FIXED ASSET REPLACEMENTS
AND ACQUISITIONS IN A TYPICAL
FIRM IN THE AEROSPACE INDUSTRY

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SUMMARY

Capital investment decisions are among the most important decisions that must be made by the top management of any company. Such decisions assume additional importance in the aerospace industry due to the dynamic nature of the business and the influential role played by the Federal government. The basic problem analyzed in this paper is that of the optimization of the decisions in the area of fixed asset acquisition and replacement as these decisions are made in the aerospace industry. Basic external and internal factors which influence decision making are defined, and the planning, acquisition, budgeting, and replacement activities which affect such decisions are discussed.

The purpose of this study is to set forth those factors that influence decisions concerning the replacement and acquisition of machinery and equipment, as these decisions are made in a typical firm in the aerospace industry. Models for these decision-making processes are developed and solution procedures are established.

The objectives of this research are: (1) definition of significant factors affecting fixed asset decisions facing a typical firm in the aerospace industry, (2) the development of realistic models in the relatively unique environment of an aerospace industry, (3) the establishment of solution procedures for optimizing the criteria of effectiveness, subject to various constraints and period linking requirements.

The present decision-making process concerning fixed asset investments as it occurs in TASCO, a typical aerospace firm, is presented to

provide a realistic background for the proposed models. Also presented are discussions of centralization and decentralization of decisions at TASCO, relations with government, and the details of the planning and budgeting of fixed asset acquisitions.

After discussing the existing operations, the significant external and internal factors to be included in the proposed system are discussed. The external factors are defined as those over which the system does not have control capability, whereas the internal ones are those over which the system does have control. Relations between TASCO and its parent company, competition, technological improvement, and type of contracts are shown to be some of the influential external elements. Grouping of fixed assets, modes of acquisition of capacity and criteria for decision-making are some of the internal factors.

These investigations lead to a classification of fixed asset models into seven types, based on a number of characteristics. To achieve clarity of structure the models progress from the simple to the more complex, and are developed in that order. The models reflect the sequential decision-making process and involve a number of inputs and states. The states reflect the condition of the fixed asset structure of TASCO at the beginning or at the end of a stage. The inputs reflect the external factors in the form of capacity, deterioration, obsolescence, functional requirements, and budgets. It is also shown that a considerable advantage is gained by combining fixed assets into "groups," classified by the functions that they perform. Existing government-owned and TASCO-owned assets are considered jointly in meeting "group functional requirements," but were segregated as to the types of decisions that can apply to each. The acquisition, replacement, salvage, lease, rent or subcontract decisions are made with respect to "typical" assets for each group.

Due to the combinatorial nature of some of the problems, enumeration of all possible decisions causes the dimensionality of the models to expand in a nonlinear manner. However, consideration of fixed asset capabilities concentrated in groups eliminates the need to consider each fixed asset item separately, and thereby reduces the dimensionality of the problem, allowing programming techniques to be used for the solutions. Two decomposition techniques are investigated in detail to assist in the solution of problems in the case that the number of groups to be considered are in excess of present computer capabilities.

Since investment in fixed assets is a form of capital investment, financial factors as well as physical factors have an effect on decision-making. Certain financial considerations are analyzed by treating TASCO as an autonomous company. Such treatment allows investigation of absolute limits on debt, changing supply schedule of funds, and optimization of certain decisions regarding equity financing. Interpretation of the dual aspects of models constructed with the addition of financial considerations is found to yield valuable information concerning marginal return on additional investments.

The practical implementation of the models as an information system with feedback is also discussed. Methods of obtaining the data and interpretation of the results are shown with actual computer runs.

This research identifies and defines a number of unique and highly significant factors that affect the fixed asset decisions of firms in the aerospace industry, and demonstrates that realistic and practical models can be developed which combine basic acquisition and replacement decisions concerning these assets. Furthermore, it is shown that the present state of the art in computer technology allows the use of linear programming solution techniques to treat simultaneously a number of interacting problems previously handled as independent areas of study.

CHAPTER I

INTRODUCTION

Definition of the Problem

Among the most important decisions which must be made in any company are those concerning capital investment. These decisions, which are made by top management, involve large sums of money and are influenced by plans for future product diversification, expansion or decentralization.

Decisions concerning capital investment are usually made periodically and involve planning, budgeting, and funding activities. In general, the numerous factors which must be considered in these decisions may be classified as either external or internal. External factors are those over which the management of the firm has minimal control, such as competition and economic environment. Internal factors are those over which it has a good measure of control, and among these are budgeting practices, long and short-term objectives, and numerous funding arrangements.

In the following discussion, fixed asset type capital investment decisions are analyzed as exemplified in the aerospace industry since this industry is one of the most important industries in the United States, and the dynamic nature of its business makes it particularly challenging for the type of research undertaken. For the purposes of this study, the aerospace industry is defined as those companies that are involved in the research and development, and production of aircraft, missiles and spacecraft, their propulsion systems, and numerous electronic, hydraulic and mechanical components.(1) The basic problem is concerned with the optimization of decisions in the area of capital investment in the aerospace industry, taking into account basic external and internal factors and the planning, acquisition, budgeting, and replacement activities that affect such decisions. The problem is studied in light of long and short range objectives of the industry, and considers investments related to given contractual requirements.

Many aspects of fixed asset investment decision-making have been previously analyzed; however, basically due to the state of the art of computer technology in treating multi-dimensional problems, the conclusions reached have not been adequately synthesized into a whole. Therefore, the importance of the problem studied stems from the fact that it provides a "system" or a synthesized approach within which the numerous factors that affect decisions in an aerospace firm can be analyzed.

Purpose and Objectives

The purpose of this research is to achieve an in-depth understanding of the factors that influence decisions about the replacement and acquisition of fixed assets, as these decisions are made in a typical firm in the aerospace industry. Models for these decision-making processes are then developed, and solution procedures for such models established.

The objectives of this research are stated below:

1. To investigate and define significant factors affecting fixed asset decisions facing a typical firm in the aerospace industry. These factors involve the advanced planning for aerospace programs, the establishment of budgets, the role of government, technological improvement,

and related aspects.

2. To develop realistic models of replacement and acquisition decisions concerning fixed assets in the relatively unique environment of an aerospace industry. The uniqueness is due to the interaction of a large number of factors involving budgets, financial restrictions, the planning horizon, and the numerous replacement and acquisition modes involved, such as buying, renting, leasing, and salvaging.

3. To establish solution procedures for optimizing the criteria of effectiveness subject to various constraints and period linking requirements. Due to the size of the decision space, extensions of present programming methods and approximation techniques will need to be investigated.

Brief History, Scope and Limitations

Government assistance to the aerospace industry in the form of providing fixed assets has been considerably reduced over the past several years.(2) This reduction creates the need for determining the optimal policies an aerospace firm can use in the replacement of government equipment, or in declaring such equipment as surplus, while at the same time providing for an expansion of capacity dictated by its long-term goals. Such a determination requires an analysis of the various factors that affect such decisions, and a systematic method of taking into account the interactions of such factors.

The MAPI (Machinery and Allied Products Institute) approach of George Terborgh,(3) published in 1949, provided a method of determining a basis upon which to base equipment analysis, specifically, the timing of replacements. However, his methods are plagued by difficulties in an appropriate choice of "defender" and "challenger." Possibilities of providing additional capacity through methods other than purchase are not considered. Furthermore, fluctuations in the need for assets over the years, and interaction of such fluctuations with the possibility of short-term rental, or subcontracting, the capability of handling multi-year budgets, and the possible financial implications of various replacements, have no way of being taken into account.

In 1962 H. M. Weingartner's[4] mathematical programming approach to solving a multi-year capital investment and budgeting problem broadened the basis of application of integer and linear programming problems to include major project type investments with possible interrelationships. This outstanding work, however, does not approach the specific questions of machine replacement and acquisitions, and related problems.

A. Charnes, W. W. Cooper and M. H. Miller's[5] analysis in 1959 of the problems of financial planning through the use of linear programming considered the liquidity constraints, borrowing and lending activities, and a number of other financial considerations. This work and others that followed still have not adequately merged the replacement and acquisition aspects of investment questions with the financial aspects.

Richard Bellman and Stuart Dreyfus'[6] dynamic programming approach to questions of replacement suffers from the inability to solve problems with more than one or two budgetary constraints, due to increases in the dimensionality of the state space.

The research reported in this paper is directed toward the synthesis of a number of fields of analysis with the purpose of providing a workable programming tool for optimal fixed asset related decisions in a typical aerospace firm. The system concepts used for such unification and model building are described in the following chapters. The fields that were synthesized for this purpose consist of capital investment and budgeting, replacement theory and practice, and modes of acquisition of fixed asset resources, such as rental, leasing, and subcontracting.

Basically, the synthesizing of a number of fields that have been thus far analyzed separately has been made possible by the increased capability of modern electronic computers. Since the present-day trend is to build faster and higher memory capacity computers, it is felt that such efforts toward synthesis will increase.

The scope of this study covers a typical company in the aerospace industry and its fixed assets of machinery, equipment, and buildings. The typical company concept is, later on in the study, enlarged to include the parent company for some of the financial analyses.

The limitations of the study are based on its being most applicable to the typical company* with which the author is most familiar. Computer capability is found to be still a major restriction in applying the system in its entirety.

Background of the Aerospace Industry

Before beginning the analysis of capital investment decisions in the aerospace industry, it is appropriate to consider briefly the history of the development of this industry, its general structure, and its objectives and goals.

History

Before World War I no real aircraft "industry" existed. Instead a

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*Lockheed Aircraft Corporation, Georgia Division type of "backyard" production process existed.[7] World War I provided the U.S. aircraft industry with major momentum and production increased considerably but dropped drastically with the end of the war. The development of the airplanes used in the war and the facilities in which their production took place were privately financed, with the manufacturers, actively competing for sales to the government.[8] The creation of passenger-carrying airlines, in 1927, provided a new market for the aircraft manufacturers. During this period success of the manufacturers depended on their ability to succeed in competing for sales in the commercial market. The industry during the 1914-1939 period grew in dollar sales from less than one million to close to a quarter of a billion; employment increased from 222 in 1914 to 63,994 in 1939.[9]

American aircraft manufacture expanded quite rapidly during World War II. The 1939 production of 5,836 airplanes rose to 96,318 in 1944,[10] and this expansion required the construction of new facilities and the purchase of new equipment. New plants were largely financed by the federal government since the industry did not have the financial resources.

After the war ended, the industry's sales decreased rapidly, from a peak of $16 billion to about $1 billion in 1947.[11] As a result of this fall, the aircraft industry experienced an over-capacity, but due to the government having financed a large part of the expansion the financial burden on the firms was not great.

The Korean conflict required expanded aircraft production. By then, the facilities of World War II were partly obsolete, and new facilities were needed. The government again provided the largest portion of the financing. In this period, however, the industry provided 34 per cent of the expansion cost as compared with the 10 per cent it had financed during World War II.[12] After the Korean conflict, production in the industry remained at a high peak, but after 1957 the production of military aircraft decreased as missiles achieved an important role.

Successful development of the hydrogen bomb marked the rapid increase in funds expended on the longer-range ballistic missiles. The missile and the airplane became interchangeable carriers by which similar missions could be performed. The introduction of the missile caused much larger expenditures for research and development, and also caused a tremendously larger demand for electronics and related equipment. It was estimated that as much as 50 per cent of the cost of a missile went into its electronic equipment, whereas 13 to 20 per cent was normal for an aircraft. This meant that airframe manufacturers had to develop capability in the electronics field.

The transition of the industry to the space age in 1957 was a significant event. It resulted in the creation of NASA (National Aeronautics and Space Administration) in 1959. Since the industry was already working on missiles, the transition to development of space satellites and related equipment was not too difficult. The industry at this stage was renamed and called "the aerospace industry." The growth of research and development expenditures, as well as the use of electronic equipment is a characteristic of this period. The additional facilities needed in this period were primarily financed by the firms themselves.

Summarizing the above historical remarks concerning the aerospace industry, we may make the following observations:

1. The changes and fluctuations in the industry with respect to

the technology and its products occur at a rapid pace.

2. During World War I the industry was an important sector of the economy but it faded afterwards until civilian air travel began on a large scale.

3. The aerospace industry became the nation's greatest industry in World War II. It experienced a decline after the war but again rose to prominence in the 'sixties.

4. Approximately 50 per cent of the industry's sales are of products non-existent ten years ago.

5. The industry has expanded into the electronics field, and has shifted emphasis from production to research and development.(13)

General Structure

A useful definition of the aerospace industry is given by Herman Stekler. He says that an aerospace industry is one that "would develop and manufacture vehicles, subsystems, and parts essential for both atmospheric and space flight, whether manned or instrumented, or necessary for effective operation in flight or space."(14)

For the purposes of this paper we concentrate on the relations between government and the aerospace industry, because government is the buyer for a large percentage of the dollar volume of aerospace industry sales.

A substantial number of sellers operate within the industry, but evidence indicates that the sales of most aerospace products are concentrated in the hands of twenty or fewer firms.(15) Classification of aerospace firms by the type of activity in which they are involved is not easy, though a classification could be based on whether the firm is a prime contractor, an associate prime contractor, or one of the subcontractors which manufacture systems, and subsystems.

Since the government does its buying through several agencies, it cannot be effectively considered a single buyer. The interservice rivalry for particular missions have a beneficial competitive effect. Competition in the aerospace industry occurs in the initial stages of the procurement process. After the awarding of the research and development contract, competition is not always present. In such an absence of competition it is up to the buyer to introduce institutional arrangements, such as competitive bidding for fixed price contracts, and to impose standards of performance to protect his interests.

Basically four types of contracts exist. These are: fixed price, cost reimbursement, special incentive and special purpose. Armed Services Procurement Regulations, Section III, Pt. 4.10 U.S.C., Chpt. 137, Sect. 2306 lists the type of contracts that may be used. The trend has been toward a greater awarding of cost-reimbursement contracts, this being due to emphasis on research and development. Also, lately, the emphasis is on including incentive clauses in these contracts.

Until 1956 the government carried a large percentage of the risk and cost associated with the ownership of the industry's facilities since these facilities had been financed by the government. Even though the industry's risk in this area has increased as a result of a decrease in this type of financing by the government, the government still bears the cost of working capital through progress payments it makes to the industry.

<u>Functions of the Industry</u>

The following statements of the functions of the aerospace industry proposed by the Labor Department provides a background for understanding the objectives of this industry.

The aerospace industry performs the functions of,

> manufacture and assembly of airplanes, lighter-than-air craft, gliders, drones, guided missiles, aircraft type engines, guided missile propulsion systems, aircraft and guided missile airframes, aircraft propellers and parts, and accessories especially designed for use with or on the above mentioned products . . . and specialized aircraft and guided missile servicing equipment.(16)

The AIA (Aerospace Industries Association) indicated that the following be also included in the above functions.

> . . . electronic, hydraulic, electrical, pneumatic, and mechanical systems for purposes such as flight control, guidance, airborne intelligence, telemetering and navigation; and/or major assemblies for use in such systems for such vehicles, which are especially designed for and perform specific functions in such vehicles; and specialized ground support servicing equipment which is especially designed for and performs specific functions in such vehicles, engines and systems.(17)

The above two quotations describing functions also indicate the numerous products which are the outputs of this industry. On the resource side a brief summary indicates the following,

> Marketing - Limited primarily to sales of complex systems to government agencies or small numbers of high value items to few customers. Very limited industrial or consumer sales, distribution, or promotion capability.
>
> Production - Limited to small quantities of high quality, high value items incorporated advanced engineering and scientific design. Very limited capability to meet stringent price competition.
>
> Engineering - Strong capability to perform state of the art research as well as complex engineering design, limited capability to design commercial products for mass production.
>
> Management - Unique capability to manage integration of large complex systems and large scale research and development organizations.

Finance - Limited financial resources, low capitalization, low profit on sales but high return on investment.(18)

Objectives and Goals

The purposes and objectives of a firm in the aerospace industry can be stated as follows:

1. To be outstanding in the design, development, and production of aircraft, missile, and space systems which will aid the nation in maintaining scientific and military superiority.

2. To be outstanding in the design, development, and production of aircraft for commercial airlines.

3. To be outstanding in all research, both technical and managerial.

4. To enter such other lines of business as may be required to perform the above roles and to attain . . . growth objectives.

5. To achieve a rate of growth and a product structure which, on a long term basis, will maximize profit on the investment of . . . stockholders.(19)

A prominent aerospace company states its basic purposes as follows:

1. To be the major company satisfying in the highest technical sense the national security needs of the United States and its allies in space, air, land, and sea.

2. To employ technical resources in meeting the nondefense needs of governments and the requirements of commercial markets.

3. To achieve continuous growth of profits at a rate needed to attract and retain stockholder investment.

4. To recognize and appropriately discharge our responsibilities for the welfare of our employees, the communities in which we do business, and society as a whole.

5. To maintain a large proportion of sales in advanced technical products bearing our name.

6. To maintain continuity of the enterprise by holding relatively low rates of change of ownership, management, and employees.

The influence of goals on company actions is well-recognized,[20] and conversion of the above purposes and objectives to yield consonant fixed asset goals and objectives will be accomplished in later chapters.

CHAPTER II

LITERATURE SEARCH

Introduction

The discussion of the literature search presented in this chapter is organized in accordance with the discussions presented in the chapters that follow it. Since the initial chapters provide the background for the development of later quantitative models, the earlier parts of this chapter concentrate on the authors who have pointed out the basic characteristics of the aerospace industry. A number of approaches to capital investment questions by various authors, are also discussed, in chronological order, since fixed assets are an integral part of the general field of capital investments.

Basically, the literature search for this paper explored four areas. First, the pertinent literature on the background of the aerospace industry, with emphasis on the industry's long-range planning activities, is discussed. This section also includes references to the literature on the fixed asset problems of this industry.

The second area is concerned with the general areas of capital investment and capital budgeting. Both these areas have been researched in detail, and the literature is quite voluminuous; therefore, only highly pertinent writings have been discussed.

The third area is concerned with the alternative approaches taken by a number of researchers with respect to questions concerning fixed assets. This section also refers to certain writings that have assisted the author in his "system" approach to model building.

Discussions of the writings which deal with solution techniques are presented in the fourth section.

Each of the above areas has been researched in depth by many competent authors; therefore, the uniqueness of this dissertation lies basically in its description of actual problems that arise in a typical aerospace firm, and the actual solution of these problems through the models to be described in the following chapters.

The Aerospace Industry and Long-Range Planning

A number of written sources provide ample background about the aerospace industry. These writings examine the industry from almost all aspects that are of interest. Basically, the sources can be designated as government publications, Rand publications, books, articles, and certain general references.

Merton Peck and Frederic Scherer[21] discuss the provision of capital and facilities by the government and private firms, and show the impact of the changing requirements for various types of production factors upon the industry.

Herman Stekler's analysis of the structure and performance of the aerospace industry reveals the development of government and aerospace industry relations. One type of government assistance, that of acquiring the fixed assets for the firms, is analyzed, and the effects on the productive capabilities and finances of the industry are noted.[22]

The expansions of facilities enabled by private and government funds during World War II are shown in "Facts for Industry."(23) More recent government publications dealing with facilities are "Report on the Management of Capital Plant Equipment,"(24) "Industrial Plant Equipment Report,"(25) and the "Annual Report of Industrial Plant Equipment."(26)

LMI (Logistics Management Institute) and others have analyzed the possible incentives the government could use to have the private firms buy their own fixed assets.(27)(28)(29)

The aeronautical "Production and Exports" section in the Federal Aviation Agency's Statistical Handbook of Aviation(30) provides a convenient source of historical data that was used to assist in evaluating the progress and the trends, and to estimate further activity in the industry.

"Aerospace Facts and Figures"(31) is an annual publication that furnishes statistical data for activities in the industry. "Aviation Forecasts"(32) and a yearly special issue of "Aviation Weekly and Space Technology"(33) provide forecasts of certain important indices of the performance of the aerospace industry.

Long-Range Planning in Aerospace Industry

Murray Weidenbaum(34) discusses the objectives, long-range plans, and related questions in economics, for an aerospace company. The importance of setting objectives and planning so as to make effective utilization of all the resources, including fixed assets, is emphasized. In their text entitled, Science, Technology, and Management(35) Fremont Kast and James Rosenzweig edit the conference papers of a number of people concerned with the problems of managing very large complex programs, from their inception to the operation of the end product. The effect of government on private industries in the form of centralization versus decentralization of decision-making, and control mechanisms are discussed.

George Steiner's[36] "Managerial Long Range Planning" also emphasizes the importance of long-range planning. In Steiner's text Stewart and Lipp[37] discuss implications of long-range planning in one of the major aerospace firms. The outlining of numerous planning activities over a number of future years is, of course, shown to extend into the basic resources of the firms. The necessity of making projections of government policies into the future, and of decision-making based on such forecasts of policies, and of the impact of these forecasts on any modeling of fixed asset decisions in a firm are brought out clearly.

Since one of the major features of aerospace industry is change, more specifically, technological change, certain observations of Brian Scott are worth noting. He states that, "Among the many assumptions about the future which are necessary in developing strategic long-range plans, none are potentially more important than those attempting to anticipate technological change."[38] According to Scott, technological change is a complex force which affects, and, in turn, is affected by, a number of forces in the economy. Most important of these are technical considerations, but competitive aspects of the industry, the availability of investment capital, and the economic feasibility of a proposed change also have strong influences. The special problem associated with a high pace of technological change in fixed assets is that, unless a methodical and systematic approach is used, effective long-range planning cannot be done. As Scott says, ". . . an investment in fixed assets for a given undertaking commits a company to a fairly rigid level of capability for that undertaking over a considerable period of time."(39) Therefore, any modeling of fixed asset related decision-making must be made to reflect such changes to be an effective planning tool.

Capital Investment, Capital Budgeting and Related Topics

The above section concentrated on literature that deals with the long-range planning aspects of the aerospace industry, and the role the government plays as an important external influence on such planning activities. This section concerns itself with the examination of the state-of-the-art on some of the fundamental concepts that underlie any model-building related to the fixed asset replacement type decisions. These basic concepts fall into two areas that have been extensively researched. The areas are: capital investment and capital budgeting. Only selected literature is discussed since the emphasis of this paper is on the development of a sound system framework for optimizing the fixed asset decisions in an aerospace firm, using the tools provided by operations research.

In addition to the two areas mentioned, the literature search also examines certain writings on depreciation, purchasing and procurement, fixed assets, accounting, taxation, replacement, and the basic model decisions of lease, rent, salvage and subcontract. This section closes with the literature related to basic model-building using sound system concepts.

Capital Investment

Robert Eastman and Clifton Anderson(40) have investigated the effect of business cycles on capital equipment policies by the use of a mathematical model. By varying the parameters of a theoretical business cycle they determined the effects on an index of fluctuation, which in this case was the dollar difference between the gross income and the operating costs of an investment in productive capital equipment.

Robert Eisner[41] conducted an interview study in which the officials of a number of manufacturing organizations were asked for determinants of the level of capital expenditures. He learned that most of the firms made some effort to determine the long-term demand for their products when considering capital expenditures, and that the investments for replacement increase during periods of expansion, and decrease when expansion slackens.

In a monograph published in 1956, the Management Sciences Research Group of Purdue University[42] attempt to put together the elements of a unified theory of investment in replacement based on least cost function. In order to make the models more realistic, the utilization rates and a number of related factors, such as salvage revenues and operating and maintenance expenses are taken into account. The use of such a sophisticated model was a step forward in the analysis of replacement decisions.

An empirical study of how firms make their investment decisions was conducted by John Meyer and Edwin Kuh.[43] Their findings are important because of the shaky ground upon which they leave some of the classical theories. For the purposes of this paper their "senility" effect is noteworthy. They say that, "The firms which have old equipment and low investment rates will, in general, continue so."[44] Aerospace industries could easily fall into this category if the deterioration and obsolescence of assets bought with government assistance many years ago are not recognized. Another finding with financial implications is that the short-run investment is determined by the excess of earnings over the dividends, whereas the long-run investments are usually based on estimates of future technological needs.

An article by Franco Modigliani and Merton Miller[45] created considerable controversy in the area of "cost of capital." Their theory essentially states that "the average cost of capital to any firm is completely independent of its capital nature." They later modified their point of view on this subject and accepted the traditional point of view which held that the cost of capital is weighted by the capital structure of the firm.

An article by John McLean in Harvard Business Review[46] discusses the advantages of one method of evaluating capital investments, the discounted cash-flow-return on investment method, and shows how it was implemented at the Continental Oil Company. The article is interesting because it shows how a large company was made aware of the time value of money and how this resulted in changing from the "payout period" to the discounted cash flow method.

Discussion of yardsticks to evaluate investments occurs in numerous articles and books, and examination of the literature shows that there are differing opinions on the subject.[47][48][49][50][51]

Herbert Schweyer[52] discusses the macro and micro aspects of investment in a chapter that is concerned with the economic state and investment. He shows the breakdown of total capital investment, and its relations to various types of financial statement analyses. Also his discussion of fixed asset investments and the "six-tenths" factor relationship between price and capacity is interesting.

Analysis of short-term financing is done by A. Robicheck,

D. Teichroew and J. Jones[53] in a certainty environment by the use of a mathematical model which is solved by linear programming. The model includes various forms of cash transactions, lines of credit, pledging of accounts receivable, term loans and stretching of accounts payable. Requirements for cash at various periods are taken as given parameters.

The joint technological and financial aspects of investments in capital projects are discussed and analyzed by A. Merrett and Allen Sykes.[54] Theirs is one of the first texts to emphasize the importance of considering these two concepts jointly.

Two books on finance, Readings On Finance[55] and Managerial Finance[56] point out the significant changes in the early part of the 1960's in the area of finance. The changes are mainly in the shifting of emphasis from acquisition of funds to the effective use of funds, which quite clearly indicates considerations that new investment models must take into account.

David Chambers[57] developed a model that was used to select investments and that took into account certain published financial results (return on gross assets, current ratio) in addition to cash flows. He also developed criteria for the amount and timing of debt; and solved the model using regular linear programming.

William House's[58] sensitivity analysis in making capital investment decisions reveals that such decisions are affected most by estimates of sales prices and sales volume. This indicates the need for an analysis of certain critical parameters of any investment model for sensitivity, since the errors in estimates could easily sway the decisions.

James Mao's[59] article on the application of linear programming to making short-term financing decisions in a greeting-card manufacturing firm is based on the model developed by Robicheck, Teichroew, and Jones.

Capital Budgeting

The earlier writings on capital investment concentrate on the basic proposition that under an optimal program, investment should be carried out to the point at which the marginal internal rate of return is equal to the market rate of interest. Imposition of budgeting on investment plans complicates the problem somewhat; however, this complication has not prevented a number of texts and articles from appearing in literature. Certain of these articles that indicate the trend of thought in this area are discussed below.

Joel Dean(60) contributed greatly to the capital budgeting procedures of firms by indicating that the alternative investments be ranked according to their internal rate of return and those projects that have an equal or greater rate than the firm's cost of capital be selected. J. Lorie and L. Savage(61) improve upon Dean's approach, specifically in the area of dependent projects.

Using linear programming, A. Charnes, W. Cooper and M. Miller(62) explore the ways in which funds may be allocated within a firm. They consider the simultaneous problems of financial planning, such as the problem of liquidity constraints and they also study the operating policies of a number of facilities. Borrowing and lending arrangements, trade credit, and the impact of changed liquidity requirements are analyzed within the framework of the linear programming approach.

The text written by Harold Bierman and Seymour Smidt(63) has a number of discussions clarifying concepts related to capital budgeting.

Specifically, chapters on the cost of capital and on capital budgeting under capital rationing are quite detailed and help clarify some of the confusion that exists in these areas. Robert Vandell and Richard Vancil[64] show the wide scope of the capital budgeting problem in the 34 cases they have analyzed.

H. Weingartner's[65] dissertation broke important ground in the application of mathematical programming to investment planning under capital rationing and with imperfect models. By using linear and integer programming techniques he developes a systematic approach to this type of budgeting problem, and thus points the way to the solution of period budgeting type problems.

A. Kalaba, A. Kent and M. Prestud[66] using a dynamic programming approach, model the pressures of technological improvement and competition on the replacement policies of a firm. This study is a theoretical model that was used to determine the sensitivity of optimal decision policies to changes in basic assumptions concerning the physical situation, and is quite novel in the manner that competition is incorporated as a factor.

In 1968 Joseph Moder and James Nickl[67] discuss a sequential procedure for evaluating the comparative worth of interesting alternatives in a capital budgeting analysis based on the maximization of their combined effectiveness. Open discussion, subjective opinion, intuitive insight, and competitive bargaining are ingredients of the proposed systematic procedure. Mathematical formulations aid the individuals who are involved in solving the problem, which essentially is mathematically intractable. Even though final solution may not be mathematically optimal, it represents an interesting approach to large problems involving many variables.

Depreciation, Taxation and Investment Credits

E. Grant and P. Norton's[68] text on depreciation, and George Terborgh's[69] "Realistic Depreciation Policy" are valuable references on types of depreciation and calculation of various economic indicators using various methods of depreciation allowances. John Ryan's[70] analysis of depreciation allowances, the trends in investment, depreciation and prices shows clearly that depreciation allowances, in general, fail to provide the necessary funds for plant replacement. He examines the consequences of this failure to recover, tax-free, the equivalent purchasing power during the life of the asset, and shows that this in effect is a concealed tax on capital that can produce technological stagnation. He also discusses methods of solving this problem.

Fundamentals of depreciation accounting and methods of depreciation authorized under the tax laws in the United States are best explained briefly by Eugene Grant and W. Ireson.[71] Depreciation and Replacement Policy, edited by J. Meij,[72] brings together a variety of viewpoints on different aspects of the problem, and also attempts to combine theoretical research with an analysis of management behaviour. Since our research is primarily application of linear programming to fixed asset replacement and acquisition models, not much emphasis will be placed on the intricacies of various forms of depreciation or taxation which are complex areas in themselves. Numerous engineering economy texts have discussed tax and depreciation aspects of investments in detail.

Government's assistance to industry in the form of investment credits is analyzed and discussed by D. Corner and A. Williams.[73]

Decision Alternatives, Model and System Development

This section surveys the literature that exists in the area of possible alternative decisions the proposed models in this paper should be capable of making. Basically, these decisions fall into the area of replacements, types of acquisitions available, and other related decisions, such as salvage or surplus. The criteria for making such decisions are also investigated.

In 1960 Vernon Smith[74] discussed the problem of productive capacity and its interdependencies with the replacement problem, and recognized that a cost minimization model could provide a unification of these heretofore separate concepts. He also was one of the first to see that replacements are not "machines" but actually are "productive machine capacities." In the models developed in later chapters of this paper this concept is brought out fully, and the "typical fixed asset" is used basically as a unit measuring a certain productive capacity.

The timing of replacements has been investigated for some time by numerous authors. A. Alchian[75] studied some of the basic aspects of the problem and its formulation, and Stuart Dreyfus and Richard Bellman[76][77] provided computational methods for solution.

Frank Sinden[78] considers the operation of a certain facility providing a service for a growing population, and finds that to meet given capacity requirements the facility must expand and replace its equipment from time to time.

A number of texts [79][80][81] on engineering economy provide the fundamentals of replacement theory and discuss the type of decisions such models can make. Pierre Masse'[82] discusses the replacement decisions from a capital investment point of view, and develops a number of mathematical models based on various assumptions.

The MAPI approach of George Terborgh[83] is outlined in his "Dynamic Equipment Policy" and is based on the policy that minimizes the sum of operating imperfection, or inferiority, and capital cost. The explicit cognizance of deterioration and obsolescence is one of the important features of the models proposed. The "urgency" rating of Terborgh is a method of ranking the proposed projects, and measures the urgency of a project as compared to keeping the old equipment for one more year. The more recent text[84] of George Terborgh improves upon his earlier work and raises a number of pertinent questions in capital budgeting as applied to replacement decisions.

A number of writings on leasing indicate it to be an alternate mode of acquisition of fixed assets to that of purchasing. J. Treynor and R. Vancil[85], Fred Weston and Rupert Craig[86], R. Vancil[87] and Donald Taylor[88] shed light on this method, and indicate that it has become quite popular in recent years, specifically, in the area of computer equipment.

C. Schwingle[89] discusses the problems associated with the valuation of industrial assets. Individual equipment manufacturing companies publish catalogues that contain resale values of old equipment.

Franco Modigliani and Franz Hohn[90] analyze the problem of the planning horizon in the environment of production. S. Reiter[91] and H. Weingartner[92] have developed models for analyzing interrelated or interdependent projects, since a number of conclusions based on independence of investment alternatives could be proven wrong if such interactions are not considered.

Solution and Implementation Aspects

The general approach of this paper is to build a model of a system operable in a specific environment.

A. Hall and R. Fagen define a system as, ". . . a set of objects together with relationships between the objects and between their attributes."[93] The various objects in our system are the data components, management budgeting activities, and the numerous outputs, some of which are used as feedback to modify the inputs.

E. Johnson[94] discusses the role of an operations research worker in studying a large system. He says,

> The problem of the operations researcher, then, is to understand the system he is studying, discover the laws that govern its behaviour, construct a model describing its operations, and then manipulate the variables of the model so that the objective desired in the actual operation can be optimized.

The input-state-output concepts of systems, canonical equations, and classification of systems is discussed by Lotfi Zadeh and Charles Desoer.[95] This approach to system definition and classification has been of great assistance in the development of the fixed asset models.

Development of period-linking constraints in the models developed has been handled by flow graphs. Theory of graphs and combinational theory is discussed by S. Vajda[96] in his text titled "Mathematical Programming." Further detailed discussion of flow graphing is by L. Ford and D. Fulkerson.[97] Salah Elmagraphy[98] discusses the theory of network models in a production system environment. He shows that the graphic representation of a system assists greatly in the development of the structure, and in understanding the interaction of the components of such a system. Saul Gass[99] and George Dantzig[100] discuss the theory and various applications of linear programming. A. Charnes and W. Cooper[101] emphasize the application of linear programming to numerous practical problems. Novel variations in linear programming formulations and how they can be used to solve unique problems are explained with great insight in the two volumes of these authors.

A significant step forward was taken with R. Gomory's[102] publication of the method of solving integer problems in 1958. He relies on a method of reshaping the problem to "force" out the solution, whereas the method proposed by A. Land and A. Doig[103] is a direct and a systematic search for an optimum. Their method could also solve the mixed-integer programming solution due to the process of progressive inclusion of integers. Computational experience in using the "Branch and Bound Technique" is discussed by A. P. G. Brown, and Z. A. Lomnicki.[104] Other methods of integer solutions are discussed by A. Geoffrion[105] in his Rand Corporation publications.

Numerous writings discuss decomposition techniques. Among these, the outstanding one is that developed by G. Dantzig and P. Wolfe,[106] and discussed with respect to applications by William Baumol and Tibor Fabian.[107]

The details of implementation aspects of fixed asset replacement and acquisition systems are referred to by many authors. The implementation phase has numerous considerations, and mentioned below is a sampling of writings that touch on different aspects of this problem.

Two manuals on MAPI approach are "MAPI Replacement Manual"[108] and

"Company Procedural Manual on Equipment Analysis."[109] Realf Otbesen[110] and C. Schwingle[111] have certain comments on machine tool cost analysis, and salvage values, respectively. J. Mathews[112] discusses the administration of capital spending in a firm.

The government specification concerning the charge for use of military property for commercial purposes is ASPR (Armed Services Procurement Regulations.[113]

CHAPTER III

PRESENT DECISION-MAKING AT TASCO CONCERNING FIXED ASSET TYPE INVESTMENTS

Introduction

This chapter discusses existing fixed asset decision-making systems in a typical firm in the aerospace industry. The particular firm presented is one of the major divisions of a corporation and hereafter is referred to as TASCO (Typical Aerospace Company). The background provided here assists in the realistic development of a proposed system. This system is then investigated and optimized, with respect to the available fixed asset decision alternatives, by the application of operations research techniques.

Background of the Parent Company

The parent company of TASCO was organized a half-century ago and grew gradually until the 1950's when its diversification and growth accelerated appreciably. At that time, it started operating a government aircraft factory, later to be called TASCO, and to become one of its major divisions. It also formed another division to handle missiles and space operations. Later additions included an electronics company, a propulsion company, an air terminal facility, and an aircraft service company, as well as diverse foreign industrial interests.

Today the company is well known for its long and broad experience in science and engineering, and for its technical and management competence across a wide range of defense systems and industrial programs. The interests of the corporation include missiles, satellites, and space exploration and communication systems; military and civilian aircraft; electronics; propulsion; shipbuilding; ground support; heavy construction; air, ground, and shipboard materials handling; underseas warfare; oceanography; bionics; nuclear products and services; military base operations; maintenance and servicing; airport management; international business developments; automated systems; tracking base operations; and general industrial development and manufacture.

The nine domestic divisions of the Parent Company, including TASCO, cover the entire aerospace field and extend into such areas as ocean systems, shipbuilding, propulsion, speciality electronics, communications, command and control, and heavy construction. Its approximately 76,000 employees include 25,000 in scientific and engineering programs and supporting work. Approximately 15,000 professional research scientists and engineers in the company conduct basic and applied research in every major field of the physical sciences, and maintain an expanding research program in the life sciences, vital to space travel.

In the beginning of 1955 the company facilities covered 12.1 million square feet of floor area, devoted principally to commercial and military aircraft research, development, manufacture, and testing. As the company broadened its missile-space work, and stepped up its diversification program, floor area increased 78 per cent in ten years, to 21.5 million square feet. The company spent 223 million--almost $58 million in 1963-1964 alone--to expand and modernize buildings, fixtures, and furnishings and to acquire additional acreage for space, propellant, nuclear, maritime, and other testing. Also in the 1955-1964 period, TASCO paid nearly $127 million in rent for use of private and government-owned facilities and equipment.

Today the company has plants and bases at 43 locations, in 17 states, where large-scale projects are researched and developed. Formulations and activities relevant to long-range plans to add plants, laboratories, and test bases are presently underway.

Background of TASCO

Organizational Structure

Figure 1 illustrates the organization of the company. The four vice presidents have project responsibilities; the branch managers have functional responsibilities and report directly to the president of the company. The Property Administration and Accounting Department, under the Finance Branch, administers the procedures and is responsible for the accounting for the fixed assets.

Company Posture

The company's management capability for large military aircraft systems is one of its strongest points. The company has an unequaled reputation in the aerospace industry for producing a quality product, according to specifications, on schedule, and at a comparatively low cost, for the military service. However, since commercial aircraft volume has been low, its reputation in the commercial field is still being established.

Geopolitical considerations give TASCO competitive advantages to the extent that these considerations affect contract awards.

