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Contract#: AFOSR-91-0047   Mod #: AMENDMENT A   Document : GRANT
Prime   #:   Contract entity: GTRC

Subprojects ? : N   CFDA: 12.800
Main project #:   PE #:

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    NEITZEL G P   MECH ENGR   (404)894-3200

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Sponsor amount   New this change   Total to date
     Contract value   0.00   41,670.00
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Title: COMPUTATIONAL STUDIES OF VORTEX BREAKDOWN

                        PROJECT ADMINISTRATION DATA

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Security class (U,C,S,TS) : U   ONR resident rep. is ACO (Y/N): N
Defense priority rating   :   N/A   N/A supplemental sheet
Equipment title vests with: Sponsor   GIT X

Administrative comments -
   AMENDMENT A EXTENDS PERFORMANCE ENDING DATE TO 11/30/91. ALL OTHER TERMS
   AND CONDITIONS REMAIN UNCHANGED.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 06/16/92

Project No. E-25-M73_____   Center No. 10/24-6-R7082-0A0_

Project Director NEITZEL G P_____ School/Lab MECH ENGR_____

Sponsor AIR FORCE/BOLLING AFB, DC_____

Contract/Grant No. AFOSR-91-0047_____ Contract Entity GTRC

Prime Contract No. _____

Title COMPUTATIONAL STUDIES OF VORTEX BREAKDOWN_____

Effective Completion Date 911130 (Performance) 920131 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	920212
Final Report of Inventions and/or Subcontracts	Y	_____
Government Property Inventory & Related Certificate	N	_____
Classified Material Certificate	N	_____
Release and Assignment	N	_____
Other _____	N	_____

Comments_____

Subproject Under Main Project No. _____

Continues Project No. _____

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other _____	N
_____	N

NOTE: Final Patent Questionnaire sent to PDPI.

COMPUTATIONAL STUDIES OF

VORTEX BREAKDOWN

AFOSR 91-0047

November 1, 1990 - November 30, 1991

Final Report

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I. INTRODUCTION

"Vortex breakdown" is the term used to describe a phenomenon in which sudden changes are observed in the structure of vortex cores embedded in flow fields with velocity gradients along the axis of the vortex. Among these abrupt changes is the appearance of an on-axis stagnation point which is followed by a separation bubble embedded in the vortex or spiraling of the vortex core. Vortex breakdown is often observed in flows of technological interest, such as in the trailing vortices on a delta wing at a significant angle of attack to the oncoming free stream or in swirling flow through pipes. In the second instance, combustors often employ vortex breakdown to fix the location of ignition. Although the phenomenon of vortex breakdown has been the subject of numerous experimental and computational studies for several years, no consensus has been reached as to the underlying causes.

A significant experimental study of a special type of vortex breakdown was undertaken by Escudier (1984) which has served as a benchmark in flow visualization work concerning the phenomenon. Escudier's experiment involved an enclosed circular cylinder of height H and radius R completely filled with a viscous fluid of kinematic viscosity ν. An endwall of the cylinder was then rotated at a fixed angular speed. The two governing dimensionless parameters are a Reynolds Number ($Re = \Omega R^2/\nu$) and the cylinder aspect ratio ($\alpha = H/R$). As pointed out by Greenspan (1990), there are other dimensionless parameters which can be used to characterize such flows. The first of these is the Ekman number defined as $E = \nu/\Omega L^2$, where L is a characteristic length of the flow. The Ekman number is a measure of the relative effects of viscous forces and Coriolis forces. The second dimensionless parameter suggested by Greenspan is the Rossby number defined as $Ro = W/L\Omega$, where W is an axial velocity. The Rossby number is an indication of the relative sizes of the axial velocity and the

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rotational velocity of the flow and has been used by some investigators along with the Reynolds number to establish a criterion for the occurrence of vortex breakdown. Such vortex breakdown criteria will be discussed shortly.

Physically, the formation of an Ekman layer on the lower, rotating endwall occurs with an accompanying "Ekman suction", meaning that low-momentum fluid is drawn from the interior of the flow to be replenished by fluid with higher angular momentum from locations closer to the container sidewall. The thickness of the Ekman layer formed is $O(E^{1/2})$. Ekman layers form within just a few radians of angular motions of the cylinder endwall. Stewartson layers of thicknesses $O(E^{1/3})$ and $O(E^{1/4})$ form along the cylinder sidewall.

Escudier found that this flow exhibited a characteristic on-axis stagnation point, along with an accompanying region of flow reversal, for certain combinations of Re and α. Escudier produced a breakdown map in parameter space showing regions where one or more breakdowns were observed. An important ramification of Escudier's work is that the regions of reversed flow observed were found to be *axisymmetric* and *steady* over the range of parameters examined.

The problem investigated by Escudier is an attractive one from a computational standpoint. The system is mathematically well-posed and as such requires no "ad hoc" assumptions of boundary conditions. More importantly, however, Escudier established experimentally that this flow does exhibit axisymmetric vortex breakdown in an environment free of nonaxisymmetric disturbances. A difficulty arises in that, for fixed geometry, there is only a single dynamical parameter (the rotational Re), making it impossible to independently vary core vorticity and axial suction. This dilemma served as partial motivation for the current work.

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Several numerical investigations of the problem of vortex breakdown as it occurs in an enclosed circular cylinder have been performed. Among these are works by Lugt & Abboud (1987), Lopez (1990), and Brown & Lopez (1990). These extended earlier studies by Lugt & Haussling (1973), Dijkstra & Heijst (1983) and other investigators who conducted similar studies in a more restrictive parameter range. The study by Lugt & Abboud includes temperature effects as cases with varying Rayleigh numbers were considered. Brown & Lopez (1990) have undertaken the development of a theory of the underlying physical mechanism of the breakdown process.

Several surveys have appeared covering not only the case of vortex breakdown as it occurs in the closed flow mentioned above, but also for the case of swirling flow in a tube. Among the more notable are reviews by Hall (1972), Leibovich (1978, 1983), and Escudier (1988). Leibovich discusses, among other things, the admissibility of axisymmetric solutions to the Navier-Stokes equations, criticality conditions, and various stability criteria of the phenomenon for several flow profiles. Leibovich (1978) also reviews experimental and computational work on the subject.

Brown & Lopez (1990) have studied the problem of vortex breakdown in an enclosed circular cylinder using numerical experimentation, solving the axisymmetric Navier-Stokes equations. Their observations indicate that the flow remained axisymmetric and laminar throughout the flow regime examined. For the cases examined by Brown & Lopez, excellent agreement was seen with the experimental results of Escudier (1984).

