

"In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

---

HEAT TRANSFER IN PULSATING FLOW

A THESIS

Presented to  
the Faculty of the Graduate Division  
by  
Roy Allen Wells

In Partial Fulfillment  
of the Requirements for the Degree  
Master of Science in Mechanical Engineering

Georgia Institute of Technology

June, 1960

328  
127

HEAT TRANSFER IN PULSATING FLOW

Approved:

\_\_\_\_\_  
Thesis Advisor

\_\_\_\_\_  
\_\_\_\_\_  
Date Approved by Chairman June 10, 1960

## ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to Dr. C. W. Gorton for the suggestion of the problem and for his guidance and encouragement during the course of the work, and to Dr. T. W. Jackson and Dr. H. C. Ward for their assistance in the presentation. He also wishes to gratefully acknowledge the assistance of his wife, Evelyn, in the preparation of this work.

## TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS . . . . .	ii
LIST OF TABLES . . . . .	iv
LIST OF ILLUSTRATIONS . . . . .	v
LIST OF SYMBOLS . . . . .	vi
SUMMARY . . . . .	vii
Chapter	
I. INTRODUCTION . . . . .	1
II. EQUIPMENT . . . . .	7
III. PROCEDURE . . . . .	13
IV. RESULTS . . . . .	15
V. CONCLUSIONS AND RECOMMENDATIONS . . . . .	20
APPENDIX I	
ANALYTICAL ANALYSIS OF HEAT TRANSFER IN PULSATING FLOW . .	21
APPENDIX II	
VELOCITY VARIATION DATA AND SAMPLE CALCULATIONS . . . . .	25
APPENDIX III	
DETAILS OF ROTATING VALVE AND FLAT PLATE . . . . .	42
REFERENCES . . . . .	45

## LIST OF TABLES

Table	Page
1. Heat Transfer Results . . . . .	26
2. Variation of Velocity Pressure with Time - $\Delta U/U_{\infty \text{ steady}} = 0.5$ . . . . .	27
3. Variation of Velocity Pressure with Time - $\Delta U/U_{\infty \text{ steady}} = 1.4$ . . . . .	28
4. Variation of Velocity with Time - $\Delta U/U_{\infty \text{ steady}} = 0.5$ . . . . .	31
5. Variation of Velocity with Time - $\Delta U/U_{\infty \text{ steady}} = 1.4$ . . . . .	32

## LIST OF ILLUSTRATIONS

Figure	Page
1. Schematic Diagram of Equipment . . . . .	11
2. Variation of $\bar{h}/h_{\text{steady}}$ with Frequency . . . . .	19
3. Variation of Velocity with Time Low Amplitude - 92 Cycles per Minute . . . . .	35
4. Variation of Velocity with Time Low Amplitude - 245 Cycles per Minute . . . . .	36
5. Variation of Velocity with Time High Amplitude - 92 Cycles per Minute . . . . .	37
6. Variation of Velocity with Time High Amplitude - 240 Cycles per Minute . . . . .	38
7. Variation of Velocity with Time High Amplitude - 386 Cycles per Minute . . . . .	39
8. Details of Plate Construction . . . . .	43
9. Rotating Valve Design . . . . .	44

## LIST OF SYMBOLS

$c_p$	specific heat
$d$	inside pipe diameter
$h_x$	local heat transfer coefficient
$h_{\text{steady}}$	average heat transfer coefficient without pulsations
$\bar{h}$	average heat transfer coefficient with pulsations
$k$	thermal conductivity
$Nu$	$\frac{h d}{k}$ - Nusselt number without pulsations
$\overline{Nu}$	Nusselt number with pulsations
$Re$	$\frac{U_{\infty \text{ steady}} d}{\mu}$ - Reynolds number without pulsations
$\overline{Re}$	Reynolds number with pulsations - based on average mass flow rate
$U_{\infty}$	Free stream velocity at a particular time with pulsations
$U_{\infty \text{ steady}}$	Free stream velocity without pulsations
$\Delta U$	Total change in velocity during one pulsation
$\mu$	dynamic viscosity
$\rho$	density



## SUMMARY

The purpose of this investigation was to determine the effect of pulsations in a turbulent airstream upon the average heat transfer coefficient from an electrically heated flat plate placed in the airstream.

For these experiments air was forced through a six-inch steel pipe by a large blower. About two feet from the downstream end of the pipe a four-inch square by  $3/8$ -inch thick plate was secured parallel to the airstream by two wires. This plate was constructed of three pieces of  $1/8$ -inch thick brass sheet fastened together at the corners with small screws. The center of the middle sheet was removed and an electric heating element, which was made by wrapping 30-gage Nichrome wire around a piece of sheet mica, was inserted. Four thermocouples were soldered to the upper surface of the plate.

Power was supplied to the plate through a variable transformer connected to a voltage regulator. A wattmeter was connected in the plate circuit to indicate the power dissipated.

Pulsations were produced in the airstream by a butterfly valve in the downstream end of the six-inch pipe. Two different sizes of butterfly valves were used to give two different pulsation amplitudes.

Three total pressure probes and one static pressure probe were placed one and one-half inches in front of the plate and spaced equally between the centerline and edge of the plate. By using these, together with a

rotating valve and three manometers, it was possible to obtain instantaneous velocity readings at three points along the front of the plate.

A total of 16 runs of different frequency were made at the high amplitude. The frequencies ranged from 92 to 448 cycles per minute. Five runs were made at the low amplitude at frequencies ranging from 92 to 245 cycles per minute. The Reynolds number of the air in the pipe, based on the average velocity past the plate, was approximately 250,000 for all runs, and the power input to the plate was approximately 50 watts.

There was an increase in the average heat transfer coefficient from the plate with increasing frequency. At the higher amplitude this increase was about ten per cent of the steady state value. At the lower amplitude the maximum increase for the data taken was five per cent of the steady state value.

The variation of velocity with time as obtained from the velocity probes is plotted in the appendix (see Figs. 3 to 12).

An analytical analysis of this problem is presented in which the assumption is made that the instantaneous heat transfer coefficient may be obtained from the instantaneous air velocity over the plate and this value integrated over one complete pulsation cycle to obtain an average heat transfer coefficient. Velocities used are those found experimentally with the probes. Values of heat transfer coefficients obtained from this analysis increase up to five per cent at the high amplitude and up to three per cent at the low amplitude.

## CHAPTER I.

### INTRODUCTION

A large portion of the theoretical and experimental work in convective heat transfer has been concerned with steady flow phenomena, but numerous applications arise in which the velocity of the fluid involved in the heat transfer varies periodically with time. The object of the present experiment is to determine the effect of the pulsation of a turbulent airstream upon the average heat transfer coefficient from a heated flat plate.

The results obtained in those few cases where an investigation was made of the average heat transfer rate to a fluid whose velocity varied periodically with time have varied widely and have sometimes been contradictory. In order to establish the present status of this problem, a brief review of the literature which deals with pulsating flow heat transfer will be given.

Martinelli and Boelter (1)\* experimentally and theoretically investigated the effects of vertical vibrations of a horizontal cylinder upon the free convection heat transfer coefficient. They found that within the range of their experiments the free convective heat transfer was not altered by the vibrations.

---

\*The numbers in parentheses refer to items in the bibliography.

Martinelli and coworkers (2) investigated the rate of heat transfer from a vertical tube heated by condensing steam to water pumped through it by a reciprocating pump operating at speeds from 13 to 265 revolutions per minute. They obtained data for average values of Reynolds number,  $\overline{Re}$ , between 2,660 and 77,300 and compared the average measured Nusselt number,  $\overline{Nu}$ , for the periodic flow to the steady flow Nusselt number,  $Nu$ , computed from the average Reynolds number. This average Reynolds number was based on the average mass flow rate with pulsations. Within the range of frequencies studied, the frequency had apparently no effect on the Nusselt number. When both average and steady flow values of the Nusselt number were plotted against Reynolds number, all points fell nearly on the same line.

At the end of a paper by Martinelli (3), J. H. Marchant presents a discussion which contains experimental data for the heat transfer between steam condensing on the outside of a tube and water flowing through the tube in steady flow and at pulsation rates of 10, 25, and 60 cycles per minute. The value of the Reynolds number varied between 400 and 100,000. For these tests there was no apparent deviation of the heat transfer coefficient for pulsating flow from that for steady flow except for  $\overline{Re} < 2000$ , where some of the values for pulsating flow were as much as 30 per cent higher than the corresponding steady state values.

Webb (4) and Morris (5) separately reported the results of heat transfer coefficient measurement for the laminar pulsating flow of oil in a pipe. The average Reynolds number varied between 26 and 1375. Pulsations were produced by a reciprocating pump. Both report no increase of heat transfer rate for pulsations; in fact, many of the values fell slightly below the steady state values.

Mueller (6) presents experimental results for air flowing turbulently inside a vertical brass tube in a Reynolds number range of 53,000 to 76,000. The air was pulsated by a valve placed upstream of the test section. Pulsation rates varied over a frequency range of 2.3 to 14.9 cycles per minute. The average Nusselt number was found to be slightly less than the steady state Nusselt number for the same average flow conditions. He then made an analysis to show that the heat transfer rate should be slightly less for those pulsating flows which can be considered as quasi-steady and suggests a criterion by which a flow may be judged to determine if it is quasi-steady.

Kubanski (7) reported the results of his tests in which he placed a 1.5 centimeter diameter brass tube perpendicular to the air flow in the open section of a wind tunnel at a Reynolds number in the wind tunnel of about 2500. By using certain frequencies of sound waves, it was possible for him to increase the heat transfer coefficient up to 50 per cent.

Romie (8) performed an experimental investigation of the variation of time average heat transfer coefficient of a pulsating airstream flowing through a one-inch diameter stainless steel tube at a Reynolds number of 5000. A 25-diameter section of the tube was electrically heated. Fluctuations were imposed on the airstream upstream of the test section and varied in frequency from 3.3 to 133 cycles per second. It was found that the ratio of the heat transfer coefficient with pulsating flow to the heat transfer coefficient measured in steady flow first increased with increasing pulsation rate, then decreased to a value of

less than one, and lastly, slowly increased to greater than one at the highest frequencies tested. He then gave a brief qualitative discussion in which he described the reasons for these results.