Although the company's relations with various Department of Defense

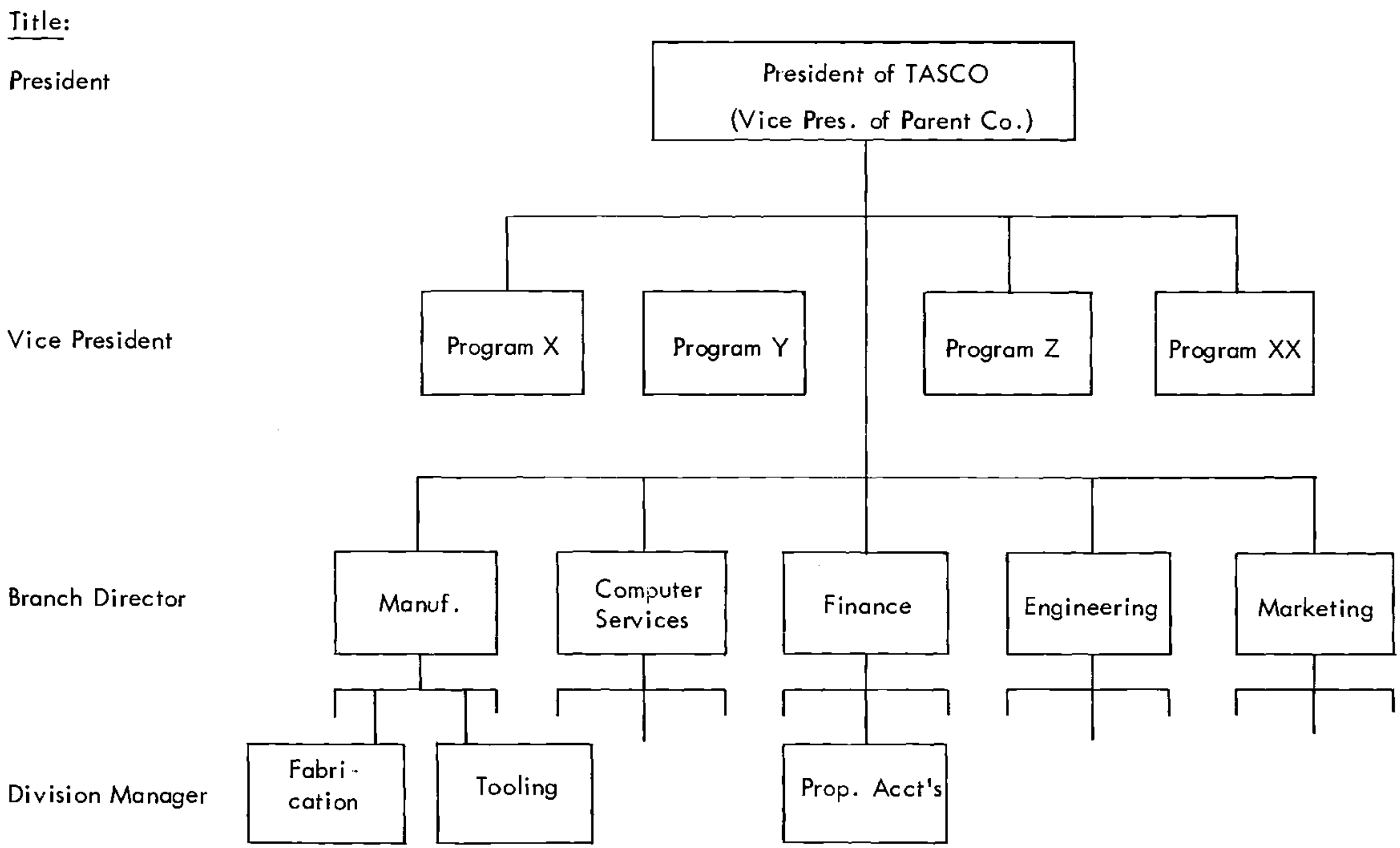


Figure 1. TASCO's Organization Chart

customers are very good, TASCO tries constantly to improve its image. The necessity for building good relations with commercial customers has also been recognized, and rapid progress is being made in this area.

Centralization and Decentralization

The parent company has decentralized TASCO in internal operations; however, through its centralized financial organization the Parent Company controls a number of key factors that have important bearing on TASCO's operations. There is a "ten year plan" that must be prepared, in compliance with the Parent Company's direction, by each of its divisions. The plan provides a statement to the Parent Company of TASCO's long-range objectives, business environment, strategies, resource requirements, and plans for achievement, and represents a major element of the master plan of the Parent Company. This plan is equally important internally, since it provides the primary basis for major planning decisions and actions of TASCO's management.

In preparation of the plan, TASCO's financial goals and marketing objectives are aligned with those of the Parent Company. Sales volume goals are derived by application of probability factors to the forecasted sales potential of a number of feasible new business prospects.

The Parent Company approves the sales goals of TASCO, and the related new business ventures that will provide such sales potential, or has them modified appropriately. The Parent Company has virtually complete control over all financial aspects of TASCO, through the use of budgets, and through centralized capital sources. The yearly fixed asset budget allows purchase of necessary fixed assets to meet contractual obligations of

TASCO. The total fixed asset budget is established by the Parent Company, but TASCO has a large measure of control over specific acquisition decisions.

It should be noted that, for all practical purposes, the Parent Company acts as the source of capital for TASCO, and through this fact, as well as through the management hierarchy, is able to exercise strong control over TASCO.

Relations With Government

The competitive advantage of a company that has government facilities is a strong incentive for aerospace companies to try to persuade the government to furnish these facilities. With government facilities, lower costs can be quoted, (no depreciation included), less need for profit on a contract becomes necessary, (less investment), and the contractor can have adequate capacity with no need to obtain depreciation charges or to obtain a return on investment when the facilities are not being utilized. The reduction in profit shown by the contractor is not adequate to adjust for this advantage.

In 1956, the DOD (Department of Defense) restated the limitation on the furnishing of government facilities:

> . . . . the provision of Government-owned industrial facilities will be authorized only when it can clearly be demonstrated that private enterprise is unable, unwilling or not organized to perform the service or provide the products necessary to meet current and mobilization requirements . . .

The DOD's policy, expressed numerous times since 1956, basically has been for the government to completely withdraw from the facilities-furnishing activity. Since 1955, the government has adhered to this policy with very few exceptions, and government facilities sold to contractors and new government purchases have consequently been small in relation to total inventory.

A study by the LMI (Logististics Management Institute)[114] attempts to provide quantitative criteria for motivating the defense contractors to acquire the needed equipment on their own, as less and less government facilities are furnished to them. The model that is developed in the following pages thus needs to show the schedule of replacements for facilities from TASCO's funds, assuming that the government is unwilling to provide assistance. Of course, the model also needs to take into account those facilities forecasted to be furnished by the government.

Planning and Budgeting

The planning for fixed asset requirements is handled differently from other functional plans, since the responsibilities do not fit within a single organization. This joint effort involves the planning, finance, and utilizing organizations. Long-range fixed asset plans are prepared and coordinated by the Property Administration and Accounting Department of the Finance Branch. As will become apparent from Figure 2, decisions regarding fixed assets are made in committee action by TASCO's top management.

The basic objectives of the fixed asset plans the company prepares yearly (ten year plan) are to insure the constant availability, suitability, and adequacy of the physical plant and equipment to meet operating needs, and to establish a competitive capability for the pursuit of new business.

Definitions

Fixed assets at TASCO are defined as property or buildings, and equipment. To be considered a fixed asset, equipment must meet the following criteria:

1. It must cost more than $500.
2. Its expected useful life is of two years or more.
3. It is economically controllable.
4. It is not directly chargeable to the performance of a contract.

Fixed assets are acquired for the following reasons:

1. As replacements
2. To increase capacity
3. To improve methods
4. For research and development
5. Because of new program commitments.

The fixed assets are obtained with TASCO or United States Air Force funds. Fixed assets obtained from the Air Force are covered by facilities contracts with TASCO. The basic modes of acquisition, purchase, or lease are discussed in detail later. Another mode of capacity acquisition, subcontracting, is also briefly dealt with in this study.

Fixed assets include any one of several accounts in TASCO's books:

1. Land
2. Building and building fixtures
3. Machinery and equipment machine tools
4. Other additions to government property
5. Other automotive and material handling costs.

It should be noted that fixed asset type equipment, when purchased, becomes part of the "fixed assets" of the company, and can be used to serve the same purpose as leased or government acquired items. It is important to understand the difference between project type equipment and fixed asset type equipment. The former is charged to a certain specific program, whereas the latter cannot be directly charged since it consists of multi-project type items with a life normally in excess of a year.

Budget Cycles

Figure 2 illustrates the information flow associated with decisions on fixed assets. Essentially, three major cycles exist. These are the purchase budget cycle, the government facilities cycle, and the lease budget cycle.

Purchase Budget Cycle. As can be seen from Figure 2, the cycle starts with TASCO's finance branch forwarding a premise report to the Parent Company. The figures in this report are established by judgment, past experience, future plans, and expansion goals. Specifically, the fixed assets portion of the premise report contains the following:

1. New major program requirements, broken down into major fixed asset accounts, such as land, buildings, machinery and equipment, and other, for the next ten years.

2. All other requirements. A total figure for the next ten years.

3. Estimate of yearly cash expenditures.

4. Estimated depreciation.

(The numbers in the following discussions refer to parts of Figure 2.) The premise report is conservative in that its emphasis is on near-future firm contracts. This report is sent to the Parent Company and after its approval (1) is returned to Finance. Then Finance allocates portions

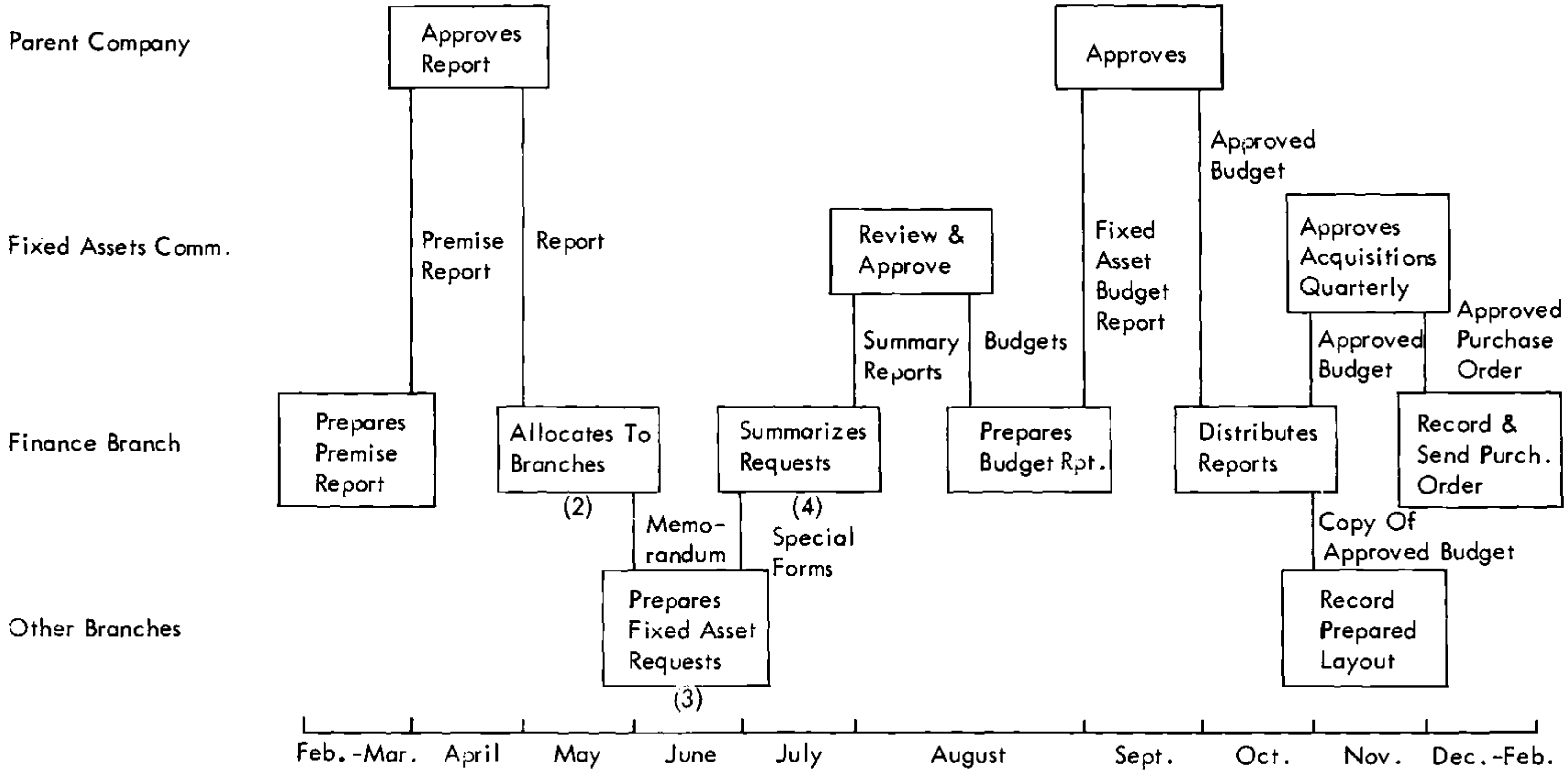


Figure 2. Purchase Budget Cycle

of this budget to various branches (2) and asks for fixed asset requests. By the end of June, these estimates are collected (3) on special justification forms and are summarized by Finance (4) for presentation to the Fixed Assets Committee, made up of branch managers and the President of TASCO.

The sum total requested by the branches usually is in excess of what was originally reflected in the premise report. After considerable negotiation, a final budget value for the following year is decided upon (5). Normally, this value is between the original premise budget total and the total of budgets requested by the various branches. The process of arriving at a final total usually involves each branch's listing of its requests according to priority of need. Selections are then made from the list of priorities until a budget value based on agreement of the committee members is reached.

The finance branch summarizes the final conclusions in a given format known as the "fixed asset budget report" (6), and forwards it to the Parent Company for approval (7). The fixed asset budget report consists of the fixed asset budget for the coming year and the following nine year fixed asset forecasts. The first section of the report includes items such as grand summary by account and major project, detailed breakdown by each major branch, listing and justification of budget items over $5,000, cash expenditures, and a schedule of proposed leases. The second section consists of a ten-year fixed asset forecast that includes a grand summary by account and major project, a listing and justification of budget items, and a schedule of proposed leases.

After approval by the parent organization (7), the budget is forwarded to the finance branch from which a copy is sent to each of the seven other branches (8). In the meantime, the Fixed Assets Committee approves (9) acquisition of budgeted items by quarters. Items approved for purchase are then purchased, and the required location cards and transactions are maintained in files. The cycle then begins again, with the preparation of premise reports to be sent to the Parent Company.

The Parent Company approves the total yearly budget requests rather than each item above $5,000. Approximately 45 per cent of the expenditures for fixed assets occur in the same year as the year the budget applies, and 55 per cent of the expenditures occur in the following year. This is due to lead time needed in ordering assets.

Government Facilities Cycle

Since a majority of TASCO's contracts are military, based on previous established regulations, it is possible to obtain funds from the DOD to purchase fixed assets, called "facilities" by the government. These fixed assets are used in fulfilling military contracts and records are kept indicating their use, as well as their utilization level. If at any time such fixed assets are to be used on nonmilitary business, then approval from DOD is requested and a certain amount of rent is paid. As previously noted, the number of fixed assets purchases by the government has been decreasing steadily over the last several years.

The first phase of the budget cycle for government facilities is connected with establishing a six-year forecast, for the purpose of aiding the particular agencies in planning. Then detailed justifications for each item are forwarded, and those items that are approved have their purchase funds totaled in a pool and kept by the DOD agency for use by TASCO.

Of course, the significant effect of this is to reduce the requirements for such funds from TASCO's own budgets. Recently, however, more and more such funds are coming out of TASCO, both because of DOD cutbacks and because of TASCO's plans for increasing its commercial business.

Lease Budget Cycle

The following policy statement by the Parent Company determines the guidelines applicable to lease agreement entered by TASCO:

Approval from the Parent Company shall be required prior to entering into a binding commitment initially, and at each renewal point of a proposed lease that qualifies under any of the conditions below:

1. A lease that involves total rental, including renewal options, in excess of $25,000; or,

2. which involves a total period in excess of five years; or,

3. under which the contracted payments, including renewal options, are equal to or in excess of the purchase price of the asset being leased.

Since a lease payment falls under the classification of overhead expense, the overhead budget department, under the finance director, must determine what effect such a lease has on the overhead budget of TASCO.

CHAPTER IV

EXTERNAL AND INTERNAL FACTORS IN PROPOSED SYSTEM

Introduction

The previous chapter discusses the background of TASCO and its relation to the Parent Company and illustrates the planning and budgeting activities as they are presently performed. This chapter discusses certain key factors that are basic to the formulation of the system models in the next chapter.

The fixed asset system analyzed is part of a number of other systems that make up the operational framework for TASCO. Each of these systems is influenced by a number of external and internal factors. Some of the external factors, of course, are the other systems with which it must interact.

The fixed asset system in operation in TASCO, as are the company's other systems, is affected by various factors. The external factors are the external conditions, parameters or assumptions upon which control cannot be exercised by the system. The internal factors are those with respect to which the proposed system will have control and will be able to optimize a certain criterion.

The external factors that affect the fixed assets decision system are best explained by reference to Figure 3.

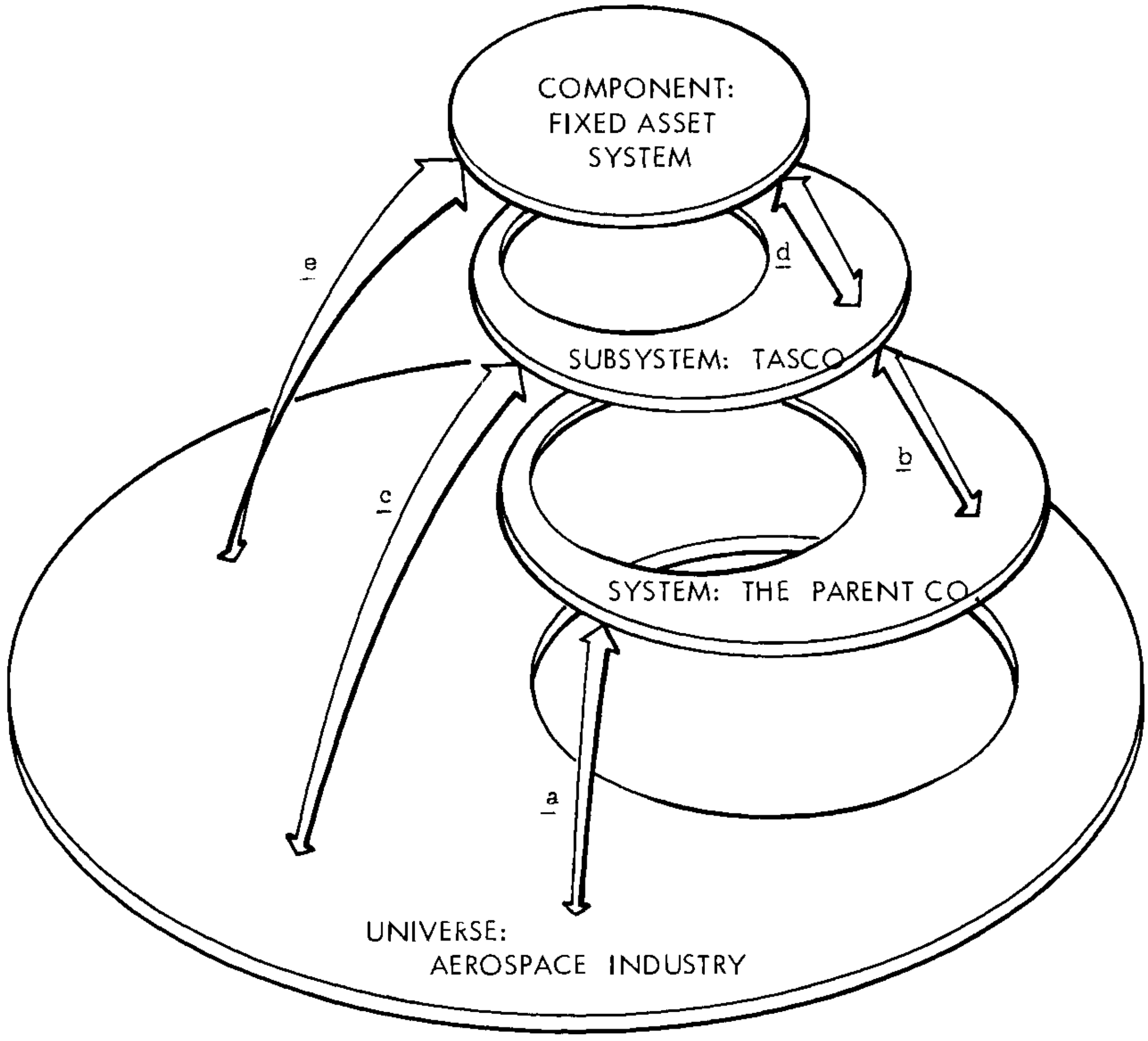


Figure 3. Fixed Asset System Interactions

External Factors

Figure 3,a indicates the external effects on the Parent Company, imposed by the national economy and by other firms in the aerospace industry. Competition, the military and cold-war conditions, and financial creditors all exert varying forms of external pressures.

The relation between TASCO and the Parent Company that is the extent of TASCO's decentralization is reflected by b. Imposition of yearly fixed asset budgets and coordination of ten-year detailed plans are certain forms of control by the Parent Company that are pertinent to TASCO's fixed asset decisions. Details of centralization and decentralization and of budget cycles and plans of TASCO have already been discussed in Chapter III.

The interaction between other aerospace industries and TASCO is indicated by c. Competition in the market for particular types of aircraft designs in which TASCO specializes is exemplified by c. This factor also is taken into account in determining the long-range plans of TASCO.

Other TASCO systems that have a bearing on the fixed asset system are shown as d. These are the engineering, manufacturing, and other branches of TASCO which, through competition for fixed asset budgets, exert pressure on each other, and on TASCO's top management. Other TASCO systems that influence fixed assets are the manpower, load leveling and smoothing, and production planning systems. Of course, the ten year long-range plans also have a direct effect on the fixed asset system.

External factors such as technological improvement in fixed assets and various costs are indicated in e. Assets furnished to TASCO by the government are also an important external factor.

Some of the external factors to be further discussed are aircraft programs and deliveries, contracts, functional requirements, technological improvement, and costs.

Aircraft Programs and Deliveries

The factor b indicates the imposition of certain goals, such as sales volume, on TASCO by the Parent Company. Factor c indicates the effect of competition on TASCO in the particular aircraft market in which it specializes. These factors are taken into account in preparing the ten-year plans of TASCO.

As indicated in Chapter III, the long-range plans of TASCO set down certain premises that indicate the course of the company for the next ten years. It is understood that the plans for the initial years are quite firm since they are based on contractual delivery of aircraft. However, new business estimates become more prevalent in the latter years of the plan, as present contracts run out. Research and development efforts of the company provide the state-of-the-art knowledge of the design and manufacturing capabilities of future aircraft. Based on the most recent plans it is possible to select a "most likely" mix of programs and quantity of aircraft deliveries for each program. (See Table 1.)

Table 1. Aircraft Programs and Quantities - Premises for Long-Range Planning (5 Years)

	1968	1969	1970	1971	1972
A/C Program (Firm Contract)	A	A	A	A	-
Quantity	20	20	15	5	-
A/C Program (Firm Planned)		B	B	B	B
Quantity		2	7	12	15
A/C Program (Planned)				C	C
Quantity				1	4

Cost Sharing, and Progress Payments

Another major external influence on TASCO, indicated by c, is the type of contractual agreements undertaken with its military and commercial customers. The type of contractual arrangements influences the cash flows associated with fixed asset decisions as a result of the "cost sharing" and "progress payments" provisions written into these contracts.

Armed Services Procurement Regulations[115] specify the types of contracts that may be used in military business. The three basic types of military contracts in which TASCO has been involved are the "firm fixed price," the "cost-plus-a-fixed-fee," and the "fixed-price incentive." The firm-fixed-price contracts do not allow any adjustments to occur in the price of a product once the contract is signed. In the cost-plus-a-fixed-fee contract (CPFF), the government and TASCO estimate the total cost of a project and establish the fixed fees which will be paid the firm. If the costs go above the original estimate, the government assumes the costs. In the fixed-price incentive contracts a target cost, a target profit, a ceiling price, and a profit/cost sharing formula are negotiated. If the final cost is less than the target cost, TASCO's profit increases by a certain ratio, as, for example, 70-30. If the costs are greater than target costs, then again the government shares part of the increased costs. Commercial contracts of TASCO normally fall into the firm fixed-price category. At any point of time it is possible to determine an average percentage of a cost dollar that will be recovered, and some forecast of the fluctuation of this figure can be made for TASCO.

The government, through progress payments, provides a major part of the working capital needs of an aerospace firm. The payments to TASCO are made weekly, and represent a specified percentage of the costs incurred in manufacturing the aircraft. A similar situation exists with respect to commercial contracts, in the form of advances and deposits. Contrary to military contracts however, these payments do not depend on the costs incurred.

Both cost sharing, and progress payments have significant influence on fixed asset acquisitions. The former provides a method of recovering part of the cost of fixed assets, and the latter provides TASCO an opportunity to earn interest on the depreciation portion of progress payments without having to wait for the completion of the contract. Both these concepts will be further discussed in Chapter VIII.

Functional Requirements

Manpower requirements in each branch are established by load analysis personnel, who determine the impact of aircraft programs, delivery rates, and schedules, future planned programs, and related factors. In the fabrication division of the manufacturing branch, where the operations are cyclical, mechanized systems show the standard hour* load as a function of delivery rate, which is weighted by a performance factor in order to take into account the "learning effect" on various programs. Other divisions in manufacturing, and the engineering branch, estimate the manpower requirements based on forecasting of loads, budgets, and a number of intangible factors. Fixed asset requirements are geared to these estimates,

*"Standard hours" for an operation may be defined as the time an employee should spend to finish a job. This employee is assumed to have a certain skill level, exerts normal effort, is familiar with the job, uses tools, material, and facilities planned, and follows a prescribed shop method.

and are affected by existing capacity, age and performance of the old assets, uniqueness of the needed assets; possibility of acquiring additional capacity on existing assets through a second or third shift operation, or subcontracting the work, and estimates of subjective priorities placed on individual items of equipment. Such needs are of course, influencing factors upon the budget planning cycles.

Technological Improvement

Technological improvement of any fixed asset is an ever present factor, and can, of course, cause to become obsolescent equipment already purchased. Deterioration of assets due to wear also depends on the state of technology that existed at the time the asset was manufactured. However, normally, it also is a function of the level of utilization, and the type and level of maintenance action carried out. The maintenance activity at TASCO is assumed to be a factor over which the fixed asset system does not have control, and therefore will be considered an external factor. If a relatively constant level of utilization is also assumed, then the deterioration of an asset can be associated with its age, and its acquisition date.

Deterioration is broken down by George Terborgh into two components: first, it is the amount by which the operating cost of the machine in service exceeds the cost obtainable from the same unit new; second, it is the amount by which the value of its service is below that obtainable from the same unit new.

Obsolescence, similarly, consists of two components: the amount of which the operating cost of a new replica of the machine exceeds the cost of the best alternative currently available; and the amount by which the value of the service rendered by this replica is below the value of the service of the best alternative.

Deterioration and obsolescence could also be reflected in terms of the output rate of the machine or equipment. Machine hours per year for a lathe, or ton-miles per year for a truck, can reflect the effects of deterioration and obsolescence, and what is more, could be used to determine the quality of the particular type of asset that will be needed to meet functional requirements for that year. Since requirements are established as a function of long-range plans of the company, a means of measuring the need for assets could be quantitatively established. By defining the output capability of a "typical" asset in appropriate units, the number of assets needed can be found in terms of typical units. These typical units can be chosen as the most likely items to replace the ones currently in use, and, in their capacity figures, they would also reflect the technological improvements or deterioration effects over the years.

The choice of a typical asset must be based on knowledge and experience of characteristics of the particular fixed asset group. Empiric functions, or tables prepared by analysts in the field, could be of assistance. Chapter VIII contains further discussion and a numerical example of this concept.

Costs

The costs related to the fixed asset system are numerous, and are by nature considered external factors. Various types of acquisition costs, operating and maintenance costs, salvage revenues and discount rates all fall into this category. Further cost fluctuations can be estimated, and should be considered in a good quantitative model.

Internal Factors

The internal factors that affect the fixed asset system are those that, within limits, the system is capable of controlling and making decisions about. Functional groups, alternative modes of acquisition of capacity and criteria for decision making fall into this category and will be discussed below.

Functional Groups

In order to be able to introduce sound quantitative criteria for evaluating fixed asset decisions, the idea of functional groups is introduced.

The functional requirements, or needs for fixed assets, of each aircraft system in the process of being built, or that is to be built, can be established by processes similar to the ones that establish manpower requirements. By using appropriate units, such functional requirements could be stated in terms of standard or actual lathe hours per year or cubic feet of chem-milling tank facility per year, or ton-miles of truck or fork lift capacity needed per year, etc. . . .

Based on these functional requirements, it is possible to establish distinct, and, for all practical purposes, mutually exclusive, functional groups that include all fixed assets performing the same function. Actual determination of what really constitutes a homogeneous group will be based on the combined judgment of numerous people who are involved with the use, loading, budgeting, and procurement of these assets. For our purposes, we could define a functional group as, "a set of fixed assets characterized by the fact that they perform the same function, and that are homogeneous to a degree defined by the combined judgment of personnel who have an interest in that fixed asset."

Alternate Capacity Acquisition Modes

Ability to formulate functional groups and to associate attributes to these groups allows decisions to be made with respect to these groups, and to the "typical" assets that are represented by these groups. The types of acquisition modes reflect some of the decisions that could be made to obtain additional capacity. As Vernon Smith[116] indicates, the interest we have is in acquiring a certain capacity to meet some functional requirements imposed on the system by long-range goals. Alternative modes of acquisition of this capacity allow most feasible and least costly (higher profit) acquisition modes to be selected.

The modes coincide with the decisions the system is capable of making, and involve purchasing, replacement with trade-in, or subcontracting. It can be seen that acquisition of capacity involves decisions of the type thus far usually made by replacement type analysis, (buy, keep, in addition to rental, or subcontracting). Straight purchase for increased capacity (no replacement involved) is feasible, and yet at times the need for meeting increased functional requirements could be met through replacement with assets of higher productivity.

Acquisition of capacity through subcontracting is a short term procurement of production assistance from outside sources (non-TASCO). Usually, the cost of this mode is double or triple the in-plant cost and is used only during peak load periods.

Extra shift and overtime are also possible ways of providing extra capacity, but will not be considered as decision-modes within the model. They will be considered as parameters of the model and their effects could be determined through a sensitivity analysis. TASCO management plans an extra shift and allows overtime only under exceptional circumstances. Therefore, normal operating conditions will be assumed in this study.

Leasing is another form of acquisition of capacity, and is a practicable alternative to ownership by the company. Whether to buy or lease is determined by financial, rather than operating conditions.

The opposite of acquisition of capacity occurs as a result of disposition of assets. The disposition of assets could occur for reasons other than load considerations. The salvage of assets in the used machine tool market could occur due to purely economical reasons. It may be more economical to salvage assets in the market than to obtain a trade value on the old assets from the manufacturer.

Criterion for Fixed Asset Decisions

As we have previously discussed, the long-range (ten year) plans of TASCO include a number of firm contracts for the first several years, and have estimates of the most likely mix of sales for the future years. The plans are formulated based on the quantitative goals that the Parent Company has set for TASCO, (see Chapter III for details) and include a breakdown of the amount and cost of resources needed to meet sales commitments and expansion goals. Capital, manpower, and fixed asset requirements are calculated by reference to past experience with similar aircraft systems, with modifications brought in whenever the new designs make it necessary. Within this framework, the expected cash flows and profits, and numerous operating and financial indices, are determined. Alternative major aircraft programs are also analyzed, and most likely ones are further investigated.

Upon an indication of the most likely course of events, various branches, and organizations within these branches, plan the details of a number of activities that will allow the realization of these established future plans.

This study assumes that the functional requirements, or needs for various types of machinery and equipment, can be determined as a function of aircraft sales forecasts upon which long-range plans are based. The need is expressed in some convenient units of measure and becomes a basis for which capacity must be acquired. The lowest discounted cash flow, or the minimum cost that will allow meeting the fixed asset requirements by taking advantage of a number of alternative modes of capacity acquisitions and replacements (subject to budgetary constraints), spread over a planning horizon, is the criterion adopted.

Such a criterion allows replacement decisions to be combined with other modes of acquiring capacity, and also allows the long-range plans, and the impact of changes in these plans, to be determined quite effectively.

Of course, a cost minimization criterion like the one stated can be accurate only if it can be assumed that these decisions, made from an absolute cost point of view, do not interact with profitability considerations. This assumption can be justified since the model developed assumes that the first phase of planning, that of choosing from alternate programs, has already been completed, and the time has come to determine the answers to a number of specific questions regarding fixed assets.

The use of cost minimization based on the absolute costs of a number of alternative modes allows a number of decisions, heretofore considered separately, to be made in one framework. Replacement analysis, in the literature, normally is concerned with the timing aspects, where a uniform need for the asset is assumed through its life. Capital budgeting and investment analysis choose from a number of investment alternatives and optimizes a criterion. Lease versus buy studies follow similar paths. A combination of a number of fixed asset decisions in a system framework using cost minimization criterion is a novel approach.

CHAPTER V

CLASSIFICATION AND DEVELOPMENT OF FIXED ASSET MODELS

Classification of Models

Prior to developing the fixed asset models that are capable of explaining the decision-making process of TASCO, and to providing optimization algorithms for the models, a certain classification is necessary. Classification is needed because the numerous characteristics of the problem demand an orderly development of the final model that will be of most use to the decision-maker. Therefore, a classification scheme that will gradually become more complex, but more realistic, is proposed. The next chapter will provide the formulations of the models for solution purposes, using notation already provided. Such an orderly classification will also indicate the areas where additional work needs to be done.

The fixed asset models are sequentially described in seven classifications that are distinct but not necessarily mutually exclusive. The table below indicates the models, which increase in complexity as one goes down the table; it also indicates the order in which the models will be developed in the following pages.

Various combinations of the following eight characteristics have been created in order to produce the classes of models that will be developed. The characteristics are described below:

1. System Level: This denotes the range of application of the

Table 2. Classification of Fixed Asset Models

MODEL	(1) SYSTEM LEVEL			(2) DECISION		(3) TIMING		(4) PLANNING HORIZON		(5) FINANCIAL CONSID.		(6) BUDGETS		(7) TERMIN.V.		(8) CENTRALIZE	
	CO.	BCH.	GP.	R	A	W	W/O	SY	MY	W	W/O	W	W/O	W	W/O	W	W/O
1			X	X			X	X			X		X		X		X
2			X	X			X		X		X		X		X		X
3			X	X	X		X		X		X		X		X		X
4		X		X	X		X		X		X	X			X		X
5	X			X	X		X		X		X	X			X	X	X
6	X			X	X	X			X		X	X		X		X	X
7	X			X	X	X			X	X		X		X		X	X

particular model. GP indicates that the model only applies to one functional group; if the groups of fixed assets in a branch are combined, then the model applies to a branch; and if several branches are combined it will constitute the entire TASCO model.

2. Decision: This characteristic indicates whether the analysis is basically for replacement, or for acquisition mode selection. The replacement models basically involve replace and keep variables, whereas the replacement and acquisition models, when combined, involve whether to replace, keep, buy without replacement, salvage, rent or subcontract, and take into account lease decision variables.

3. Timing: This indicates whether lead-times are involved between the budgeting and the actual acquisition of the asset.

4. Planning Horizon: This indicates a single year or a multi-year model.

5. Financial Considerations: This indicates the financial considerations, if TASCO is assumed to be an autonomous company.

6. Budgets: This indicates whether fixed asset models include budgets.

7. Terminal Values: These values are used to take into account the finite nature of the planning horizon.

8. Centralization: This refers to the possibility of pooling machinery and equipment in certain branches rather than having identical items appear in several branches.

Model Development

The Decision System

Following are a number of models that incorporate the phenomena associated with the fixed assets at TASCO. The models are basically sequential in nature in that the decisions to buy, subcontract, lease, etc. have effects on the decisions to be made the following year. Figure 4 illustrates the general sequential model.

The inputs $X(1)$, the decisions $A(1)$ and the initial state $Y(0)$ combine to create the output $Y(1)$, and the cost $C(1)$ of the first stage. For the second and following stages, the output of the preceding stage, in combination with the new inputs, and space of decisions for the particular stage form the new output, and cost of that stage. The X vector gives all the information about inputs to the particular stage and consists of existing old TASCO and government assets, functional requirements, and other constraint values that change yearly. The following notation will be used in describing the generalized equations:

$X(n)$: The input vector in year n; $X(n)$ is a subset of input space, $S(X)$.

$A(n)$: The decision matrix of replacement and acquisition variables, ordered according to the age of the items, in year n; $A(n)$ is a subset of decision space, A.

$Y(n)$: The output or solution vector, also called the state vector since it gives a description of the system at the end of stage n, $Y(n)$ is a subset of $A(n)$.

$C(n)$: The cost associated with the solution $Y(n)$, is a subset of $S(C)$.

In symbolic form we have:

First year, $n = 1$

$$Y(1) = f_1(X(1),Y(0);A(1)) \tag*{(5-1) (a)}$$

$$C(1) = g_1(Y(1) = G(X(1),Y(0);A(1)) \tag*{(b)}$$

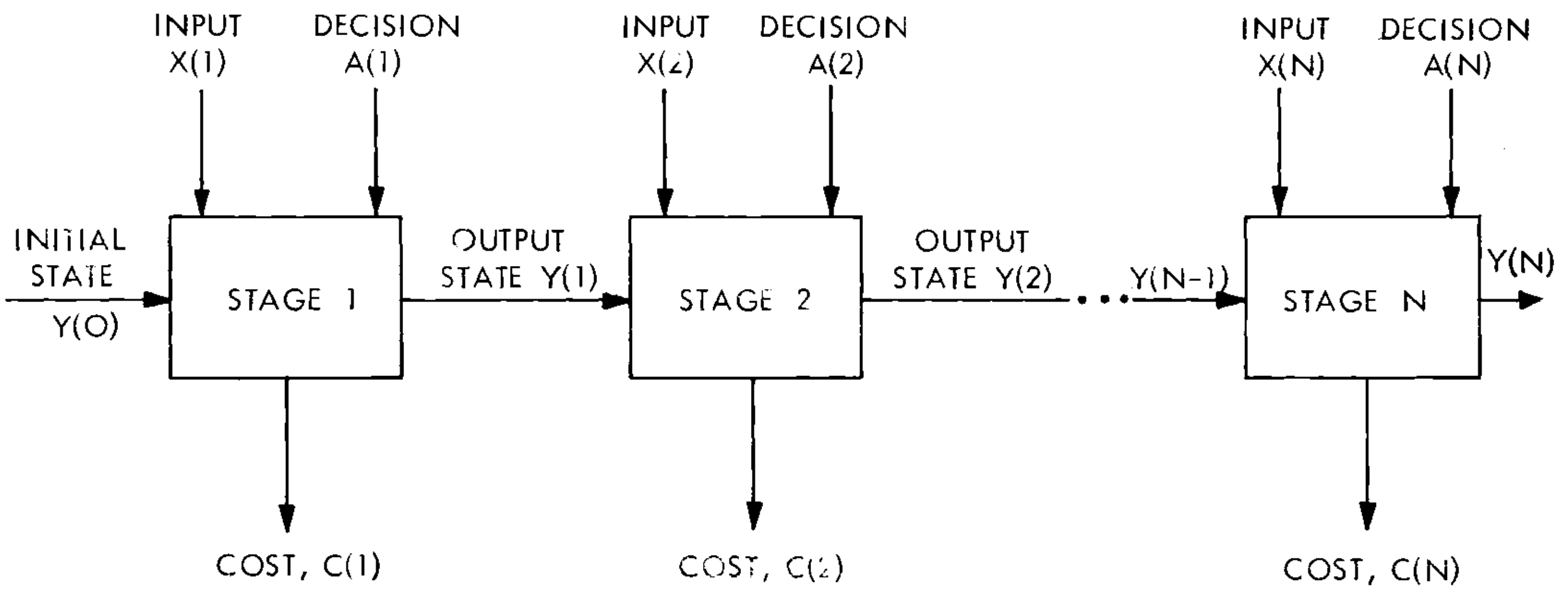


Figure 4. The Decision System

Second year, n = 2

$$Y(2) = f_2(X(2),Y(1);A(2)) \tag{5-2 a}$$

$$C(2) = g_2(X(2),Y(1);A(2)) \tag{b}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

Nth year, n = N

$$Y(n) = f_N(X(N),Y(N-1);A(N)) \tag{5-3 a}$$

$$C(N) = g_N(X(N),Y(N-1);A(N)). \tag{b}$$

Following sections will discuss further details of the models and will introduce additional notation.

General Description of the Model

The sequential model described in Figure 4 represents the real-world situation as it exists at TASCO. The initial state consists of the fixed assets in existence, at time 0, contributing toward meeting some load or functional requirement. New requirements placed on the fixed assets during the year 1 results in certain decisions regarding acquisition of capacity. Such decisions, upon implementation, change the original state to what is called the output state. The next year the same process is repeated.

The decision space S(A) consists of all possible decisions that may be made with respect to fixed assets, and includes, basically, replacement and acquisition decisions. Salvage or surplus decisions are assumed to be part of the acquisition decisions, since they are "negative" acquisitions.

