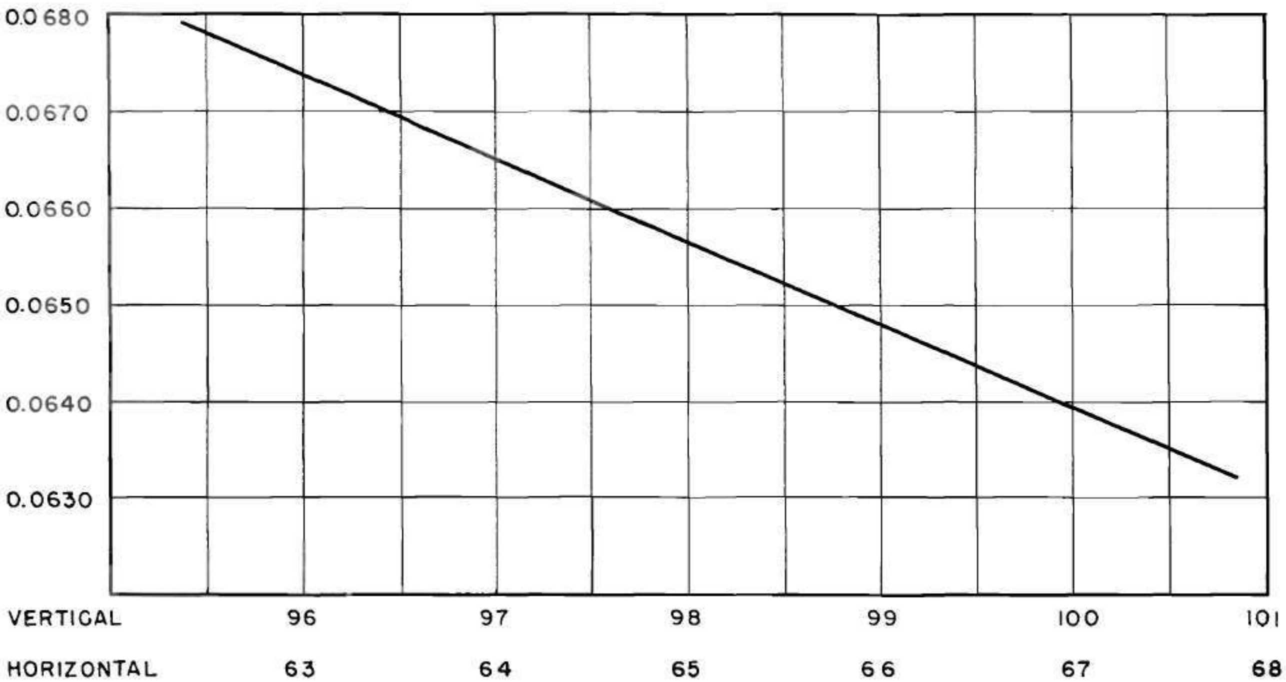
The results of the plate load tests were checked by two unconfined compression tests on a gelatin cube cast at the same time as the gelatin for test number 15. The tests were made immediately after the plate load tests in a universal testing machine of 0.2-pound sensitivity. The results of these tests agreed within 1.4 per cent of the corresponding plate tests. (See Appendix.)

The Lateral Load Test.--The test box was first marked for test identification. Three eccentricities of load of 2, 3 and 4 inches above the surface of the gelatin were marked on the pile, and the pile clamp was initially set at the smallest eccentricity. An approximate maximum load that would not exceed the elastic limit of the gelatin was determined and divided into three increments of load. The camera was set at a range of the distance from the focal point of the lens to a point half way between the pile and the outer grid lines. Photographs were then made of the zero load condition and each of the three loaded increments. The eccentricity was changed and the procedure repeated until a total of twelve photographs had been made. Three pile lengths of 4, 7 and 10 inches were tested with each of the five pile thicknesses, making a total of fifteen individual tests.

Constant Temperature Check.--It is assumed throughout this work that the modulus of elasticity of the gelatin is constant with depth. Since a small change in temperature will effect the value of this modulus, a check was made after each test in order to determine just how constant the temperature was throughout the depth of the gelatin. The check was accomplished by plunging a thermometer into the gelatin and visually reading it at various depths. The average maximum difference in temperature for an individual test proved to be 3.2 degrees Fahrenheit. Since this value very nearly coincides with the change in temperature of the gelatin before and after the lateral load tests, it is deduced that the average maximum difference in E_s throughout the depth would be no more than six per cent.

Measurement of Pile Deflections. --All deflections and points of rotation were measured directly from 5- by 7-inch prints of the test photographs. The photographs made from test number 2 are illustrated in small scale by Fig. 12. It was necessary to make two corrections before accurate measurements could be made. First, it was found after printing the photographs that some of the prints varied in their individual scale. This scale was easily determined from the grid lines. Second, it was necessary to apply a correction factor that would compensate for the parallactic effect of photographing the grid lines and pile, which were not equidistant from the camera lens. Since the pile was set in the center of the box, a parallax correction of 1.07 was readily obtained directly from the photographs. This correction for parallax was independent of scale and remained constant for all tests owing to the constant relative positions of the test box and camera. Both of these corrections were combined and are presented in Fig. 4 as a convenient graph. An ordinary engineers' scale and reading glass were used in scaling off the deflections. In order to obtain a check on the scaled deflections, several physical measurements of the pile's deflection above the surface were made during the first six tests. A comparison of these recorded measurements with the corresponding scaled deflections showed an average difference of approximately two per cent, which was presumably caused by errors in the physical measurements.

Templates, made by cutting away a portion of the photograph along the leading edge of the pile and at the intersections of the grid lines, were made from the zero load photographs. Tracing along the templates with a fine pointed pencil transferred the zero positions of the piles to the loaded pile photographs. The points of pile rotation were in this manner pin-pointed, and any desired deflection could be directly measured.



Instructions For Use

1. Using the 60 engineers scale, measure the distance between the vertical or horizontal grid lines in 1/60 inches.

2. Find the corresponding factor on the graph and multiply by any measured distance in 1/60 inches to obtain the true distance in inches in the plane of the pile.

Figure 4. Photograph Correction Factors

CHAPTER IV

ANALYSIS OF RESULTS

The Deflected Pile.--The photographed curves of the deflected piles were all similar in general shape. This similarity was embodied in the visual recognition of points of rotation and reversals of deflection at these points. Fig. 5 illustrates the general shape of the deflected pile. The theory of beams on an elastic foundation, which presents a *situation analogous to that of the loaded piles, predicts a sinusoidal* curve of diminishing amplitude (12). This assumption would theoretically imply that more than one point of rotation would exist in a pile of sufficient length. The majority of the tests made disclosed deflected piles developing only one point of rotation. Fig. 3 shows a pile that actually did have two points of rotation. The second point is seen to be at the pile's lower end.

Point of Pile Rotation.--The following variable factors, which are common to the general case of laterally loaded, free-head piles in a homogeneous elastic medium, control the point of rotation.

E_p = modulus of elasticity of the pile (psi)

I = moment of inertia of the pile about axis of bending (in^4)

e = eccentricity of the lateral load measured from surface of medium (in)