It should be stated that the phenomenon of vortex breakdown is not well understood physically. There has been much debate over whether the observed stagnation and consequent recirculation of the flow is a manifestation of an instability or is the result of some criticality

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condition, i.e. similar to a "hydraulic jump." Escudier (1987) points out that much evidence exists in favor of the latter, i.e. vortex breakdown is *not* the result of an instability. Escudier contends that the upstream flow in such cases is, at best, only marginally stable and that vortex flows are more stable to axisymmetric disturbances. Hence, the observed phenomenon cannot be the result of the instability of the approach flow. Secondly, the breakdown has been seen to appear quite suddenly rather than growing with time or spatially, which would be more characteristic of an instability.

Adding to the controversy is the fact that more than one type of breakdown has been observed experimentally. There is discussion concerning whether or not the so-called axisymmetric bubble-type breakdown observed by Escudier (1984) in his experiments is distinct from the spiral-type breakdown observed by Lambourne & Bryer (1962) for delta-wing flows and by Escudier & Zehnder (1982) for swirling pipe flows. Escudier (1987) maintains that the bubble-type breakdown is the principal form and that the spiral form results due to the instability of the bubble form. Neitzel (1988) suggests that the two forms *may* not be independent but rather the differences observed may be an artifact of the visualization process. Brown & Lopez (1990) suggest that for the confined swirling flow at the values of Re and α examined, the spiral form of breakdown observed in other geometries is perhaps not evident because the downstream boundary condition is not free, i.e. the rotating endwall gives rise to the condition $v = 0$ at $r = 0$, thereby fixing the vortex core to the center of the downstream boundary.

Leibovich (1978), who has contributed much to the study of the subject, maintains that, while the Navier-Stokes equations do permit solutions which are axially symmetric and contain regions of closed stream surfaces, the experimental evidence shows that these solutions must also be unstable to nonaxisymmetric disturbances. Leibovich (1984) goes on

to say that, while vortex breakdown may or may not be the result of an instability, the instabilities help to shape the global structure , and hence, the aerodynamic characteristics of the flow.

II. PROPOSED BREAKDOWN CRITERIA

Some criteria for the occurrence of vortex breakdown have been proposed by different investigators. One such proposal has been put forth by Spall, Gatski, and Grosch (1987), which is related to wing-tip vortices and is dependent on the definition of an appropriate Rossby number. They discuss the differing types of explanations for the occurrence of vortex breakdown, including hydrodynamic instability, a phenomenon analogous to boundary-layer separation, and the possibility of critical states. The stability argument is again discredited because, as pointed out by Leibovich (1978), breakdown may occur with no sign of instability, and also flows that are unstable may not experience vortex breakdown. The idea proposed by Hall (1972) is that the occurrence of vortex breakdown is the result of a failure of the quasicylindrical approximation, meaning that axial gradients in the vortex become large compared to radial gradients. As pointed out by Spall, *et al.*, this is analogous to the occurrence of boundary layer separation, indicating that the boundary layer equations have failed.

The route pursued by Spall, *et al.* (1987) is that of critical states. This argument is based on the realization that a flow with a significant swirl component may sustain an axisymmetric standing wave. They point out that the significant nondimensional parameter is the Rossby number, and that the parameter space of interest is Rossby number-Reynolds number space. Their criterion is that, for flows with $Re > 100$, the critical Rossby number is $Ro = 0.65$. They point out that for smaller values of Re, the critical Rossby number is reduced due to increased viscous damping effects. Spall, Gatski, and Grosch also review previous theoretical and computational studies of vortex breakdown. Of particular interest is

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their discussion of the importance in numerical work of selecting the "appropriate" upstream boundary and initial conditions.

Brown & Lopez (1990) note that the specification of both upstream and downstream boundary conditions is a major factor affecting the results obtained in a numerical simulation of the vortex-breakdown problem for swirling pipe flows. They maintain that the upstream condition is the one of the greatest importance. Since only the "test section" of the flow of interest may be faithfully simulated due to limitations on the number of grid points which may be employed, the boundary condition specified must be that the upstream flow is steady and locally cylindrical. Experiments conducted by Faler & Leibovich (1978), among others, indicates that this is a reasonable assumption for the flow region well upstream of the location of breakdown.

Downstream and radial conditions represent less of a concern but should also be addressed, as indicated by Brown & Lopez (1990). The downstream condition for swirling pipe flows is set so as not to affect the simulation of the flow in the test section. This is equivalent to assuming uniform outflow and implies that no flow enters the domain through this boundary. When a bubble forms near enough to the downstream boundary, the condition of uniform outflow is violated. A somewhat crude remedy would be to move the location of the exit condition further downstream from the location of the breakdown bubble.

Acceptable numerical results for the swirling pipe flow may be obtained when the radial boundary condition is set at a location at least two core diameters from the region of breakdown. Brown & Lopez (1990) report that, for their numerical simulation of confined swirling flow, the radial boundary was placed at a distance equal to five core radii from the cylinder axis. Enforcement of this boundary condition places a limiting streamline in the flow domain.

The breakdown mechanism proposed by Brown & Lopez (1990) is related to the production of vorticity through stretching and tilting of vortex lines. They contend that it is the production of a negative component of azimuthal vorticity that is responsible for on-axis flow stagnation. They argue that such a negative component of vorticity is produced initially by the divergence of axial vortex lines. This divergence can be brought about by an adverse pressure gradient, as in the external flow over a delta-wing, or by inertial rebound, as in the case of flow in a cylindrical container with a rotating lid. Once the lines begin to diverge, an inviscid positive feedback mechanism causes divergence to continue until the necessary component of vorticity is produced to bring the axial flow to rest. Brown & Lopez have also developed a vortex breakdown criterion for inviscid flows. This criterion states that a necessary condition for vortex breakdown to occur is that the tangent of the helix angle of the velocity exceed that of the vorticity on some stream surface.