The inputs describe the external factors on the model and consist of functional requirements, budgets, and a number of cost and capacity parameters. The input space S(X) consists of all such external factors that influence the decisions.

The cost space is a set of scalar functions that associate a value with each decision in the decision space. A particular value depends on the particular decisions that make up the output state.

The problem associated with the present system is that decisions are influenced by short-term priority considerations, and do not take into account the long-term effects on costs. The first year budgets are the only ones considered in fixed asset purchases, even though future budgets could significantly affect this year's decisions. This concept is brought out clearly by J. H. Lorie and L. J. Savage,[117] who show the effect on decisions when the sum total of capital expenditures is constrained or limited in more than one time period. H. M. Weingartner[118] also analyzes multi-year budgeting and its effects on capital investment decisions, using mathematical programming techniques. These authors illustrate the importance of the need to consider multi-year budgets in investment problems.

The sequential model described above needs to be optimized so that an ordered set of decisions, or a policy, can be selected that will minimize the sum of cost functions for each stage. It is obvious that, due to interactions between stages, the problem cannot be solved stage by stage.

This optimization can be expressed as:

Find:

$$C = \min_{\{Y\}} \sum_{n=1}^{N} g_n(X(n), A(n), Y(n-1)) \qquad \begin{matrix}(5\text{-}4)\\(a)\end{matrix}$$

Subject to:

$$h_1(X(1),A(1),Y(0)) \gtreqless h^*(1) \qquad \text{(b)}$$

$$h_2(X(2),A(2_1Y(1)) \lesseqgtr h^*(2)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$h_N(X(N),A(N),Y(N-1) \gtreqless h^*(N)$$

where $h^*(n)$, for $n = 1, 2, \ldots, N$, is a vector of constraints.

The following sections describe the various elements of the system, and the seven classes of models.

The Inputs, X(n)

The inputs to the basic model are functional requirements, FR(n); capabilities of the assets to meet functional requirements, CP(n); budgets, BT(n); and costs, CT(n). The notation established in this section will be in general terms since further subscripts will be added under each model developed later.

$$X(n) = (FR(n),CP(n),BT(n),CT(n);n=1,2,3,\ldots,N). \qquad (5\text{-}5)$$

Functional Requirements, FR(n):

As was explained in detail in Chapter III and IV the long range plans of TASCO set down certain premises that indicate the course of the company for the next ten years. It is understood that the plans for the initial years are quite firm since they are based on contractual deliveries. However, new business estimates become more prevalent for the following years as present contracts run out. In terms of actual aircraft programs, as well as the quantity of deliveries projected, these new businesses are probable estimates. Based on the most recent plans, it is possible to select a "most likely" mix of programs and quantity of aircraft deliveries for each program.

The functional requirements of fixed assets of each aerospace system being built, or that is to be built in the future, is established by using appropriate units. Functional requirements may be stated as standard lathe hours, or square feet of office space, or cubic feet of chem-milling tank facility, etc. Based on these functional requirements, it is possible to establish distinct, and for all practical purposes, mutually exclusive "functional groups" that include all fixed assets performing the same function. Actual determination of what really constitutes a homogeneous group will be based on combined judgment of numerous people who are involved with the use, loading, budgeting, and procurement of these assets. Definition of appropriate functional groups, and determination of units of measurement of their function, as well as the load or requirements to be placed on them for the duration of the planning horizon, is the first step in the implementation of the proposed model.

Capability, CP(n)

"Capability" refers to the ability of a unit of fixed asset to satisfy the functional requirements imposed on it. Such capability is herein expressed in the same units as are the functional requirements, e.g. standard hours for lathes, square feet of office space, cubic feet of chem-milling facility.

The capability of most assets is subject to obsolescence and deterioration; therefore such time and age dependent aspects needs to be forecasted and input to the model.

In our model the functional obsolescence and deterioration is a slightly different concept in that it is expressed in terms of some easily measurable unit, such as, for example, standard hours for machine tools. This allows determination of acquisitions and replacements to meet yearly functional requirements. As the item ages, due to longer and more frequent down-times, the capacity, in hours, of the item decreases. However each year, due to technological improvement, newer machines can provide a higher number of hours, or capacity, within a given period of, for instance, one year. The basis of comparison is the new typical asset capability, in hours, in the first year of the study. The "typical asset" must be one chosen based on knowledge and experience of characteristics of the particular fixed asset group. The values for obsolescence and deterioration may be generated by some empiric functions, or tables could be prepared by analysts in the field. An example below should make this clear. (Table 3.)

Table 3. An Example Illustrating Functional Obsolescence and Deterioration of a Machine (Standard Hours)

Age, t	1	2	3
0	CP(i,1,0)* = 1000	CP(i,2,0) = 1100	CP(i,3,0) = 1200
1	-	CP(i,2,1) = 900	CP(i,3,1) = 900
2	-	-	CP(i,3,2) = 700

*CP(i,n,t): Capacity, or capability, in appropriate units, of fixed asset i, at beginning of period n, of age t.

In Table 3 the increase from an initial 1000 hours to 1100 hours and 1200 hours represents technological improvement in the capacity of the fixed asset. This could also be stated as being the obsolescence of the asset purchased in year one, with an output of 1000 hours in that year. The diagonal row of 1000, 900, and 700 represents the deterioration of the fixed asset that was new in year one.

Budgets, BT(n)

The primary budget of concern is the Fixed Asset Purchase Budget, imposed on TASCO by the Parent Company. About 50 per cent of the fixed assets are budgeted and bought within the same year. Certain machinery and equipment due to long lead times involved in their procurement cannot be budgeted and procured within the same year. This aspect will need to be taken into consideration during the development of the model.

Costs, CT(n)

Basic cost parameters are:

p: purchase or replacement costs

s: salvage values

r: rental costs

sc: subcontract costs

l: lease payments

om: operating and maintenance costs

Purchase Costs, Salvage Values. Purchase costs of fixed assets normally increase from year to year, and also vary among vendors. Since the study does not cover the selection of vendors, it is assumed that the vendor has already been chosen in each case. Table 4 illustrates the notation to describe such price changes. Table 4 also includes replacement costs of assets of different ages purchased in different years.

Table 4. Purchase and Replacement Costs

Age	1	2	3	
0	p(i,1,0)*	p(i,2,0)	p(i,3,0)	---
1	-	p(i,2,1)	p(i,3,1)	---
2	-	-	p(i,3,2)	---

*p(i,n,t): Purchase or replacement cost in dollars of fixed asset i, in year n, where t is the age of the old asset. If t = 0, then it is a purchase without replacement; if t ≠ 0, then it is a replacement cost reflecting the difference between the cost of the new asset and the value of the old one traded in.

A similar table below illustrates the notation used for salvage values.

Table 5. Salvage Values of Old Fixed Assets

		Year of Study		
Age	1	2	3	
0	-	-	-	---
1	-	s(i,2,1)*	s(i,3,1)	---
2	-	-	s(i,3,2)	---

*s(i,n,t): Salvage cost of items i, in year n, of age t.

It should be noted that in both Table 4 and Table 5 the age of the asset is given as 0, 1, or 2, and older assets are not illustrated. This is because the notation is similar; for example, the cost of replacement of a fixed asset of 12 years of age, being replaced in the second year of the study, would be shown as p(i,2,12); the salvage value of a 15 year old asset in the third year study would appear as s(i,3,15).

Rental, Lease, and Subcontract Costs. Rental of fixed assets is assumed to occur on a year by year basis. Each time a rental is made the asset will be in new condition and will have all the latest technological improvements incorporated into it. The installation cost, if incurred, is part of the rental cost, and is assumed to recur yearly.

Subcontracting the work outside of TASCO is one method of acquiring additional capacity. Rental and subcontracting are treated as mutually exclusive methods in the models built. TASCO's Computing Branch normally employs the rental mode, whereas Manufacturing Branch normally uses subcontracting. A table describing subcontract variables would look similar to Table 6 (Rental Costs).

Lease costs are similar to rental, in that they are paid yearly, but once a lease contract is signed the payments become a contractual obligation until the expiration of the contract. Table 6 and Table 7, respectively, show the notation for rentals and leases.

Table 6. Rental Costs

Year of Study 1	2	3	
r(i,1)	r(i,2)	r(i,3)	---

*r(i,n): Rental cost of item i, in beginning of year n.

Table 7. Lease Costs

Year of Study 1	2	3
l(i,1,3)*	l(i,2,3)	l(i,3,3)*

*l(i,n,t): Lease cost associated with item i, in year n, with last payment occurring in beginning of year t.

Operating and Maintenance Costs. These costs vary with the technological improvement and age of the assets, as discussed below.

Let om(i,n,t): operating and maintenance costs in dollars, for fixed asset i, in year n. It should be noted that t is the age of the asset at the beginning of year n. If t = 0, then the asset is new and this cost is incurred in the first year of the life of the asset.

The operating and maintenance cost structure follows the pattern shown in Tables 3 and 4.

Initial State, Y(0)

The initial state refers to the TASCO and government owned fixed assets that provide an initial capability to meet functional requirements. Since these items may have been acquired at different times in the past, and may also be of different capacities, the initial state consists of a wide spectrum of assets of different ages and sizes.

The notation appears as follows:

$$Y(0) = (TE(1,1), TE(1,2), \ldots, TE(1,\alpha); GE(1,1), GE(1,2), \ldots, GE(1,\beta))$$

where

$$TE(1,1), TE(1,2), \ldots, TE(1,\alpha)$$

are the existing TASCO owned fixed assets of ages 1, 2, ..., α years,

$$GE(1,1), GE(1,2), \ldots, GE(1,\beta)$$

existing government owned fixed assets of ages 1, 2, ..., β years.

<u>Decision Space, A(n)</u>

The basic decision variables are:

TB(n,t) = Number of TASCO items to buy in beginning of year n, by replacing same number of units of age t, t = α, α - 1, ..., 0.

TK(n,t) = Number of TASCO items to keep in year n, of age t years at beginning of year n,

TS(n,t) = Number of TASCO items to be salvaged at beginning of year n, that are t years old,

TR(n) = Number of items to be rented in new condition at beginning of year n,

TSC(n) = Number of equivalent asset capacities to be subcontracted in year n,

TL(n,t) = Number of items leased in year n, of age t at the beginning of year n,

GN(n) = Number of government items that will be received at beginning of year n,

GS(n,t) = Number of government assets to be salvaged, that are t years old.

t = Age of the asset, and varies from α to 0 for TASCO assets, and β to 0 for government assets.

The decision matrix appears as follows:

$$A = \begin{bmatrix} TB \\ TK \\ TS \\ TR \\ TSC \\ TL \\ GN \\ GK \\ GS \end{bmatrix} \tag{5-6}$$

It should be noted that output state is a particular subset of A.

Cost, C(n)

The criterion to be optimized is the cost over the planning horizon and consists of the product of optimal decisions for each stage or year, A(n), and the costs discussed above. The cost values will be adjusted to reflect after-tax costs, and will include discounting. Each cost criterion will be discussed in detail under model in the following section.

The Models

The description of the sequential model in the preceding pages illustrates the framework within which decisions are presently made. The model presented is necessarily a simplified illustration of reality since a number of factors, such as lags between budgeting and acquisition, and decentralization, have not been shown. Prior to these factors being brought into the model, it is necessary to further describe some assumptions around which the following models will be developed.

The building blocks of the models proposed are "fixed asset groups."* The groups are considered as an assemblage of capacity, existing at a point of time, (the start of the planning horizon). The items within a group are considered to be equivalent to each other, all having been acquired at a given point of time in the past; therefore, all have the same age. Since decisions differ with respect to government assets, (e.g. TASCO cannot "sell" or "trade-in" a government asset) the items in one group are segregated, the TASCO owned separated from the government owned. This segregation allows the capacity of these assets to be combined to meet functional loads; however, each subgroup is made up of equivalent items, and could have different equivalent ages. Therefore, we may speak of the "Saddle-type Turret Lathe" fixed asset group and mean a number of TASCO turret lathes, and a number

*See page 50 for a definition of a fixed asset group.

of government turret lathes each having its own equivalent age. This group can consist of 12 government equivalent lathes, of age 9 years; and 14 TASCO equivalent lathes, of age 5 years. If a decision to replace an existing, old TASCO asset is made, then the output refers to one or more of the 14 equivalent lathes, each 5 years old. Obviously, a more detailed analysis, most probably based on experience of the shop personnel is needed to determine exactly which one of the 14 would need to be replaced.

The following pages contain the seven models shown in Table 2. The notation developed in the earlier parts of this chapter will be used and additional notation will be introduced where needed. The models progress from the simple ones to more generalized and complex ones.

This chapter represents, symbolically, the various models and does not attempt to illustrate the solution or optimization aspects. The next chapter discusses the optimization of the sequential process, the individual stages of which are discussed here.

Model 1

Model 1 is a one stage decision model. The initial input, Y(0) consists of available TASCO and government assets; the inputs consist of functional requirements, FR(1); and cost parameters related to each decision, CT(1); the decision, A(1) consists of a number of replacement decisions; and the criteria function is the overall cost to be minimized, C(1).

Symbolically, the relationship may be shown as:

$$Y(1) = f_1(X(1),Y(0);A(1)) \tag{5-7 (a)}$$

$$C(1) = g_1(X(1),Y(0);A(1)) \tag{b}$$

MODEL 1

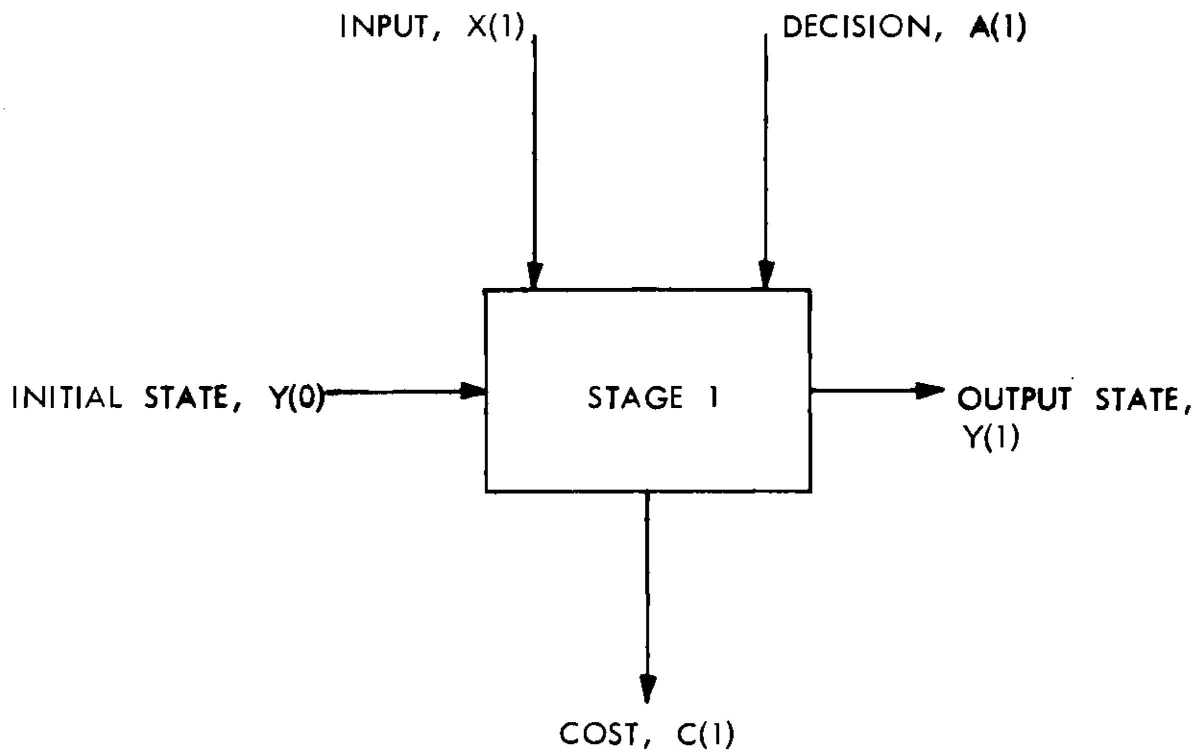


Figure 5. Model 1 Schematic

or

$$C(1) = g_1(Y(1)) \qquad (c)$$

Decision Variables A(1)*

$$A(1) = \begin{array}{c} \text{Age, t:} \\ \begin{bmatrix} TB(1) \\ TK(1) \\ TS(1) \\ GN(1) \\ GK(1) \\ GS(1) \end{bmatrix} \end{array} = \begin{array}{ccc} \alpha & \beta & 0 \\ \multicolumn{3}{c}{\begin{bmatrix} TB(1,\alpha) & 0 & TB(1,0) \\ TK(1,\alpha) & 0 & 0 \\ TS(1,\alpha) & 0 & 0 \\ 0 & 0 & GN(1,0) \\ 0 & GK(1,\beta) & 0 \\ 0 & GS(1,\beta) & 0 \end{bmatrix}} \end{array} \qquad (5\text{-}8)$$

The only independent variable is t, time, and the matrix of decision variables is formed according to age, decreasing from left to right.

Inputs, X(1). The inputs consist of a number of parameters. They are functional requirements, FR(1); capabilities, CP(1); cost values CT(1); and new government assets to be acquired.

$$X(1) = (FR(1),CP(1),CT(1),GN(1)) \qquad (5\text{-}9)$$

The functional requirements are an upper limit that the capacity of all replaced equipment needs to meet, and is symbolized as:

*α and β hereafter refer to the equivalent, average age of the TASCO and government assets in a group.

FR(1): functional requirements, in appropriate units during the first year.

Capability, CP(1) is a matrix similar to A(1), and is shown below:

$$CP(1) = \begin{bmatrix} CPTB(1) \\ CPTK(1) \\ CPTS(1) \\ CPGN(1) \\ CPGK(1) \\ CPGS(1) \end{bmatrix} = \begin{bmatrix} CPTB(1,\alpha) & 0 & CPTB(1,0) \\ CPTK(1,\alpha) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & CPGN(1,0) \\ 0 & CPGK(1,\beta) & 0 \\ 0 & CPGS(1,\beta) & 0 \end{bmatrix} \quad (5\text{-}10)$$

The cost matrix CT(1) associated with the decisions bears a resemblance to both the A(1), and CP(1) matrices, and is shown below:

$$CT(1) = \begin{bmatrix} CTTB(1) \\ CTTK(1) \\ CTTS(1) \\ CTGN(1) \\ CTGK(1) \\ CTGS(1) \end{bmatrix} = \begin{bmatrix} CTTB(1,\alpha) & 0 & CTTB(1,0) \\ CTTK(1,\alpha) & 0 & 0 \\ CTTS(1,\alpha) & 0 & 0 \\ 0 & 0 & CTGN(1,0) \\ 0 & CTGK(1,\beta) & 0 \\ 0 & CTGS(1,\beta) & 0 \end{bmatrix} \quad (5\text{-}11)$$

The details of calculation of values in CT(1) are shown in Appendix A. GN(1) indicates a specific number of new government assets to be acquired by TASCO.

Initial State, Y(0)

$$Y(0) = (TE(1,\alpha), GE(1,\beta))$$

Output State, Y(1). This is a subset of the A(1) matrix shown above, and represents a feasible solution, or the solution to the optimization problem if optimization has been undertaken. If a second stage existed, then this would be essentially the input state to it.

The Model Equation. Cost,

$$MC(1) = \sum_r \sum_s c_{r,s}(N) \tag{5-12 (a)}$$

where

$$c_{r,s}(1) = CT(1)A(1)' ,$$

r and s are the row and column numbers, respectively, of the elements of the matrix, CT(1)A(1)'.

Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(1) \geq FR(1) \tag{b}$$

where

$$cpa_{r,s}(1) = CP(1)A(1)' .$$

Government Acquisitions:

$$GN(1) = GN(1)* \tag{c}$$

where * indicates a numerical value.

Initial Inputs:

$$TB(1,\alpha)+TS(1,\alpha) = TE(1,\alpha) \quad \text{(See Appendix A)} \tag{d}$$

$$GK(1,\beta)+GS(1,\beta) = GE(1,\beta) \quad \text{(See Appendix A)}$$

Model 2

This model incorporates a planning horizon of more than one year. All other characteristics of this model are similar to those of Model 1. Figure 4 illustrates the decision system for this model.

Decision Matrix, $A(n)$; $n = 1, 2, 3, \ldots, N$. $A(1)$ is same as the one shown in Model 1. $A(2)$ is illustrated below:

$$\begin{array}{ccccccc} & \text{Age:} & & \underline{\alpha+1} & \underline{\beta+1} & \underline{0} & \underline{1} \end{array} \tag{5-13}$$

$$A(2) = \begin{bmatrix} TB(2) \\ TK(2) \\ TS(2) \\ GN(2) \\ GK(2) \\ GS(2) \end{bmatrix} = \begin{bmatrix} TB(2,\alpha+1) & 0 & TB(2,0) & TB(2,1) \\ TK(2,\alpha+1) & 0 & 0 & TK(2,1) \\ TS(2,\alpha+1) & 0 & 0 & TS(2,1) \\ 0 & 0 & GN(2,0) & 0 \\ 0 & GK(2,\beta+1) & 0 & GK(2,1) \\ 0 & GS(2,\beta+1) & 0 & GS(2,\beta+1) \end{bmatrix}$$

. . . .

Inputs, $X(1), X(2), \ldots, X(N)$

$$X(1) = (FR(1),\ CP(1),\ CT(1),\ GN(1)) \tag{5-14 (a)}$$

$$X(2) = (FR(2), CP(2), CT(2), GN(2)) \quad \text{(b)}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$X(N) = (FR(N), CP(N), CT(N), GN(N)) \quad \text{(c)}$$

The capacity and cost inputs for the Nth stage are illustrated on pages 79 and 80.

The elements of CT(N) are computed as shown in Appendix A. GN(N) indicates the number of government assets to be acquired by TASCO.

Output, Y(N). The output represents a feasible solution to the Nth stage. If optimization has been done then it would represent the set of optimal values to the space of decision variables represented by A(N).

The Model Equations. Cost for Nth stage,

$$C(N) = \sum_r \sum_s c_{r,s}(N) \quad \text{(5-18) (a)}$$

where

$$c_{r,s}(N) = CT(N)A(N)'$$

and where r,s are the row and column numbers of elements of the resulting matrix CT(N)A(N)'.

Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(N) \geq FR(N) \quad \text{(b)}$$

where

$$A(N) = \begin{array}{c} \underline{\text{Age:}} \\ \begin{bmatrix} TB(N) \\ TK(N) \\ TS(N) \\ GN(N) \\ GK(N) \\ GS(N) \end{bmatrix} \end{array} = \begin{array}{ccccccc} \underline{\alpha+N-1} & \underline{\beta+N-1} & \underline{0} & \underline{1} & \underline{2} & \underline{\cdots} & \underline{N-1} \\ \left[\begin{matrix} TB(N,\alpha+N-1) \\ TK(N,\alpha+N-1) \\ TS(N,\alpha+N-1) \\ 0 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ GK(N,\beta+N-1) \\ GS(N,\beta+N-1) \end{matrix} & \begin{matrix} TB(N,0) \\ 0 \\ 0 \\ GN(N,0) \\ 0 \\ 0 \end{matrix} & \begin{matrix} TB(N,1) \\ TK(N,1) \\ TS(N,1) \\ 0 \\ GK(N,1) \\ GS(N,1) \end{matrix} & \begin{matrix} TB(N,2) \\ TK(N,2) \\ TS(N,2) \\ 0 \\ GK(N,2) \\ GS(N,2) \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} & \left.\begin{matrix} TB(N,N-1) \\ TK(N,N-1) \\ TS(N,N-1) \\ 0 \\ GK(N,N-1) \\ GS(N,N-1) \end{matrix}\right] \end{array} \tag{5-15}$$

(5-16)

$$
\begin{array}{rcc}
 & \underline{\text{Age:}} & \begin{array}{ccccccc} \underline{\alpha+N-1} & \underline{\beta+N-1} & \underline{0} & \underline{1} & \underline{2} & \underline{\ldots} & \underline{N-1} \end{array} \\
CP(N) = & \begin{bmatrix} CPTB(N) \\ CPTS(N) \\ CPTS(N) \\ CPGN(N) \\ CPGK(N) \\ CPGS(N) \end{bmatrix} = &
\begin{bmatrix}
CPTB(N,\alpha+N-1) & 0 & CPTB(N,0) & CPTB(N,1) & CPTB(N,2) & \ldots & CPTB(N,N-1) \\
CPTK(N,\alpha+N-1) & 0 & 0 & CPTK(N,1) & CPTK(N,2) & \ldots & CPTK(N,N-1) \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & CPGN(N,0) & 0 & 0 & \ldots & 0 \\
0 & CPGK(N,\beta+N-1) & 0 & CPGK(N,1) & CPGK(N,2) & \ldots & CPGK(N,N-1) \\
0 & 0 & 0 & CPGS(N,1) & CPGS(N,2) & \ldots & CPGS(N,N-1)
\end{bmatrix}
\end{array}
$$

The generalized cost matrix is shown below: (5-17)

$$
CT(N) = \begin{bmatrix} CTTB(N) \\ CTTK(N) \\ CTTS(N) \\ CTGN(N) \\ CTGK(N) \\ CTGS(N) \end{bmatrix} =
\begin{bmatrix}
CTTB(N,\alpha+N-1) & 0 & CTTB(N,0) & CTTB(N,1) & CTTB(N,2) & \ldots & CTTB(N,N-1) \\
CTTK(N,\alpha+N-1) & 0 & 0 & CTTK(N,1) & CTTK(N,2) & \ldots & CTTK(N,N-1) \\
CTTS(N,\alpha+N-1) & 0 & 0 & CTTS(N,1) & CTTS(N,2) & \ldots & CTTS(N,N-1) \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
0 & CTGK(N,\beta+N-1) & CTGK(N,0) & CTGK(N,1) & CTGK(N,2) & \ldots & 0 \\
0 & CTGS(N,\beta+N-1) & 0 & 0 & 0 & \ldots & 0
\end{bmatrix}
$$

$$\text{cpa}_{r,s}(N) = CP(N)A(N)' .$$

Government Acquisitions:

$$GN(N) = GN(N)^* \qquad (c)$$

where * indicates a numerical value.

Consistency Requirements: (See Appendix A)

$$TB(N-1,0)+TB(N-1,\alpha+N-2) = TB(N,1)+TK(N,1)+TS(N,1) \qquad (d)$$

$$TK(N-1,1) = TB(N,1)+TK(N,2)+TS(N,2)$$

$$TK(n-1,2)+TB(N,3)+TK(N,3)+TS(N,3)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$TK(N-1,N-2) = TB(N,N-1)+TK(N,N-1)+TS(N,N-1)$$

$$TK(N-1,\alpha+N-2) = TB(N,\alpha+N-1)+TK(N,\alpha+N-1)+TS(N,\alpha+N-1)$$

$$GK(N,\beta+N-1)+GS(N,\beta+N-1) = GK(N-1,\beta+N-2)$$

$$GK(N,N-1)+GS(N,N-1) = GK(N-1,N-2)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$GK(N,2)+GS(N,2) = GK(N-1,1)$$

$$GK(N,1)+GS(N,1) = GN(N-1,0)$$

Model 3

This model introduces rental and lease as new acquisition models. The decision matrix is expanded to take into account the additional variables. Subcontracting is a method of satisfying functional requirements that is becoming quite prevalent; however, it is basically similar to the rental mode of acquiring capability, and therefore would be treated in the same way in the model expounded below.

Rental of an asset is assumed to be made on a year by year basis, any installation or set-up costs therefore are repeated each year. Leasing is employed to free working capital, and the type of leasing considered is the "full payout" type, obligating the company to pay the full cost of the lease over the contract term.

Only those parts of Model 2 that are changed by the two acquisition modes are modelled, since the two models are similar in all other areas.

The following notation is introduced:

TR(n): Number of equivalent units of the asset to be rented/subcontracted at the beginning of nth year.

CR(n,o): Cost of renting and installing a unit of the asset at beginning of nth year, or subcontracting an equivalent amount of work,

TL(n,t): Number of equivalent units of the asset leased in year n, of age t,

CTL(n): Cost of leasing in year n of an asset of age t; it is assumed that length of lease period is same in all cases, and yearly lease payments are equal.

Capacities of rental or subcontract arrangements are assumed to be the same as new assets purchased at beginning of each year. Lease capacities change similar to purchased equipment that is kept. CPR(n) and CPL(n) refer, respectively, to capacities of rental/subcontract and capacities of leased assets.

Decision Matrix,

$$A(n); n = 1,2,3,\ldots,N$$

The Nth stage decision space is illustrated below:

(5-19)

$$\begin{array}{cccccccccc}
 & \underline{\text{Age}} & & \underline{\alpha+N-1} & \underline{\beta+N-1} & \underline{0} & \underline{1} & \underline{2} & \underline{\ldots} & \underline{N-1} \\
A(N) = & \begin{bmatrix} TB(N) \\ TK(N) \\ TS(N) \\ TR(N) \\ TL(N) \\ GN(N) \\ GK(N) \\ GS(N) \end{bmatrix} & = & \multicolumn{7}{l}{\left[\begin{array}{ccccccc} \multicolumn{7}{l}{\text{Same as Model 2}} \\ \multicolumn{7}{l}{\text{" " " "}} \\ \multicolumn{7}{l}{\text{" " " "}} \\ 0 & 0 & TR(N,0) & 0 & 0 & 0 & 0 \\ 0 & 0 & TL(N,0) & 0 & 0 & 0 & 0 \\ \multicolumn{7}{l}{\text{Same as Model 2}} \\ \multicolumn{7}{l}{\text{" " " "}} \\ \multicolumn{7}{l}{\text{" " " "}} \end{array}\right]}
\end{array}$$

<u>Inputs, X(1), X(2), ---, X(N)</u>.

(5-20)

$$X(1) = (FR(1),\ldots,TL(1)^*) \quad \text{(a)}$$

$$X(2) = (FR(2),\ldots,TL(2)^*)$$

$$\cdots\cdots\cdots\cdots$$

$$X(N) = (FR(N),\ldots,TL(N)^*) \quad \text{(c)}$$

GN(N)*, and TL(N)* are the specific quantity of government assets to be acquired and the number of leases outstanding at beginning of period N.

$$TL(N)^* = (TL(N,1)^*, TL(N,2)^*,\ldots,TL(N,N-1)^*) \quad (5\text{-}21)$$

It is assumed that there are no outstanding leases at the beginning of Period 1; by suitable modification of the TL(1)* vector, lease agreements made prior to period n = 1 could also be introduced.

Output, Y(N). This is similar to Model 2 except for the inclusion of rental and lease modes of acquisition.

Model Equations. Cost for Nth stage,

$$MC(N) = \sum_r \sum_s c_{r,s}(N) \tag{5-22 (a)}$$

where

$$c_{r,s}(N) = CT(N)A(N)'$$

Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(N) \quad FR(N), \tag{b}$$

where

$$cpa_{r,s}(N) = CP(N)A(N)'$$

Government Acquisitions: Same as in Model 2

Consistency Requirements: Same as in Model 2 except that the following equations reflecting lease arrangements are added. (See Figure 6)

$$TL(N,1) = TL(N-1,0) \tag{c}$$

$$TL(N,2) = TL(N-1,1) = TL(N-2,0)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$TL(N,d-1) = TL(N-1,d-2) = TL(N-2,d-3) = \ldots = TL(N-d+1,0)$$

In Figure 6 let length of lease contract be d years, and N indicate present year.

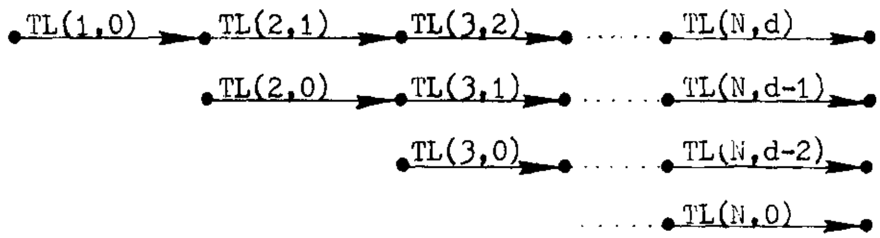


Figure 6. Network Flow for Leases

Model 4

This model introduces two significant changes over and above the previous ones. Budgetary constraints are introduced, primarily for yearly fixed asset expenditures; and since these budgets apply to branches of TASCO, the model is expanded to include the fixed assets in a branch.

Model 3 essentially includes all the needed information to make decisions on a group by group basis. Budgets, however, have the effect of connecting the groups, since a given overall branch budget has to be optimally allocated among the groups. Figure 7 clarifies some of the changes.

The groups within the Branch are designated by the letter i where $i = 1, 2, 3, \ldots, I$.

e.g. $X(i,1)$ indicates input to group i in year 1.

$Y(i,2)$ indicates output of group in year 2.

$TB(i,n,t)$ indicates group i assets of age t, purchased or replaced by TASCO in year n.

Decision Matrix, $A(n)$; $n = 1, 2, 3, \ldots N$. The Nth stage decision space is illustrated below:

$$A(n) = \begin{bmatrix} A(1,n) \\ A(2,n) \\ \ldots \\ A(I,n) \end{bmatrix} \quad \text{where } A(i,n) = \begin{bmatrix} TB(i,n) \\ TK(i,n) \\ TS(i,n) \\ TR(i,n) \\ TL(i,n) \\ GN(i,n) \\ GK(i,n) \\ GS(i,n) \end{bmatrix} = \begin{bmatrix} \text{Same} \\ \text{as} \\ \text{in} \\ \text{Model 3} \end{bmatrix} \tag{5-23}$$

Inputs, $X(1),X(2),X(3),\ldots,X(N)$.

(5-24)

$$X(1) = (X(1,1),X(2,1),\ldots,X(I,1);BT(1)) \tag{a}$$

$$X(2) = (X(1,2),X(2,2),\ldots,X(I,2);BT(2)) \tag{b}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$X(N) = (X(1,N),X(2,N),\ldots,X(I,N);BT(N)) \tag{c}$$

where $X(i,n)$ is the set of inputs associated with year or stage n, and group i. $BT(n)$ is the budget applied to the fixed assets of the branch in year n.

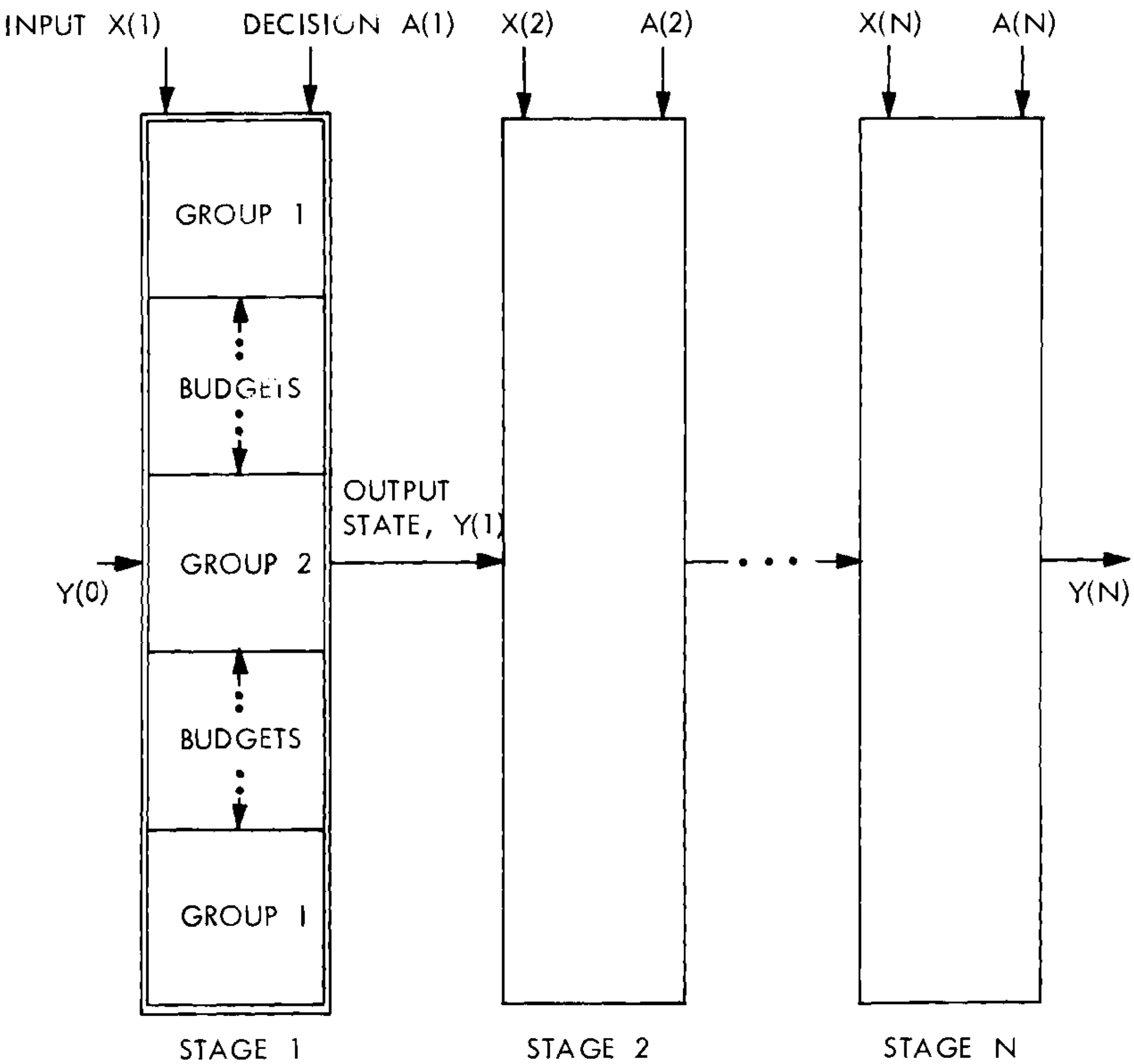


Figure 7. Model 4 Schematic

$$X(i,n) = (FR(i,n), CP(i,n), CT(i,n), GN(i,n)^*, TL(i,n)^*) \tag{5-25}$$

Model Equations. Cost for the Nth stage,

(5-26)

$$MC(N) = \sum_i \sum_r \sum_s c_{r,s}(i,N) \qquad \text{(a)}$$

where i is the group number (i = 1, ...I), and

$$c_{r,s}(i,N) = CT(i,N) \cdot A(i,N)'$$

Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(1,N) \geq FR(1,N) \qquad \text{(b)}$$

$$\sum_r \sum_s cpa_{r,s}(2,N) \geq FR(2,N)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$\sum_r \sum_s cpa_{r,s}(I,N) \geq FR(I,N)$$

Government acquisitions are the same as in Model 2 for each group.

Consistency requirements are the same as in Model 2 for each group, except the following are added to each group: i = 1, 2, 3, ..., I

$$TL(i,N,1) = TL(i,N-1,0) \qquad \text{(c)}$$

$$TL(i,N,2) = TL(i,N-1,1) = TL(i,N-2,0)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$TL(i,N,d-1) = TL(i,N-1,d-2) = TL(i,N-2,d-3) = \ldots = TL(i,N-d+1,0)$$

Budgetary Constraints:

$$\sum_i \sum_r \sum_s b_{r,s}(I,N) \leq BT(N) \qquad \text{(d)}$$

where

$$b_{r,s}(i,N) = B(i,N)A(i,N)'$$

and $B(i,N)$ is a matrix, similar to (5-16) and (5-17), that has as its elements the purchase or replacement costs, $p(i,N,t)$.

Model 5

The changes in this model are concerned with further expansion of the model to include more than one branch of TASCO; and to consider the possibility of more than one group, physically quite distinct from each other, performing the same function. The first change is handled in a way similar to Model 4, by generating additional subscripts without changing the fundamental pattern of the equations for a group. The second change affects the structure more significantly, and is discussed in detail below.