Some of the first data to show that heat transfer rates for turbulent pipe flow may be significantly increased in certain cases due to fluctuations in the flow velocity is due to West and Taylor (9). For their experiment the pulsations were produced by a single cylinder, reciprocating pump operating at a constant speed of 100 revolutions per minute. The water from the pump passed through a surge chamber where the pulsations could be damped to any desired degree by varying the amount of air in the chamber. The ratio of maximum to minimum volume of air in this tank was called the pulsation ratio and used as a measure of the severity of the pulsations. For the tests the mass flow rate of the water was nearly constant. The temperature of the water entering the heated test section was controlled to two different values giving average Reynolds numbers of about 35,000 and 75,000 in two series of heating tests. A third run in which the water was cooled was made with a Reynolds number of about 45,000. The ratio of average heat transfer coefficient for pulsating flow,  $\bar{h}$ , to average heat transfer coefficient,  $h$ , computed from average flow conditions was found to increase with pulsation ratio to a maximum value of nearly two at a pulsation ratio of about 1.4. The ratio began to decrease at higher pulsation ratios.

Since no measurement of surface temperature was made by West and Taylor, the heat transfer coefficient of the water could not be directly calculated. An average heat transfer coefficient was obtained by measuring the average total resistance based on steam or cooling fluid to

water temperatures and subtracting from this a computed resistance for the pipe and the outside fluid. Since the value for the pulsating water heat transfer coefficient depends largely upon the calculated thermal resistance of the outside fluid, the results contain a large element of uncertainty. However, since the same method was used for all of the runs, the trend indicated by the test demonstrates a definite increase in the heat transfer rate due to the pulsations. The general conclusion of West and Taylor was that it would frequently be possible to increase the capacity of heat exchangers by pulsating the fluid flowing through them. Since the frequency and Reynolds numbers of these tests were the same as for some of the experiments by Martinelli (2), and both pulsations were produced in tubes by single cylinder reciprocating pumps, there is a definite contradiction that awaits explanation.

Havemann and Rao (10) carried out investigations of air near atmospheric pressure flowing through a horizontal pipe one inch in diameter. Heat transfer coefficients were measured both at steady flow and with fluctuations produced by a poppet valve in the flow. Wave form and amplitude were measured for a series of tests in which Reynolds number varied from 5,000 to 35,000 and frequency of pulsations ranged from 5 to 33 cycles per second. It was found that the average Nusselt number changed up to about 30 per cent under different conditions of frequency, amplitude, wave form and Reynolds number. In general, the change was negative below a certain frequency and positive above it. This frequency was a function of wave form and, to some extent, Reynolds number. At low amplitudes, little change in the average Nusselt number was found, but at large amplitudes the change in average Nusselt number was not

proportional to amplitude, but depended also on frequency and Reynolds number.



## CHAPTER II

### EQUIPMENT

In this experiment air was forced through the system (see Fig. 1) by a large American Blower Company centrifugal blower. This blower was powered by a 15 horsepower, 3-phase Allis-Chalmers induction motor.

Air entered the system through a five-foot length of six-inch steel pipe which had a three-inch thin plate orifice placed 42 inches from the entrance. The orifice was placed according to ASME orifice standards (11) and the discharge coefficient taken from a table in the standards. A 48-inch U-tube manometer containing water was connected across the orifice to flange pressure taps one inch from each face. This manometer was graduated in tenths of an inch and could be read to  $\pm 0.05$  inches of water. The air discharged from the pipe into a chamber which helped to dampen the pulsations in the air so that accurate readings could be obtained with the orifice. The air then passed through a metering valve, which controlled the flow rate of air in the system, and into the blower.

The air discharged from the blower into a nine-foot length of six-inch diameter pipe. A single thickness of window screen was placed over the entrance of this pipe and a bank of straightening tubes made of one-inch steel tubes, nine inches long, was placed in the pipe to make the air velocity distribution over the pipe nearly constant and

straighten the flow. The velocity was later checked and found to be constant within five per cent across the plate in steady flow. The flat plate used in the tests was secured parallel to the air flow about two feet from the exit end of this pipe, and a butterfly valve, which produced the pulsations, was placed six inches from the end. Variation of the amplitude of pulsation was accomplished by changing the size of the disk used for this valve. Two different disks were used in this experiment; one for low pulsation amplitude with a diameter of four and one-half inches, and one for high pulsation amplitude with a diameter of five and one-half inches. The inside pipe diameter was six inches.

The butterfly valve was driven by a timing belt which was turned by a Vickers hydraulic transmission. This transmission could be adjusted to give any desired frequency of pulsation.

The flat plate used for this experiment (see Fig. 13) was constructed of three pieces of 1/8-inch thick by four inches square brass plate. The brass plates were placed flat on top of each other and secured together at each corner by a small screw. The leading edge of the plate was flat. The center section of the middle plate was removed to leave a hollow in the plate. In this hollow was placed a heating element made by wrapping a thin sheet of mica with about thirty turns of 30-gage Nichrome wire. Other sheets of mica were placed around this heating element to shield it from the plate.

Power was supplied to the plate through a voltage regulator connected to a variable transformer, which in turn was connected to the plate. The voltage regulator was supplied by a standard 110 volt power

outlet. A 0-100 watt General Electric wattmeter was connected directly in the circuit with the plate to give readings of the power dissipated by the plate to the air. This meter was accurate to  $\pm 0.25$  watts. At the end of the first operating period (see Table 1), this meter was damaged and was replaced with a 0-500 watt Weston wattmeter which was accurate to  $\pm 1.25$  watts.

Four small holes were drilled in the upper surface of the plate (see Fig. 13) and thermocouples made from 30-gage copper and constantan wire were brought up through these holes and soldered to the surface of the plate. Two of these thermocouples were on the right side of the plate and, of these, one was  $1/2$  inch in front of the middle of the plate and the other was  $1/2$  inch behind the middle. The other two were on the left edge of the plate with one  $1/2$  inch from the front and the other  $1/2$  inch from the rear of the plate. All thermocouples were about  $5/8$  inch in from the edges of the plate. The thermocouple wires ran inside the plate and out a small hole in the back of the plate. The cold junction of the thermocouples was placed in an ice bath. The voltage was read with a Leeds and Northrup Model 8662 potentiometer. The measured temperatures are probably accurate within  $\pm 0.5^{\circ}$  Fahrenheit.

Three total pressure probes and one static pressure probe were spaced across the pipe one and one-half inches in front of the plate. One of the total pressure probes was at the center line of the plate, one at the edge, and the third midway between the two. These static pressure probes were made from 0.072 O. D. stainless steel hypodermic tubing. A  $90^{\circ}$  bend was made in the tubing one inch from the end and

this short section faced into the airstream. The static pressure across the tube was checked both in steady state and with pulsation rates of 350 and 190 cycles per minute and found to have no variation greater than 0.05 inches of water. Because of this, only one static pressure probe was used. It was centered between two of the other probes. For this static pressure probe a small pitot tube was used with the total pressure probe left unconnected. These probes were connected to three manometers through a rotating valve (see Fig. 14) which opened only for a short time at a set point in the pulsation cycle. This valve consisted of a fixed steel plate through which four pairs of holes were drilled, and a rotating disk with four slots cut in its surface. For each pair of holes there was a tube from a pressure probe connected to one hole and a tube leading to a manometer connected to the other. As the disk rotated, a slot would come behind a pair of holes and connect the probe and manometer for a short period of time. This shaft was geared through a small differential and turned by the timing belt from the Vickers hydraulic transmission. By changing the position of the differential, the valve could be made to open at any point in the pulsation cycle. With this arrangement it was possible to read velocity pressure at a certain time in the cycle and from this to calculate both velocity as a function of time and the velocity profile in the pipe at any time.

