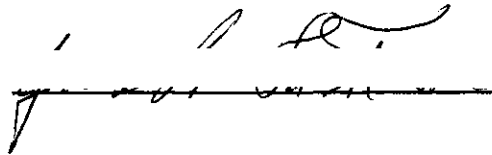


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A handwritten signature, possibly "J. R. ...", is written above a horizontal line. The line is mostly straight but has a small, sharp downward hook at the left end.

7/25/68

A COST-EFFECTIVENESS MODEL FOR
EXPENDABLE COUNTERMEASURES

A THESIS

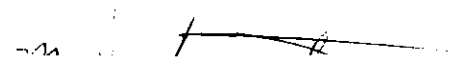
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The Faculty of the Graduate Division
by
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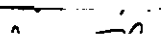
In Partial Fulfillment
of the Requirements for the Degree
Master of Science
in the School of Industrial and Systems Engineering

Georgia Institute of Technology

December, 1970


A COST-EFFECTIVENESS MODEL FOR
EXPENDABLE COUNTERMEASURES

Approved: 

Chairman 

Date approved by Chairman:  Dec 11, 1970

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SUMMARY

The object of this thesis was to present a general methodology to be used in the comparative evaluation of expendable countermeasures. From this comparison, the most optimal mix of countermeasures for a particular mission can be selected. The decision criterion on which selection is based is cost-effectiveness.

The measure of cost effectiveness used is the expected cost resulting from the use of the particular mix of countermeasures under consideration. The expected cost is defined as the sum of a deterministic cost and a stochastic cost. The deterministic cost is expressed in terms of the unit cost per item, and includes such costs as research and development, manufacture, and deployment. In order to determine stochastic costs, possible outcomes of the mission under consideration are defined, and costs are attached to each of these possible outcomes. These costs are then weighted by the probabilities of the respective outcomes resulting from the use of the particular mix of countermeasures being evaluated. These weighted costs are then summed in order to obtain the stochastic portion of the expected cost.

A situation in which a mission is to be run over the same threat/target many times is discussed. This situation is different from a single mission for the reasons that the continued missions allow the enemy operators to learn from their experience, as well as the fact that the threat itself has probably been changed from the effect of the prior missions. A discrete-state Markov chain was used to analyze this situation.

CHAPTER I

INTRODUCTION

Background

The question of costs in a military environment has always been one of great concern. This is especially true of equipment costs relative to improving survivability of, say, tactical aircraft. It is generally conceded that when the nation is in a state of national emergency, it is desirable to employ, or develop in the shortest period of time, whatever equipment is necessary to counter the threat. However, in a peacetime economy, the cost in dollars becomes the governing factor in the selection of competitive equipment. Most military equipment must not only be effective so as to allow achievement of the objectives of the specific mission which might utilize such equipment, but must also be cost-effective so that the monetary expenditure does not outweigh the value of the mission.

The particular class of equipment with which this thesis is concerned is electronic countermeasure equipment that possibly could be used in the improvement of survivability of aircraft. For example, consider an aircraft or flight of aircraft having a mission over enemy-held territory. In the course of its mission, the aircraft will be expected to fly to the target, accomplish its mission with respect to the target, and return to its home base. In the course of this mission, the aircraft will probably meet a defensive array of weaponry, including such

items as radar of different types, anti-aircraft artillery, missiles, and possibly enemy fighter aircraft. Given the parameters of these defensive reaction capabilities, the purpose of the mission, the terrain over which the mission is to be flown, and any other relevant data, what type of penetration aid, or what mix of penetration aids, should be carried by the aircraft? A penetration aid (penaid), for purposes of this thesis, will be defined as any piece of equipment which confuses, delays, or in any way inhibits the enemy from the most effective use of their defensive reaction capabilities.

One classification of penetration aids is that of expendable and non-expendable pen aids. Expendable penetration aids are either non-recoverable after use or have a small probability of recovery. Non-expendable penetration aids are generally those carried by the aircraft and are lost only if the aircraft itself is lost. This research deals primarily with expendable pen aids. It considers non-expendables only when there is an opportunity cost directly related to mission success associated with their use. The results of the techniques presented herein may therefore be used as a basis for comparison of expendable and non-expendable penetration aids as well as comparison among different expendable pen aids. Non-expendables are often more effective (this need not always be the case), and they are also usually more expensive. Thus, in a tactical situation, a cost-effectiveness comparison would be of value to the military planner.

