AN INVESTIGATION OF MOMENT DISTRIBUTION METHODS OF ANALYZING RIGID FRAMES SUBJECTED TO SIDESWAY

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LIST OF ABBREVIATIONS

1.	M	Moment
2.	F.E.M.	Fixed end moment
3.	Dist.	Distribution
4.	Kc	Member stiffness coefficient at a joint
5.	C.O.	Carry over
6.	I	Moment of inertia
7.	L	Length of horizontal member
8.	h	Height of vertical member
9.	H	Horizontal reaction at the base of a column
10.	V	Vertical reaction at the base of a column
11.	x	Imaginary force to prevent sidesway
12.	P	Concentrated load
13.	k	Constant relating horizontal reactions
14.	n, N B, C	Variables taken from the reference book, "Rahmenformeln", by A. Kleinlogel

EXPLANATION OF SIGN CONVENTION

The sign convention used is known as the member end convention as it is applied in connection with moment distribution analyses. If a member is cut away from the joint, the sign is positive if a clockwise moment on the end of the member is necessary to hold it in equilibrium, or it is negative if a counterclockwise moment is necessary.

Thus it can be seen that, for a single span fixed ended beam, the fixed end moments due to a downward load on the span will be prefixed with a minus sign on the left end of the span and a plus sign on the right. Both, however, are what is usually considered negative moments according to beam convention since there is tension on the upper fibers of the beam.

Where moments are determined by ordinary statics, the beam convention for signs is used. On all bending moment diagrams, positive moments are plotted on the bottom side of horizontal members and on the inside of vertical members. Negative moments are naturally plotted the reverse of positive moments.

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ABSTRACT

The investigation undertaken was limited to single span rigid frames. An attempt was made to illustrate the conventional method of analyzing single span rigid frames subjected to sidesway by presenting the solutions of ten frames with various types of loading, make-up of members, and with both fixed and hinged ends. It was the purpose of the author to choose among these examples certain conditions which are not readily found in textbooks, and to explain their solutions sufficiently so that they might serve as examples for other problems of a similar nature.

A simplified method of solution for frames with hinged ends, still utilizing the moment distribution method, was developed and presented. There are four propositions for this simplified or modified method, with the accompanying proofs, stated in the thesis. Each proposition is applicable for frames with certain specific properties and types of loads. After each proposition and its proof, a modified solution of one of the examples which was solved in the conventional manner is presented.

The modified solution is a repetition of the conventional solution in the first phase of the analysis where it is assumed that fixed end moments on a frame may be distributed if no sidesway is

permitted. The second phase of the conventional analysis, in which corrections for sidesway are calculated, is eliminated in the modified solution and is replaced by some very simple formulae applied to the results of the first phase of the analysis to complete the solution.

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INTRODUCTION

The investigation undertaken in this thesis is limited to single span rigid frames. There is no attempt to go into the theory underlying the moment distribution method of analysis. Many books have been published which adequately explain the theory, and it is felt that a repetition here would serve no useful purpose.[1]

The objective of this thesis is twofold: (1) To present the solutions of actual problems using the conventional or standard moment distribution method of analysis, thus enabling the reader to learn more readily to solve problems of a similar character, and (2) To eliminate the second phase of the standard moment distribution analysis in which it is necessary to make corrections for sidesway.

To obtain the first objective, the complete solutions of ten rigid frames with different makeups of members and different type loadings are presented using the conventional method of analysis.

The second objective is the one considered most important in this thesis. The standard procedure, using the moment distribution method of analysis in a rigid frame with sidesway, requires two separate analyses which are ultimately combined to get the actual forces. In the first of these two, an imaginary force must be assumed as acting in such

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[1]For example, Cross, Hardy and Morgan, N.D., Continuous Frames of Reinforced Concrete, John Wiley & Sons, Inc. 1932.

a manner that no movement of the frame or sidesway can occur. However, the horizontal reactions obtained from the first of these two analyses will not satisfy the requirement that the summation of horizontal forces on a structure in equilibrium must equal zero. Therefore, corrections to the reactions and moments obtained initially must be obtained in order to get the actual values. This requires a second analysis which is often intricate and subject to errors of interpretation. An attempt is made in this thesis to get the actual moments and reactions by the use of some simple formulae after the first of the two analyses has been completed. In the cases where this has been found possible, a proposition is stated with the accompanying proof. The formulae derived in these proofs are then used in applying a modified solution to the frames which were solved in the conventional manner.

Example I of the text contains a more detailed explanation of the conventional procedure than does subsequent examples. If the reader should have any difficulty in understanding any of them, it is believed that a reference back to Example I may be helpful.

PROCEDURE

An analytical approach was used throughout in the preparation of this thesis. A laboratory approach was considered, but it did not seem suited to the investigation being made.

In the introduction it was stated that the purpose of the thesis was twofold. First, it was desired to clarify certain peculiarities in analyzing specific frames by presenting their complete solutions. A variety of examples with varied shapes, types of loadings, and with fixed and hinged bases were chosen and solved. The explanations given with each solution are intended to make the method of applying the standard solution apparent to the reader.

The principal objective, that of eliminating the second phase of the standard moment distribution method of analysis, was approached in this manner:

For each example given in the thesis, and many more, a complete analysis of the frame was made in the conventional way. Then, an attempt was made to correlate the values of the actual forces and those obtained at the end of the first phase of the analysis. The first discovery was made in the case of a symmetrical frame with vertical columns, hinged bases, and an unsymmetrically placed vertical load. It was noticed that the final moments at the top of the columns were equal to the average of the moments obtained at those points after the first phase of the analysis had been completed. Other similar frames were checked and in each case it was found that this relation held true.

This led to the belief that, under similar loadings in a frame of this type, the same relation would always be true. A general proof for this case was successfully attempted and is presented under Proposition I of this thesis. In establishing this proof, a procedure was evolved which appeared to be applicable to other frames for the purpose of obtaining a correlation between the forces obtained at the end of the first phase of the moment distribution analysis and the actual forces.

It was in this manner that the proofs for Propositions II, III, and IV were established.

EXAMPLE I- SYMMETRICAL FRAME, HINGED ENDED, WITH VERTICAL COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Conventional Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 1.

<u>Procedure</u>

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 2. Calculate fixed end moments at BC and CB and stiffness coefficients at joints B and C. Moment distribution is then carried out as shown in Fig. 2.

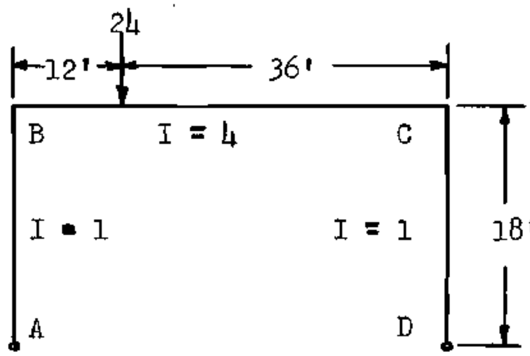

Fig. 1

Kc	A 1/3	B 2/3	2/3 C	1/3 D
F.E.M.		-162.0	54.0	
1st Dist.	54.0	108.0	-36.0	-18.0
C.O.		-18.0	54.0	
2nd Dist.	6.0	12.0	-36.0	-18.0
C.O.		-18.0	6.0	
3rd Dist.	6.0	12.0	-4.0	-2.0
C.O.		-2.0	6.0	
4th Dist.	0.7	1.3	-4.0	-2.0
C.O.		-2.0	0.7	
5th Dist.	0.7	1.3	-0.5	-0.2
C.O.		-0.2	0.6	
6th Dist.	0.1	0.1	-0.4	-0.2
Moments	67.5	-67.5	40.4	-40.4

Fig. 2

From Fig. 2 it is seen that the moments at B and C are 67.5 and 40.4 respectively. They are indicated in Fig. 3 as they would act on the columns drawn as a pair of free bodies. The horizontal shear in the columns, which equals the moment divided by the column length, is 3.75 and 2.24 acting in opposite directions. Evidently, there must be an assumed force of 1.51 acting to the left at C to justify the distribution of Fig. 2 in which it was assumed that no movement occurred in the frame.

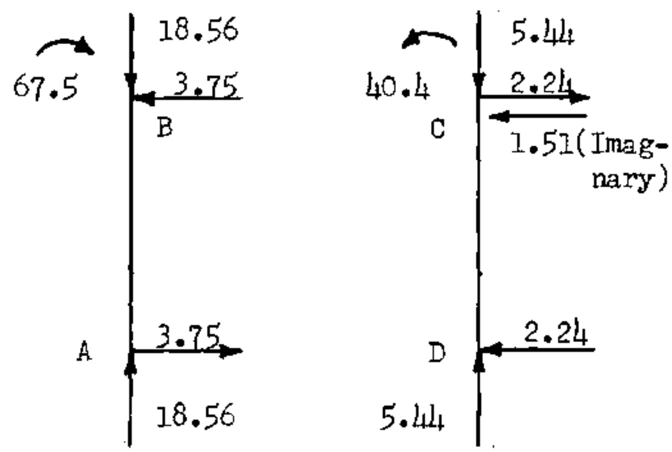

Fig. 3

Since the force of 1.51 does not exist, the actual moments and reactions may be found by adding algebraically to the results of Fig. 3 the moments and reactions due to a force of 1.51 applied at B and acting horizontally to the right. It is apparent that this force would produce horizontal reactions at A and D which would satisfy the requirement that the summation of horizontal forces on a frame in equilibrium must equal zero.

A convenient method of determing the moments and reactions due to the force of 1.51 is to allow an arbitrary force P, acting to the right at B, to produce fixed end moments of 100.0 in the columns. Then when P is determined, the moments and reactions due to a force of 1.51 will simply be proportional to the ratio between P and 1.51 multiplied by the moments and reactions due to P. In Fig. 4 the moment distribution caused by fixed end moments of 100.0 in the columns is carried out. It was possible to use a modified stiffness factor in Fig. 4 because of anti-symmetry of the moments[2], and to analyze only half of the frame.

The free body of Fig. 5 indicates the resulting moments and shears on the columns. Thus it is seen that a force P of 4.16 applied at B and acting horizontally to the right would produce fixed end moments of 100.0 in the columns. Accordingly, the results of Fig. 5 must be multiplied by the ratio of 1.51 to 4.16. These values become 13.55 for moment, 0.755 for horizontal shear, and 0.56 for vertical shear with proper regard for signs.

Final Moments

BA = 67.5 - 13.55 = 53.95

BC = -67.5 + 13.55 = -53.95

CB = 40.4 + 13.55 = 53.95

CD = -40.4 - 13.55 = -53.95

Kc	A	1/4 B	3/4 ℄
F.E.M.	-100.0	-100.0	
1st Dist.	100.0	25.0	75.0
C.O.		50.0	
2nd Dist.		-12.5	-37.5
Moments		-37.5	37.5

Fig. 4

[2] Cross, Hardy and Morgan, N.D., Continuous Frames of Reinforced Concrete, John Wiley & Sons, Inc. 1932.

Final Reactions

$H_A = 3.75 - 0.755 = 2.995$

$V_A = 18.56 - 0.56 = 18.00$

$H_D = 2.24 + 0.755 = 2.995$

$V_D = 5.44 + 0.56 = 6.00$

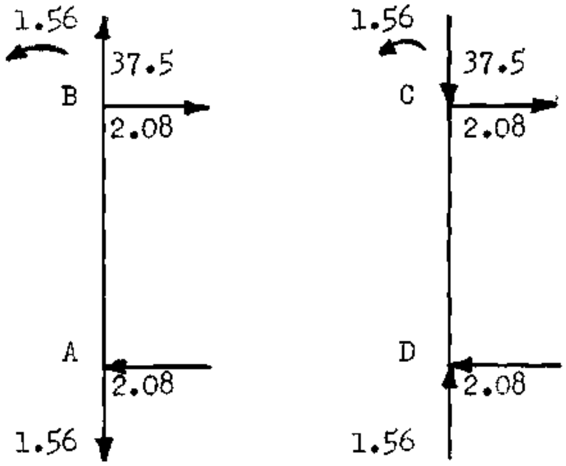

Fig. 5

The moment diagram for the frame of Fig. 1 is shown in Fig. 6.