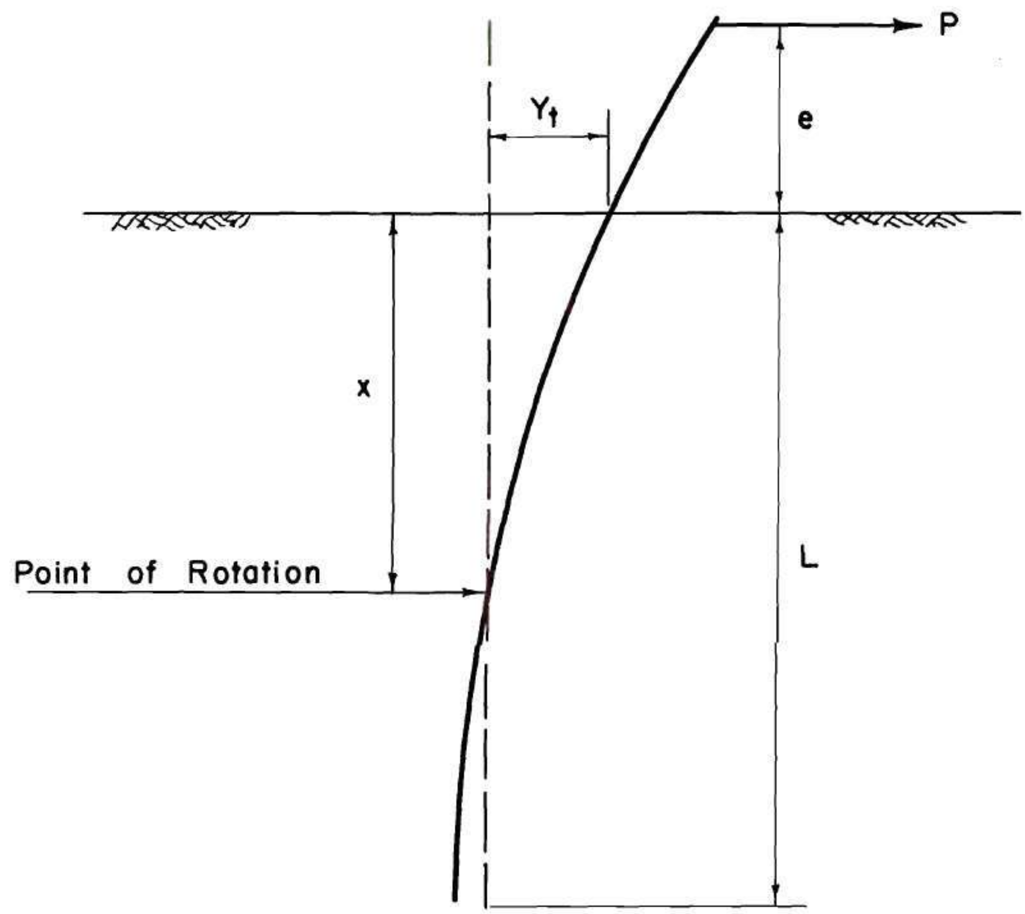


Figure 5. General Shape of Deflected Pile

L = embedded length of pile (in)

E_s = modulus of elasticity of the elastic medium (psi)

b = width of pile parallel to axis of bending (in)

The magnitude of the load and the amount of deflection should have no effect on the point of rotation as long as the elastic limit of the materials is not exceeded. The stiffness, or resistance to bending, of the pile and the stiffness, or resistance to deformation, of the elastic medium are also controlled by these factors. Letting Q_p and Q_s represent the pile and elastic medium stiffnesses respectively, it follows that:

$Q_p = f(E_p, I, L, e, b)$

$Q_s = f(E_s, L, b)$

A logical combination of the Q_p variables based on dimensional analysis would be

$$Q_p = K \frac{E_p I}{(e + L)\, b}$$

where K is a constant. This expression is similar to the familiar one for beam stiffness as defined in structural mechanics, excepting the inclusion of b. The units of this expression would be pounds of resistance. It is readily seen that Q_s should be expressed as $K(E_s Lb)$ where the units would also be pounds of resistance. This expression is analogous to the total force of resistance to deflection offered by a plane area, Lb, subjected to a uniform pressure, KE_s.

A ratio of pile stiffness to elastic medium stiffness can be expressed as

$$\frac{Q_p}{Q_s} = \frac{E_p I}{E_s L b^2 (e + L)} \, .$$

This is a dimensionless ratio that contains all the variables controlling the depth of rotation. The magnitude of $\frac{Q_p}{Q_s}$ may be conveniently referred to as the flexibility factor of the pile and elastic medium acting together. It will be hereafter referred to as S. The depth to the point of rotation, x, may be expressed, dimensionless, as a ratio of L.

A graph, Fig. 6, plotted with S as the independent variable and $\frac{x}{L}$ as the dependent, reveals a well defined relationship between these two ratios. Presenting the test results as dimensionless ratios is necessary in order to be perfectly general in the analysis. The data may be applied to any size pile or any soil, provided that the assumed condition of E_s and I are met and the stiffness ratios fall within the limits of the test results.

It can be proven that the value of $\frac{x}{L}$ can never be greater than 2/3. (See Appendix.) It is interesting to note from Fig. 6 that no value of $\frac{x}{L}$ exceeded 2/3, although it may be seen that the curve is approaching that limiting value as the flexibility factor increases and approaches infinity. It may be said that x is a hyperbolic function of the various influencing variables with an asymptote equal to the maximum value of

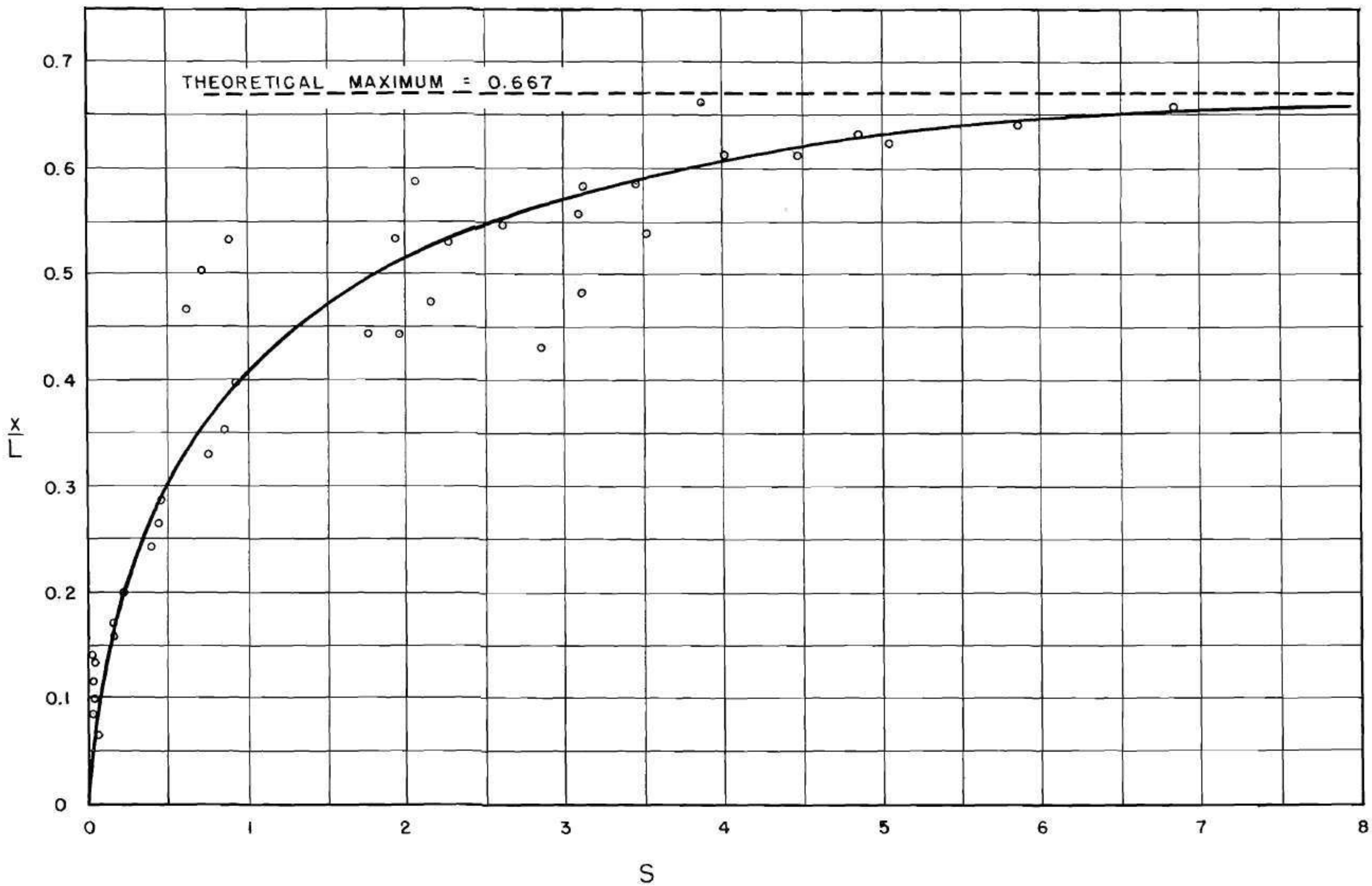


Figure 6. Plot of Flexibility Factor, S, Against Point of Rotation, $\frac{x}{L}$.

$\frac{x}{L}$. Another point of practical interest is the fact that for any situation with S equal to or greater than six, the point of rotation will be located approximately at the lowest 1/3 point of the embedded length, and the pile may be considered as being infinitely stiff.