It should be noted that one of the prime reasons for treating the branches as separate entities having distinct group numbers (even though some of the groups may be physically the same) is due to their organizational independence, which TASCO management wishes to preserve. A centralized fixed asset system numbering all groups in TASCO from 1 to I might have a lower overall cost to TASCO, but it could result in conflicts with other TASCO policies. The lower cost would be due to the possible salvaging of some assets by one branch while another branch might be purchasing some of

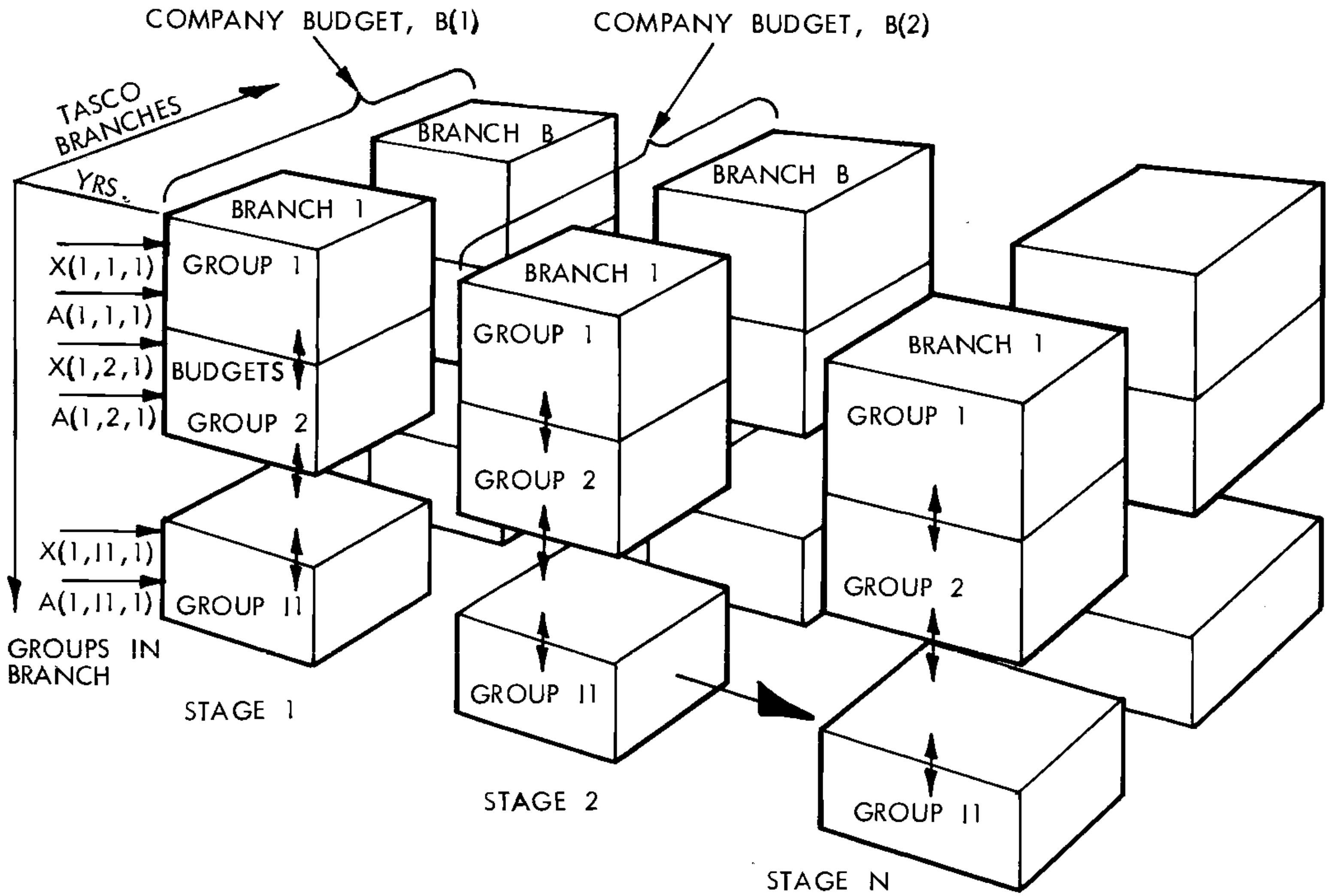


Figure 8. Decision Making at TASCO Level

the same assets. This, of course, could be effectively reconciled during the implementation phase of the approach. The models being built have the inherent capability to perform this type of an integrated approach, but we will pursue the independent branch concept.

The budgets in this model are treated as being applied to the whole company rather than to a single branch.

Model 5 is the same as Model 4 except that a subscript for branches must be introduced.

e.g. A(i,n,b) indicates the decision matrix associated with the fixed asset group i of branch b. Similar notation could be followed for all other input parameters.

The model equations are similar to the equations of Model 4 except that the groups are summed over all the branches instead of any one particular branch.

If two groups are quite different physically, or in some other characteristic, yet perform the same function, then the following modification could be made of the two or more groups.

Model Equations; nth Stage. If groups d_1, and d_2 functionally differ, then, d_1:

Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(d_1,n) \geq FR(d_1,n) \tag{5-27 (a)}$$

d_2: Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(d_2,n) \geq FR(d_2,n) \tag{b}$$

If groups d_1 and d_2 are distinct but perform the same function, then,

$$\sum_r \sum_s (cpa_{r,s}(d_1,n)+cpa_{r,s}(d_2,n)) \geqslant FR(d_1+d_2,n) \qquad (c)$$

Model 6

This model improves the realism of decisions regarding individual group models by incorporating terminal values and a time delay between the budgeting and actual procurement of the asset. Terminal values are discussed first.

Terminal Values. Since the model considers a finite horizon, terminal values of the assets at the end of the planning horizon will need to be taken into account. Terminal values apply only to TASCO assets at the end of the planning horizon. If N is the number of years in the planning horizon there will be a set of decisions to be made at the beginning of the period N (already discussed in previous models), and a set of salvage decisions to be made at the end of the Nth period. The following development is made for a typical fixed asset group outlined in Model 2 since the following models do not affect the basic concept discussed.

Let,

$TT(N,t)$ be the number of equivalent units to be salvaged at the end of the period N, of age t at beginning of period N,

and

$s(N,t)$ be the salvage value of a unit to be salvaged at the end of period N, of age t at beginning of period N.

Decision Matrix, $A(N)$. Figure 9 illustrates the decisions associated

(5-28)

$$
\begin{array}{r}
\text{Age:}\quad \underline{\alpha+N-1}\quad \underline{\beta+N-1}\quad \underline{0}\quad \underline{1}\quad \underline{2}\quad \underline{\cdots}\quad \underline{N-1}\quad \underline{N}\quad \underline{\alpha+N}
\end{array}
$$

$$
A(N) = \begin{bmatrix} TB(N) \\ TK(N) \\ TS(N) \\ TT(N) \\ GN(N) \\ GK(N) \\ GS(N) \end{bmatrix} = \begin{bmatrix}
 & \cdots & \multicolumn{3}{c}{\text{Same as in Model 2}} & \cdots & 0 & 0 & 0 \\
 & \cdots & \multicolumn{3}{c}{\text{" " " " "}} & \cdots & 0 & 0 & 0 \\
 & \cdots & \multicolumn{3}{c}{\text{" " " " "}} & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & TT(N,1) & TT(N,2) & \cdots & TT(N,N-1) & TT(N,N) & TT(N,\alpha+N) \\
 & \cdots & \multicolumn{3}{c}{\text{Same as in Model 2}} & \cdots & 0 & 0 & 0 \\
 & \cdots & \multicolumn{3}{c}{\text{" " " " "}} & \cdots & 0 & 0 & 0 \\
 & \cdots & \multicolumn{3}{c}{\text{" " " " "}} & \cdots & 0 & 0 & 0
\end{bmatrix}
$$

with the last stage, Year N, and includes terminal decisions.

Inputs, X(N).

$$X(N) = (FR(N), CP(N), CT(N), GN(N)^*)$$

The CP(N), and CT(N) matrices of Model 2 need to be similarly expanded to include the corresponding rows and columns due to TT(N). Since no capacity is introduced by TT(N) there is the following change to CP(N).

$$CP(N) = \begin{bmatrix} CPTB(N) \\ CPTK(N) \\ CPTS(N) \\ CPTT(N) \\ CPGN(N) \\ CPGK(N) \\ CPGS(N) \end{bmatrix} = \begin{array}{c} \begin{array}{cccccccc} \underline{\alpha+N-1} & \underline{\beta+N-1} & \underline{0} & \underline{1} & \underline{\ldots} & \underline{N-1} & \underline{N} & \underline{\alpha+N} \end{array} \\ \left[\begin{array}{cccccccc} & & & & & & & 0 \\ & & & \text{Same as Model 2} & & & & 0 \\ & & & & & & & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 0 \\ & & & & & & & 0 \\ & & & & & & & 0 \end{array}\right] \end{array} \tag{5-29}$$

The model equations are similar in form to Model 2, except for the expanded decision matrix, and therefore, the corresponding capacity and cost matrices need to be modified accordingly. The changes in the consistency requirements entail the addition of several equations to take care of the end points. These are best understood by referring to the network flow diagram of Figure 9.

$$TT(N+1,1) = TB(n,0)+TB(N,1)+TB(N,2)+\ldots+TB(N,N-1)+TB(N,\alpha+N-1) \tag{5-30}$$

TT(N+1,2) = TK(N,1)

TT(N+1,3) = TK(N,2)

TT(N+1,4) = TK(N,3)

. . . .

TT(N+1,N) = TK(N,N-1)

TT(N+1, +N) = TK(N, +N-1)

Time Delay. The second modification brought about by Model 6 is the time lag between the budgeting and the procurement of the asset. This situation arises due to the long lead-times involved in the procurement of certain machinery and equipment. The actual procurement date is important since the asset starts meeting the functional requirements as of that date rather than the prior date of budgeting assumed to be approximately equal to the installation date. Most TASCO assets are budgeted and procured within the same year; however, exceptions arise in certain areas, notably in buildings and highly complex machinery such as numerically controlled mills. Buildings may be budgeted and progress payments made over several years only at the end of which they begin to meet their functional requirement. Since we only consider machinery and equipment in this analysis such aspects will not be considered; however, they could be incorporated into the model without much difficulty if desired.

Table 10 illustrates the basic concept discussed.

The dashes (-) in the diagonal blocks of Table 10 indicate that the budgeting and acquisition have been done in the same year. $TB_j(n,t)$ indicates the number of items to be acquired in year n, by replacement of units of age t, this acquisition having been budgeted in year j. The

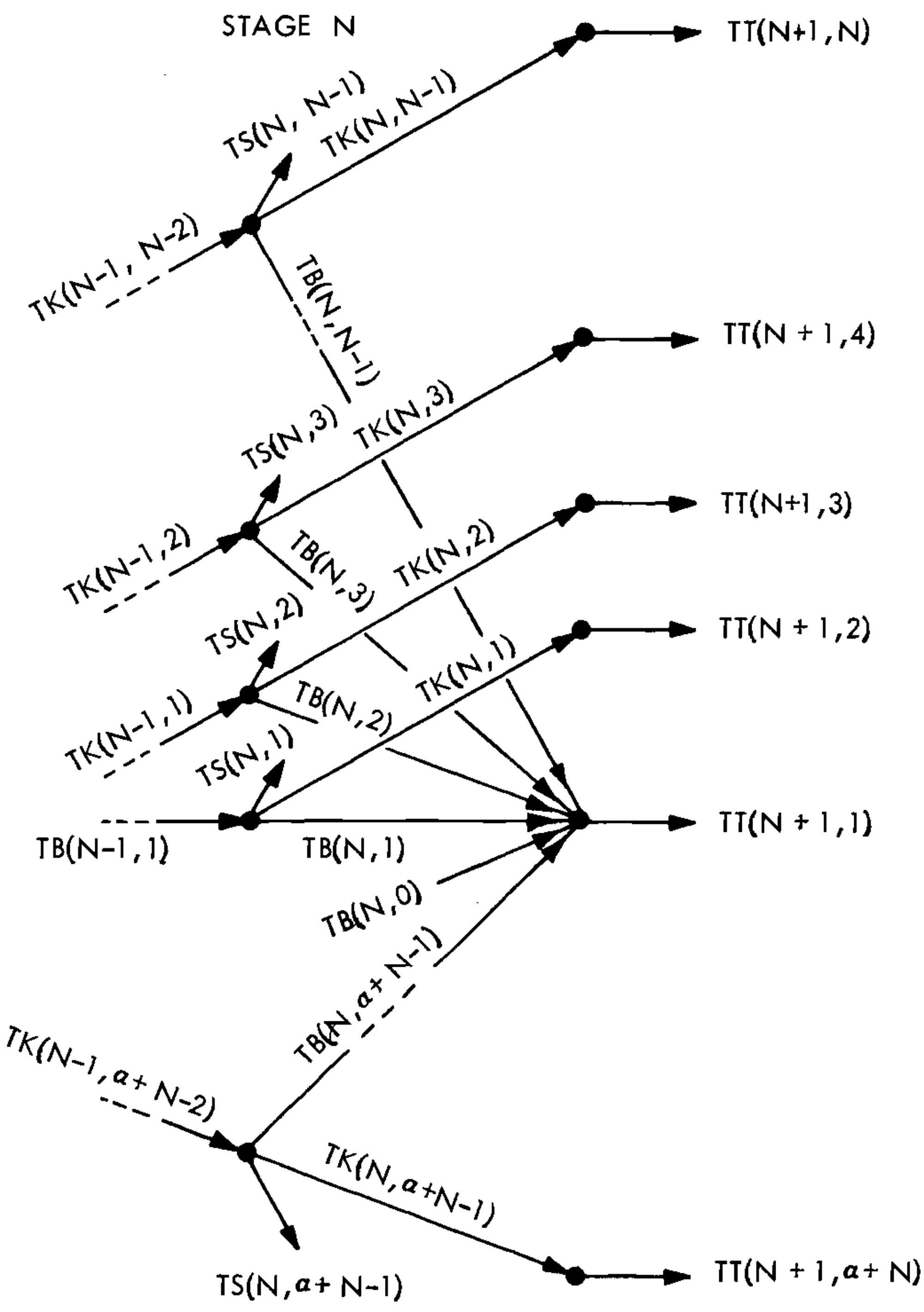


Figure 9. Network Diagram for Terminal Decisions

effect of such a time-lag will cause the cash flows to start when the asset is acquired; therefore, the model cannot make any decisions during such a period, e.g. if an item has a one year lead time, then the model will make decisions starting the second year, since all ordering has been already done for the first year.

<u>Model 7</u>

This model will be discussed in detail in Chapter VII.

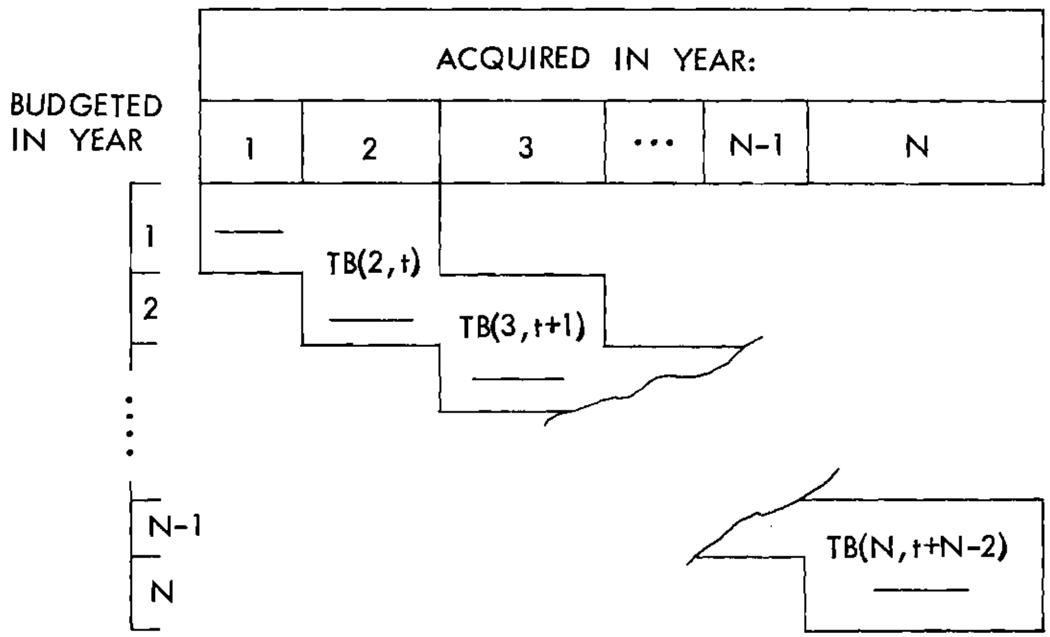


Figure 10. Lag Between Budgeting and Acquisition for an Item t Years Old

CHAPTER VI

SOLUTION OF MODELS I - VI

Introduction

The preceding chapter was devoted to developing models that represent the acquisition and replacement of fixed assets at TASCO. This chapter discusses methods of solving these problems. Illustrative examples are to be found in the appendices.

The models presented thus far have a sequential nature with a number of input and state variables at each stage. Since the state variables are obtained by a process of enumeration, linear programming solution techniques offer an efficient means of solving this, essentially dynamic, problem. Due to the large dimensionality of the models three decomposition techniques are also discussed.

Discussion of decomposition techniques follows a section that illustrates the linear programming structure of the models. The first decomposition technique discussed was proposed by Dantzig-Wolfe,[119] the second is one that uses a search technique based on Lagrange multipliers; and the third is a decomposition algorithm that arrives at the solution by successive approximations.

Solution by Linear Programming

Linear Programming Structure

Figure 11, 12, and 13 illustrate the basic linear programming structure for Models 1 through 6.

Let a(r,s) and ct(r,s) be the elements corresponding to the decision matrix A_{mxn}, and matrix CT_{mxn} respectively.

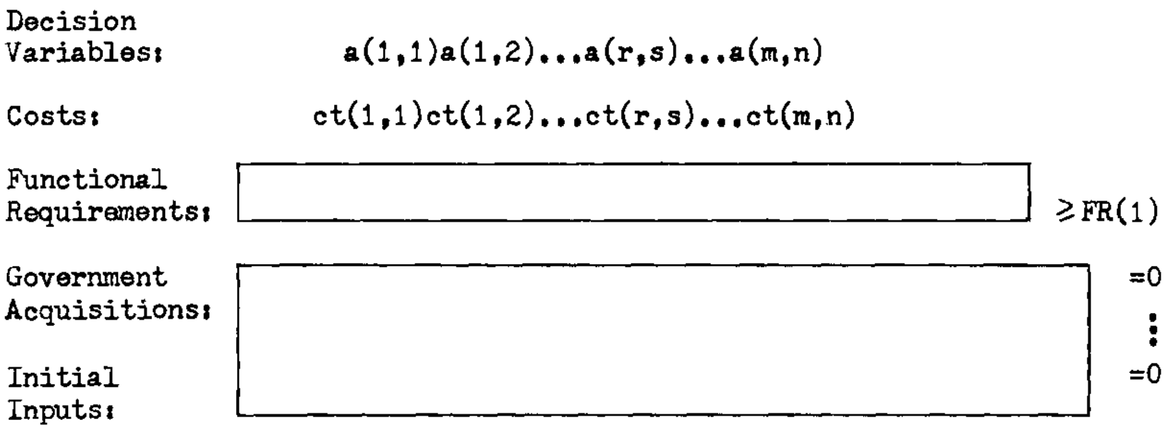


Figure 11. Linear Programming Structure Model 1

Models 4 through 7 (Model 7 will be discussed in Chapter VII) are similar from a linear programming structure point of view. The basic structure, as can be seen below, is a diagonalization of certain constraint equations pertaining to individual groups, with budgetary and other constraints providing a "tie," along a horizontal row, between the groups. Appendix B contains illustration of the coefficient matrix for Model 6, with N = 5.

The linear programming formulation appears below:

Minimize

$$C = \sum_i \sum_n C(i,n) = \sum_i \sum_n \sum_r \sum_s c_{r,s}(i,n) \qquad \text{(6-1)(a)}$$

where

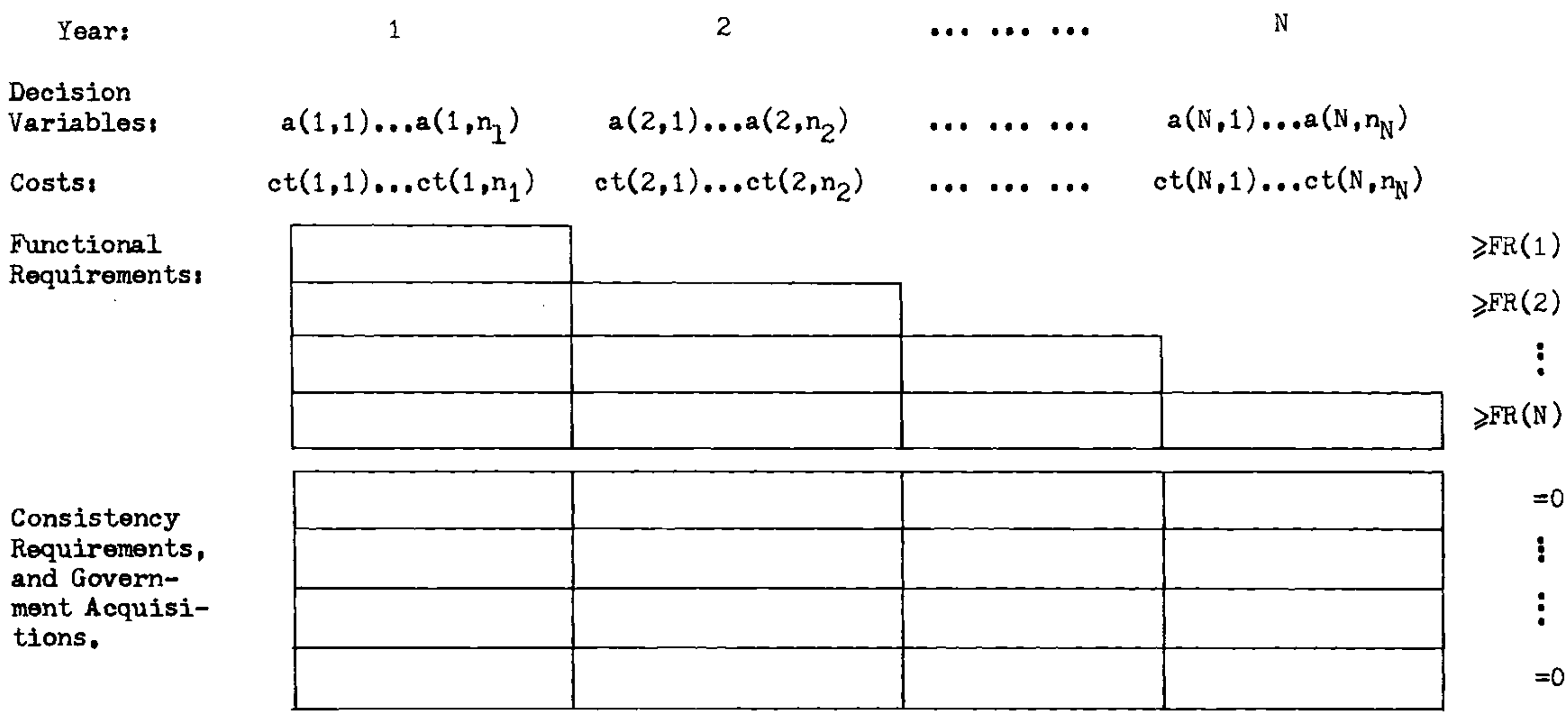


Figure 12. Linear Programming Structure - Models 2 and 3

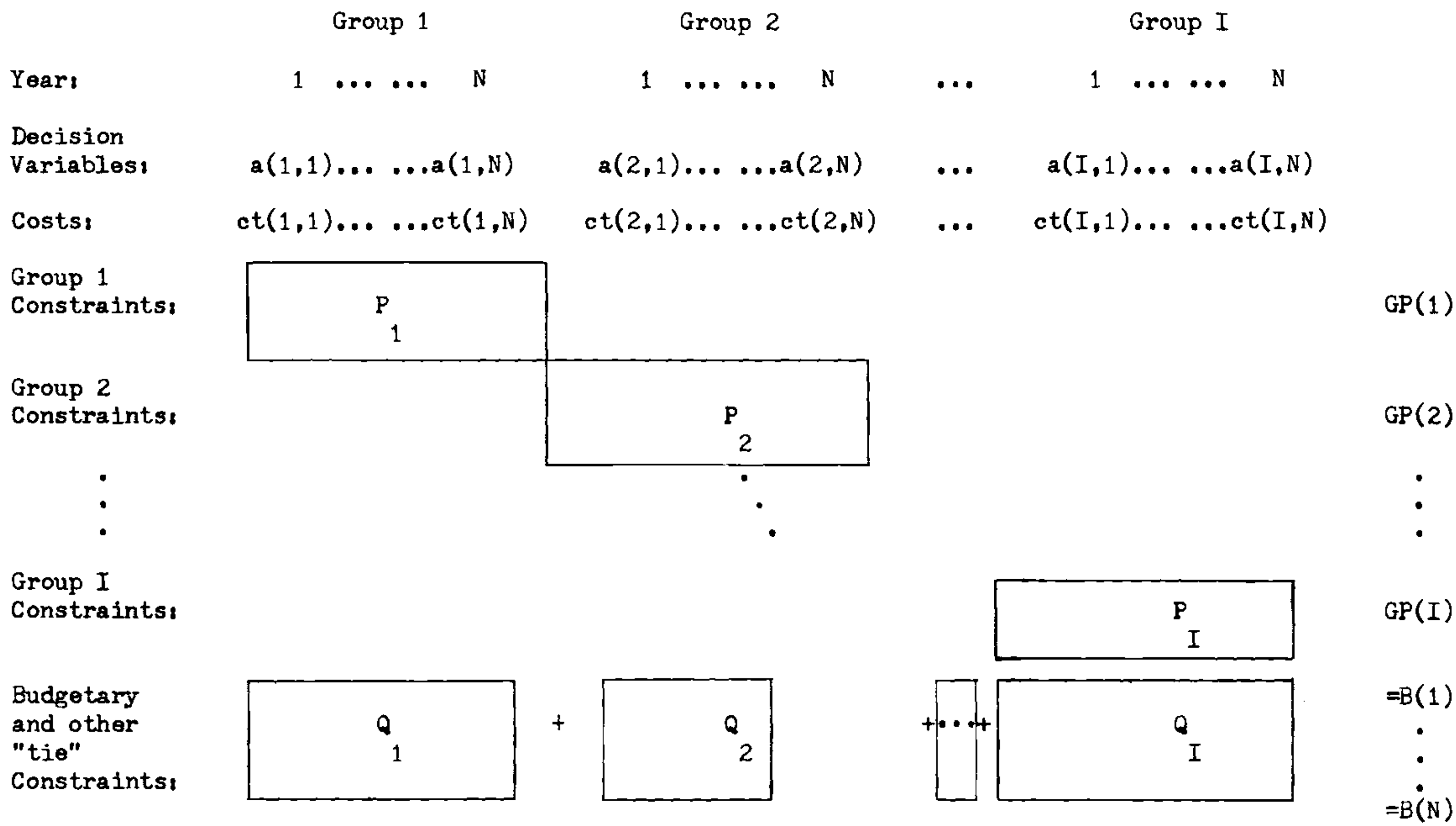


Figure 13. Linear Programming Structure - Models 4 - 7

$$c_{r,s}(i,n) = CT(i,n)A(i,n)' . \qquad (b)$$

Group 1 constraints $\lesseqgtr GP(1)$

Group 2 constraints $\lesseqgtr GP(2)$

. . . .

Group I constraints $\lesseqgtr GP(I)$

$$\sum_{i=1}^{I} \text{(Budgetary and other Constraints in Year 1)} \lesseqgtr BT(1) \qquad (c)$$

$$\sum_{i=1}^{I} \text{(Budgetary and other Constraints in Year 2)} \lesseqgtr BT(2)$$

. . . .

$$\sum_{i=1}^{I} \text{(Budgetary and other Constraints in Year N)} \lesseqgtr BT(N)$$

Non-negativity Requirements, $a_{r,s}(i,n) \geq 0$

Evaluation of the Linear Programming Approach

The evaluation of the linear programming approach to Models 1 through 7 can be approached from several points of view. The approaches presented below consider the integer versus continuous variable solutions, the interpretation of slack variables in the primal solution, the dual problem and its interpretation of slack variables in the primal solution, the dual problem and its interpretation, the planning horizon, uncertainty and, finally, the dimensionality considerations. The last item, dimensionality, leads to consideration of possible methods of decomposing the problem into smaller subproblems.

Integer Versus Continuous Solutions. The solution values of the linear programming algorithm structures must be integers to be realistic. However, the existence of noninteger values in the linear programming solutions is alleviated by a number of factors. Basically, these factors are the interpretation of fractional fixed assets in the context of the model formulation, and the ability to interpret the dual solution to the problem and thereby obtain a capability to decompose the large linear programming problem into smaller subproblems. The fractional values of fixed assets are an obvious advantage of the linear programming solution, since the decisions are made with respect to "typical" assets. A typical truck in one of the fixed asset groups may have a capability of 50,000 ton miles per year and the functional requirements for the group may be 150,000 ton miles in a given year. A fractional solution of buying 2.4 trucks and renting 0.6 trucks indicates a smaller truck should be rented, one typical truck should be bought, and another truck--1.4 times the typical capacity--should be purchased.

The integer optimal solution is obtained by solving a linear programming problem augmented by a number of constraints. These artificial constraints carry dual prices which may be interpreted as marginal returns of the integer values in the solution. Since the imputation of dual prices to the budget rows of the original problem is not a straightforward process, the decomposition of a large integer-programming presents practical difficulties. This limitation of integer programming prevents the use of the decomposition algorithm proposed since the algorithm is primarily based on the interpretation of the dual variables of the linear programming problem.

Furthermore, the rounding-off of solution values could be employed to derive approximate integer solutions. Since the lower bound of objective function is known (linear programming optional objective function value), by recomputing objective function values using rounded off integer values, an estimate of the error incurred could be obtained.

Another practical limitation of the use of integer programming is associated with the lack of core space in existing computers to solve large dimensional integer problems. This is due to the number of artificial constraints introduced to obtain integer solution values.

Slack Variables in the Primal Solution. Slack variables in functional requirement rows introduce certain additional capability into the linear programming formulation of the models. Slack in functional requirements rows indicates excess capacity, and costs could be associated with this additional investment in capability as shown below:

Model 1

Objective function:

$$C(1) = \sum_r \sum_s c_{r,s}(1) + ce(FR1) \cdot e(FR1) \qquad \begin{matrix}(6\text{-}2)\\(a)\end{matrix}$$

Functional Requirements:

$$\sum_r \sum_s cpa_{r,s}(1) - e(FR1) = FR(1) \qquad (b)$$

ce(FR1) refers to the cost associated with a unit of excess capacity.

e(FR1) refers to the slack variable in the functional requirements row.

Similar association of costs to slacks in functional requirements

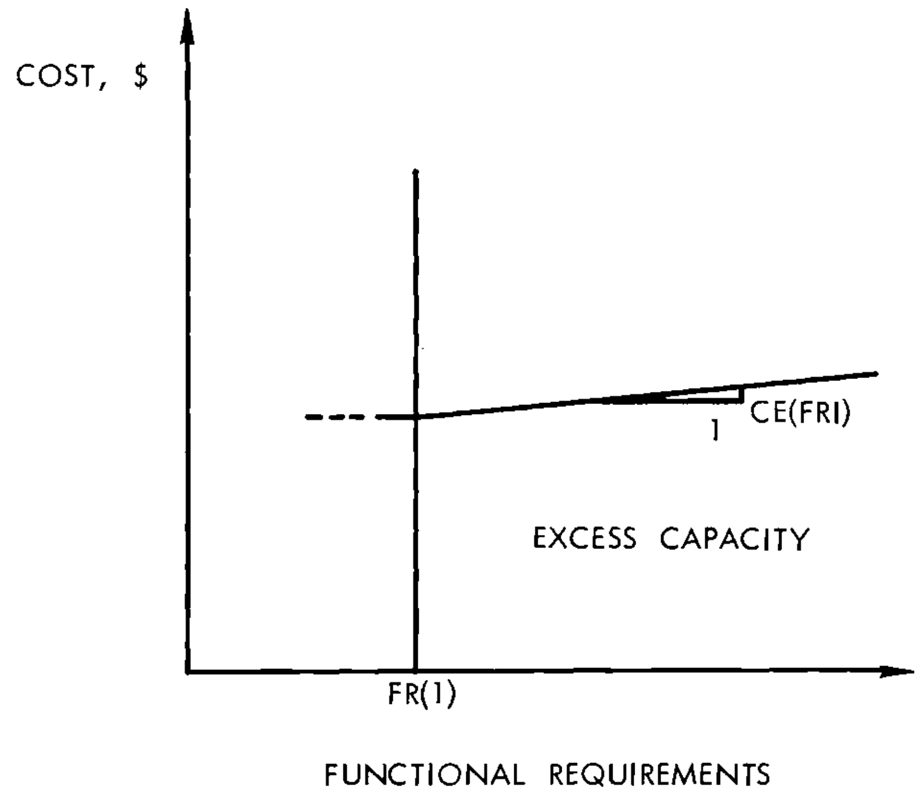


Figure 14. Slack in Functional Requirements

could be accomplished for the remainder of the models.

Slack variables in budget constraints of Models 4 through 7 indicate excess budgets available to meet the functional requirements. However, since no shifting of TASCO funds from one year to another is allowed the models thus far developed need not be revised.

The Dual Problem and Its Interpretation. Associated with every linear programming problem is a corresponding optimization problem called the dual problem. The original problem is called the primal, and the optimum solution of either one yields information concerning the optimum solution of the other.

The dual of the primal problem for Models 2 and 3 can be expressed as:

Maximize

$$\sum_n W_{CP}(n)FR(n) + \sum_n W_{GN}GN(n) + \sum_n W_{CY(n)}CY(n) \tag{6-3 (a)}$$

subject to

$$
\begin{aligned}
&W_{CP}(1)A_{CP(1)}+W_{GN(1)}A_{GN(1)}+W_{CY(1)}CY(1) \leqslant CT(1) \qquad \text{(b)}\\
&W_{CP(2)}A_{CP(2)}+W_{GN(2)}A_{GN(2)}+W_{CY(2)}CY(2) \leqslant CT(2)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&W_{CP(n)}A_{CP(n)}+W_{GN(N)}A_{GN(N)}+W_{CY(N)}CY(N) \leqslant CT(N)
\end{aligned}
$$

where $A_{CP(n)}$, $A_{GN(n)}$, and $A_{CP(n)}$ refer to the columns of the decision matrix A(n) that correspond to the CT(1) in the objective function of the primal.

The W is the vector of dual variables, and the subscripts CP(n), GN(n), and CY(n) are used to determine the particular W elements. The elements of W may be considered to be the "opportunity value" of the budget constraints in the primal problem. $W_{CP(n)}$ is the "value" of a unit of functional requirements in year n; $W_{GN(n)}$ is the value of a unit of asset acquired by the government; and $W_{CY(n)}$ may be termed the value of the consistency requirements.

The dual can be interpreted as maximizing the value of a number of inputs, subject to limitations on their costs.

The dual of Models 4 through 6 appears below:

Maximize

$$\sum_i \sum_n W_{CP(i,n)}FR(i,n)+W_{GN(i,n)}GN(i,n)+W_{CY(i,n)}CY(i,n)+U(n)BT(n) \tag{6-4 (a)}$$

subject to

$$W(1,1)P(1)+U(1)Q(1) \leqslant CT(1,1) \tag{b}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$W(1,N)P(1)+U(N)Q(1) \leqslant CT(1,N)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$W(I,1)P(I)+U(1)Q(I) \leqslant CT(I,1)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$W(I,N)P(I)+U(N)Q(I) \leqslant CT(I,N)$$

The P(i) and Q(i) are the matrices that appear in Figure 13, and the U(n) refers to the "opportunity value" of a budget dollar in year n.

The Planning Horizon. Weingartner[120] indicates a planning horizon to be ". . . a value T such that the set of accepted projects having outlays or reserves in year T or sooner are exactly the same whether the model is built with an infinite horizon or a horizon set at T." In the models developed, a planning horizon could be defined as a value N such that the set of decisions made up to year N or sooner are exactly the same whether the model is built with an infinite horizon or a horizon set at N.

The truncation of the problem by the imposition of horizons has been already modeled (see Models 4 through 7). Model 6 shows how terminal values may be taken into account. The series formed by partial summation of sequences of the inner products of the cost and solution vectors for each year can be shown to converge under certain assumptions. It is assumed that the discount factor is constant over the years, that there exists an upper bound, K_n, to the cash outlay that can occur in any given year, n, and that K_n increases linearly reflecting the growth of the firm.

Let

$$K_n = K_o + Kn, \text{ for } n = 1, 2, \ldots,$$

K_n: Upper bound for cash outlays in any year, n,

K_o: A constant,

K : Growth associated with the particular cash outlays,

C_n: Vector of costs in year n,

X_n: Vector of solution variables, for year n,

i : Interest rate,

C_nX_n: A scalar indicating the value of the objective function for year n,

and

$$C_nX_n \leqslant K_n.$$

Proof:

Total cash outlay,

$$\sum_{n=1}^{N} C_nX_n$$

Discounted total cash outlay,

$$DTC(N) = \sum_{n=1}^{N} \frac{C_nX_n}{(1+i)^{n-1}}$$

but

$$\sum_{n=1}^{N} \frac{C_nX_n}{(1+i)^{n-1}} \leqslant \sum_{n=1}^{N} \frac{K_o+Kn}{(1+i)^{n-1}}, \tag{6-5}$$

and since

$$\lim_{N\to\infty} \sum_{n=1}^{N} \frac{K_o}{(1+i)^{n-1}} = \frac{K_o}{1-\frac{1}{1+i}} \tag{6-6}$$

and

$$\lim_{N\to\infty} \sum_{n=1}^{N} \frac{Kn}{(1+i)^{n-1}} = \frac{K}{(1-\frac{1}{1+i})^2}, \qquad (6\text{-}7)$$

therefore, discounted total cash outlay DTC(N), converges.

The convergence of

$$\sum_{n=1}^{N} \frac{C_n X_n}{(1+i)^{n-1}}$$

shows that the advantage of longer horizons diminishes rapidly due to the discounting effect. This fact is the basis for truncating the horizon at 5 or 10 years. This is fortunate since, as can be seen in the following sections, the increase in dimensionality of the problem varies in a non-linear fashion with the length of the planning horizon.

Uncertainty. The models developed assume that deterioration, obsolescence, functional requirements, and various cost parameters are all known with certainty. The justification for this is that there still exist numerous areas to be investigated in large scale, multi-dimensional capital investment problems. These areas are basically associated with dimensionality, and usually have thus far prevented further research into such problems. With the advent of electronic computers of larger memory capacity, further investigation of such investment problems has become feasible. However, the memory capacities of some present day computers still becomes inadequate, due to the combinational nature of the decisions. This limitation indicates the need for further research into decomposition techniques that are practical and theoretically sound.

Trygve Haavelmo, in his text titled A Study in the Theory of Investment, expresses the necessity for more study into the need for further building of investment models under certainty, particularly from the point of building insight into the problems. He says, "We are . . . far from having exhausted the amount of clarification and insight that can be gained from the study of exact models. We shall find more than enough to do even in a hypothetical world of non-stochastic models."(121)

Dimensionality. The greatest problem with formulating the fixed assets problem as a direct linear programming approach lies in its dimensionality. Without overall budgetary constraints that tie the groups together, dimensionality presents no problem, since individual groups can be solved independently. Following is a table showing the dimensions of Models 1 through 6 in terms of N, the planning horizon.

Table 8 shows the number of rows and columns of the linear programming matrix for the seven models described above. Model dimensionality indicates the number of rows and columns of the matrix. Figure 15 indicates the group row and column dimensions as a function of N.

The run-time for a single module on the Univac's 1108 took 26 seconds for a 2 year, and 42 seconds for a 5 year model. The run-time for 14 fixed asset groups was approximately 4 minutes and 38 seconds.

