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OPTIMUM DEPLOYMENT OF COUNTERMEASURES

A THESIS

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SUMMARY

The objective of this thesis was to present a procedure to be used in determining which expandable countermeasures a penetrating aircraft should use against a given air defense system. It is shown that an optimum deployment strategy, which specifies the type and number of countermeasures to be dispensed at various points along the aircraft's route, can be found. The decision criterion on which the optimum strategy is based include probability of aircraft survival and cost-effectiveness.

In order to determine the optimum deployment strategy, a mathematical model was developed to relate the parameters of the radar system and the variables associated with the penetrating aircraft as well as the countermeasures. The output of the model is the probability of the aircraft surviving a section of the flight route. The probability of survival, the decision criterion, and a capacity constraint are used to determine the optimum strategy. The cost-effectiveness criterion is an expected cost expression which is the sum of the deterministic cost of the countermeasures and the cost associated with the possible loss of the aircraft. The optimization technique used is dynamic programming.

CHAPTER I

INTRODUCTION

The Problem Statement

Background

When tactical air forces fly missions over enemy-held territory, they usually encounter some type of defensive system. It may be a complex system, consisting of radars of different types, antiaircraft artillery, missile sites, and manned interceptors, or it may be a simple system consisting of one missile site and its associated radar.

Radar is used by the air defense system to aid in detecting and tracking the penetrating forces. It is also used in directing weapons, such as missiles and manned interceptors, into position to intercept and destroy the penetrating forces. The radar is an essential link in the control element of the defensive system, and anything that reduces its effectiveness, reduces the effectiveness of the entire system.

The various techniques that electronically interfere with radar performance are called electronic countermeasures (ECM) (4,11,12,13,24). Countermeasures can be divided into two classes, active ECM and passive ECM. Active countermeasures are called *jammers*. They either create false targets or mask the aircraft by radiating electromagnetic energy on the radar's frequency. Passive countermeasures do not generate electromagnetic energy. They act in a passive manner, creating false targets or masking the aircraft by reflecting large amounts of transmitted energy back to the radar. Thus countermeasures in both classes attempt to nullify the efficient operation of the radar system by either deceiving the radar operators with false targets, or by saturating the radar display with sufficient clutter to prevent the operators from detecting the aircraft.

Another classification of countermeasures is that of expendable and non-expendable countermeasures. Expendable countermeasures are considered non-recoverable after use, and all passive countermeasures are in that class. Non-expendable countermeasures are generally those carried on the aircraft and are lost only if the aircraft is lost. Until recently all active countermeasures were considered non-expendable. However, recent advances in solid state circuitry and battery power sources have made it feasible to develop expendable jammers (6).

The Problem

Passive countermeasures have been ineffective when used against radars equipped with modern electronic counter-countermeasures (4,14). The development of expendable jammers promises to regain the effectiveness of expendable countermeasures. As more expendable countermeasures become available, the mission planner is faced with the problem of deciding which of these countermeasures to carry on a given mission and the optimum location at which to use them. Factors affecting his decision include the capacity of the aircraft, the location of the threat radars with respect to the mission route, and the characteristics of the individual countermeasures. The objective, or criterion, used in making this decision is usually related to mission accomplishment and to the survival of the penetrating aircraft.

The availability of expendable jammers has created a similar problem for military personnel who are responsible for the evaluation and procurement of countermeasures. In evaluating competing new designs or modifications of existing countermeasures, the primary factors to be considered are cost and operational effectiveness. The strategy used in deploying the countermeasures can greatly influence their effectiveness. Therefore, the evaluator must determine the optimum timing and location for dispensing the countermeasures being considered. If such an optimum is practical to implement, then he should use it as the deployment strategy in his evaluation. A cost-effectiveness criterion that has been used in comparative evaluations is an expected cost expression which includes the deterministic cost of the countermeasures and the operational effectiveness expressed as probability of mission success (16).

Objectives

The primary purpose of this research is to develop a procedure for determining optimum deployment strategies for expendable countermeasures. A deployment strategy should specify the type and number of countermeasures to be dispensed at various points along a mission route. The criteria to be optimized are related to the problems discussed. In the mission planner's problem it is probability of survival since aircraft survival and mission accomplishment are of primary concern. In the countermeasure evaluator's problem cost-effectiveness is the criterion to be used since costs as well as effectiveness are of primary importance. A mathematical model must be formulated to relate the parameters of the radar system with the variables associated with the penetrating aircraft and its countermeasures. The technique for finding the optimum strategy for use in a given mission will be dynamic programming. A computer program will be developed and run for some example missions.

Literature Search

Although there are many articles on models for weapon systems, there are few on employing expendable countermeasures because of the relative newness of the concept of expendables. Some of these articles and others which relate to various aspects of the problem will be discussed, along with an indication of how they relate to this work.

Electronic Countermeasures

Descriptions of passive countermeasures can be found in references by Skolnik (24), Klass (12) and Dil Pare (6). The common passive ECM devices are chaff and decoys. Chaff is the oldest method of confusing an enemy aircraft. It consists of a large number of dipole reflectors, usually in the form of metallic foil strips packaged as a bundle. After they are released by the aircraft, the foil strips are scattered by the wind and form a highly reflecting cloud. A relatively small bundle can form a cloud with a radar cross section comparable to that of a large aircraft. When a single bundle is used it creates a false target and is called *spot chaff*. When an aircraft, or a forward fired chaff rocket, continuously releases chaff, a corridor-like cloud is formed and any following aircraft that fly through it are masked on the radar much like a smoke screen. Chaff deployed in this manner is called *corridor chaff*. Once the chaff is in the atmosphere, its velocity is due only to wind and gravity, thus it is a very slow moving target compared to an aircraft. Radar engineers recognized this fact and designed moving target indicators to eliminate chaff from the operator's display, making it ineffective.

Decoys are small aircraft-like vehicles fitted with radar signal enhancement devices such as corner reflectors to make them appear as large as an aircraft on the radar display. They must have a power supply to give them a speed similar to the speed of an aircraft. The power supply adds extra weight which limits the number of decoys that can be carried conveniently. It also makes them relatively expensive, further limiting their usefulness.

Active countermeasures are discussed in references by Skolnik (24), Klass (13), Dax (4), Holahan (11), Kovit (14), and Day (5). Active jammers include noise jammers which seek to hide the target by saturating the radar receiver with noise and repeater jammers which create false targets.

Noise jammers usually radiate white Gaussian noise covering the bandwidth of the radar receiver to be jammed. Spot jammers radiate large amounts of energy on a relatively narrow band and are highly effective when radiating on the correct bandwidth. However, many modern radars have frequency "agility" which allows them to change frequency rapidly. Jammers that can radiate noise over a relatively wide band of frequencies are *barrage* jammers. They usually can cover the entire tuning range of a particular class of radar transmitters, thus rapidly changing frequencies offers no relief from jamming. However, the power available is spread over a wide band resulting in less noise power within the radar receiver passband than if the same power were radiated by a spot jammer. Sweepthrough jamming is another way to jam over a frequency band wider than that of a spot jammer. The jammer "sweeps" the carrier frequency of a tunable transmitter over the radar band. Thus it radiates large amounts of energy on each frequency for a very short period of time. The effectiveness of the sweepthrough jamming depends on obtaining a noise modulation in which the time taken by the sweeping carrier to traverse the receiver band is approximately equal to the receiver response time.

The effect of weak noise jamming is to paint a single strobe on the face of the display at the azimuth angle of the jammer. As the power increases the strobe becomes wider and more strobes appear at different azimuth angles. If the jammer is powerful enough the scope will be completely blocked so that no targets appear.

Repeater jammers generate false echoes by delaying the received radar signal and retransmitting it at a slightly later time. The delay causes the repeated signal to appear at a range and/or azimuth different from that of the jammer. A transponder repeater plays back a stored replica of the radar signal after it is triggered by the radar.

S. Kownacki (15) has proposed a method of utilizing chaff and a transponder to create multiple moving targets. In the proposed method, chaff is deposited in the form of clumps, ejected backwards from the dispenser, with the relative velocity roughly equal to that of the true velocity of the dispenser. (This prevents it from burning up in a dense atmosphere.) It is then irradiated by an airborne transponder, located on the vehicle dispensing the chaff, so as to simulate desired radar echoes in range and Doppler, making returns indistinguishable from echoes from the aircraft. The transponder is triggered by the pulses from the ground radar. This will be one of the expendable countermeasures included in the set of countermeasures to be evaluated by the proposed procedure.

Expendable jammers first appeared as decoys equipped with active repeaters to help enhance the radar signal (12). Decoys were also equipped with small jammers to mimic jammers on the target aircraft in order to make them appear more realistic. Dil Pare (6) describes expendable jammers in detail in his article. They may be dispersed by dropping them from aircraft, or from precursor missiles or rockets. They usually require parachutes to remain aloft a sufficient amount of time. However, it is possible to deploy expendables that radiate signals while on the ground or in tree tops. Some expendable jammers radiate a noise-modulated or cw signal tuned to the radar's frequency automatically. Others are pre-set prior to deployment. Often the parachute shrouds serve as the antenna for the expendable jammer. One difficulty Dil Pare mentions is in developing an efficient power source that is cheap enough to be considered expendable. The various dispensing techniques described here are considered in the development of the model.

Radar Equations

In order to relate the various parameters of the radar, aircraft and jammers, radar engineers have developed the radar equations. A widely referenced text which covers the basic theory behind the equations has been written by Skolnik (24). The signal power obtained by a receiving system from reflected energy is

$$S = C_1 \left(\frac{P_r G_r^2 \lambda^2 \sigma}{(4\pi)^3 R_t^4} \right) \tag{1-1}$$

where

S = power at receiver emanating from radar transmitter.

P_r = generated radar power.

G_r = radar antenna gain.

λ = receiver center frequency wave length.

σ = effective radar target radar cross section.

R_t = range from radar to target.

C_1 = constant that includes radar losses and signal processing gain.

The power obtained by the receiver from a jammer is

$$J = C_2 \left(\frac{P_j G_j G_r' \lambda^2 B_r}{(4\pi)^2 R_j^2 B_j} \right) \tag{1-2}$$

where

J = power at receiver emanating from jamming source.

P_j = generated jamming power.

G_j = transmit antenna gain of jammer.

G_r' = receiver antenna gain, antenna (main or side) lobe receiving jammer signal.

B_r = receiver band width.

B_j = jammer band width.

R_j = range from jammer antenna to receiving antenna.

C_3 = constant that includes losses in the jammer and the receiver.

The parameters to be used in this research will be described in greater detail in the next chapter.

Friedman (9) shows how the equations for J and S can be combined to form a signal-to-noise ratio (SNR). He defines SNR as the ratio of information carrying signal to noise from which intelligence can be extracted. Based upon the SNR, a threshold is established at the receiver output. The signal-plus-noise energy must exceed this threshold for the signal to be detected. If the noise alone exceeds this threshold, a false alarm results. For a given false alarm rate the SNR can be related directly to probability of detection.

Receiver output noise power is defined as N where

$$N = KT_eB_r. \tag{1-3}$$

Noise power is identified as antenna and amplifier noise as well as usual system losses. It is customary to reference such receiver noise to T_e, the effective noise temperature of the receiving system. K is Boltzmann's constant.

In the presence of jamming SNR is

$$\mathrm{SNR} = \frac{\text{signal}}{\text{self noise} + \text{jamming}}$$

$$\mathrm{SNR} = \frac{S}{N + J}$$

$$\mathrm{SNR} = \frac{C_1\left(\dfrac{P_r G_r^2 \lambda^2 \sigma}{(4\pi)^3 R_t^4}\right)}{KT_e B_r + C_2\left(\dfrac{P_j G_j G_r' \lambda^2 B_r}{(4\pi)^2 R_j^2 B_j}\right)} . \qquad (1\text{-}4)$$

In a correction to Friedman's article, Miamidian (19) shows that for N jammers

$$\mathrm{SNR} = \frac{C_1\left(\dfrac{P_r G_r^2 \lambda^2 \sigma}{(4\pi)^3 R_t^4}\right)}{KT_e B_r + \dfrac{\lambda^2 B_r}{(4\pi)^2}\left[\sum_{n=1}^{N}\left(\dfrac{P_{jn} G_{jn} G_{rn}' C_{2n}}{R_{jn}^2 B_{jn}}\right)\right]} . \qquad (1\text{-}5)$$

Friedman notes that the SNR equations may be simplified for other than maximum range target locations. To calculate the required jamming power to screen a target not near radar maximum range, the radar receiver self-noise, KT_eB_r may be neglected.

Another reference which discusses radar equations is Barton's article (1). He derives radar equations to calculate the performance of radar in a clutter environment such as chaff. Chaff is a passive countermeasure that is relatively easy to counter with ECCM. Thus it will not be included in the model formulated in this thesis. The equations derived by Barton make it easy to extend the model to include chaff.