Lateral Deflections. --It was pointed out in Chapter III that lateral pile deflections could be scaled from the test photographs within approximately two per cent of their actual value. This is necessarily true only for the deflections of greatest magnitude in any individual test, which happen to be the most important ones. The only deflection of any practical importance is the one at the surface of the soil. If by some means this surface deflection could be predetermined, the most important part of the deflection curve problem would be solved. A rough approximation of this surface deflection, Y_t, may be obtained by assuming a hinge at the point of rotation and statically computing a Y_t that would balance the external moment. In this computation it would be necessary to approximate the modulus of earth reaction, k, from E_s and to assume a shape of the deflection curve above the point of rotation. The experimental results of this research are quite conclusive, however, in disclosing the existence of a comparatively large bending moment in the pile at the point of rotation.

A relationship between E_s and k was determined experimentally by measuring areas defined by the deflected pile, back figuring to obtain k, and then comparing with the corresponding E_s. Fig. 7 presents this relationship and the Appendix contains representative computations made for k. It is to be understood that this relationship is applicable only within the limits of this series of tests, since the value of b remained constant and E_s did not cover a wide range.

The results of the computations made in arriving at the k and E_s relationship were further utilized to determine the effect the flexibility factor has on the bending moment existing at the point of rotation. Fig. 8 shows the results of plotting S as the independent variable against

$$\frac{M_t}{M_t + M_b}$$

as the dependent. Here, M_t and M_b are respectively the resisting moments about the point of rotation due to the resistive elastic medium forces above and below this point. The relationship is supported by two important facts. First, it is readily seen that the bending moment, M_b, at the point of rotation decreases as the flexibility factor increases. This important relationship may likewise be extracted from the $\frac{x}{L}$ and S relationship by visualizing the approach to a hinged condition at the point of rotation as this point moves further down. Second, it can be seen that M_t is approaching a theoretical maximum value of 89 per cent of the total resisting moment when S becomes approximately six. A mathematical proof of this statement is included in the Appendix. It is

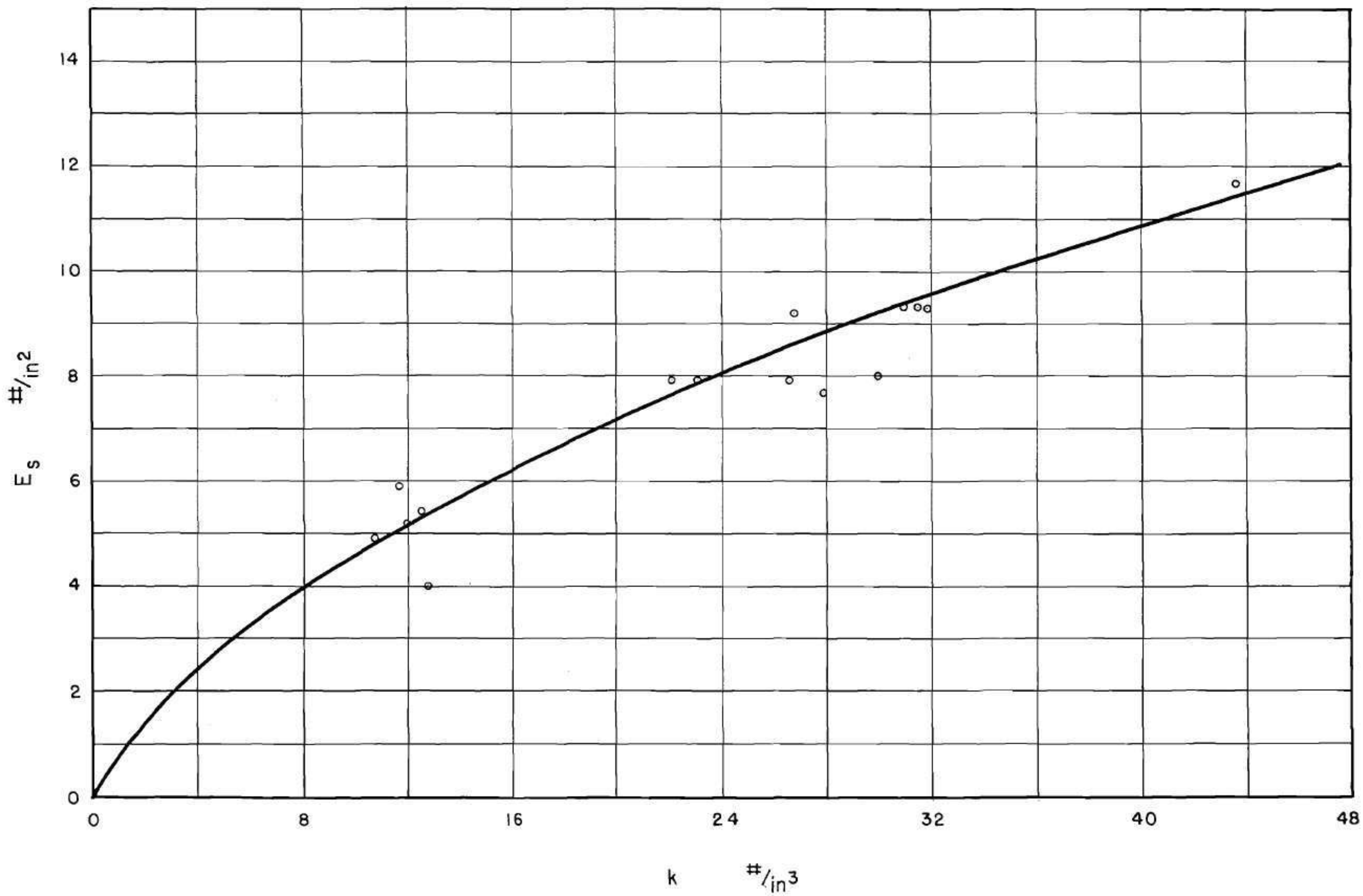


Figure 7. Plot of Computed Modulus of Foundation, k, Against Experimental Modulus of Elasticity, E_s.

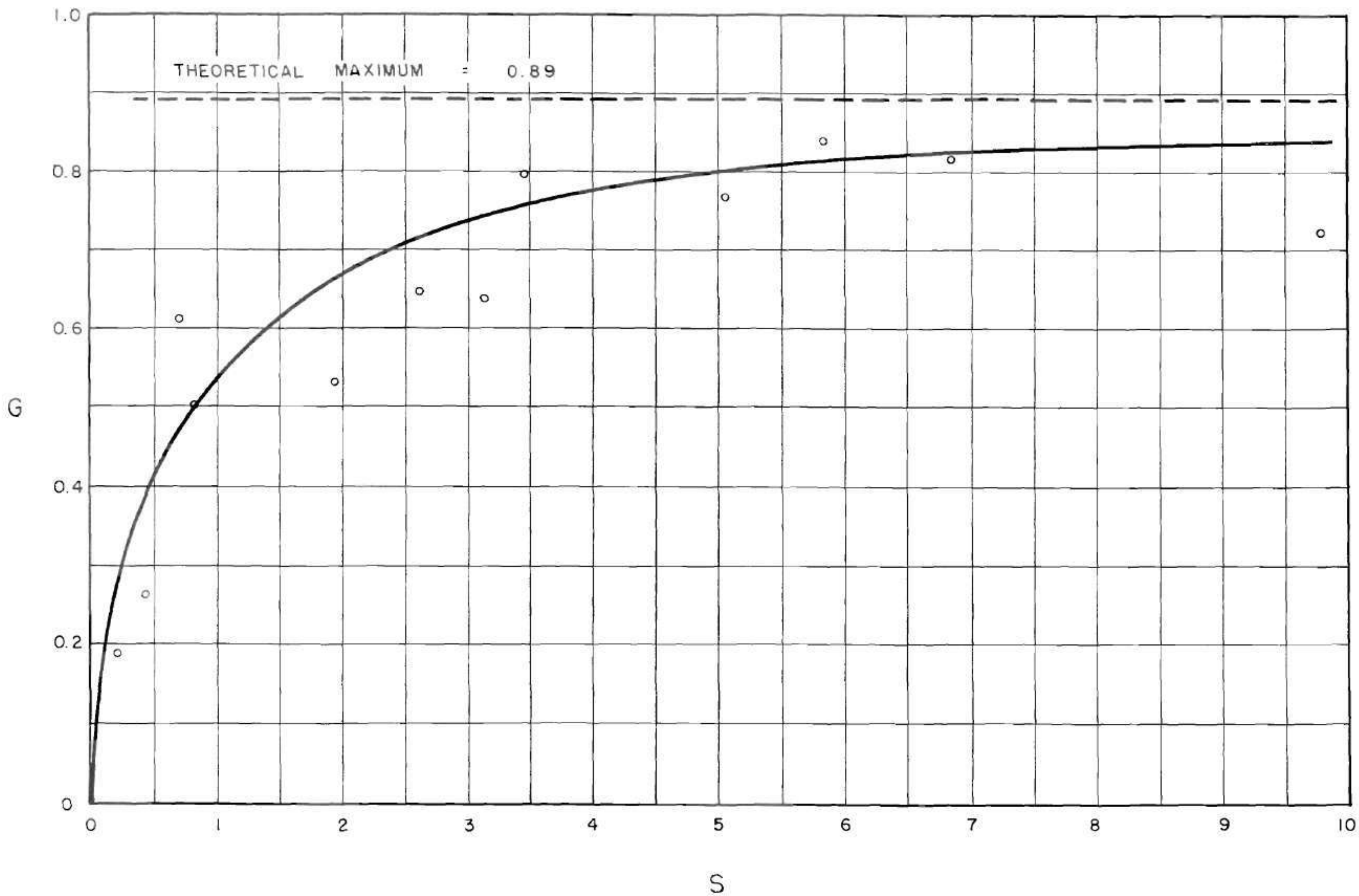


Figure 8. Plot of Flexibility Factor, S, Against Ratio of Resisting Moments, G.

interesting to recall from Fig. 6 that as S approaches this same value of six, the point of rotation approaches a depth of 2/3 L and the pile is for all practical purposes infinitely stiff.

