"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential
financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.


## A THESIS

Presented to

# the Faculty of the Graduate Division 

by
Donald Ross Pitts

```
In Partial Fulfillment of the Requirements for the Degree Master of Science in Mechanical Engineering
```

Georgia Institute of Technology
May, 1960

# LAMIHAR FLOW IN 

SCREWED PIPE FITTINGS


Date Approved By Chairman: Ineng 26,1960

## ACKNOWLEDGEMENTS

This problem was undertaken as a result of a suggestion by Dr. C.W. Gorton. His valuable guidance and assistance throughout the study and the preparation of this thesis is appreciated. Acknowledgements are also made to Mr. B. F. Barfield and Mr. J. M. Spurlock who served on the reading committee and rendered helpful suggestions. Finally, appreciation is expressed to my wife, Bettie, whose understanding and encouragement have made this thesis possible.

TABLE OF CONTENTS
Page
ACKNOWLEDGEMENTS ..... ii
LIST OF TABLES ..... v
LIST OF FIGURES ..... vi
LIST OF SYMBOLS ..... vii
SUMMARY. ..... $1 x$
Chapter
I. INTRODICTION. ..... 1
II. INSTRUMENTIATION AND EQUIPMENT: ..... 3
III. TEST PROCEDURE. ..... 5
IV. CORRELATION OF DATA ..... 7
Fluid Properties
Pressure Loss Data Correlation for IndividualFittings
Pressure Loss Data Correlation for Similar Fittings
v. CONCLUSIONS ..... 11
VI. RECOMMENDATIONS ..... 12
Application of Data
Further Investigation
APPENDIX
A. APPARATUS ..... 15
B. OIL DATA. ..... 20
C. SAMPLE CALCULATIONS . . . . . . . . . . . . . . . . . . . 24
D. TABULATED EXPKRIMENTAL AND COMFUTED DATA. . . . . . . . . 29
E. EXPERIMENTAL ERROR. . . . . . . . . . . . . . . . . . . . 39

BIBLIOGRAPHY . . . . . . . . . . . . . . . . . . . . . . . . . . . 42

## LIST OF TABLES

Table Page

1. Lengths of Pipe Used in the Test Systems ..... 18
2. Pipe Inside Diameters for the Test Systems. . . . . . . . . ..... 18
3. Fitting Inside Diameters for the Test Systems ..... 18
4. Fi'sting Length to Inside Diameter Ratios ..... 19
5. Fitting Inside Diameter to Pipe Inside Diameter Ratios. . . ..... 19
6. Experimental Data for Three-Eighths Inch System ..... 30
7. Computed Data for Three-Eighths Inch System ..... 37
8. Experimental Data for One-Half Inch System. . . . . . . . ..... 32
9. Computed Data for One-Half Inch System ..... 32

## LIST GF FIGURES

Figure ..... Page

1. Recirculating, Constant Head Flow System Schematic..... ..... 16
2. Test Section Schematic. ..... 17
3. Variation of Specific Gravity with Eemperature. ..... 21
4. Variation of Kinematic Viscosity with Temperature ..... 22
5. Variation of Equivalent liength with Reynolds Number for 90 Degree Bends ..... 33
6. Variation of Equivalent Length with Reynolds Number for Tees. ..... 34
7. Variation of Equivalent Length with Reynolds Number for 45-Degree Bends ..... 35
8. Variation of Equivalent Length Bivided by Diameter Squared with Reynolds Number for 90. Degree Bends. ..... 36
9. Variation of Equivalent Length Divided by Diameter Squared with Reynolds Number for Tees ..... 37
10. Variation of Equivalent Length Divided by Diameter Squared with Reynolds Number for 45 Degree Bends. ..... 38
$\qquad$

## LIST OF SYMBOLS

A pipe cross-sectional area
d measured value of pipe inside diameter
$d_{f} \quad$ measured value of fitting inside diameter
$d_{n} \quad$ catalog value of pipe inside diameter
f friction factor
$h_{L}$ calculated head loss using diameter d (feet of oil)
$h_{\text {Ln }}$ calculated head loss using diameter $d_{n}$ (feet of oil)
$h_{1}$ measured head loss in $X$ inches of straight pipe upstream. of test specimen
$h_{2}$ measured head loss for 90 degree bend plus $X$ inches of straight pipe
$h_{3}$ measured head loss for right angle flow through a tee plus $X$ inches of straight pipe
$h_{4}$ measured head loss for 45 degree bend plus $X$ inches of straight pipe