Evaluating Expendable Countermeasures

In a recent thesis, La Force (16) developed a general methodology to be used in the comparative evaluation of expendable countermeasures. From a comparison, the optimal mix of countermeasures to be carried on a particular mission can be selected. The decision criterion on which the selection is based is cost-effectiveness.

The measure of cost-effectiveness used is the expected cost resulting from the use of the particular mix of countermeasures under consideration. The expected cost is defined as the sum of a deterministic cost and a stochastic cost. The deterministic cost is the cost of the countermeasures used in the mission. The stochastic cost are determined by defining possible outcomes of the mission and assigning cost to these outcomes. These costs are then weighted by the probabilities of the respective outcomes resulting from the use of the countermeasures being evaluated. Summing these weighted costs yields the stochastic portion of the expected cost.

This research will make use of that cost-effectiveness criterion. However, the outcomes used will not be the same as those used by La Force. The outcomes used in this thesis are mission success and mission failure. The mission is considered a success if the penetrator survives the mission, otherwise it is a failure.

In his work La Force uses integer programming to determine the optimum aircraft load for various mixes of countermeasures. He then discusses the importance of using an optimum strategy in deploying the countermeasures. He suggests that a set of strategies be specified and an evaluation be conducted using each strategy with each mix of countermeasures. Clearly, the number of evaluations conducted will increase very rapidly when using that procedure. This work will show how dynamic programming can be used to determine the optimum deployment strategy and optimum aircraft load with regard to probability of success. This probability of success could then be used in La Force's procedure to determine the most cost-effective set of countermeasures. However, it will be shown that an alternative formulation of the dynamic programming procedure can be used to obtain this result directly.

La Force suggests using computer simulation as the technique to determine the probabilities associated with the outcomes. He concludes that it may be the only technique available that can handle the many parameters which must be considered. He states that due to the high expense involved in a simulation study it is important to obtain as much information as possible from pre-simulation studies. One application of the procedure developed in this research would be to determine the deployment strategy to be used in a detailed simulation study.

Fukuda et al. (10) consider an expendable countermeasure problem in a slightly different context. Their penetrator is a ballistic missile warhead and their countermeasures are decoys. The objective is to enhance survivability of the warheads against terminal defenses. The number and size of decoys that can be carried in a missile payload is subject to constraints. The effectiveness of the decoys depends on their size (compared to the warhead) and the number used. The strategy of the offense is a selection of a decoy specification which, under size and weight constraints, uniquely determines the number, say N, of decoys to be contained in a missile payload together with a fixed number, say K (possibly greater than one), of warheads. The problem is viewed as a game of strategy between the offense and the defense and dynamic programming is used to find the optimal strategies.

In determining the probabilities of survival of the warhead, they consider a cloud of L = K + N objects which is subjected to defense action. The defense correctly identifies a warhead and kills it with probability, P_k. The probability of survival is given by:

$$\frac{K}{L}(1-P_k). \qquad (1\text{-}6)$$

A similar expression is used in this thesis when dealing with decoys.

Brodheim et al. (3) have developed a general model of air defense/offense interaction on a basis of a dynamic program. The attacking force consists of aircraft, tactical missiles or ICBM's, and the defense system consists of control centers and interceptor missiles. The objective of the offensive force is to destroy a given set of targets with the least possible cost. The defense objective is to maximize the offensive cost of an attack. Hence, the measure of effectiveness they use is the expected value of the total cost of an attack. One application suggested is to determine the effectiveness of using decoys during various stages of the attack. Since their attacks are repeated missions until the target is destroyed or until the defense runs out of weapons, it cannot be applied to the problem considered in this report.

Tactics Selection

Fawcett (8) applies dynamic programming and some notions in decision theory to the problem of making a rational selection of tactics for an air-to-ground attack when faced with uncertainty. In his work he discusses a user problem and a weapon system designer's problem that are similar to our mission planner's problem and countermeasure evaluator's problem. He relates the user and designer problems by considering them as parts of a two stage decision problem. The designer's decisions are those that determine the characteristics of the weapon system. The user's decisions are depicted as second stage decisions that are made subject to constraints imposed by the system design. When evaluating the operational effectiveness of the various designs, the designer must consider the second stage decisions. These decisions are concerned with determining the best tactic (strategy) for a given design in a particular situation. That, he notes, can be a major constrained optimization problem where the objective function and constraints must be carefully formulated. He concludes that the tactics optimization problem seems to be inseparable from the design evaluation problem whenever the user has some latitude of choice as to the manner of system employment. This then provides the primary motivation for determining the optimal deployment strategy for expendable countermeasures.

Dynamic Programming

In the problems considered, sequential decisons must be made concerning how many countermeasures of a certain type to allocate along each section of the flight route. The number of countermeasures available is limited by the capacity of the aircraft. Each allocation is considered to have a given effect on an objective function which is to be optimized. Dynamic programming is a very powerful approach to solving this type problem.

Bellman (2) invented the name dynamic programming and developed much of the theory behind the technique. His "principle of optimality" is the starting point for developing the recursive relationships which make it possible to convert a sequential decision process containing many variables into a series of single-stage problems containing only a few variables. Stated in his words, "An optimal policy has the property that whatever the initial state and decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

In his text, Numhauser (21) discusses the techniques for applying the principle of optimality to solve sequential decision problems. He develops recursive relationships using serial multi-stage decision systems. He defines such a system as "a set of stages joined together in series so that the output of one stage becomes the input to the next stage." (See Figure 1.) In order to have the optimal return as the function of the initial state, X_N, calculations are done by a "backward" recursion and the stages are numbered backward.

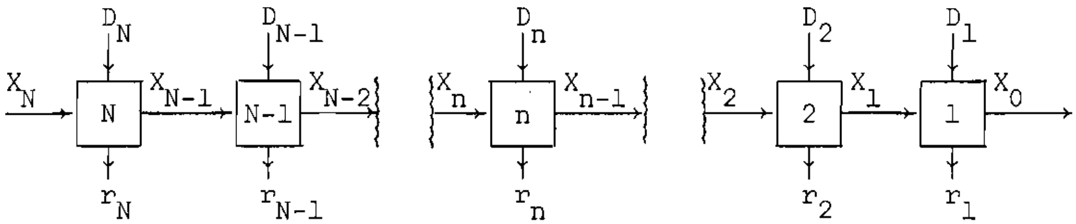

Figure 1. Diagram of Multi-Stage Process

For the general stage n (n=1,2,...,N) of the N-stage system the state of the system is completely described by the state vector X_n. The decision made at that stage is designated by the decision vector D_n the stage transformation is

$$X_{n-1} = t_n(X_n, D_n) \tag{1-7}$$

and the stage return is

$$r_n = r_n(X_n, D_n) \tag{1-8}$$

Denoting $f_N(X_N)$ as the maximum N-stage return we have

$$f_N(X_N) = \max_{D_N,\ldots,D_1} g[r_N(X_N,D_N), r_{N-1}(X_{N-1},D_{N-1}),\ldots,r_1(X_1,D_1)] \tag{1-9}$$

subject to

$$X_{n-1} = t_n(X_n, D_n) \qquad n = 1,\ldots,N .$$

In order to develop the recursive relations, we must be able to decompose a general function g as given in Equation (1-9) so that the maximization with respect to $D_{N-1},\ldots,D_1$ can be moved inside the N-stage return. A sufficient condition for making the important change in the position of the maximization has been given by Mitten (20). Nemhauser (21) calls it the *monotonicity* condition. If the function g satisfies the monotonicity condition and a condition he calls *separability*, it can be decomposed. The conditions Nemhauser requires are given. If

1. *Separability*

$$g[r_N(X_N,D_N),r_{N-1}(X_{N-1},D_{N-1}),\ldots,r_1(X_1,D_1)] \tag{1-10}$$

$$= g_1\{r_N(X_N,D_N),g_2[r_{N-1}(X_{N-1},D_{N-1}),\ldots,r_1(X_1,D_1)]\}$$

where g_1 and g_2 are real valued functions, and

2. *Monotonicity*

g_1 is a monotonically nondecreasing function of g_2 for every r_N. Then

3. *Decomposition*

$$\max_{D_N,D_{N-1},\ldots,D_1} g[r_N(X_N,D_N),r_{N-1}(X_{N-1},D_{N-1}),\ldots,r_1(X_1,D_1)] \tag{1-11}$$

$$= \max_{D_N} g_1\ r_N(X_N,D_N),\ \max_{D_{N-1},\ldots,D_1} g_2[r_{N-1}(X_{N-1},D_{N-1}),\ldots,r_1(X_1,D_1)]\}$$

It is easily shown that if the total return function is a sum of the individual stage returns or a product of the stage returns where the stage returns are non-negative real numbers, then the problem can be decomposed.

CHAPTER II

THE SCENARIO MODEL

General

The purpose of this chapter is to discuss the formulation of the model to be used to calculate the probability of survival of the penetrating aircraft. A more detailed description of the problem will be presented. A modified version of the signal-to-noise ratio equation (1-5) forms an important part of the model. The parameters of this equation and their relationship to the model will be described. Assumptions that are made to limit the complexity of the model will be discussed. The model developed will not be a complex one since its primary purpose is to aid in demonstrating the procedure for selecting the optimum deployment strategy. However, the discussion presented here will indicate how the model can be enlarged.

Tactical Missions

The Penetrating Force

Tactical missions over enemy held territory have one or more of three goals, air-superiority, interdiction, or reconnaissance. On an air-superiority mission the penetrating aircraft either attempt to engage enemy interceptor aircraft in the air, or they have a specific target that is part of the air defense system such as airfields, missile sites, or radars. The targets in interdiction missions are lines of communication or supply routes. On a reconnaissance mission the objective is to obtain information concerning the location of enemy installations and troops. A special type of reconnaissance mission is an *elint* mission (17,18). Here the goal is to obtain electronic intelligence concerning enemy radar locations, range, and signal characteristics. This information is then used in the planning of countermeasure employment for future missions.

Since the tactical aircraft usually have specific targets they plan their mission routes in advance. Modern navigational equipment and control from friendly radar makes it possible for the aircraft to stay on route, locate the target, and often avoid enemy weapons. Thus, in this model a fixed aircraft flight route and altitude will be used. It is assumed that the penetrator knows the position of enemy radars with respect to this route. Elint aircraft, satellites and other new equipment designed for pinpointing the location of enemy radar make this feasible.

For air-superiority and interdiction missions several aircraft are generally employed. For reconnaissance missions generally only a single aircraft is required although it may be escorted by a fighter for protection. Whenever more than one aircraft is used on a mission, they fly in close formation until the target is engaged. At this time they are often at low altitude and out of radar coverage if the target is not a radar site. While flying in close formation several aircraft usually appear as a single target on radar displays unless they are very close to the radar site. For this reason it is assumed that a single penetrating aircraft is used in the development of this model.

Examination of the SNR equation (1-5) shows that the only parameters associated with the aircraft are range of the aircraft from the radar R_t and the aircraft's radar cross section σ. Both of these will be variables in the model. Range is a common term that does not need explanation. Radar cross section provides a measure of the effective aircraft echoing area. It is defined as that equivalent area which would intercept the radar signal and, if scattered equally in all directions, produce an echo at the radar equal to that received from the target. It is dependent on the aspect angle from which the target is viewed as well as the radar frequency. It is a product of the illuminated surface area and the reflectivity of that area. For complex targets, such as moving aircraft, these parameters vary continuously, making analysis difficult. Thus, in the calculations only equivalent nominal values of radar cross section will be used.

The Radar System

The defensive systems encountered by tactical air forces are generally not as complex as the systems encountered by strategic air forces. Several factors contribute to this. Tactical forces are usually not employed until large portions of the defensive system have been destroyed by the strategic forces. Reconnaissance and interdiction missions do not require deep penetration into enemy territory. Thus they encounter only the border defense system. Tactical missions are also used extensively in limited wars where the opposing forces do not have the technology to build and operate complex systems.

The defensive system considered in this thesis will be a single surface-to-air missile site with its associated radar. All of the parameters associated with the radar and missiles are assumed to be known. The radar parameters are the power radiate P_r, radar antenna gain G_r, receiver antenna gain G_r', receiver bandwidth B_r and the radar frequency λ. In the model the values of these terms will be constants and will appear only in a general constant term. The missile parameters include range of the missile, probability of missile kill, and the number of missiles that can be controlled at the same time. The probability of missile kill will be assumed to be a function of the target range. It will be assumed that the missile site can control only one missile at a time.

From Equation (1-2) it is seen that only the receiver bandwidth and receiver antenna gain effect the power received from the jammer. The bandwidth appears in the ratio B_r/B_j and is used to accommodate the analysis of jammer spectra that exceed receiver bandwidth. The receiver antenna gain term represents gain from the main beam or sidelobes. When expendable jammers are deployed in close proximity to the radar, sidelobe jamming becomes important and must be included in the model.