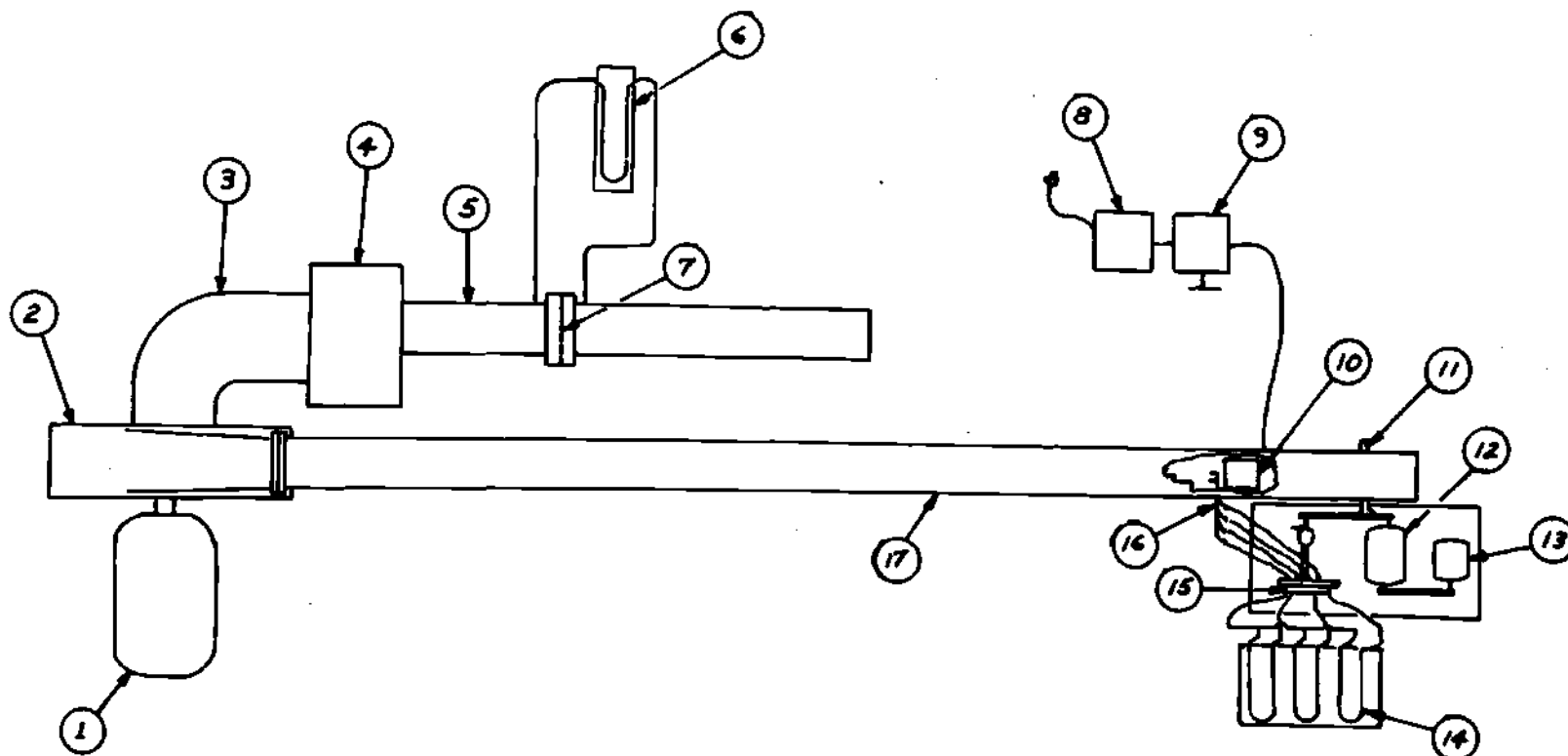


Figure 1. Equipment Layout

(See next page for key.)

## KEY TO FIGURE 1

## EQUIPMENT

1. Blower Drive Motor
2. Blower
3. Blower Inlet Pipe
4. Pulsation Damping Chamber
5. Air Metering Pipe
6. Manometer
7. Orifice
8. Constant Voltage Transformer
9. Variable Transformer
10. Test Plate
11. Butterfly Valve
12. Hydraulic Transmission
13. Electric Motor
14. Velocity Pressure Manometers
15. Rotating Valve
16. Pressure Probes
17. Six-inch Pipe
18. Potentiometer

### CHAPTER III

#### PROCEDURE

The blower was started and allowed to run until the temperature of the air leaving it was changing no faster than one degree in 30 minutes. The inlet valve to the blower was adjusted until the manometer across the orifice in the inlet pipe read 22.4 inches of water. With pulsations, the manometer readings varied as much as  $\pm 0.2$  inches of water, so an average reading had to be taken. This value was maintained throughout the tests.

After the air temperature had stabilized, the heater element in the plate was turned on and adjusted to a power output of fifty watts. This value was set at the start of each operating period of the equipment and not changed for that series of tests.

During the tests several runs were made without pulsations to serve as a reference for the runs with pulsations. The equipment was allowed to run until no change of plate temperature in a ten minute period could be measured. The temperatures of the four thermocouples on the surface of the flat plate and of the thermocouple in the airstream just ahead of the plate were recorded.

The butterfly valve was then rotated in the pipe to produce pulsations in the flow. With the air flow rate and plate power output kept constant as mentioned above, readings of plate and air temperature were again taken when they became steady. Then the pulsation rate was

increased and the procedure repeated. During the first operating period of the equipment, one steady-state run and runs at frequencies of about 90, 125, 160, 190, and 240 cycles per minute at both high and low amplitude were made. The heater in the plate burned out during a 245 cycles per minute, low amplitude run and had to be repaired. Because of the error involved in replacing the plate in exactly the same position in the pipe and in setting the power input to the plate, the steady state heat transfer changed about 12 per cent. To allow for this, the heat transfer coefficient for pulsating flow was compared to the heat transfer in steady flow as obtained only from those runs made during the same operating period of the equipment.

In the final two operating periods of the equipment, two steady state runs were made; one at the beginning and one at the end of the series of runs.

Velocity data were not taken during the first operating period of the equipment. A series of runs at the same frequency and amplitude were made later and only velocity data were taken then.

Both velocity and heat transfer data were taken during the second and third operating periods.



## CHAPTER IV

## RESULTS

At low amplitude for which  $\Delta U/U_{\infty \text{ steady}}$ , the ratio of the total change in instantaneous velocity to the average velocity, was about 0.5, runs were made for four different values of frequency varying between 92 and 187 cycles per minute. While making a run at 245 cycles per minute, the plate heater burned out. Since the variation of heat transfer coefficient with pulsations had only been about five per cent at maximum to this point, it was felt that it was not worthwhile to continue tests at this amplitude. Therefore, after the plate was repaired, all runs were made at the high amplitude.

At the high amplitude the ratio  $\Delta U/U_{\infty \text{ steady}}$  was about 1.4. Data were taken for 16 runs with frequencies varying between 92 and 448 cycles per minute.

An arithmetic average of the temperatures on the plate was used to obtain the heat transfer coefficient for the plate. At the larger pulsation amplitude the ratio of average heat transfer coefficient with pulsations to steady state heat transfer coefficient increased with increasing frequency until it reached a maximum of 1.10 at about 250 cycles per minute. At higher frequencies than this it decreased very slightly, finally reaching a value of 1.09 at 448 cycles per minute, the highest frequency for which tests were made (see Fig. 2).

At the smaller amplitude the ratio of  $\bar{h}/h_{\text{steady}}$  increased with frequency to a value of 1.05 at 187 cycles per minute, the highest frequency used for low amplitude. The value of  $h_{\text{steady}}$  for these tests was approximately 26 BTU/hr ft<sup>2</sup> °F. A rough calculation using an equation for a flat plate in turbulent flow gave a value of about 20 BTU/hr ft<sup>2</sup> °F.

It is realized that the variation of heat transfer coefficient is small compared to the error possible in the system. Since the values obtained for heat transfer coefficient are compared only to values obtained for steady state during the same operating period and the power input to the plate kept constant during the operating period, it is felt that the trends indicated by the results are valid.

Curves of the time variation of velocity through one cycle at various frequencies and both amplitudes have been plotted and are included in the appendix (see Figs. 3-7). Only the values of velocity at the edge and centerline of the plate are plotted. The velocity at the third probe did not vary more than five per cent from the velocity indicated by the probe at the edge of the plate.

At the higher amplitude the velocity pressure manometers gave negative readings for some points in the cycle. This probably indicates reversed flow, but these readings have no significance in computing velocity. These points were disregarded and the shape of the time variation of velocity curves estimated for these points, assuming that the negative velocity pressure indicates at least some reversed flow.

As the butterfly valve closed, the velocity near the center of the plate decreased more rapidly than the velocity near the edge. At the high amplitude and certain frequencies it had sometimes dropped to zero, while the velocity near the edge was over 60 feet per second. This was probably caused by the annular space left around the butterfly valve.

The resonant frequencies of the air in the pipe were calculated by considering the pipe as open at both ends, and one of these was seen to occur at 220 cycles per minute. It can be seen from an examination of the curves of velocity against time (see Figs. 3-7) that the velocity at the center of the plate and the velocity at the edge of the plate are nearly in phase at 240 cycles per minute and high amplitude, while they are somewhat out of phase at other frequencies and high amplitude. This would seem to indicate a resonant frequency. The heat transfer rate is seen to reach its maximum value at this point also. This is probably due to the fact that the air is in resonance.

An analytical analysis of this problem (see Appendix I) is made in which it is assumed that the instantaneous heat transfer coefficient may be obtained from the instantaneous air velocity over the plate and this value integrated over one complete pulsation cycle to get an average heat transfer coefficient. Velocities used are those found experimentally with the probes. Values of heat transfer coefficient obtained from this analysis increase up to five per cent at the high amplitude and three per cent at the low amplitude.

All velocities used were calculated from the readings of the pressure probes. The readings from the orifices were used only to keep the flow through the system constant while the runs were made. The orifice did not indicate the correct air flow through the discharge pipe since only a part of the total flow passed through the orifice.