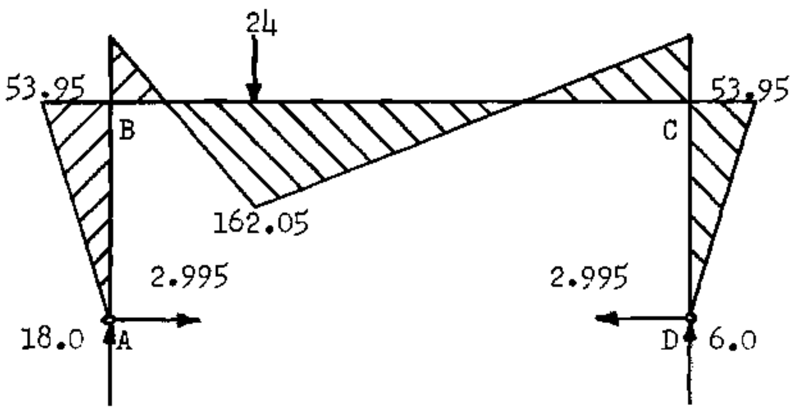

Fig. 6

PROPOSITION I - HYPOTHESIS - IN A HINGED ENDED SINGLE SPAN SYMMETRICAL RIGID FRAME WITH VERTICAL COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD, THE MOMENTS AT THE TOP OF THE COLUMNS ARE EQUAL TO THE AVERAGE OF THE MOMENTS OBTAINED AT THOSE POINTS WHEN THE FRAME IS ANALYZED BY THE MOMENT DISTRIBUTION METHOD ASSUMING THAT NO SIDESWAY CAN OCCUR.

Proof

In the frame of Fig. 7, it is evident that the horizontal reactions at A and D must be opposite and equal. The moments at B and C, therefore, must be equal and have a value of Hh.

The moment distribution method of analyzing frames is based on the premise of non-yielding supports. Accordingly, an analysis of the frame may be made assuming that a force x at C, as shown in Fig. 8, exists and is sufficient to prevent sidesway.

For statical equilibrium, x must equal $H_1 - H_2$, assuming that H_1 is greater than H_2.

However, x does not exist and in order to determine corrections to H_1 and H_2, we must determine the effect of a force x at C acting as shown in Fig. 9. The correctness of this procedure is readily seen by superposing Fig. 9 upon Fig. 8 and noting that the loaded frame then is the same as that of Fig. 7.

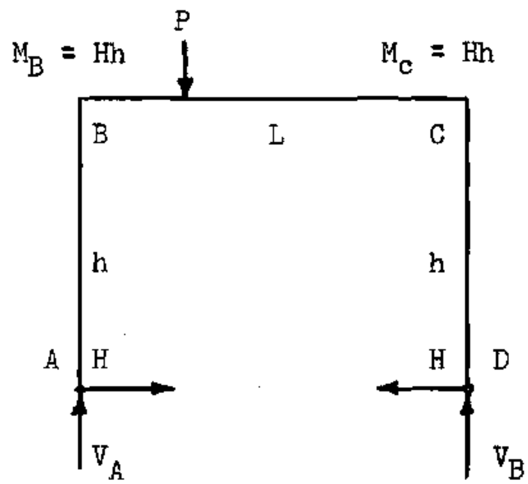

Fig. 7

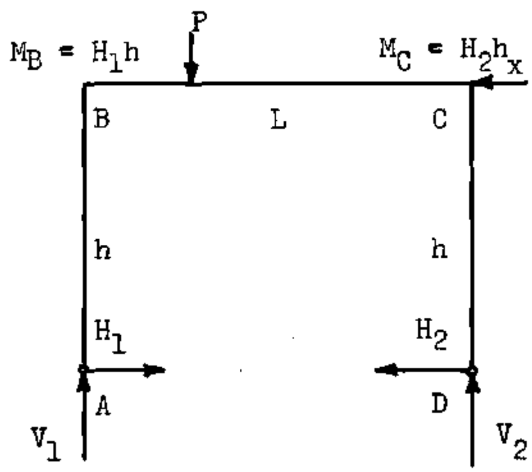

Fig. 8

The reactions at A and D due to the force x at C in Fig. 9 are x/2.[3]

Now, to determine the value of H, superpose Fig. 9 on Fig. 8.

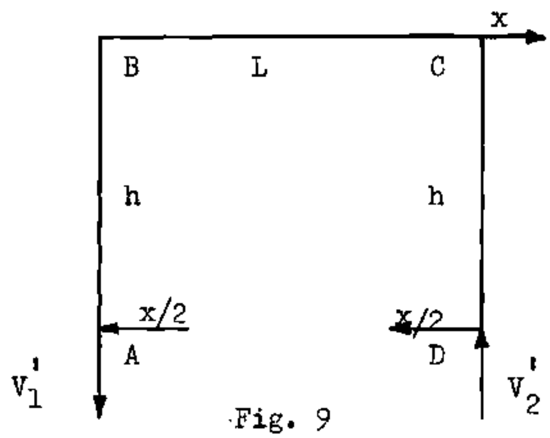

Fig. 9

$$H = H_1 - x/2$$

$$H = H_2 + x/2$$

$$2H = H_1 + H_2$$

or,

$$H = \frac{H_1 + H_2}{2}$$

It follows from the value obtained for H that the moment at B and C is the following:

$$M_A = M_B = \frac{H_1 h + H_2 h}{2} \qquad \text{Formula (1)}$$

The average of the moments at B and C as determined in Fig. 8 with the restraining force x at C is exactly equal to the value obtained by the use of Formula (1) above. This is, therefore, the proof of the hypothesis as stated.

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[3]Cross, Hardy and Morgan, N.D., Continuous Frames of Reinforced Concrete, John Wiley & Sons, Inc., 1932, p. 106.

EXAMPLE I- SYMMETRICAL FRAME, HINGED ENDED, WITH VERTICAL COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Modified Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 1.

Procedure

Proceed exactly in the same manner as in the conventional solution until the results shown in Fig. 3 are obtained. Then apply Proposition I to obtain the moments at B and C. The sum of the moments at B and C in Fig. 3 is $67.5 + 40.4 = 107.9$. Divide by two to obtain an average value of 53.95, the actual value of the moments at B and C.

The horizontal reactions at A and D are obtained by dividing 53.95 by 18, the length of the column, with a resulting value of 2.995.

The vertical reactions are determined by taking moments of all the external forces on the frame at points A and D and solving directly. They are 18.00 at A and 6.00 at D.

EXAMPLE II- SYMMETRICAL FRAME, FIXED ENDED, WITH VERTICAL COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Conventional Solution)

It is required to find the moments at A, B, C, and D and the reactions at A and D in the frame of Fig. 10.

<u>Procedure</u>

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 11. Carry out moment distribution as shown in Fig. 11.

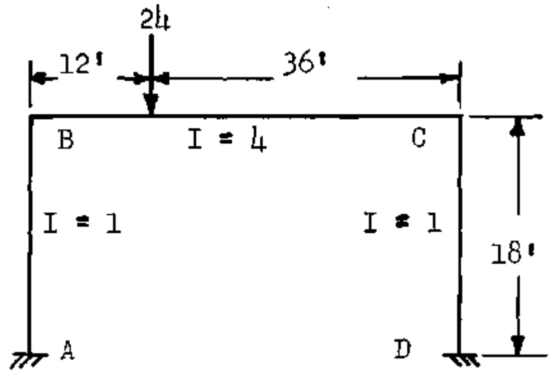

Fig. 10

Kc	A	2/5 B	B 3/5	3/5 C	C 2/5	D
F.E.M.			-162.0	54.0		
1st Dist.		64.8	97.2	-32.4	-21.6	
C.O.	32.4		-16.1	48.6		-10.8
2nd Dist.		6.4	9.7	-29.2	-19.4	
C.O.	3.2		-14.6	4.9		-9.7
3rd Dist.		5.8	8.8	-2.9	-2.0	
C.O.	2.9		-1.4	4.4		-1.0
4th Dist.		0.6	0.8	-2.6	-1.8	
C.O.	0.3		-1.3	0.4		-0.9
5th Dist.		0.5	0.8	-0.2	-0.2	
C.O.	0.3		-0.1	0.4		-0.1
6th Dist.			0.1	-0.2	-0.2	-0.1
Moments	39.1	78.1	-78.1	45.2	-45.2	-22.6

Fig. 11

Fig. 12 indicates the frame members as free bodies from which it is seen that the horizontal forces are unbalanced by 2.74.

An arbitrary force is applied acting horizontally to the right at B which produces fixed end moments of 100.0 in the columns. The resulting moment distribution is carried out in Fig. 13.

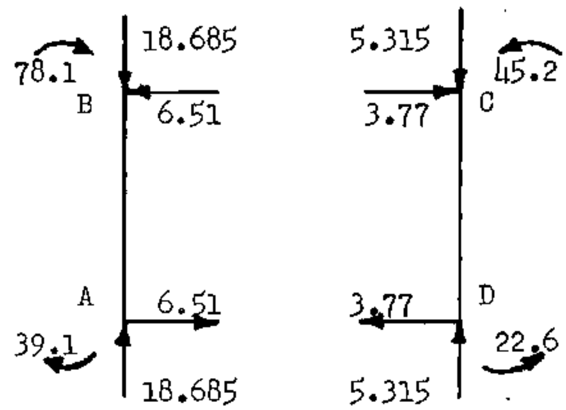

Fig. 12

Kc	A	4/13 B	9/13
F.E.M.	-100.0	-100.0	
1st Dist.		30.8	69.2
C.O.	15.4		
2nd Dist.			
Moments	-84.6	-69.2	69.2

Fig. 13

In Fig. 14 the moments and shears are shown for the arbitrary force which is determined to be 17.08. To obtain the moments and shears due to a force of 2.74, the values in Fig. 14 must be multiplied by the ratio of 2.74 to 17.08.

These values become 0.46 for vertical shear, 1.37 for horizontal shear, 13.6 for moment at A and D, and 11.1 for moment at B and C.

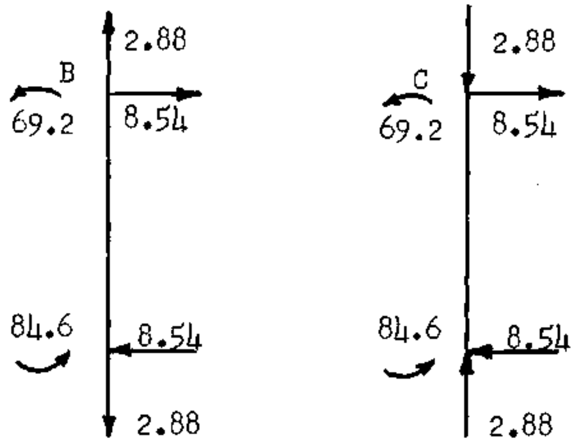

Fig. 14

Final Reactions

H_A = 6.51 - 1.37 = 5.14

H_D = 3.77 + 1.37 = 5.14

V_A = 18.685 - 0.46 = 18.225

V_D = 5.315 + 0.46 = 5.775

Final Moments

AB = 39.1 - 13.6 = 25.5

BA = 78.1 - 11.1 = 67.0

BC = -78.1 + 11.1 = -67.0

CB = 45.2 + 11.1 = 56.3

CD = -45.2 - 11.1 = -56.3

DC = -22.6 - 13.6 = -36.2

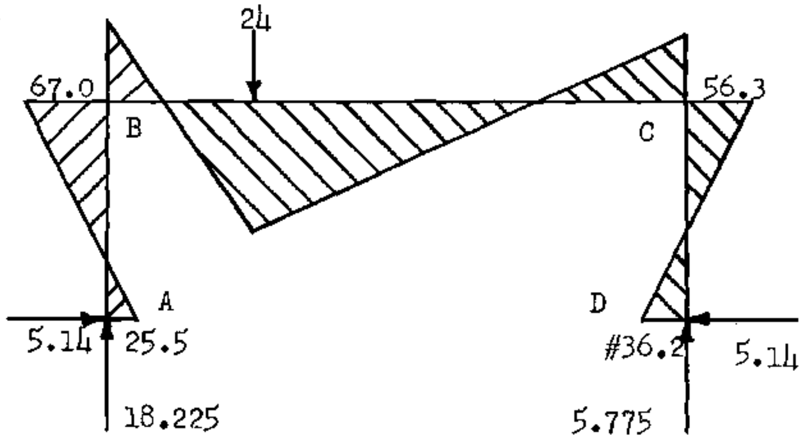

Fig. 15

The moment diagram for the frame of Fig. 11 is shown in Fig. 15.