Radars use directive antennas for transmission and reception. On transmission the directive antenna channels the radiated energy into a beam to enhance the energy concentrated in the direction of the aircraft. A measure of the ability of an antenna to concentrate energy in a particular direction is called the gain. Two different, but related definitions of antenna gain are the directive gain and the power gain. The directive gain is descriptive of the antenna pattern, but the power gain is more appropriate for use as a value in radar equations.

The power gain G_r is a measure of the power radiated in a particular direction by a directive antenna to the power which would have been radiated in the same direction by an omnidirectional antenna with 100 per cent efficiency. It is proportional to the area of the antenna. In the model it will be a constant.

The antenna pattern is a plot of the antenna gain as a function of the direction of radiation. A typical antenna pattern has the shape of sin X/X curve as shown in Figure 2.

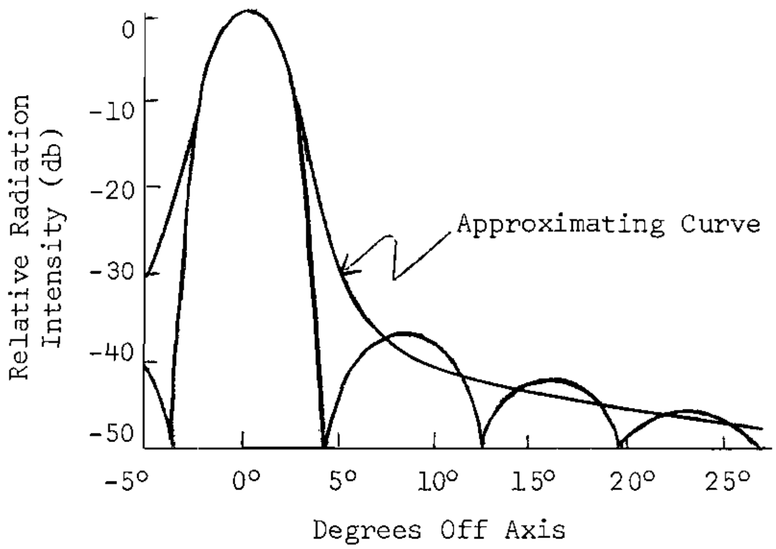

Figure 2. Antenna Radiation Pattern

The pattern has a main beam or lobe which carries the signal that is of primary interest and sidelobes that are radiated energy which is not properly directed. Values of receiver sidelobe antenna gain G_r' are frequently difficult to obtain. The antenna pattern may be unknown and even when it is known it has nulls and widely-varying sidelobes. The absolute sidelobe gain can vary with elevation, azimuth, frequency and mode of operation. Consequently a nominal average value of sidelobe gain is generally used. This may be approximated by

$$G_r' \doteq \frac{G_r}{1 + (\theta_j - \theta_t)^2} \tag{2-1}$$

where $(\theta_j - \theta_t)$ is the angle between the jammer and aircraft in degrees.[1] This is an approximation to the sin X/X shaped curve the radar pattern often resembles and is shown as the smooth curve on Figure 2.

Another characteristic of the antenna pattern is the shape of the main beam. Antenna beam shapes most commonly employed in radar are the pencil beam and the fan beam. The pencil beam is approximately axially symmetric and has a beam width of a few degrees or less. It is commonly used in target tracking radars where it is necessary to measure continuously the angular position of a single target in both azimuth and elevation. The fan beam is broad in one dimension and narrow in the other. For example, a long-range search radar may have a beamwidth of 1° in azimuth and 60° in elevation. The beamwidth determines the number of angular resolution cells that must be scanned in order for a radar to cover certain regions. If the search radar is required to give complete circular coverage, it must scan 360° in azimuth. The scanning region may be considered as being divided into 360 azimuth resolution cells of

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[1]This approximation was suggested to the author by Mr. Ron Pearl, Radar Branch of the Electronic Engineering Station, Georgia Institute of Technology.

1° each. When using expendable jammers of relatively low power, a minimum number is usually needed in each azimuth resolution cell to properly screen the aircraft. Thus the number of resolution cells, and hence the beamwidth, affect the number of jammers required.

An important set of parameters that should be included in a detailed model of a defense system are the parameters related to the radar operators. These parameters include the manner in which the operators utilize available equipment, and the various communications passing between the operators. A model that includes all of these parameters would be a rather complex model such as a simulation model. The model will consider only the operator's decision to fire a missile.

The Countermeasures

The countermeasures available for use by tactical air forces include those in the expendable and non-expendable classes. When employing non-expendable countermeasures, or on-board jammers as they are usually called, relatively little preplanning is required. They can be used at any time during the mission, and the decision to use them is often based on detecting the hostile radar signal (11). It is anticipated that effective employment of expendable countermeasures requires preplanning. The number and type of countermeasures that are to be used should be known prior to the mission in order to load the aircraft. A deployment strategy is needed to prevent inadvertently using all the countermeasures during an encounter at the beginning of the mission. A plan is needed to aid the pilot in selecting the correct time and direction to fire any precursor rockets or missiles that are used to disperse the countermeasures. It should be noted that this preplanning may require accurate intelligence on the enemy defensive system, and a fixed mission route.

From the SNR equations it is seen that the parameters associated with the jammers include the power of the jammer P_j, jammer antenna gain G_j, range of the jammer R_j and jammer bandwidth B_j. Personnel responsible for evaluating new countermeasure designs would be interested in comparing the cost-effectiveness of various changes in any of these parameters. For example, a new design may require a selection from several different antennas. They may vary in price, size, and antenna gain. In order to evaluate the effectiveness, the model would need a variable for antenna gain. However, from the viewpoint of the mission planner the antenna gain and bandwidth will be fixed by the designs available. He will be able to vary only the jammer range from the radar by selecting different delivery techniques, and the power by selecting the number to be deployed. In formulating the model, the jammer antenna gain and bandwidth will be considered constants and included in a common constant term. This will allow the demonstration of the procedure for both the mission planner's problem and the evaluator's problem to be accomplished with a less complex model.

The power of the jammer used is an important variable. In Equation (1-5), the power received at the radar from the jammers is a sum of the powers of the individual jammers. Thus increasing the numbers of jammers deployed in a certain location increases the power. Also Equation (2-1) indicates that the jammers deployed in several consecutive azimuth resolution cells contribute to the total power received by the radar. The power radiated by the individual jammer is limited by its size and its power source. The number that can be used is restricted by the capacity of the aircraft or the dispenser employed to disperse the countermeasures.

The jammer range is also an important variable. It is influenced primarily by the delivery method. From Equations (1-1) and (1-2), it can be seen that the jammer power at the radar varies inversely as the square of the distance between the radar and jammer while the radar echo power varies with distance inversely as the fourth power. Thus, for expendable jammers dispersed directly from the aircraft (and for on-board jammers), there will always be some distance below which the radar echo will be larger than the jammer signal. This is called the self-screening range or the *burn-through* range. Dispersing the jammers directly from the aircraft will require a large number of jammers because the aircraft and jammers separate rapidly due to the aircraft speed. Hence, other dispersion techniques appear superior.

From the SNR equations, the range of the jammer varies directly as a square term. Thus, the signal-to-noise ratio and the probability of detection will be lowered by deploying the jammers closer to the radar. However, the range from the radar at which the jammers can be deployed depends on the accuracy of the dispenser and the pilot's knowledge of the exact location of the radar with respect to his aircraft. To be of use, the jammer must be located between the radar and the aircraft. For a precursor rocket or missile dispenser, a known circular miss distance is assumed. From this miss distance, an average range from the radar can be found, such that the jammer will be on the correct side of the radar with a very high probability. This average range will be used in the model as the range of the jammers from the radar.

The Model

Probability of Detection

In Chapter I it was noted that the probability of a detection of an aircraft by a radar is related to the signal-to-noise ratio. In order to calculate the probability of detection A, a modified form of the SNR Equation (1-5) will be used. The expression for self noise, KT_eB_r, is dropped since it is needed only near maximum range. The terms P_r, λ, G_r, G_r', B_r and B_j are considered as constants as discussed in the previous sections. They and the constants C_1 and C_2 form a new constant C. The expression given by Equation (2-1) is used for G_r' and the signal-to-noise ratio for our model is[2]

$$SNR = \frac{C \dfrac{\sigma}{R_t^4}}{\sum_{n=1}^{N} \dfrac{P_{jn}}{R_{jn}^2} \cdot \dfrac{1}{1 + (\theta_t - \theta_{jn})^2}} . \qquad (2\text{-}2)$$

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[2]This equation was suggested to the author by Mr. Ron Pearl, Radar Branch Electronic Engineering Station, Georgia Institute of Technology.

For this thesis the probability of detection will be assumed to be directly proportional to the signal-to-noise ratio. The probability of detection of an aircraft using N expendable jammers is

$$A = \begin{cases} (SNR)\cdot C' & \text{for } (SNR)\cdot C'<1 \\ 1 & \text{for } (SNR)\cdot C'\geq 1 \end{cases} \quad (2\text{-}3)$$

where C' is a constant.

Probability of Aircraft Loss

The probability of the aircraft being destroyed L is dependent on the probability of detection, tracking error and the probability of the missile destroying the aircraft. The tracking error will also be considered dependent on the probability of detection. When the probability of detection is small, the tracking error will be large, if the aircraft is being tracked at all. When the probability of detection is high, the tracking error will be small. A threshold of the probability of detection A' will be used to determine whether the operator is tracking the target well enough to fire a missile. For example, if A' = 0.5, then the probability of detection must be greater than 0.5 for a missile to be fired.

The probability of the missile destroying the aircraft once it is fired will be a function of the range of the aircraft. It will be expressed as an equation or read into the computer as a table. For example, if the maximum missile range is 39 miles, the probability of the missile's destroying the aircraft at a range greater than 39 miles would be zero. Assuming the probability of the missile's destroying the aircraft at very close range to be 0.4, and the probability for other ranges as being directly proportional to the distance from the radar, the equation for probability of missile kill M will then be

$$M = \begin{cases} 0.4 - \dfrac{R_t}{100} & \text{for } R_t \leq 40 \\ 0 & \text{for } R_t \geq 40. \end{cases} \tag{2-4}$$

The probability of the aircraft being destroyed will increase as the tracking error decreases, or as the probability of detection increases. It will be expressed as a product of the probability of detection and the probability of missile kill. Using the probability of missile kill from this example, the probability of aircraft loss can be expressed as

$$L = \begin{cases} A \cdot M & \text{for } A \geq A', \ R_t < 40 \\ 0 & \text{otherwise.} \end{cases} \tag{2-5}$$

After the missile is fired, a period of time will elapse while the missile travels to the target. In this period of time the aircraft will also travel a certain distance. It can be seen from Equation (2) that the probability of detection will vary during the travel time. Therefore, we will use an average probability of detection for A in Equation (2-5).

When the expendable countermeasure is chaff irradiated by a transponder, a different expression is needed for probability of aircraft loss. The radar will detect n moving targets due to the chaff and one due to the aircraft. If these n+1 moving targets cannot be distinguished by the radar operator, the probability of identifying the aircraft correctly will be $\frac{1}{n + 1}$. The probability of the aircraft being destroyed will be a product of the probability of identifying the correct target and the probability of missile kill. That is

$$L = M \cdot \frac{1}{n + 1} . \tag{2-6}$$

This expression is analogous to Equation (1-6).

CHAPTER III

SELECTING THE OPTIMAL STRATEGY

General

In this chapter a procedure to determine optimal deployment strategies for the countermeasures will be presented. A mathematical formulation of both the mission planner's problem and the countermeasure evaluator's problem will be described. The solution procedure used to solve these problems will make use of the model developed in the previous chapter.

Application of the Model

The scenario model can be used to determine the probability of aircraft loss at different locations along the flight route. In order to calculate the probability that the aircraft survives the mission, the flight route must be divided into sections. The probability of aircraft loss is then calculated for each section of the route and these probabilities are combined to form the probability of aircraft surviving the mission or probability of success as it was defined in Chapter I.

Selecting the length of the sections of the flight route can be a problem. If they are very long, the average probability of detection used in the model may be exceedingly different from that at various points along the section. Also the number of missiles that could be fired may be greater than one, requiring a slight modification of the model. If they are shorter than the aircraft travel distance determined from missile flight time, using the model could result in having more missiles being fired than is actually possible. The length of the sections used in this report will be determined from the aircraft travel distance based on the time it takes for a missile site to lock-on, fire, and guide a missile to a target at maximum missile range. It is also assumed that the probability of aircraft loss obtained from the sections of this length is independent of the other sections.

Mission Planner's Problem

Mathematical Formulation

The problem facing the mission planner is to determine the optimal countermeasure deployment strategy that gives maximum probability of aircraft survival. The mission route will be divided into N sections of equal length. At each of the sections the mission planner can deploy a number and type of countermeasure resulting in a probability of aircraft loss for that section. The total number of countermeasures used in the mission is limited by the available capacity of the aircraft.