The following formulae were derived and used in the computations.

Assuming straight line deflection curve,

$$Y_t = 3 \frac{GP(e + x)}{x^2 bk}$$

Assuming parabolic deflection curve,

$$Y_t = 4 \frac{GP(e + x)}{x^2 bk}$$

In these expressions, G equals the dimensionless ratio

$$\frac{M_t}{M_t + M_b}$$

All other variables are as have been previously defined. A careful study of all the deflection curves disclosed a useful method of utilizing S to determine which equation should be used. The results of the study showed the following to be consistently true.

For S = 0 to 2, use parabolic equation (constant = 4)

For S = 2 to 4, use average of parabolic and straight line equation (constant = 3.5)

For $S = 4$ to ∞, use straight line equation (constant = 3)

The steps necessary in computing Y_t in any general situation that falls under the limitations of this research are summarized below.

1. Determine E_s and k
2. Compute S
3. Determine x from Fig. 6
4. Determine G from Fig. 8
5. Select deflection equation from value of S
6. Solve for Y_t from selected equation.

An approximate evaluation of Y_t may now be made, since the problem has been reduced to one involving only simple statics. Random photographs of tests were picked and the deflections computed. The computed results agreed with the measured results with an average error not greater than 13.5 per cent. A tabulation of these results is included in the Appendix.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions are confined to the limiting conditions under which this research was made.

A laterally loaded pile in an elastic medium will deflect in a curve approximating a sinusoidal curve of diminishing amplitude. The magnitude of the lateral load has no effect on the point of rotation within the elastic range of the materials involved.

The point of pile rotation may be predetermined from the physical characteristics of the pile and the elastic medium. As the ratio of pile stiffness to elastic-medium stiffness increases, the point of pile rotation moves downward. The depth to rotation approaches a limiting value of 2/3 the embedded pile length as the stiffness ratio approaches infinity.

The bending moment existing in the pile at the point of rotation may be expressed as a percentage of the total resistive moment from a knowledge of the involved stiffnesses. At the point of rotation the bending moment in the pile will not be less than 11 per cent of the total resistive moment and should never be neglected.

The maximum lateral deflection of a pile occurs at the surface of the elastic medium, and its value may be predetermined from the lateral load and existing conditions of stiffness.

It is recommended that a series of additional tests be made before the detailed results of this thesis are used for any purpose which is not purely academic. Such tests should include piles with a varying width. This is necessary in order to verify the conclusions drawn from the test results of this thesis, since the pile width remained constant throughout this entire project. A full-scale load test would be highly desirable. Most full-scale tests that have been heretofore made do not include sufficient information about the existing soil conditions. A deep deposit of homogeneous saturated clay would be a soil having properties closely resembling those of the gelatin used in this thesis. The deflection at the soil surface is the only one that need be considered in a test of the pile prototype.

More work should be done to determine the relationship existing concerning the soil's modulus of elasticity, the pile's geometry and the resulting modulus of foundation.

APPENDIX I

MIXING, REMELTING AND CURING PROCEDURES

Preparation of an Initial Gelatin Mix. --

1. Compute and weigh out proportions of gelatin and glycerine.
2. Bring five gallons of water to boil in a container.
3. Mix gelatin and glycerine thoroughly.
4. Slowly add approximately two gallons of boiling water.
5. Place container and mix on previously tared scales and bring to desired weight with boiling water.

Curing Procedure. --

1. Allow hot mix to cool to 104 degrees Fahrenheit and pour.
2. Place mass into refrigerator adjusted at 55 degrees Fahrenheit.
3. Allow mix to cure in refrigerator for at least 24 hours.

Remelting Procedure. --

1. Remove congealed gelatin from test box in one piece by pouring a small amount of hot water down the inside faces and turning upside down.
2. Cut gelatin mass into cubes two-inches square.
3. Place entire mass, plus small addition for waste, into a

container and subject to a hot-water bath kept at approximately 130 degrees Fahrenheit until liquid state is reached.

4. Cool mass to 104 degrees Fahrenheit and continue as previously outlined.

Table 1. Mix Proportions

Mix No.	Per Cent by Weight of Total Mix Gelatin	Glycerine	Water	Total Weight in Pounds
1	8.0	11.2	80.8	59.51
2	10.0	15.0	75.0	48.46
3	11.0	17.0	72.0	48.46

Table 2. Relationship of Mix, Remeltings, E_s and Temperature

Test No.	Mix No.	Remeltings	E_s $\#/in^2$	Average Mass Temperature in Degrees Fahrenheit
1	1	2	2.2	--
2	2	0	5.2	--
3	2	1	4.9	64
4	2	2	4.0	68
5	2	3	5.9	64
6	2	4	5.4	65
7	3	0	7.7	59
8	3	1	8.0	58
9	3	2	7.9	59
10	3	3	8.3	57
11	3	4	10.9	53
12	3	5	14.0	44
13	3	0	9.2	55
14	3	1	11.7	47
15	3	2	9.3	59

APPENDIX II

COMPUTATION OF E_s

Plate Load Tests.--The theoretical expression for the settlement of a circular plate supported by a semi-infinite elastic medium may be written as follows. (13)

$$P = 1.58 \frac{(1 - u^2)}{E_s} qR$$

where p = settlement of the plate

q = uniform pressure on the plate

R = radius of the plate

E_s = modulus of elasticity of the elastic medium

u = Poisson's ratio.

In this work, let u equal 0.5 (assuming the gelatin to be incompressible) and let Q equal the total load on the plate. Then

$$q = \frac{Q}{A} = \frac{4Q}{\pi d^2}$$

and

$$P = 1.58 \times \frac{.75}{E_s} \times \frac{4Q}{\pi d^2} \times \frac{d}{2} = \frac{.75Q}{E_s d}$$

or

$$E_s = \frac{.75Q}{pd} .$$

The results of plotting the data from test number 15 are shown by Figs. 9 and 10.

Unconfined Compression Tests.--The results of the unconfined compression tests for test number 15 are shown by Fig. 11. Here, E_s is computed as it is defined, the ratio of stress to strain.

For d = 5/8", $E_s = \dfrac{.75Q}{pd} = \dfrac{.75Q}{.050 \times .625 \times 454} = .05286 \times 180 = 9.53$ psi