The primary purpose of this research is to develop and discuss techniques to be used in a comparative evaluation of expendable penetration aids when the criterion for selection is cost-effectiveness.

For purposes of this research, the most cost-effective penaid or mix of penaid for a particular situation will be defined as that mix which, when both the deterministic costs and the weighted possible costs resulting from the use of that particular mix are considered, yields the minimum expected cost. These techniques are to be general enough to allow evaluation of both developed and proposed penetration aids, using either actual or anticipated costs and relevant measures of effectiveness. Effectiveness will be expressed in the form of certain probability statements, and sensitivity studies will be made to indicate the importance of these probabilities.

The ultimate objective of the tactical mission, of course, is mission success. While there exist alternative definitions, mission success is used here in the sense of getting to the target, accomplishing the goal with respect to the target, and safely returning to home base. For a given set of input parameters, what penetration aids, or what mix of penetration aids, should be used? For example, a mix for one particular mission may give a high probability of success while the same mix would give a low probability of success for different input parameters. The large number of possible combinations makes it imperative that the model be able to handle these, and that it give the maximum information possible from the fewest possible design situations.

The model must include the loss due to failure along with the physical cost of the devices. Each cost must also be related to a probability of success. Costs and success probabilities will then be combined to determine the optimum cost-effective decision for each of the pertinent situations.

Literature Search

The concept of cost-effectiveness as a quantifiable decision parameter has been in existence for quite some time, and has been used extensively in both the military and civilian realms. Most of the work in this area done in and for the military is classified, and hence cannot be referenced here. In the specific area of penetration aids, Cornell Aeronautical Laboratories has done much research involving effectiveness, and some work involving costs. However, this work is classified, and hence details cannot be discussed in this thesis.

One reference in this area which is unclassified is a Ph.D. dissertation by C. D. Fawcett (1). This work treats the problem of the selection of tactics for an air-to-ground attack when uncertain conditions prevail. The approach is one of dynamic programming, with some use of decision theory. Fawcett considers two major types of "duels." The first is the situation where the success criterion is the expected value of the number of hits made on the target. The second is the case where at least a certain number of hits must be made on the target. Specific cases and generalizations are given for both instances.

This research will make use of two concepts discussed by Fawcett. The primary one is the emphasis on the omnipresent uncertainty when dealing with a military situation. This uncertainty is what makes research in this field interesting, and must be accounted for in any work done. In fact, the techniques to be presented in this thesis are structured around this uncertainty. The other concept is that of having a minimum number of hits on a target. This latter concept is altered to be the minimum numbers of weapons per mission.

The following references do not deal explicitly with cost-effectiveness of expendable penetration aids. However, they do deal either directly or indirectly with cost-effectiveness models or methodology for possible use in development of cost-effectiveness models.

Dantzig's (2) purpose is to minimize the expected cost when some factor in the problem is known only through an assumed or known probability distribution function. He demonstrates the practicality of linear programming in cases where there is a multi-stage decision problem using the minimum cost diet problem with unknown prices and the shipment problem with stochastic demand as examples. The main theme of his paper is to propose and prove several convexity theorems to be used in the solution procedure.

Dantzig and Ferguson (3) use the theory developed in Dantzig's paper to solve the resource allocation problem of the allocation of (commercial) aircraft to routes when demand is uncertain in order to maximize expected profits. Basically, the paper demonstrates the procedures to be used in the solution of such a problem.

The method proposed and demonstrated in the above papers were considered for use in this research, but rejected for three reasons, the validity of which will become obvious in Chapter III. First, our problem becomes an integer-valued problem, and linear programming does not guarantee an optimal integer solution. Second, our problem may require the solution of feasibility and optimality in separate stages, not simultaneously. Third, their assumption of a known probability distribution function was found to be inapplicable.

Reismann and Buffa (4) present a general mathematical model to

be used in the economic evaluation of investment of equipment. The general tactic is to find the present worth of the value of several factors, including purchase price, salvage value, non-periodic replacement costs, operating expenses, and revenue received from the investment. Obviously, purchase price, operating expenses, and replacement costs (in a sense) are the only factors applicable here. These will be discussed in Chapter II.