Solutions Through Decomposition

The two decomposition techniques proposed to cope with the problem of dimensionality have the advantage of simplicity at the cost of the approximate answers they provide. Chapter II of this thesis discusses

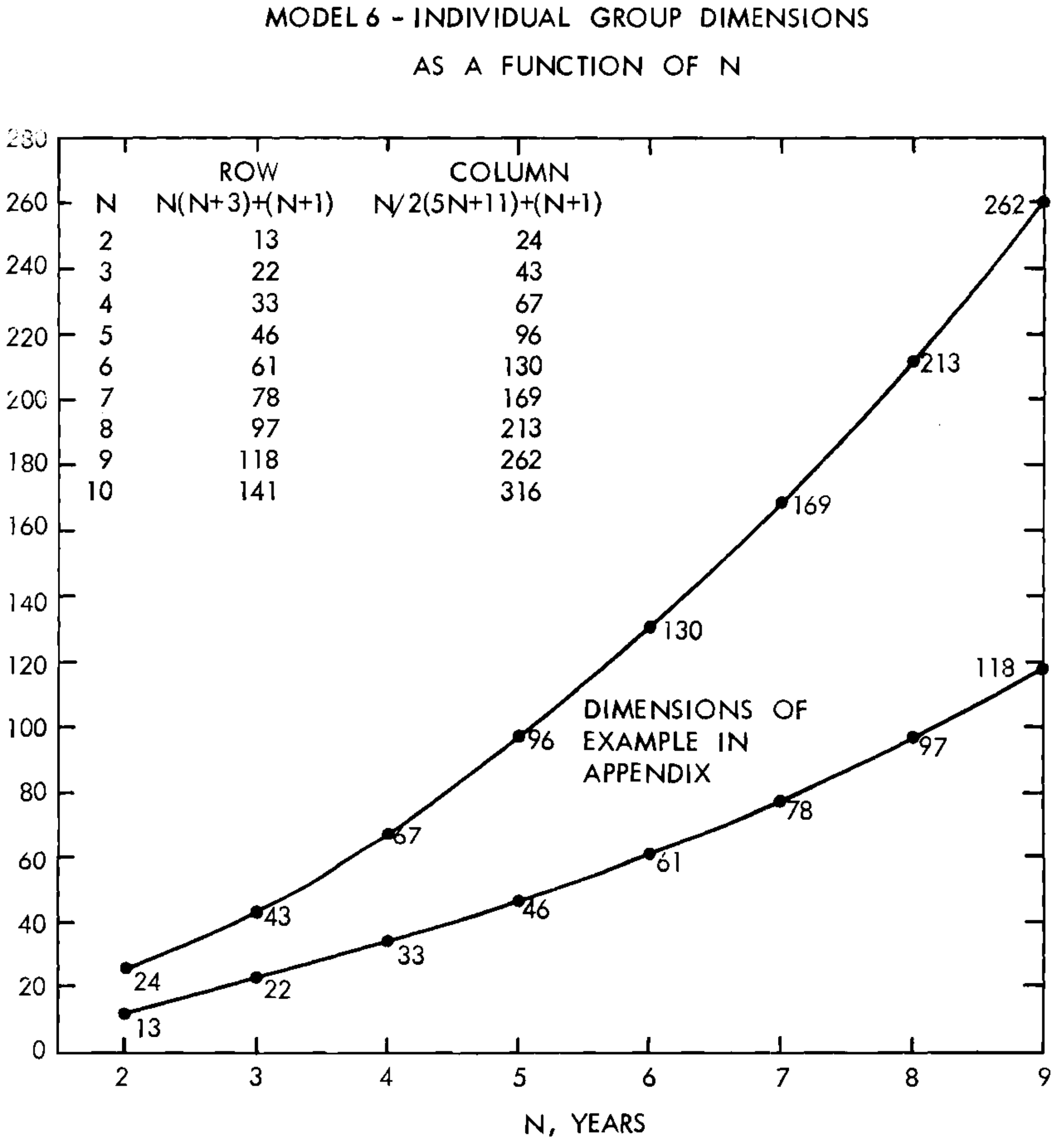


Figure 15. Model 6 - Individual Group Dimensions as a Function of N

Table 8. Dimensionality of the Models

Model	Horizon (Years)	No. of Budget Equations	Model Dimensionality Rows	Columns
1	1	0	4	7
2	N	0	$N(N+3)$	$^{N}/_{2}\ (5N+11)$
3*	N	0	"	"
4	N	N	$(I_i)(N)(N+3)+N$	$(I_i)(\frac{N}{2})(5N+11)$
5	N	N	$\sum_i^B I_i((N)(N+3))+N$	$\sum_i^B I_i((\frac{N}{2})(5N+11))$
6	N	N	$\sum_i^B I_i(N(N+3)+(N+1))+N$	$\sum_i^B I_i((\frac{N}{2})(5N+11)+(N+11))$
7**	See Chapter VII			

I_i: Number of fixed asset groups in branch i; $i = 1, \ldots, B$

*Additional dimensionality due to lease type decisions have been left out, but could easily be obtained by referring to the discussion in Chapter V.

**Dimensionality here depends on the financial considerations.

some of the decomposition techniques in existence. Dantzig-Wolfe's decomposition,[122] which is "exact," is first discussed as it relates to the linear programming problem we have in Models 4 through 6.

Dantzig-Wolfe Decomposition

Models 4 through 7 have a structure that yields itself to decomposition, since they are composed of separate linear programming problems tied together by a number of constraints considerably smaller than the total number imposed on the problem. Figure 15 displays the basic structure. The original problem is expressed in the Model 4 equations. This original problem is reformulated so that the new problem is made up of the extreme points of the sets defined by the equations of Model 4.

If $W_i = (a_{i1},\dots,a_{ik})$ is the set of all extreme points of the convex polyhedron S_i defined by a_i non-negative and meeting group i constraints; i.e.,

$$a_i \geqslant 0, \quad P_i a_i = GP_i;$$

and

$$\left.\begin{aligned} R_{ik} &= Q_i a_{ik} \\ C_{ik} &= C_i a_{ik} \end{aligned}\right. \quad \text{for } k = 1, \dots, K \tag{6-8}$$

then the extremal program may be defined as:

Find numbers $s_{ik}(i = 1,\dots,I;\ k = 1,\dots,K)$ satisfying

$$\sum_k s_{ik} = 1 \text{ (all i)} \tag{6-9}$$

that minimize

$$\sum_{ik} c_{ik} s_{ik}. \tag{6-10}$$

The extremal problem is the original problem restated in terms of a convex combination of the latter's points, and since $s_i = \sum_k a_{ik} s_{ik}$, the solution of the extremal problem provides the solution for the original problem. The extremal problem is shown in the following figure.

$s_{11} \ldots s_{ik}$	$s_{21} \ldots s_{2k}$	,....,	$s_{11} \ldots s_{Ik}$	
$c_{11} \ldots c_{ik}$	$c_{21} \ldots c_{2k}$	,....,	$c_{11} \ldots c_{Ik}$	
Columns	Columns		Columns	
$R_{11} \ldots R_{1k}$	$R_{21} \ldots R_{2k}$	,....,	$R_{I1} \ldots R_{Ik}$	=BT
1...1				=1
	1...1			=1
				⋮
			1...1	=1

Figure 16. The Extremal Problem

The constraint equations for the extremal problem are N + I in number; the N joint constraints (three types of budgetary constraints) of the original problem have become the N constraints of the extremal problem and the constraints of ith subproblem have become the single constraints.

$$\left(\sum_k s_{ik} = 1, \text{ (all } i\text{)} \right) \tag{6-11}$$

The reduction in the total number of constraints is sizeable. The reduction is accomplished by enlarging the number of variables, since K extreme points exist for each variable in the original problem. Dantzig and Wolfe show an effective method of reducing this large number of variables to a solution algorithm that handles them one at a time.

Search Using Lagrange Multipliers

A decomposition algorithm that proposes to solve the large linear programming problems associated with Models 4 through 7 is discussed below. The results obtained by application of the algorithm to three fixed asset groups is illustrated with a numerical example in Appendix B. Compared to the Dantzig-Wolfe exact method discussed above, this and the following techniques are approximations that have the advantage of simplicity. The regular linear programming routines need not be changed, since changes can be introduced manually during successive runs of the program.

Since the fixed asset groups can be solved as independent linear programs without budgets, a lower bound of the objective function for both decomposition techniques exists and can serve as a check on the progress of successive stages of approximations.

Formulation of the Problem. The original problem (Models 4 through 7) has a set of diagonal submatrices and several horizontal rows representing budgets, as shown below.

Minimize

$$C(X) = C_1X_1+C_2X_2+\ldots+C_IX_I \tag{6-12 (a)}$$

subject to

$$P_1X_1 = GP(1) \qquad \text{(b)}$$

$$P_2X_2 = GP(2)$$

$$\cdots\cdots$$

$$P_IX_I = GP(I)$$

$$Q_1(1)X_1(1)+Q_2(1)X_2(1)+\ldots+Q_I(1)X_I(1) = B(1) \qquad \text{(c)}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$Q_1(N)X_1(N)+\ldots+\ldots+Q_I(N)X_I(N) = B(N)$$

Lagrange multipliers will be used to absorb the last budget rows in the objective function to form the integrated problem:

Minimize

$$C(X,\lambda) = C_1X_1+C_2X_2+\ldots+C_IX_I+\lambda_1(Q_1(1)X_1+\ldots+Q_1(1)X_1-B(1)) \qquad \text{(6-13) (a)}$$
$$+\ldots+\lambda_N(Q_1(N)X_1+\ldots+Q_I(N)X_I-B(N))$$

subject to

$$P_1X_1 = GP(1) \qquad \text{(b)}$$

$$P_2X_2 = GP(2)$$

$$\cdots\cdots$$

$$P_IX_I = GP(I)$$

$$\lambda_i, X_{ij} \geqslant 0 \text{ for all i, j.}$$

Now the integrated problem may be solved by formulating subproblems and conducting a search over the λ_i, where ($i = 1, \ldots, N$), and the remainder of this section discusses this procedure.

The decomposition technique could be further clarified as follows: Let

$$C_1X_1 = C_1(0)X_1(0)+C_1(1)X_1(1)+C_1(2)X_1(2)+\ldots+C_1(N)X_1(N) \quad (6\text{-}14)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_IX_I = C_I(0)X_I(0)+C_I(1)X_I(1)+C_I(2)X_I(2)+\ldots+C_I(N)X_I(N)$$

where $C_i(0)X_i(0)$ indicates the cost of group i decisions not affected by budgets, and $C_i(n)X_i(n)$ the costs of decisions that are affected by budgets in the nth year. The λ values apply to those components of C_1X_1 that are constrained. The objective function may be written in a more detailed form as:

$$C(X,\lambda) = C_1(0)X_1(0)+(C_1(1)+\lambda_1Q_1(1))X_1(1)+(C_1(2) \quad (6\text{-}15)$$
$$+\lambda_1Q_1(2))X_1(2)+\ldots+(C_1(N)+\lambda_NQ_1(N))X_1(N)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_I(0)X_I(0)+(C_I(1)+\lambda_1Q_I(1))X_I(1)+(C_I(2)+\lambda_2Q_I(2))X_I(2)$$
$$+\ldots+(C_I+\lambda_NQ_I(N))X_I(N)$$

The constant terms B(n) have been dropped since they will not affect the decisions and can be later added to the objective function. This objective function can be searched for the λ_1* by solving a sequence of decomposed problems.

The decomposed problem approach is as follows: Assume an arbitrary value of λ_1, e.g. between 3 and 5. Find λ^*_1 by solving the following set of linear programming problems:

Group 1:

Minimize

$$C_1(0)X_1(0)+(C_1(1)+\lambda_1 Q_1(1))X_1(1)+(C_1(2)X_1(2))+\ldots+(C_1(N)X_1(N)) \quad \text{(6-16) (a)}$$

subject to

$$P_1X_1 = GP(1) \quad \text{(b)}$$

Group 2:

Minimize

$$C_2(0)X_2(0)+(C_2(1)+\lambda_1 Q_2(1))X_2(1)+(C_2(2)X_2(2)+\ldots+(C_2(N)X_2(N)) \quad \text{(6-17) (a)}$$

subject to

$$P_2X_2 = GP(2) \quad \text{(b)}$$

. . . . . .

Group I:

Minimize

$$C_I(0)X_I(0)+(C_I(1)+\lambda_1 Q_1(1))X_I(1)+(C_I(2)X_I(2))+\ldots+(C_2(n)X_2(N)) \quad \text{(6-18) (a)}$$

subject to:

$$P_I X_I = GP(I) \tag{b}$$

This results in a solution vector, $X_1{}^k(1)$ where k is the iteration number, (here k = 1).

The process is repeated with a new λ, value and the difference between the two successive budget values determines the level of approximation.

If

$$\sum_{i=1}^{I} Q_i(1)(X_i{}^k(1)-X_i^{k+1}(1)) \leq \epsilon*(1) \tag{6-19}$$

where

$$\epsilon*(1)$$

is a small number, then the process stops with that value of $\lambda_1 = \lambda_1^*$.

This process is repeated for λ_2^* by a search over the $X_i(2)$ for i = 1, 2, 3, ..., N. The coefficients of $X_i(1)$ are changed from $Q_i(1)$ to λ_1^* $(Q_i(1))$ during the search for λ_2. This process of multidimensional search by one dimension at a time is continued until all $\lambda*_n$ are found, and the solution satisfied the budgetary restrictions B(n) within the errors (n). This process may need to be repeated for i = 1, ..., N more than once to obtain stabilized valued of λ_n^*.

Figure 13 illustrates the relationship between the λ_i and B(n).

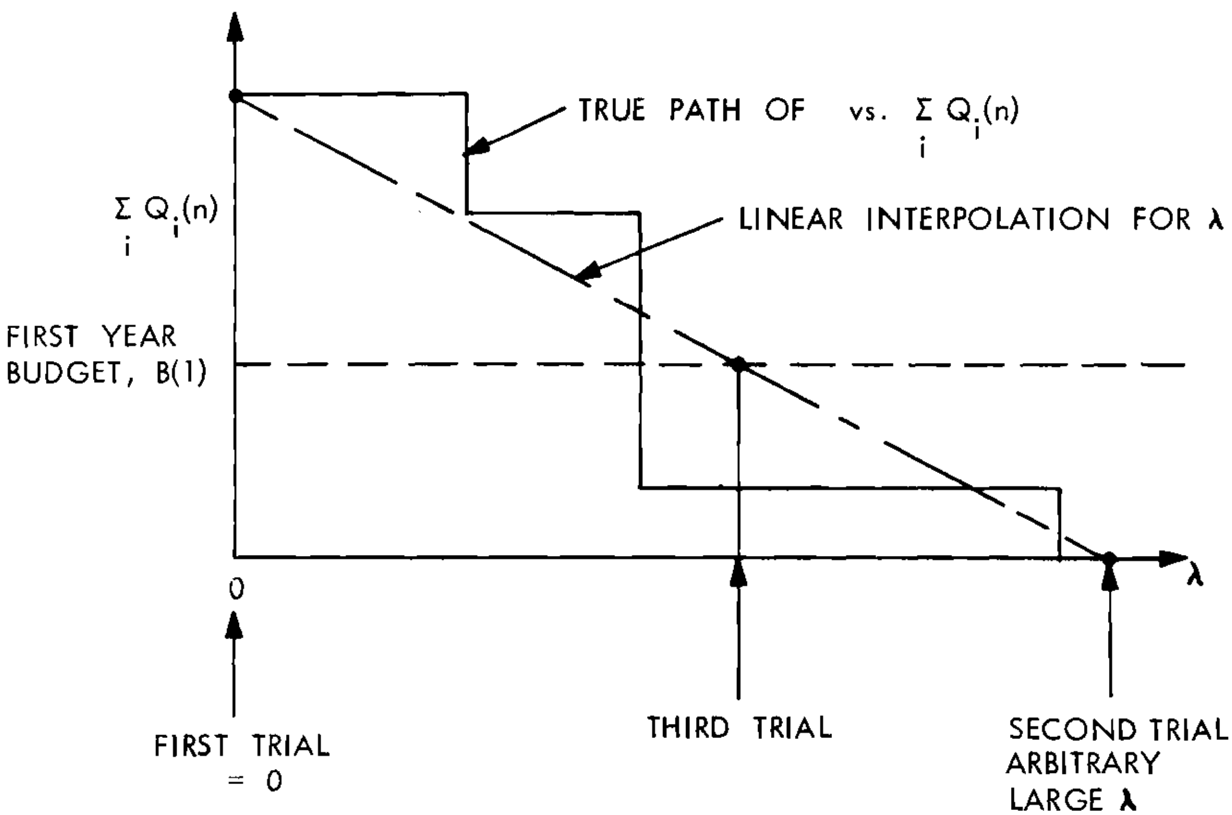


Figure 17. Search Using Lagrange Multipliers

The Lagrange multipliers, λ_n, can be interpreted as the dual prices, and represent the value of a budget dollar for each of the years, $n = 1, \ldots, N$. Solution of a programming problem by absorbing the constraint into the functional has been illustrated by R. Bellman(123) and H. Everett.(124) Everett shows that while the use of Lagrange multipliers does not guarantee a solution in all cases, the simplicity of the method makes it well suited to the solution of problems of allocating limited resources among a number of activities. J. Cord(125) discusses the dual nature of the Lagrange multipliers obtained by search techniques in a dynamic programming (investment allocation) type problem. However, the interpretation of the Lagrangian multipliers is ambiguous due to the discrete nature of the variables in Cord's problem.

A Successive Approximation Algorithm

This algorithm makes use of the dual values of each group in such a way that the budgets that apply to all the groups are allocated in an efficient way. At each stage of the algorithm the dual variable values of each group for the particular year are ranked and the budget for that year is allocated from the lowest ranking to the highest ranking group until a limit is reached such that further change will cause a basis change. At this stage, a new optimization is made, resulting duals for the next year's budgets are ranked within the groups, and budgets for that year are reallocated. At the end of Nth year, allocation started by the magnitude of improvement in the objective function determines if a new cycle should be perturbing the right hand sides of those groups that are at their limit, and the dual values are ranked again and the process repeats itself. Since the lower bound of each group is a solution with unlimited budgets, it serves as a good approximation for initial allocations.

The process described makes sure that the successive values of the objective function decrease monotonically. Existence of a lower bound corresponding to unrestricted budgets allows determination of the improvement achieved at each iteration, and assures convergence of the algorithm.

<u>The Structure of the Decomposable Problem</u>. For simplicity, a two group integrated problem is illustrated below:

Minimize

$$C(X) = C_1X_1+C_2X_2 \qquad \text{(6-20) (a)}$$

subject to

$$P_1X_1 = GP(1) \qquad \text{(b)}$$

$$P_2X_2 = GP(2)$$

$$Q_1(1)X_1(1)+Q_2(1)X_2(1) = B(1) \qquad \text{(c)}$$

$$Q_2(2)X_2(2)+Q_2(2)X_2(2) = B(2)$$

The two groups are "tied" together by the budgetary constraints for two years. X_1, and X_2 are the decision vectors applicable to the first and second groups. The set of decisions applicable to the first and second year are shown as:

1st Group (X_1): 2nd Group (X_2):

1st year ,$X_1(1)$ 1st year ,$X_2(1)$

2nd year ,$X_2(2)$ 2nd year ,$X_2(2)$

It should be noted that even though decomposition applies only to two groups, it will be equally effective in dealing with decompositions applied to more groups or branches. This is due to the basic structure of the branches in a diagonal manner with the budget equations, which provides the "tie" that connects the branches into a company.

Outline of the Decomposition Procedure. The basic idea can be envisioned as follows:

Top management asks each branch manager to calculate the budgets he needs. These are determined by allowing each branch to run the linear programming problem including their groups without the imposition of any budgets. The optimal program inputs a budget level for each year, which are the budgets needed if branches could get funds.

This information is supplied to the top management, and they determine new budget levels for each branch. These changed budgets are always tighter than, or, at most, equal to, the funds initially requested by the branches. This information is conveyed to the branches. Using the tighter budgets, the branches determine new constrained optima, corresponding plans, dual values and certain range information.

Top management, based on the information provided to them by branches, determines new budget allocations, based on a ranking of dual values, to be explained below.

This reallocation process is repeated a number of times until the overall objective function either cannot be improved upon or the improvements are below a present magnitude.

Intermediate Dual Prices. Optimal values of the ordinary variables of the dual problem are denoted by $W_1(1)$, $W_2(1)$, $W_1(2)$, and $W_2(2)$.

The dual variable represents the shadow price, or the marginal value of budget input. That is, $W = \frac{\partial C}{\partial B}$, where C represents the present value of the total cash outlay and B represents the budgets. It should be emphasized that these variables are interpreted in terms of the optimal basic solution of the primal and dual problems, and by intermediate dual prices we mean the dual prices that correspond to the optimal solutions of certain problems that appear in the proposed algorithm.

The budgets in TASCO are almost always binding, which indicates that normally there will be no zero values for these prices except at the end of the algorithm. This is because, based on optimal allocation of budgets in certain years, a branch may have all its demands for budgets satisfied.

Description of the Decomposition Procedure. The budgets, $B(n)$, apply to the "integrated" problem. The integrated problem could be made up of several groups, representing branches, or a number of branches, representing TASCO. The groups in the branch, or the branches in the company are referred to as the "subproblems." Here GP(1) and GP(2) are the constraints that apply to subproblems 1 and 2.

Initially, each subproblem is solved independently. In our illustration, this would be:

Subproblem 1:	Subproblem 2:
Minimize C_1X_1	Minimize C_2X_2

$$\text{subject to } P_1X_1 = GP(1) \qquad \text{subject to } P_2X_2 = GP(2)$$

Let the optimal solutions be X_1^o and X_2^o. These solutions give TASCO management a first indication of the funds requested for the lowest overall cash outlay.

Let

$$X_1^o = (X_1^o(1), X_1^o(2)) \text{ and } X_2^o = (X_2^o(1), X_2^o(2)) \tag{6-21}$$

be first and second year decisions.

The budgets needed to support the optimal subproblem solutions are:

(6-22)

$$Q_1^o(1)X_1^o(1)+Q_2^o(1)X_2^o(1) = b_1^o(1)+b_2^o(1) = B^o(1) \qquad \text{(a)}$$

$$Q_1^o(2)X_1^o(2)+Q_2^o(2)X_2^o(2) = b_1^o(2)+b_2^o(2) = B^o(2) \qquad \text{(b)}$$

The management makes an initial allocation of the B(1) and B(2); ($B(1)<B^o(1)$, and $B(2)<B^o(2)$) determine by some rule of thumb, such as by making them proportional to "optimal" budgets of each group, $B^o(1)$, and $B^o(2)$. In other words,

$$b_i^m(n) = B(n)\,\frac{b_i^o(n)}{B^o(n)} \tag{6-23}$$

or,

$$b_1^1(1) = B(1)\,\frac{b_1^o(1)}{B^o(1)}\,;\; b_2^1(1) = B(1)\,\frac{b_2^o(1)}{B^o(1)}$$

and,

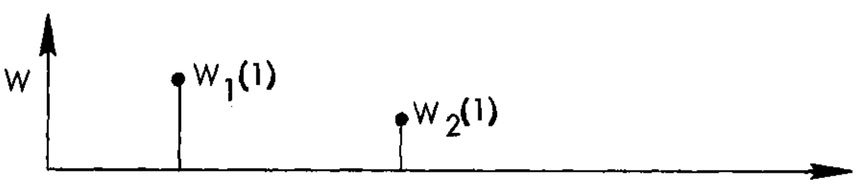
$$b_1^1(2) = B(2)\,\frac{b_1{}^{o}(2)}{B^{o}(2)};\quad b_2{}^1(2) = B(2)\,\frac{b_2{}^{o}(2)}{B^{o}(2)}, \qquad (6\text{-}24)$$

where $b_i^m(n)$ indicates the budget allocation to the ith subproblem, in nth year, in mth cycle. Each cycle consists of a reallocation of each of the N years. Since it is assumed that $B(1)<B^{o}(1)$, and $B(2)<B^{o}(2)$ all the budgets will be binding.

Next these $b_i^m(n)$ values are given to the managers of the organizations with the appropriate subproblems. Each solves the subproblem, using the $b_i^m(n)$ values as constraints. In addition to the primal solution, he calculates the duals and performs a dual ranging. The dual ranges indicate the range over which each right hand side element may be varied without requiring a change of basis. Certain theoretical aspects of these statements are discussed in the following section.

Let $\Delta_i^{m}\pm(n)$ indicate the upper, (+), and lower, (-), range within which the particular right hand side can vary without a change in basis.

Management obtains the dual values and ranges for each subproblem and ranks them from the highest to the lowest first for year 1. Let us assume the order appears as shown in Figure 18.



Subproblem i, Year 1

Figure 18. Dual Ranking, Year 1

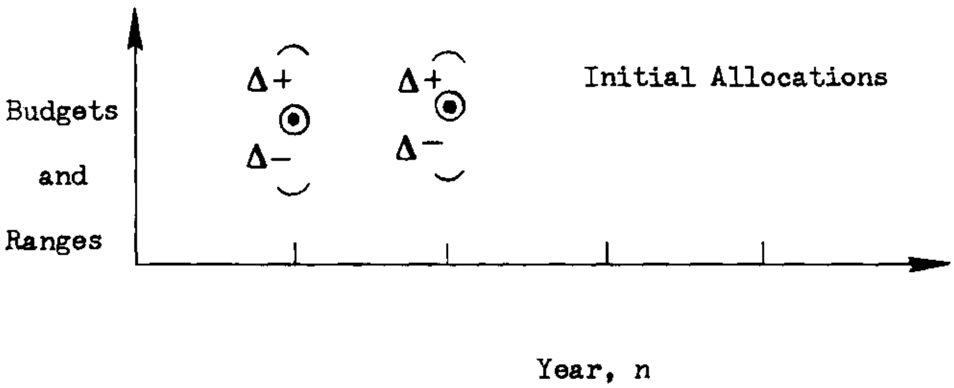


Figure 19. Right Hand Side Ranges

Since within the Δ limits the same duals apply, we can allocate up to the limit Δ - of the $b_2(1)$ to the $b_1(1)$ until the upper limit $b_1(1) + \Delta_1+(1)$ is reached. We can go on allocating in this manner until there is no more to allocate.

At this point the management is able to calculate the new objective function as follows:

$$C^{m+1} = C^{m} - \sum_{i,1} \alpha_i^m(n) \cdot W_i^m(n) \cdot \Delta_i^{+m}(n) + \sum_{i,1} \beta_i^m(n) \cdot W_i^m(n) \cdot \Delta_i^{-m}(n) \qquad (6\text{-}25)$$

C^m refers to the previous objective function of the integrated sub-problem, α and β are the fraction of Δ utilized,

$$0 \leq \alpha, \beta \leq 1$$

A constraint that applies to this solution is,

$$\sum_i \Delta_i+(1) - \sum_i \Delta_i-(1) = 0 \qquad (6\text{-}26)$$

This constraint assures that the sum of money reallocated in year 1 is the same in each cycle. Now the same process is repeated for the reallocation of $B(2)$ to $b_1(2)$ and $b_2(2)$, and this ends one cycle.

Next, a test regarding the improvement is made, and, if successive iterations produce objective functions within $(\epsilon > 0)$ of each other, then the process is terminated. i.e. if $C^m - C^{m+1} \leq \epsilon$, then stop.

If after the allocation there is still room for improvement, i.e. $C^m - C^{m+1} > \epsilon$, then those right hand sides that have been allocated budgets up to their limits are perturbed. This causes a new basis with resulting new dual variables and ranges. The ranking process is again utilized on a yearly basis.

A simplified flow chart of the algorithm appears below:

Steps of the Algorithm	Flow Chart
Step 1: Solution of Each Group with Unlimited Budgets	Solve: Min. C_iX_i S.T. $P_iX_i = GP(i)$ for $i = 1, 2, \ldots, I$
Step 2: Calculation of Optimal Budgets for Each Year (All Groups)	Calculate: $B^o(1) = \sum_i b_i^{\,o}(1) = \sum_i Q_i(1)X_i(1)$ . . . . $B^o(N) = \sum b_i^{\,o}(N) = \sum Q_i(N)X_i(N)$
Step 3: Initial Allocation	Allocate $B(n)$ $b_i^1(1) = B(1)\cdot(b_i^{\,o}(1)/B^o(1))$ . . . . $b_i^{\,1}(N) = B(N)\cdot(b_i^{\,o}(N)/B^o(N)$ for all i

Step 4: Solve with New Allocations

Solve: Min. $C_i X_i$

S.T.: $P_i X_i = GP(i)$

$Q_i(1) = b_i^m(1)$

. . . .

$Q_i(N) = b_i^m(N)$

for all i

Step 5: Calculate Duals, and Limits of Constraint

Determine:

W_i values, and

$\Delta_i^{\pm}(n)$ for all i and the particular n

Step 6: Rank and Allocate by Duals

Rank and Reallocate Nth Year

(n = 1, ..., N)

$W_1(1) \ldots W_I(1)$

. . . .

$W_1(n)^* \ldots W_I(N)^*$

$W_1(N) \ldots W_I(N)$

*indicates ranked values of groups

Step 7: Calculate Improvement in Objective Function Value

Calculate:

$$C^{m+1} = C^m - \sum_{i,n} {}_i^m(n) \cdot W_i^m(n) \Delta_i^{+m}(n) + \sum_{i,n} \beta_i^m(n) W_i^m(n) \Delta_i^{-m}(n)$$

Step 8: Test for Improvement

Test:

$C^m - C^{m+1} \leq \epsilon$?

no: stop

Yes

Perturb:

Cause basis of Step 4 to be changed by ϵ to be added (subtracted) from $b_i^m(n)$

Appendix C includes a numerical example that illustrates this procedure.

Derivation of Certain Theorems Related to Decomposition. What needs to be shown is that the method of decomposing suggested will converge to the optimal solution. In order to be able to do this, it is convenient to discuss some preliminary concepts associated with the duals.

Assume a general linear program exists such that it includes slack variables among the variables so that the structural constraints are equations.

Minimize

$$C = C_1X_1+\dots+C_{m+n}X_{m+n} \tag{6-27 (a)}$$

subject to:

$$\begin{aligned} a_1X_1+\dots+a_{1,m+n}X_{m+n} &= B_1 \\ \cdots\cdots\cdots\cdots & \\ a_{m,1}X_1+\dots+a_{m,m+n}X_{m+n} &= B_m \end{aligned} \tag{b}$$

$$\text{all } X_i \geqslant 0$$

If it is assumed that the basic, non-degenerate variables $X_1, \ldots,$ X_m are the current optimal solution, then we have $X_i \geq 0$ for $i = 1, \ldots, m$, and $X_i = 0$ for $i = m+1, \ldots, m+n$.

Solving the resulting equations for X, a solution in the form of

$$AX = B, \text{ or } X = A^{-1}B$$

is obtained.

Here A is an m by m square matrix. Now if a variable was to be introduced whose value was zero to begin with (i.e. X_p is a nonbasic variable) then some reduction in the nonzero outputs would be necessary. If this change is indicated by X_i, then the constraint equations would become:

$$a_{1,1}(X_1-\Delta X_1)+\ldots+a_{1,m}(X_m-\Delta X_m)+a_{1,p} = B_1, \qquad (6\text{-}28)\ (a)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m,1}(X_1-\Delta X_1)+\ldots+a_{m,m}(X_m-\Delta X_m)+a_{m,p} = B_m, \qquad (b)$$

or in matrix terms,

$$A(X-\Delta X)+A_p = B, \qquad (6\text{-}29)$$
$$\text{or } AX-A\Delta X+A_p = B$$

where A_p is the vector of the pth column of coefficients.

Subtracting $AX = B$ the following is obtained,

$$A_p = A\,X, \text{ or } X = A^{-1}A_p. \tag{6-30}$$

If $\overline{C} = (C_1, C_2, \ldots, C_m)$, then the dual values

$$W = (W_1, W_2, \ldots, W_m)$$

are given by:

$$W = \overline{C}A^{-1}. \tag{6-31}$$

The expression shown above is a definition and a calculation rule for dual values.[126]

It should be noted that A is the original column coefficient corresponding to the final basis. This expression indicates that as long as the same basis is maintained the same dual values will remain.

Proof: $W = \overline{C}A^{-1}$ shown above contains $\overline{C}$ which represents the costs corresponding to the optimal basis, and A^{-1} which is the inverse of the final optimal basis. Therefore, as long as the basis is not changed, then the dual values will not change.

$W_i \epsilon W$ can be interpreted as the marginal cost values with respect to the budget dollars. If the B_i apply to the fixed asset budgets, then W_i is the rate of change of the objective function (minimum total cash outlay) per budget dollar.

Proof:

$$WB = \overline{C}A^{-1}B \tag{6-32}$$

$$= \bar{C}X = C$$

$$WB = C$$

But, $\frac{\partial C}{\partial B} = W_i$, therefore W_i is the rate of change of total cash outlay per dollar budget outlay.

This indicates that improvement in the objective function of a decomposed problem can be brought about by allocating additional dollars to one or more of the constraints. The scheme presented above attempts to do this in an efficient way for the smaller subproblems. Of course there is a limit to the extent to which an allocation can be made. There may not be enough funds to allocate, or during the reallocation, the budgets may either be cut in one problem or increased in another subproblem to such an extent that an infeasibility may be created. The following illustrates the limits within which the reallocation may range.

The limits for reallocation of budgets must have the following range:

$$\max - \frac{X_i}{(a_{i,k})^{-1}} \leqslant \Delta B_k \leqslant \min - \frac{X_i}{(a_{i,k})^{-1}} \qquad (6\text{-}32)$$

Proof: Let the optimal solution vector be stated as:

$$X^o = A^{-1}B \geqslant 0 \qquad (6\text{-}34)$$

Let the change in B be B_k where $B = (B_1,\dots,B_k,\dots,B_m)$, then if B^* is the new right hand side,

$$X^* = A^{-1}B^* = (X_1 + (a_{i,k})^{-1}\Delta B_k) \geqslant 0 \tag{6-35}$$

for all i in the basis and where $(a_{i,k})^{-1}$ is the element in the ith row and kth column of A^{-1}.

Therefore, solving above inequality for B_k, the following is obtained.

For

$$(a_{i,k})^{-1} > 0 \tag{6-36}$$

$$\Delta B_k \geqslant \frac{-X_i}{(a_{i,k})^{-1}}$$

and for

$$(a_{i,k})^{-1} < 0 \tag{6-37}$$

$$\Delta B_k < \frac{-X_1}{(a_{i,k})^{-1}}$$

The algorithm takes advantage of the existence of limits to reallocate budgets among subproblems so that at each allocation funds go to those that can make the most use of it (high ranking dual values) from those that have not as much "profitability" associated with the funds at their disposal.

CHAPTER VII

CAPITAL BUDGETING, CAPITAL MARKET IMPERFECTIONS, AND FIXED ASSET MODELS

Introduction

The six models developed thus far have basically dealt with the operational, or the physical, aspects of fixed asset acquisitions. The seventh model classification illustrated in Table 2 of Chapter V, introduces certain financial considerations that improve the realism of the models already developed.

Investment in fixed assets is a form of capital investment, and, therefore, is not only affected by the physical aspects influencing decision-making as we have thus far analyzed them, but also by the financial aspects. The financial aspects include the analysis of the problem in the light of various capital rationing situations and capital market imperfections. The importance of the interaction between the financial and physical investment (acquisition) decisions has been noted by many authors. N. H. Jacoby and J. F. Weston say that,

> The two types of decisions (types of financing and the determination of how much to invest) clearly are interdependent. The particular forms and variants of financing that are available at any time to business concerns have an influence of considerable importance upon the amount of the current demand for funds.(127)

Ezra Solomon redefined the scope of financial management to cover both the use and the acquisition of funds. He states three questions as being of fundamental importance in this relation. They are:

1. What specific assets should an enterprise acquire?
2. What total volume of funds should an enterprise commit?
3. How should the funds required be financed?(128)

The first question has been dealt with within the framework of TASCO's fixed assets problem in earlier chapters. The answer to the second question, concerned with commitment of certain amounts of dollars to fixed assets, has already been indicated through the budget prescribed for TASCO by the Parent Company. In this chapter, the third question is analyzed as it related to fixed asset acquisition decisions being made in an imperfect capital market environment.

Solution of the models developed without budgetary restrictions, and with the assumption of a constant cost of capital is based on the assumption of perfect capital markets. Capital rationing by the Parent Company and the existence of different interest rates and costs of capital imply an imperfect capital market under which our analysis will proceed.

If the fixed asset problem had been formulated for the Parent Company, then it would have been quite realistic to take into account the forms of capital financing and their respective costs with a view toward optimizing not only the physical investments but the financial aspects as well. Since, in reality, TASCO does not have any autonomy regarding its finances, certain assumptions contrary to the actual conditions must be made.

The first analysis is based on the assumption that TASCO is allowed to loan or borrow money to finance its own investments in fixed assets. The loans that are considered are those which are made when a surplus of funds, for instance from the sale of used equipment, becomes available and

and no other use for it is seen.

The second analysis considers a situation in which limits are placed on the amounts that could be borrowed each year. The third analysis assumes a sloping supply schedule of funds where the interest rate varies with the amount of debt. A discussion of optimization of decisions regarding equity financing is also included.

The approach taken treats uncertainty through consideration of the attitudes of suppliers of capital toward risk. The analyses are performed for the purpose of showing that financial considerations can be incorporated into the models and meaningful inferences can be drawn through the primal and dual formulations. No claim is made that the methods proposed could be put to use without further refinement.

An approach to formulating and solving interrelationships between financial and physical flows within a firm was first made by A. Charnes, W. Cooper and M. Miller.[129] Weingartner[130] improved upon this analysis, using a similar approach. It is felt that the discussion following is along the same lines as Weingartner's, yet with certain noteworthy differences that are brought out below.

Borrowing and Lending Without Limits - Model 7

The model developed here is similar to the one illustrated in Appendix A of Chapter V. Normally, Model 7 would consist of a number of fixed asset groups in each branch, all tied together by certain restrictions. Because of the excessive number of subscripts that would have to be carried through, which would add little or nothing to the discussion of finances, Model 7 is shown as an extension of Model 3. Budgets or any other restrictions are added directly to a single group. In actuality, of course, a number of branches, each made up of numerous groups, would need to be considered.

In this and the following models of this chapter, similar to previous developments, the present worth of a number of alternative decisions is minimized. The cash outflows that occur are current, not present, values, except where they are considered in the objective function. A basic difference between the earlier models and these models is the consideration of a new set of "cash throw-off" decisions.

A number of variables are redefined: however, similarity with previous definitions is maintained as closely as possible.

n: year, $n = 1,\ldots,N$,

N: horizon year,

CT(n): row vector of present worth of costs associated with each decision A(n) in year n,

A(n): column vector of decisions associated with various modes of replacement and acquisition of assets in year n,

v(n): amount loaned in year n,

d(n): amount borrowed in year n,

r: interest rate at which borrowing and lending are done,

cp(n): capacity row associated with decisions A(n),

cs(n): consistency coefficient row associated with decision A(n),

b(n): purchase and replacement cost row of assets associated with A(n),

FR(n), CS(n), BT(n): functional requirements consistency, and budget requirement constraint values in year n.

Additional notation will be introduced as needed. Lending and borrowing are assumed to be done on a yearly basis, setting them apart from long-term considerations of these variables.

The following model illustrates the basic concepts:

Minimize

$$\sum_n CT(n)A(n) - \frac{v(N)}{(1+r)^{N-1}} + \frac{d(N)}{(1+r)^{N-1}} \qquad (7\text{-}1)\ (a)$$

subject to:

$$cp(1)A(1) \geqslant FR(1) \qquad (b)$$
$$\vdots$$
$$cp(N)A(N) \geqslant FR(N)$$
$$cs(1)A(1) = CS(1) \qquad (c)$$
$$\vdots$$
$$cs(N)A(N) = CS(N)$$
$$b(1)A(1)+v(1)-d(1) \leqslant BT(1) \qquad (d)$$
$$b(2)A(2)+v(2)-(1+r)v(1)+(1+r)d(1)-d(2) \leqslant BT(2) \qquad (e)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$b(N)A(N)+v(N)-(1+r)v(N-1)+(1+r)d(N-1)-d(N) \leqslant BT(N) \qquad (f)$$
$$\text{All elements of } A(n) \geqslant 0,\ n = 1,\ldots,N. \qquad (g)$$

The $1/(1+r)^{N-1}$ terms in the objective function convert v(N) and d(N) to present value. Since the objective function minimizes costs, the last year's debt is entered with a positive sign. The reason for inserting v(N) and d(N) in year N is that since horizon is at N, no borrowing or lending can occur thereafter.