EXAMPLE III- SYMMETRICAL FRAME, HINGED ENDED, WITH VERTICAL COLUMNS AND A HORIZONTAL LOAD ON ONE OF THE COLUMNS (Conventional Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 16.

<u>Procedure</u>
Rotate members AB and CD until they are in a horizontal position as shown in Fig. 17. Proceed with the moment distribution of Fig. 17.

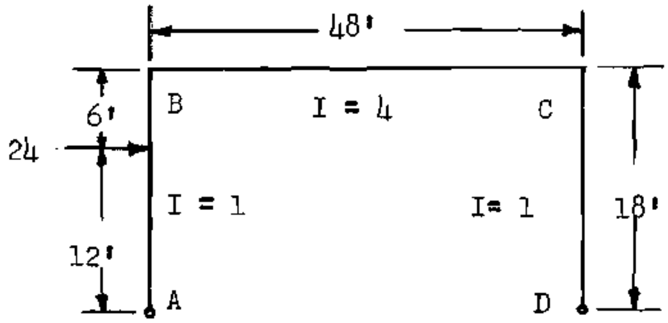

Fig. 16

Kc	A	1/3 B	2/3	2/3 C	1/3	D
F.E.M.	-32.0	64.0				
1st Dist.	32.0	-21.3	-42.7			
C.O.		16.0		-21.3		
2nd Dist.		-5.3	-10.7	14.2	7.1	
C.O.			7.1	-5.4		
3rd Dist.		-2.4	-4.7	3.6	1.8	
C.O.			1.8	-2.4		
4th Dist.		-0.6	-1.2	1.6	0.8	
C.O.			0.8	-0.6		
5th Dist.		-0.3	-0.5	0.4	0.2	
C.O.			0.2	-0.2		
6th Dist.		-0.1	-0.1	0.1	0.1	
Moments		50.0	-50.0	-10.0	10.0	

Fig. 17

In Fig. 18 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 24.00 + 0.56 -5.22 = 19.34. Since the frame of Fig. 16 is the same as the one used in Example I, the results of Figs. 4 and 5 are also applicable to this example. Therefore, the moments and reactions determined in Fig. 5 must be multiplied by the ratio of 19.34 to 4.16 to determine the corrections to be used in this example.

These become 7.25 for vertical shear, 9.67 for horizontal shear, and 174.3 for moment with proper regard for signs.

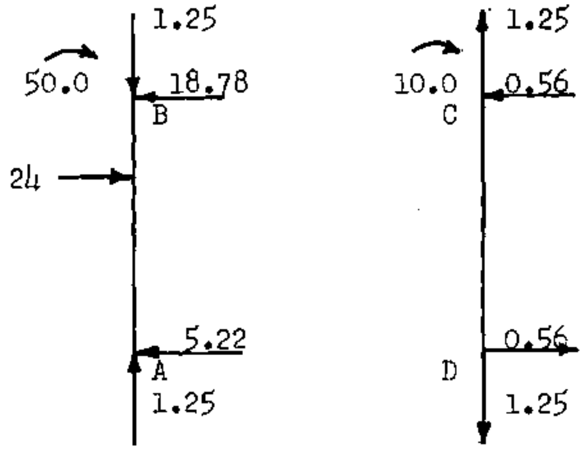

Fig. 18

Final Reactions

$H_A = 5.22 + 9.67 = 14.89$

$H_D = -0.56 + 9.67 = 9.11$

$V_A = 1.25 - 7.25 = -6.00$

$V_D = -1.25 + 7.25 = 6.00$

Final Moments

BA = 50.00 - 174.30 = -124.30 BC = -50.00 + 174.30 = 124.30

CB = -10.00 + 174.30 = 164.30 CD = 10.00 - 174.30 = -164.30

The moment diagram for the frame of Fig. 16 is shown in Fig. 19.

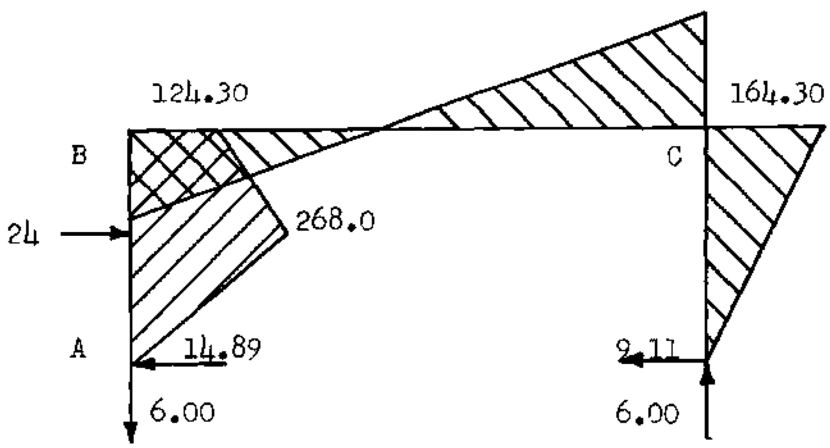

Fig. 19

PROPOSITION II- HYPOTHESIS- IN A HINGED ENDED, SINGLE SPAN, SYMMETRICAL RIGID FRAME WITH A HORIZONTAL LOAD ON ONE OF THE COLUMNS, THE HORIZONTAL REACTIONS ON THE COLUMN BASES CAN BE OBTAINED DIRECTLY FROM THOSE REACTIONS WHICH ARE DETERMINED IN A MOMENT DISTRIBUTION ANALYSIS ASSUMING THAT NO SIDESWAY CAN OCCUR.

Proof

In the frame of Fig. 20, it is evident that $H_1 + H_2 = P$. A moment distribution analysis of the frame may be made if there is an assumed force x at C of sufficient magnitude to prevent sidesway as shown in Fig. 21.

For statical equilibrium, x must equal $P - H_1 - H_2$.

Since the force x does not actually exist, corrections must be determined for H_{1A} and H_{2D} of Fig. 21. This is accomplished by applying a force x at C as shown in Fig. 22.

The reactions at A and D due to the force x in Fig. 22 are x/2. (See Proposition I)

By superposing Fig. 22 on Fig. 21, the following is determined:

$$H_1 = H_{1A} + x/2$$

$$H_2 = H_{2D} + x/2$$

In the equations above, H_2 may be replaced by its equivalent of $P - H_1$. The two equations can then be added to get a value for H_1.

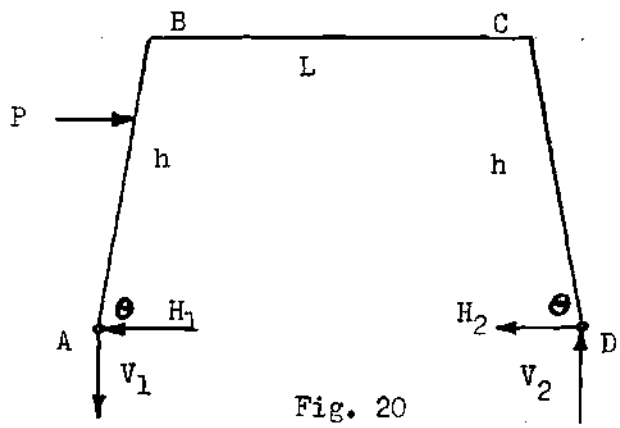

Fig. 20

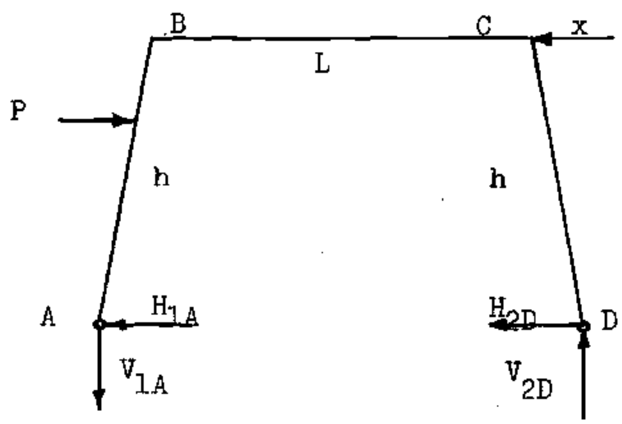

Fig. 21

$$H_1 = H_{1A} + x/2$$

$$P - H_1 = H_{2D} + x/2$$

$$-P + 2H_1 = H_{1A} - H_{2D}$$

or,

$$H_1 = \frac{P + H_{1A} - H_{2D}}{2} \quad \text{Formula (2)}$$

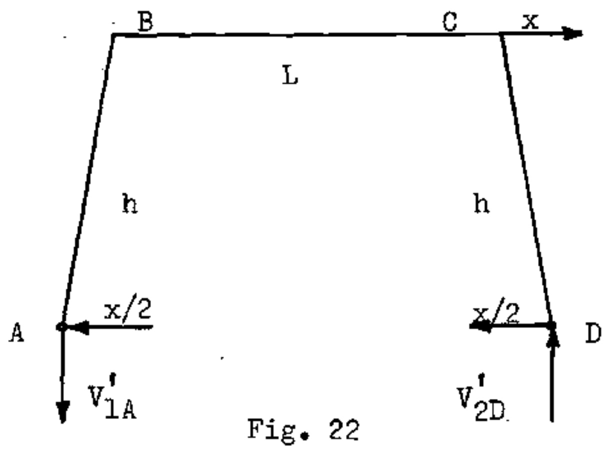

Fig. 22

By a similar procedure H_2 can be obtained, and is given in Formula (3).

$$H_2 = \frac{P - H_{1A} + H_{2D}}{2} \quad \text{Formula (3)}$$

By the use of Formulae (2) and (3) it is possible to obtain the final horizontal reactions in a frame of the type shown in Fig. 20 immediately after a moment distribution analysis in which it was assumed that no sidesway occurred. The vertical reactions can be obtained by statics. The moments are easily determined once the vertical and horizontal components of the reactions are found.

EXAMPLE III- SYMMETRICAL FRAME, HINGED ENDED, WITH VERTICAL COLUMNS AND A HORIZONTAL LOAD ON ONE OF THE COLUMNS (Modified Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 16.

Procedure

Proceed exactly in the same manner as in the conventional solution until the results in Fig. 18 are obtained. Then apply formulae developed in Proposition II to obtain horizontal reactions.

$$H_2 = \frac{24.00 - 5.22 + (-0.56)}{2} = 9.11 \qquad \text{Formula (3)}$$

$$H_1 = \frac{24.00 + 5.22 - (-0.56)}{2} = 14.89 \qquad \text{Formula (2)}$$

The vertical reactions of minus 6.00 at A and plus 6.00 at D are found by taking moments at D and A respectively.