L length
$L_{1}$ equivalent length for a 90 degree lyend expressed in feet based on catalog value of inside diameter for schedule 40 pipe
$\mathrm{Le}_{2}$ equivalent length for right angle flow through a tee expressed in feet based on catalog value of inside diameter for schedule 40 pipe
$\mathrm{Le}_{3}$ equivalent length for a 45 degree bend expressed in feet based on catalog value of inside diameter for schedule 40 pipe
m fluid mass flow rate
Q volumetric flow rate
R Reynolds number based on measured pipe inside diameter
$R_{n} \quad$ Reynolds number based on catalog value of schedule 40 pipe inside diameter
$t_{1}$ temperature of fluid flowing, degrees F'ahrenheit
${ }^{t}{ }_{2}$ ambient temperature at the menometer board, degrees Fahrenheit
W a pipe length defined in Appendix A
$x \quad$ a term defined in Appendix $A$
X a pipe length defined in Appendix A
Y a pipe length defined in Appendix A
Z a pipe length defined in Appendix A
$\rho_{a}$ fluid density, manometer oil
$\rho_{f} \quad$ fluid density, oil flowing
$\downarrow$ fluid kinematic viscosity

SUMMARY

The problem investigated is that of experimentally determining pressure losses for laminar flow through screwed iron pipe fittings. A literature survey has indicated that this problem has previously been investigated only for elbows. The published data from this previous work indicates large experimental deviations from the equivalent length versus Reynolds number curve presented as a correlation of data obtained with one-half to four-inch elbows.

The present investigation included tees, 45 degree bends, and 90 degree bends in three-eighthsand one-half inch iron pipe. A piping system incorporating each of the above fitttings and pressure instrumentation was constructed for each of the listed pipe sizes. Data were obtained over a Reynolds number range of 300 to 1000 . These were presented as graphs of fitting equivalent length versus Reynolds number.

The results indicate that a single curve cannot directly represent equivalent length variation with Reynolds number for several sizes of similar fittings. However, a correlation is obtained when the equivalent length'divided by the square of the fitting inside diameter is plotted as a function of the flow Reynolds number.

As previousiy expected, the results indicate the pressure loss for right angle flow through a tee to be of greater magnitude than for flow through either a 45 or 90 degree bend. One unexpected fact was uncovered; the loss through a 45 degree bend is greater than the loss through a 90 degree bend.

It is recommended that the correlation of data from more than one size fitting (by presenting equivalent length divided by the square of the diameter as a function of Reynolds number) be more thoroughly investigated. It is not deemed advisable to employ this method to extend the range of application of the present data.

## CHAPTER I

## INIRODUCTITON

The problem under investigation is to determine frictional pressure losses for laminar incompressible flow through screwed iron pipe fittings. The lack of published information concerning this problem prevents accurate analyses for design of laminar flow systems. A wealth of published pressure drop data is available for turbulent flow, but the less frequently encountered laminar flow regime has apparently created little interest. Wilson, et al., (1) *have obtained data for 90 degree screwed elbows only, and this information is not necessarily applicable to other fittings. Beck (2) investigated the case of laminar flow in straight pipes, bends, and fittings for flanged or welded connections. The data from this work are not applicable to screwed connection fittings because of the absence of geometrical similitude. This point is expressly stated by Beck. No other pressure loss information for the laminar flow regime was revealed by the literature survey conducted as a part of the present investigation.

The analytical solution for head loss for laminar flow through a 90 degree bend is yet to be discovered. Consequently, a practical
*Numbers in parentheses refer to the references listed in the Bibliography.
approach to the problem under investigation would appear to be an experimental one. A desirable result would be data correlation in a readily useable form, and such a correlation is suggested as a part of this problem.

## CHAPIER II

## INSIRUMENTAATION AND EQUIFMENI

The test apparatus is illustrated schematically in Figs. 1 and 2 on pages 16 and 17. White mineral oil was the test fluid used throughout the program. The constant head supply provided gravity flow free from pump fluctuations. Oil flow rates were controlled with a manually operated gate valve upstream of the test specimen. The oil sump was provided with a steam coil heat exchanger to permit heating of the oil. Heating resulted in lowered oil viscosity and consequently increased maximum flow rates.

Piezometers for static pressure mearurement were located five diameters upstream and 76 and 58 diameters downstream of the threeeighths and one-half inch fittings respectively.

It can be shown that the length required to obtain fully developed laminar flow in a tube with a well designed entry section originating in a large fluid reservoir is given by:

$$
\frac{L}{d}=0.058 \mathrm{R}
$$

It is assumed that the distance required to obtain fully developed lamingr flow is less for the fittings than that given by the above equation. Since it was initially decided to restrict the investigation to Reynolds numbers below 1000, it was further assumed that pressure measurement 58 diameters downstream of the test specimen would include all pressure losses attributable to the fitting.

Each piezometer consisted of four 0.062 diameter holes, perpendicular to the pipe centerline and at 90 degree angulap spacing on the pipe circumference, surrounded by a collection ring. Each hole was de-burred to minimize errors in static pressure measurement. Pressure readings were obtained with single tube vertical manometers and a cathetometer with 0.005 centimeter graduations.