Brown and Lopez (1990, Part 2) have developed an inviscid breakdown criterion for the Escudier problem as described below. They use as a starting point the following equation involving streamfunction Ψ, circulation Γ, and total head $\mathcal{H}$;

$$\eta = (\Gamma/r)\, d\Gamma/d\Psi - d\mathcal{H}/d\Psi,$$

where η is the azimuthal component of vorticity and head is defined as

$$\mathcal{H} = p/\rho + 1/2(u^2 + v^2 + w^2).$$

They have proceeded to develop a relationship between the azimuthal component of vorticity, the stream surface radius σ and the helix angles γ and β of velocity and vorticity, respectively. This relationship is given as

$$\eta = \gamma_0 \zeta_0 (\sigma_0/\sigma - \sigma/\sigma_0) \quad \text{for } \eta_0 = 0,$$

$$\text{or} \quad \eta/\eta_0 = \sigma_0/\sigma(\gamma_0/\beta_0) - \sigma/\sigma_0(\gamma_0/\beta_0 - 1) \quad \text{for } \eta_0 \neq 0,$$

where the subscripted values indicate a reference state. Specifically, the helix angle tangents are defined as $\gamma_0 = v_0/w_0$ and $\beta_0 = \eta_0/\zeta_0$, where v and w represent the azimuthal and axial components of velocity, respectively, and η and ζ represent the azimuthal and axial components, respectively, of vorticity. Again, the subscripted quantities indicate values at a reference state. Thus, it is seen that for $\eta_0 > 0$, the only way that negative η can be developed is that $\alpha_0/\beta_0 > 1$. When σ/σ_0 is increased sufficiently, meaning that the stream surface has diverged sufficiently from the reference value, the axial flow is brought to rest.

The proposed Brown & Lopez breakdown mechanism has served to motivate the current study. One of the objectives of this work is to test this proposed mechanism using numerical simulation. The flow examined differs from that studied by Escudier (1984) in that, in addition to the one rotating endwall, the remainder of the cylinder is rotated at a smaller, fixed angular velocity. This sidewall rotation allows independent control of axial suction, related to the differential speed, and core vorticity. Independent rotation of the sidewall facilitates introduction of yet another parameter (another Re) with which to examine this flow. Several flow cases corresponding to various combinations of the above dimensionless parameters are examined. The proposed Brown & Lopez breakdown mechanism is tested quantitatively by comparison with the numerical results obtained here.

III. THE MODIFIED ESCUDIER PROBLEM AND NUMERICAL APPROACH

The original problem treated experimentally by Escudier (1984) and numerically by others, as chronicled above, "suffers" from having but a single dynamical dimensionless parameter, the Reynolds number

$$Re = \Omega R^2 / \nu,$$

so that independent control of swirl and axial speed is impossible. The proposed modification which was investigated in this research consisted of rotating one endwall with speed Ω_1, as in the Escudier problem, and rotating the other endwall and sidewall together with speed $\Omega_2 < \Omega_1$. The sidewall and endwall rotating together with Ω_2 serve to modify the overall vorticity in the flow, while the differential speed, to a degree, controls the axial flow. A sketch of the geometry is shown in Figure 1.

In addition to the aspect ratio α defined above, there are now *two* dynamical parameters which may be constructed in a number of ways. Since most of the computations which we have performed were done for different, but *fixed*, values of $\Delta\Omega = \Omega_1 - \Omega_2$, we shall present results using the dimensionless parameters

$$Re_\Delta = R^2 \Delta\Omega / \nu,$$

a Reynolds number, and

$$\omega = \Omega_2 / \Omega_1,$$

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a speed ratio with the requirement that $0 \leq \omega < 1$. Notice that, *for a fixed value* of Re_Δ, the limit $\omega = 0$ corresponds to the standard Escudier problem with $Re_\Delta = Re$, while the limit $\omega \to 1$ corresponds to $Re = (1 - \omega)^{-1} Re_\Delta \to \infty$.

One of the interesting possibilities investigated in this research was the occurrence of a situation which we term *incipient vortex breakdown*. For a value of Re_Δ which exhibits a single (steady-state) breakdown bubble for $\omega = 0$ (the Escudier case), as the speed ratio ω is increased from zero, the bubble shrinks in size until a value of ω is reached at which it disappears entirely. This is the condition of incipient vortex breakdown. Such a state is useful indeed for testing any breakdown hypothesis, since the flow upstream of the eventual bubble is ripe for breakdown to occur, yet unaffected by the presence of on-axis stagnation.

The numerical solution of the governing unsteady, axisymmetric, Navier-Stokes equations was accomplished using a modified version of a code which has previously demonstrated success in calculating the Escudier case (Neitzel 1988) as well as problems in hydrodynamic stability (Neitzel 1984, Chen, Neitzel & Jankowski 1987) and spin-up (Kitchens 1980). The code solves the equations in stream-function/vorticity/circulation form employing a predictor-corrector multiple-iteration (PCMI) procedure for the vorticity and circulation equations, treating the radial direction implicitly and sweeping in the axial direction until convergence is attained at each time step. The stream function equation is solved at each iteration of the vorticity solution using successive line over-relaxation (SLOR) and the results are used to update the vorticity boundary conditions (Roache 1982). In addition to modifications necessary to implement the boundary conditions of the modified Escudier problem, a code had to be constructed to compute the pressure for use in calculating the total head of Brown & Lopez (1990). This was done by solving the pressure-Poisson equation derived from the momentum and continuity equations using a standard procedure outlined by

Roache. The details of the equations and solution techniques will not be presented here, but may be found in the papers referenced above and in Watson (1992).

IV. RESULTS AND DISCUSSION

Since our primary motivation was to examine the Brown & Lopez (1990) breakdown hypothesis, we confined our attention to a few cases which exhibit a *single* breakdown bubble at steady state in the Escudier ($\omega = 0$) problem. Experiments by Escudier (1984) yielded instances of multiple bubbles and these have also been computed by Lopez (1990), but these were not of interest in the present study. A case examined experimentally by Escudier and numerically by Neitzel (1988) corresponds to $Re_\Lambda = 1748$, $\alpha = 1.75$, $\omega = 0$. Figure 2 shows a sequence of steady-state meridional streamline patterns for this Re_Λ and α as ω is increased from zero, through the point of incipient vortex breakdown, and beyond. The axis of symmetry corresponds to the left-hand boundary of each plot and the faster-rotating endwall is at the bottom. Notice that the waviness (and divergence) of the streamlines upstream of the bubble location exists even at the point of incipient breakdown, yet is diminished greatly as the value of ω is increased beyond $\omega_{incipient}$. The divergence of *vortex* lines, in the theory of Brown & Lopez, is responsible for the appearance of a negative component of azimuthal vorticity responsible for breakdown.

As one measure of bubble size, the bubble radius (scaled on R) was computed from the stream-function data and is plotted as a function of the speed ratio ω for this case in Figure 3. Although the radius appears to be monotonically decreasing with ω, similar computations for other cases indicate that the bubble may actually first *increase* in size, reaching a maximum, before beginning to decrease to the point of incipient breakdown. Such a situation is shown in Figures 4 and 5 for the case $Re_\Lambda = 1500$, $\alpha = 2$, although the increase in size is too small to be distinguishable from the streamline plots.