The moments at B and C are obtained by taking moments at each of those points as shown below.

Final Moments

1. At joint B- $14.89 \times 18 - 24.00 \times 6 = 124.30$
2. At joint C- $- 9.11 \times 18 = -164.30$

EXAMPLE IV- SYMMETRICAL FRAME, FIXED ENDED, WITH VERTICAL COLUMNS AND A HORIZONTAL LOAD ON ONE OF THE COLUMNS (Conventional Solution)

It is required to find the moments at A, B, C, and D, and the reactions at A and D in the frame of Fig. 23.

Procedure

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 24. Proceed with the moment distribution of Fig. 24.

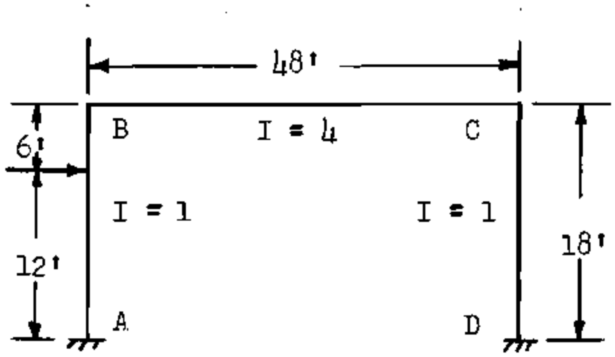

Fig. 23

Kc	A	2/5 B	3/5	3/5 C	2/5	D
F.E.M.	-32.0	64.0				
1st Dist.		-25.6	-38.4			
C.O.	-12.8			-19.2		
2nd Dist.				11.5	7.7	
C.O.			5.8			3.8
3rd Dist.		-2.3	-3.5			
C.O.	-1.1			-1.7		
4th Dist.				1.0	0.7	
C.O.			0.5			0.4
5th Dist.		-0.2	-0.3			
C.O.	-0.1			-0.2		
6th Dist.				0.1	0.1	
Moments	-46.0	35.9	-35.9	-8.5	8.5	4.2

Fig. 24

In Fig. 25 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 24.00 + 0.71 - 8.56 = 16.15. Since the frame of Fig. 23 is the same as the one used in Example II, the results of Figs. 13 and 14 are also applicable to this example. Therefore, the moments and reactions determined in Fig. 14 must be multiplied by the ratio of 16.15 to 17.08 to determine the corrections to be used in this example.

These become 8.07 for horizontal shear, 2.72 for vertical shear, 80.0 for moment at joints A and D, and 65.4 for moment at joints B and C with proper regard for signs.

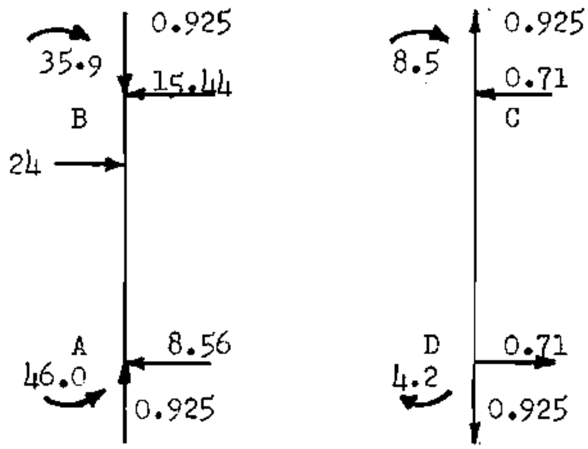

Fig 25

Final Reactions

$H_A = 8.56 + 8.07 = 16.63$

$H_D = -0.71 + 8.07 = 7.36$

$V_A = 0.925 - 2.72 = -1.795$

$V_D = -0.925 + 2.72 = 1.795$

Final Moments

AB = -46.0 - 80.0 = -126.0

BA = 35.9 - 65.4 = -29.5

BC = -35.9 + 65.4 = 29.5

CB = -8.5 + 65.4 = 56.9

CD = 8.5 - 65.4 = -56.9

DC = 4.2 - 80.0 = -75.8

The moment diagram for the frame of Fig. 23 is shown in Fig. 26.

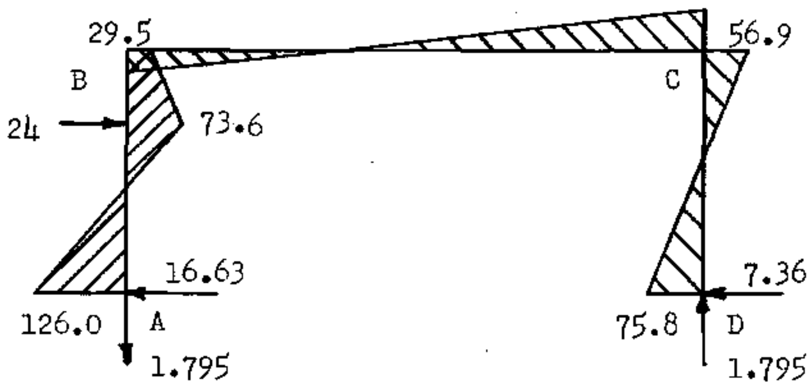

Fig. 26

EXAMPLE V- SYMMETRICAL FRAME, HINGED ENDED, WITH INCLINED COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Conventional Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 27.

<u>Procedure</u>

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 28. Proceed with the moment distribution of Fig. 28.

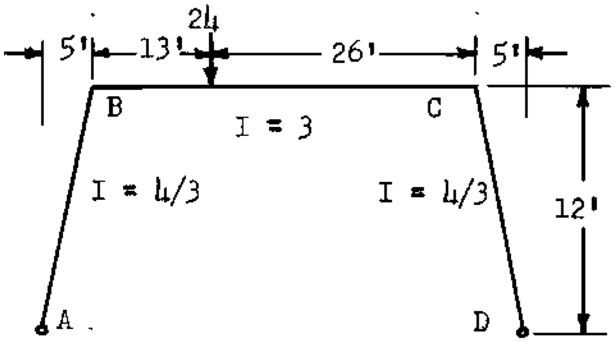

Fig. 27

Kc	A 1/2	B 1/2	1/2 C	1/2 D
F.E.M.		-138.6	69.3	
1st Dist.	69.3	69.3	-34.6	-34.7
C.O.		-17.3	34.6	
2nd Dist.	8.7	8.6	-17.3	-17.3
C.O.		-8.7	4.3	
3rd Dist.	4.3	4.4	-2.2	-2.1
C.O.		-1.1	2.2	
4th Dist.	0.6	0.5	-1.1	-1.1
C.O.		-0.5	0.2	
5th Dist.	0.2	0.3	-0.1	-0.1
C.O.			0.2	
6th Dist.			-0.1	-0.1
Moments	83.1	-83.1	55.4	-55.4

Fig. 28

In Fig. 29 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 13.89 - 7.65 = 6.24. An arbitrary force is applied acting horizontally to the right at B which produces fixed end moments of 100.0 in the columns and 19.2 in the girder. Using these fixed end moments, the moment distribution is carried out in Fig. 31.

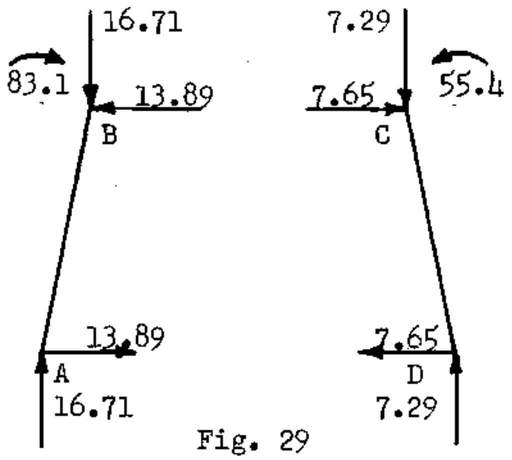

Fig. 29

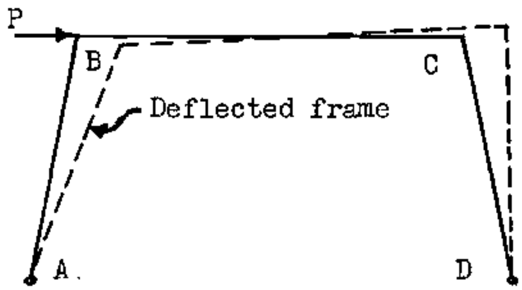

Fig. 30

From Fig. 30 it can be seen that, in the frame with inclined columns, a force P will effect a deflection in the girder as well as the columns. The deflection produces fixed end moments in the frame members in proportion to the deflection multiplied by the moment of inertia of the member and divided by its length squared.[4] It was in this manner that the fixed end moment of 19.2 was obtained for the girder BC. Due to anti-symmetry of the moments, a modified stiffness factor for the girder was used in the moment distribution of Fig. 31.

The pair of free bodies shown in Fig. 32 indicate the moments and shears resulting from the arbitrary force P which is determined to be 7.88. In order to get the moments and shears due to a force of 6.24, the values obtained in Fig. 32 must be multiplied by the ratio of 6.24 to 7.88.

These become 1.53 for vertical shear, 3.12 for horizontal shear, and 29.8 for moment with proper regard for signs.

Kc	A	2/5 B	3/5
F.E.M.	-100.0	-100.0	19.2
1st Dist.	100.0	32.3	48.5
C.O.		50.0	
2nd Dist.		-20.0	-30.0
Moments		-37.7	37.7

Fig. 31

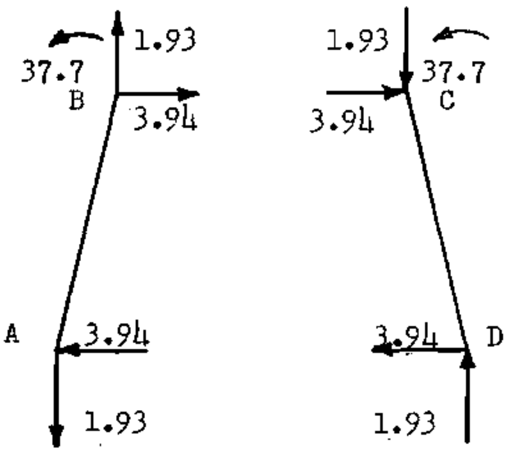

Fig. 32

[4]Cross, Hardy and Morgan, N.D., Continuous Frames of Reinforced Concrete, John Wiley & Sons, Inc., 1932, p. 113, fig. 33, statement 2.

Final Reactions

$H_A = 13.89 - 3.12 = 10.77$ $\qquad$ $H_D = 7.65 + 3.12 = 10.77$

$V_A = 16.71 - 1.53 = 15.18$ $\qquad$ $V_D = 7.29 + 1.53 = 8.82$

Final Moments

BA = 83.1 - 29.8 = 53.3 $\qquad$ BC = -83.1 + 29.8 = -53.3

CB = 55.4 + 29.8 = 85.2 $\qquad$ CD = -55.4 - 29.8 = -85.2

The moment diagram for the frame of Fig. 27 is shown in Fig. 33.