Let d_n represent the type countermeasure that can be used at the n*th* section of the mission route. Associated with each type of countermeasure is a set of parameter values that are used as the variables in Equation (2-2) of the scenario model. The function $L_n(d_n,e_n)$ denotes the probability of aircraft loss and is calculated from the model when a number e_n of countermeasures of type d_n are used in the n*th* section.

Tactical aircraft are designed to carry ordnance mounted internally and externally in pods located below the wings and fuselage. Extra fuel is also carried in externally-mounted pods. Any countermeasures that are to be used must be mounted in the same manner. The number of countermeasures that can be carried is often limited more by the number of pods available than by volume or weight constraints. The capacity function $X_n(d_n,e_n)$ represents the capacity needed when e_n units of countermeasures type d_n are to be used. The value of $X_n(d_n,e_n)$ will normally be the capacity per unit times the number of units.

A fixed capacity constraint X is assumed to be known. This may not be the case when the mission planner is deciding the relative merit of adding countermeasures at the expense of displacing ordnance or fuel. However, it will be seen that the procedure to be used to determine the optimum deployment strategy also gives the optimum strategy for a set of subproblems which have capacity constraints less than the fixed constraint X. If the problem is solved for a maximum conceivable capacity constraint, the mission planner can use the solutions to the subproblems in making his decisions.

Having selected the capacity constraint X, the mission planner's problem is to determine the number and type of countermeasure to use at each section of the route to obtain the greatest probability of survival of the aircraft while keeping the total capacity within the constraint. The probability of the aircraft surviving the mission is denoted S_N since it is a product of the probability of the aircraft surviving each of the N sections of the mission route. The mathematical formulation of the mission planner's problem is maximize

$$S_N = \prod_{n=1}^{N} [1-L_n(d_n,e_n)] \tag{3-1}$$

subject

$$\sum_{n=1}^{N} X_n(d_n,e_n) \leq X \tag{3-2}$$

where

$$d_n = 1,2,\ldots,\bar{d}_n$$

$$e_n = 1,2,3,\ldots$$

and

$$X_n(d_n,e_n) \geq 0$$

for

$$n = 1,2,\ldots,N .$$

It should be noted that the problem formulated here can be thought of as a serial multi-stage decision system as described in Chapter I. The sections of the mission route correspond to the stages of Figure 1. At each stage n the decisions are how many countermeasures to use, e_n, and what type countermeasure to use, d_n. This can be expressed as the decision vector $D_n = (d_n,e_n)$. The state vector and stage transformation can be obtained from the constraint relation. The return at each stage has the same functional form and the total N-stage return S_N is expressed in an equation similar to Equation (1-9).

The usual solution procedure for problems of this type are either optimization techniques based on the calculus or direct enumeration methods. The optimization methods utilize algorithms or search procedures which converge on a solution in a finite number of steps. However, to prove convergence for an algorithm that solves this class of problems, some rather restrictive assumptions are often made about the form of the problem. Typical assumptions are the existence of a continuously differentiable objective function, a concave objective function, differentiable constraints, and convex sets of constraints. The usual methods based on calculus are not easy to apply to this problem because the existence of various countermeasure types d_n causes discontinuity in the functions $X_n(d_n,e_n)$ and $L_n(d_n,e_n)$.

In the direct enumeration procedure all feasible sets of non-negative integers d_n and e_n are used to evaluate the objective function S_N. The set or sets of d_n and e_n which give the maximum value of the objective function is then selected as the optimum deployment strategy. In utilizing this procedure, a very large number of sets must be examined in order to determine the optimum for even a modest size problem. Consider for example, a problem involving four different countermeasure types that can be deployed at any of ten sections of the route. If each variable e_n can take on five different values for each type of countermeasure, then there are $(4\times5)^{10}$ different sets to investigate in order to find the optimum. It would take an extremely long time to evaluate each of these sets even with the aid of a modern computer. Therefore, a better computation procedure is needed.

Dynamic programming provides an excellent computational procedure for this type optimization problem. It allows the mission planner's N section decision problem containing many variables to be solved by a series of one-section problems containing relatively few variables. When the conditions given by Equation (1-10) have been met, the problem can be decomposed into N subproblems. Then, the solutions to the subproblems can be combined to obtain the solution to the original problem. Thus, instead of solving one optimization problem in which all of the decisions are interdependent, the optimal decisions are determined almost one at a time.

The computational advantage of dynamic programming over direct enumeration is considerable. This is due to two reasons. At each stage for the given capacity constraint all non-optimal combinations of decision variables d_n and e_n are eliminated from further consideration and are not used in any of the following stages. For the above example, the dynamic programming approach need only consider $(4 \times 5) \times 10$ sets of decision variables whereas direct enumeration required the consideration of $(4 \times 5)^{10}$ sets of decision variables. The second computational advantage is in the number of multiplications required to evaluate the objective function S_N. Dynamic programming requires only a single multiplication to compute the value of the objective function which is compared at each stage. The direct enumeration requires n multiplications to compute the value of the objective function for each set of decision variables.

Dynamic Programming Recursion Equations

The procedure to be used in solving the mission planner's problem must be able to examine all feasible sets of the decision variable without missing the optimal or violating the capacity constraint. The recursion equations of dynamic programming allow us to accomplish this one stage at a time. These recursion equations are obtained by decomposing the problem.

The problem to be decomposed can be written in a form similar to Equation (1-9) by denoting the maximum of S_N as $f_N(X)$. This reflects the fact that the maximum of the function S_N over the decision space determined by the feasible values of d_n and e_n $(n=1,\ldots,N)$ depends upon N and X. The problem to be decomposed is

$$f_N(X) = \max_{\substack{d_1,\ldots,d_N \\ e_1,\ldots,e_N}} \left\{ \prod_{n=1}^{N} [1-L_n(d_n,e_n)] \right\} \tag{3-3}$$

subject to

$$\sum_{n=1}^{N} X_n(d_n,e_n) \leq X \tag{3-4}$$

where the maximization is over all feasible nonnegative integers d_n and e_n $(n=1,2,\ldots,N)$.

The separability and monotonicity conditions required for decomposition are easily verified. Since S_N represents the product in Equation (3-3). That is

$$S_N = \prod_{n=1}^{N} [1-L_n(d_n,e_n)]. \tag{3-5}$$

By Equation (1-10) the separability condition is satisfied since Equation (3-5) can obviously be written as

$$S_N = [1-L_N(d_N,e_N)] \cdot \left\{ \prod_{n=1}^{N-1} [1-L_n(d_n,e_n)] \right\} = [1-L_N(d_N,e_N)] \cdot S_{N-1}.$$

Since $[1-L_n(d_n,e_n)]$ is a probability, its values are nonnegative real numbers only. Then if $S'_{N-1} \geq S''_{N-1}$, the inequality

$$S'_N = [1-L_N(d_N,e_N)] \cdot S'_{N-1} \geq [1-L_N(d_N,e_N)] \cdot S''_{N-1} = S''_N$$

holds for all values of $[1-L_N(d_N,e_N)]$. This is the desired monotonicity property. Thus, the problem may be decomposed. This means that the position of maximization with respect to d_n and e_n for $n=1,\ldots,N-1$ can be moved inside the *Nth* stage function with no possibility of missing the optimal solution.

Thus the problem may be written

$$f_N(X) = \max_{\substack{d_N \\ e_N}} \left\{ [1-L_N(d_N,e_N)] \cdot \max_{\substack{d_1,\ldots,d_{N-1} \\ e_1,\ldots,e_{N-1}}} \left[\prod_{n=1}^{N-1} [1-L_n(d_n,e_n)] \right] \right\} \tag{3-6}$$

subject to

$$\sum_{n=1}^{N} X_n(d_n,e_n) \leq X, \tag{3-7}$$

where the maximization is over all feasible nonnegative integers d_n and e_n $(n=1,\dots,N)$. Since the function

$$\max_{\substack{d_1,\dots,d_{N-1}\\ e_1,\dots,e_{N-1}}} \left\{ \prod_{n=1}^{N-1} [1-L_n(d_n,e_n)] \right\}$$

depends on the values of d_N and e_N only through the constraint, it can be maximized separately for any given value of d_N and e_N. From the definition of $f_N(X)$ it follows that

$$f_{N-1}[X-X_N(d_N,e_N)] = \max_{\substack{d_1,\dots,d_{N-1}\\ e_1,\dots,d_{N-1}}} \left\{ \prod_{n=1}^{N-1} [1-L_n(d_n,e_n)] \right\} \qquad (3\text{-}8)$$

subject to

$$\sum_{n=1}^{N-1} X_n(d_n,e_n) \le X - X_N(d_N,e_N) \qquad (3\text{-}9)$$

where

$$d_n = 1,2,\dots,\bar{d}_n$$

$$e_n = 1,2,3,\dots$$

$$n = 1,2,\dots,N-1.$$

Equation 3-6 may now be written

$$f_N(X) = \max_{\substack{d_N \\ e_N}}\{[1-L_N(d_N,e_N)] \cdot f_{N-1}[X-X_N(d_N,e_N)]\}. \qquad (3\text{-}10)$$

The original N-stage problem can now be considered as two smaller optimization problems. First, $f_{N-1}\,[X-X_N(d_N,e_N)]$ is calculated for all feasible values of d_N and e_N. Second, Equation (3-10) is used to select the optimal decision variables d_N^* and e_N^*. By treating $f_{N-1}[X-X_N(d_N,e_N)]$ and then $f_{N-2}[X-X_{N-1}(d_{N-1},e_{N-1})],\ldots,f_2[X-X_1(d_1,e_1)]$, for arbitrary X, in the same manner as $f_N(X)$, the original N stage problem can be decomposed into N one-stage optimization problems. The recursion equations are

$$f_n(X) = \max_{\substack{X_n(d_n,e_n)\le X \\ d_n=1,2,\ldots,\bar{d}_n \\ e_n=1,2,3,\ldots}} \{[1-L_n(d_n,e_n)] \cdot f_{n-1}[X-X_n(d_n,e_n)]\} \quad (3\text{-}11)$$

for arbitrary X and n=2,3,...,N

and

$$f_1(X) = \max_{\substack{x_1(d_1,e_1)\le X \\ d_1=1,2,\ldots,\bar{d}_1 \\ e_1=1,2,3,\ldots}} [1-L_1(d_1,e_1)] \qquad (3\text{-}12)$$

for arbitrary X.

Their recursive solution, starting with n=1 and continuing through n=N, yields the optimal value of $f_N(x)$, and the optimal decisions.

Computational Procedure

In solving the recursion equatins, $f_n(k)$ is calculated for specified values of k and for each section of the route n=1,2,...,N. Since k and $X_n(d_n,e_n)$ could take on all nonnegative real numbers less than or equal to X, it would be difficult, if not impossible, to evaluate all values of $f_n(k)$. Therefore, these functions will be evaluated using only integer values of k, $X_n(d_n,e_n)$, and X.

The computational procedure begins by calculating $f_1(k)$ for each k=0,1,...,X by using Equation (3-12). It is assumed that if e_1, countermeasures of type d_1 are available at the last section of the route (stage 1 in Figure 1), then they are all deployed in that section. In computing $f_1(k)$ for a given capacity k only the maximum capacity $X_1(d_1,e_1) \leq k$ is considered, and the maximizations are carried out for the combinations of d_1 and e_1 which are restricted to the nonnegative integers that satisfy this constraint. The values of d_1 and e_1 which maximize $f_1(k)$ are denoted $d_1'(k)$ and $e_1'(k)$.

Having computed $f_1(k)$ for k=0,1,...,X the recursion equation (3-11) is used to compute $f_n(k)$ for k=0,1,...,X and for n=2,3,...,N-1. Beginning with n=2, the values of $f_2(k)$ are obtained by carrying out the maximizations over all possible combinations of d_2 and e_2 which are restricted to the nonnegative integers that satisfy $X_2(d_2,e_2) \leq k$. To carry out the calculations for each k Equation (3-11) can be expressed as

$$f_2(k) = \max \begin{bmatrix} \max\limits_{\substack{X_2(1,e_2)\le k \\ e_2=1,2,3...}} \{[1-L_2(1,e_2)]\cdot f_1[k-X_2(1,e_2)]\} \\ \\ \max\limits_{\substack{X_2(2,e_2)\le k \\ e_2=1,2,3...}} \{[1-L_2(2,e_2)]\cdot f_1[k-X_2(2,e_2)]\} \\ \vdots \\ \max\limits_{\substack{X_2(\bar{d}_2,e_2)\le k \\ e_2=1,2,3...}} \{[1-L_2(\bar{d}_2,e_2)]\ f_1[k-X_2(\bar{d}_2,e_2)]\} \end{bmatrix}$$

Since d_2 assumes only the values $1,2,\ldots,\bar{d}_2$, a maximization can be carried out over the feasible values of e_2 for a given d_2. When the maximums have been calculated for each feasible value of d_2, the largest of these maximums is selected as the value of $f_2(k)$. The values of d_2 and e_2 that maximize $f_2(k)$ are denoted $d_2'(k)$ and $e_2'(k)$. A similar procedure is then used to compute $f_3(k)$ for $k=0,1,\ldots,X$ and is repeated until the N*th* stage is to be evaluated.