Mass flow rates were measured with an electric timer with 0.1 second graduations and a balance type scale with 0.01 pound graduations. Fluid properties were measured with a $0-200$ degree Fahrenheit themometer, two hydrometers, and two Saybolt viscosimeters. Specific gravity and viscosity data are presented as finctions of temperature in Figures 3 and 4 on pages 21 and 22.

## CHAPMER III

## TEST PROGEHBRE

Oil flow was established in the system by energizing the pump motor and opening the flow control valve. When so desired, steam was supplied to the heat exchanger. The system was allowed to stabilize With respect to temperature which was indicated by decreasing magnitude pressure fludtuations in the manometers; Fhring the system warm-up period, the manometers were bled to renove trapped or entrained air from the piezometer collection rings and tubing.

When the pressure fluctuations had decreased until they were no longer visually observable, the flow was considered to be stable and data were recorded. This was accomplished by recording the fluid discharge timperature, marking the oil meniscus in each manometer, collecting a tivied mass of fluid, again marking the meniscus in each manometer and again recording the fluid temperature. A small pressure change usually occurred while the mass flow rate was being measured. These pressure changes averaged approximately one-sixteenth. of an inch of oil, and during several tests, no observable pressure changes occurred. The temperature of the fluid flowing varied no more than one-half of one degree Fabrenheit during any single test. In addition to the previously mentioned data, the ambient air temperature at the manometer boace was recorded for each flow rate. This was later used to comrect the pressure data for the difference in specific gravity between the oil in the manometers and the
oil flowing through the system. The previously described system warmup usually required approximately two hours which allowed the oil in the manometers to cool. It was assumed that the manometers were in thermal equilibrium with the ambient air.

The average of the two meniscus marks on the manometer board was later measured with a cathetometer and recorded. This afforded maximum time for accurate leveling and adjustment of the cathetometer. Since all other data had been previously obtained in a minimum time period thereby reducing errors due to minute flow changes, this was considered to be the best procedure possible with the existing instrumentation.

## CHAPrer IV

## CORREJATION OF EXPERIMENTAL DATA

Fluid Properties.--The variation of specific gravity with temperature for petroleum products is normally linear for temperatures to approximately 400 degrees Fahrenheit. The measured values obtained as a part of this problem comply with this, and consequently are presented as Figure 3, page 2l, on a rectangular coordinate graph.

The variation of viscosity with temperature for petroleun filuids is not normally linear. The American Society For Testing Materials has developed standard viscosity-temperature charts for liquid petroleum products which provide linearization of such data. Figure 4 includes a portion of the Society's standard viscosity-temperature chart for 1iquid petroleum products (D341-43), chart $C$, kinematic viscosity, high range. The data obtained during this investigation and data obtained by others using a different viscosimeter during a concurrent investigation are presented on this chart. At the higher temperatures, the experimental data deviate markedly from the linearized extrapolation, however, the extrapolation is considered to be the best interpretation of the bulk of these data.

Pressure Loss Data Correlation for Individual Fittings.--As previously stated in the introduction, the primary aim of this research is to present pressure loss data in a useful form. A generally accepted presentation for turbulent flow pressure losses in fittings is a relationship between fitting equivalent length and Reynolds number. This affords the
user the convenience of simply adding values to piping lengths and then performing a single pressure loss calculation for the complete system. Consequently, this form of data presentation is employed herein.

The pressure loss attributable to a particular fitting must include effects due to the downstream flow disturbance created as the fluid passes through that article, but not the loss attributable to the downstream pipe without the fitting. Since the test systems were designed for the upstream pipe length identical with the combined pipe lengths between plezometers for each fitting, the pressure loss measured for the straight plpe alone was deducted from the pressure loss measured for the pipe and fitting. The difference is the loss attributable to the fitting.

The piping used in both systems differed in inside diameter with standard or catalog values of pipe dimensions. Again for reasons of data applicability, the pressure loss correlation is made with Reynolds numbers based on the catalog values of inside pipe diameters. The pressure loss attributable to the fitting divided by the pressure loss per foot of length of straight pipe is defined as the fitting equivalent length. The measured pressure loss per foot of straight pipe was corrected for the small deviation from the catalog value of inside diameter by the method presented in Appendix C. The correlations thus obtained are presented as Figures 5, 6, and 7 on pages 33, 34, and 35 for 90 degree bends, tees, and 45 degree bends respectively.

The correlations for the two tees, as determined from Figure 6, are:

$$
\begin{equation*}
\mathrm{Le}=0.408 \times 10^{-3}\left(\mathrm{R}_{\mathrm{n}}\right)^{1.25} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{Le}=0.616 \times 10^{-3}\left(R_{n}\right)^{1.25} \tag{2}
\end{equation*}
$$

for the three-eighths and one-half inch sizes respectively. Unfortunately, the 90 and 45 degree bend data correlations do not permit the formulation of similar simple empirical equations.