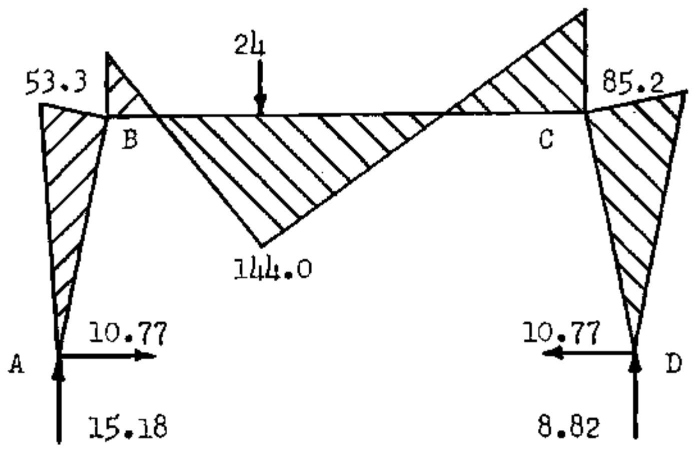

Fig. 33

PROPOSITION III- HYPOTHESIS- IN A HINGED ENDED, SINGLE SPAN, SYMMETRICAL RIGID FRAME WITH AN UNSYMMETRICALLY PLACED VERTICAL LOAD, THE HORIZONTAL REACTIONS ON THE COLUMN BASES ARE EQUAL TO THE AVERAGE OF THE HORIZONTAL REACTIONS OBTAINED AT THOSE POINTS WHEN THE FRAME IS ANALYZED BY THE MOMENT DISTRIBUTION METHOD ASSUMING THAT NO SIDESWAY CAN OCCUR.

Proof

In the frame of Fig. 34, it is evident that the horizontal reactions at A and D must be opposite and equal. A moment distribution analysis of the frame may be made if there is an assumed force x at C of sufficient magnitude to prevent sidesway. Let it be assumed that the force x in Fig. 35 fulfills this requirement. Then x must equal to $H_1 - H_2$, assuming that H_1 is greater than H_2.

Since the force x does not actually exist, corrections for H_1 and H_2 of Fig. 35 are determined by applying a force x at C as shown in Fig. 36.

The reactions at A and D due to the force x at C in Fig. 36 are x/2. (See Proposition I).

By superposing Fig. 36 on Fig. 35, the following is determined:

$$H = H_1 - x/2$$

$$H = H_2 + x/2$$

$$2H = H_1 + H_2$$

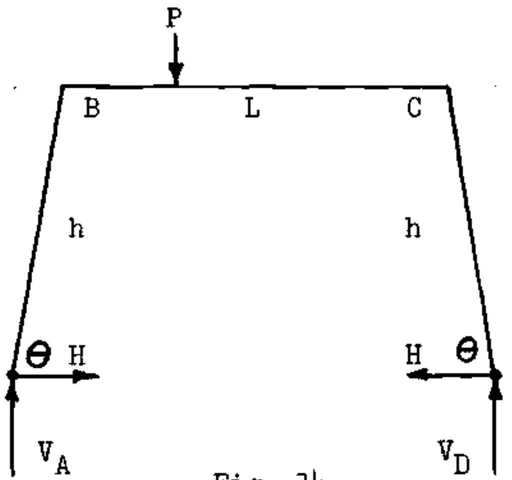

Fig. 34

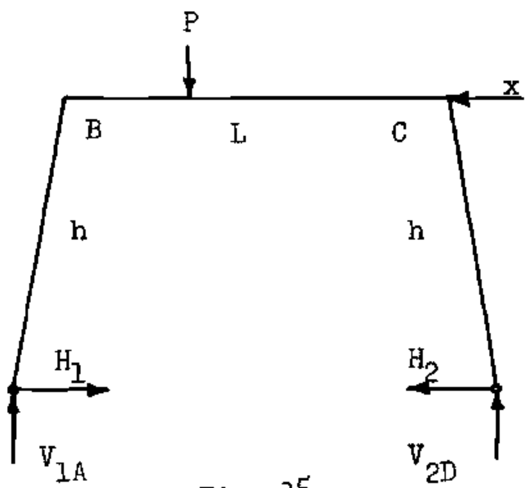

Fig. 35

Divide both sides of the equation on the preceding page by two to get an expression for H.

$$H = \frac{H_1 + H_2}{2} \qquad \text{Formula (4)}$$

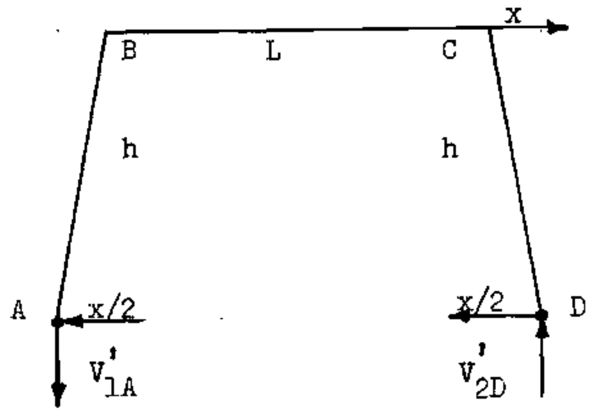

Fig. 36

Since H_1 and H_2 are the horizontal reactions which would be obtained if there was a force x at C as indicated in Fig. 35, it can be seen that the actual horizontal reactions at A and D are equal to the average of those values by an inspection of Formula (4). This is, therefore, the proof of the hypothesis as stated.

EXAMPLE V- SYMMETRICAL FRAME, HINGED ENDED, WITH INCLINED COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Modified Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 27.

Procedure

Proceed exactly in the same manner as in the conventional solution until the results in Fig. 29 are obtained. Then apply Formula (4) of Proposition III to obtain the horizontal reactions at A and D. The sum of the horizontal reactions at A and D in Fig. 29 is 13.89 + 7.65 = 21.54. Divide by two to obtain an average value of 10.77, the actual value of the horizontal reactions at A and D.

The vertical reactions of 15.18 at A and 8.82 at D are found by taking moments at D and A respectively of all the external forces acting on the frame.

The moments at B and C are obtained by taking moments at each of those points as shown below.

Final Moments

1. At joint B- $15.18 \times 5 - 10.77 \times 12 = -53.3$
2. At joint C- $8.82 \times 5 - 10.77 \times 12 = -85.1$

EXAMPLE VI- SYMMETRICAL FRAME, FIXED ENDED, WITH INCLINED COLUMNS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Conventional Solution)

It is required to find the moments at A, B, C, and D, and the reactions at A and D in the frame of Fig. 37.

Procedure

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 38. Proceed with the moment distribution of Fig. 38.

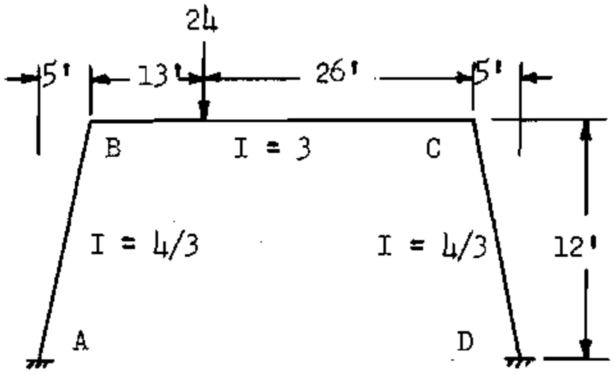

Fig. 37

Kc	A	4/7 B	3/7	3/7 C	4/7	D
F.E.M.			-138.6	69.3		
1st Dist.		79.2	59.4	-29.7	-39.6	
C.O.	39.6		-14.9	29.7		-19.8
2nd Dist.		8.5	6.4	-12.7	-17.0	
C.O.	4.2		-6.3	3.2		-8.5
3rd Dist.		3.6	2.7	-1.4	-1.8	
C.O.	1.8		-0.7	1.4		-0.9
4th Dist.		0.4	0.3	-0.6	-0.8	
C.O.	0.2		-0.3	0.1		-0.4
5th Dist.	0.1 ←	0.2	0.1		-0.1	
Moments	45.9	91.9	-91.9	59.3	-59.3	-29.6

Fig. 38

In Fig. 39 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 18.50 - 10.40 = 8.10. An arbitrary force is applied acting horizontally to the right at B which produces fixed end moments of 100.0 in the columns and 19.2 in the girder. Using these fixed end moments, the moment distribution is carried out in Fig. 40.

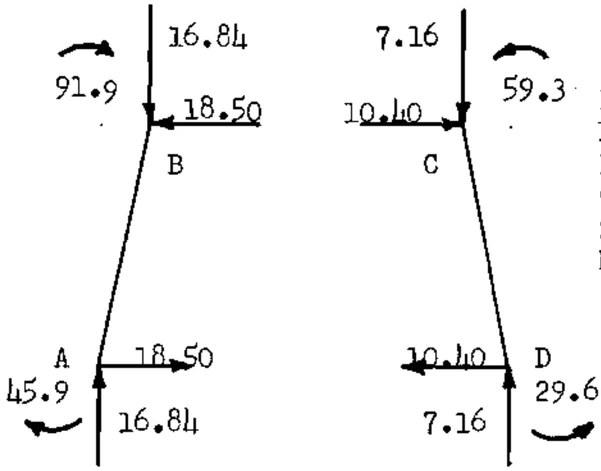

Fig. 39

Kc	A	8/17 B	9/17
F.E.M.	-100.0	-100.0	19.2
1st Dist.		38.0	42.8
C.O.	19.0		
2nd Dist.			
Moments	-81.0	-62.0	62.0

Fig. 40

The pair of free bodies shown in Fig. 41 indicates the moments and shears resulting from the arbitrary force which is determined to be 26.48. In order to get the moments and shears due to a force of 8.10, the values obtained in Fig. 41 must be multiplied by the ratio of 8.10 to 26.48.

These become 0.97 for vertical shear, 4.05 for horizontal shear, 24.8 for moment at A and D, and 19.0 for moment at B and C with proper regard for signs.

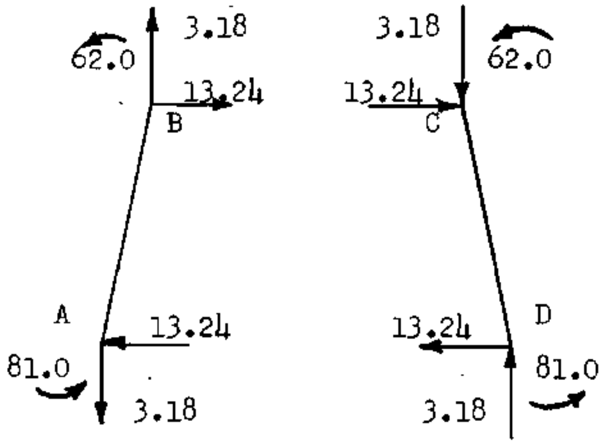

Fig. 41

Final Reactions

$H_A = 18.50 - 4.05 = 14.45$ $\qquad$ $H_D = 10.40 + 4.05 = 14.45$

$V_A = 16.84 - 0.97 = 15.87$ $\qquad$ $V_D = 7.16 + 0.97 = 8.13$

Final Moments

AB = 45.9 - 24.8 = 21.1 $\qquad$ BA = 91.9 - 19.0 = 72.9

BC = -91.9 + 19.0 = -72.9 $\qquad$ CB = 59.3 + 19.0 = 78.3

CD = -59.3 - 19.0 = -78.3 $\qquad$ DC = -29.6 - 24.8 = -54.4

The moment diagram for the frame of Fig. 37 is shown in Fig. 42.

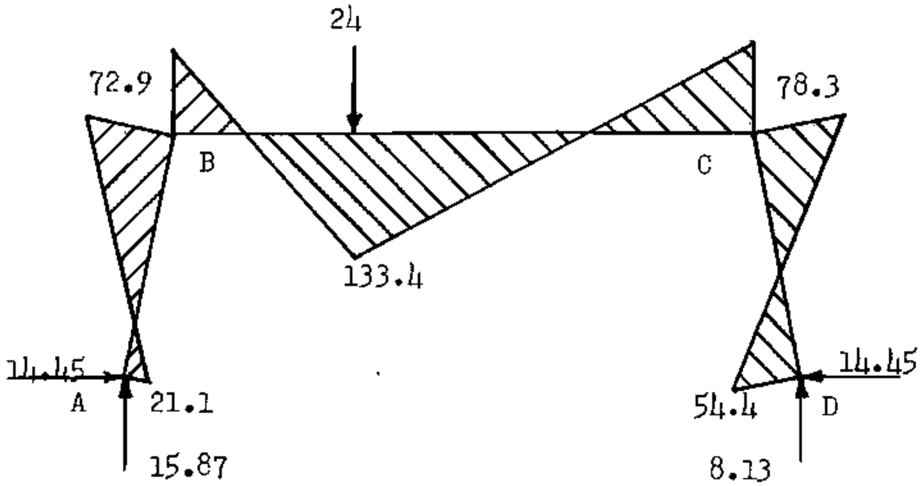

Fig. 42

EXAMPLE VII- SYMMETRICAL FRAME, HINGED ENDED, WITH INCLINED COLUMNS AND A LOAD PLACED NORMAL TO ONE OF THE COLUMNS

(Conventional Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 43.