At stage N one often needs only evaluate $f_N(X)$ to determine the optimum. The objective function for the mission planner's problem allows this. However, it will be seen that the objective function for the countermeasure evaluator's problem has a form that requires the evaluation of $f_N(k)$ for $k=0,1,\ldots,X$. The values of $f_N(k)$ are calculated using Equation (3-11) and the procedure described above. Then the relation

$$f_N(X) = \max_k f_N(k) \tag{3-13}$$

is used to determine the optimum. Finding the values for $f_N(k)$, $k=0,1,\ldots,X$, provides information the mission planner can use in making the decision concerning the merit of replacing ordnance or fuel with countermeasures. The values of $d_N'(X)$ and $e_N'(X)$ that determine the optimum value $f_N(X)$ are denoted d_N^* and e_N^*. This is the optimum deployment strategy for section N of the mission route.

When carrying out the computations, it is convenient to use tables, such as Table 1, to store the maximum values $f_n(k)$ and the decisions $d_n'(k)$ and $e_n'(k)$ for all values of k. It should be noted that the maximum, $f_n(k)$, may have more than one optimal decision set $d_n'(k)$ and $e_n'(k)$. The alternative decisions may also be conveniently recorded in the tables.

Table 1. Stage n Decision Table

k	$e_n'(k)$	$d_n'(k)$	$f_n(k)$
0	$e_n'(0)$	$d_n'(0)$	$f_n(0)$
1	$e_n'(1)$	$d_n'(1)$	$f_n(1)$
2	$e_n'(2)$	$d_n'(2)$	$f_n(2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
X	$e_n'(X)$	$d_n'(X)$	$f_n(X)$

When calculating the values of $f_n(k)$ for $n=2,3,\ldots,N$, the value of $f_{n-1}[k-X_n(d_n,e_n)]$ is needed for all feasible combinations of d_n and e_n. Since the values of $f_{n-1}(k)$ have already been calculated for $k=0,1,\ldots,X$, the value of $f_{n-1}[k-X_n(d_n,e_n)]$ can be obtained from the tables for stage n-1, by using the value of $f_{n-1}(k)$ corresponding to the number $k-X_n(d_n,e_n)$ in the k column. Note that in calculating $f_N(k)$ the values of $f_{N-1}(k)$ may not be needed for every value of k. If the capacity required by the countermeasures is unity, $f_{N-1}(k)$ must be obtained for all $k=0,1,\ldots,X-1$. However, if the values of X and $X_N(d_N,e_N)$ assume only even numbers, $f_{N-1}(k)$ is needed for only even numbers of k. But, in order to calculate $f_{N-1}(k)$ for each k, the values of $f_{N-2}[k-X_{N-1}(d_{N-1},e_{N-1})]$ must be obtained for all the combinations of d_{N-1} and e_{N-1} that satisfy the constraint $X_{N-1}(d_{N-1},e_{N-1}) \leq k$. Thus, for a large number of variables, it is usually easier to calculate $f_n(k)$ for every integer value of k than to trace back through each stage to determine the values of k for which $f_n(k)$ will be needed.

After the optimum deployment strategy is obtained for section N of the route, the relation (3-9), d_N^* and e_N^* can be used to determine the optimum strategy for the rest of the sections. In determining the maximum in Equation (3-8), the relation

$$\sum_{n=1}^{N-1} X_n(d_n,e_n) \leq X - X_N(d_N^*,e_N^*)$$

must hold for the remaining variables. It is obvious that this maximum is $f_{N-1}[X-X_N(d_N^*,e_N^*)]$. The decision variables, d_{N-1} and e_{N-1}, that give this maximum are $d'_{N-1}[X-X_N(d_N^*,e_N^*)]$ and $e'_{N-1}[X-X_N(d_N^* e_N^*)]$. They determine the optimum deployment strategy of section N-1 which are denoted

$$d^*_{N-1} = d'_{N-1}[X-X_N(d^*_N,e^*_N)]$$

$$e^*_{N-1} = e'_{N-1}[X-X_N(d^*_N,e^*_N)].$$

These values may be obtained directly from the table for stage N-1 in the row for $k=X-X_N(d^*_N e^*_N)$. In a similar manner the strategy for section N-2 is obtained from

$$d^*_{N-2} = d'_{N-2}[X-X_N(d^*_N,e^*_N)-X_{N-1}(d^*_{N-1},e^*_{N-1})]$$

$$e^*_{N-2} = e'_{N-2}[X-X_N(d^*_N,e^*_N)-X_{N-1}(d^*_{N-1},e^*_{N-1})].$$

In general the recursive relationships

$$\begin{aligned} d^*_{N-n} &= d'_{N-n}[X - \sum_{i=0}^{n-i} X_{N-i}(d^*_{N-i},e^*_{N-i})] \\ e^*_{N-n} &= e'_{N-n}[X - \sum_{i=0}^{n-i} X_{N-i}(d^*_{N-i},e^*_{N-i})] \end{aligned} \tag{3-14}$$

can be used to find the optimum strategies for the remaining sections of the route.

The Countermeasure Evaluator's Problem

Cost-Effectiveness Criterion

The problem facing the countermeasure evaluator is similar to the mission planner's problem. He is interested in the probability of survival of the penetrator, but he must also consider the monetary cost associated with the probability of survival. Thus the criterion he uses is cost-effectiveness.

The primary problem facing the countermeasure evaluator is that of determining the relative effectiveness of new countermeasure designs or modifications to existing countermeasures. For countermeasures in the expendable class improvements are usually made in such items as the antenna, power source and the delivery system. The cost of these improvements is often in terms of increased volume and weight as well as monetary value. Trade-offs are available between the cost of delivery systems that put low-powered jammers close to the radar and the cost of increasing the jammer power while using a less accurate delivery system.

In order to evaluate the various countermeasure designs, the manner in which the countermeasures can be employed must be considered. Countermeasures deployed in a non-optimal manner will not appear as effective as they would if they were deployed using an optimal deployment strategy. Thus, in evaluating the countermeasures, the evaluator needs to determine the optimum deployment strategy for each countermeasure. Then, if the optimum strategy is practical to implement, it should be used in the evaluation. Even if the optimum strategy is impractical to implement, the procedure developed here provides information that the evaluator can use in determining the best, practical strategy.

As noted in Chapter I, the cost-effectiveness criterion often used by the countermeasure evaluator is expected cost. The costs associated with the mission are the cost of the countermeasures and the cost associated with the possible loss of the aircraft. The cost of the countermeasures is a deterministic cost that includes such costs as research, development, manufacture and shipment. The cost associated with the loss of the aircraft is the weighted cost calculated by multiplying the value of the mission aircraft by the probability of aircraft loss. The function $C_n(d_n,e_n)$ represents the cost of using e_n units of countermeasure type d_n in the n*th* section of the mission route. This will usually be the cost per unit times the number of units, however, it could be the cost of a precursor rocket or missile plus the cost of the expendable jammers. Let K denote the cost of the aircraft and E_N the expected cost of the mission.

When using the expected cost of the mission as the criterion, the evaluator is interested in determining the deployment strategy that minimizes it. One method that could be used to determine this minimum would be to use the deployment strategy employed by the mission planner. The procedure presented in the previous section would be used to solve the mission planner's problem, calculating $f_N(k)$ for $k=0,1,\ldots,X$ and determining the optimal strategies associated with each value of k. The minimum expected cost could then be found from the relation

$$E_N = \min_k \{ \sum_{n=1}^{N} C_n(d_n^*,e_n^*) + K\cdot[1-f_N(k)]\} \tag{3-15}$$

where d_n^* and e_n^* is the optimum deployment strategy associated with each k. However, the optimum E_N determined by this procedure may not be the minimum expected cost of the mission, since this method does not consider all possible sets of d_n and e_n. It considers only the sets d_n^* and e_n^*, the optimum deployment strategy for the mission planner's problem. Since the expected cost determined by this method does reflect the actual strategy that would be employed by the user of the countermeasures, it may be of interest to the evaluator.

An objective function that can be used to determine the minimum expected cost of the mission can be formulated by considering the sections of the mission route as stages depicted in Figure 1. Note that stage 1 is the last section of the mission route while stage N is the first section. For a penetrator arriving at section 1, the expected cost of this section is the cost of the countermeasures employed in this section plus the weighted cost associated with the penetrator not surviving this section. This is expressed as

$$C_1(d_1,e_1) + K \cdot L_1(d_1,e_1). \tag{3-16}$$

Now considering stage 2 as depicted in Figure 1, the expected cost of the mission at this section is the sum of the cost of the countermeasures employed in this section, the weighted cost associated with the penetrator not surviving the section with probability $L_2(d_2,e_2)$ and the weighted cost associated with the penetrator surviving the section with probability $1 - L_2(d_2,e_2)$. This is expressed as

$$C_2(d_2,e_2) + K \cdot L_2(d_2,e_2) \tag{3-17}$$

$$+ [1-L_2(d_2,e_2)] \cdot [C_1(d_1,e_1)+K \cdot L_1(d_1,e_1)].$$

Proceeding in a similar manner through each section, the N-section expected cost equation is

$$E_N = C_N(d_N,e_N) + K \cdot L_N(d_N,e_N) + [1-L_N(d_N,e_N)] \cdot \Big\{ C_{N-1}(d_{N-1},e_{N-1}) +$$

$$K \cdot L_{N-1}(d_{N-1},e_{N-1}) + [1-L_{N-1}(d_{N-1},e_{N-1})] \cdot$$

$$\Big[\ldots [C_1(d_1,e_1) + K \cdot L_1(d_1,e_1)] \Big] \Big\} . \tag{3-18}$$

Mathematical Formulation and the Recursion Equations

The mathematical formulation of the countermeasure evaluator's problem can now be stated. Let $h_N(X)$ denote the minimum expected cost of the mission when X units of capacity are available. The problem statement is

$$h_N(X) = \min_{\substack{d_1,\ldots,d_N \\ e_1,\ldots,e_N}} \Big\{ C_N(d_N,e_N)+K \cdot L_N(d_N,e_N)$$

$$+ [1-L_N(d_N,e_N)] \cdot \Big[C_{N-1}(d_{N-1},e_{N-1}) + K \cdot L_{N-1}(d_{N-1},e_{N-1})$$

$$+ [1-L_{N-1}(d_{N-1},e_{N-1})] \cdot \Big[\ldots [C_1(d_1,e_1)+K \cdot L_1(d_1,e_1)] \Big] \Big] \Big\} \tag{3-19}$$

subject to

$$\sum_{n=1}^{N} X_n(d_n,e_n) \le X \tag{3-20}$$

where

$$d_n = 1,2,\ldots,\bar{d}_n$$

$$e_n = 1,2,3,\ldots$$

and

$$X_n(d_n,e_n) \ge 0$$

for

$$n = 1,2,\ldots,N.$$

To verify the separability and monotonicity conditions required for decomposition, rewrite Equation (3-18), expressing it in the form of Equation (1-10). This gives the equation

$$E_N = C_N(d_N,e_N) + K\cdot L_N(d_N,e_N) + [1-L_N(d_N,e_N)]\cdot E_{N-1}. \tag{3-21}$$

Since all the costs in Equation (18) are positive, K, $C_n(d_n,e_n)$, $L_n(d_n,e_n)$, and $1-L_n(d_n,e_n)$ are defined so that their range consists of the nonnegative real numbers only. Thus, if $E'_{N-1} \ge E''_{N-1}$, the inequality

$$E'_N = C_N(d_N,e_N) + K\cdot L_N(d_N,e_N) + [1-L_N(d_N,e_N)]\cdot E'_{N-1} \ge$$

$$C_N(d_N,e_N) + K\cdot L_N(d_N,e_N) + [1-L_N(d_N,e_N)]\cdot E''_{N-1} = E''_N$$

holds for all values of K, $C_N(d_N,e_N)$, $L_N(d_N,e_N)$ and $1-L_N(d_N,e_N)$. Hence, the separability and monotonicity properties hold, and the position of the maximization with respect to d_n and e_n for n=1,2,...,N-1 can be moved inside the N*th* stage function allowing the problem to be solved using recursive equations, one stage at a time.

The recursive equations follow immediately from Equation (3-21) and the definition of $h_N(X)$. They are

$$h_n(X) = \min_{\substack{X_n(d_n,e_n)\leq X \\ d_n=1,2,\ldots,\bar{d}_n \\ e_n=1,2,3,\ldots}} \{c_n(d_n,e_n) + K\cdot L_n(d_n,e_n) + [1-L_n(d_n,e_n)]\cdot h_{n-1}[X-X_n(d_n,e_n)]\} \tag{3-22}$$

for arbitrary X and n=2,3,...,N

and

$$h_1(X) = \min_{\substack{X_1(d_1,e_1)\leq X \\ d_1=1,2,\ldots,\bar{d}_n \\ e_1=1,2,3,\ldots}} [C_1(d_1,e_1) + K\cdot L_1(d_1,e_1)]. \tag{3-23}$$

for arbitrary X.