For d = 3/4", $E_s =$ " $=$ $= .04406 \times 219 = 9.65$

For d = 7/8", $E_s =$ " $=$ $= .03776 \times 249 = 9.40$

Average $= 9.53$ psi

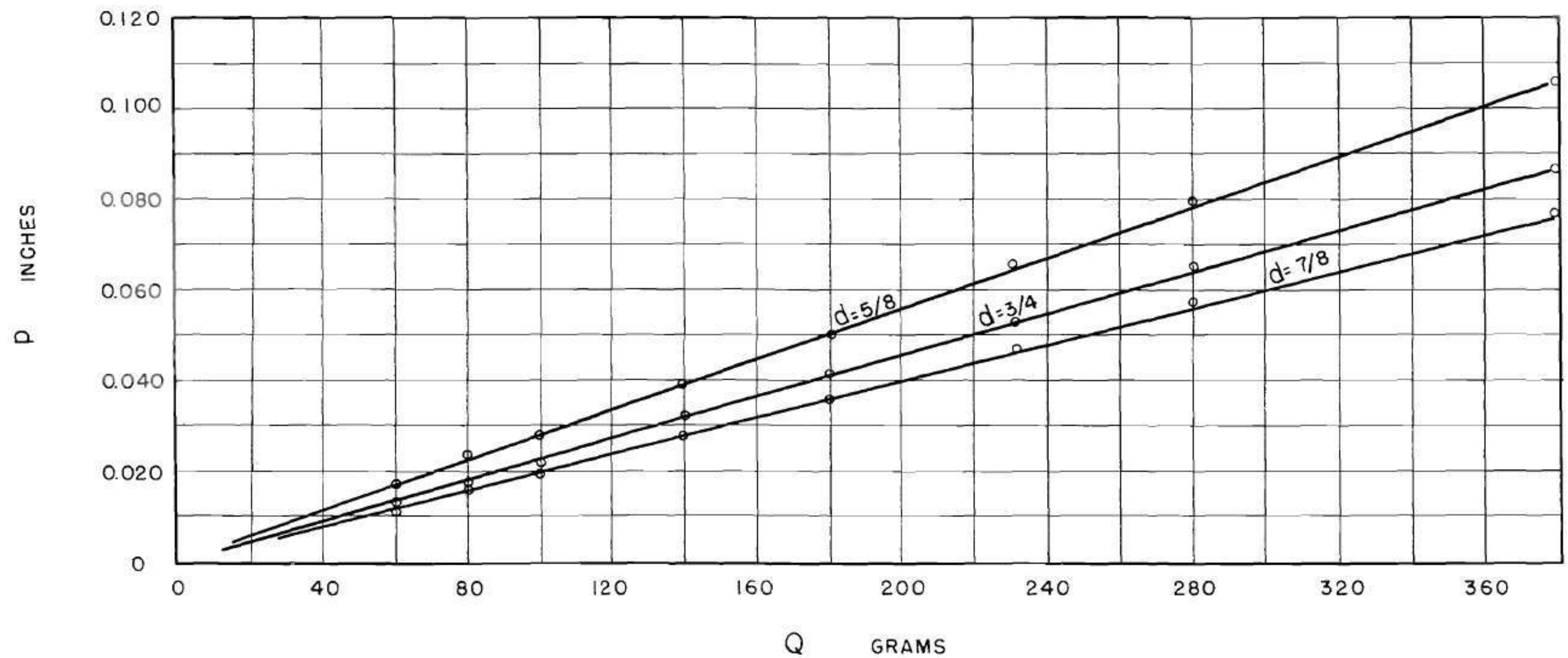


Figure 9. Plate Load Test For E_s Preceeding Test No. 15.

For d = 5/8", $E_s = \frac{.75Q}{pd} = \frac{.75Q}{.050 \times .625 \times 454} = .05286 \times 169 = 8.93$ psi

For d = 3/4", $E_s =$ " $= .04406 \times 207 = 9.12$

For d = 7/8", $E_s =$ " $= .03776 \times 237 = 8.95$

Average = 9.00 psi

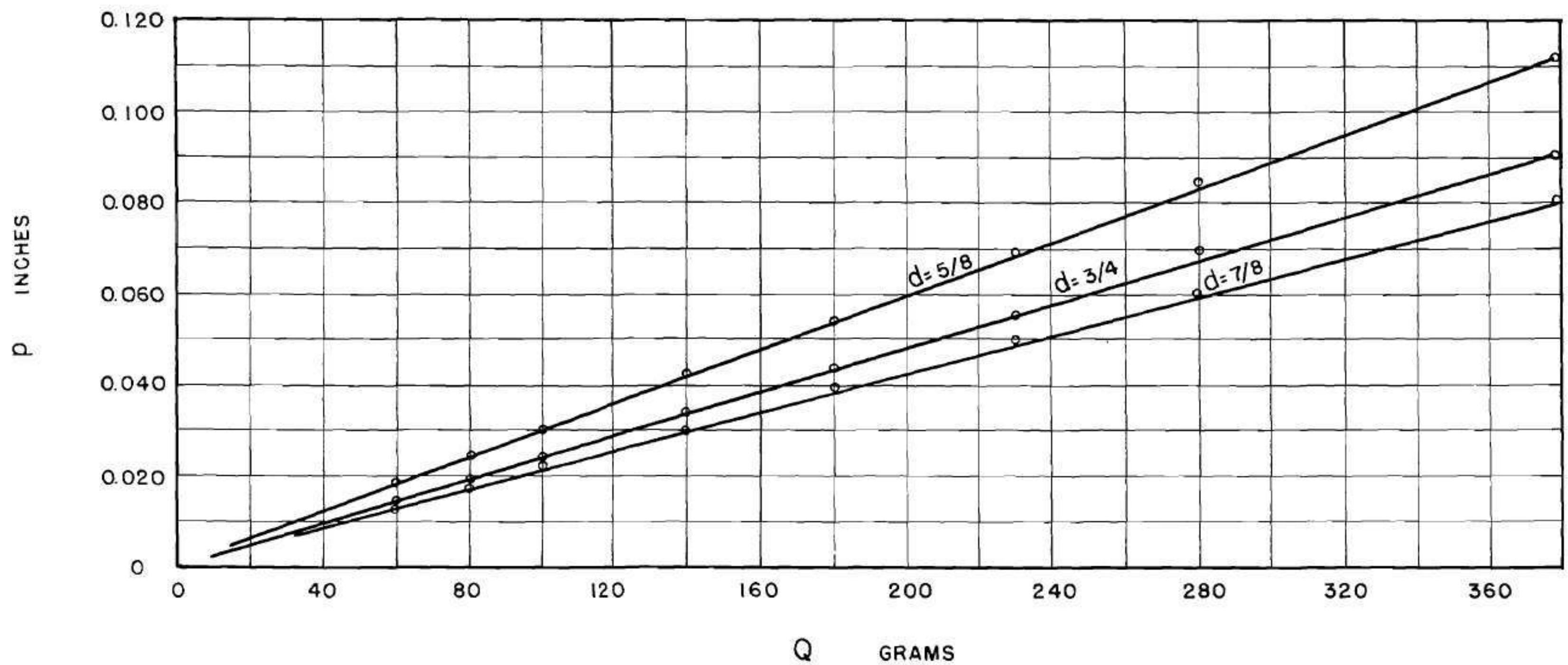
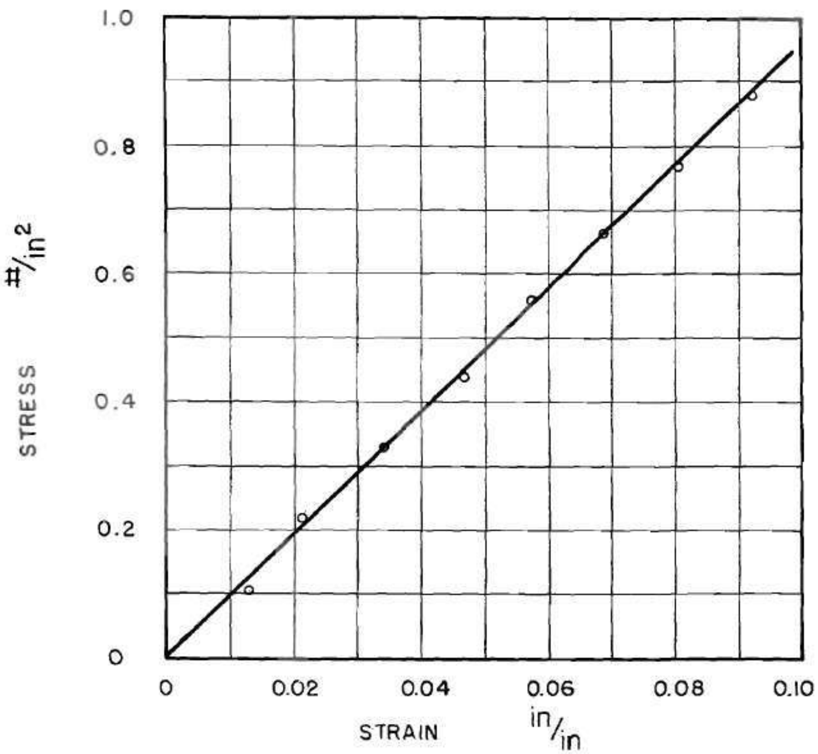