Procedure

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 44. Proceed with the moment distribution of Fig. 44.

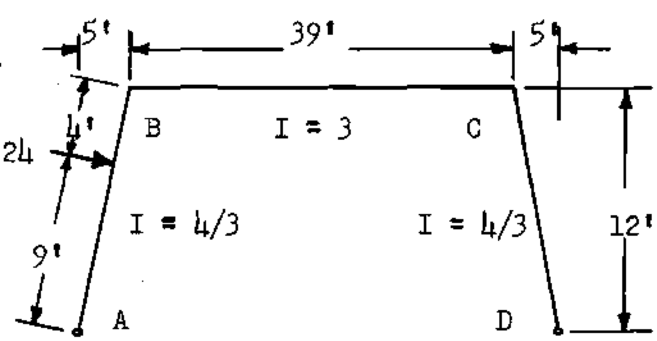

Fig. 43

Kc	A	1/2 B	1/2	1/2 C	1/2	D
F.E.M.	-20.4	46.0				
1st Dist.	20.4	-23.0	-23.0			
C.O.		10.2		-11.5		
2nd Dist.		-5.1	-5.1	5.7	5.8	
C.O.			2.9	-2.6		
3rd Dist.		-1.5	-1.4	1.3	1.3	
C.O.			0.6	-0.7		
4th Dist.		-0.3	-0.3	0.4	0.3	
C.O.			0.2	-0.2		
5th Dist.		-0.1	-0.1	0.1	0.1	
Moments		26.2	-26.2	-7.5	7.5	

Fig. 44

In Fig. 45 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 22.15 + 0.98 - 1.61 = 21.52. Since the frame of Fig. 43 is the same as the one used in Example V, the results of Figs. 31 and 32 are also applicable to this example. Therefore, the moments and reactions determined in Fig. 32 must be multiplied by the ratio of 21.52 to 7.88 to determine the corrections to be used in this example.

These become 5.27 for vertical shear, 10.76 for horizontal shear, and 103.0 for moment with proper regard for signs.

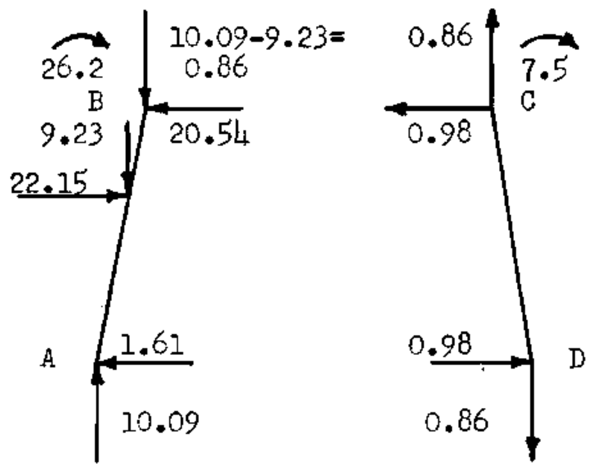

Fig. 45

Final Reactions

$H_A = 1.61 + 10.76 = 12.37$

$H_D = 10.76 - 0.98 = 9.78$

$V_A = 10.09 - 5.27 = 4.82$

$V_D = 5.27 - 0.86 = 4.41$

Final Moments

BA = 26.2 - 103.0 = -76.8 BC = -26.2 + 103.0 = 76.8

CB = -7.5 + 103.0 = 95.5 CD = 7.5 - 103.0 = -95.5

The moment diagram for the frame of Fig. 43 is shown in Fig. 46.

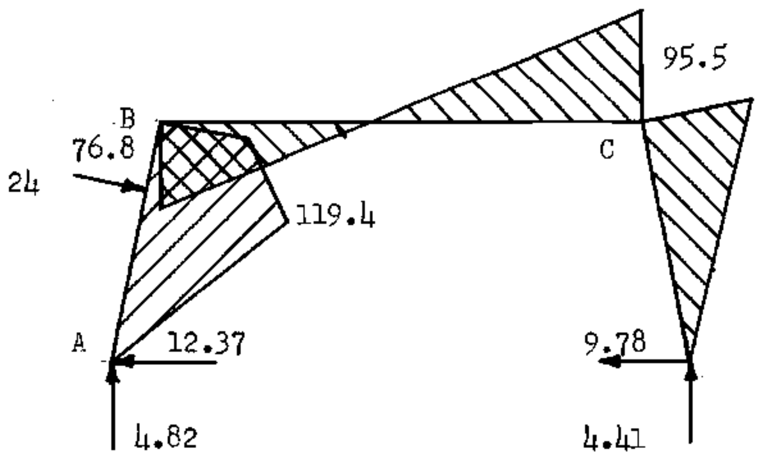

Fig. 46

EXAMPLE VII- SYMMETRICAL FRAME, HINGED ENDED, WITH INCLINED COLUMNS AND A LOAD PLACED NORMAL TO ONE OF THE COLUMNS (Modified Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 43.

Procedure

Proceed exactly in the same manner as in the conventional solution until the results in Fig. 45 are obtained. Then apply formulae developed in Proposition II to obtain horizontal reactions.

$$H_D = \frac{22.15 - 1.61 + (-0.98)}{2} = 9.78 \qquad \text{Formula (3)}$$

$$H_A = \frac{22.15 + 1.61 - (-0.98)}{2} = 12.37 \qquad \text{Formula (2)}$$

The vertical reactions of 4.82 at A and 4.41 at D are found by taking moments at D and A respectively of all the external forces acting on the frame.

The moments at B and C are obtained by taking moments at each of those points as shown below.

Final Moments

1. At joint B- $4.82 \times 5 + 12.37 \times 12 - 24 \times 4 = 76.8$
2. At joint C- $4.41 \times 5 - 9.78 \times 12 = -95.5$

EXAMPLE VIII- SYMMETRICAL FRAME, FIXED ENDED, WITH INCLINED COLUMNS AND A LOAD PLACED NORMAL TO ONE OF THE COLUMNS

(Conventional Solution)

It is required to find the moments at A, B, C, and D, and the reactions at A and D in the frame of Fig. 47.

Procedure

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 48. Proceed with the moment distribution of Fig. 48.

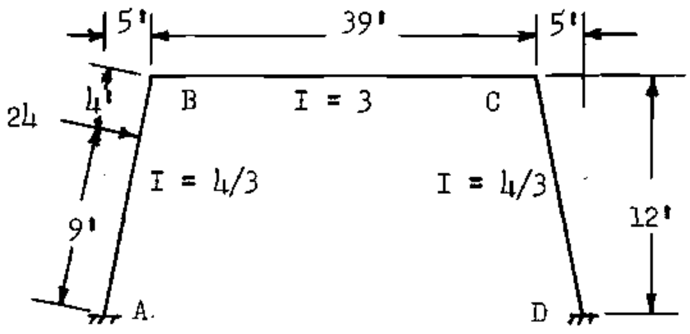

Fig. 47

Kc	A	4/7 B	3/7	3/7 C	4/7	D
F.E.M.	-20.4	46.0				
1st Dist.		-26.3	-19.7			
C.O.	13.1			-9.9		
2nd Dist.				4.2	5.7	
C.O.			2.1			2.8
3rd Dist.		-0.9	-1.2			
C.O.	-0.5			-0.6		
4th Dist.				0.3	0.3	
C.O.			0.1			0.2
5th Dist.		-0.1				
Moments	-34.0	18.7	-18.7	-6.0	6.0	3.0

Fig. 48

In Fig. 49 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 22.15 + 1.01 - 5.17 = 17.99. Since the frame of Fig. 47 is the same as the one used in Example VI, the results of Figs. 40 and 41 are also applicable to this example. Therefore, the moments and reactions determined in Fig. 41 must be multiplied by the ratio of 17.99 to 26.48 to determine the corrections to be used in this example.

These become 2.16 for vertical shear, 8.99 for horizontal shear, 55.0 for moment at A and D, and 42.1 for moment at B and C with proper regard for signs.

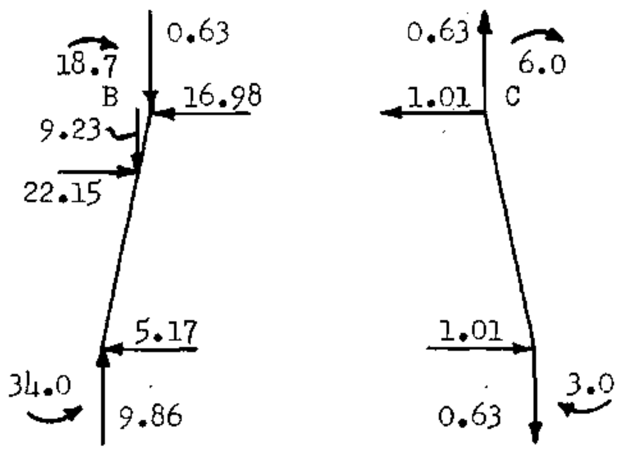

Fig. 49

<u>Final Reactions</u>

$H_A = 5.17 + 8.99 = 14.16$

$H_D = 8.99 - 1.01 = 7.98$

$V_A = 9.86 - 2.16 = 7.70$

$V_D = 2.16 - 0.63 = 1.53$

<u>Final Moments</u>

$AB = -34.0 - 55.0 = -89.0$

$BA = 18.7 - 42.1 = -23.4$

$BC = -18.7 + 42.1 = 23.4$

$CB = -6.0 + 42.1 = 36.1$

$CD = 6.0 - 42.1 = -36.1$

$DC = 3.0 - 55.0 = -52.0$

The moment diagram for the frame of Fig. 47 is shown in Fig. 50.

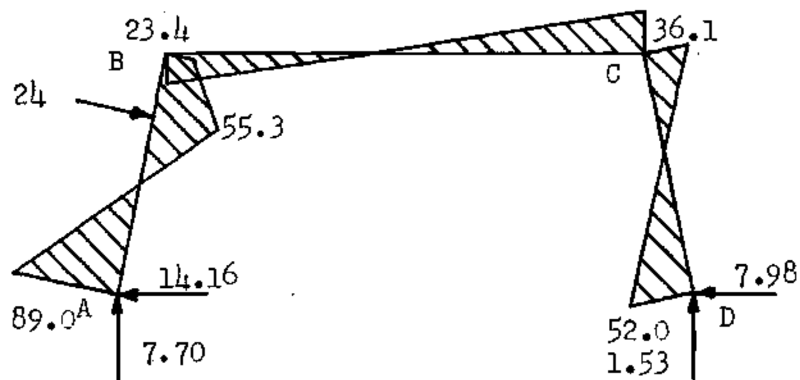

Fig. 50

EXAMPLE IX- UNSYMMETRICAL FRAME, HINGED ENDED, WITH VERTICAL COLUMNS OF UNEQUAL HEIGHTS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Conventional Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 51.

Procedure

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 52. Proceed with the moment distribution of Fig. 52.