A survey of the techniques used in the evaluation of competing items is given by Terry (5). He points out that there are three major reasons for making a comparative evaluation: (1) Selection (e.g., of personnel, purchasing projects); (2) Assignment (e.g., personnel, markets, usage); (3) Improvement (e.g., personnel, equipment, methods). He is primarily concerned with the situation in which a single evaluator is used to make a determination among several competing items, but only one item may be chosen. Since the research done in this paper involves choosing a particular set of penetration aids (i.e., Terry's "Selection"), a review of the different techniques for selection is informative. As quoted from Terry, they are:

1. Average Score: add ratings and obtain average.
2. Weighted average: add weight ratings and obtain average.
3. Multiplication Method: multiply ratings to obtain "score."
4. Exponentially Weighted Multiplication Method: multiply exponentially "weighted" ratings to obtain "score."
5. Loss Control (pessimistic): determine factors with lowest rating for each item.
6. Game Theoretic Approach: treat rating matrix as a zero-sum, two-person game.
7. Dominance: determine existence of any dominant item (i.e., dominant in every category considered).

It seems almost trite to state that if a penetration aid which costs less and is more effective than any other being tested is found,

that it should be used (i.e., "Dominance"). This situation seems almost too good to be true. However, it is possible that the research may find some particular penetration aid which will dominate some other aid in every category, and hence the dominated aid can be dropped from consideration. In the main, we will use technique (3) in conjunction with technique (1) in this research. The Average Score Method will be used in the deterministic portion of the model (Chapter II) and the Multiplication Method will be used for the stochastic portion (Chapter III). The two methods will be combined to make the final decision.

CHAPTER II

DETERMINISTIC COSTS

Introduction of the General Model

The purpose of this chapter is to introduce the cost-effectiveness model to be used in this research. Some notation will be presented, and a portion of the model will be discussed at length.

When considering the cost of an expendable penetration aid, two major areas present themselves. The first is that of the physical cost of the device itself, e.g., research and development, manufacture, and deployment. Note that this type of cost is (theoretically) deterministic and can be found exactly. The symbol "D" will be used to denote the deterministic cost of all devices used for a particular mission.

The second major area of expenditure is that cost involved in the usage of the device(s). It is in this area that uncertainty plays a large role. The proposed method of dealing with this uncertainty is through certain probability statements about possible outcomes resulting from the usage of the device(s). In order to determine the expected cost of usage, these probabilities will be used as weighting factors in association with the cost of the respective possible outcomes. This cost will be represented by the symbol "S."

Thus the decision criterion for the selection of a particular set of expendable penetration aids will be to minimize the expected cost; that is,

$$\text{Min. } E(C) = D + S$$

where

$E(C)$ = expected cost

$$D = \sum_j d_j X_j$$

d_j = deterministic cost of device j on a "per item" basis

X_j = amount (integer-valued) of device j used for the mission under consideration.

The methods used in determining both S and X_j will be discussed in the next chapter. The rest of this chapter will discuss the determination of d_j .

Considerations for Deterministic Costs

Figure 1 is a basic flow chart to be used in the evaluation of costs of the deterministic type. It represents the general procedures and processes through which a new concept for an expendable penetration aid must pass before its use. Since the methods and consequences of the use of these devices will be discussed in the next two chapters, they are not shown here.

Table 1 is a codification of the general types of costs to be used in conjunction with Figure 1. While most of these costs are self-explanatory, some merit further discussion.

The U. S. Air Force and the U. S. Navy are the primary governmental departments in this country which would have interest in expendable penetration aids. For convenience, Air Force Systems Command, which is usually the contractor for U. S. Air Force research and development projects, is shown in cost block #1. For the sake of simplicity, it will