Their recursive solution is carried out in the same manner as the recursive solution to the mission planner's problem as described in the section on computational procedure.

Numerical Examples

As a numerical example to illustrate the application of the recursion equations, consider a countermeasure evaluation problem. The countermeasures include three different types of expendable jammers and two different types of transponder-equipped chaff dispensers. Their parameters are listed in Table 2.

Table 2. Expendable Countermeasure Parameters

Parameters	Countermeasure Types				
	1	2	3	4	5
Power	1	1	40	False Target	False Target
Number/Dispenser	10	20	1	25	15
Cost	1500	5000	8000	8000	5000
Minimum Range	7	5	4	Aircraft Range	Aircraft Range
Capacity Required	1	2	10	2	1

Type 1 is a rocket dispenser that disperses expendable jammers along a straight line path parallel to the aircraft flight route as depicted in Figure 3. Type 2 is a rocket dispenser that disperses expendable jammers at an approximately uniform range from the radar. Type 3 is a rocket dispenser that carries a single jammer which is dispensed close to the radar. Types 4 and 5 are chaff dispensers equipped with transponders that create 25 and 15 moving targets, respectively. These false targets appear in the vicinity of the aircraft, and cannot be

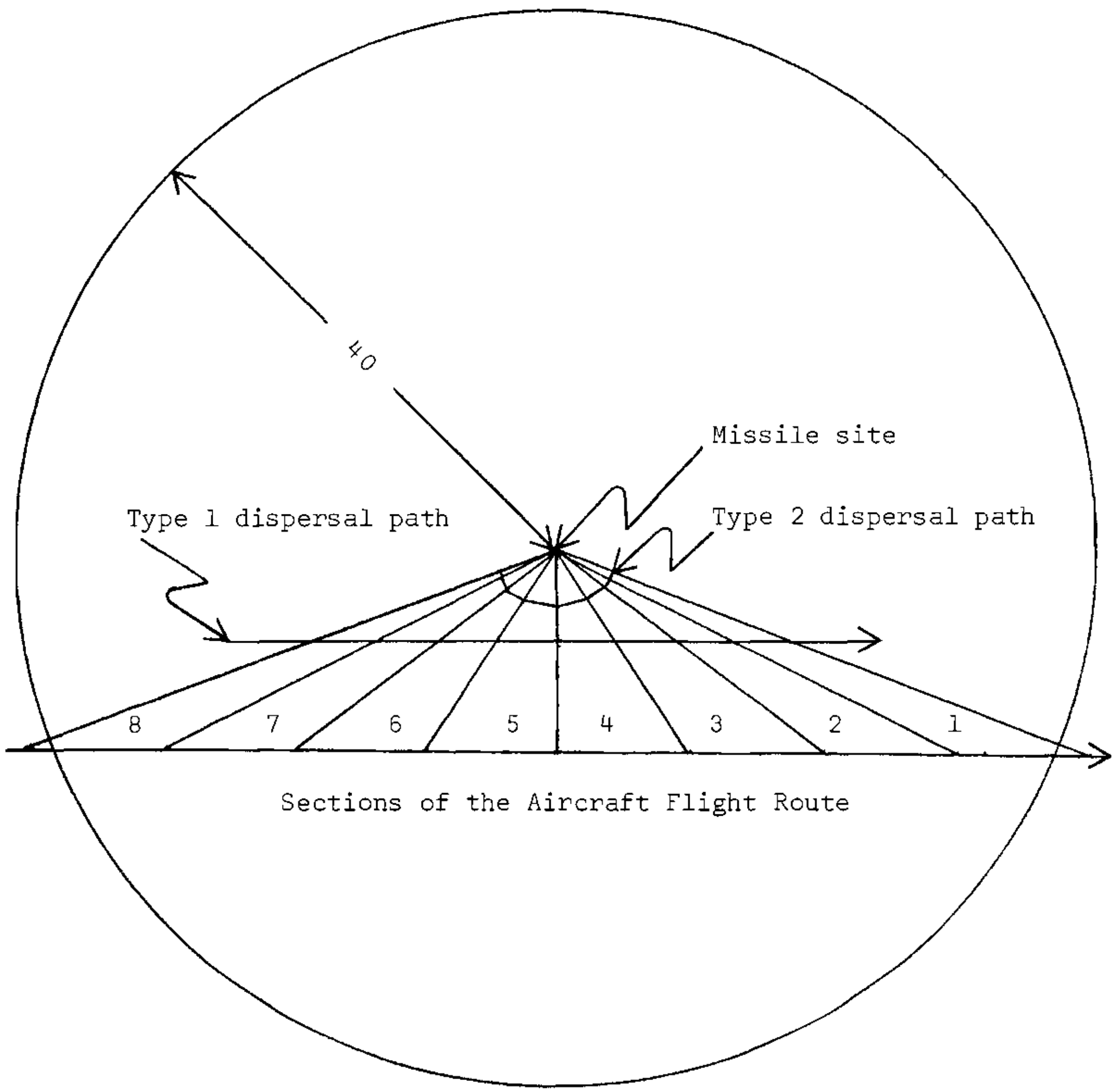

Figure 3. Diagram of Problem Statement

distinguished from the aircraft by the radar operator. Each type countermeasure is considered available for use at each section of the route, but only one type can be used in a section.

The defensive system is a missile site with a radar and missile range of 40 miles. The route of the aircraft is a straight-line route and the closest penetration to the missile site is 15 miles. The route is divided into eight sections which are each ten miles long. The cost of the aircraft is $2,500,000 and the capacity available for countermeasures is 30 units. The aircraft radar cross sections vary from 2 to 30.

Table 3 shows the results of the computation for section 8 of the route.

Table 3. Section 8 Decision Table

Capacity X	Number	Type	Expected Cost	Capacity X	Number	Type	Expected Cost
0	0		1958743.87	16	1	1	72293.06
1	0		1791927.81	17	1	1	73778.39
2	0		1573757.23	18	1	1	75263.72
3	0		1330614.09	19	1	1	67812.50
4	0		1025876.95	20	1	1	54032.26
5	0		764116.66	21	1	1	52500.00
6	0		459187.43	22	1	1	52176.47
7	0		318140.60	23	1	1	53669.12
8	1	1	168983.85	24	1	1	55161.76
9	1	1	136212.92	25	1	1	56478.26
10	1	1	103188.48	26	1	1	56156.86
11	1	1	89564.01	27	1	1	57649.51
12	1	1	75950.83	28	1	1	32000.00
13	1	1	74410.06	29	2	1	33500.00
14	1	1	72931.83	30	3	1	35000.00
15	1	1	72605.82				

The values of the function $h_8(X)$ and the decisions $d_8(X)$ and $e_8(X)$ are shown. From this table the minimum expected cost is $32,000.00, and the optimum capacity required is 28 units, which is slightly less than the 30 units available. Table 4 shows the optimum deployment strategies for this problem.

Table 4. Optimum Deployment Strategy for the Evaluation Problem

Section of Route	Number Required	Type Required
1	1	1
2	1	1
3	1	2
4	1	3
5	1	3
6	1	2
7	1	1
8	1	1

To provide a comparative example the mission planner's objective function was used in calculating the results to the problem described above. Table 5 shows the probability of the aircraft's surviving the mission and the expected cost associated with each capacity from zero to 30 units for both the mission planner's objective function and the countermeasure evaluator's objective function. The expected cost associated with the mission planner's objective function was calculated using Equation (3-15).

The optimum solution to the problem is the same when the available capacity is 30 units. However, it is interesting to determine what the solutions are when the available capacity is considered to be 25 units. When using the mission planner's objective function the optimum capacity needed is 25 units, whereas the optimum capacity needed is 22 units when using the countermeasure evaluator's objective function.

Table 5. Results Using Different Objective Functions

Capacity	Mission Planner Probability of Survival	Mission Planner Expected Cost	Countermeasure Evaluator Probability of Survival	Countermeasure Evaluator Expected Cost
0	.21650245	1958743.89	.21650245	1958743.87
1	.28415947	1794601.33	.28415947	1791927.81
2	.37295930	1577601.73	.37295930	1573757.23
3	.47141889	1336452.78	.47141889	1330614.09
4	.59587137	1030321.60	.59587137	1025876.95
5	.70102514	768937.17	.70102514	764116.66
6	.82473545	461161.38	.82473545	459187.43
7	.88168320	320292.00	.88168319	318140.60
8	.94256317	169592.10	.94256317	168983.85
9	.95568728	136781.79	.95568728	136212.92
10	.96899414	103514.65	.96899414	103188.48
11	.97643650	89908.76	.97643650	89564.01
12	.98393602	76159.97	.98393602	75950.83
13	.98654453	74638.66	.98654453	74410.06
14	.98915997	73100.07	.98915997	72931.83
15	.99048913	74777.19	.98968989	72605.82
16	.99182008	76449.79	.99022011	72293.06
17	.99262566	79435.87	.99022011	73778.39
18	.99343190	82420.26	.99022011	75263.72
19	.99397232	86069.21	.98437500	67812.50
20	.99451303	89717.41	.99193549	54032.26
21	.99490074	93748.17	.99456522	52500.00
22	.99590164	54245.89	.99509804	52176.47
23	.99671054	57223.65	.99509804	53669.12
24	.99725275	60868.13	.99509804	55161.76
25	.99764152	64896.20	.99456522	56478.26
26	.99793389	69165.26	.99509804	56156.86
27	.99816178	73595.55	.99509804	47649.51
28	1.00000000	32000.00	1.00000000	32000.00
29	1.00000000	35500.00	1.00000000	33500.00
30	1.00000000	35000.00	1.00000000	35000.00

Note that if the mission planner's problem was solved and the decision was based on minimum expected cost, then the optimum capacity would also be 22 units. Table 6 presents the deployment strategies for the two objective functions for 22 and 25 units of capacity.

Table 6. Optimum Deployment Strategies Using Different Objective Functions

Section of Route	Mission Planner				Countermeasure Evaluator			
	22 Units		25 Units		22 Units		25 Units	
	Number	Type	Number	Type	Number	Type	Number	Type
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
3	1	2	1	2	1	2	6	1
4	4	5	7	5	1	3	1	3
5	1	3	1	3	2	4	3	5
6	1	2	1	2	1	2	1	2
7	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1

A Fortran program was written and used to make the calculations for these examples. It is found in the Appendix along with the optimum strategies for the countermeasure evaluation problem associated with each capacity from zero to 30 units. The computer time used in calculating each of these problems was 15 seconds. The computer time needed for a problem with 200 units of capacity and 8 stages was 2 minutes 20 seconds.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The objective of this research was to develop a procedure that could be used to determine the optimum deployment strategies for expendable countermeasures. The need for such a procedure arises from two related problems. First, new expendable countermeasure designs are being developed at a rapid pace, and the military services, being the primary user of these devices, must be able to evaluate their relative cost-effectiveness. Second, the operational user of expendable countermeasures must plan for their employment prior to the mission in order to insure maximum probability of mission success.

The procedure presented in this thesis should be considered as a method of obtaining information on which to base decisions rather than as an "automatic strategy selector." The countermeasure evaluator usually considers more factors than cost-effectiveness in choosing the best design. These factors include additional workload on the pilot, maintenance requirements, logistics implications and delivery schedules. The mission planner must also consider the fuel and ordnance required for the mission as well as the countermeasures.

The model developed in Chapter II was purposely made simple to more easily carry out the computations. However, by using the parameters that were incorporated into the constant term of the signal-to-noise ratio equation (2-2) and the exact relationship between SNR and probability of detection, the model would yield more accurate values for probability of detection. This, along with a detailed model of the operator response would provide a more exact figure for probability of aircraft loss. Also a complex model could be developed to include several radars and multiple aircraft. The interaction between the radars would require a model that is able to handle a very large number of parameters such as a simulation model.

The dynamic programming approach used to solve these sequential decision problems appears to be very efficient. The computational procedure based on the recursion equations is easy to implement and will always yield the optimum solution to the problem. By using this procedure the optimum deployment strategy is determined not only for the capacity constraint specified, but for each integer capacity constraint less than the constraint specified. In fact, for each section of the route $n<N$, the optimum decisions are available for every integer value up to the specified capacity. Therefore, a large amount of information that can be used for sensitivity analysis is available with no additional calculations.

In order to determine the optimum deployment strategies for expendable countermeasures employed in conjunction with on-board jammers, the scenario model can be changed to include the possibility of using on-board jammers at each section of the route. The model would need to determine for each section of the route whether it is better to use the on-board jammer only, the expendable countermeasures only, or both the expendable and on-board jammers together. Only a slight modification of the computer program is needed to include this, since the SNR equation of the model can already handle the power term associated with the on-board jammer. By including the on-board jammer in this manner, it is not considered as one of the countermeasures in the set of countermeasures being evaluated. However, the optimum deployment strategies for the on-board jammer and the expendable countermeasures can be obtained using this procedure.