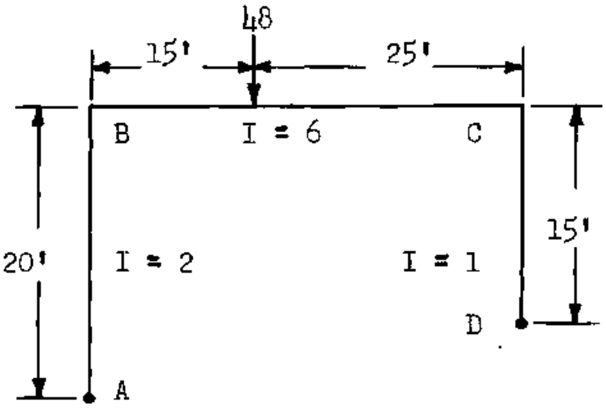

Fig. 51

	A	AB	B	BC	CB	C	CD	D
Kc		1/3		2/3	3/4		1/4	
F.E.M.				-281.2	168.8			
1st Dist.		93.7		187.5	-126.6		-42.2	
C.O.				-63.3	93.8			
2nd Dist.		21.1		42.2	-70.4		-23.4	
C.O.				-35.2	21.1			
3rd Dist.		11.7		23.5	-15.8		-5.3	
C.O.				-7.9	11.7			
4th Dist.		2.7		5.2	-8.8		-2.9	
C.O.				-4.4	2.6			
5th Dist.		1.5		2.9	-1.9		-0.7	
C.O.				-1.0	1.5			
6th Dist.		0.3		0.7	-1.1		-0.4	
C.O.				-0.5	0.3			
7th Dist.		0.2		0.3	-0.2		-0.1	
Moments		131.2		-131.2	75.0		-75.0	

Fig. 52

In Fig. 53 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 6.56 - 5.00 = 1.56. An arbitrary force is applied acting horizontally to the right at B which produces fixed end moments of 100.0 in column AB and 88.9 in column CD. Using these fixed end moments, the moment distribution is carried out in Fig. 54.

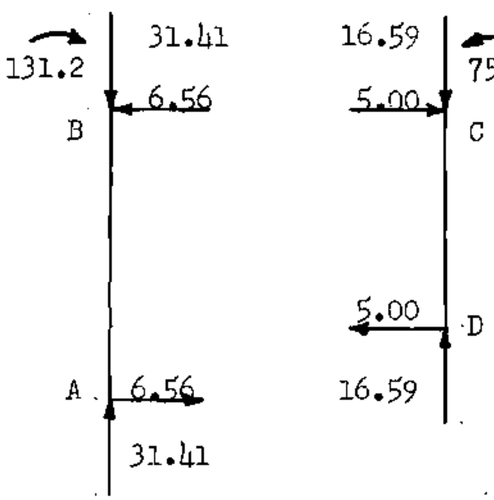

Fig. 53

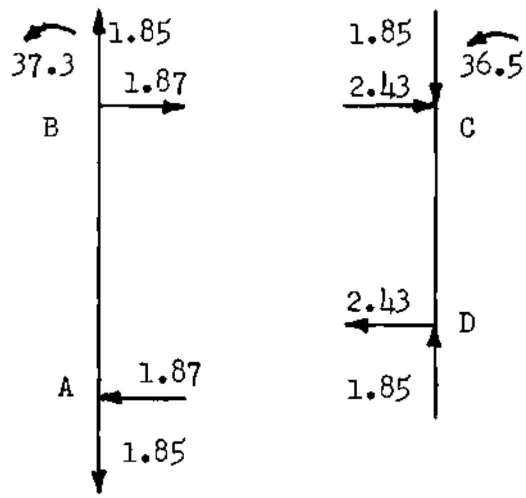

Fig. 55

Kc	A	1/3	B 2/3	3/4 C	1/4	D
F.E.M.	-100.0	-100.0			-88.9	-88.9
1st Dist.	100.0	33.3	66.7	66.7	22.2	88.9
C.O.		50.0	33.4	33.4	44.4	
2nd Dist.		-27.8	-55.6	-58.3	-19.5	
C.O.			-29.1	-27.8		
3rd Dist.		9.7	19.4	20.8	7.0	
C.O.			10.4	9.7		
4th Dist.		-3.5	-6.9	-7.3	-2.4	
C.O.			-3.7	-3.5		
5th Dist.		1.2	2.5	2.6	0.9	
C.O.			1.3	1.2		
6th Dist.		-0.4	-0.9	-0.9	-0.3	
C.O.			-0.5	-0.5		
7th Dist.		0.2	0.3	0.4	0.1	
Moments		-37.3	37.3	36.5	-36.5	

Fig. 54

The moments and shears produced by the force chosen arbitrarily are shown on the column free bodies of Fig. 55, from which it is seen that this force was 4.30. In order to get the moments and shears due to a force of 1.56, the values obtained in Fig. 55 must be multiplied by the ratio of 1.56 to 4.30.

These become 0.67 for vertical shear, 0.68 for horizontal shear at A, 0.88 for horizontal shear at D, 13.5 for moment at B, and 13.2 for moment at C with proper regard for signs.

Final Reactions

$H_A = 6.56 - 0.68 = 5.88$ $H_D = 5.00 + 0.88 = 5.88$

$V_A = 31.41 - 0.67 = 30.74$ $V_D = 16.59 + 0.67 = 17.26$

Final Moments

BA = 131.2 - 13.5 = 117.7 BC = -131.2 + 13.5 = -117.7

CB = 75.0 + 13.2 = 88.2 CD = -75.0 - 13.2 = -88.2

The moment diagram for the frame of Fig. 51 is shown in Fig. 56.

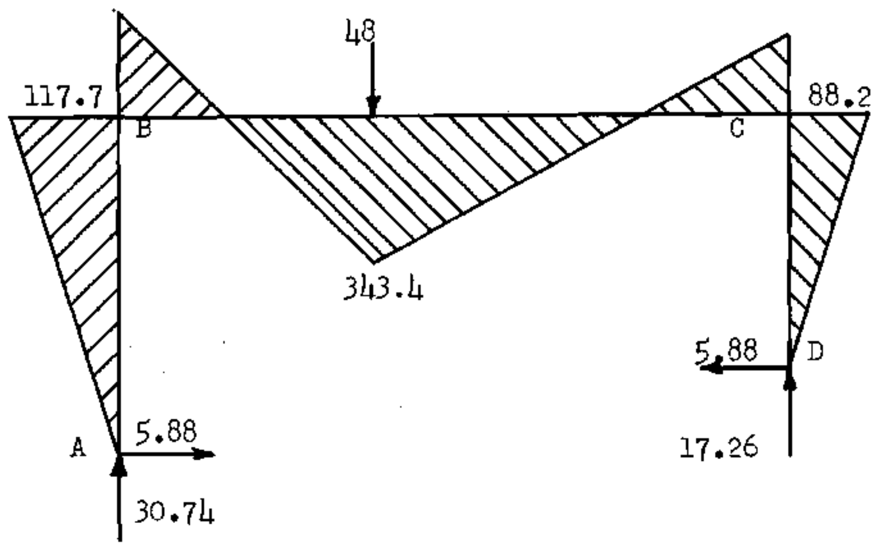

Fig. 56

PROPOSITION IV- HYPOTHESIS- IN A HINGED ENDED, SINGLE SPAN, RIGID FRAME WITH VERTICAL COLUMNS OF UNEQUAL HEIGHTS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD, THE HORIZONTAL REACTIONS CAN BE OBTAINED DIRECTLY FROM THOSE REACTIONS WHICH ARE FOUND IN A MOMENT DISTRIBUTION ANALYSIS ASSUMING THAT NO SIDESWAY CAN OCCUR.

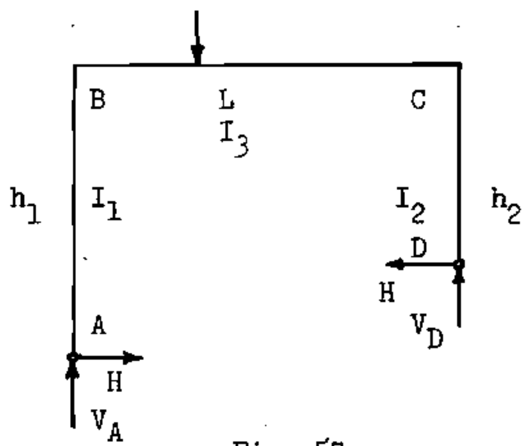

Fig. 57

Proof

In the frame of Fig. 57 it is evident that the horizontal reactions at A and D must be opposite and equal. A moment distribution analysis of the frame may be made if there is an assumed force x at C of sufficient magnitude to prevent sidesway. Let it be assumed that the force x in Fig. 58 fulfills this requirement. Then x must equal to $H_1 - H_2$, assuming that H_1 is greater than H_2.

Since the force x does not actually exist, corrections for H_1 and H_2 of Fig. 58 are determined by applying a force x at C as shown in Fig. 59. The horizontal reactions at A and D due to this force are designated as x_1 and kx_1 whose sum must be equal to x.

The value of k is determined from the following formulae, presented by Kleinlogel,[5] which are applicable to the frame of Fig. 59:

$$H_A = x \frac{nC}{N} \qquad H_D = x \frac{B}{N}$$

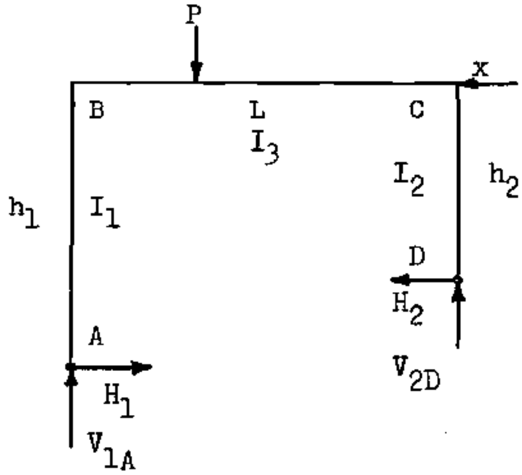

Fig. 58

[5]Kleinlogel, Adolf, Rahmenformeln, Verlag Von Wilhelm Ernst and Sohn, Berlin, 1944, p. 174 - 175.

Using the two formulae just given, the following expression is arrived at for a value of k:

$$k = \frac{H_D}{H_A} = \frac{B}{nC} \qquad \text{or,}$$

$$k = \frac{2\left\{\frac{I_3}{I_1}\cdot\frac{h_1}{L} + 1\right\} + \frac{h_2}{h_1}}{\frac{h_2}{h_1}\left\{1 + 2\,\frac{h_2}{h_1}\left(1 + \frac{I_3}{I_2}\cdot\frac{h_2}{L}\right)\right\}}$$

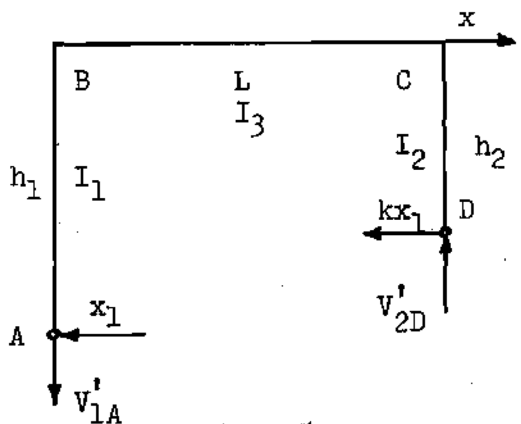

Fig. 59

Formula (5)

By superposing Fig. 59 on Fig. 58, the following is obtained:

$$H = H_1 - x_1$$
$$H = H_2 + kx_1$$

or,

$$kH = kH_1 - kx_1$$
$$H = H_2 + kx_1$$
$$\overline{kH + H = kH_1 + H_2}$$

The first of the two equations above is multiplied through by k as shown at the right and then added to the second. Both sides of the resulting equation are then divided by $1 + k$ to obtain a value for H which is given in Formula (6).

$$H = \frac{kH_1 + H_2}{1 + k} \qquad \text{Formula (6)}$$

Therefore, as stated in the hypothesis, it is possible to obtain the horizontal reactions in a frame of the type shown in Fig. 57 by applying Formulae (5) and (6) and using the horizontal reactions which are obtained in a moment distribution analysis assuming that no sidesway can occur.