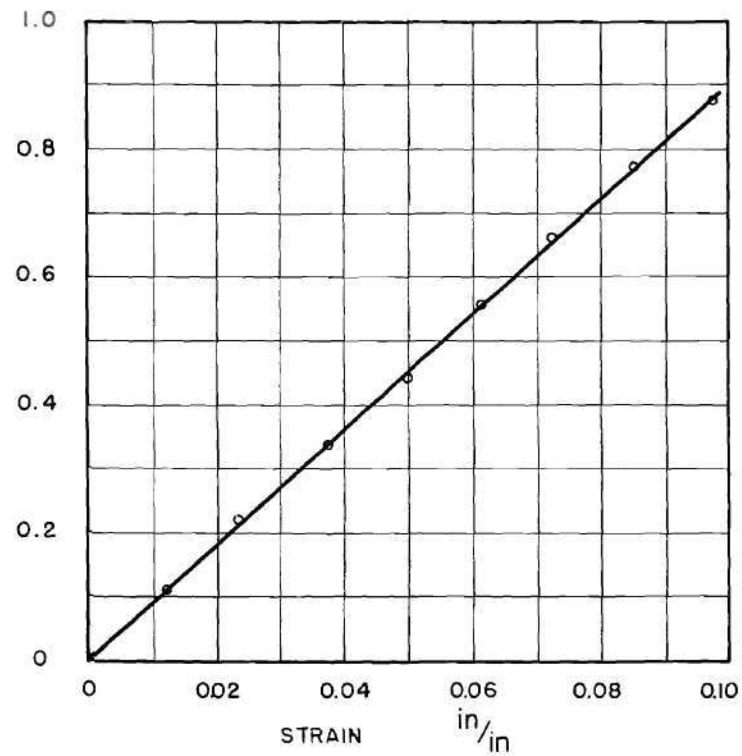
Figure 10. Plate Load Test For E_s Following Test No. 15.

$$E_s = \frac{\text{stress}}{\text{strain}} = \frac{0.963}{0.10} = 9.63 \text{ psi}$$



Preceeding Test No. 15

$$E_s = \frac{\text{stress}}{\text{strain}} = \frac{0.895}{0.10} = 8.95 \text{ psi}$$



Following Test No. 15

Figure 11. Unconfined Compression Tests for E_s Preceeding and Following Test No. 15.

APPENDIX III

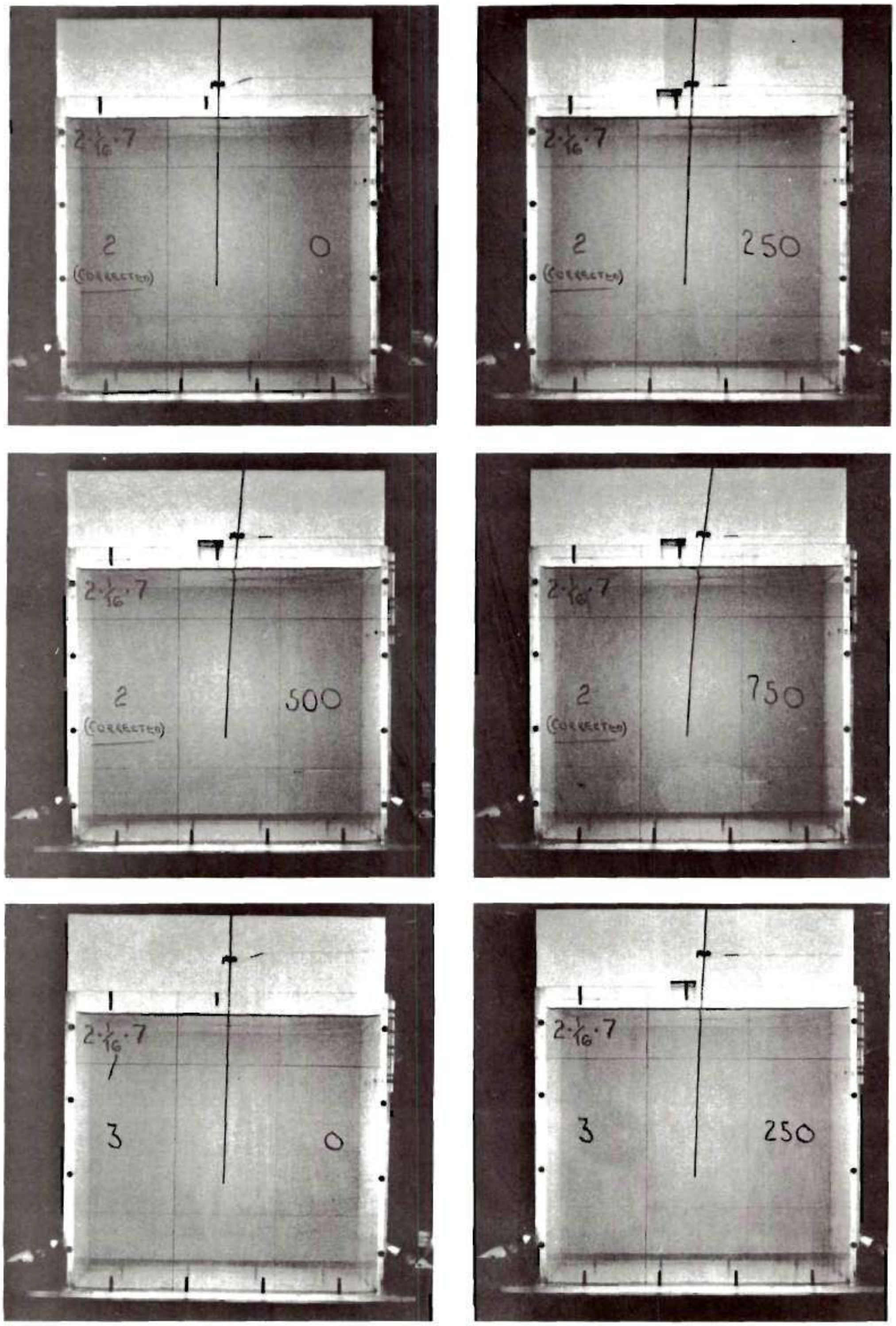


Figure 12. Photographs of Test No. 2.

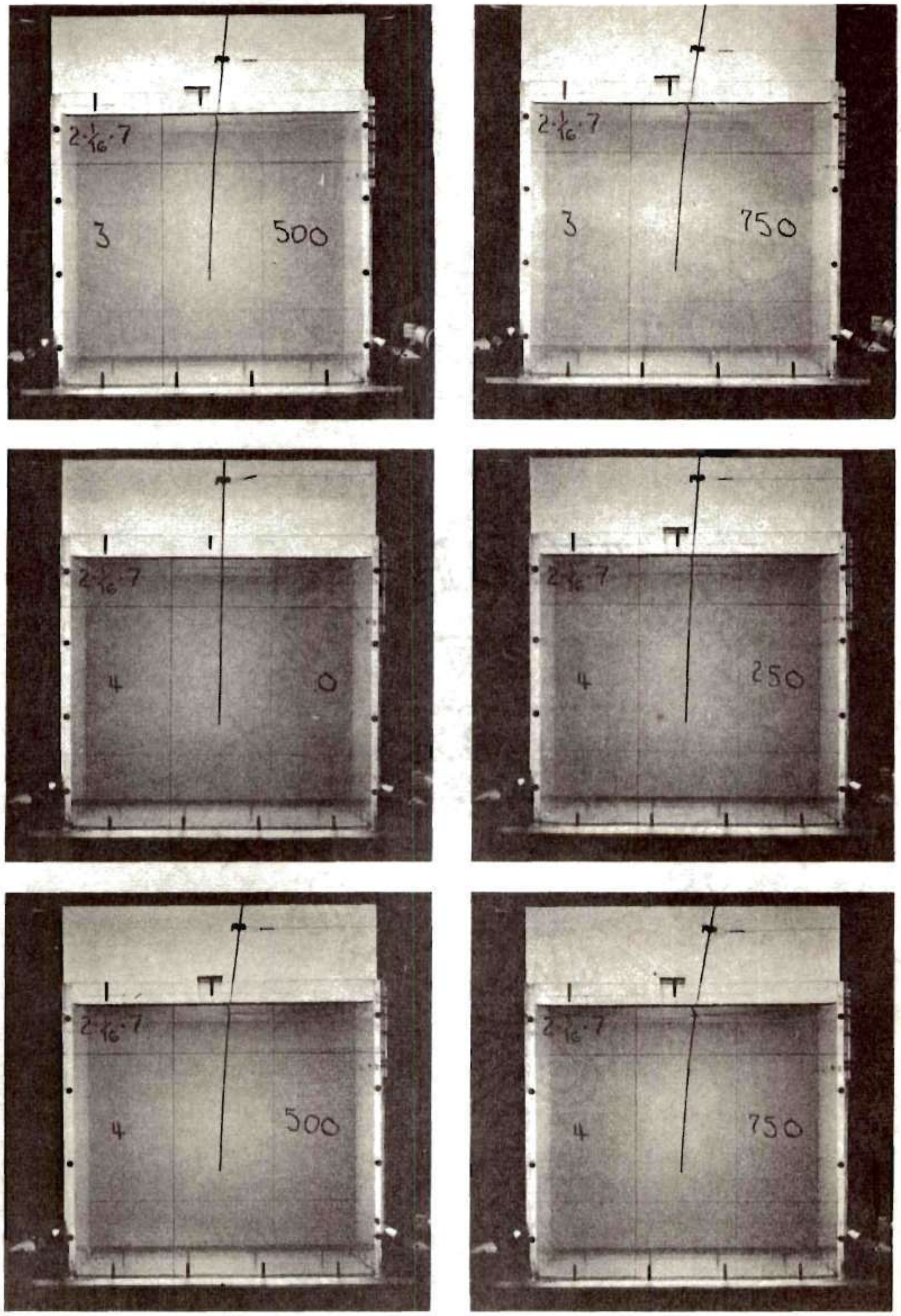


Figure 12. (Cont.)