The dual of the above primal problem is stated below:

Maximize

$$\sum_n W_{FR}(N)FR(N) + \sum_n W_{CS}(n)CS(n) - \sum_n W_{BT}(n)BT(n) \tag{7-2 a}$$

subject to:

$$W_{FR}(1)cp(1)+W_{CS}(1)cs(1)-W_{BT}(1)b(1) \leq CT(1) \tag{b}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$W_{FR}(N)cp(N)+W_{CS}(N)cs(N)-W_{BT}(N)b(N) \leq CT(N)$$

$$W_{BT}(N) \leq \frac{1}{(1+r)^{N-1}} \tag{c}$$

$$-W_{BT}(N) \leq \frac{-1}{(1+r)^{N-1}} \tag{d}$$

$$W_{BT}(n-1)-(1+r)W_{BT}(n) \leq 0, \quad n = 2,\ldots,N \tag{e}$$

$$-W_{BT}(n-1)+(1+r)W_{BT}(n) \leq 0, \quad n = 2,\ldots,N \tag{f}$$

$$W_{FR}(n), W_{CS}(n), W_{BT}(n) \geq 0, \quad n = 1,\ldots,N \tag{g}$$

The dual variables represent marginal, or opportunity, values associated with the resources. W_{BT} could be viewed as the marginal return from the additional investment of a budget dollar. W_{FR} is the marginal cost of a unit of functional requirements. The exact meaning and interpretation of W_{CS} is however, quite difficult, since the sign of cs(n) can be positive or negative depending on the particular equation.

Inequalities (2c) and (2d) can be written as

$$\frac{1}{(1+r)^{N-1}} \leqslant W_{BT}^{*}(N) \leqslant \frac{1}{(1+r)^{N-1}} \qquad (7\text{-}3)$$

This implies that

$$W_{BT}^{*}(N) = \frac{1}{(1+r)^{N-1}}$$

Inequalities (7-2e) and (7-2f) indicate that

$$W_{BT}^{*}(n-1)-(1+r)W_{BT}(n) = 0, \quad n = 2,\ldots,N$$

Therefore,

$$W_{BT}^{*}(n-1)-(1+r)W_{BT}^{*}(n) = 0,$$

$$\frac{W_{BT}^{*}(n-1)}{W_{BT}(n)} = 1+r. \qquad (7\text{-}4)$$

If we combine result (7-3) with (7-4) we have

$$W_{BT}^{*}(n) = (1+r)W_{BT}^{*}(n+1) = (1+r)W_{BT}^{*}(n+2) \qquad (7\text{-}5)$$

$$= \ldots = (1+r)^{N-n}W_{BT}^{*}(n)$$

$$= \frac{(1+r)^{N-n}}{(1+r)^{N-1}} = \frac{1}{(1+r)^{n-1}}$$

Equation (7-4) indicates the annual incremental rate of interest

*An asterisk indicates a particular value of the variable.

at which borrowing and lending takes place. Equation (7-5) shows that the present value of a budget dollar in year n is equivalent to that of the present value of a dollar borrowed in year n.

Weingartner(131) is able to make a number of additional interpretations, basically due to the fact that he analyzes a number of investment alternatives where each investment is constrained to be between 0 and 1. Since no such relationship can have a meaningful part in the present formulation, a number of otherwise meaningful interpretations cannot be done.

Absolute Limits on Debt - Model 7

Placing absolute limits on the debt that TASCO can carry at any time and considering the implications of such limits is the basis of the following investigation. If expenditures for assets are generally made from internally generated funds (retained earnings, depreciation), reflecting the attitude of management toward debt, then absolute limits on debt would be a conservative policy, in accordance with this type of management thinking.

The only additional notation introduced is D(n), which is the absolute upper limit TASCO can borrow from external sources at an effective interest rate, r. Concerning the interest rate H. Bierman and S. Smidt point out,

> The appropriate rate of interest rates in future time periods are relevant to decisions made in the present because they affect the profitability of funds reinvested at those times. Cash flows expected in each future time period should be discounted at the rate of interest that will apply in that period. But how is this to be predicted? Generally, it will not be difficult to predict future lending and borrowing rates. Given these predictions, it will be safe to assume that the appropriate

rate of discount for cash future will be somewhere between these upper and lower limits.(132)

A model that can be proposed in order to study the topics at issue is

Minimize

$$\sum_{n} CT(n)A(n) - \frac{v(N)}{(1+r)^{N-1}} + \frac{d(N)}{(1+r)^{N-1}} \qquad (7\text{-}6)\ (a)$$

subject to:

$$cp(1)A(1) \geq FR(1) \qquad (b)$$
$$\vdots$$
$$cp(N)A(n) \geq FR(N)$$
$$cs(1)A(1) = CS(1) \qquad (c)$$
$$\vdots$$
$$cs(N)A(N) = CS(N)$$
$$b(1)A(1)+v(1)-d(1) \leq BT(1) \qquad (d)$$
$$\cdots\cdots\cdots\cdots \qquad (e)$$
$$b(N)A(N)+v(N)-(1+r)v(N-1)+(1+r)d(N-1)-d(N) \leq BT(N) \qquad (f)$$
$$d(1) \leq D(1) \qquad (g)$$
$$\vdots$$
$$d(N) \leq D(N)$$
$$\text{All elements of } A(n) \geq 0,\ n = 1,\ldots,N. \qquad (h)$$

If the debt constraints are active, then the effect will be to reduce the marginal, or opportunity, value at which the various decisions are evaluated. The formulation of the dual can help explain this point. Maximize

$$\sum_{n} (W_{FR}(n)FR(n)+W_{CS}(n)CS(n)-W_{BT}(n)BT(n)-W_{D}(n)D(n)) \tag{7-7 a}$$

subject to:

$$W_{FR}(1)cp(1)+W_{CS}(1)cs(1)-W_{BT}(1)b(1) \leq CT(1) \tag{b}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$W_{FR}(N)cp(N)+W_{CS}(N)cs(N)-W_{BT}(N)b(N) \leq CT(N) \tag{c}$$

$$-W_{BT}(N) \leq \frac{-1}{(1+r)^{N-1}} \tag{d}$$

$$-W_{D}(N)+W_{BT}(N) \leq \frac{1}{(1+r)^{N-1}}$$

$$-(1+r)W_{BT}(n)+W_{BT}(n-1) \leq 0; \; n = 2,\ldots,N \tag{e}$$

$$(1+r)W_{BT}(n)-W_{BT}(n-1)-W_{D}(n-1) \leq 0; \; n = 2,\ldots,N \tag{f}$$

$$W_{FR}(n), W_{CS}(n), W_{ST}(n), W_{D}(n) \geq 0; \text{ for } n = 1,\ldots,N \tag{g}$$

As long as the amount borrowed, d*(n), is less than D(n), this model is similar to the one presented under a no limit case, with $W_D^*(n) = 0$. Therefore, equations (7-6e) and (7-6f) can be written as

$$0 \leq -(1+r)W_{BT}^*(n)+W_{BT}^*(n-1) \leq 0 \tag{7-8}$$

or

$$\frac{W_{BT}^*(n-1)}{W_{BT}^*(n)} = 1+r$$

which is the incremented annual marginal return from a budget dollar.

Condition (7-7<u>c</u>) seems to allow values of $W_{BT}(N)$ to be greater than $1/(1+r)^{N-1}$, in contrast to the previous unlimited borrowing capability case. By combining (7-7<u>c</u>) and (7-7<u>d</u>) we have

$$\frac{1}{(1+r)^{N-1}} \leqslant W_{BT}^*(N) \leqslant \frac{1}{(1+r)^{N-1}} + W_D(N). \tag{7-9}$$

This indicates that the marginal return at which alternatives in Nth year are being evaluated can be greater than $1/(1+r)^{N-1}$ by an amount equal at most, to the marginal return associated with a borrowed dollar. However, that in fact this cannot be so is illustrated by the following argument.

Inequalities (7-7<u>e</u>) and (7-7<u>f</u>) can be written as

$$(1+r)W_{BT}^*(n+1) - W_D^*(n) \leqslant W_{BT}^*(n) \leqslant (1+r)W_{BT}^*(n+1).$$

This expression indicates that $W_{BT}^*(n)$ at most can be equal to $(1+r)W_{BT}^*(n+1)$. Therefore this indicates that $W_D^*(N)$ in (7-9) must be zero. Therefore it can be seen that in year N borrowing up to the limit cannot take place, and that

$$W_{BT}^*(N) = \frac{1}{(1+r)^{N-1}}$$

If borrowing is done up to the limit, then $W_D^*(n) > 0$ and the expression (7-8) is replaced by the more complicated

$$W_{BT}^{*}(n) = \frac{1}{(1+r)^{N-1}} - \sum_{\eta=1}^{n} (1+r)^{1-\eta} W_{D}^{*}(\eta) \qquad (7\text{-}10)$$

Expression (7-9) shows that factors other than r affect the choice of alternatives when borrowing limits are imposed. The opportunity value, or return on investment, of a budget dollar is no longer a simple expression but involves an additional term due to borrowing. This indicates that as long as some borrowing limits are active, then the value of a budget dollar is less than the lending and borrowing rate r.

Imperfections Introduced by a Changing Supply Schedule of Funds - Model 7

A more realistic approach to the risk factor than placing absolute limits on the amount to be borrowed is a funds supply curve that shows a rise in the interest rate as the borrower's debt rises in proportion to his equity. The increased rate of interest is due to the increased risk incurred when larger amounts are borrowed. Brigham and Smith[133] indicate that the supply schedule of funds is not only dependent on the amount borrowed but also on the size of the company that is doing the borrowing.

In our analyses we will assume that the size of the Parent Company is such that it can effectively be considered a "large" company, with income well in excess of the $25,000 that was indicated by Brigham[134] to be the line that separates large from small.

Figure 20 shows the average and marginal costs of debt as a function of debt.

The sloping supply schedule of funds is taken into account by specifying the rate applicable to the marginal debt amount. The interest

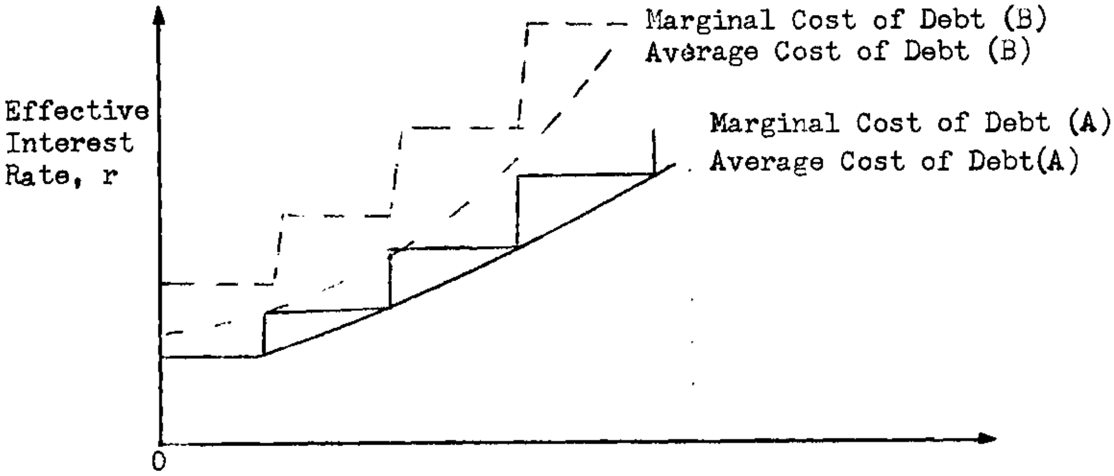


Figure 20. Effective Interest Rate as a Function of Debt.

rate applicable to the jth increment d_j is denoted by r_j.

Minimize

$$\sum_n CT(n)A(n) - \frac{v(N)}{(1+r_N)^{N-1}} + \frac{\sum_{j=1}^{J} d_j(N)}{(1+r_N)^{N-1}} \qquad \text{(7-11) (a)}$$

subject to: (same as in (7-8b)) (b)

(same as in (7-8c)) (c)

$$b(1)A(1)+v(1)-\sum_j d_j(1) \leqslant BT(1) \qquad \text{(d)}$$

$$b(2)A(2)+v(2)-(1+r)v(1)+\sum_j(1+r_j)d_j(1)-\sum_j d_j(2) \leqslant BT(2) \qquad \text{(e)}$$

. . . .

$$b(N)A(N)+v(N)-(1+r)v(N-1)+\sum_j(1+r_j)d_j(N-1)-\sum_j d_j-(N) \leqslant BT(N) \qquad \text{(f)}$$

$$\sum_j d_j(n) \leqslant D(n), \text{ for } j = 1,\ldots,J;\ n = 1,\ldots,N \qquad \text{(g)}$$

$$\text{All elements of } A(n) \geqslant 0 \qquad \text{(h)}$$

Since $r_{j-1} \leq r_j \leq r_{j+1}$, and the objective function is being minimized; borrowing will be done using, first, all the funds with the lowest interest rate, then, as these funds are used, those of the next higher rate, and so on. The dual of this problem has implications similar to the ones discussed under the previous section that dealt with a single interest rate.

It was mentioned in the introduction to this study that the aerospace industry presently is being pressured into spending more of its own funds for facilities. This fact, combined with increased incentive type contracts, places a heavier burden of risk on the firms. (A) in Figure 20 indicates the interest cost for a firm in a less risky position than (B). Numerous authors in the field indicate that leverage (debt to equity ratio) is a good measure of the riskiness of the business, so this ratio is watched quite closely by financial analysts.

Optimization of Decisions Regarding Equity Financing

The amount and timing of equity financing, and optimization of decisions in this area, can be suitably handled by models described thus far.

As compared to debt, which is the amount of funds owed to the creditors, equity is the amount of funds contributed by the stockholders, or owners, of the business. Owners of the firm view additional stock issues desirable only if the future earnings brought about by an issue is more than the decrease in the total earnings which will accrue to them.

The holders of common stock rank last in the priority of claims on liquidation, which indicates that the capital they contribute provides a cushion for creditors if losses occur on liquidation. The ratio of equity to total assets indicates the percentage by which assets may shrink in value before the creditors sustain a loss. Leverage ratios also indicate the risk associated with the enterprise, and measure the contributions of owners as compared with the financing provided by the firm's creditors.

Weston and Brigham indicate certain implications of leverage ratios:

> First, creditors look to the equity, or owner-supplied funds, to provide a margin of safety. If owners have provided only a small proportion of total financing, the risks of the enterprise are borne mainly by the creditors. Second, by raising funds through debt the owners gain the benefits of maintaining control of the firm with a limited investment. Third, if the firm earns more on the borrowed funds than it pays in interest, the return to the owners is magnified.(135)

From the point of view of the firm, stocks are regarded as loans which call for no repayment. In the model developed below we assume that the firm follows a divident policy of paying out v dollars per share per year.

Flotation costs associated with debt and with stocks are indicated by f_d and f_s respectively, and will be assumed to be proportional to the size of the issue. The rates are greater for smaller issue sizes, and decrease as the size of the issue increases.[136] Another imperfect market consideration that is added to the model to make it more realistic is variation of interest rates from year to year.

Specifically, the model proposed has four significant improvements that help make it more realistic. These are: consideration of dividends, interest rates on debt as a function of the size and year the debt is incurred, flotation costs, and leverage limits.

The solution to optimizing the amount and the timing of stock issues, and incurring debt over a planning horizon of N years is solved in two stages. First the present value of the per share value of cash outlays is determined without incremental stock issues, and then the model is solved with stock issues. The particular timing and amount of stock issued needs to be determined parametrically.

The additional notation to be used in the model is illustrated below:

f_d: a fixed flotation cost for debt,

$f_s(q(N))$: stock flotation cost that varies as a function of size,

k: a fixed dividend value per share,

q(n): number of incremental stock shares issued in year n,

e(n): forecast of earnings per share in year n,

l: an upper limit for leverage,

C^o: present value of total cash outlay without additional issues of stock,

C': present value of total cash outlay with additional issues of stock,

Q^o: number of outstanding shares at beginning of study, without issuing additional stock.

The model is represented as

Minimize

$$\sum_n CT(n)A(n) - \frac{v(N)}{(1+r)^{N-1}} + \frac{\sum_{j=1}^{J} d_j(N)}{(1+r)^{N-1}} \qquad \text{(7-12)(a)}$$

subject to: (same as in (7-8b) and (7-8c)) (b)(c)

$$b(1)A(1)+v(1)-\sum_j d_j(1)(1-f_d)-q(1)(e(1)-k-f_s(q(1))) \leqslant BT(1) \qquad (d)$$

$$b(2)A(2)+v(2)-(1+r)v(1)+\sum_j(1+r_j(1))d_j(1)-d_j(2)(1-f_d) \qquad (e)$$

$$-q(2)(e(2)-k-f_s(q(2))) \leqslant BT(2)$$

. . . .

$$b(N)A(N)+v(N)-(1+r)v(N-1)+\sum_j(1+r_j(N-1)d_j(N-1)-d_j(N)(1-f_d) \qquad (f)$$

$$-q(N)(e(N)-k-f_s(q(N))) \leqslant BT(N)$$

$$d(1)(1-f_d) \leqslant 1(BT(1)+q(1)(e(1)-k-f_s(q(1))) \qquad (g)$$

. . . .

$$d(N)(1-f_d) \leqslant 1(BT(N)+q(N)(e(N)-k-f_s(q(N))) \qquad (h)$$

All elements of $A(n)$ are nonnegative (i)

Solution of above model to determine C^o must be done parametrically since the interest rate on the loan (no stock issue considered for C^o) is a function of the year and the size of the loan. The initial run could be made by letting $r(N,d(N))-r_j(N)$, where * indicates specific values of $d(n)$. Based on the resulting $d(1)$, $d(2)$,..., $d(N)$ values, a new run with new r values could progressively result in increased accuracy of C^o. (Note: If f_d also depended on the size of the debt, a similar parametric procedure could converge on the proper value.)

Once the C^o is found, then per share present worth of cash outlays for N years can be expressed by C^o/Q^o.

It is assumed that for purposes of this study, additional stock issues will become necessary whenever the per share cost of the above solution, including stock issues, is less than (over N years) the per share cost of the above solution without stock issues. This can be expressed as:

$$\frac{C^o}{Q^o} \geq \frac{C'}{Q^o \sum_n q(n)}$$

Equation (7-10) could be expressed as:

$$C' - (C^o/Q^o) \sum_n q(n) \leq C^o.$$

The solution of the left hand side of the inequality depends on C^o and Q^o, which are parameters corresponding to the no stock issue case. That particular $\sum_n q(n)$ that results in the lowest left hand side can be found by a parametric search.

i.e.

$$\sum_n q(n) \leq \beta$$

where β is some arbitrary number greater than zero.

By searching over various values of β this minimum value can be found.(137) Once the minimum value is found, another parametric analysis could be conducted, similar to determining $r(n,d(N))$ values, to determine the $f_s(q(N))$ values.

Conclusion

This chapter has shown a possible extension of the six models already developed to a seventh one, in order to include a number of financial considerations under various market imperfections.

In formulating Model 7 the basic assumption was that TASCO is autonomous and is capable of entering financial arrangements on its own. The financial considerations discussed were made under the assumptions of the company's ability to borrow and lend without limit, the imposition of absolute limits on debt, and a changing supply schedule of funds. Optimization of decisions regarding equity financing was also discussed briefly within the context of the models developed. It was shown that meaningful inferences can be drawn from the duals of the models as to the marginal returns on fixed asset investments, thereby allowing financial and physical aspects of the problems to be considered jointly in making better decisions.

CHAPTER VIII

IMPLEMENTATION CONSIDERATIONS

Introduction

This chapter discusses some of the details of implementation of the previously proposed models. First, the cash flow calculations for Model 6, in the context of the aerospace industry, are illustrated with an example. Appendices D through G include the raw data for an actual case, together with its solution.

Second, the approach to solving fixed assets problem is formulated as an information system with data inputs, and a report output. The point of view taken is that of a typical aerospace firm that is interested in implementing such a system.

Finally, the organizational aspects and the question of independent, contingent, and mutually exclusive fixed assets are discussed as they relate to the proposed approach.

Cash Flow Considerations

Since the models developed have as their objective function cost minimization, all cash outlays are considered. Table 9 shows the factors that affect the cash flows associated with the acquisition of a new fixed asset through TASCO funds.

It is assumed that the planning horizon is for a period of five years, the asset has an initial acquisition cost of $10,000, and a salvage revenue of $4,000 at the beginning of the sixth year. The life of the asset for depreciation purposes is assumed to be ten years, and a double declining balance depreciation method that converts to a straight line after mid-life is assumed.

The description of costs are shown below:

1. Acquisition Cost, $p(n)$: This refers to the purchase and installation cost of the new asset in year n. Here, $n = 1$.

2. Operating Costs, $op(n,t)$: These costs include the direct labor and pertinent overhead cost items. t refers to the age of the asset.

3. Progress Payments, $ppo(n,t)$: These are that portion of the operating costs that are paid to TASCO to support its working capital needs. The fraction of the total investment for the next N years can be forecast, and the product of these factors, $PP(n)$, with the operating cost figures, yields the $ppo(n,t)$ figures.
e.g.

$$ppo(1,0) = (PP(1))(op(1,0)) = (0.764)(1000) = \$764.$$

4. Investment Credit, ic: This is assumed to be 7 per cent of the original acquisition cost.
e.g.

$$ic = (0.07)(10,000) = \$700.$$

5. Depreciation, $d(t)$: Depreciation is not a cash flow item, therefore, does not affect before-tax cash flow calculations. Based on a ten year life, the double declining balance rate is 20 per cent, and

Table 9. Cash Flows Associated With A Fixed Asset

	Costs:	Year, n 1	2	3	4	5	6
1.	Acquisition Cost	+10000.					
2.	Operating Costs	+ 1000.	+1000.	+1000.	+1000.	+1000.	
3.	Progress Payments	- 764.	- 732.	- 694.	- 681.	- 626.	
4.	Investment Credit	- 700.					
5.	Depreciation	+ 2000.	+1600.	+1280.	+1024.	+ 819.	
6.	Progress Payments	- 1528.	-1171.	- 888.	- 697.	- 513.	
7.	Maintenance	+ 200.	+ 200.	+ 200.	+ 200.	+ 200.	
8.	Progress Payments	- 153.	- 146.	- 139.	- 136.	- 125.	
9.	Rental Fee to Government (Applies only to Government Assets)*	+ 299.	+ 346.	+ 664.	+ 806.	+ 667.	
10.	Return of Progress Payments** (3)+(6)+(8)		+2445.	+2049.	+1721.	+1514.	
11.	Sharing Portion of Cost** SP(n)((2)+(5)+(7))		-1850.	-1946.	-1701.	-1610.	
12.	Tax Impact on Costs** 0.528((2)+(5)+(7)-(11))		- 713.	- 502.	- 282.	- 276.	
13.	Salvage Revenue Impact on Cash Flow						-3618.
14.	Net Cash Flow (1)+(2)+(3)+(4)+(6)+(7)+(8) +((10)+(11)+(12))+(13)	+ 8055.	- 967.	- 920.	- 576.	- 436.	-3618.

*This cost only applies to government assets, therefore, is not included as part of the Net Cash Flow calculations of (14).

**Lag One Year

the following illustrates the yearly changes.

Year	Depreciation Charge
1	$2000.
2	1600.
3	1280.
4	1024.
5	819.
6-10	655.

6. Progress Payments, ppd(n,t): This item is similar to (3) in that a portion of the depreciation charges against the asset are recovered through progress payments.

e.g.

$$ppd(1,0) = (d(1,0))(PP(1)) = (2000)(0.764) = \$1528$$

7. Maintenance, m(n,t), and (8) Progress Payments, ppm(n,t): These two items are similar to Operating Costs and the Progress Payments associated with them as illustrated in (2) and (3).

9. Rental Fee to government, gr(n,t): If the asset in question is a government asset then TASCO must pay the government a rental fee for use of these assets on nongovernment business. This fee depends on the acquisition cost of the asset and a rate schedule that takes into account the age of the asset. Another factor that must be taken into account is the per cent of time the asset is used for commercial business.

Lag One Year: A lag of one year is introduced to take into account the time lag between the actual incurring of a cost, and certain cost recoveries that take place upon delivery of an aircraft. A typical example of this is the tax benefits due to cost that can only be recovered upon sale of the aircraft.

10. Return of Progress Payments: Upon delivery of product of the progress payments are returned to the customer by application of a stated percentage to the invoice price of the products.

11. Sharing Portion of Cost, cs(n,t): This refers to the cost overruns or underruns that are shared between TASCO and the customer according to certain contractual rules. It is possible to forecast a factor for each year, up to the planning horizon, that represents the fraction, SP(n) of a cost dollar that would be recovered.

e.g.

$$cs(2,1) = SP(1)(oc(1,0)+d(1,0)+m(1,0)) = (0.578)(3200) = \$1734.$$

12. Tax Impact on Costs: This item is the tax benefit that results due to costs that were charged in the preceding year.

13. Salvage Revenue Impact on Cash Flow, s(n,t): This represents the impact of the salvage revenue of the asset at the end of the planning horizon, and for purposes here has been also assumed to be the terminal value of the asset. The calculations for the five year old asset at beginning of the sixth year can be shown as follows:

$$s(6,5) = s(6,5)-(p(1)-\sum_{t=1} d(t))(0.528) = 4000-382 = \$3618$$

where s(n,t) is the salvage revenue associated with an asset of age t years, at beginning of year n.

14. Net Cash Flow, NCF: This is the sum of the rows as shown in Table 9.

It should be noted that since progress payments are made weekly and a number of costs are incurred continuously (or weekly) throughout the year, appropriate discounting must be applied to such costs. Since the concept of discounting does not present undue difficulty it will not be further discussed here. The linear programming objective function coefficients for other decisions, such as "Keep, Replace" are computed, using rules similar to the ones illustrated in Table 9.

Implementation Considerations

The System

The linear programming fixed asset investment models discussed in previous chapters could best be implemented if considered as part of an overall system. Such a system is illustrated in Figure 21.

The system portrayed treats fixed assets decision making similar to a process control. At certain points in time data in the form of functional requirements, capacities, costs, budgets, and initialization values are fed into a centralized data bank. This data is then fed into the computer and a solution is obtained by a linear programming routine. The solution consists of a number of decisions that are in turn fed into a decision evaluator. A decision evaluator basically consists of a number of managers who evaluate the soundness of the decisions. A number of factors, some intangible, affect their final evaluation. These extraneous

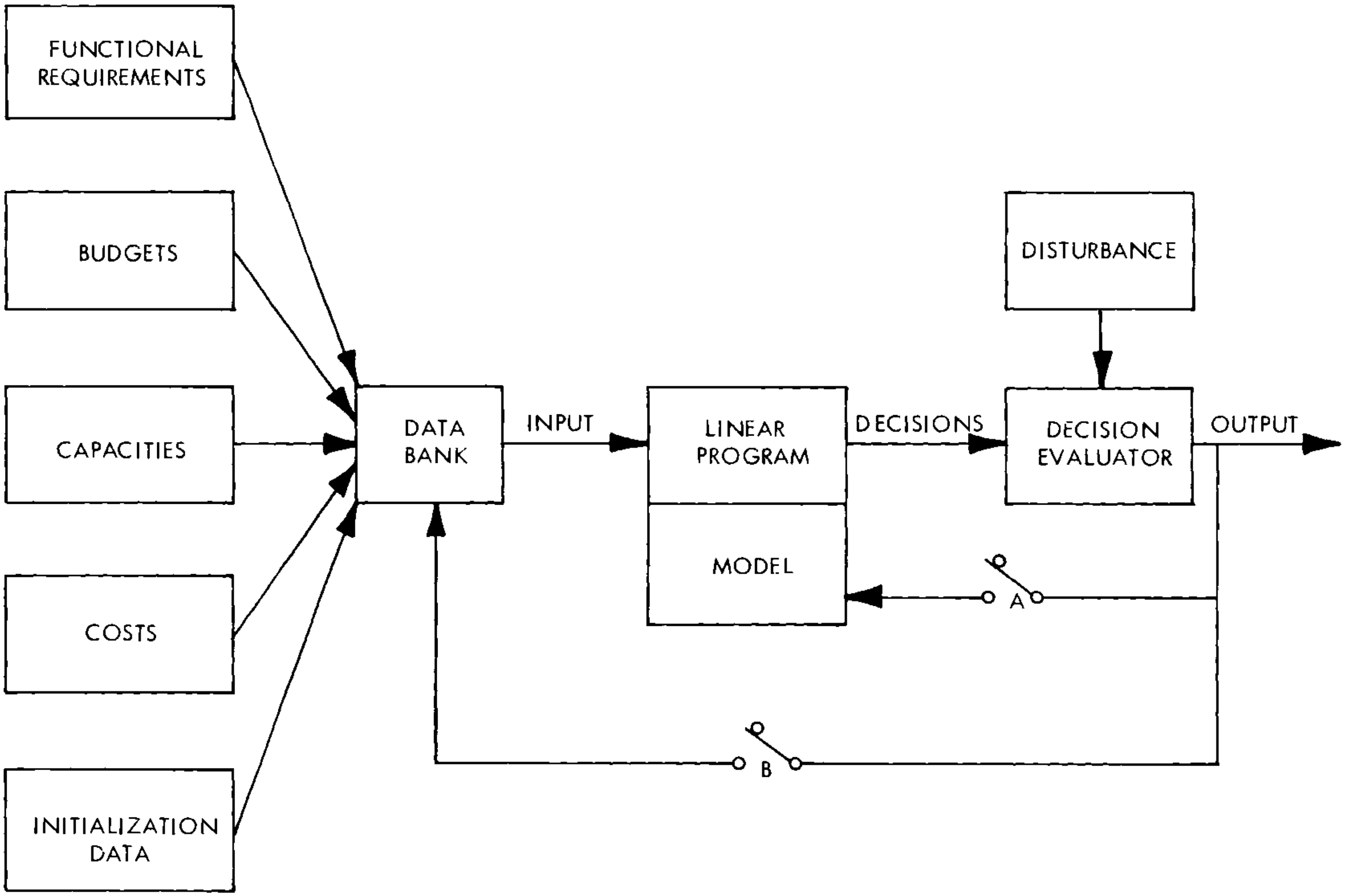


Figure 21. The Fixed Asset Decision System

factors are shown as "Disturbance." The final decisions of the managers became the output. This output could then be used to change the basic model (Switch A) or some of the input data (Switch B). Impacts of various budget levels, functional requirements, or other factors could be determined by changing certain input data and recycling the system. Ability of the system to minimize the amount of effort at the evaluation stage is highly desirable, and the feedback through A and B will insure the smooth operation of the system. The response of the system to disturbances and alternative evaluation queries depends on the time it takes to quantify them.

Chapter III discussed the "Fixed Asset Budget Cycle" of TASCO. The system proposed could be made a part of this cycle and can be made to yield decisions on a routine basis. Since the response time is quite short, quarterly or biannual runs could be easily implemented.

Figure 22 illustrates the details of the block shown as "Linear Program" in Figure 21. Output from the data bank is shown as being fed into a matrix generator. The matrix generator, as the name implies, creates a matrix with the data in appropriate places for solution by linear programming. The generator has N as a parameter and can create a matrix for any N, until limited by memory size of the computer. The generator has the capability of blocking off TASCO, or government decisions; including or excluding terminal values, and including I fixed asset modules.

The report generator consists of a program that makes up a tabular report, complete with decisions, related budgets and cash flows. See Appendix H.

Inputs to the System

This section will discuss in further detail the manner of obtaining the inputs to the data bank.

Functional Requirements. This is the most important data since it directly influences decisions which concern the acquisition of adequate capacity to meet the long range plans of the company. This data is also more liable to change than any other data. There is no single method of determining this data since a number of intangibles affect it, and each method has its advantages and disadvantages.

It may be best to describe the activities related to fixed assets as being either related to present business (firm aircraft delivery schedules) or to some future business (research and development). The functional requirements related to present business can be arrived at quite accurately; however, as the time span is increased, the present contracts will decrease in number. This, of course, is offset by new business in which the firm will hope to get involved at an ever increasing rate. The sum of yearly firm business requirements and expected new business requirements will result in the needed data.

If utilization of fixed asset groups can be obtained for the past several years, multiple regression analysis relating utilization to aircraft deliveries, sales, or direct labor hours can be performed, and these values can then be cross-checked with other methods of determining this data. Appendix E includes an illustration of this approach to the determining of functional requirements.

Capacity Data. After having decided on the grouping of fixed assets by consideration of TASCO owned and government owned existing assets, a

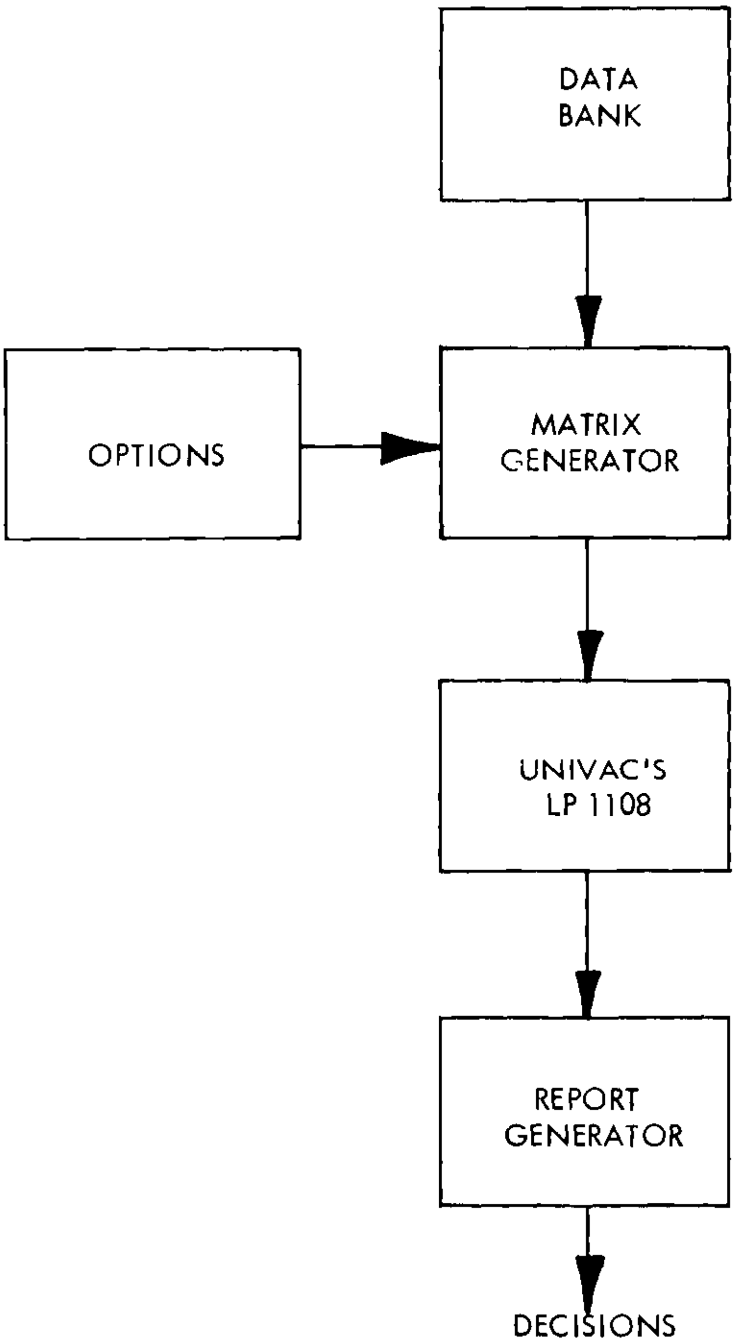


Figure 22. Basic Elements of the Computer Program

typical asset for each group is selected. Based on the age and capacity distribution of the particular group of assets, a number of typical assets having a certain age are assumed to be equivalent to the existing assets. The yearly output capacity of the typical asset is recorded for each of the years in the planning horizon. Deterioration in the output of the asset plays a large part in the fluctuation of this data.

Capacity data for each typical new asset is also recorded and is influenced by the possible technological improvements forecasted. (The capacities of assets purchased in later years but kept from one year up to the planning horizon are also recorded.) Of course the fact that all these capacities are expressed in typical units results in decisions being made in terms of such units.

Appendix F includes this data for two sample fixed assets. It also would be sound to limit the number of groups to, for instance, those having purchase prices of $5,000 or more, in the initial runs. This will limit the scope of applicability of the model; however, it is felt that such screening will make the initial data collection easier.

Cost Data. Cost data consists of purchase, replacement, rental or subcontracting, salvage, operating and maintenance costs. Of these, operating cost forecasts are hardest to come by, due to the influence of the number of components that make them up. Each one of these costs will be discussed below.

Purchase and replacement costs relate to the purchase or replacement costs of a new item defined as being "typical" for this study. An outright purchase occurs when no specific item is being replaced, and the cost includes the basic acquisition cost, including the normal complement of tools, accessories, jigs, motors, freight, sales tax, etc. Also included is the cost of installation, such as the cost of foundation, wiring, etc. Replacement cost also includes these items; however, the trade-in value of the old asset that is being replaced is subtracted from the purchase cost. In effect, therefore, the replacement cost is the net sum of money the firm has to pay to obtain the asset, have it installed and have the old one removed. It is quite important to include all related costs of shipping and, if necessary, disassembling the old asset, since without the inclusion of all these costs the replacement value would be a biased estimate. Similar to the other data being collected, these values will need to be forecasted for new assets every year, for the duration of the horizon, and for all ages of assets acquired during consecutive years. Appendix G shows a numerical example of how this data could be developed. Data could be developed by the firm making the fixed assets study, by "feeling out" the market and the manufacturers of various machinery and equipment. Another method could be to employ the services of certain firms whose business it is to obtain such data.*

Salvage revenues are values obtained from selling a typical used asset in the open market, and differ from the trade-in values which, as the name implies, are sums the manufacturer of the assets allows to be subtracted from the purchase price, based upon his receiving the specified "old" asset. There are possible occasions when the trade-in value is lower than would be a sale in the open market.

Operating and maintenance costs include a number of components;

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*One of the firms involved in this type of activity is American Appraisers, Inc.

however, some may be much harder to estimate than others. The components may be listed as:[138]

Direct labor, including overtime and shift premiums

Set-up time

Indirect labor

"Fringe" labor costs

Ordinary maintenance

Special repairs

Tool costs

Supplies

Defective material - rework

Spoilage - scrap

Downtime - outage

Power consumption

Floor space, if usable

Property taxes and insurance

Other

Direct labor includes straight time rate as well as overtime and shift premiums if that is a normal pattern of operation, and also is taken into account in the capacity data section. It is necessary to examine variables such as speeds and feeds, make-ready and set-up time, inspection, stock-supply, stock-loading, moving or cleaning.

If set-up time is significant and rates for set-up men vary from those of the operators, then this must be calculated.

Indirect labor should not be based on overhead rates but must be investigated for each fixed asset group.

Fringe labor costs include paid vacations, social security tax, insurance, and other benefits paid to the employee by the company.

Ordinary maintenance and special repairs fall under the maintenance costs. Ordinary maintenance is the preventive type of maintenance involved in oiling, greasing, and making minor adjustments. Special repairs are the unscheduled, random failures, such as major adjustments, trial runs and experimentation.

Tool costs will change with the fixed asset groups, and with the typical assets chosen for replacement purposes. If tool life is shorter than the machine or equipment, then a new tool purchase cost must be shown together with the operating costs.

Supplies refer to the costs associated with the operation of the equipment, such as saw blades, flux, etc. Defective material (rework) and spoilage (scrap) refer to the direct, indirect, and material costs of reworking or scrapping parts. It is quite difficult to isolate the cause of the problem to lack of operator training, or carelessness of the operator, or fault of the machine. Therefore, good judgment must be used in this area to isolate the causes, and to predict such costs.

Downtime (outage) occurs during operation of the equipment. In certain cases such occurrences are quite costly, due to schedule problems created downstream; in other instances the job could easily be routed to another station with no loss of time. Each groups' load forecasts, as well as historic load and schedule data, must be analyzed prior to associating costs to these outages. Power consumption, floor space, and taxes and insurance must also be taken into account on an individual group basis.