EXAMPLE IX- UNSYMMETRICAL FRAME, HINGED ENDED, WITH VERTICAL COLUMNS OF UNEQUAL HEIGHTS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD

(Modified Solution)

It is required to find the moments at B and C and the reactions at A and D in the frame of Fig. 51.

Procedure

Proceed exactly in the same manner as in the conventional solution until the results in Fig. 53 are obtained. Then apply formulae developed in Proposition IV to obtain horizontal reactions.

$$k = \frac{2\left\{\frac{6}{2} \cdot \frac{20}{40} + 1\right\} + \frac{15}{20}}{\frac{15}{20}\left\{1 + 2 \cdot \frac{15}{20}\left(1 + 6 \cdot \frac{15}{40}\right)\right\}} = 1.3047 \qquad \text{Formula (5)}$$

$$H = \frac{5.00 + 1.3047 \times 6.56}{2.3047} = 5.88 \qquad \text{Formula (6)}$$

The vertical reactions of 30.74 at A and 17.26 at D are found by taking moments at D and A respectively of all the external forces acting on the frame which will include the horizontal reaction value of 5.88 determined above.

The moments at B and C are obtained by taking moments at each of those points as shown below.

Final Moments

1. At joint B- $-5.88 \times 20 = -117.6$
2. At joint C- $-5.88 \times 15 = -88.2$

EXAMPLE X- UNSYMMETRICAL FRAME, FIXED ENDED, WITH VERTICAL COLUMNS OF UNEQUAL HEIGHTS AND AN UNSYMMETRICALLY PLACED VERTICAL LOAD (Conventional Solution)

It is required to find the moments at A, B, C, and D, and the reactions at A and D in the frame of Fig. 60.

<u>Procedure</u>

Rotate members AB and CD until they are in a horizontal position as shown in Fig. 61. Proceed with the moment distribution of Fig. 61.

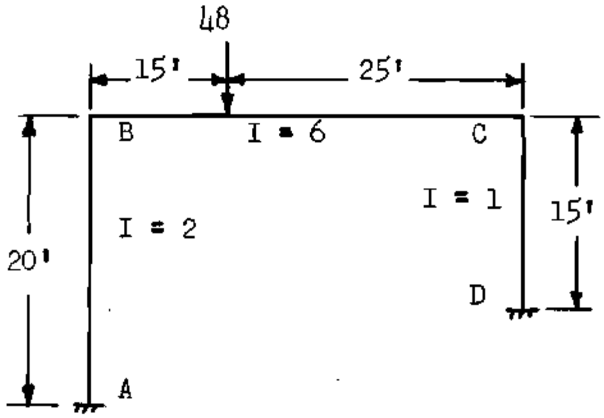

Fig. 60

Kc	A	2/5 B	3/5	9/13 C	4/13	D
F.E.M.			-281.2	168.8		
1st Dist.		112.5	168.7	-116.9	-51.9	
C.O.	56.3		-58.4	84.3		-26.0
2nd Dist.		23.4	35.0	-58.4	-25.9	
C.O.	11.7		-29.2	17.5		-12.9
3rd Dist.		11.7	17.5	-12.1	-5.4	
C.O.	5.8		-6.1	8.8		-2.7
4th Dist.		2.4	3.7	-6.1	-2.7	
C.O.	1.2		-3.0	1.8		-1.4
5th Dist.		1.2	1.8	-1.2	-0.6	
C.O.	0.6		-0.6	0.9		-0.3
6th Dist.		0.2	0.4	-0.6	-0.3	
C.O.	0.1		-0.3	0.2		-0.1
7th Dist.		0.1	0.2	-0.1	-0.1	
Moments	75.7	151.5	-151.5	86.9	-86.9	-43.4

Fig. 61

In Fig. 62 the frame columns are shown as a pair of free bodies with an unbalanced horizontal force of 11.36 - 8.69 = 2.67. An arbitrary force is applied acting horizontally to the right at B which produces fixed end moments of 100.0 in column AB and 88.9 in column CD. Using these fixed end moments, the moment distribution is carried out in Fig. 63.

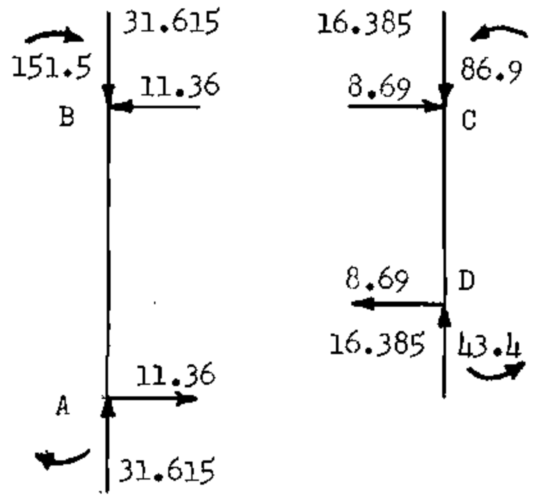

Fig. 62

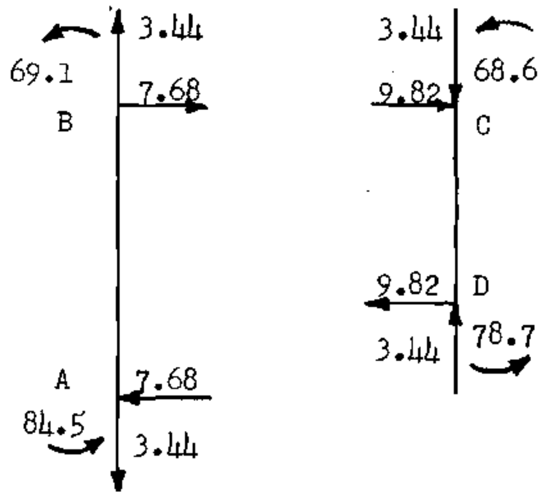

Fig. 64

Kc	A	2/5 B	3/5	9/13 C	4/13	D
F.EM.	-100.0	-100.0			-88.9	-88.9
1st Dist.		40.0	60.0	61.5	27.4	
C.O.	20.0		30.8	30.0		13.7
2nd Dist.		-12.3	-18.5	-20.8	-9.2	
C.O.	-6.2		-10.4	-9.2		-4.6
3rd Dist.		4.2	6.2	6.4	2.8	
C.O.	2.1		3.2	3.1		1.4
4th Dist.		-1.3	-1.9	-2.2	-0.9	
C.O.	-0.6		-1.1	-1.0		-0.5
5th Dist.		0.4	0.7	0.7	0.3	
C.O.	0.2		0.3	0.3		0.1
6th Dist.		-0.1	-0.2	-0.2	-0.1	
Moments	-84.5	-69.1	69.1	68.6	-68.6	-78.7

Fig. 63

The moments and shears produced by the force chosen arbitrarily are shown on the column free bodies of Fig. 64, from which it is seen that this force was 17.50. In order to get the moments and shears due to a force of 2.67, the values obtained in Fig. 64 must be multiplied by the ratio of 2.67 to 17.50.

These become 0.52 for vertical shear, 1.17 for horizontal shear at A, 1.50 for horizontal shear at D, 12.9 for moment at A, 10.5 for moment at B and C, and 12.0 for moment at D with proper regard for signs.

Final Reactions

$H_A = 11.36 - 1.17 = 10.19$ $\qquad$ $H_D = 8.69 + 1.50 = 10.19$

$V_A = 31.615 - 0.52 = 31.095$ $\qquad$ $V_D = 16.385 + 0.52 = 16.905$

Final Moments

AB = 75.7 - 12.9 = 62.8	BA = 151.5 - 10.5 = 141.0
BC = -151.5 + 10.5 = -141.0	CB = 86.9 + 10.5 = 97.4
CD = -86.9 - 10.5 = -97.4	DC = -43.4 - 12.0 = -55.4

The moment diagram for the frame of Fig. 60 is shown in Fig. 65.

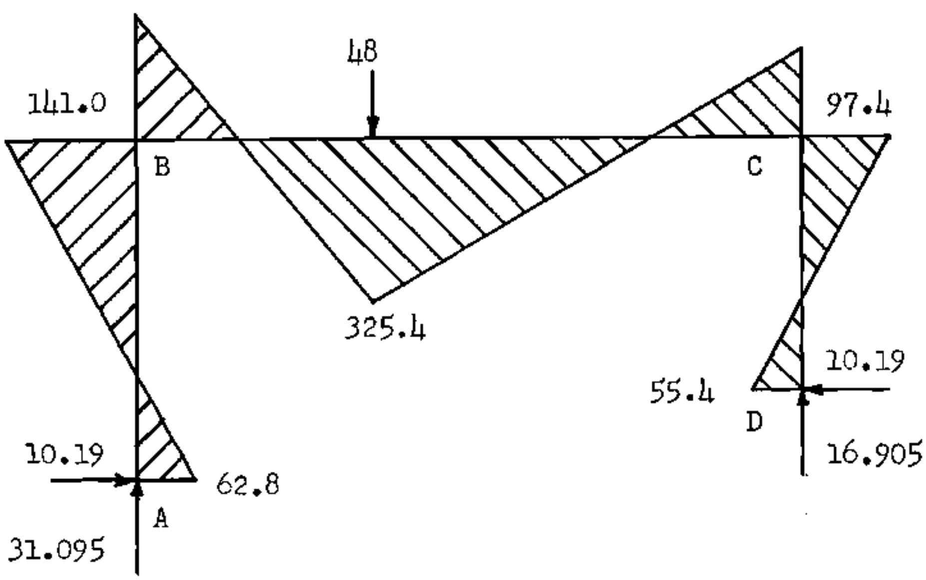

Fig. 65

CONCLUSIONS

The following conclusions can be drawn from this investigation:

1. (Applicable to single span, symmetrical, hinged ended rigid frames with either horizontal or vertical loading.) It is possible to eliminate the second phase of the conventional moment distribution analysis , in which corrections are made for sidesway, and to replace it with some simple formulae to complete the solution. Proofs of these formulae are embodied in this thesis.

2. (Applicable to single span, hinged ended, rigid frames with vertical columns of unequal heights with vertical loading on a horizontal girder which spans from column to column.) It is possible to eliminate the second phase of the conventional moment distribution analysis, in which corrections are made for sidesway, and to replace it with formulae developed in this thesis to complete the solution. The use of these formulae for this unsymmetrical case requires considerable arithmetic and it is questionable whether or not it is an improvement upon the conventional moment distribution analysis. It does have merit in that it may be used as a method of checking solutions obtained by the conventional method.

RECOMMENDATIONS

It was possible in this thesis to present a modified solution for the determination of moments and reactions in single span, symmetrical rigid frames with hinged bases using the moment distribution method. It is believed that a further study of the problem in which the bases are fixed might lead to a simplified or modified solution for that condition also. The problem may be attacked in the same manner as for the hinged base condition to get the actual horizontal reactions. The difficulty apparently lies in being able to determine the values of the moments at the fixed bases.

For the unsymmetrical frame with the vertical columns and hinged bases, it is believed that a good approximation to the actual values can be obtained by using a formula less complicated than the one developed in Proposition IV. It is suggested that an approximately correct formula be obtained for this case in which it is assumed that the girder is perfectly rigid and that the shears developed in the columns are directly proportional to their moments of inertia divided by the cube of their lengths.

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