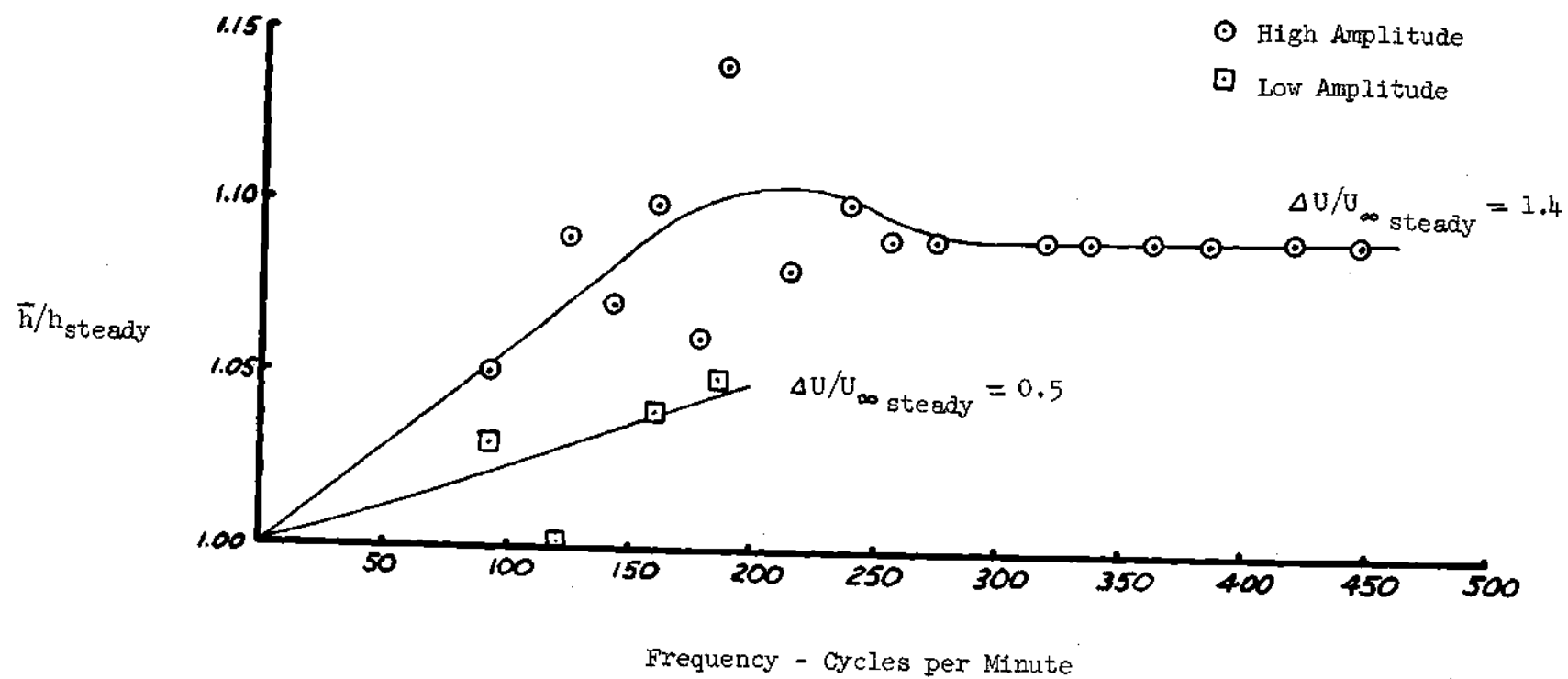


Figure 2. Variation of  $\bar{h}/h_{\text{steady}}$  with Frequency

## CHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS

It was found that the rate of heat transfer from a flat plate to an airstream flowing over it could be changed by periodically pulsating the airstream. The magnitude of this change increased with both frequency and amplitude of pulsation at low frequencies, but did not change further with increasing frequency over 250 cycles per minute.

The analytical work done on this problem indicates that the waveform of the pulsations may well be one of the important considerations in dealing with a pulsating flow.

Much more work must be done on this problem before it will be completely solved. It is recommended that in future work this problem be extended by increasing the range of frequency and amplitude, by varying the Reynolds number of the air flow and by changing the waveform of the pulsations.

## APPENDIX I

## ANALYTICAL ANALYSIS OF HEAT TRANSFER IN PULSATING FLOW

# ANALYTICAL ANALYSIS OF HEAT TRANSFER IN PULSATING FLOW

Karlsson (12) performed an experimental investigation of the effects of pulsation of a turbulent airstream upon the boundary layer formed on a flat plate. To do this, he sinusoidally pulsed air inside a square duct and used a hot wire anemometer to measure the instantaneous values of velocity in the boundary layer. He found that in all of his tests, which include the frequencies and amplitudes used in this series of tests, the instantaneous velocity profile in the boundary layer was almost exactly the same as would be found in the steady flow boundary layer that would be formed at the instantaneous Reynolds number.

It is therefore supposed that the time average heat transfer coefficient at some point on the plate might be obtained by integrating over one pulsation cycle one of the equations used in steady flow to relate heat transfer to fluid flow conditions. This is admittedly a crude analysis of the problem, but it should help to give some insight into it. A similar study was made by Mueller (6) in which he assumes a sinusoidal pulsation. For this case he shows that the average heat transfer coefficient will decrease.

For moderate temperature difference and fully developed turbulent flow, an equation which relates the Coburn heat transfer factor,  $j$ :

$$j = \frac{h}{\rho U_c C_p} \left( \frac{\mu C_p}{K} \right)^{2/3}$$



to the properties of the fluid flowing over a flat plate was found by an experimental procedure to be:

$$j = \frac{h_x}{\rho U_{\infty} C_p} \left( \frac{\mu C_p}{K} \right)^{2/3} = \frac{0.0296}{\left( \frac{\rho U_{\infty} C_p}{\mu} \right)^{0.2}}$$

Rearranging to get an expression for  $h_x$ :

$$h_x = \frac{0.0296 \rho C_p}{\left( \frac{\rho U_{\infty} C_p}{\mu} \right)^{0.2}} \left( \frac{\mu C_p}{K} \right)^{2/3} U_{\infty}^{0.8}$$

And from the assumption made in this analysis:

$$\bar{h}_x = \frac{1}{2\pi} \int_0^{2\pi} \frac{0.0296 \rho C_p}{\left( \frac{\rho U_{\infty} C_p}{\mu} \right)^{0.2}} \left( \frac{\mu C_p}{K} \right)^{2/3} U_{\infty}^{0.8} d\theta$$

To get an average value for the entire plate, this value could be integrated over the length of the plate.

$$\bar{h} = \int_0^L \frac{1}{2\pi} \int_0^{2\pi} \frac{0.0296 \rho C_p}{\left( \frac{\rho U_{\infty} C_p}{\mu} \right)^{0.2}} \left( \frac{\mu C_p}{K} \right)^{2/3} U_{\infty}^{0.8} d\theta dL$$

Since the integrations are independent, the integration over the length may be performed first, giving:

$$\bar{h} = \frac{1}{2\pi} \int_0^{2\pi} \frac{0.037 \rho C_p}{\left( \frac{\rho U_{\infty} C_p}{\mu} \right)^{0.2}} \left( \frac{\mu C_p}{K} \right)^{2/3} U_{\infty}^{0.8} d\theta$$

Or, for a given problem in which the flow velocity is the only variable:

$$\bar{h} = \frac{C_1}{2\pi} \int_0^{2\pi} U_{\infty}^{0.8} d\theta$$

$$\text{Where } c_1 = \frac{0.037 \rho C_p}{\left(\frac{L}{\mu}\right)^{0.2}} \left(\frac{\mu C_p}{k}\right)^{-2/3}$$

From the same type of analysis, the steady state heat transfer coefficient would be a function of the average flow velocity without pulsation:

$$h_{\text{STEADY}} = C_1 U_{\infty}^{0.8}$$

So, the ratio  $\bar{h}/h_{\text{steady}}$  would be:

$$\bar{h}/h_{\text{STEADY}} = \frac{\int_0^{2\pi} U_{\infty}^{0.8} d\theta}{2\pi U_{\infty, \text{STEADY}}^{0.8}}$$

Or:

$$\bar{h}/h_{\text{STEADY}} = \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{U_{\infty}}{U_{\infty, \text{STEADY}}} \right)^{0.8} d\theta$$

The values of  $U_{\infty}$  and  $U_{\infty, \text{steady}}$  were taken from the test data obtained with the velocity probes and the integral evaluated by using the trapezoidal rule for approximate integration. The results obtained from this are as follows:

Frequency Cycles per Minute	$\Delta U/U_{\infty, \text{steady}}$	$\bar{h}/h_{\text{steady}}$ Computed	$\bar{h}/h_{\text{steady}}$ Experimental
92	0.5	1.02	1.03
160	0.5	1.05	1.04
190	0.5	1.04	1.05
92	1.4	0.98	1.04
160	1.4	1.05	1.08
240	1.4	1.05	1.10
320	1.4	1.03	1.08
386	1.4	1.03	1.08

## APPENDIX II

## VELOCITY VARIATION DATA AND SAMPLE CALCULATIONS

Table 1  
Heat Transfer Results

Frequency Cycles Per Minute	$\Delta U/U_{\infty}$ steady	Operating Period	Plate Temp. ° F				Air Temp. ° F	$\bar{h}/h_{\text{steady}}$
0	0	1	137	138	140	140	118	---
92	0.5	1	140	141	142	142	121	1.03
120	0.5	1	143	144	146	146	124	1.00
162	0.5	1	143	145	146	147	125	1.04
187	0.5	1	144	145	147	147	126	1.05
92	1.4	1	139	140	142	142	121	1.05
124	1.4	1	143	145	147	146	126	1.09
162	1.4	1	144	146	147	147	127	1.10
189	1.4	1	146	147	148	148	129	1.14
240	1.4	1	147	148	149	149	129	1.10
0	0	2	151	151	152	152	125	---
145	1.4	2	152	153	154	154	129	1.07
178	1.4	2	153	154	154	154	130	1.06
218	1.4	2	154	154	155	155	131	1.08
258	1.4	2	153	154	155	155	131	1.09
276	1.4	2	153	154	155	155	131	1.09
320	1.4	2	153	154	155	155	131	1.09
0	0	3	150	151	152	152	126	---
0	0	3	148	148	149	149	124	---
338	1.4	3	150	151	151	152	129	1.09
364	1.4	3	150	150	151	152	129	1.09
386	1.4	3	150	151	151	152	129	1.09
420	1.4	3	150	151	151	152	129	1.09
448	1.4	3	150	151	151	152	129	1.09
0	0	3	149	150	150	151	125	---

Table 2

Variations of Velocity Pressure with Time

$$\Delta U/U_{\infty \text{ steady}} = 0.5$$

		Pressure (inches of water)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N* 92		1.6	1.8	1.9	2.0	2.3	1.7	0.8	0.6
	*1	1.7	1.9	2.0	2.1	2.4	2.0	1.2	0.8
	**2	1.6	1.8	1.9	2.0	2.3	2.0	1.2	0.8
	***3								
N 120		1.0	1.4	1.7	1.9	2.1	2.0	1.2	0.5
	1	1.1	1.5	1.9	2.1	2.4	2.5	1.8	0.7
	2	1.1	1.5	1.9	2.1	2.4	2.6	1.6	0.8
	3								
N 162		1.2	1.6	1.8	1.9	1.8	1.7	1.2	0.9
	1	1.3	1.8	2.0	2.1	2.2	2.2	1.6	1.1
	2	1.3	1.7	1.9	2.0	2.1	2.1	1.6	1.1
	3								
N 187		1.0	1.4	1.6	1.8	1.7	1.6	1.4	0.9
	1	1.2	1.6	1.8	2.0	2.1	2.2	1.6	1.0
	2	1.2	1.5	1.8	1.9	2.1	2.2	1.6	1.1
	3								
N 245		0.9	1.2	1.6	1.8	1.9	2.2	1.7	1.0
	1	1.0	1.3	1.8	2.0	2.3	2.5	1.8	1.0
	2	0.9	1.3	1.7	2.0	2.3	2.5	1.8	1.0
	3								