The Use of Outputs

Appendix H shows the details of an output based on one fixed asset group without any budgetary limits imposed.

It should be understood that even though a number of decisions will be recommended by the model, the final decision rests with the top management of the company. A number of intangibles that are hard to quantify always exist and could reverse even the best of the analyses. Even though an action may not be forthcoming, an important aim will have been achieved--that of making management aware of which equipment needs to be acquired or replaced.

A common occurrence in a number of initial runs of Model 6 indicated a tendency to replace or salvage assets that were relatively new. In such cases a number of such "undesirable" variables were excluded from the basis. Comparison of the objective function values with an without these variables indicates how much it "costs" to exclude them. An illustration of this approach is shown also in Appendix H.

Organizational Aspects

A firm that is interested in implementing the proposed system will need, during the initial implementation phases, to form a team and have this team report to a level of management high enough to allow inter-branch decision-making. The team should be composed of representatives from the branches that will contribute data, a programmer, and an operations research analyst.

It would be best to implement the system on a pilot study basis, present the results to the management and then enlarge the scope of the study to cover the remaining fixed assets. Once the pilot study is over, efforts should be spent to have a particular branch, such as Finance, assume full responsibility for the system's implementation on a periodic basis. Each branch would have a group that collects and modifies the data pertaining to their assets on a continuous basis. As coordinator, Finance Branch would provide assistance and guidance where needed, and would make the decisions as to when the model would need to be cycled, and who would receive the output.

Some authorization must be given to the Finance Coordinator so that he is able to direct fixed asset counterparts in each branch as to certain details. The operations research analyst should be available to answer any specific questions and to determine if any improvements are needed. The management will also need to decide levels of approval for the expenditures. The same levels that existed prior to the proposed system would seem to be adequate.

Mutually Exclusive, Independent, and Dependent Fixed Assets

If undertaking one investment completely eliminates the need for an other investment, then the two investments are known as mutually exclusive. This problem does not arise in the models that have been developed since the alternatives are not defined in terms of substitution of one item for another. The typical fixed asset for a group is never compared with another type of item that can be substituted for it. This question, though important, is beyond the scope of this research. Therefore, questions, such as, "Should A or B be considered to satisfy a certain fixed asset requirement?", do not arise. Decisions as to A or B are made outside the model, and once having been made, the model determines least cost replacement, acquisition, or salvage policy with respect to the item.

Of course, the model could be enlarged to include choices between mutually exclusive investments; however, this could seriously impair the practical utility of the model, due to the increase in dimensionality. For all practical purposes, it would seem desirable to have this analysis precede the use of the model, or to exercise the model using one alternative at a time.

An investment is said to be independent of another if the cash flows of one investment are not affected by either accepting or rejecting the other investment. If the cash flows of one investment are affected by the other, then the two investments are dependent. Such dependence could also exist between more than two investments.

In the models proposed, the fixed asset groups are treated as being independent. Dependent fixed assets need to be combined into compound projects, and related data must take this combination into account. For this reason interactions between investments discussed by Reiter[139] and Weingartner[140] do not apply to this paper since they were exclusively concerned with the problem of selecting from a number of interacting projects. Here the concern is essentially one of selecting from alternate modes of acquisition of capacity.

It is quite true that the assumption of independence between projects in certain instances may not be accurate. Two dependent fixed assets within the same group may not have the same replacement intervals, and considering them as a compound investment could lead to suboptimization. In those cases, the alternative approach would be to consider the two investments as being independent and to define the capacity requirements in a consistent manner.

If a number of contingent projects <u>must</u> be analyzed, then the "contingent chains" discussed by Weingartner[141] could be used. Then interactions could be built into the model; however, it is felt that such constraints could make the model (already quite large) impractical, depending on the number of such relationships imposed.

CHAPTER IX

CONCLUSIONS AND EXTENSIONS

Conclusions

This research has analyzed fixed asset type capital investment decisions as exemplified in the aerospace industry. The analysis proceeded by examining the decision-making framework concerning fixed asset type investments in a typical aerospace firm. This examination consisted of investigating the organizational structure, centralization and decentralization aspects, relations with government, and the planning and budgeting as it is presently accomplished. Analysis of the present system led to defining a number of external and internal factors that would influence the system to be proposed. Certain aspects of the aerospace industry that set it apart from other industries were brought out.

Following the analysis of the real world aspects of the problem, the classification and development of a number of decision making models were undertaken. The sequential nature of the decisions, the system states, and inputs and outputs of the system were illustrated. The decision variables, parameters, and constraints were formulated consecutively, from the simplest to the most complex models.

The development of the models was followed by the application of linear programming solution techniques to these models. Certain decomposition methods that allow handling larger scale problems were investigated and illustrated in the appendices.

A discussion of the financial considerations led to the investigation of the effects of debt and equity on the marginal returns of investments. This was accomplished through the use of analysis of the duals of the problems. Such extensions helped show the capabilities of the fixed asset solution schemes, and introduced additional realism into the models developed.

Finally, the implementation aspects of the models proposed were discussed in light of the experience gained through a limited application at the Georgia Division of Lockheed Aircraft Corporation.

The pilot run consisted of developing the system on Univac's 1108 computer, gathering data for 65 metal working machines, and subsequently exercising the computer with the actual data. This trial demonstrated the practicality of the system, and showed that further efforts should be aimed toward the improvement of data acquisition aspects.

This research has accomplished three objectives. First, it has investigated and defined the significant factors affecting the fixed asset decisions that face a typical firm in the aerospace industry. Second, realistic models of fixed asset replacement and acquisition decisions in the relatively unique environment of an aerospace industry have been developed. It has been shown that the ability to formulate functional groups and to associate attibutes to these groups allow decisions to be made with respect to these groups, and to the typical assets that are represented by these groups. These decisions involve purchase, lease, rent, replacement, salvage, and subcontract variables. The constraints are the functional requirements that are influenced by the long-range goals of the company, yearly budgets, and a number of consistency requirements that assure proper flow through the system. The objective function was defined as the discounted cash flow values associated with each variable, and takes into account the effect of progress payments, and cost sharing arrangements prevalent in the aerospace industry. The models developed have covered the physical as well as the financial aspects of the problem. Third, this research has solved the models formulated, by using linear programming techniques, and has discussed decomposition as a means of solving problems that exceed the capacity of available computers. Discussion of the implementation considerations has shown that the models can be feasibly installed and operated as an information system.

Extensions

Further research can progress along a number of lines. These are:

1. Investigation of the stochastic aspects of the models. Cost parameters and functional requirements are most likely candidates to be treated as random variables.

2. Determination of the implications of other alternative modes of acquiring additional capacity. Two such additional modes could be overtime and third shift operations.

3. Determination of methods of handling the age dispersion of existing assets without increasing the dimensionality of the models.

4. Further investigation of decisions regarding fixed assets not included in this research. These assets are land, buildings, and special purpose research and development facilities acquired due to competitive pressures.

APPENDICES

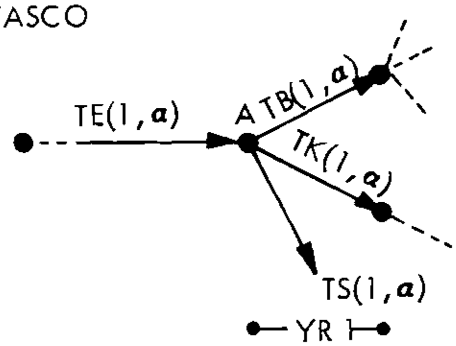
APPENDIX A

CONSISTENCY REQUIREMENTS

These equations establish the relation between certain decision variables of one stage and those of the preceding stages. The clearest manner of structuring these equations are network flow diagrams, therefore, ample use of them will be made in the following discussion.

The network flow diagrams assure that the sum of inputs at a node equal those leaving it. In our model this corresponds to assuring that the number of units to be kept, salvaged and replaced at the beginning of a year are equal to the quantity there was to start with.

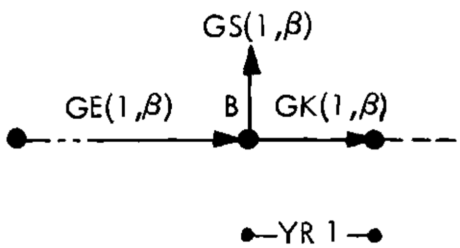
TASCO



at node A we have,

$$TB(1, \alpha) + TK(1, \alpha) + TS(1, \alpha) = TE(1, \alpha)$$

GOVERNMENT



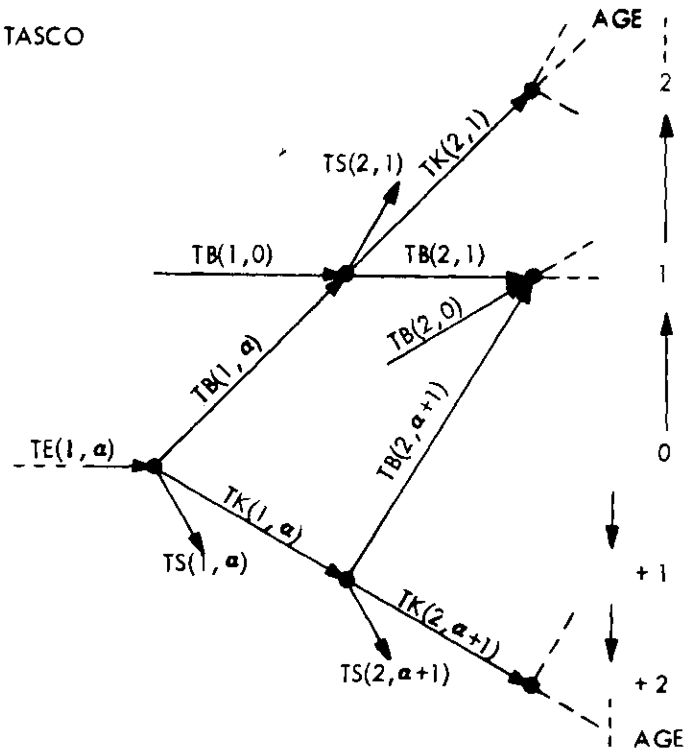
at node B, we have,

$$GK(1, \beta) + GS(1, \beta) = GE(1, \beta)$$

Figure 23. Flow Diagram, n = 1

n = 2

TASCO

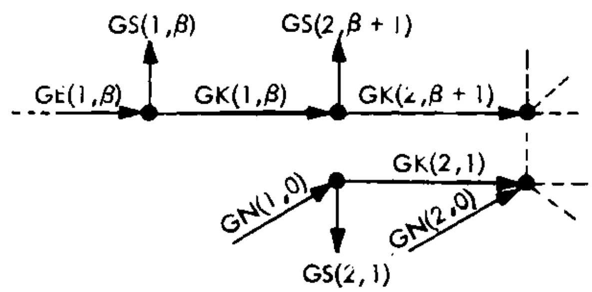


TB(1,0) + TB(1,α) = TS(2,1) + TK(2,1) + TB(2,1)

and

TK(1,α) = TK(2,α+ 1) + TS(2,α+ 1)

GOVERNMENT



GS(1,β) + GK(1,β) = GE(1,β)

GS(2,β+ 1) + GK(2,β+ 1) = GK(1,β)

GS(2,1) + GK(2,1) = GN(1,0)

Input . GN(1,0) = GN(1,0)*
GN(2,0) = GN(2,0)*

Figure 24. Flow Diagram, n = 2

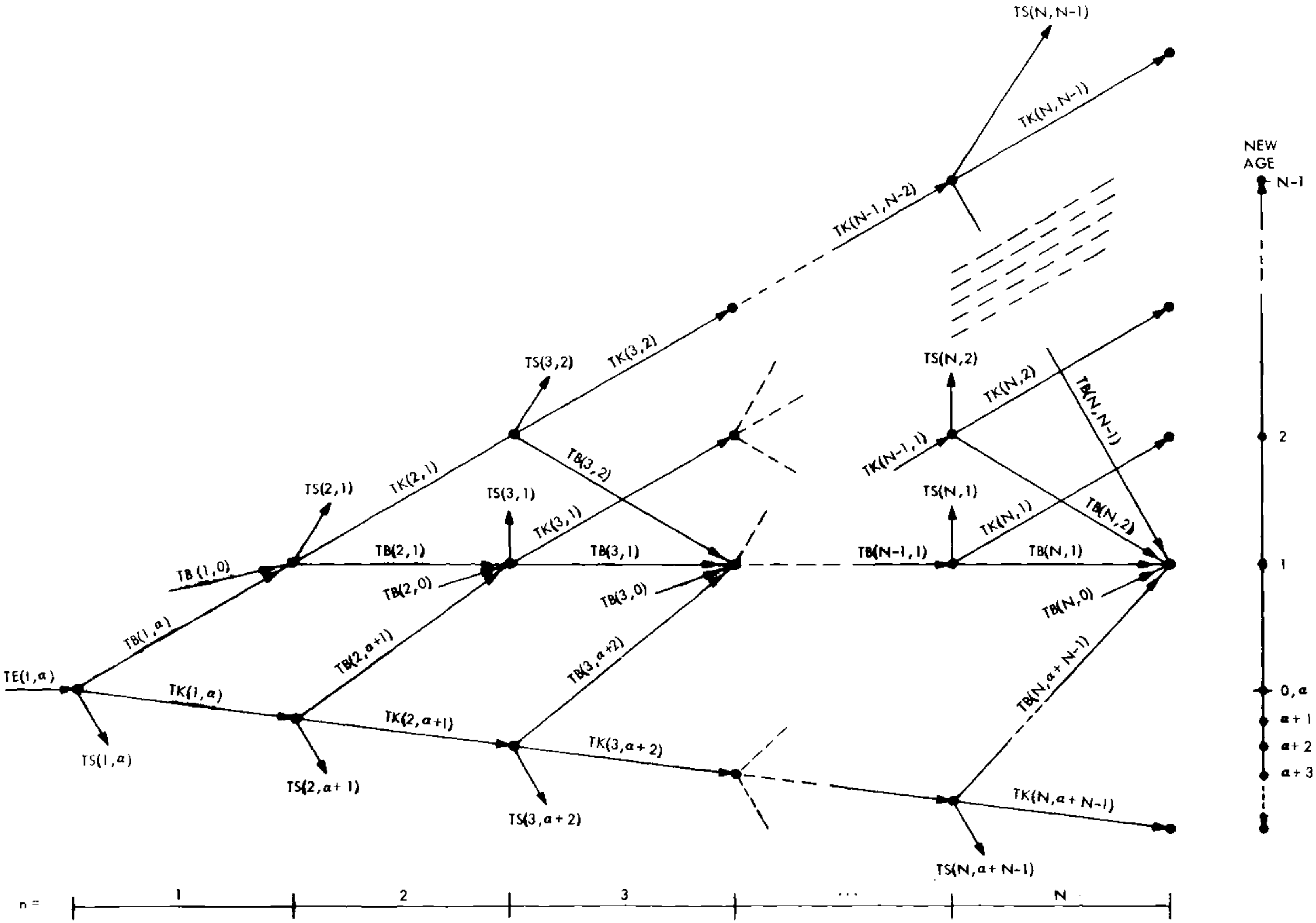


Figure 25. Flow Diagram, n = N

APPENDIX B

LINEAR PROGRAMMING STRUCTURE

The illustration in this appendix shows the linear programming structure of Model 6 as applied to a single fixed asset group.

Numbers 1 through 60 apply to the capacity figures in appropriate units. Numbers 61 through 95 refer to the costs associated with the purchase, replacement or salvage of assets.

Decisions related to leasing have been omitted from this illustration. However, Chapter V contains illustrations of the equations that could easily be incorporated into this structure.

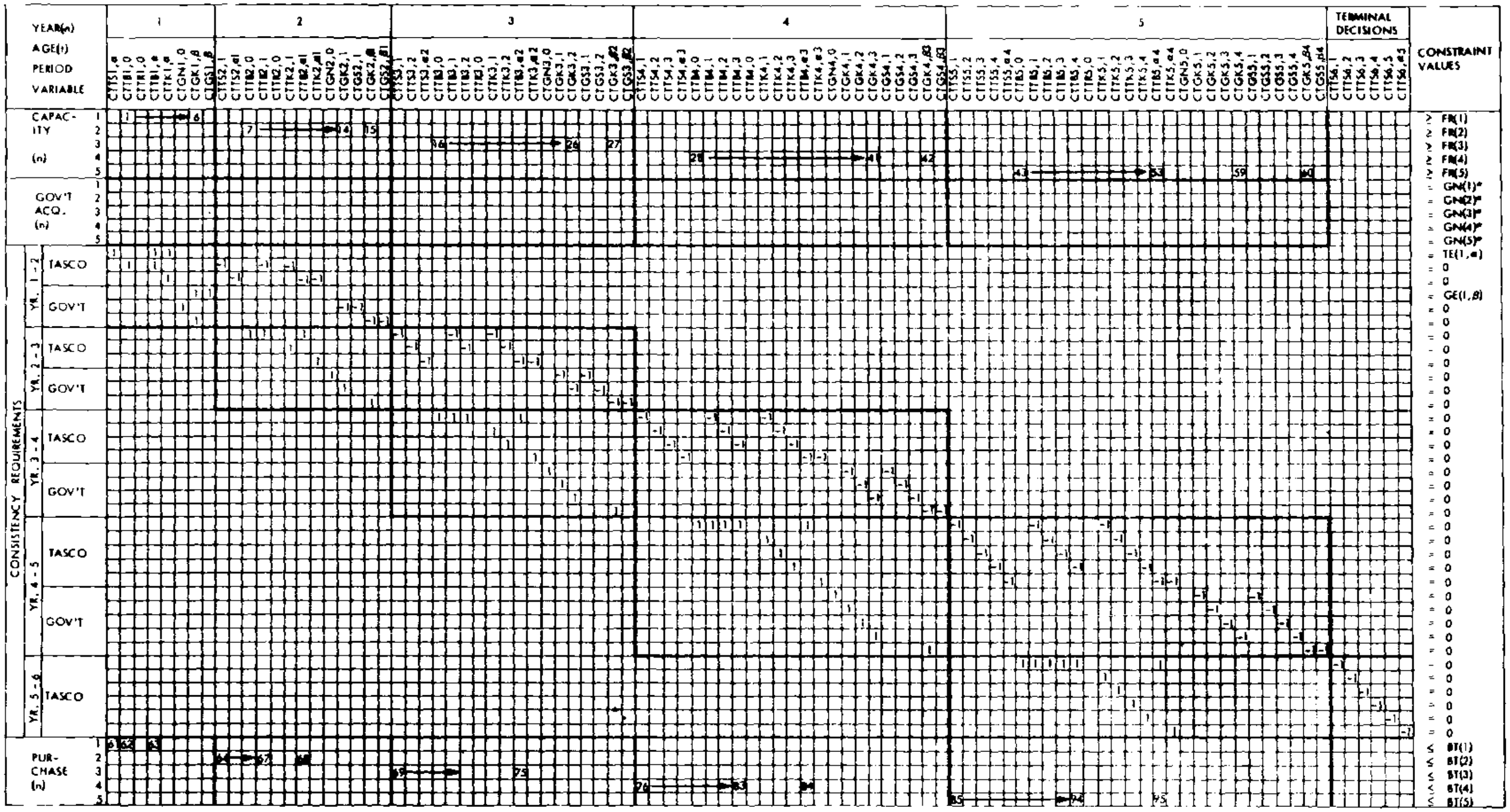


Figure 26. Linear Programming Structure - Model 6

APPENDIX C

DECOMPOSITION USING LAGRANGE MULTIPLIERS

This appendix numerically demonstrates the decomposition of a problem by using a search technique based on Lagrange Multipliers.

The problem is to determine the allocation of a fixed budget of $200,000 to each of three fixed asset groups for each year of the planning horizon. The three groups consist of gasoline fork lifts, grinders, and diagrammers.

The data presented in the first part follows the description given in Chapter V, and the three groups are assumed to represent three branches "tied" by budget constraints.

Each cycle represents a computer run, and the estimated value of λ_n is obtained by the linear interpolation scheme discussed in the text.

The structure illustrated in Appendix B has been used in solution of the models. Table 11 illustrates in detail the cycle 3 and cycle 4 results summarized in Table 12.

Table 10. Data for Three Groups

Group 1 - Gasoline Fork Lifts

Capacity Data

On Hand

19. Units of TASCO Average Age 10. Years.

8. Units of Government Average Age 20. Years.

Functional Requirements Data (hrs.)

Year, n				
1	2	3	4	5
77709.	85480.	94028.	103431.	113774.

Year, n	Age, t: 0	1	2	3	4	α	β
Unit Capacity Data							
1	2878.					2158.	1295.
2	2878.	2878.				2050.	1230.
3	2878.	2878.	2590.			1948.	1168.
4	2878.	2878.	2590.	2446.		1851.	1110.
5	2878.	2878.	2590.	2446.	2302.	1758.	1055.

Cost Data

Year, n	0	1	2	3	4	α	β
Operation and Maintenance Cost							
1	7418.					6964.	6964.
2	7418.	6739.				6964.	6964.
3	7418.	6739.	6744.			6964.	6964.
4	7418.	6739.	6744.	6744.		6964.	6964.
5	7418.	6739.	6744.	6744.	6744.	6964.	6964.
Purchase and Replacement Costs							
1	6638.					3000.	
2	6638.	1000.				3000.	
3	6638.	1000.	2000.			3000.	
4	6638.	1000.	2000.	3000.		3000.	
5	6638.	1000.	2000.	3000.	3000.	3000.	

Age, t:

Year, n	0	1	2	3	4	α	β
	Salvage Revenues						
1	0.					3638.	
2	5638.					3638.	
3	5638.	4638.				3638.	
4	5638.	4638.	4638.			3638.	
5	5638.	4638.	3638.	3638.		3638.	
6	5638.	4638.	3638.	3638.	3638.	3638.	

Rental or Subcontract Costs. (If rental operating costs must be included)

7800.	7800.	7800.	7800.	7800.
6532.	6532.	6532.	6532.	6532.

Group 2 - Grinders

Capacity Data

On Hand

16. Units of TASCO Average Age 7. Years.

32. Units of Government Average Age 11. Years.

Functional Requirements Data (hrs.)

Year, n

1	2	3	4	5
100000.	120000.	140000.	160000.	180000.

Year, n	Age, t: 0	1	2	3	4	α	β
Unit Capacity Data (hrs.)							
1	2000.					1920.	1920.
2	2040.	1930.				1920.	1920.
3	2080.	2020.	1960.			1920.	1920.
4	2120.	3060.	2000.	1940.		1920.	1920.
5	2160.	2100.	2040.	1980.	1920.	1920.	1920.
Cost Data							
Operation and Maintenance Data							
1	28300.					31056.	41056.
2	27360.	28330.				31056.	41056.
3	26620.	27390.	28390.			31056.	41056.
4	25780.	26650.	27450.	21420.		31056.	41056.
5	24840.	25810.	26710.	27580.	28420.	31056.	41056.
Purchase and Replacement Costs							
1	10500.					6500.	
2	11000.	7000.				7000.	
3	11500.	7250.	7250.			7500.	
4	12000.	7000.	7250.	7500.		8000.	
5	12500.	7000.	7250.	7500.	7750.	8500.	
Salvage Revenues							
1	0.					4000.	
2	4000.					4000.	
3	4500.	4250.				4000.	
4	5000.	4750.	5400.			4000.	

Year, n	Age, t: 0	1	2	3	4	α	β
5	5500.	5250.	5000.	4750.		4000.	
6	6000.	5750.	5500.	5250.	5000.	4000.	

Rental or Subcontract Costs. (If rental, operating costs must be included)

5250.	5250.	5250.	5250.	5250.
28000.	27160.	26320.	25480.	24640.

Group 3 - Diagrammers

Capacity Data

On Hand

7. Units of TASCO Average Age 2. Years.

0. Units of Government Average Age 0. Years.

Functional Requirements Data (hrs.)

Year, n: 1	2	3	4	5
21000.	21000.	22000.	23000.	24000.

Unit Capacity Data (hrs.)

Year, n	Age, t: 0	1	2	3	4	α	β
1	3000.					2500.	2500.
2	3100.	2800.				2500.	2500.
3	3200.	2900.	2700.			2500.	2500.
4	3300.	3000.	2800.	2600.		2500.	2500.
5	3400.	3100.	2900.	2700.	2500.	2500.	2500.

Age, t:

Year, n	0	1	2	3	4	α	β
Cost Data							
Operation and Maintenance Cost							
1	25000.					29000.	29000.
2	25000.	26000.				29000.	29000.
3	25000.	26000.	27000.			29000.	29000.
4	25000.	26000.	27000.	28000.		29000.	29000.
5	25000.	26000.	27000.	28000.	29000.	29000.	29000.
Purchase and Replacement Costs							
1	41000.					24000.	
2	43000.	18000.				26000.	
3	45000.	18000.	22000.			28000.	
4	47000.	18000.	22000.	26000.		30000.	
5	49000.	18000.	22000.	26000.	30000.	32000.	
Salvage Revenues							
1	0.					17000.	
2	25000.					17000.	
3	27000.	23000.				17000.	
4	29000.	25000.	21000.			17000.	
5	31000.	27000.	23000.	19000.		17000.	
6	33000.	29000.	25000.	21000.	17000.	17000.	

Rental or Subcontract Costs. (If rental, operating costs must be included)

20500.	21500.	22500.	23500.	24500.
25000.	25000.	25000.	25000.	25000.

Table 11. Cycle 3 and 4 Details

Cycle 3

$\lambda_1 = 6.36, \lambda_2 = 1., \lambda_3 = 1., \lambda_4 = 1., \lambda_5 = 1.$

	Objective Function	Budgets 1	2	3	4	5
GP 1	1.361785	0.	128.05	19.73	238.57	23.87
GP 2	10.137881	0.	583.07	117.10	121.03	0.
GP 3	1.172429	0.	62.43	164.43	33.05	27.01
Total	$12.672095	0.	773.55	301.26	392.65	46.88

Cycle 4

$\lambda_1 = 6.36, \lambda_2 = 5., \lambda_3 = 1., \lambda_4 = 1., \lambda_5 = 1.$

	Objective Function	Budgets 1	2	3	4	5
GP 1	1.496796	0.	0.	147.77	238.57	23.87
GP 2	10.158823	0.	0.	774.05	120.84	0.
GP 3	1.185432	0.	0.	204.44	34.73	26.91
Total	$12.841051	0.	0.	1126.27	394.14	50.78

The λ_2 = value to be used in the following cycle is found by linear interpolation.

$$\lambda_2 = (773.55-200.00)\,\frac{(5-1)}{(773-0)} + 1 = 3.96$$

Table 12. Decomposition Using Lagrange Multipliers

Cycle	λ_1	λ_2	Values λ_3	λ_4	λ_5	Objective Function ($\$10^6$)	Year 1	Year 2	Year 3	Year 4	Year 5	Estimated λ_n
1	1.	1.	1.	1.	1.	12.511896	618.94	175.27	65.44	239.18	50.78	
2	10.	1.	1.	1.	1.	12.674095	0.	770.50	301.57	392.65	50.88	λ_1=6.36
3*	6.36	1.	1.	1.	1.	12.672095	0.	773.55	301.26	392.65	46.88	
4*	6.36	5.	1.	1.	1.	12.841051	0.	0.	1126.27	394.14	50.78	λ_2=3.96
5	6.36	3.96	1.	1.	1.	12.841051	0.	0.	1126.27	394.14	50.78	
6	6.36	3.96	5.	1.	1.	13.140479	0.	0.	0.	1297.96	59.26	λ_3=4.29
7	6.36	3.96	4.29	1.	1.	13.140479	0.	0.	0.	1416.96	59.26	
8	6.36	3.96	4.29	5.	1.	13.542114	0.	0.	0.	0.	301.41	λ_4=4.44
9	6.36	3.96	4.29	4.44	1.	13.542114	0.	0.	0.	0.	301.41	
10	6.36	3.96	4.29	4.44	5.	13.760414	0.	0.	0.	0.	0.	λ_5=2.34
11	1.	3.96	4.29	4.44	2.34	13.026488	636.92	0.	0.	0.	0.	
12	2.	3.96	4.29	4.44	2.34	13.581700	110.13	0.	0.	0.	0.	λ_1=1.831
13	1.831	1.	4.29	4.44	2.34	12.933881	0.	915.02	0.	0.	0.	
14	1.831	2.	4.29	4.44	2.34	13.504798	110.13	17.94	0.	0.	0.	λ_2=1.796
15	1.831	1.796	1.	4.44	2.34	12.947137	84.68	0.	1088.80	0.	0.	

Cycle	Values λ_1	λ_2	λ_3	λ_4	λ_5	Objective Function ($\$10^6$)	Year 1	Year 2	Year 3	Year 4	Year 5	Estimated λ_n
16	1.831	1.796	2.	4.44	2.34	13.419115	84.68	14.57	134.30	0.	0.	λ_3=1.466
17	1.831	1.796	1.466	1.	2.34	13.087955	84.58	22.67	0.	1235.73	0.	
18	1.831	1.796	1.466	1.5	2.34	13.385632	84.68	14.57	134.38	27.40	0.	λ_4=1.423
19	6.831	1.796	1.466	1.423	1.	13.327775	84.68	14.57	57.02	27.40	196.25	
20	1.831	1.796	1.466	1.423	1.	13.327775	84.68	14.57	57.02	27.40	196.25	λ_5=1.
21	1.0	1.796	1.466	1.423	1.0	12.926376	569.07	0.	39.46	31.57	206.51	
22	1.2	1.796	1.466	1.423	1.0	13.124215	157.98	17.94	37.67	32.46	177.33	λ_1=1.182
23	1.182	1.	1.466	1.423	1.	12.799681	84.68	832.86	19.63	41.43	123.84	
24	1.182	1.2	1.466	1.423	1.	13.092998	157.98	35.06	37.67	32.46	158.84	λ_2=1.159
25	1.182	1.159	1.	1.423	1.	12.725227	157.98	17.94	1054.93	21.70	149.12	
26	1.182	1.159	1.2	1.423	1.	13.056586	157.98	35.12	54.61	32.46	146.22	λ_3=1.171
27	1.182	1.159	1.771	1.	1.	12.833795	157.98	35.06	54.61	1188.06	69.76	
28	1.182	1.159	1.171	1.1	1.	13.025852	157.98	35.06	54.61	78.35	111.79	λ_4=1.089
29	1.05	1.159	1.171	1.089	1.	12.907740	619.00	35.06	54.61	224.67	125.94	
30	1.1	1.159	1.71	1.089	1.	12.991218	157.88	35.01	54.61	54.61	125.94	λ_1=1.096

*See Table 11 for details.

Table 13 summarizes the results of the decomposition procedure.

Table 13. Summary of Decomposition Using Lagrange Multipliers

	1 = 1.096, 2 = 1.137, 3 = 1.10, 4 = 1.089, 5 = 1.0					
	Objective Function	Budgets, B(n)				
		1	2	3	4	5
GP 1	1.3146	1100	18	38	32	91
GP 2	10.1771	0	0	0	0	0
GP 3	1.1759	48	17	164	52	32
Total	12.6676	158	35	202	84	123

APPENDIX D

SUCCESSIVE APPROXIMATION ALGORITHM

This appendix represents a numerical example of the "Successive Approximation Algorithm" of Chapter VI. The same data as was used in Appendix C is used here. This example also employs the construction illustrated in Appendix B.

Similar to Appendix C, it is assumed that three groups are to be allocated a total budget of $200,000 each year, and the problem is to determine a procedure for calculating each group's share so that the overall objective function is minimized.

<u>Steps 1 and 2: Solution of Each Group with Unlimited Budgets</u>

	Objective Function (10^6)	Optimal Budgets, $b_i^o(n)$, (10^3)				
		1	2	3	4	5
GP 1	1,274.	110.	18.	73.	239.	24.
GP 2	10,078.	525.	113.	447.	121.	0.
GP 3	1,160.	48.	17.	280.	34.	27.
Total	12,512.	683.	148.	800.	394.	51.

<u>Step 3: Initial Allocation</u>

$B(n) = \$200,000.$ for $n = 1,\ldots,5$

$b_1^1(1) = B(1) . \dfrac{b_1^o(1)}{B^o(1)} = 32.2$ $\qquad b_2^1(1) = 153.7$ $\qquad b_3^1(1) = 14.1$

$b_1^1(2) = B(1)\cdot\frac{b_2^o(1)}{B^o(1)} = 24.3$ $\qquad$ $b_2^1(2) = 152.7$ $\qquad$ $b_3^1(2) = 23.0$

$b_1^1(3) = B(1)\cdot\frac{b_3^o(1)}{B^o(1)} = 18.2$ $\qquad$ $b_2^1(3) = 111.8$ $\qquad$ $b_3^1(3) = 70.0$

$b_1^1(4) = B(1)\cdot\frac{b_4^o(1)}{B^o(1)} = 121.3$ $\qquad$ $b_2^1(4) = 61.4$ $\qquad$ $b_3^1(4) = 17.3$

$b_1^1(5) = B(1)\cdot\frac{b_5^o(1)}{B^o(1)} = 94.2$ $\qquad$ $b_2^1(5) = 0$ $\qquad$ $b_3^1(5) = 105.8$

Step 4: Solution Using

Allocations of Step 3 (m = 1)

	Objective Function (10^6)	Budgets, B(n) 1	2	3	4	5
GP 1	1,480.	32.2	24.3	18.2	121.3	94.2
GP 2	10,247.	153.7	152.7	111.8	61.4	0.
GP 3	1,211.	14.1	23.0	70.0	17.3	104.8
Total	12,938.	200.	200.	200.	200.	200.

Step 5: Dual Values

and Limits of Constraint Constants

Year N	Group No. 1 Duals	Right Side Limits	2 Duals	Right Side Limits	3 Duals	Right Side Limits
1	0.002713	32.2+7.61	0.000327	153.7+210.	0.000615	14.1+19.9
		-6.5		-67.7		-14.1

Step 6: Rank and Allocate by Duals

The new budgets, or right sides are calculated as follows:

Group 1: 32.2 + 7.6 = 39.8

Group 2: 153.7 - 7.6 = 146.1; 1461. - 19.9 = 126.2

Group 3: 14.1 + 19.9 = 34.0

Groups Ranked by Duals

	Group No.					
	1		3		2	
Year n	Dual	Right Side	Dual	Right Side	Dual	Right Side
1	0.002713	39.8	0.000615	34.0	0.000327	126.2

The linear programming solution for the three groups with the new, first year reallocated budget (refer to Step 4 above) yields the following objective function.

Objective function:	Group 1:	\$ 1,459.
	Group 2:	10,256.
	Group 3:	1,199.
		\$12,914.

The improvement in objective function is, (Step 7)

\$12,938. - \$12,914. = \$24.

Now, the second year budgets are reallocated using the same procedure as in Steps 5 and 6.

The second year duals and limits appear as:

Year n	Group No. 1 Dual	Right Side	2 Dual	Right Side	3 Dual	Right Side
2	0.002207	24.3+0 -24.2	0.000357	152.7+302. -40.6	0.000496	23.0+0. -23.

No improvement of second year budgets by reallocation is possible, since both Groups 2 and 3 have upper limits of 0. Following table shows the third year duals and limits:

Year n	Group No. 1 Dual	Right Side	2 Dual	Right Side	3 Dual	Right Side
3	0.001141	18.2+0 -11.9	0.000320	111.7+311.9 -111.7	0.000293	70.+0 -70.

Ranking by duals and calculation new right hand sides yield,

Groups Ranked by Duals

Year n	Group No. 1 Dual	Right Side	2 Dual	Right Side	3 Dual	Right Side
3	0.001141	18.2	0.000320	145.2	0.000293	36.5

The linear programming solution for the three groups with the new, third year reallocated budgets, and with the first, second, fourth and fifth year budgets the same yields, Objective function: Group 1: $ 1,459.

Group 2: 10,245.

Group 3: 1,208.

$12,913.

The improvement in the objective function is,

$12,914. - $12,913. = $1.

The same process is repeated for years 4 and 5, and yields the following.

Year n	Group No. 1 Dual	Group No. 1 Right Side	Group No. 2 Dual	Group No. 2 Right Side	Group No. 3 Dual	Group No. 3 Right Side
4	0.000083	121.3+0	0.000260	61.4+28.7	0.000359	17.3+1.2
		-11.9		-61.4		-17.3

Ranking and reallocation yields,

Year n	Group No. 3 Dual	Group No. 3 Right Side	Group No. 2 Dual	Group No. 2 Right Side	Group No. 1 Dual	Group No. 1 Right Side
4	0.000359	18.5	0.000260	72.0	0.000083	109.4

Corresponding objective functions are:

Group 1: $ 1,460.

Group 2: 10,242.

Group 3: $\frac{1,208.}{\$12,910.}$

The improvement in objective function is, therefore,

$$\$12,913. - \$12,910. = \$3.$$

The fourth year, reallocated optimization indicated that the fifth year duals all have a value of 0, indicating slacks within the budgets.

At this stage a decision was made to repeat the five year process once more, but with a different basis. The perturbation was accomplished by changing the right hand side values (budgets) beyond their limits. The results of the second five year cycle is presented in Table 14.

Table 14. Summary of the Second Five Year Cycle

	Objective Function	Budgets, B(n)				
		1	2	3	4	5
GP 1	1,312	84.6	35.6	21.4	109.4	94.2
GP 2	10,266	69.7	141.4	146.3	72.1	0.0
GP 3	1,204	45.7	23.0	32.3	18.5	105.8
Total	12,782	200.0	200.0	200.0	200.0	200.0

The decomposition analysis was stopped at the second cycle, however, additional cycles could be used to determine if further improvement was possible.

APPENDIX E

FUNCTIONAL REQUIREMENTS

The following is a sequence of steps for arriving at a forecast of functional requirements. It should be noted that this approach is one of the many approaches that could be taken to derive the requirements data. This and the following data are derived for turret lathes in the Tooling Division of the Manufacturing Branch.

The data provided are hypothetical since emphasis is on the method rather than the actual numbers.

1. The Tooling Division has historic records of the past ten years, as well as the forecasts of direct labor hours for the following five years. Future estimates are made by taking the sum of firm business orders at hand, and the expected value of new business orders. The following table illustrates the data:

Table 15. Total Direct Labor Hours in Tooling (Historic and Forecast)

Year	Firm Business	New Business	Total
1959	54,590		54,593
1960	57,000		57,071
1961	61,050		61,050
1962	70,250		70,296
1963	81,050		81,059

Year	Firm Business	New Business	Total
1964	79,620		79,627
1965	74,400		74,475
1966	80,300		80,391
1967	91,000		91,056
1968	92,250		92,250
1969*	104,700	10,000	114,700
1970	80,000	25,000	105,000
1971	10,000	80,000	90,000
1972	5,000	115,000	120,000
1973	5,000	135,000	140,000

Table 16. Direct Labor Hours

(Turret Lathes - Standard)

Year	Hours
1959	5267
1960	5571
1961	5951
1962	6724
1963	7135
1964	6915
1965	6990
1966	7113
1967	7930
1968	7977

*Start of Forecasts

The linear regression equation based on above data is:

$$Y = 1727.1 + .0678\ X$$

3. Correlation coefficient = 0.987; Standard Error = 15.Y represents the yearly turret lathe hours, and X is the yearly tooling direct labor. Using this equation for forecasting purposes, the functional requirements for the next five years appear as:

Table 17. Five Year Forecast of Functional Requirements (Turret Lathes - Standard)

Year	Hours
1969	9,504
1970	8,846
1971	7,819
1972	9,863
1973	11,219

Table 17, as indicated above, could be derived using a number of different approaches, depending on the accuracy and amount of historic data available.

APPENDIX F

CAPACITY DATA

Initial Capacity: One TASCO owned and two government owned turret lathes make up the initial capacity in Tooling. The government owned lathes are each 15 years old, Model 4; and the TASCO owned lathe is 12 years old, Model 4; and each has a power rating of 10 horsepower.

Typical Item: This is a Standard Turret Lathe No. 4, 10 horsepower.