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One of the motivations for the modified Escudier problem was to allow for independent control of the axial velocity and vorticity in the flow. Since the boundary condition applied to the meridional-plane stream function ψ at solid surfaces is $\psi = 0$, and since differences in stream function are related to volume flow rates between streamlines, the value ψ_{max} which is attained at the center of the large circulation is a measure of the axial pumping provided by the differential rotation $\Delta\Omega$. A plot of ψ_{max} (scaled by $R^3\Delta\Omega$) as a function of ω for the case depicted in Figure 2 is shown in Figure 6. The fact that ψ_{max} is nearly independent of speed ratio confirms the suspicion that the axial flow is dominated by $\Delta\Omega$.

The influence of aspect ratio on the speed ratio for incipient breakdown is illustrated in Figure 7, which shows results for $\omega_{incipient}$ as a function of α for $Re_\Delta = 1500$. For the limited results available, $\omega_{incipient}$ increases sharply with increasing aspect ratio, i.e., the cylindrical sidewall and attached lid must be rotated at a higher rate for larger aspect ratios to cause the bubble to disappear.

As mentioned in section II, Brown & Lopez (1990) have developed a breakdown criterion which states that a necessary condition for vortex breakdown to occur is that the helix angle, γ, of velocity exceed that, β, of the vorticity on some stream surface in the flow. The difference $\gamma - \beta$ is computed from the numerical solutions as

$$\gamma - \beta = v/w - (u_z - w_r)/(v_r - v/r).$$

We have chosen to present the results for $\gamma - \beta$ not along a stream surface, but along an axial line which is one grid node away from the axis $r = 0$. These are shown in Figure 8 for the parameter values of the case in Figure 2. For every case with a bubble, a positive value of $\gamma -$

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β is noted at a position which nearly coincides with the location of the upper edge of the bubble. For the case of incipient breakdown, although there is upstream divergence of the flow, the helix-angle difference remains negative along the entire z-axis. The criterion of Brown & Lopez (1990) asserts that a positive value occurring *somewhere* for this difference is a *necessary* criterion for a breakdown. It seems logical that, to be useful as a predictive tool, this sign change should occur immediately upstream of the eventually appearing bubble. For the cases in Figures 2 and 8, this does not always appear to be the case. It would also have been desirable to observe a sign change for the case of incipient vortex breakdown. However, the claim of Brown & Lopez that this sign change is a necessary condition for breakdown does appear to be true. More study of this aspect of the problem seems warranted.

In summary, the modified Escudier problem seems well suited for the evaluation of vortex-breakdown criteria. The independence provided by the additional dimensionless parameter and the relative ease with which cases of incipient vortex breakdown can be generated make it an attractive model of the vortex-breakdown process. Numerical experimentation is unhampered by the difficulties associated with computing flows in *open* systems, such as swirling flow through pipes of trailing vortices above delta wings. In these systems, the application of *ad hoc* upstream and downstream boundary conditions is routine, rendering the results suspect. Computational schemes which do not rely upon such boundary conditions are needed to be able to treat these problems of more practical relevance.