Pressure Loss Data Correlation for Similar Fittings.--The figures obtained by the preceding method for similar fittings suggest that further correlation to eliminate the difference due to size is possible. Geometrical similitude is established by two dimensionless parameters, the inside unthreaded length divided by the inside diameter and the ratio of the fitting inside diameter to the discharge pipe inside diameter. These ratios based on measured fitting and pipe inside diameters and design data lengths are listed in Tables 4 and 5.

The major pressure loss difference between two similar fittings of different size is probably due to differences in energy dissipation at the restriction imposed by the discharge pipe extending into the fitting. Obviously, for flow through similar fittings at the same Reynolds number, temperature, and visocisty, the velocity is greater in the three-eighths than in the one-half inch size fittings. Since the velocity is a function of the inside diameter squared, this parameter would be expected to partially correlate the data for similar fittings. The values of equivalent lengths for each fitting, as indicated by the
curves of Figures 5, 6, and 7, were divided by the square of the fitting inside diameter and plotted versus Reynolds numbers based on the catalog value of pipe inside diameter. The correlations thus obtained are presented as Figures 8, 9, and 10 on pages 36,37 , and 38.

## CHAPIER V

CONCLUSIONS

Pressure losses in laminar flow due to fittings used with schedule forty, three-eighths and one-half inch iron pipe sizes are greater for right-angle flow through a tee'than through a forty-five or ninety degree bend of the same pipe size, and are greater for a forty-five degree bend than for a ninety degree bend of identical pipe size.

Laminar flow pressure loss data for both sizes of each of the three types of fittings tested can be presented as a single function of Reynolds number. This can be accomplished by plotting the equivalent length divided by the square of the fitting inside diameter versus the flow Reynolds number.

## CHAPTER VI

RECOMMENDATIONS

Application of Rata.--The results of this investigation should be considered as tentative and should be substantiated by statistical experiments. The data obtained for elbows ( 90 degree bends) appears to indicate greater pressure losses than those reported by Wilson, et al., (l) which were obtained.by measuring the pressure drop across two close-connected elbows and dividing this measurement by two. That apparatus was apparently designed to measure system pressure losses, and elbow data were obtained as a sideline investigation. It appears unlikely that such an approach could result in data as accurate as could be obtained with a single fitting and pressure instrumentation sufficiently far downstream to measure the total effect of the fitting.

In the absence of other data, Figures 8,9 , and 10 may be applicable to other sizes of fittings provided that geometrical similarity is maintained. It should be noted that the length to diameter ratios for forty-five degree bends employed in this problem were appreciably different; however, the correlation of data as shown in Figure 10 indicates that this parameter has little effect. More caution is advised concerning the fitting diameter to pipe diameter ratio. The data were obtained with schedule 40 pipe and are not recommended for use with pipe of markedly different wall thickness.

Extrapolation of these data is not advised except in the case of the tees. In the absence of other data, a small extrapolation
to lower Reynolds number values of Figures 6 or 9 appears reasonable due to the linear characteristic evidenced. Extrapolation to higher Reynolds numbers is not recommended since other investigations have indicated that a maximum fitting equivalent length for elbows is obtained at a Reynolds number appreciably below 2000.

These results should not be directly applied to designs employing fittings closely located to each other. The results presented are total pressure losses for a single fitting with sufficient straight pipe downstream to allow the fluid to return to the fully developed laminar flow condition.

Further Investigation.--Should further investigation of this problem be undertaken, it is recommended that a size and pipe thickness be selected to permit investigation of the previously mentioned fitting length to diameter and fitting internal diameter to pipe internal diameter ratios. The latter of these two is suspected of being more prominent in influencing pressure loss, consequently a system employing schedule 160 pipe is recommended. A refinement of the test apparatus to include insulation is desirable. This should reduce experimental error indicated by deviation from the isothermal, fully developed, laminar flow friction factor.

An investigation of the transition length required to allow the fluid to return to the fully developed condition would be useful. This would define the limitations on the data already obtained and permit more accurate application to system designs. This could
possibly be accomplished by multiple pressure instrumentation downstream of the fitting investigated.

APPENDIX A

APPARAIUS


Figure 1. Recirculating, Constant Head Flow System Schematic


Note: 1) Letter dimensions are tabulated in Table 1 .
2) Dimensions include threaded lengths extending into fittings.