\* N cycles per minute

\*\* 1 velocity probe located at plate centerline

\*\*\* 2 velocity probe located one inch from edge of plate

\*\*\*\* 3 velocity probe located at edge of plate

Table 3

## Variation of Velocity Pressure with Time

$$\Delta U/U_{\infty \text{ steady}} = 1.4$$

		Pressure (inches of water)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N*	92								
	***1	1.1	2.1	2.2	2.4	2.0	0	-1.5	-1.5
	****2	0.4	1.8	2.1	2.5	3.2	2.2	0.8	-0.6
	*****3	0.2	1.8	2.1	2.5	3.2	2.1	0.7	-0.6
N	124								
	1	1.3	2.4	3.1	3.1	2.1	0.1	-2.1	-2.3
	2	0	1.1	1.6	2.3	3.3	2.3	1.1	0
	3	-0.1	1.0	1.6	2.3	3.4	2.5	1.3	0.1
N	162								
	1	0.9	1.3	1.9	2.1	2.1	0.5	-1.8	-2.4
	2	0.9	1.4	1.9	2.2	2.9	2.0	1.0	-0.3
	3	0.8	1.3	1.9	2.2	2.9	2.1	1.3	-0.3
N	189								
	1	0.8	1.4	2.0	2.2	2.0	0	-1.5	-2.5
	2	0.7	1.5	2.0	2.4	2.9	1.8	0.2	0.6
	3	0.7	1.5	2.0	2.4	2.9	1.8	0.2	0.6
N	240								
	1	0.2	0.5	1.6	2.1	2.6	1.7	0.7	-0.6
	2	0.4	0.6	1.8	2.4	3.1	2.2	0.8	-0.3
	3	0.4	0.6	1.7	2.3	3.2	2.3	0.8	-0.5

\* N cycles per minute

\*\* 1 velocity probe located at plate centerline

\*\*\* 2 velocity probe located one inch from edge of plate

\*\*\*\* 3 velocity probe located at edge of plate

(Continued)

Table 3 (Continued)

		Pressure (Inches of water)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N 145	1	0.0	0.9	1.4	1.8	2.0	1.9	-0.6	-1.3
	2	0.0	0.8	1.3	1.8	2.0	2.7	2.2	1.3
	3	0.1	0.8	1.3	1.7	2.1	2.8	2.3	1.5
N 178	1	0.1	0.4	2.1	2.4	2.5	1.9	0.0	-0.9
	2	0.1	0.4	1.7	2.1	2.4	2.6	2.0	0.8
	3	0.2	0.5	1.7	2.1	2.3	2.7	1.9	0.9
N 218	1	-0.3	0.5	2.0	2.7	3.0	2.3	0.0	-1.1
	2	-0.5	0.3	1.7	2.5	3.1	3.1	2.0	0.0
	3	-0.5	0.3	1.8	2.6	3.0	3.1	2.1	0.0
N 258	1	-0.6	0.4	1.3	2.1	3.0	2.9	0.5	-1.3
	2	-0.5	0.2	1.0	1.9	2.9	3.2	2.4	0.6
	3	-0.5	0.2	1.1	2.0	2.9	3.3	2.3	0.5
N 276	1	-0.4	0.3	1.4	2.4	2.9	2.8	0.6	-1.1
	2	-0.4	0.1	1.1	2.0	2.8	3.2	2.4	0.8
	3	-0.3	0.1	1.2	2.1	2.9	3.3	2.5	0.7
N 320	1	-0.4	0.3	1.3	2.1	2.8	3.0	1.0	-1.3
	2	-0.3	0.1	1.0	1.8	2.7	3.3	3.0	0.9
	3	-0.3	0.1	1.1	1.9	2.8	3.3	2.9	0.9
N 338	1	-0.1	0.0	0.6	1.2	1.6	2.8	2.0	0.2
	2	-0.2	0.1	0.8	1.3	2.0	3.1	3.0	1.1
	3	-0.2	0.0	0.7	1.4	2.1	3.0	3.0	1.1
N 364	1	-0.2	0.4	1.0	1.9	2.7	2.9	2.0	0.2
	2	-0.2	0.5	1.0	1.9	2.5	3.1	3.1	1.2
	3	-0.3	0.4	0.9	1.9	2.6	3.1	3.2	1.1

(Continued)

Table 3 (Continued)

		Pressure (Inches of water)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N	386								
	1	-0.4	0.3	1.1	2.1	2.7	2.8	1.8	0.2
	2	-0.2	0.7	1.1	1.9	2.5	3.1	3.0	1.4
	3	-0.2	0.2	1.0	1.9	2.6	3.1	3.0	1.4
N	420								
	1	-0.6	0.1	1.1	2.0	2.8	2.9	1.9	0.4
	2	-0.4	0.3	1.0	1.8	2.7	3.1	3.1	1.7
	3	-0.5	0.2	1.0	1.8	2.7	3.1	3.1	1.6
N	448								
	1	-0.3	0.2	1.0	1.7	2.4	2.9	1.9	0.3
	2	-0.3	0.2	0.9	1.6	2.2	3.1	3.1	1.8
	3	-0.4	0.0	0.9	1.6	2.3	3.2	3.1	1.8
N	0								
	1	1.9							
	2	1.9							
	3	1.9							



Table 4  
Variation of Velocity with Time

$$\Delta U/U_{\infty \text{ steady}} = 0.5$$

		Velocity (Feet per second)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N*	92								
	**1	89.0	94.5	97.2	99.5	106.8	91.9	62.9	54.5
	***2	91.9	97.2	99.5	102.0	109.0	99.5	77.0	62.9
	****3	89.0	94.5	97.2	99.5	106.8	99.5	77.0	62.9
N	120								
	1	70.3	83.4	91.9	97.2	102.0	99.5	77.0	49.8
	2	73.8	86.3	97.2	102.0	109.0	111.3	94.5	58.8
	3	73.8	86.3	97.2	102.0	109.0	113.7	89.0	62.9
N	162								
	1	77.0	89.0	94.5	97.2	94.5	91.9	77.0	66.8
	2	80.3	94.5	99.5	102.0	104.3	104.3	89.0	73.8
N	187								
	1	70.3	83.4	89.0	94.5	91.9	89.0	84.3	66.8
	2	77.0	89.0	94.5	99.5	102.0	104.3	89.0	70.3
	3	77.0	86.3	94.5	97.2	102.0	104.3	89.0	73.8
N	245								
	1	66.8	77.0	89.0	94.5	97.2	104.3	91.9	70.3
	2	70.3	80.3	94.5	99.5	106.8	111.3	94.5	70.3
	3	66.8	80.3	91.9	99.5	106.8	111.3	94.5	70.3

\*N Cycles per minute

\*\*1 Velocity probe located at plate centerline

\*\*\*2 Velocity probe located one inch from edge of plate

\*\*\*\*3 Velocity probe located at edge of plate

Table 5  
Variation of Velocity with Time

$$\Delta U/U_{\infty \text{ steady}} = 1.4$$

		Velocity (Feet per Second)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N* 92	**1	73.8	102.0	104.3	109.0	99.5	0.0	--	--
	***2	44.5	94.5	102.0	111.3	126.0	104.3	62.9	--
	****3	44.5	94.5	102.0	111.3	126.0	102.0	58.8	--
N 124	1	80.3	109.0	123.9	123.9	102.0	22.3	--	--
	2	0	73.8	89.0	106.8	129.1	106.8	73.8	0
	3	--	70.3	89.0	106.8	132.3	111.3	80.3	22.3
N 162	1	66.8	80.3	97.2	102.0	102.0	49.8	--	--
	2	66.8	83.4	97.2	104.3	119.9	99.5	70.3	--
	3	62.9	80.3	97.2	104.3	119.9	102.0	80.3	--
N 189	1	62.9	83.4	99.5	104.3	99.5	0	--	--
	2	58.8	86.3	99.5	109.0	119.9	94.5	31.4	54.5
	3	58.8	86.3	99.5	109.0	119.9	94.5	31.4	54.5
N 240	1	31.4	49.8	89.0	102.0	113.7	91.9	58.8	--
	2	44.5	54.5	94.5	109.0	123.9	104.3	62.9	--
	3	44.5	54.5	91.9	106.8	126.0	106.8	62.9	--

\*N Cycles per minute

\*\*1 Velocity probe located at plate centerline

\*\*\*2 Velocity probe located one inch from edge of plate

\*\*\*\*3 Velocity probe located at edge of plate

(Continued)

Table 5 (Continued)

		Velocity (Feet per second)							
		Point in Cycle							
		1	2	3	4	5	6	7	8
N 145	1	0	66.8	83.4	94.5	99.5	97.2	--	--
	2	0	62.9	80.3	94.5	99.5	115.6	104.3	80.3
	3	22.3	62.9	80.3	91.9	102.0	117.8	106.8	86.3
N 178	1	22.3	44.5	102.0	109.0	111.3	97.2	0	--
	2	22.3	44.5	91.9	102.0	109.0	113.7	99.5	62.9
	3	31.4	49.8	91.9	102.0	106.8	115.6	97.2	66.8
N 218	1	--	49.8	99.5	115.6	122.0	106.8	0	--
	2	--	38.5	91.9	111.3	123.9	123.9	99.5	0
	3	--	38.5	94.5	113.7	122.0	123.9	102.9	0
N 258	1	--	44.5	80.3	102.0	122.0	119.9	49.8	--
	2	--	31.4	70.3	97.2	119.9	126.0	109.0	54.5
	3	--	31.4	73.8	99.5	119.9	127.9	106.8	49.8
N 276	1	--	38.5	83.4	109.0	119.9	117.8	54.5	--
	2	--	22.3	73.8	99.5	117.8	126.0	109.0	62.9
	3	--	22.3	77.0	102.0	119.9	127.9	111.3	58.8
N 320	1	--	38.5	80.3	102.0	117.8	122.0	70.3	--
	2	--	22.3	70.3	94.5	115.6	127.9	122.0	66.8
	3	--	22.3	63.7	86.2	117.8	127.9	119.9	66.8
N 338	1	--	0	54.5	77.0	89.0	117.8	99.5	31.4
	2	--	22.3	62.9	80.3	99.5	123.9	122.0	73.8
	3	--	0	58.8	83.4	102.0	122.0	122.0	73.8
N 364	1	--	44.5	70.3	97.2	115.6	119.9	99.5	31.4
	2	--	49.8	70.3	97.2	111.3	123.9	123.9	77.0
	3	--	44.5	66.8	97.2	113.7	123.9	126.0	73.8