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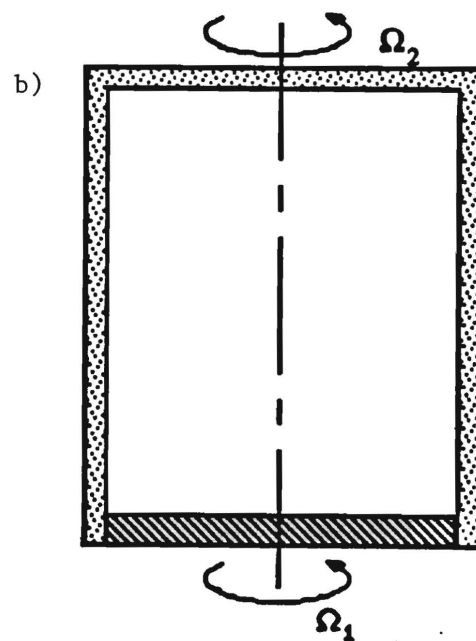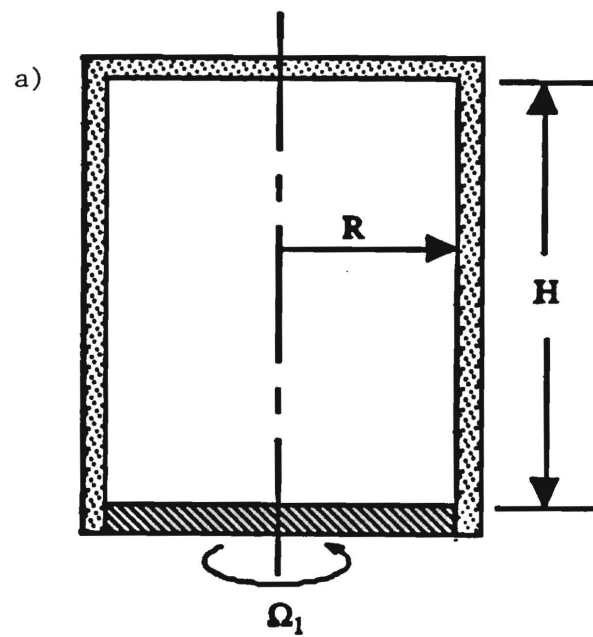
Figure 1.   Sketch of the flow domain with indicated boundary motions: a) standard Escudier experiment; b) modified Escudier problem.
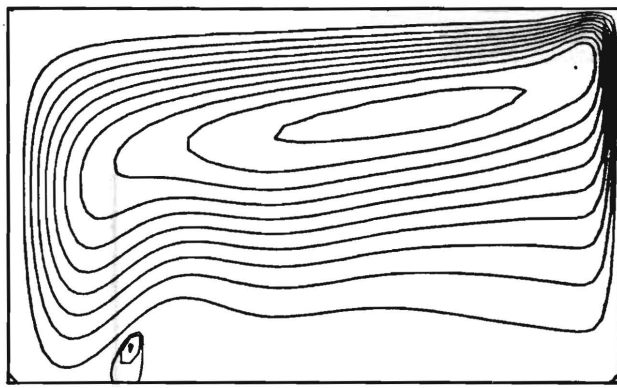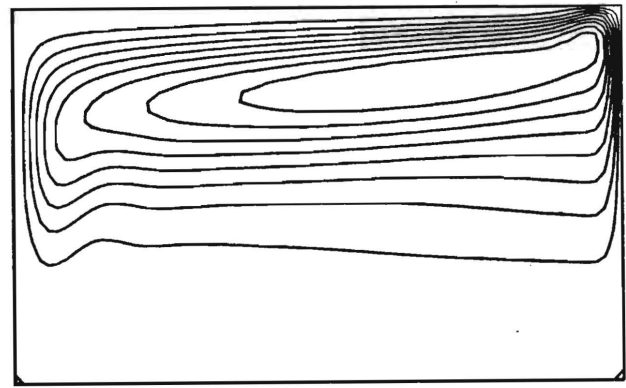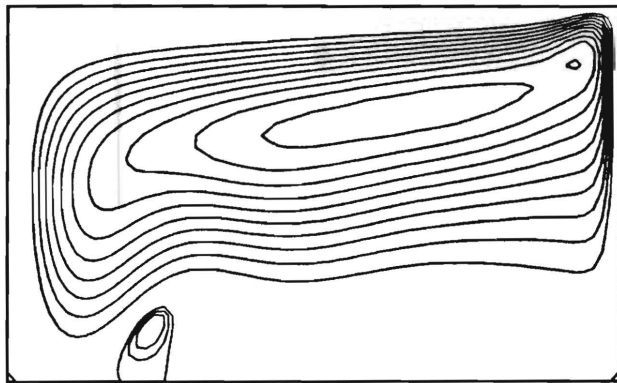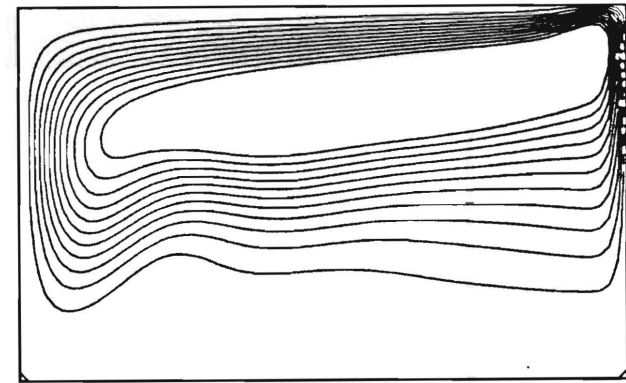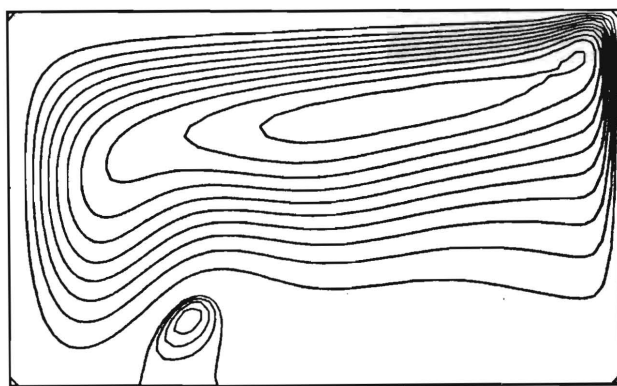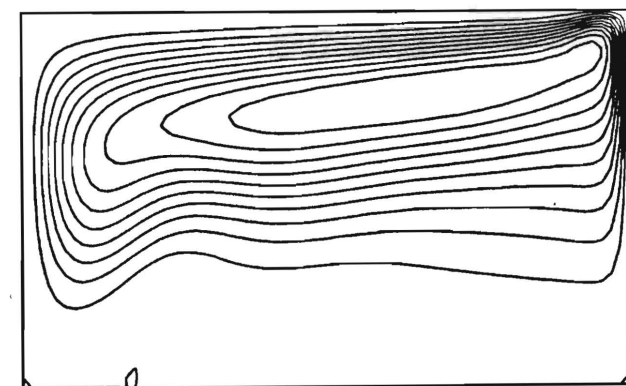
Figure 2. Selected steady-state meridional-streamline plots for the cases $Re_\Delta = 1748$, $\alpha = 1.75$ and: a) $\omega = 0$; b) $\omega = 0.0124$; c) $\omega = 0.0207$; d) $\omega = 0.0256$; e) $\omega = 0.0262$ (incipient vortex breakdown); f) $\omega = 0.1260$. Streamline contour increments differ inside and outside of the bubble.
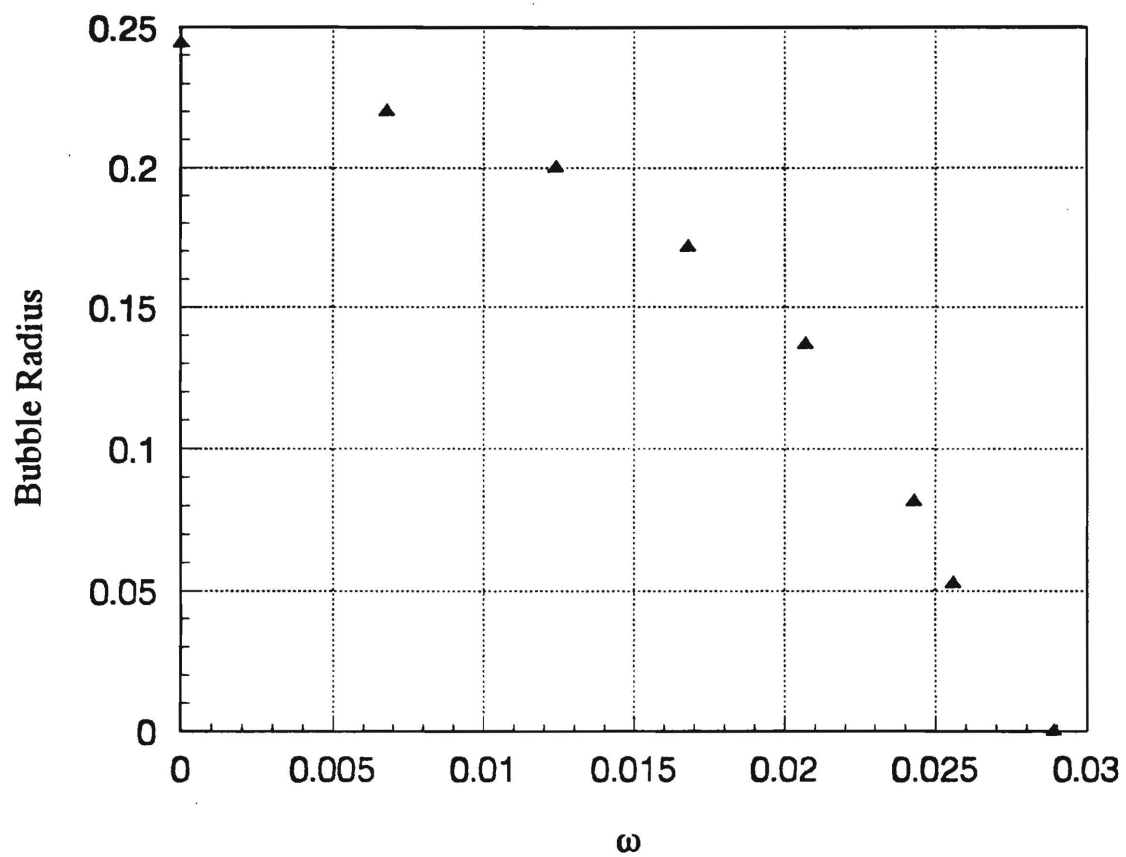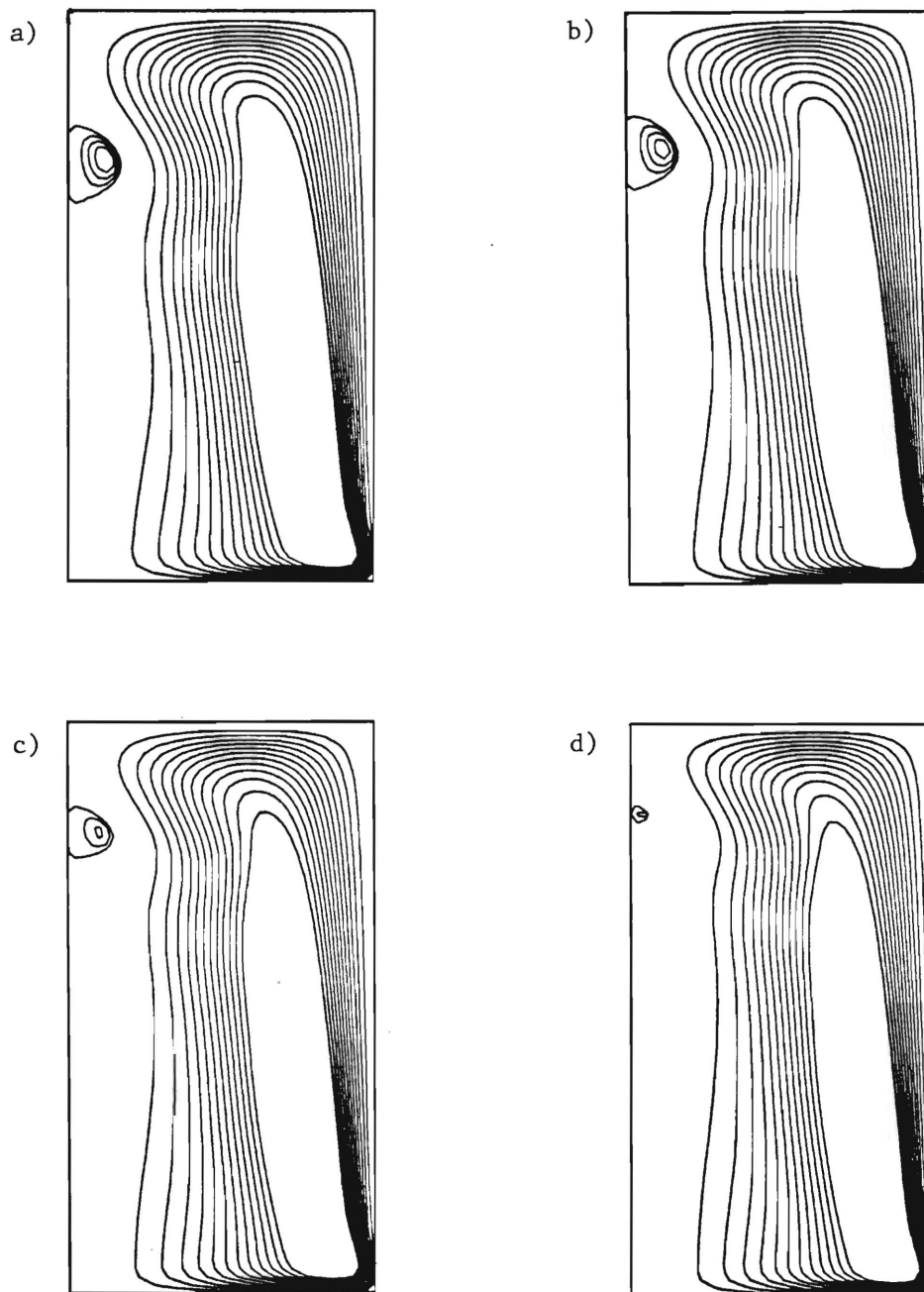
Figure 3   Dimensionless bubble radius versus ω for $Re_\Delta = 1748$ and $\alpha = 1.75$.