Capacity Estimates: These estimates are based on the downtime, operator performance and related factors that affect the productivity of the machine. Downtime is directly affected by the age of the asset, whereas working conditions and other variables influence the operator performance. The following relationship shows the manner in which capacity may be estimated.

Capacity in year n, age t = (80 X 52 - Downtime (n,t)) Operator Performance (n)

Capacity (n,t) = (4160 - d (n,T)) P(n)

As technologically improved assets become available, or as the age of an already available asset increases downtime, figures change. Performance is assumed to be affected by the year rather than by the asset.

Initial data about capacity needs to be based on judgment, and the best representation would be a tabular format. As historic data is collected, curve fitting techniques could be used to forecast capacity levels.

Estimates of downtime as a function of age and year could be compared with certain empirical formulas. One such formula may be:

$$\text{Downtime } (n,t) = Ae^{-Z(n-t)} + B(1-e^{-wt})u^{n-t}$$

where A is the minimum downtime, due to maintenance, and A + B is the maximum downtime hours (1). u is the fractional increase in maintenance as the asset ages.

The following table is based on a first year downtime estimate of 100 hours, with a maximum of 600 hours over the five years.

Table 18. Downtime Estimates, D(n,t) in Hours

	Age, t						
n	0	1	2	3	4	Old TASCO	Old Govt.
1	100					600	600
2	37	416				600	600
3	13	194	532			600	600
4	4	92	252	590		600	600
5	2	44	121	181	599	600	600

Performance ratings of 0.70, 0.72, 0.75, 0.75 and 0.75 are assumed respectively for each of the five years to be forecast. An example below will clarify the computation of capacities.

Let

$$n = 3 \text{ and } t = 2$$

$$D(3,2) = 100e^{-(3-2-1)} + 500(1-2^{-2})0.5^{3-2-1}$$

$$= 100 + 500(1-e^{-2})$$

$$= 532$$

Capacity, (3,2) = (4160-532) 0.72 = 2612 hours.

Based on the assumed performance ratings and the method of determining capacities, the following data is developed for a typical turret lathe.

Table 19. Yearly Capacity Data (Actual 4160 Hours)

2 Shift, 5 Day Week

n	Age, t 0	1	2	3	4	Old TASCO	Old Govt.*
1	2842					2492	2492
2	2968	2696				2492	2492
3	2986	2855	2612			2492	2492
4	3117	3051	2931	2677		2492	2492
5	3118	3087	3029	2908	2671	2492	2492

*Operator performance for old assets is assumed to remain at 0.70 level.

APPENDIX G

COST DATA

The cost data will be developed in this order:

1. Salvage revenues
2. Purchase and replacement costs
3. Maintenance costs
4. Operating costs
5. Rental/subcontract costs

It should be obvious that the accuracy of the above cost data depends to a large extent on the knowledge of personnel who develop such data. Intimate knowledge of fixed asset markets, the manufacturers and their capabilities, special conditions of the general economy, as well as a deep understanding of the technical intricacies of the assets and the processes that use such assets is a must for reliable estimates. Certain rules of thumb or empiric formulas may be used; however, they should not be taken at face value, and should be closely scrutinized by experienced personnel. The following techniques for acquiring the data should therefore, serve only as guides.

1. Salvage Revenues:

Book values based on straight line or other types of depreciation accounting do not, in general, reflect the market salvage value of equipment. A demand and supply relationship, especially during national crisis, (e.g. the Vietnam War) seems to influence the salvage values and lead times for ordering of equipment, so that a ten year old machine tool could be selling above its original purchase price.

Based on published catalogues of a number of companies in the used equipment business, the resale value of a twelve year old turret lathe No. 4 is \$24,000. Compared to the cost of a new lathe of \$30,000, the resale value is quite high, indicating that the used tool and equipment market is very high. Salvage values for old assets should remain high according to the judgment of a number of knowledgable people, and assuming that the present hostilities in the political arena continue so that \$24,000 should remain as a fairly constant value for the next five years.

For new turret lathes, the salvage values can be forecast by fitting an exponential curve implied by Dreyfus.[142]

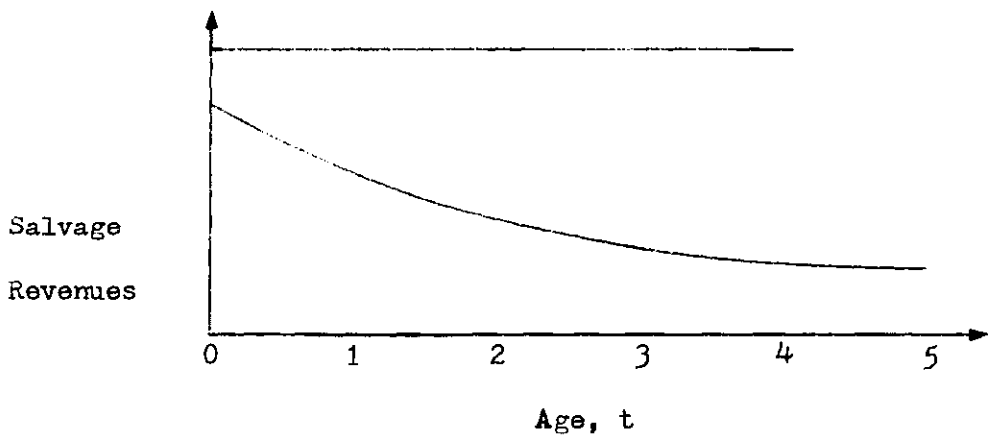


Figure 27. Salvage Revenue

Let

$s(t)$ = Salvage value of lathe t years old

c = Cost of new lathe

p = Reduction in value of asset during the initial year expressed as a fraction.

s = Rate of reduction in salvage value every year.

Making the assumption that \$24,000 resale value is obtained during the twelfth year and using an exponential decay function, we have, c = \$30,000, $p = 0.01$, $s(12)$ = \$24,000, $p = 0.01$, $s(t) = c(1-p)e^{-st}$.

$$s(12) = 24,000 = 30,000\ (1-0.01)e^{-s(12)}$$

Therefore,

$$s = 0.0177$$

Based on an unchanging \$30,000 initial cost and the use of a constant rate of reduction in the resale values, the following table was prepared.

Table 20. Salvage Values

			Age			
n	0	1	2	3	4	Old TASCO
1	-					24,000
2	-	29,700				24,000
3	-	29,700	29,168			24,000
4	-	29,700	29,168	28,164		24,000
5	-	29,700	29,168	28,164	27,670	24,000

A similar table will need to be prepared for each of the groups, taking into account the judgment of concerned personnel with experience in this area.

2. Purchase and Replacement Costs

Purchase cost of the typical No. 4 turret lathe is \$30,000 and is assumed to be constant over the next five years. Since salvage values vary (See Section 1), the replacement costs will also vary. The relationship for replacement costs over age may be shown as:

$$pc(n,t) = pc(t) \approx c(1-(1-p)e^{-st})$$

when pc(n,t) is the Purchase or Replacement Cost in year n for age t.

Table 21 shows the results for the turret lathe in question:

Table 21. Purchase and Replacement Costs

			Age			
n	0	1	3	3	4	Old TASCO
1	30,000					6,000
2	30,000	300				6,000
3	30,000	300	832			6,000
4	30,000	300	832	1,836		6,000
5	30,000	300	832	1,836		6,000

The assumption of \$30,000 will need to be developed for each group, and whenever the typical assets technologically superior to the present one, then the corresponding costs must be reflected in the data.

3. Maintenance Costs

These costs depend on a number of factors, such as age, year the equipment was manufactured, complexity, the level of utilization, and the frequency and intensity of the preventive maintenance.

Most complex devices follow a period, initially after purchase of high maintenance (debug) activity. Then this slackens off to a constant level, and during the last phase, due to wearout, the maintenance activity creeps up.

Table 22. Data for 12 and 15 Year Old Lathes

12 Yr. Old	1962	1963	1964	1965	1966	Average
Item 1	$198.	$ 88.	$122.	$ 264.	$145.	$163.40
Item 2	75.	134.	556.	1989.	532.	324.25
Item 3	35.	303.	175.	443.	143.	219.80
Item 4	96.	298.	294.	292.	97.	215.40
15 Yr. Old						
Item 1	68.	502.	68.	473.	148.	215.80
Item 2	70.	63.	82.	194.	284.	138.60
Item 3	98.	183.	163.	556.	131.	226.20

The debug phase for a conventional turret lathe does not normally exist. This is due to the fact that these tools have reached a level of technical maturity in being manufactured so that an initial phase of high maintenance does not exist. The same cannot be said of some of the more exotic tools, such as the numerically controlled milling machines, due to the relatively new "state of the art" situation prevailing in this area.

Based on D. Davis'[143] analysis of failure distributions, and data shown in Table 22, exponential distribution of time to failure can adequately describe the failure characteristics of a number of devices. This constant failure rate is assumed for the lathe sample for which data is being developed. It is also assumed that the wearout phase for the lathes in question occurs beyond twenty years, which is the age of the government items now fifteen years old.

The downtime figures that appeared under Capacity (Appendix F) refer to the actual production time that was lost due to maintenance. However, there are quite a number of situations where maintenance could be scheduled so as not to interfere with production. Therefore, the downtime figures usually represent a fraction of the total maintenance activity on an asset.

Based on the constant failure rate assumption for new and old lathes, a fixed sum of $220.00 (average of column 7 of Table 22) is found to be spent every year on maintenance.

4. Operating Costs

Ordinary and special maintenance that appears under Operating Costs has been discussed above. The remaining costs will need to be developed for new typical and old assets. Most of the rates used for operating costs have been developed by concerned organizations and are based on history and experience.

Direct labor cost for old and new machines is:

80 hrs./wk. X 52 wks./yr. X $4.20/hr. = $17,472./yr.

Indirect labor cost for old and new lathes is the same and is 47 per cent of direct labor costs, 0.50 X \$17,472. = \$5,241./yr.

Supplies are estimated at \$25. per year for old and new assets.

Scrap and rework costs are assumed to vary approximately in proportion (10 per cent) to the downtime, at a historically established rate of \$6.00 that includes labor and material costs.

Table 23. Scrap and Rework Costs

	Age						
n	0	1	2	3	4	Old TASCO	Old Govt.
1	60.					360.	360.
2	22.	250.				360.	360.
3	8.	116.	319.			360.	360.
4	2.	55.	151.	354.		360.	360.
5	1.	26.	73.	169.	359.	360.	360.

Downtime costs are difficult to establish; therefore, assumed rates will be used. A parametric analysis could, however, be performed to determine sensitivity of final results to such rate variations. For machinery, downtime cost is influenced heavily by the load, as can be seen by the following figure.

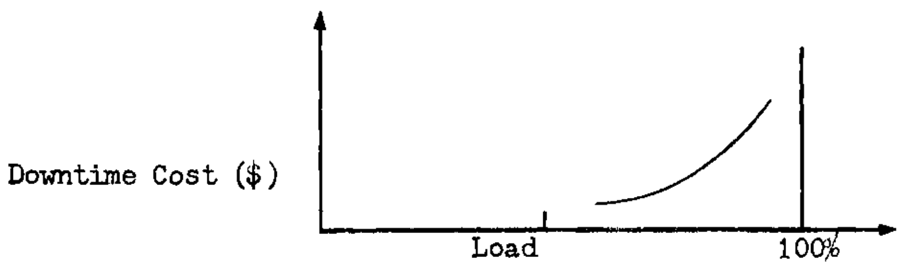


Figure 28. Load on Turret Lathes

Downtime above a certain level will cause work to be subcontracted, even higher levels will cause rerouting and rescheduling the work. Therefore, costs rise in an exponential manner. Downtime costs during 100 per cent load could be quite high in certain machinery and equipment, seriously affecting delivery schedules. Setting downtime cost at 60 per cent load to four times the direct labor cost of approximately \$16 (the approximate subcontract cost) we derive the following relationship:

Downtime Cost, DC = $e^{\lambda(1)}$ where 1 is the load in fractions, is a parameter to be empirically determined.

$$DC(1 = 0.6) = \$16 = e^{\lambda(.6)}$$

Therefore,

$$\lambda = 4.6$$

At the present 80 per cent load, the downtime cost is:

$$DC(1 = 0.8) = e^{4.6(0.8)} = e^{3.68} = \$39.$$

Due to better planning of equipment needs, this year's 80 per cent utilization level is expected to increase to 85, 85, 87, 88 per cent for the next five years, respectively. These load levels generate downtime costs of \$49.9, \$49.9, \$54.6 and \$57.5.

Based on above estimates, the downtime costs appear as follows:

Table 24. Downtime Costs

	Age						
n	0	1	2	3	4	Old TASCO	Old Govt.
1	3,900					23,400	23,400
2	1,846	20,758				29,940	29,940
3	649	9,681	26,547			29,940	29,940
4	218	5,023	13,759	32,214		32,760	32,760
5	115	2,530	6,957	16,215	34,442	34,500	34,500

Power costs for the old and the typical asset are approximately $100 per year. Set-up time is included as part of the direct labor charge. Floor space, property taxes and insurance are excluded due to their small value.

The following table totals the components of operating and maintenance costs discussed above.

Table 25. Operating and Maintenance Costs

	Age						
n	0	1	2	3	4	Old TASCO	Old Govt.
1	35,629					55,429	55,429
2	33,537	52,677				61,969	61,969
3	32,326	41,466	58,535			61,969	61,969
4	31,889	36,747	45,579	64,237		64,789	64,789
5	31,785	34,225	38,699	48,470		66,529	66,529

5. Rental Costs

Since this type of equipment could not be rented, subcontract costs for an equivalent amount of work was estimated and are shown in the table below.

Table 26. Subcontract Costs

n	Cost
1	$51,000
2	51,000
3	51,000
4	51,000
5	51,000

APPENDIX H

COMPUTER OUTPUTS

The computer outputs presented in this appendix have made use of the linear programming structure presented in Figure 26. The outputs consist of four types of analyses. These are (1) no budget limit, free (2) no budget limit, frozen (3) budgets set at $5,000, free and (4) budgets set at $5,000, frozen. The no budget limit, free, case will be illustrated in detail, the remaining three outputs will be presented as computer outputs without the detailed description given in Case (1).

Case (1): No Budget Limit, Free

Input Data

This analysis was accomplished by introducing large right hand side values for the fixed asset budget constraint values. Free, refers to the unfrozen nature of the analysis where all the decision variables appear without any being artifically frozen out at the start. Figure 29 refers to the raw input data that is fed into the matrix generator.

In this figure NYR is the planning horizon; NRHS is the number of right hand sides; JGOVT refers to the fact that government assets are included in the study; JDEBUG, MATRIX, JFIN are certain options used for a possible diagnosis of the program code, and PCTCOM is the fraction of the time the government assets are used on commercial business.

The row below the title, RAM TURRET LATHES SAMPLE, in Figure 29, refers to a number of parameters of the program. The .0 in the initial field is the lead time, the 12.0 in the second field is the average age of TASCO lathes, the 15.0 in the third field is the average age of government lathes. The 10.0 and the 30000.0 in the fourth and fifth fields refer to the life of a typical asset for depreciation purposes, and a representative initial acquisition cost of a typical government asset for rental fee calculations, respectively.

Additional data in Figure 29 follows the pattern for capacities, and costs described in Chapter VIII. First five fields of the last row indicate functional requirements, and the last two indicate the typical number of TASCO and government assets.

The analyses assumes a minimum cost of capital of 15 per cent per year.

Linear Programming Output

The output of the linear programming routine appears in a number of optional forms. Figure 30 shows two sets of labels for identifying the matrix elements since the primal solutions, and the coefficients of the cost rows are number coded. The first five row labels correspond to the FR(1),...,FR(5), and the second five labels refer to the BT(1),...,BT(5) rows. The RNT and O/M refer to ten rows that were inserted for a possible use as rental and operation and maintenance cost budgets. The remaining rows refer to the consistency requirements.

The column labels, or names of the variables used in this program were different than those presented earlier in this paper. The conversion from one to another is accomplished by changing the N and L appearing as the first letter of a variable to T, and the O appearing as the first letter to G.

Figure 31 shows the primal output. Since the objective function is of a minimization type all the cost values appear with negative signs. It should also be noted that all cost values have been scaled down by a factor of 10^6, and all budget and functional requirements have been scaled down by a factor of 10^3. Therefore, the objective function here has a value of $8,943.

Figure 32 illustrates the cost coefficients associated with each of the variables.

Report Generator

Figure 33 shows the report output corresponding to this case. In the table at the top, GELAC should be replaced by TASCO. The second column of the table indicates the type of decision, third column indicates the year, fourth indicates the activity level, fifth indicates the age of the asset to which the decision applies, and the sixth column indicates the cash flow associated with the decision.

Cases (2), (3) and (4)

Figures 34 and 35 illustrate the primal and the corresponding report output, respectively, for Case (2) (unlimited budget, frozen). Figures 36 and 37 illustrate the primal and the report output, respectively, for Case (3) ($5,000 yearly budgets, free). Figures 38 and 39 illustrate the primal and the report output, respectively, for Case (4) ($5,000 yearly budgets, frozen).

```
NYR     NRHS    NCST    JGOVT   JOPTN   JDEBUG MATRIX JFIN
5       5       5       0       1       0       0       0
PCTCOM .0830   .0960   .2770   .3360   .3710

RAM   TURRET   LATHES   SAMPLE   PROBLEM

.0          12.0        15.0        10.0        30000.0
2842.0                                                      2492.0      2492.0
2968.0      2696.0                                          2492.0      2492.0
2968.0      2855.0      2612.0                              2492.0      2492.0
3117.0      3051.0      2931.0      2677.0                  2492.0      2492.0
3118.0      3087.0      3029.0      2908.0      2671.0      2492.0      2492.0
30000.0                                                     6000.
30000.0     300.0                                           6000.0
30000.0     300.0       300.0                               6000.0
30000.0     300.0       300.0       1836.0                  6000.0
30000.0     300.0       832.0       1836.0      2330.0      6000.0
51000.0
51000.0
51000.0
51000.0
51000.0
35629.0                                                     55429.0     5542.9
33537.0     52677.0                                         61969.0     6196.9
32326.0     41466.0     58535.0                             61969.0     6196.9
31889.0     36747.0     45579.0     64237.0                 64789.0     6478.9
31785.0     34225.0     38699.0     48053.0     66470.0     66529.0     6652.9
                                                            -24000.
-29700.                                                     -24000.
-29700.     -29168.                                         -24000.
-29700.     -29168.     -28164.                             -24000.
-29700.     -29169.     -28164.     -27670.                 -24000.
-29700.     -29169.     -28164.     -27670.     -24000.
9504.       8846.       7829.       9863.       11219.      1.0         2.0
```

Figure 29. Raw Data for Matrix Generator

ROW LABELS

1)CAP1	2)CAP2	3)CAP3	4)CAP4	5)CAP5
6)PUR1	7)PUR2	8)PUR3	9)PUR4	10)PUR5
11)RNT1	12)RTN2	13)RTN3	14)RNT4	15)RTN5
16)O/M1	17)O/M2	18)O/M3	19)O/M4	20)O/M5
21)IN1	22)IN2	23)IN3	24)IN4	25)IN5
26)IN6	27)IN7	28)IN8	29)IN9	30IN10
31)IN11	32)IN12	33)IN13	34)IN14	35)IN15
36)IN16	37)IN17	38)IN18	39)IN19	40)IN20
41)IN21	42)IN22	43)IN23	44)IN24	45)IN25
46)IN26	47)IN27	47)IN28	49)IN29	50)IN30
51)IN31	52)IN32	53)IN33	54)IN34	55)IN35
56)IN36	57)IN37	58)IN38	59)IN39	60)IN40
61)IN41				

COLUMN LABELS

1)LS	1,12	2)NB	1,0	3)NR	1,0	4)LB	1,12
8)OS	1,15	9)NS	2,1	10)LS	2,13	11)NB	2,0
15)LB	2,13	16)LK	2,13	17)GN	2,0	18)GK	2,1
22)NS	3,1	23)NS	3,2	24)LS	3,14	25)NB	3,0
29)NK	3,1	30)NK	3,2	31)LB	3,14	32)LK	3,14
36)GS	3,1	37)GS	3,2	38)OK	3,17	39)3,1	3,17
43)LS	4,15	44)NB	4,0	45)NB	4,1	46)NB	4,2
50)NK	4,2	51)NK	4,3	52)LB	4,15	53)LK	4,15
57)	4,3	58)GS	4,1	59)GS	4,2	60)GS	4,3
64)NS	5,2	65)NS	4,3	66)NS	5,4	67)LS	5,15
71)NB	5,3	72)NB	5,4	73)NR	5,0	74)NK	5,1
78)NB	5,16	79)LK	5,16	80)GN	5,0	81)GK	5,1
85)GS	5,1	86)GS	5,2	87)GS	5,3	88)GS	5,4
92)NS	6,2	93)NS	6,3	94)NS	6,4	95)NS	6,5

5)LK	1,12	6)GN	1,0	7)OK	1,15
12)NB	2,1	13)NR)	2,0	14)NK	2,1
19)GS	2,1	20)OK	2,16	21)OS	2,16
26)NB	3,1	27)NB	3,2	28)NR	3,0
33)GN	3,0	34)GK	3,1	35)GK	3,2
40)NS	4,1	41)NS	4,2	42)NS	4,3
47)NB	4,3	48)NR	4,0	49)NK	4,1
54)GN	4,0	55)GK	4,1	56)GK	4,2
61)OK	4,18	62)OS	4,18	63)NS	5,1
68)NB	5,0	69)NB	5,1	70)NB	5,2
75)NK	5,2	76)NK	5,3	77)NK	5,4
82)GK	5,2	83)GK	5,3	84)GK	5,4
89)OK	5,19	90)OS	5,19	91)NS	6,1
96)LS	6,17				

Figure 30. Row and Column Labels for the LP Matrix

DATE :

10 DEC 68 RAM TURRET LATHES (FREE) (UNLIMITED BUDGETS) .000.005.001

PRIMAL OUTPUT

CASE ITERATION 50 OBJECTIVE VALUE -.008943

	LABEL		COST	ACTIVITY		LABEL		COST	ACTIVITY
E	A 1	0	.000000	-.008943	E	A 2	0	.000000	-.008943
E	A 3	0	.000000	-.008943	E	A 4	0	.000000	-.008943
E	A 5	0	.000000	-.008943	Z	R27	1	.000000	.000000
Z	R33	1	.000000	.000000	Z	R34	1	.000000	.000000
Z	R41	1	.000000	.000000	Z	R43	1	.000000	.000000
Z	R51	1	.000000	.000000	Z	R53	1	.000000	.000000
	R 6	0	.000000	9976.277200		R 7	0	.000000	10008.189200
	R 8	0	.000000	10010.052500		R 9	0	.000000	9981.329000
	R10	0	.000000	9986.488500		R11	0	.000000	9999.989900
	R12	0	.000000	9999.989900		R13	0	.000000	9999.989900
	R14	0	.000000	9999.989900		R15	0	.000000	9999.989900
	R16	0	.000000	9932.238800		R17	0	.000000	9943.957300
	R18	0	.000000	9956.796600		R19	0	.000000	9937.116700
	R20	0	.000000	9923.124200		C 2	1	-.030114	.590429
	C 4	1	-.006114	1.000000		C 5	1	-.010357	.000000
	C 6	1	-.006266	.000000		C 7	1	-.000938	2.000000
	C 9	1	.021724	.289216		C12	1	-.000259	1.301213
	C14	1	-.003160	.000000		C16	1	-.007580	.000000
	C17	1	-.003710	.000000		C20	1	-.000660	2.000000
	C22	1	.018891	.348433		C26	1	-.000267	.952780
	C29	1	-.001762	.000000		C30	1	-.004220	.000000
	C32	1	-.006868	.000000		C33	1	-.002597	.000000
	C34	1	-.003610	.000000		C38	1	-.000440	2.000000
	C44	1	-.017968	.612507		C45	1	.000180	.952780
	C49	1	-.000671	.000000		C50	1	-.001939	.000000
	C51	1	-.003956	.000000		C53	1	-.005611	.000000
	C54	1	-.001722	.000000		C55	1	-.002143	.000000
	C57	1	-.004870	.000000		C61	1	-.000301	2.000000
	C68	1	-.015454	.434392		C69	1	.000326	1.565287
	C74	1	-.000227	.000000		C75	1	-.000985	.000000
	C76	1	-.002002	.000000		C77	1	-.003593	.000000
	C79	1	-.004725	.000000		C80	1	-.001259	.000000
	C89	1	-.000223	2.000000		C91	1	.012421	1.999679

END PRIMAL OUTPUT

Figure 31. The Primal Output for Case (1): Free, Unlimited Budgets

```
                                                                        DATE  1(
10 DEC 68 RAM TURRET LATHES        (FREE)    (UNLIMITED BUDGETS)         .000.011.001

   CASE                                  VECTOR OUTPUT

   ORIGNL FORM OF EQUATION A 1   0 COST =      .000000

        .000000 =
   +   1.00000(A 1   0) -     .01298(C 1   1) +     .03011(C 2   1) +     .00953(C 3   1)
   +    .00611(C 4   1) +     .01036(C 5   1) +     .00627(C 6   1) +     .00094(C 7   1)
   -    .02172(C 9   1) -     .01129(C10   1) +     .02426(C11   1) +     .00026(C12   1)
   +    .00624(C13   1) +     .00316(C14   1) +     .00339(C15   1) +     .00758(C16   1)
   +    .00371(C17   1) +     .00605(C18   1) +     .00066(C20   1) -     .01889(C22   1)
   -    .01701(C23   1) -     .00981(C24   1) +     .02114(C25   1) +     .00027(C26   1)
   +    .00067(C27   1) +     .00565(C28   1) +     .00176(C29   1) +     .00422(C30   1)
   +    .00299(C31   1) +     .00687(C32   1) +     .00260(C33   1) +     .00361(C34   1)
   +    .00550(C35   1) +     .00044(C38   1) -     .01643(C40   1) -     .01479(C41   1)
   -    .01327(C42   1) -     .00853(C43   1) +     .01797(C44   1) -     .00018(C45   1)
   +    .00017(C46   1) +     .00083(C47   1) +     .00442(C48   1) +     .00067(C49   1)
   +    .00194(C50   1) +     .00396(C51   1) +     .00219(C52   1) +     .00561(C53   1)
   +    .00172(C54   1) +     .00214(C55   1) +     .00291(C56   1) +     .00487(C57   1)
   +    .00030(C61   1) -     .01428(C63   1) -     .01286(C64   1) -     .01154(C65   1)
   -    .01098(C66   1) -     .00742(C67   1) +     .01545(C68   1) -     .00033(C69   1)
   -    .00002(C70   1) +     .00055(C71   1) +     .00043(C72   1) +     .00362(C73   1)
   +    .00023(C74   1) +     .00098(C75   1) +     .00200(C76   1) +     .00359(C77   1)
   +    .00173(C78   1) +     .00472(C79   1) +     .00126(C80   1) +     .00143(C81   1)
   +    .00175(C82   1) +     .00275(C83   1) +     .00422(C84   1) +     .00022(C89   1)
   -    .01242(C91   1) -     .01118(C92   1) -     .01004(C93   1) -     .00955(C94   1)
   -    .00800(C95   1)

   END VECTOR OUTPUT
```

Figure 32. Cost Coefficients for Case (1): Free, Unlimited Budgets

RAM TURRET LATHES SAMPLE

GELAC	BUY	1	0.590	0	17712.9
GELAC	BUY	1	1.000	11	6000.0
GELAC	SALV	2	0.289	1	-8589.7
GELAC	BUY	2	1.301	1	390.4
GELAC	SALV	3	0.348	1	-10348.5
GELAC	BUY	3	0.953	1	285.8
GELAC	BUY	4	0.613	0	18375.2
GELAC	BUY	4	0.953	1	285.8
GELAC	BUY	5	0.434	0	13031.8
GELAC	BUY	5	1.565	1	469.6
GELAC	SALV	6	2.000	1	-59390.5

DETAIL CASH FLOW ANALYSIS

YEAR	F.A.BUDG.	OTHER	TOTAL	SALVAGE
1968	$ 23713.	$167523.	$191236.	
1969	$ -8199.	$167577.	$159377.	$ -8590.
1970	$-10063.	$154738.	$144675.	$ -10348.
1971	$ 18661.	$179493.	$198154.	
1972	$ 13501.	$196618.		

Figure 33. Report Generator for Case (1): Free, Unlimited

DATE 10

10 DEC 68 RAM TURRET LATHES (FROZEN) (UNLIMITED BUDGETS) .000.005.001

PRIMAL OUTPUT

CASE ITERATION 53 OBJECTIVE VALUE -.029439

	LABEL		COST	ACTIVITY		LABEL		COST	ACTIVITY
E	A 1	0	.000000	-.029439	E	A 2	0	.000000	-.029439
E	A 3	0	.000000	-.029439	E	A 4	0	.000000	-.029439
E	A 5	0	.000000	-.029439	Z	R27	1	.000000	.000000
Z	R33	1	.000000	.000000	Z	R34	1	.000000	.000000
Z	R41	1	.000000	.000000	Z	R43	1	.000000	.000000
Z	R51	1	.000000	.000000		R 6	0	.000000	10000.073900
	R 7	0	.000000	9994.011000		R 8	0	.000000	9999.989900
	R 9	0	.000000	9981.142300		R10	0	.000000	9986.541300
	R11	0	.000000	9963.440700		R12	0	.000000	9984.449300
	R13	0	.000000	9999.989900		R14	0	.000000	9999.989900
	R15	0	.000000	9999.989900		R16	0	.000000	9933.669300
	R17	0	.000000	9954.176500		R18	0	.000000	9946.275400
	R19	0	.000000	9921.578300		R20	0	.000000	9903.048700
	C 1	1	.012981	.003503		C 3	1	-.009529	.716653
	C 4	1	-.006114	.000000		C 5	1	-.010357	.996497
	C 6	1	-.006266	.000000		C 7	1	-.000938	2.000000
	C13	1	-.006239	.304716		C14	1	-.00[illegible]160	.000000
	C15	1	-.003389	.996497		C16	1	-.007580	.000000
	C17	1	-.003710	.000000		C20	1	-.000660	2.000000
	C29	1	-.001762	.996497		C30	1	-.004220	.000000
	C31	1	-.002989	.000000		C32	1	-.006868	.000000
	C33	1	-.002597	.000000		C34	1	-.003610	.000000
	C38	1	-.000440	2.000000		C44	1	-.017968	.628253
	C49	1	-.000671	-.000000		C50	1	-.001939	.996497
	C51	1	-.003956	.000000		C53	1	-.005611	.000000
	C54	1	-.001722	.000000		C55	1	-.002143	.000000
	C56	1	-.002908	.000000		C57	1	-.004870	.000000
	C61	1	-.000301	2.000000		C68	1	-.015454	.448290
	C74	1	-.000227	.628253		C75	1	-.000985	.000000
	C76	1	-.002002	.996497		C77	1	-.003593	.000000
	C79	1	-.004725	.000000		C80	1	-.001259	.000000
	C89	1	-.000223	2.000000		C91	1	.012421	.448290
	C92	1	.011182	.628253		C94	1	.009550	.996497

END PRIMAL OUTPUT

Figure 34. Primal Output for Case (2): Frozen, Unlimited Budgets

RAM TURRET LATHES SAMPLE

GELAC	SALV	1	0.004	11	-84.1
GELAC	RENT	1	0.717	0	36549.3
GELAC	RENT	2	0.305	0	15540.5
GELAC	BUY	2	0.996	12	5979.0
GELAC	BUY	4	0.628	0	18847.6
GELAC	BUY	5	0.448	0	13448.7
GELAC	SALV	6	0.448	1	-13314.2
GELAC	SALV	6	0.628	2	-18325.5
GELAC	SALV	6	0.996	4	-27573.1

DETAIL CASH FLOW ANALYSIS

YEAR	F.A.BUDG.	OTHER	TOTAL	SALVAGE
1968	$ -84.	$202642.	$202558.	$ -84.
1970	$ 5979.	$172898.	$178877.	
1970	$ 0.	$165259.	$165259.	
1971	$ 18848.	$195032.	$213879.	
1972	$ 13449.	$216694.	$230142.	

Figure 35. Report Generator for Case (2): Frozen, Unlimited

DATE

10 DEC 68 RAM TURRET LATHES (FREE) ($5000 BUDGETS) .000.033.001

PRIMAL OUTPUT

CASE ITERATION 81 OBJECTIVE VALUE -.012081

	LABEL		COST	ACTIVITY		LABEL		COST	ACTIVITY
E	A 1	0	.000000	-.012081	E	A 2	0	.000000	-.012081
E	A 3	0	.000000	-.012081	E	A 4	0	.000000	-.012081
E	A 5	0	.000000	-.012081		R 3	1	.000000	.510269
	R 8	0	.000000	4.662900		R11	0	.000000	9968.178000
	R12	0	.000000	9990.935000		R13	0	.000000	9999.989900
	R14	0	.000000	9985.394000		R15	0	.000000	9971.087700
	R16	0	.000000	9954.462900		R17	0	.000000	9949.911800
	R18	0	.000000	9951.272500		R19	0	.000000	9946.243200
	R20	0	.000000	9941.137200		C 1	1	.012981	.033333
	C 3	1	-.009529	.623763		C 4	1	-.006114	.966667
	C 6	1	-.006266	.000000		C 7	1	-.000938	2.000000
	C11	1	-.024259	.157000		C12	1	-.000259	.966667
	C13	1	-.006239	.177546		C14	1	-.003160	.000000
	C15	1	-.003389	-.000000		C16	1	-.007580	-.000000
	C17	1	-.003710	.000000		C18	1	-.006051	.000000
	C19	1	.000000	-.000000		C20	1	-.000660	2.000000
	C26	1	-.000267	1.123667		C29	1	-.001762	.000000
	C30	1	-.004220	.000000		C32	1	-.006868	.000000
	C33	1	-.002597	.000000		C34	1	-.003610	.000000
	C36	1	.000000	-.000000		C38	1	-.000440	2.000000
	C44	1	-.017968	.155430		C45	1	.000180	1.123667
	C48	1	-.004417	.286190		C49	1	-.000671	.000000
	C50	1	-.001939	.000000		C51	1	-.003956	.000000
	C53	1	-.005611	.000000		C54	1	-.001722	.000000
	C55	1	-.002143	-.000000		C57	1	-.004870	.000000
	C60	1	.000000	-.000000		C61	1	-.000301	2.000000
	C68	1	-.015454	.153876		C69	1	.000326	1.279097
	C73	1	-.003622	.566707		C74	1	-.000227	.000000
	C75	1	-.000985	.000000		C76	1	-.002002	.000000
	C77	1	-.003593	.000000		C79	1	-.004725	.000000
	C80	1	-.001259	.000000		C81	1	-.001433	-.000000
	C82	1	-.001750	-.000000		C83	1	-.002748	-.000000
	C89	1	-.000223	2.000000		C91	1	.012421	1.432972

END PRIMAL OUTPUT

Figure 36. Primal Output for Case (3): Free, with $5,000 Yearly Fixed Asset Budgets

RAM TURRET LATHES

GELAC	SALV	1	0.033	11	-800.0
GELAC	RENT	1	0.624	0	31811.9
GELAC	BUY	1	0.967	11	5800.0
GELAC	BUY	2	0.157	0	4710.0
GELAC	BUY	2	0.178	1	9054.8
GELAC	BUY	3	1.124	1	337.1
GELAC	BUY	4	0.155	0	4662.9
GELAC	BUY	4	1.124	1	337.1
GELAC	RENT	4	0.286	0	14595.7
GELAC	BUY	5	0.154	0	4616.3
GELAC	BUY	5	1.179	1	383.7
GELAC	RENT	5	0.567	0	28902.1
GELAC	SALV	6	1.433	1	-42559.3

DETAIL CASH FLOW ANALYSIS

YEAR	F.A.BUDG.	OTHER	TOTAL	SALVAGE
1968	$ 5000.	$177111.	$182111.	$ -800.
1969	$ 5000.	$170677.	$175677.	
1970	$ 337.	$160262.	$160599.	
1971	$ 5000.	$184963.	$189963.	
1972	$ 5000.	$207507.	$212507.	

Figure 37. Report Generator for Case (3): Free, with $5,000 Yearly Fixed Asset Budgets

DATE :

10 DEC 68 RAM TURRET LATHES (FROZEN) ($5000 BUDGETS) .000.033.001

PRIMAL OUTPUT

CASE ITERATION 76 OBJECTIVE VALUE -.030302

	LABEL		COST	ACTIVITY		LABEL		COST	ACTIVITY
E	A 1	0	.000000	-.030302	E	A 2	0	.000000	-.030302
E	A 3	0	.000000	-.030302	E	A 4	0	.000000	-.030302
E	A 5	0	.000000	-.030302	Z	R27	1	.000000	.000000
Z	R34	1	.000000	.000000		R 6	0	.000000	5.000000
	R 8	0	.000000	4.144339		R11	0	.000000	9963.597300
	R12	0	.000000	9982.928000		R13	0	.000000	9999.989900
	R14	0	.000000	9976.442200		R15	0	.000000	9962.314300
	R16	0	.000000	9933.475200		R17	0	.000000	9955.177000
	R18	0	.000000	9946.590400		R19	0	.000000	9936.609600
	R20	0	.000000	9928.127500		C 3	1	-.009529	.713582
	C 5	1	-.010357	1.000000		C 6	1	-.006266	.000000
	C 7	1	-.000938	2.000000		C10	1	.011288	.033333
	C13	1	-.006239	.334546		C14	1	-.003160	-.000000
	C15	1	-.003389	.966667		C16	1	-.007580	.000000
	C17	1	-.003710	.000000		C20	1	-.000660	2.000000
	C25	1	-.021137	.028522		C29	1	-.001762	.966667
	C30	1	-.004220	.000000		C33	1	-.002597	.000000
	C34	1	-.003610	.000000		C35	1	-.005501	.000000
	C38	1	-.000440	2.000000		C44	1	-.017968	.166667
	C48	1	-.004417	.461719		C49	1	-.000671	.028522
	C50	1	-.001939	.966667		C51	1	-.003956	.000000
	C52	1	-.002187	-.000000		C53	1	-.005611	-.000000
	C54	1	-.001722	.000000		C55	1	-.002143	-.000000
	C56	1	-.002908	-.000000		C57	1	-.004870	.000000
	C61	1	-.000301	2.000000		C68	1	-.015454	.166667
	C73	1	-.003622	.738734		C74	1	-.000227	.166667
	C75	1	-.000985	.028522		C76	1	-.002002	.966667
	C77	1	-.003593	-.000000		C79	1	-.004725	.000000
	C80	1	-.001259	.000000		C81	1	-.001433	-.000000
	C82	1	-.001750	-.000000		C83	1	-.002748	-.000000
	C89	1	-.000223	2.000000		C91	1	.012421	.166667
	C92	1	.011182	.166667		C93	1	.010036	.028522
	C94	1	.009550	.966667		C95	1	.008002	-.000000

END PRIMAL OUTPUT

Figure 38. Primal Output for Case (4): Frozen, with $5,000 Yearly Fixed Asset Budgets

RAM TURRET LATHES SAMPLE

GELAC	RENT	1	0.714	0	36392.7
GELAC	SALV	2	0.033	12	-800.0
GELAC	RENT	2	0.335	0	17061.8
GELAC	BUY	2	0.967	12	5800.0
GELAC	BUY	3	0.029	0	855.7
GELAC	BUY	4	0.167	0	5000.0
GELAC	RENT	4	0.462	0	23547.7
GELAC	BUY	5	0.167	0	5000.0
GELAC	RENT	5	0.739	0	37675.0
GELAC	SALV	6	0.167	1	-4950.0
GELAC	SALV	6	0.167	2	-4861.5
GELAC	SALV	6	0.029	3	-803.3
GELAC	SALV	6	0.967	4	-26747.7

DETAIL CASH FLOW ANALYSIS

YEAR	F.A.BUDG.	OTHER	TOTAL	SALVAGE
1968	$ 0.	$202680.	$202680.	
1969	$ 5000.	$173419.	$178419.	$ -800.
1970	$ 856.	$164944.	$165799.	
1971	$ 5000.	$203549.	$208549.	
1972	$5000.	$229290.	$234290.	

Figure 39. Report Generator for Case (4): Frozen, with $5,000 Yearly Fixed Asset Budgets

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