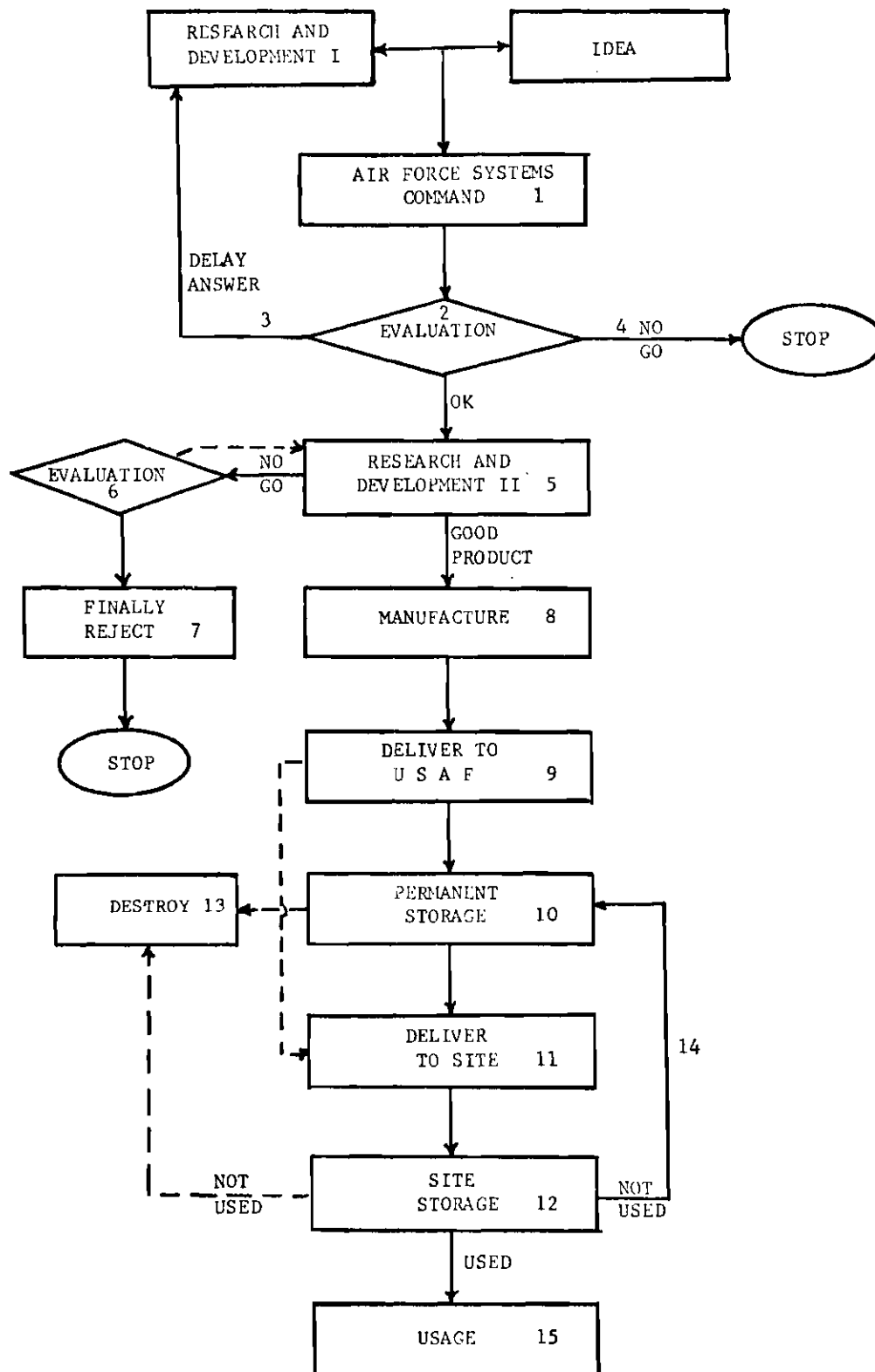


Figure 1. Flow Chart for Deterministic Costing

Table 1. General Deterministic Cost Considerations

-
- * + 1. a. Administration costs
b. Personnel required and their time
 - * + 2. a. Administration
b. Personnel
c. Equipment
 - 3. a. If later accept, two types of cost
 - i. Time value of money
 - ii. Opportunity cost due to delay
 - b. If later don't accept, cycle costs loss (ie., 1, 2, and 4; possibly twice)
 - 4. a. All costs in 1 and 2 lost
b. Possible opportunity cost loss due to rejection of a good product
 - * 5. a. Administration costs
b. Monitoring costs
 - i. On-site
 - ii. Correspondence
 - 6. a. Same as 2a, b, c
b. Perhaps several times
 - 7. a. All costs in 1, 2, 5, and 6 lost
b. 4b cost also
c. Cost of misguided anticipation
 - * 8. a. Contract letting method
 - i. Accept bidding, which implies
 - (a) Pay originator (patent rights, work accomplished, etc.)
 - (b) Include costs involved in the contract
 - ii. Dicker with the originator
 - b. Contract terms, involving, for example
 - i. Quantity
 - ii. Quality