Figure 4.   Selected steady-state meridional-streamline plots for the cases $Re_\Delta = 1500$, $\alpha = 2$
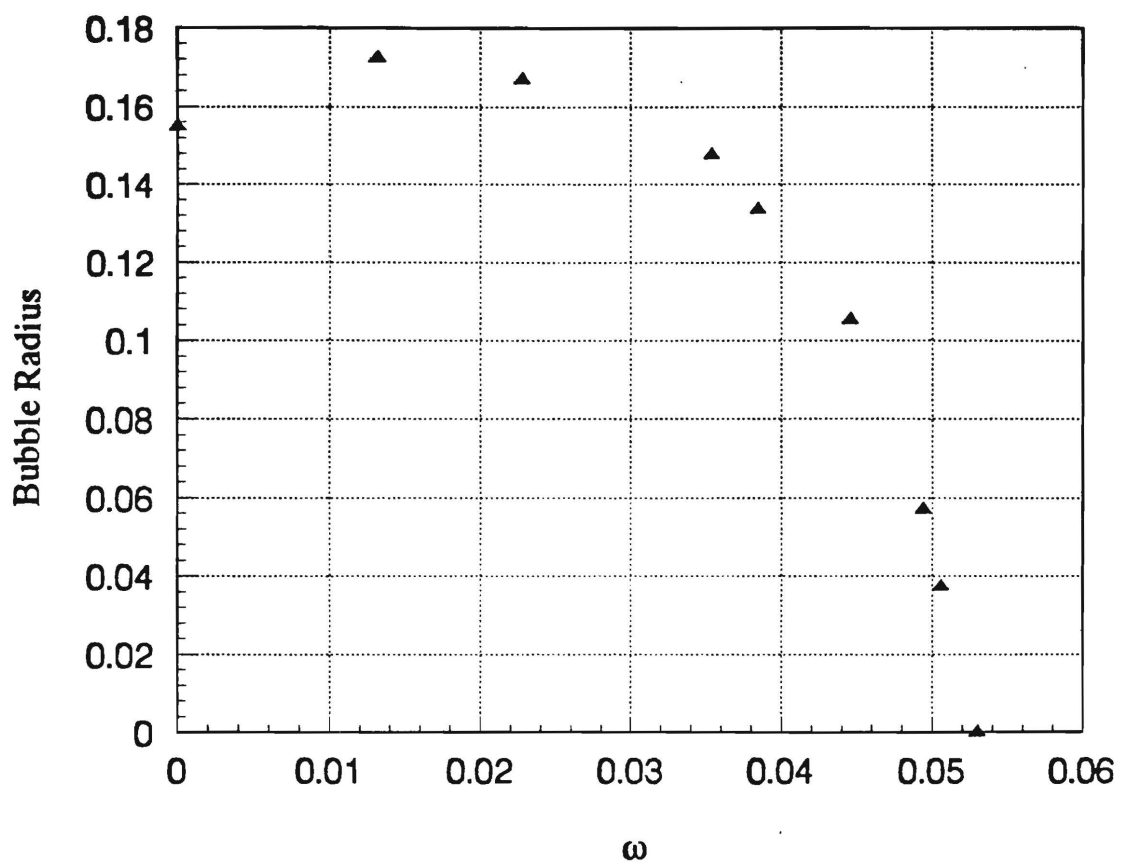and: a) $\omega = 0.0132$; b) $\omega = 0.0196$; c) $\omega = 0.0354$; d) $\omega = 0.0494$.