Figure 2. Test Section Schematic

Table 1. Lengths of Pipe Used in the Test Systems

| Nominal <br> Size | $W$ <br> (Inches) | $X$ <br> (Inches) | $Y$ <br> (inches) | $Z$ <br> (Inches) |
| :---: | :---: | :---: | :---: | :---: |
| $3 / 8$ | 47.5 | 36.00 | 2.46 | 33.54 |
| $1 / 2$ | 47.5 | 37.29 | 2.96 | 34.33 |

Table 2. Pipe Inside Diameters for the Test Systems

| Nominal <br> Size | Pipe No. 1 <br> (inches) | Pipe No. 2 <br> (inches) | Pipe No. 3 <br> (inches) | Pipe No.4 <br> (inches) |
| :---: | :---: | :---: | :---: | :---: |
| $3 / 8$ | 0.4696 | 0.4701 | 0.4764 | 0.4717 |
| $1 / 2$ | 0.5914 | 0.5938 | 0.5909 | 0.5898 |

Note: Pipe diameters are averages of four measurements made at each end of each pipe.

Table 3. Fitting Inside Diameters for the Test Systems

| Nominal <br> Size | 90Degree Bend <br> (Inches) | Tee <br> (inches) | 45 Degree Bend <br> (Inches) |
| :---: | :---: | :---: | :---: |
| $3 / 8$ | 0.6588 | 0.690 | 0.6585 |
| $1 / 2$ | 0.8165 | 0.843 | 0.8327 |

Note: The diameters were obtained by taking the average of four measurements on a cut section of each fitting tested.

Table 4. Fitting Length to Inside Diameter Ratios

| Nominal <br> Size | 90 Degree Bend | Tee | 45 Degree Bend |
| :--- | :---: | :---: | :---: |
| $3 / 8$ | 1.71 | 1.63 | 1.33 |
| $1 / 2$ | 1.68 | 1.64 | 1.05 |
| Note:The catalog value of fitting lengths minus the thread engagement <br> lengths were used to determine fitting lengths for Table 4. |  |  |  |

Table 5. Fitting Inside Diameter to Pipe Inside Diameter Ratios

| Nominal <br> Size | 90 Degree Bend | Tee | 45 Degree Bend |
| :--- | :---: | :---: | :---: |
| $3 / 8$ | 1.39 | 1.46 | 1.39 |
| $1 / 2$ | 1.38 | 1.42 | 1.41 |

Note: Diameters of pipe and fittings were measured as described under Tables 2 and 3.

APPENDIX B

OIL DAIA


Note: Specific gravity of water at $60^{\circ} \mathrm{F}$ is unity

Figure 3. Variation of Specific Gravity with Temperature

$\triangle$ - Data by Author
o - Data by Others

Figure 4. Variation of Kinematic Viscosity wfith
Temperabure

## APPENDIX C

SAMPLE CALCULATIONS

## APPENDIX C

## SAMPLE CALCULATIONS

For each set of experimental pressure loss data, corresponding Reynolds numbers based on both the average inside diameter of the pipe used in the system and the catalog value of the pipe inside diameter were computed for the same flow rate. It is convenient to establish a relationship between these two Reynolds numbers. Writing expressions for each the following is obtained:

$$
\begin{equation*}
R=\frac{d v}{\nu} \text { and } R_{n}=\frac{d_{n} v_{n}}{\nu} \tag{3}
\end{equation*}
$$

Substituting for $v$ and $v_{n}$ yields

$$
\begin{equation*}
R=\frac{d \cdot 4 Q}{\nu \cdot \pi d^{2}}=\frac{1}{d}\left(\frac{4 Q}{\pi v}\right) \tag{3a}
\end{equation*}
$$

and

$$
\begin{equation*}
R_{n}=\frac{d_{n} \cdot 4 Q}{\nu \cdot \pi d_{n}^{2}}=\frac{1}{d_{n}}\left(\frac{4 Q}{\pi \nu}\right) \tag{3b}
\end{equation*}
$$

Combining equations (3a) and (3b) yields

$$
\begin{equation*}
R_{n}=R\left(\frac{d}{d_{n}}\right) \tag{4}
\end{equation*}
$$

The measured pressure loss for each fitting is divided by the pressure loss per foot of pipe to obtain the fitting equivalent length.