APPENDIX IV

Table 3. $\frac{x}{L}$ versus $\frac{E_p I}{E_s L b^2 (e + L)} = S$

Test No.	e in	x in	L in	b in	E_s psi	E_p psi	I in^4	S	$\frac{x}{L}$
	2	1.36	9.71	0.5	8.3	$30x10^6$	$0.159x10^{-6}$	0.020	0.140
10	3	1.32						0.019	0.136
	4	1.15						0.017	0.118
	2	0.67	6.84		10.9			0.029	0.098
11	3	0.58						0.027	0.085
	4	0.58						0.024	0.085
	2	0.25	3.79		14.0			0.062	0.066
12	3	0.88						0.053	0.232
	4	0.72						0.058	0.190
	2	1.93	9.74		5.9		$1.272x10^{-6}$	0.226	0.198
5	3	1.67						0.204	0.171
	4	1.57						0.193	0.161
	2	1.96	6.84		5.4			0.468	0.287
6	3	1.80						0.442	0.263
	4	1.66						0.382	0.243
	2	2.07	3.86		8.0			0.842	0.536
8	3	1.94						0.718	0.503
	4	1.81						0.625	0.649

Table 3 (Cont.)

Test No.	e in	x in	L in	b in	E_s psi	E_p psi	I in^4	S	$\frac{x}{L}$
	2	3.91	9.89		4.9		4.300×10^{-6}	0.896	0.396
3	3	3.52						0.830	0.356
	4	3.33						0.764	0.337
	2	3.20	6.77		4.0			2.170	0.472
4	3	3.00						1.950	0.443
	4	3.00						1.760	0.443
	2	2.10	3.76		7.7			3.110	0.558
7	3	2.03						2.640	0.545
	4	2.00						2.290	0.532
	2	6.19	9.75		2.2		10.180×10^{-6}	4.830	0.635
1	3	6.00						4.460	0.615
	4	6.00						4.070	0.615
	2	4.56	6.88		5.2			3.860	0.662
2	3	4.04						3.460	0.586
	4	4.04						3.140	0.586
	2	2.54	3.86		7.9			6.830	0.658
9	3	2.48						5.850	0.642
	4	2.40						5.070	0.622

Table 3 (Cont.)

Test No.	e in	x in	L in	b in	E_s psi	E_p psi	I in^4	S	$\frac{x}{L}$
	2	6.06	9.75		9.2		19.920×10^{-6}	2.260	0.621
13	3	5.74						2.090	0.589
	4	5.22						1.940	0.535
	2	3.62	6.72		11.7			3.510	0.538
14	3	3.25						3.130	0.484
	4	2.90						2.840	0.432
	2	2.30	3.84		9.3			11.400	0.599
15	3	2.18						9.800	0.567
	4	2.05						8.540	0.534

Table 4. Summary of Computed k Values and Corresponding E_s.

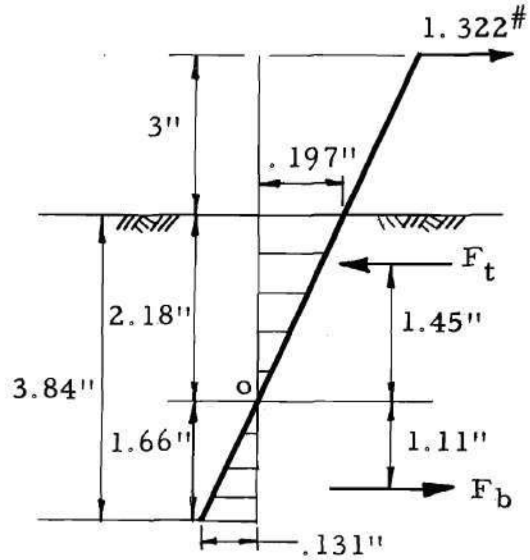
Test No.	e in	Computed k $\#/in^3$	Experimental E_s $\#/in^2$
2	3	12.0	5.2
3	3	10.7	4.9
4	3	12.7	4.0
5	3	11.7	5.9
6	3	12.5	5.4
7	3	27.9	7.7
8	3	30.0	8.0
9	2	26.5	7.9
9	3	22.1	7.9
9	4	23.1	7.9
12	3	15.6	14.0
13	3	26.7	9.2
14	3	43.6	11.7
15	2	31.5	9.3
15	3	31.8	9.3
15	4	31.0	9.3

Table 5. Comparison of Flexibility Factors and Resisting Moment Ratios

Test No.	e in	M_b/k in^4	M_t/k in^4	G	S
5	3	0.432	0.097	0.183	0.20
6	3	0.187	0.066	0.261	0.44
8	3	0.064	0.099	0.606	0.72
3	3	0.501	0.502	0.501	0.83
4	3	0.201	0.214	0.516	1.95
7	3	0.076	0.138	0.645	2.64
2	3	0.194	0.770	0.799	3.46
9	4	0.083	0.276	0.768	5.07
9	3	0.052	0.277	0.842	5.85
9	2	0.051	0.214	0.807	6.83
13	3	0.035	0.614	0.946	2.09
14	3	0.104	0.180	0.634	3.13
15	3	0.060	0.156	0.722	9.80
15	2	0.052	0.128	0.711	11.40

Sample Computations for k. --Two computations for k are as follows.

(a) From test number 15, assuming straight-line deflection and summing moments about o, it is seen that



$$5.18'' \times 1.322\# = 1.45\,F_t + 1.11\,F_b.$$

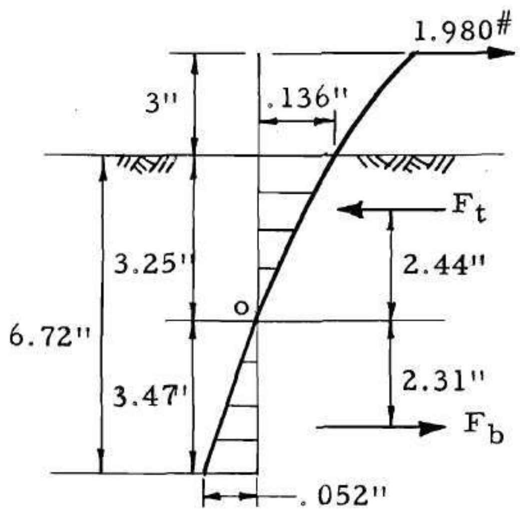
By evaluating F_t and F_b, it follows that

$$F_t = (2.18'' \times .197'' \times \frac{1}{2} \times .5'')\,k\,,$$

$$F_b = (1.66'' \times .131'' \times \frac{1}{2} \times .5'')\,k\,,$$

from which $k = 31.8\ \#/\text{in}^3$.

(b) From test number 14, assuming parabolic deflection above o, straight-line deflection below o and summing moments about o, it is seen that



$$6.25'' \times 1.980\# = 2.44\,F_t + 2.31\,F_b.$$

By evaluating F_t and F_b, it follows that

$$F_t = (3.25'' \times .136'' \times \frac{1}{3} \times .5'')\,k\,,$$

$$F_b = (3.47'' \times .052'' \times \frac{1}{2} \times .5'')\,k$$

from which $k = 43.6\ \#/\text{in}^3$.