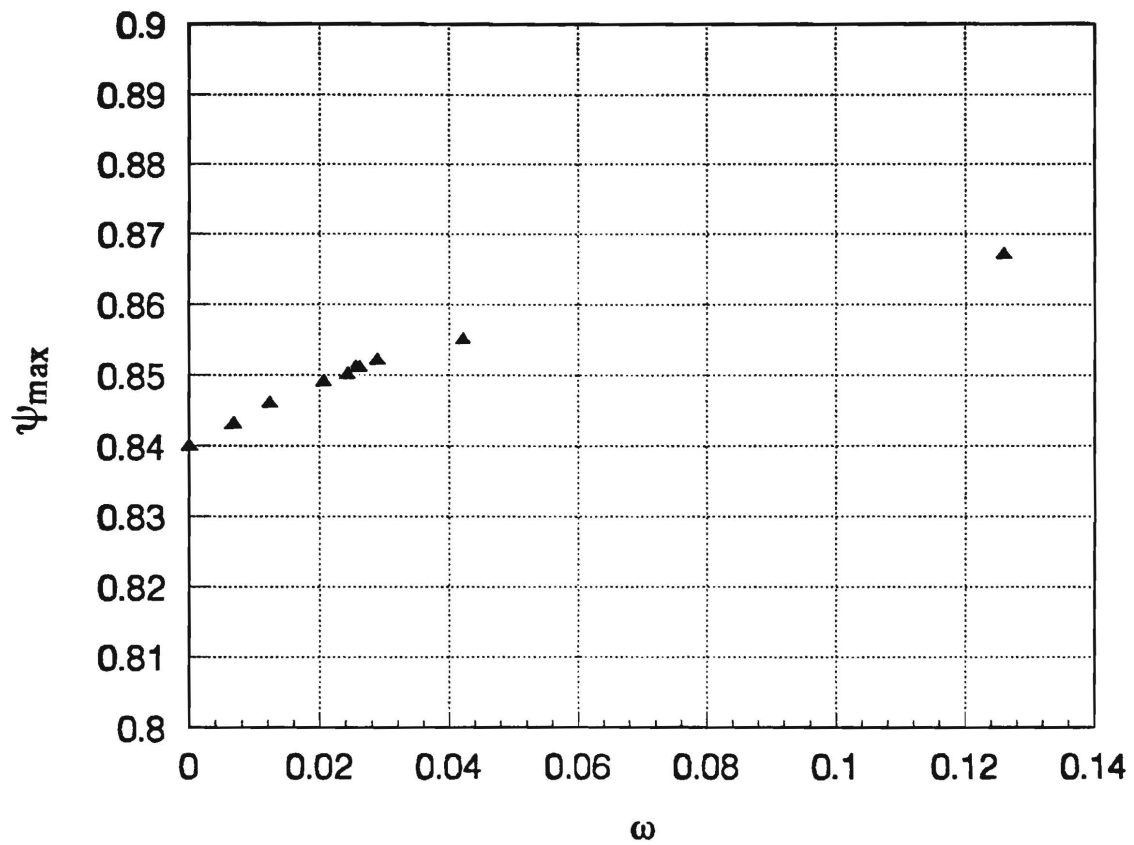Figure 5.   Dimensionless bubble radius versus ω for $Re_\Delta = 1500$ and $\alpha = 2$.