The mathematical formulations of the problems in this report consider only a single capacity constraint. They could be formulated with different constraints or with more than one constraint, and dynamic programming can still be used as the solution procedure. For example, the mission planner's problem could be solved with a constraint on volume and one on total cost of the countermeasures. This problem can be solved using two state variables and a procedure similar to the one presented in this thesis.

Recommendations

In this research a procedure was developed to determine the optimum deployment of countermeasures assuming all the parameters related to the enemy radar system were known. A similar procedure should be developed for the situation where the penetrator is faced by uncertainty as to the exact defensive system the enemy is employing. Dynamic programming may be applicable for this problem also.

An obvious extension to the problems considered here would be to determine the optimum flight route along with the optimum deployment strategy for an aircraft facing a defensive system composed of numerous radars. It should be clear that the route the penetrator uses will affect the deployment strategy and its probability of survival.

In considering the countermeasure evaluator's problem, the cost-effectiveness is found for a set of countermeasures employed against a fixed defensive system. In comparing the different designs, more than one situation should be considered. Several defensive system scenarios could be formulated and the cost-effectiveness could be determined with respect to each of these. Research could be done on how to determine the best countermeasure design from the results of many scenarios.

APPENDIX

THE COMPUTER PROGRAM AND OUTPUT

```
.AI     ANGLE INCREMENT
 ANG    COMPUTING ANGLE
 C      CONSTANT
 CD     DISTANCE USED IN CALCULATING SECTION LENGTH
 CS     CROSS SECTION FOR CALCULATING
 CSTAC  COST OF THE AIRCRAFT
 CSTJM  COST OF THE JAMMER
 D      DECISION ON NUMBER OF JAMMERS TO USE IN A SECTION
 F      RETURN FOR THE SECTION
 JTYPE  DECISION ON THE TYPE OF JAMMER TO USE IN A SECTION
 LOAD   NUMBER OF JAMMERS LOADED IN MISSILE
 MAT    MAXIMUM NUMBER OF JAMMERS THAT CAN BE USED
 MCAP   MAXIMUM CAPACITY OF THE AIRCRAFT
 NC     NUMBER OF RESOLUTION CELLS IN A SECTION
 NS     NUMBER OF STAGES
 NTYPE  NUMBER OF JAMMER TYPES CONSIDERED
 NU     NUMBER OF UNITS OF CAPACITY THE JAMMER REQUIRES
 P      PROBABILITY OF AIRCRAFT LOSS FOR THE SECTION
 PJAM   POWER OF THE JAMMER
 PROB   SUBROUTINE USED TO CALCULATE PROBABILITY OF AIRCRAFT LOSS
 PS     PROBABILITY OF AIRCRAFT SURVIVAL
 RA     MINIMUM RANGE OF THE AIRCRAFT TO THE RADAR
 RCS    RADAR CROSS SECTION
 RJAM   MINIMUM RANGE OF THE JAMMER TO THE RADAR
 RM     RANGE OF THE MISSILE
 RR     RADAR RANGE
 TD     TRAVEL DISTANCE UNDER RADAR COVERAGE

        A,AN,I,ITEMP,J,JTEMP,K,L,MCAP1,N,NO,OD,OTYPE,PK,Q,TEMPF,TEMPP
        ARE VARIABLES USED IN THE PROGRAM
      INTEGER D,OD(8,401),OTYPE(8,401)
      DIMENSION ANG(8),RCS(8),NC(8),P(8,401),F(8,401),D(8,401),NU(6),
     *CSTJM(6),LOAD(6),CS(4),JTYPE(8,401),MAT(6),NO(401),PS(401)
      COMMON PJAM(6),C,RA,AI,RJAM(6)
      AI=.026179939
      C=750.
      CS(1)=30.
      CS(2)=10.
      CS(3)=5.
      CS(4)=2.
      CSTAC=2500000.
      CSTJM(1)=1500.
      CSTJM(2)=5000.
      CSTJM(3)=8000.
      CSTJM(4)=8000.
      CSTJM(5)=5000.
      LOAD(1)=10
      LOAD(2)=20
      LOAD(3)=1
      LOAD(4)=25
      LOAD(5)=15
      NTYPE=5
       MCAP=80
      NU(1)=1
      NU(2)=2
      NU(3)=10
      NU(4)=2
      NU(5)=1
      PJAM(1)=1.
      PJAM(2)=1.
```

```
      PJAM(3)=40.
      RA=15.
      RJAM(1)=7.
      RJAM(2)=5.
      RJAM(3)=4.
      RR=40.
C     THIS PART OF THE PROGRAM CALCULATES THE NUMBER OF STAGES, THE
C     NUMBER OF RESOLUTION CELLS, THE CALCULATING ANGLES, AND THE CROSS
C     SECTIONS.
      TD=2.*SQRT(RR**2-RA**2)
      CD=(10.*(1.+AINT(TD/10.)))/2.
      NS=1+INT(TD/10.)
      N=NS/2
      DO1I=1,N
      A=I
      ANG(I)=ATAN((CD-10.*A)/RA)
          AN=ATAN((CD-10.*A+10.)/RA)-ANG(I)
      NC(I)=IFIX(AN/AI)
      J=NS+1-I
      ANG(J)=ANG(I)
      NC(J)=NC(I)
 1    CONTINUE
      IF(MOD(NS,2).NE.1)GOTO3
      N=NS/2+1
       ANG(N)=-.078540
      AN=2.*ATAN(5./RA)
      NC(N)=IFIX(AN/AI)
      RCS(N)=CS(1)
      J=N-1
      DO2I=1,J
      RCS(N-I)=CS(I+1)
 2    RCS(N+I)=CS(I+1)
      GOTO5
 3    K=N+1
      DO4I=1,N
      RCS(K-I)=CS(I)
 4    RCS(K+I-1)=CS(I)
 5    WRITE(6,6)TD,CD,NS
 6    FORMAT(/,1X,'TRAVEL DIST = ',F5.2,4X,'COMPUTING DIST =',F5.2,4X,
     *'NUMBER OF STAGES =',I3)
      WRITE(6,7)ANG
 7    FORMAT(/,1X,'ANGLES',3X,8F11.8)
      WRITE(6,8)RCS
 8    FORMAT(/,1X,'CROSS SECTIONS',3X,8F8.4)
      WRITE(6,9)NC
 9    FORMAT(/,1X,'NUMBER OF AZIMUTH CELLS',3X,8I5)
C     THIS PART OF THE PROGRAM CALCULATES THE OPTIMUM STRATEGY USING THE
C     RECURSION EQUATIONS.
      MCAP1=MCAP+1
      DO16N=1,NS
      DO15K=1,MCAP1
      NO(K)=K-1
      IF(N-1)10,10,12
 10   F(N,K)=1.0E38
      DO11I=1,NTYPE
      MAT(I)=(K-1)/NU(I)
      PK=PROB(MAT(I)*LOAD(I),NC(N),RCS(N),ANG(N),I)
      Q=CSTJM(I)*MAT(I)+CSTAC*PK
      IF(Q.GT.F(1,K))GOTO11
      F(1,K)=Q
```

```
      P(1,K)=PK
      D(1,K)=MAT(I)
      JTYPE(1,K)=I
11    CONTINUE
      GOTO15
12    F(N,K)=1.0E38
      DO14J=1,MCAP1
      IF(K-J.LT.0)GO TO15
      TEMPF=1.0E38
      DO13I=1,NTYPE
      IF(MOD(J-1,NU(I)).NE.0)GOTO13
      MAT(I)=(J-1)/NU(I)
      PK=PROB(MAT(I)*LOAD(I),NC(N),RCS(N),ANG(N),I)
      Q=CSTJM(I)*MAT(I)+CSTAC*PK+(1.-PK)*F(N-1,K-J+1)
      IF(Q.GT.TEMPF)GOTO13
      TEMPF=Q
      TEMPP=PK
      ITEMP=MAT(I)
      JTEMP=I
13    CONTINUE
18    IF(TEMPF.GT.F(N,K))GOTO14
      F(N,K)=TEMPF
      P(N,K)=TEMPP
      D(N,K)=ITEMP
      JTYPE(N,K)=JTEMP
14    CONTINUE
15    CONTINUE
16    CONTINUE
      WRITE(6,17)(NO(J),(D(I,J),JTYPE(I,J),P(I,J),F(I,J),I=1,4),J=1,81 )
      WRITE(6,17)(NO(J),(D(I,J),JTYPE(I,J),P(I,J),F(I,J),I=5,8),J=1,81 )
17    FORMAT(/,6X,4('DEC',1X,'TYPE',2X,'PROB',2X,'RETURN',4X),/(
     *I5,4(1X,I3,I2,1X,F8.7,F11.2)))
C     THIS PART OF THE PROGRAM TRACES THE OPTIMUM DECISIONS FOR THE GIVEN
C     INPUT CAPACITY.
      DO23K=1,81
      I=K
      PS(K)=1.
      DO22N=NS,1,-1
      OD(N,K)=D(N,I)
      OTYPE(N,K)=JTYPE(N,I)
      L=OTYPE(N,K)
      PS(K)=PS(K)*(1.-P(N,I))
22    I=I-OD(N,K)*NU(L)
23    CONTINUE
      WRITE(6,24)(NO(J),(OD(I,J),OTYPE(I,J),I=1,8),PS(J),F(8,J),J=1, 81)
24    FORMAT(/,8X,'STAGE 1',4X,'STAGE 2',4X,'STAGE 3',4X,'STAGE 4',4X,
     *'STAGE 5',4X,'STAGE 6',4X,'STAGE 7',4X,'STAGE 8',
     */,1X,'CAP. ',8('DEC  TYPE  ')
     *  ,'PROB SURV   EXP COST',/,  (I5,8(I4,I4,3X),F10.8,F11.2))
      END
```

```
      FUNCTION PROB(NJ,NC,CS,A,IND)
C THIS SUB-FUNCTION CALCULATES PROBABILITY OF AIRCRAFT LOSS USED IN
C THE MAIN PROGRAM
      COMMON PJAM(6),C,RA,AI,RJAM(6)
      RM=40.
      IF(RM.LT.RA/COS(A))GOTO1
      IF(NJ.EQ.0)GO TO2
      IF(IND.EQ.1)GOTO3
      IF(IND.EQ.2)GOTO6
      IF(IND.EQ.3)GOTO5
      IF(IND.EQ.4)GOTO7
      IF(IND.EQ.5)GOTO7
1     PROB=0.0
      RETURN
2     PROB=.40-(RA/COS(A))/100.
      RETURN
3     PJ=PJAM(IND)
      RJ=RJAM(IND)
      ANJ=NJ
      ANC=NC
      P=PJ*ANJ/ANC
      PROD=(C*CS/((RA/COS(A+3.*AI))**4))/((P/(21.25*(RJ/COS(A))**2))+
     *(P/(10.*(RJ/COS(A+AI))**2))+(P/(3.25* (RJ/COS(A+2.*AI))**2))+
     *(P/((RJ/COS(A+3.*AI))**2))+(P/(3.25*(RJ/COS(A+4.*AI))**2))+
     *(P/(10.*(RJ/COS(A+5.*AI))**2))+(P/(21.25*(RJ/COS(A+6.*AI))**2)))
      IF(PROD.GT.1.)GO TO2
      IF(PROD.GE.0.5)GOTO4
      PROB=0.0
      RETURN
5     ANC=NC
      RJ=RJAM(IND)
      X=(ANC*AI/2.)/.01745329
      BOT=PJAM(IND)/((X+1.)**2)*(RJ**2)
      TOP=C*CS/((RA/COS(A))**4)
      PROD=TOP/BOT
      IF(PROD.GT.1.)GO TO2
      IF(PROD.GE.0.5)GOTO4
      PROB=0.0
      RETURN
6     ANJ=NJ
      ANC=NC
      RJ=RJAM(IND)
      P=(PJAM(IND)*ANJ)/ANC
      Z=A+(ANC*AI/2.)
      TOP=C*CS/((RA/COS(Z))**4)
      BOT=(2.*P/(21.25*(RJ**2)))+(2.*P/(10.*(RJ**2)))+
     *(2.*P/(3.25*(RJ**2)))+(P/(RJ**2))
      PROD=TOP/BOT
      IF(PROD.GT.1.)GO TO2
      IF(PROD.GE.0.5)GOTO4
      PROB=0.0
      RETURN
7     ANJ=NJ
       PROD=1./(ANJ+1.)
4     PROB=PROD*(.40-(RA/COS(A))/100.)
      RETURN
      END
```