(Continued)

Table 5 (Continued)

		Velocity (Feet per second)								
		Point in Cycle								
		1	2	3	4	5	6	7	8	
N	386	1	--	38.5	73.8	102.0	115.6	117.8	94.5	31.4
		2	--	38.5	73.8	97.2	111.3	123.9	122.0	83.4
		3	--	31.4	70.3	97.2	113.7	123.9	122.0	83.4
N	420	1	--	22.3	73.8	99.5	117.8	119.9	97.2	44.5
		2	--	38.5	70.3	94.5	115.6	123.9	123.9	91.9
		3	--	31.4	70.3	94.5	115.6	123.9	123.9	89.0
N	448	1	--	31.4	70.3	91.9	109.0	119.9	97.2	38.5
		2	--	31.4	66.8	89.0	104.3	123.9	123.9	94.5
		3	--	0	66.8	89.0	106.8	126.0	123.9	94.5
N	0	1	97.2							
		2	97.2							
		3	97.2							

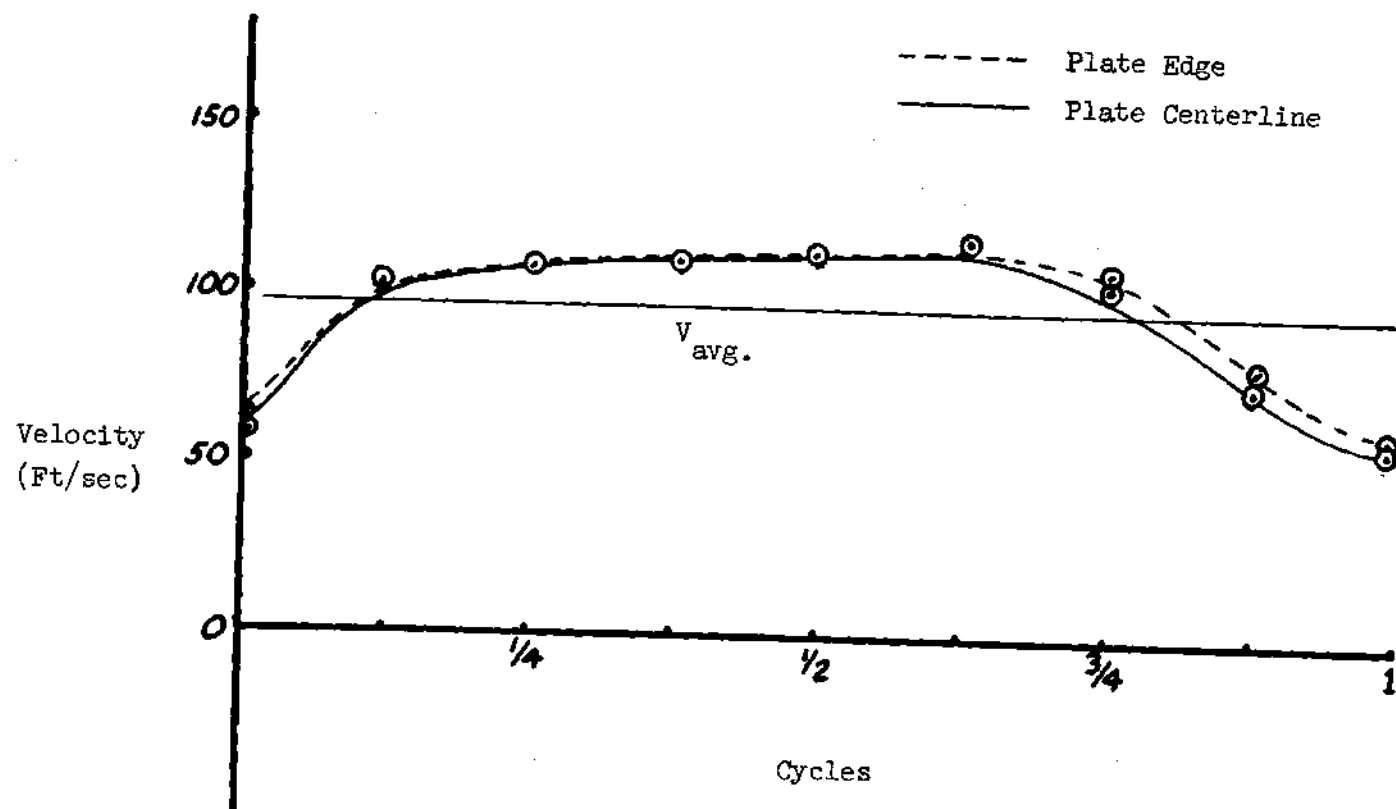


Figure 3. Variation of Velocity with Time Low  
Amplitude 92 Cycles per Minute

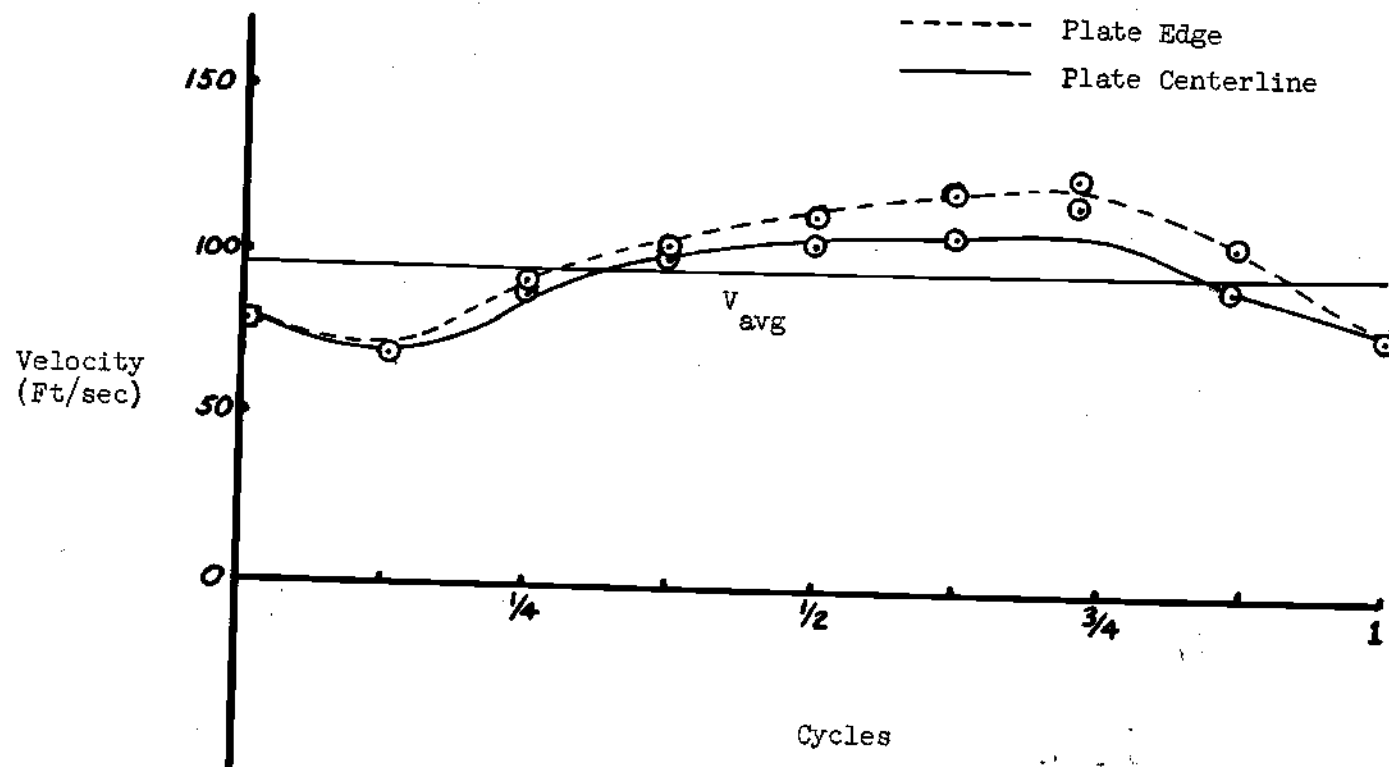


Figure 4. Variation of Velocity with Time Low  
Amplitude - 245 Cycles per Minute