It is desirable to present this data in terms of catalog values of pipe inside diameters; consequently the relationship between pipe head loss and diameter is convenient. The head loss for laminar fluid flow in circular pipes is given by

$$
\begin{equation*}
h_{L}=\frac{x}{R} \cdot \frac{v^{2}}{2 g} \cdot \frac{L}{d} \tag{5}
\end{equation*}
$$

where

$$
f=\frac{x}{R}
$$

Consequently, the head loss based on the measured average pipe diameter is

$$
h_{L}=\frac{x \nu}{d \cdot \frac{4 Q}{\pi d^{2}}}: \frac{\left(\frac{4 Q^{2}}{\pi d^{2}}\right)^{2}}{2 g} \cdot \frac{L}{d}
$$

or,

$$
\begin{equation*}
h_{L}=\frac{1}{d^{4}}\left(\frac{2 Q \times \nu_{L}}{\pi g}\right) \tag{5a}
\end{equation*}
$$

Similarly, the head loss for a pipe of catalog value inside diameter is

$$
\begin{equation*}
h_{\text {Ln }}=\frac{1}{d_{n}^{4}}\left(\frac{2 Q x-\nu_{L}}{\pi g}\right) \tag{5~b}
\end{equation*}
$$

Combining equations (5a) and (5b) yields

$$
\begin{equation*}
h_{L n}=h_{L}\left(\frac{d}{d_{n}}\right)^{4} \tag{6}
\end{equation*}
$$

The following calculations are for the last set of data and results presented in Tables 8 and 9 on page 30.

From Figure 3, the specific gravities of the oil flowing and the manometer oil, 0.837 and 0.849 respectively, are obtained. From Figure 4, the Kinematic viscosity is found to be $0.276 \times 10^{-3}$ (feet) ${ }^{2}$ per second. The volumetric flow rate is determined by

$$
Q=\frac{\dot{\mathrm{m}}}{\rho_{\mathrm{f}}}=1.851 \times 10^{-3} \mathrm{ft} .3^{3} / \mathrm{sec} .
$$

The average fluid velocity is computed by the continuity equation where the area is calculated by assuming the pipe diameter to be the average of the measured diameters of the four pipe sections. Hence,

$$
v=\frac{Q}{A}=\frac{1.851 \times 10^{-3}}{1.908 \times 10^{-3}}=0.9705 \mathrm{ft} . / \mathrm{sec}
$$

Consequentiy, the actual Reynolds number is

$$
R=\frac{d v}{\nu}=\frac{0.5914 \times 0.9705}{12 \times 0.176 \times 10^{-3}}=274
$$

By equation (4),

$$
R_{n}=274 \quad \frac{0.5914}{0.622}=261
$$

The measured head loss in feet of fluid flowing for the upstream pipe is

$$
h_{1}=\frac{6.305}{30.48} \times \frac{\rho_{\mathrm{a}}}{\rho_{\mathrm{f}}}=0.2097 \mathrm{ft}
$$

From equation (5),

$$
x=\frac{h_{1} \cdot R}{v^{2}}\left(\frac{2 g d}{L}\right)
$$

or,

$$
\begin{aligned}
& x=\left(\frac{0.2097 \times 274}{0.941}\right)\left(\frac{64.4 \times 0.5914}{37.29}\right) \\
& x=f \cdot R=62.4
\end{aligned}
$$

The equivalent length of the 90 degree bend is computed as follows:

$$
L e_{1}=\frac{\left(h_{2}-h_{1}\right)\left(\frac{\rho_{a}}{\rho_{f}}\right)}{h_{\operatorname{Ln}}}
$$

By equation (6), this becomes

$$
L e_{I}=\frac{\left(n_{2}-n_{1}\right)\left(\frac{\rho_{a}}{\rho_{f}}\right)}{n_{L}\left(\frac{d}{d_{n}}\right)^{4}}
$$

Substituting $h_{1}$ for $h_{L}$ and correcting $h_{1}$ to units of centimeters of oil flowing results in:

$$
\begin{equation*}
L e_{1}=\frac{\left(h_{2}-h_{1}\right)\left(\frac{\rho_{a}}{\rho_{f}}\right)}{h_{1}\left(\frac{\rho_{a}}{\rho_{\mathrm{f}}^{-}}\right)\left(\frac{d}{d_{n}}\right)^{4}}=\frac{h_{2}-h_{1}}{h_{1}\left(\frac{d}{d_{n}}\right)^{4}} \tag{7}
\end{equation*}
$$

Substituting values of $h_{1}$ and $h_{2}$ from Table 8 and previously listed values of $d$ and $d_{n}$ yields

$$
L e_{1}=\frac{6.795-6.305}{6.305\left(\frac{0.5914}{0.622}\right)^{4}}=0.295 \mathrm{ft}
$$

Equivalent lengths of the other fittings are obtained similarly.