Figure 6   Variation of axial volume flow rate, measured by ψ_{max}, with speed ratio ω. Flow parameters are identical to those in Figures 2 and 3.
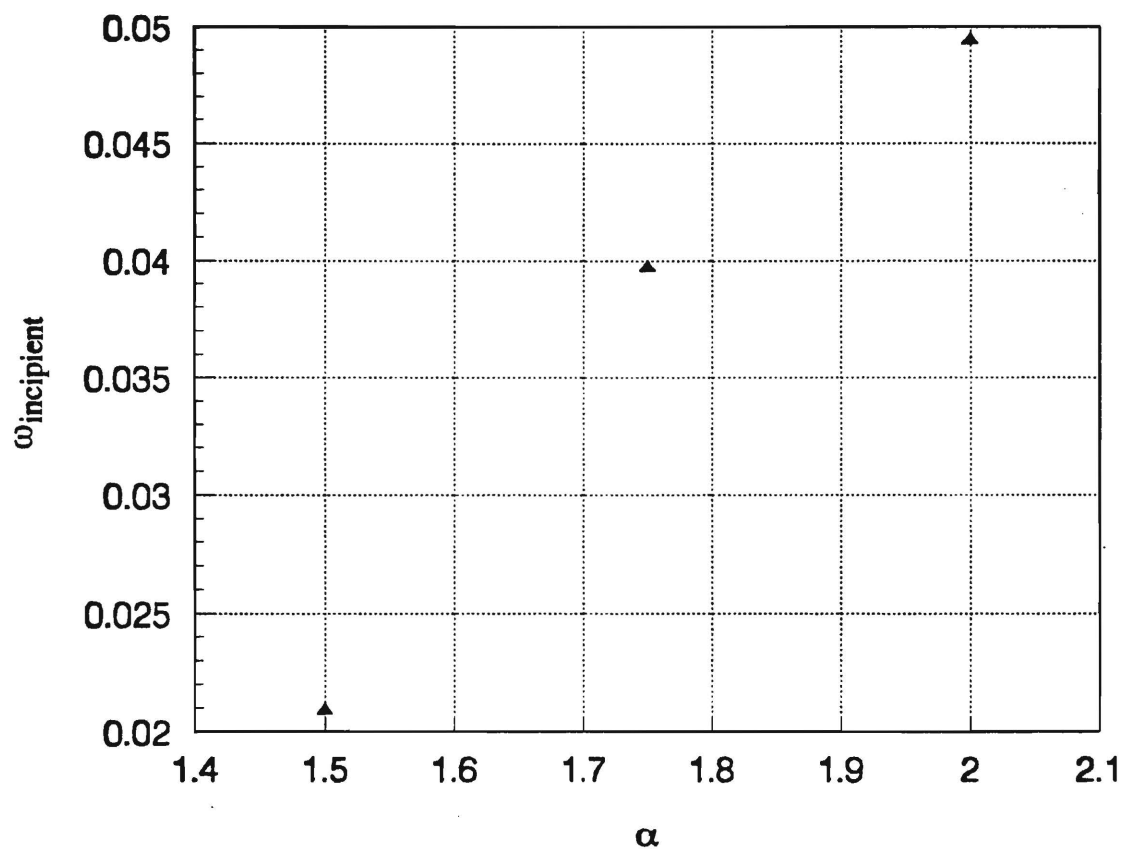
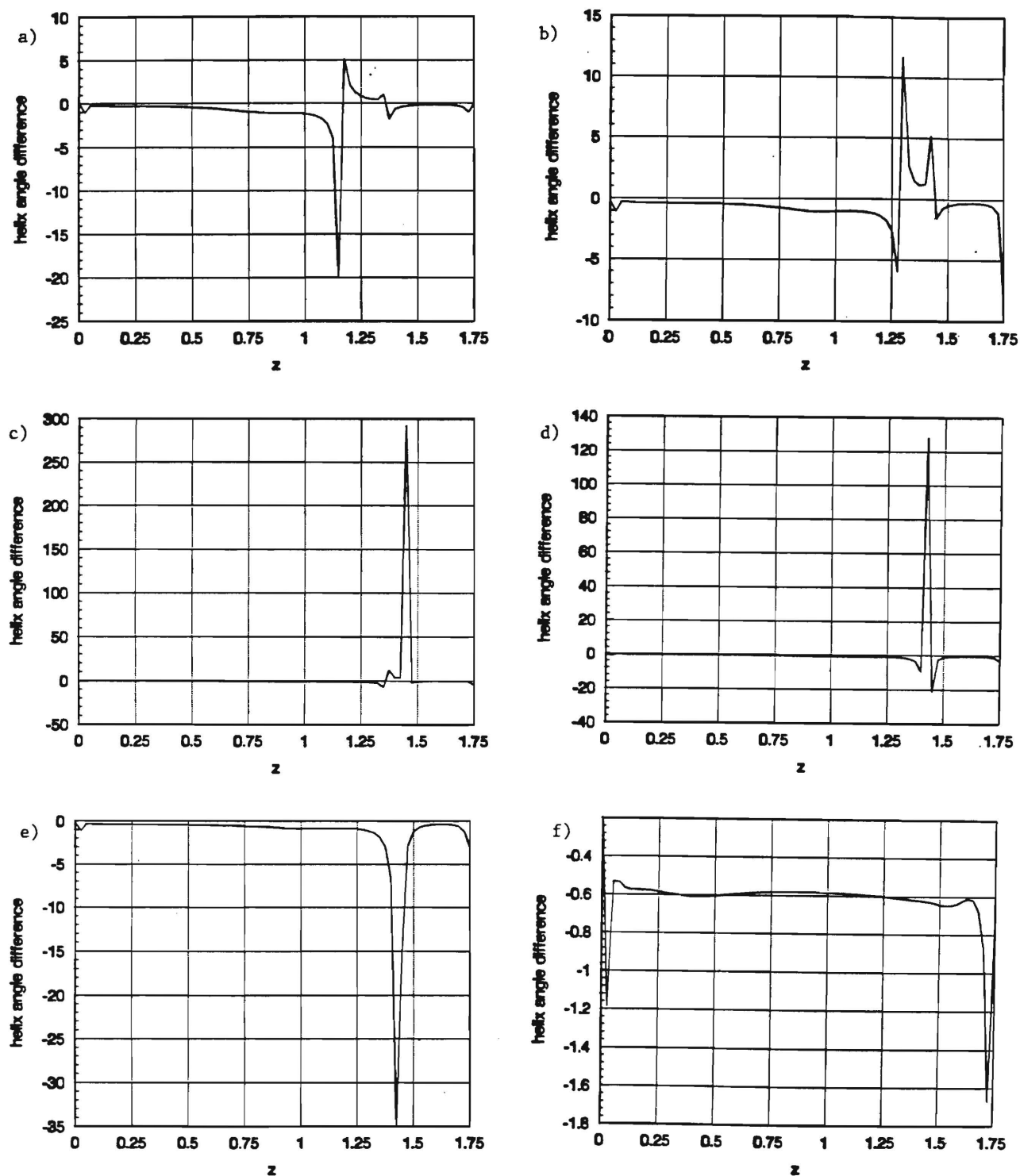Figure 7. Speed ratio $\omega_{incipient}$ for incipient vortex breakdown versus aspect ratio for $Re_{\Delta} = 1500$.

Figure 8. Helix angle difference ($\gamma - \beta$) as a function of axial distance (at $r = 0.025$) for the case shown in Figure 2: a) $\omega = 0$; b) $\omega = 0.0124$; c) $\omega = 0.0207$; d) $\omega = 0.0256$; e) $\omega = 0.0262$ (incipient vortex breakdown); f) $\omega = 0.1260$.