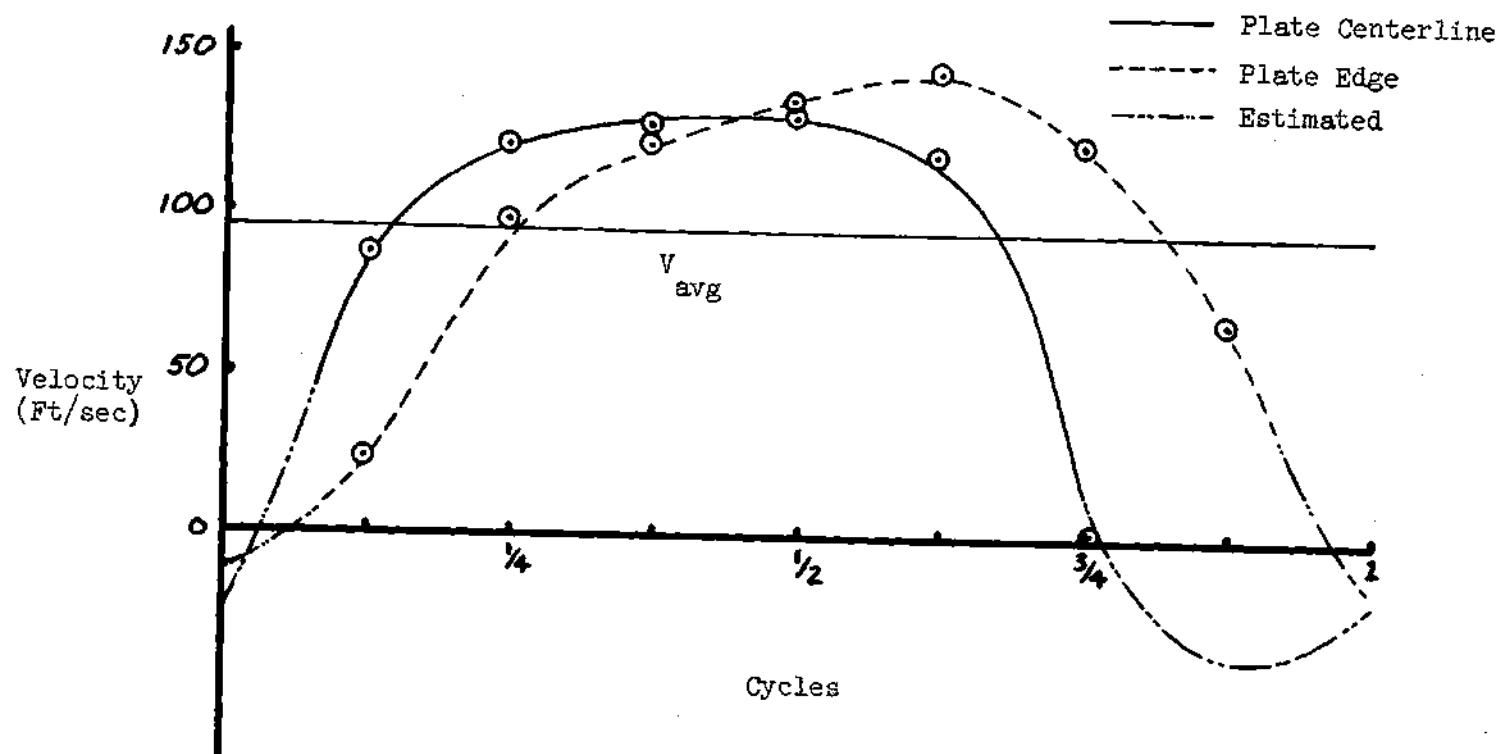


Figure 5. Variation of Velocity with Time  
High Amplitude - 92 Cycles per Minute

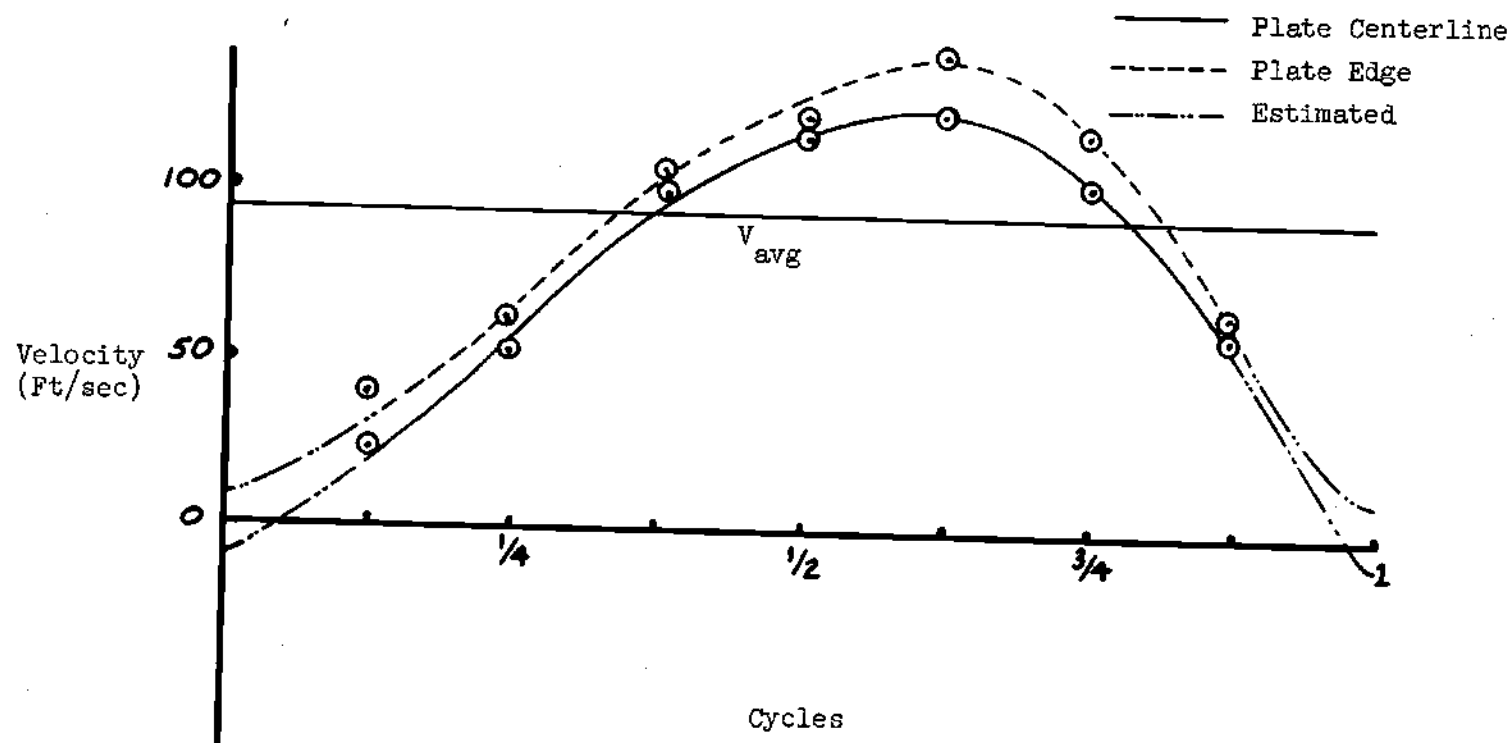


Figure 6. Variation of Velocity with Time  
High Amplitude - 240 Cycles per Minute



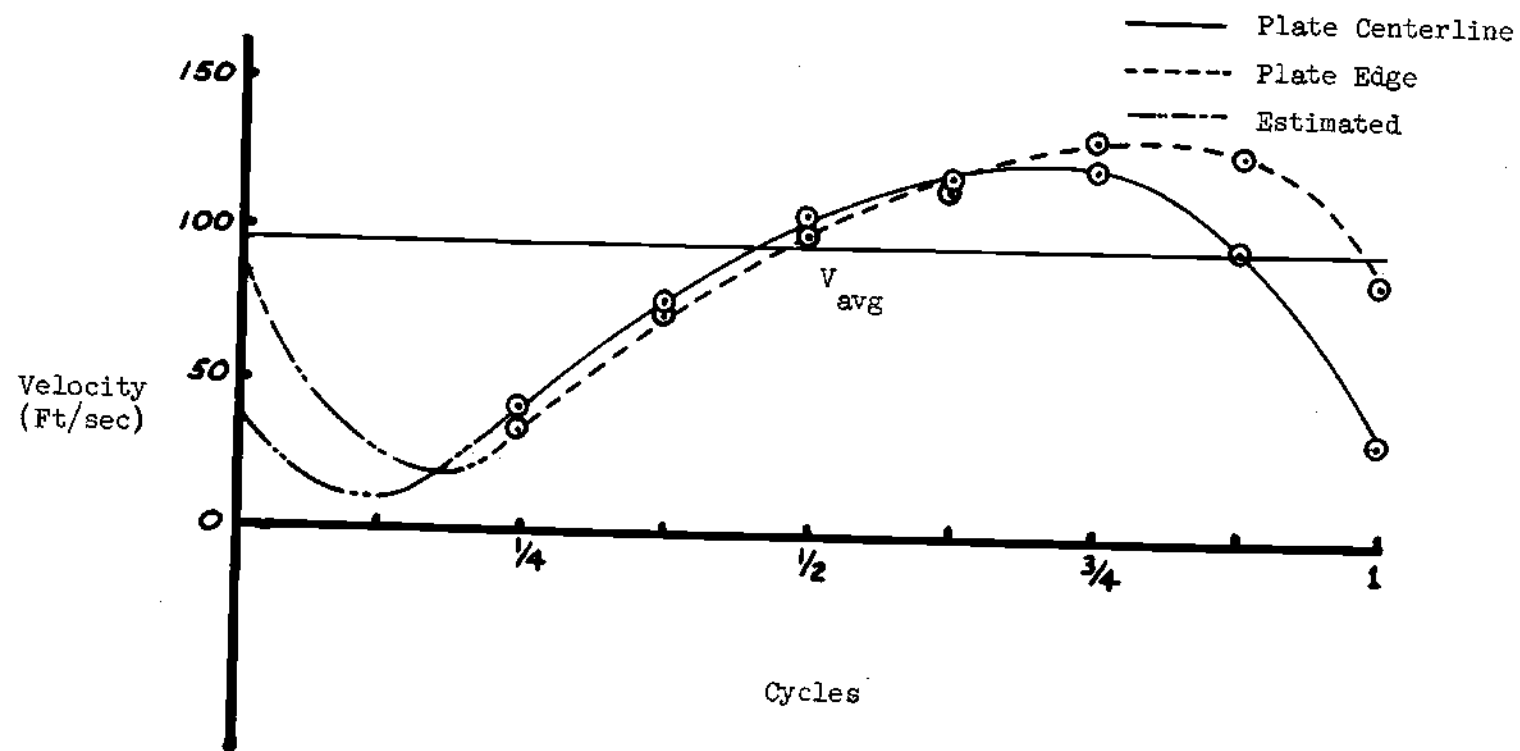


Figure 7. Variation in Velocity with Time  
High Amplitude - 386 Cycles per Minute

## SAMPLE CALCULATIONS

I. Calculation of  $\bar{h}/h_{\text{steady}}$ 

The definition of  $h$  is:

$$h = \frac{\text{heat transferred}}{\text{area} \times \text{temperature difference}}$$

For the constant heat output and area of the plate used in this experiment:

$$\frac{h}{h_{\text{steady}}} = \frac{\Delta T_{\text{steady}}}{\Delta T_{\text{pulsating}}}$$

where  $\Delta T$  is the arithmetic average of the plate temperatures minus the free stream air temperature.