## APPENDIX D

TABULAIED FXXPERTMENTAL AND COMPUIED DATA

Table 6. Experimental Data for Three-Eighths Inch System

| $\left({ }^{\mathrm{O}_{\mathrm{F}}}\right)^{\prime}$ | $\begin{gathered} t_{2} \\ \left(O_{F}\right) \end{gathered}$ | $\begin{gathered} \dot{\mathrm{m}} \\ (1 \mathrm{~b} . / \mathrm{sec} .) \end{gathered}$ | $\begin{gathered} h_{1} \\ (\text { criv. }) \end{gathered}$ | $\begin{gathered} \mathrm{h}_{2} \\ (\mathrm{~cm} .) \end{gathered}$ | $\begin{gathered} \mathrm{h}_{3} \\ (\mathrm{~cm} \cdot) \end{gathered}$ | $\begin{gathered} \mathrm{h}_{4} \\ (\mathrm{~cm} .) \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 130.0 | 72.7 | . 0.1995 | 23.025 | 30.610 | 35.745 | 32.160 |
| 127.5 | 72.0 | 0.1972 | 23.815 | 31.255 | 36.295 | 32.970 |
| 129.0 | 72.0 | 0.1932 | 22.685 | 29.715 | $34 \cdot 560$ | 31.475 |
| 129.0 | 71.5 | 0.1850 | 21.695 | 28.190 | 32.515 | 29.900 |
| 126.5 | 71.0 | 0.1714 | 20.855 | 26.725 | 29.990 | 27.800 |
| 127.0 | 75.5 | 0.1983 | 24.195 | 31.820 | 36.795 | 33.205 |
| 126.0 | 75.5 | 0.1954 | 23.930 | 31.350 | 36.085 | 32.670 |
| 125.0 | 75.5 | 0.1895 | 23.760 | 30.755 | 35.040 | 31.660 |
| 123.5 | 75.5 | 0.1633 | 20.780 | 26.265 | 28.855 | 27.800 |
| 122.5 | 76.0 | 0.1521 | 19.705 | 24.280 | 26.535 | 25.635 |
| 121.8 | 76.0 | 0.1403 | 18.515 | 22.185 | 24.140 | 23.230 |
| 121.0 | 76.5 | 0.1266 | 16.880 | 20.005 | 21.300 | 20.775 |
| 120.0 | 77.0 | 0.1150 | 15.620 | 18.105 | 19.345 | 18.815 |
| 119.5 | 78.0 | 0.1042 | 14.330 | 16.255 | 17.180 | 16.745 |
| 115.0 | 66.5 | 0.1465 | 22.530 | 26.640 | 28.555 | 27.730 |
| 118.0 | 70.0 | 0.1104 | 15.465 | 17.670 | 18.680 | 18.210 |
| 124.5 | 70.5 | 0.1287 | 16.230 | 19.295 | 20.940 | 20.255 |
| 125.0 | 71.0 | 0.1128 | 14.110 | 16.550 | 17.535 | 17.075 |
| 123.5 | 73.0 | 0.0902 | 11.420 | 12.760 | 13.595 | 13.135 |
| 123.0 | 73.0 | 0.0794 | 10.250 | 11.095 | 11.820 | 17.480 |

Note: Symbols are defined on pages vil and vili.

Table 7. Computed Data for Three-Eighths Inch System

| $R$ | $R_{n}$ | $L e_{1}$ <br> $(f t)$ | $L e_{2}$ <br> $(f t)$ | $L e_{3}$ <br> $(f t)$ | $f . R$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 927 | 888 | 1.174 | 1.970 | 1.414 | 60.15 |
| 895 | 858 | 1.113 | 1.869 | 1.370 | 61.50 |
| 885 | 848 | 1.104 | 1.865 | 1.381 | 60.40 |
| 847 | 812 | 1.067 | 1.778 | 1.348 | 60.30 |
| 755 | 724 | 1.004 | 1.561 | 1.187 | 60.25 |
| 886 | 850 | 1.171 | 1.857 | 1.327 | 61.15 |
| 854 | 818 | 1.106 | 1.810 | 1.302 | 59.85 |
| 818 | 784 | 1.049 | 1.693 | 1.186 | 60.80 |
| 690 | 660 | 0.940 | 1.384 | 1.204 | 60.30 |
| 629 | 603 | 0.828 | 1.235 | 1.072 | 60.35 |
| 568 | 544 | 0.706 | 1.082 | 0.908 | 60.05 |
| 509 | 488 | 0.660 | 0.934 | 0.823 | 60.40 |
| 458 | 439 | 0.567 | 0.850 | 0.729 | 60.85 |
| 410 | 394 | 0.479 | 0.710 | 0.601 | 60.90 |
| 538 | 516 | 0.650 | 0.954 | 0.824 | 64.03 |
| 426 | 408 | 0.508 | 0.742 | 0.634 | 61.35 |
| 548 | 525 | 0.674 | 1.034 | 0.884 | 60.50 |
| 490 | 469 | 0.616 | 0.865 | 0.749 | 61.00 |
| 380 | 364 | 0.418 | 0.679 | 0.536 | 60.20 |
| 334 | 320 | 0.294 | 0.546 | 0.428 | 61.10 |
|  |  |  |  |  |  |

Note: Symbols are defined on pages vii and vili.