TRAVEL DIST = 74.16 COMPUTING DIST =40.00 NUMBER OF STAGES = 8

ANGLES 1.10714872 .92729522 .58800260 .00000000 .00000000 .58800260 .92729522 1.10714872

CROSS SECTIONS 2.0000 5.0000 10.0000 30.0000 30.0000 10.0000 5.0000 2.0000

NUMBER OF AZIMUTH CELLS 4 6 12 22 22 12 6 4

	STAGE 1				STAGE 2				STAGE 3				STAGE 4			
	DEC	TYPE	PROB	RETURN	DEC	TYPE	PROB	RETURN	DEC	TYPE	PROB	RETURN	DEC	TYPE	PROB	RETURN
0	0	5	.0645898	161474.50	0	5	.1500000	512253.33	0	5	.2197224	949005.87	0	5	.2500000	1336754.41
1	1	1	.0000000	1500.00	1	1	.0000000	162974.50	1	5	.0137327	544550.37	1	5	.0156250	978240.16
2	2	1	.0000000	3000.00	1	1	.0000000	3000.00	1	5	.0137327	200068.06	1	5	.0156250	580104.27
3	3	1	.0000000	4500.00	2	1	.0000000	4500.00	1	5	.0137327	42290.43	1	5	.0156250	241004.50
4	4	1	.0000000	6000.00	3	1	.0000000	6000.00	1	2	.0000000	8000.00	1	5	.0156250	85692.14
5	5	1	.0000000	7500.00	4	1	.0000000	7500.00	1	2	.0000000	9500.00	1	5	.0156250	51937.50
6	6	1	.0000000	9000.00	5	1	.0000000	9000.00	1	2	.0000000	11000.00	2	5	.0080645	38096.77
7	7	1	.0000000	10500.00	6	1	.0000000	10500.00	1	2	.0000000	12500.00	3	5	.0054348	36543.48
8	8	1	.0000000	12000.00	7	1	.0000000	12000.00	6	1	.0000000	12000.00	2	4	.0049020	36215.69
9	9	1	.0000000	13500.00	8	1	.0000000	13500.00	7	1	.0000000	13500.00	2	4	.0049020	37708.33
10	1	3	.0000000	8000.00	9	1	.0000000	15000.00	8	1	.0000000	15000.00	2	4	.0049020	39200.98
11	1	3	.0000000	8000.00	1	3	.0000000	9500.00	9	1	.0000000	16500.00	3	5	.0054348	40521.74
12	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	3	.0000000	11000.00	2	4	.0049020	40196.08
13	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	3	.0000000	12500.00	2	4	.0049020	41688.73
14	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	3	.0000000	14000.00	1	3	.0000000	16000.00
15	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	2	.0000000	14500.00	1	3	.0000000	17500.00
16	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	2	.0000000	14500.00	1	3	.0000000	19000.00
17	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	2	.0000000	14500.00	1	3	.0000000	20500.00
18	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	2	.0000000	14500.00	1	3	.0000000	20000.00
19	1	3	.0000000	8000.00	1	1	.0000000	9500.00	1	2	.0000000	14500.00	1	3	.0000000	21500.00
20	2	3	.0000000	16000.00	1	1	.0000000	9500.00	1	2	.0000000	14500.00	1	3	.0000000	23000.00
21	2	3	.0000000	16000.00	2	1	.0000000	11000.00	1	2	.0000000	14500.00	1	3	.0000000	24500.00
22	2	3	.0000000	16000.00	3	1	.0000000	12500.00	1	2	.0000000	14500.00	1	3	.0000000	19000.00
23	2	3	.0000000	16000.00	4	1	.0000000	14000.00	1	2	.0000000	16000.00	1	3	.0000000	20500.00
24	2	3	.0000000	16000.00	5	1	.0000000	15500.00	1	3	.0000000	17500.00	1	3	.0000000	22000.00
25	2	3	.0000000	16000.00	1	3	.0000000	16000.00	1	3	.0000000	17500.00	1	3	.0000000	22500.00
26	2	3	.0000000	16000.00	1	3	.0000000	16000.00	1	3	.0000000	17500.00	1	3	.0000000	22500.00
27	2	3	.0000000	16000.00	1	3	.0000000	16000.00	1	3	.0000000	17500.00	1	3	.0000000	22500.00
28	2	3	.0000000	16000.00	1	3	.0000000	16000.00	1	3	.0000000	17500.00	1	3	.0000000	22500.00
29	2	3	.0000000	16000.00	1	3	.0000000	16000.00	1	3	.0000000	17500.00	1	3	.0000000	22500.00
30	3	3	.0000000	24000.00	1	1	.0000000	17500.00	1	3	.0000000	17500.00	1	3	.0000000	22500.00

	STAGE 5				STAGE 6				STAGE 7				STAGE 8			
	DEC	TYPE	PROB	RETURN	DEC	TYPE	PROB	RETURN	DEC	TYPE	PROB	RETURN	DEC	TYPE	PROB	RETURN
0	0	5	.2500000	1627565.80	0	5	.2197224	1819259.16	0	5	.1500000	1921370.30	0	5	.0645898	1958743.87
1	0	5	.2500000	1358680.11	0	5	.2197224	1609453.69	0	5	.1500000	1743035.64	0	5	.0645898	1791927.81
2	1	5	.0156250	1007017.65	0	5	.2197224	1335059.36	0	5	.1500000	1509800.45	0	5	.0645898	1573757.23
3	1	5	.0156250	615102.63	0	5	.2197224	1029256.87	0	5	.1500000	1249868.34	0	5	.0645898	1330614.09
4	1	5	.0156250	281301.30	1	5	.0137327	645987.27	0	5	.1500000	924089.19	0	5	.0645898	1025876.95
5	1	5	.0156250	128415.70	1	5	.0137327	316769.92	0	5	.1500000	644254.43	0	5	.0645898	764116.66
6	1	5	.0156250	95188.48	1	5	.0137327	165983.85	1	1	.0000000	318269.92	0	5	.0645898	459187.43
7	1	5	.0156250	81564.01	1	5	.0137327	133212.92	1	1	.0000000	167483.85	0	5	.0645898	318140.60
8	2	5	.0080645	67950.83	1	2	.0000000	100188.48	1	1	.0000000	134712.92	1	1	.0000000	168983.85
9	2	5	.0080645	66410.06	1	2	.0000000	86564.01	1	1	.0000000	101688.48	1	1	.0000000	136212.92
10	3	5	.0054348	64931.83	1	2	.0000000	72950.83	1	1	.0000000	88064.01	1	1	.0000000	103188.48
11	3	5	.0054348	64605.82	1	2	.0000000	71410.06	1	1	.0000000	74450.83	1	1	.0000000	89564.01
12	2	4	.0049020	64293.06	1	2	.0000000	69931.83	1	1	.0000000	72910.06	1	1	.0000000	75950.83
13	2	4	.0049020	65778.39	1	2	.0000000	69605.82	1	1	.0000000	71431.83	1	1	.0000000	74410.06
14	2	4	.0049020	67263.72	1	2	.0000000	69293.06	1	1	.0000000	71105.82	1	1	.0000000	72931.83
15	1	5	.0156250	59812.50	1	2	.0000000	70778.39	1	1	.0000000	70793.06	1	1	.0000000	72605.82
16	2	5	.0080645	46032.26	1	2	.0000000	72263.72	1	1	.0000000	72278.39	1	1	.0000000	72293.06
17	3	5	.0054348	44500.00	1	2	.0000000	64812.50	1	1	.0000000	73763.72	1	1	.0000000	73778.39
18	2	4	.0049020	44176.47	1	2	.0000000	51032.26	1	1	.0000000	66312.50	1	1	.0000000	75263.72
19	2	4	.0049020	45669.12	1	2	.0000000	49500.00	1	1	.0000000	52532.26	1	1	.0000000	67812.50
20	2	4	.0049020	47161.76	1	2	.0000000	49176.47	1	1	.0000000	51000.00	1	1	.0000000	54032.26
21	3	5	.0054348	48478.26	1	2	.0000000	50669.12	1	1	.0000000	50676.47	1	1	.0000000	52500.00
22	2	4	.0049020	48156.86	1	2	.0000000	52161.76	1	1	.0000000	52169.12	1	1	.0000000	52176.47
23	2	4	.0049020	49649.51	1	2	.0000000	53478.26	1	1	.0000000	53661.76	1	1	.0000000	53669.12
24	1	3	.0000000	24000.00	1	2	.0000000	53156.86	1	1	.0000000	54978.26	1	1	.0000000	55161.76
25	1	3	.0000000	25500.00	1	2	.0000000	54649.51	1	1	.0000000	54656.86	1	1	.0000000	56478.26
26	1	3	.0000000	27000.00	1	2	.0000000	29000.00	1	1	.0000000	56149.51	1	1	.0000000	56156.86
27	1	3	.0000000	28500.00	1	2	.0000000	30500.00	1	1	.0000000	30500.00	1	1	.0000000	57649.51
28	1	3	.0000000	28000.00	1	2	.0000000	32000.00	2	1	.0000000	32000.00	1	1	.0000000	32000.00
29	1	3	.0000000	29500.00	1	2	.0000000	33500.00	3	1	.0000000	33500.00	2	1	.0000000	33500.00
30	1	3	.0000000	31000.00	6	1	.0000000	33000.00	4	1	.0000000	35000.00	3	1	.0000000	35000.00

	STAGE 1		STAGE 2		STAGE 3		STAGE 4		STAGE 5		STAGE 6		STAGE 7		STAGE 8			
CAP.	DEC	TYPE	DEC	TYPE	DEC	TYPE	DEC	TYPE	DEC	TYPE	DEC	TYPE	DEC	TYPE	DEC	TYPE	PROB SURV	EXP COST
0	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	.21650245	1958743.87
1	0	5	0	5	0	5	1	5	0	5	0	5	0	5	0	5	.28415947	1791927.81
2	0	5	0	5	0	5	1	5	1	5	0	5	0	5	0	5	.37295930	1573757.23
3	0	5	0	5	1	5	1	5	1	5	0	5	0	5	0	5	.47141889	1330614.09
4	0	5	0	5	1	5	1	5	1	5	1	5	0	5	0	5	.59587137	1025876.95
5	0	5	1	1	1	5	1	5	1	5	1	5	0	5	0	5	.70102514	764116.66
6	0	5	1	1	1	5	1	5	1	5	1	5	1	1	0	5	.82473545	459187.43
7	1	1	1	1	1	5	1	5	1	5	1	5	1	1	0	5	.88168319	318140.60
8	1	1	1	1	1	5	1	5	1	5	1	5	1	1	1	1	.94256317	168983.85
9	1	1	1	1	1	2	1	5	1	5	1	5	1	1	1	1	.95568728	136212.92
10	1	1	1	1	1	2	1	5	1	5	1	2	1	1	1	1	.96899414	103188.48
11	1	1	1	1	1	2	2	5	1	5	1	2	1	1	1	1	.97643650	89564.01
12	1	1	1	1	1	2	2	5	2	5	1	2	1	1	1	1	.98393602	75950.83
13	1	1	1	1	1	2	3	5	2	5	1	2	1	1	1	1	.98654453	74410.06
14	1	1	1	1	1	2	3	5	3	5	1	2	1	1	1	1	.98915997	72931.83
15	1	1	1	1	1	2	2	4	3	5	1	2	1	1	1	1	.98968989	72605.82
16	1	1	1	1	1	2	2	4	2	4	1	2	1	1	1	1	.99022011	72293.06
17	1	1	2	1	1	2	2	4	2	4	1	2	1	1	1	1	.99022011	73778.39
18	1	1	3	1	1	2	2	4	2	4	1	2	1	1	1	1	.99022011	75263.72
19	1	1	1	1	1	2	1	3	1	5	1	2	1	1	1	1	.98437500	67812.50
20	1	1	1	1	1	2	1	3	2	5	1	2	1	1	1	1	.99193549	54032.26
21	1	1	1	1	1	2	1	3	3	5	1	2	1	1	1	1	.99456522	52500.00
22	1	1	1	1	1	2	1	3	2	4	1	2	1	1	1	1	.99509804	52176.47
23	1	1	2	1	1	2	1	3	2	4	1	2	1	1	1	1	.99509804	53669.12
24	1	1	3	1	1	2	1	3	2	4	1	2	1	1	1	1	.99509804	55161.76
25	1	1	1	1	6	1	1	3	3	5	1	2	1	1	1	1	.99456522	56478.26
26	1	1	1	1	6	1	1	3	2	4	1	2	1	1	1	1	.99509804	56156.86
27	1	1	1	1	7	1	1	3	2	4	1	2	1	1	1	1	.99509804	57649.51
28	1	1	1	1	1	2	1	3	1	3	1	2	1	1	1	1	1.00000000	32000.00
29	1	1	1	1	1	2	1	3	1	3	1	2	1	1	2	1	1.00000000	33500.00
30	1	1	1	1	1	2	1	3	1	3	1	2	1	1	3	1	1.00000000	35000.00

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