For example, at 92 cycles per minute, low amplitude:

$$\Delta T_{\text{steady}} = \frac{137 + 138 + 140 + 140}{4} - 117.8 = 21^{\circ}\text{F}$$

$$\Delta T_{\text{pulsating}} = \frac{139 + 140 + 142 + 142}{4} - 121 = 20^{\circ}\text{F}$$

$$\frac{h}{h_{\text{steady}}} = \frac{21^{\circ}\text{F}}{20^{\circ}\text{F}} = 1.05$$

## II. Calculation of Velocity

Velocity of the air in the pipe may be calculated from the velocity pressure data by using the relation:

$$V = \sqrt{2gh \frac{\rho_{\text{H}_2\text{O}}}{\rho_{\text{air}}}}$$

where  $h$  is the pressure read on the manometer in feet of water.

For example, at 92 cycles per minute, low amplitude and point 1:

$$V = \sqrt{2g \frac{1.6}{12} \frac{1.94}{2.10 \times 10^{-3}}} = 88.9 \text{ ft/sec}$$

### APPENDIX III

#### DETAILS OF ROTATING VALVE AND FLAT PLATE

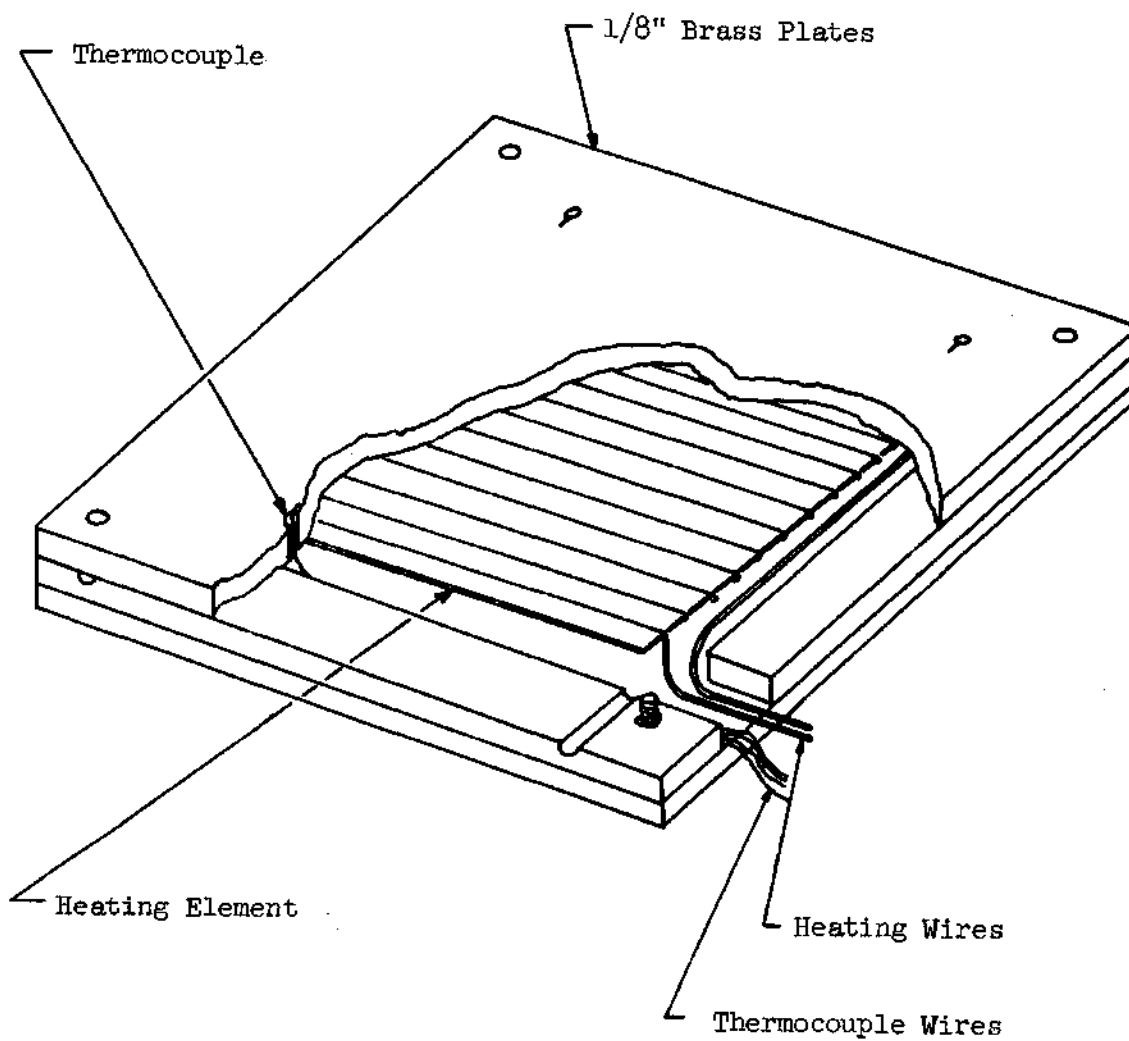


Figure 8. Details of Plate Construction

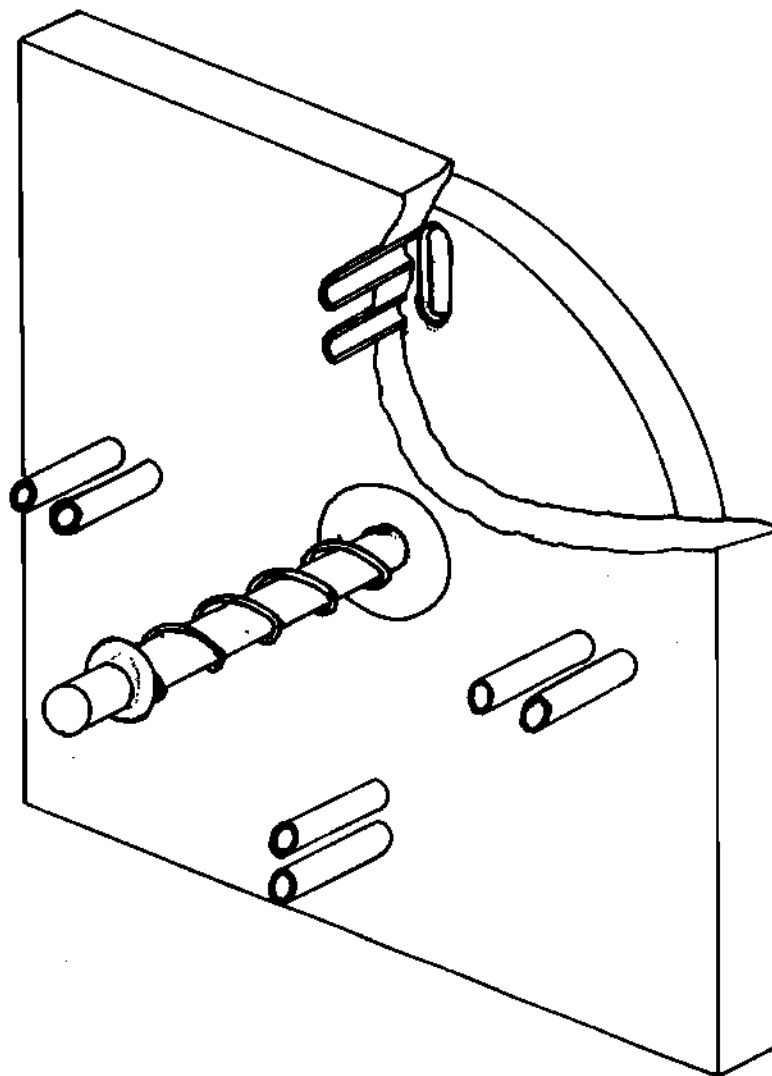


Figure 9. Rotating Valve Design

## REFERENCES

1. Martinelli, R. C. and L. M. K. Boelter, "The Effect of Vibration upon Free Convection from Horizontal Cylinders." Heating, Piping and Air Conditioning, 11, (1939), 525-527.
2. Martinelli, R. C., Boelter, L. M. K. Weinberg, E. B. and S. Yakahi, "Heat Transfer to Fluid Flowing Periodically at Low Frequencies," Transactions American Society of Mechanical Engineers, 65, (1943), 789-798.
3. Ibid., 796-798.
4. Webb, R. F., Heat Transfer in Laminar Pulsating Flow, Unpublished Ph.D. Thesis, University of Washington, 1949.
5. Morris, R. R., Heat Transfer in Laminar Pulsating Flow, Unpublished Ph.D. Thesis, University of Washington, 1950.
6. Mueller, W. K., "Heat Transfer Characteristics of Periodically Pulsating Turbulent Pipe Flow," Proceedings of the 5th Midwestern Conference on Fluid Mechanics. Ann Arbor: University of Michigan Press, 1957, pp. 146-159.
7. Kubanskii, P. H., "The Effect of Acoustic Vibrations of Finite Amplitude on the Boundary Layer," Journal of Technical Physics, U.S.S.R., 22-4, (1952), translated in Physics Abstracts, 55, No. 8731, (1952).
8. Romie, F. E., Heat Transfer to Fluids Flowing with Velocity Pulsations in a Pipe, Unpublished Ph.D. Thesis, University of California at Los Angeles.
9. West, F. B. and A. T. Taylor, "Pulsating Flow of Water Effect on Heat Transfer," Chemical Engineering Progress, 48, (1950), 39-43.
10. Havemann, H. A. and N. N. Harayan Rao, "Heat Transfer in Pulsating Flow," Nature, London, 174, (July, 1954), 41.
11. Stearns, R. F., Jackson, R. M., Johnson, R. R., and C. A. Larson, Flow Measurement with Orifice Meters. New York: D. Van Nostrand Company, 1951.
12. Karlsson, K. F., "An Unsteady Turbulent Boundary Layer, Report to Flight Propulsion Laboratory Department (Under Contract with Department of Aeronautics, Johns Hopkins University), General Electric Company, Cincinnati, (May, 1958).
13. Giedt, W. H., Principles of Engineering Heat Transfer, New York: D. Van Nostrand Company, 1957, p. 170.