| $\begin{array}{r} t_{I} \\ \left(O_{F}\right) \\ \hline \end{array}$ | $\begin{gathered} t_{2} \\ \left(0_{F}\right) \end{gathered}$ | $\begin{gathered} \dot{\mathrm{m}} \\ (\mathrm{lb} . / \mathrm{sec} .) \end{gathered}$ | $\begin{gathered} \mathrm{h}_{1} \\ \text { (em.) } \end{gathered}$ | $\begin{gathered} \mathrm{h}_{2} \\ (\mathrm{~cm} .) \end{gathered}$ | $\begin{gathered} \mathrm{h}_{3} \\ (\mathrm{~cm} .) \\ \hline \end{gathered}$ | $\begin{gathered} n_{4} \\ (c m .) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 120.5 | 82.0 | 0.3494 | 19.825 | 30.360 | 38.435 | 32.100 |
| 120.7 | 81.0 | 0.3387 | 19.230 | 28.980 | 36.590 | 30.555 |
| 121.0 | 80.0 | 0.3170 | 17.750 | 26.560 | 33.305 | 27.960 |
| 119.5 | 78.0 | 0.2765 | 16.035 | 22.850 | 27.805 | 23.895 |
| 118.5 | 78.3 | 0.2300 | 13.605 | 18.325 | 21.580 | 19.185 |
| 118.0 | 78.8 | 0.1851 | 11.125 | 14.135 | 15.815 | 14.720 |
| 116.0 | 78.5 | 0.1401 | 8.710 | 10.210 | 11.360 | 10.745 |
| 123.0 | 78.0 | 0.0966 | 6.305 | 6.795 | 7.365 | 7.070 |

Note: Symbols are defined on pages vil and viil.

$R_{n}$

Figure 5. Variation of Equivalent Length with Reynolds Number for 90 Degree Bends


Figure 6. Variation of Equivalent Length with Reynolds Number for Tees


Figure 7. Variation of iquivenient Length with Reynolds Number for 45 Degree Bends

$R_{n}$

Figure 8. Variation of Equivalent Lenicti Divided by Diameter Squared with Reynolds Number for 90 Degree Bends


Figure-9. Variattor of Equivalant Lerigth Divided by Diameter Squared with Reynolds Number for Tees

$R_{n}$

Figure 10. Variation of tquivaient Length Divided by Diameter Squared with Reynolds Number for 45 Degree Bends

## APPFINIX E

EXPERIMENTIAL ERROR

## APPENDIX E

## EXPERIMENTAL ERROR

For each test performed and corresponding Reynolds number obtained, a friction factor was calculated. This was compared with the friction factor for fully developed, isothermal, laminar flow defined by

$$
h_{L}=\frac{64}{R} \cdot \frac{v^{2}}{2 g} \cdot \frac{L}{d}
$$

For the three-eighths inch pipe, the deviation from the theoretical friction factor ranged from -6.49 to +0.47 percent. The majority of these were approximately five percent low. Deviations from the theoretical friction factor for the one-half inch pipe ranged from -5.53 to -2.19 percent.

Two factors could be primarily responsible for these deviations. The viscosity data is somewhat questionable in the range of temperatures most frequently employed, 115 to 125 degrees Fahrenheit. The data obtained at higher temperatures indicate that the linearized curve which best represents the bulk of data is too high. The second explanation is that due to deviation from isothermal flow. Much of the data were obtained with oil temperatures approximately 50 degrees Fahrenheit above the ambient air, consequently there existed a continuous heat transfer from the piping. The oil flow temperature was
measured at the discharge end only, approximately 15 feet downstream from the straight section of pipe for which the friction factors were calculated. The temperature of ofl flowing was obviously slightly higher than the measured values indicate, and this resulted in a negative value for the friction factor deviation. Preliminary checks using the lower viscosity values during the gathering of experimental data indicated the friction factor deviation to be in the range of two percent. Consequently, it was deemed unnecessary to insulate the system.

The effect of experimental error indicated by the friction factor deviation is to some extent eliminated by the method of data correlation. The pressure loss measured for the fitting has been divided by the pressure loss per foot of pipe based on measurements taken at the same time. This would eliminate any consistent error in the pressure data. Unfortunately, most of the apparent error is believed to be in Reynolds number measurement, and consequently these results should be considered as no better than the deviation from the isothermal, laminar flow friction factor indicates.

## BIBLIOGRAPHY

1. Wilson, R.E., W. H. McAdams, and M. Seltzer, "The Flow of Fluids through Commercial Pipe Lines," The Journal of Industrial and Engineering Chemistry, Vol. 14, Feb. 1922, p. II3.
2. Beck, Cyrus, "Laminar Flow Friction Losses Through Fittings, Bends, and Values," Journal of The American Society of Naval Engineers, Vol. 56, Feb. 1944, pp. 62-83.