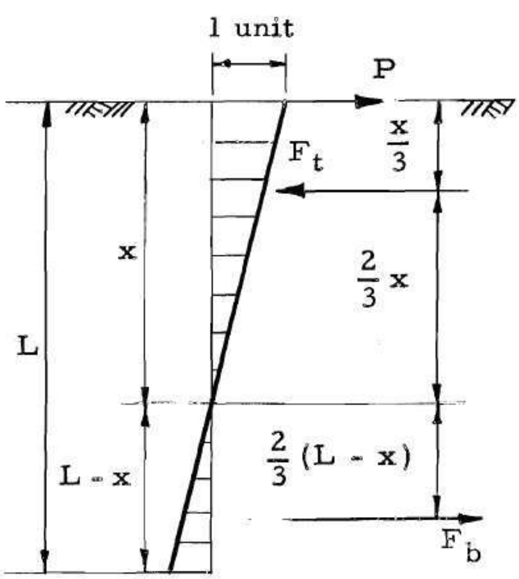
Table 6. Comparison of Y_t as Measured with Y_t as Computed

Test No.	Load Conditions e (in)	P (#)	Pile Data I (in^4)	L (in)	Gel. Data E_s (psi)	Y_t Comp. (in)	Y_t Meas. (in)	% Error
6	4	0.66	1.272×10^{-6}	6.84	5.4	0.19	0.23	17.3
10	4	0.33	0.159 "	9.71	8.3	0.09	0.10	10.0
5	2	0.55	1.272 "	9.74	5.9	0.07	0.08	12.5
5	2	1.10	" "	9.74	5.9	0.14	0.18	22.1
5	2	1.65	" "	9.74	5.9	0.21	0.26	19.2
8	3	0.33	" "	3.86	8.0	0.09	0.08	12.5
8	3	0.66	" "	3.86	8.0	0.18	0.15	20.0
3	3	1.10	4.300 "	9.89	4.9	0.14	0.18	22.2
4	4	0.44	" "	6.77	4.0	0.13	0.13	0.0
4	4	0.88	" "	6.77	4.0	0.26	0.25	4.0
4	4	1.32	" "	6.77	4.0	0.40	0.36	11.1
15	4	1.32	19.920 "	3.84	9.3	0.29	0.24	20.8

APPENDIX V

PROOFS AND DERIVATIONS

Maximum Depth of Point of Rotation, $\frac{x}{L}$. --The maximum value of x will occur when e equals zero. This may be explained in two ways. (a) A study of the experimental data contained in Table 3 reveals that x increases as e decreases. (b) In the sketch below, by summing moments about the point of rotation, it is seen that the smaller the distance to the line of action of P, then the smaller will be that part of the resisting moment that must be supplied by the soil pressure below the point of rotation. As this lower resisting moment decreases, x will necessarily increase. Maximum x will also occur when EI is infinite, assuming that the elastic medium will have some stiffness. This fact is substantiated by the experimental results of this thesis.



By summing moments about P,

$$F_t \frac{x}{3} = F_b \left[x + \frac{2}{3}(L - x) \right] .$$

Now, F_t and F_b may be expressed in terms of the volumes of the stress prisms and the modulus of earth reaction, k. Therefore,

$$F_t = (1)\frac{x}{2}\,bk$$

and

$$F_b = \frac{(L - x)^2}{2x}\,bk\,.$$

Then

$$\frac{x}{2}\,bk\,\frac{x}{3} = \frac{(L - x)^2}{2x}\,bk\left[x - \frac{2}{3}(L - x)\right]$$

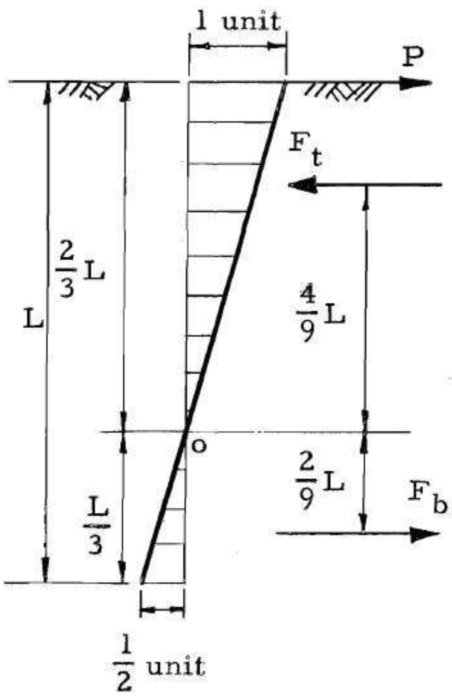
from which

$$x = \frac{2}{3}\,L$$

or

$$\frac{x}{L} = \frac{2}{3}\,.$$

Maximum Resisting Moment Ratio, G.--Start with the previously proven fact that the maximum value of x will be $\frac{2}{3}L$, the pile is infinitely stiff compared with the elastic medium, and e is zero.



By definition, M_t is the moment about o caused by F_t, and M_b is the moment about o caused by F_b. Hence,

$$M_t = \frac{4}{9} L F_t$$

and

$$M_b = \frac{2}{9} L F_b .$$

Evaluating F_t and F_b,

$$F_t = \frac{2}{3} L (1)(\frac{1}{2})(b)(k) = \frac{L}{3} bk$$

$$F_b = \frac{L}{3}(\frac{1}{2})(\frac{1}{2})(b)(k) = \frac{L}{12} bk .$$

Therefore,

$$M_t = \frac{4}{9} L (\frac{L}{3} bk) = \frac{4}{27} L^2 bk$$

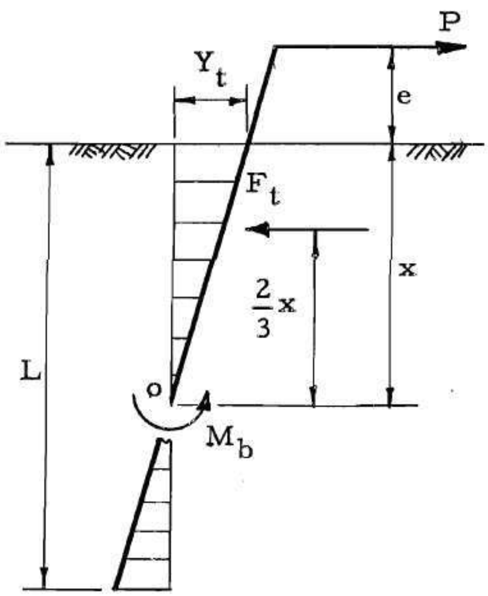
$$M_b = \frac{2}{9} L (\frac{L}{12} bk) = \frac{1}{54} L^2 bk$$

and by dividing M_t by the sum of M_t plus M_b it follows that the maximum value of the upper resisting moment ratio may be expressed as

$$\frac{M_t}{M_t + M_b} = \frac{\frac{4}{27} L^2 bk}{\frac{9}{54} L^2 bk} = \frac{8}{9} \text{ or } .89 .$$

Derivation of Deflection Formulae.--Two conditions are considered:

(a) Considering straight line deflection and summing moments about o,



$$P(e + x) = M_t + M_b = \frac{M_t}{G}$$

where

$$M_t = \frac{2}{3} x \, (F_t) = \frac{1}{2} (x)(Y_t)(b)(k) \frac{2}{3} x .$$

Hence,

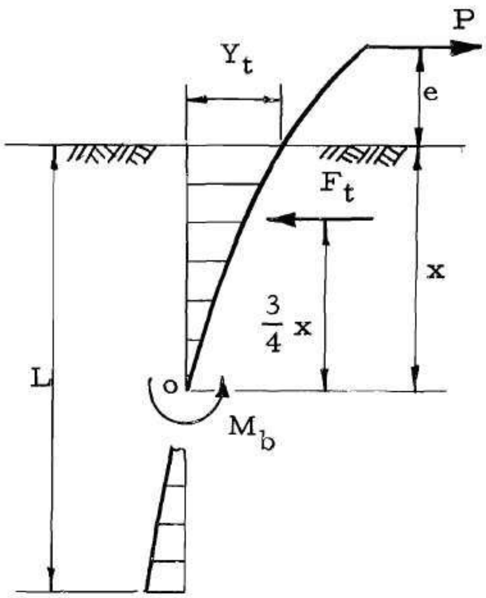
$$P(e + x) = \frac{x^2 \, bk \, Y_t}{3G}$$

from which

$$Y_t = 3 \frac{GP(e + x)}{x^2 \, bk}$$

(b) Considering parabolic deflection and summing moments about o,

$$P(e + x) = M_t + M_b = \frac{M_t}{G}$$



where

$$M_t = \frac{3}{4}x(F_t) = \frac{1}{3}(x)(Y_t)(b)(k)\frac{3}{4}x .$$

Hence,

$$P(e + x) = \frac{x^2 bkY_t}{4G}$$

from which

$$Y_t = 4\frac{GP(e + x)}{x^2 bk} .$$

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