Table 1. General Deterministic Cost Considerations (Concluded)

-
- iii. Due date
 - iv. Special equipment required (if required)
 - v. Assistance (if any) offered by the Air Force
 - vi. Labor problems
 - vii. Time value of money
 - viii. "Acts of God"
 - ix. Etc.
 - c. Monitoring and administration costs
- * 9. Point of and method of delivery may affect cost 8b
 - * 10.
 - a. Cost of shipment to permanent storage
 - b. Cost of building at permanent storage (special type?)
 - c. Personnel and administrative costs
 - d. Possible cost for equipment for (special) handling
 - * 11.
 - a. Same as 9 and 10
 - b. Special difficulties involved due to nature of use site
 - * 12.
 - a. Same as 9 and 10
 - b. Special difficulties involved due to nature of use site
 - 13.
 - a. Special equipment/location required for destruction
 - b. If not used at site, costs 10 occur again
 - 14. Same as 9 and 10
 - 15.
 - a. Special equipment required (ground or aircraft)
 - b. Special training required (ground or aircraft personnel)
 - c. Extra people required at use site because of nature of penaid?
- IN ALL PHASES:
- *
 - 1. Extra costs due to security classification, if applicable
 - 2. Printing of manuals, training of personnel, etc.
 - + → costs applicable to any proposal
 - * → costs applicable to any accepted proposal, though amounts of money may vary
-

be assumed that this agency will absorb all expenses incurred by the government in the deterministic evaluation in this study. (For example, intra-Air Force communications and transportation costs for the finished product would certainly not be charged solely to AFSC.) Which agency actually incurs the expense is unimportant to this research.

Evaluation of a proposed device may be conducted by the Air Force itself, in which case costs (2) will be as shown. If a contract is let by the Air Force for evaluation, they will have administrative as well as contract costs.

The opportunity costs mentioned in 3a and 4b lend themselves to evaluation only ex post facto. Indeed, the opportunity cost in 4b may never be able to be evaluated. Consequently, while these costs are valid theoretically, their value in a practical situation seems somewhat nebulous and they would probably not be included in a calculation of deterministic costs.

The time value of money mentioned in costs 3a and 8b are factors which are often overlooked in these type of evaluations. Since the type of devices under consideration often require several years and millions of dollars to develop and deploy, omitting these costs will ignore a potentially large expenditure. The exact application of the concept of the time value of money would vary from case to case, but should certainly be considered as a relevant factor.*

The cost of misguided anticipation in 7c is one which is very

*We will not go into more detail since doing so would involve much time and space as there are so many facets to this problem. This would detract from the main purpose of this chapter.

difficult to evaluate. In this country, the development of new hardware has become almost routine. When the "go ahead" for further research and development has been given for a new piece of gear, the capabilities the device offer are often assumed to be a fact for future planning. When the particular project fails to develop as anticipated, thinking and planning must be revamped. The exact price of this process would be very difficult to assess. The question of whether this cost should be considered a sunk cost (and therefore forgotten) or should be added to the cost of a device which eventually is accepted would be difficult to determine.

Note that this last question applies not only to misguided anticipation costs, but also must be asked about all costs involved in projects which are eventually discarded.

One brief comment should be made concerning the contract terms in 8b. The speed with which the product is manufactured may have an associated opportunity cost. If the country is engaged in an encounter in which the product can be used as soon as it can be delivered to the site, with an increased probability of mission success attendant with its use, there is obviously a cost savings involved in getting the finished product to the field as quickly as possible. If this is the case, the expected savings may be compared with the increased cost of rapid manufacture in order to arrive at the optimum delivery dates.

Note that in cost 15, only the deterministic costs are mentioned: i.e., the costs required to prepare the penetration aid for use. Since the actual device usage costs are not deterministic, they will be covered in Chapters III and IV.

The cost of security classification is difficult to evaluate, but is a very real cost. If the device carries some type of classification, not only will extra measures have to be taken in all actions involving the device itself, but all administrative action will have to be carefully handled by selected personnel. Special care would have to be taken in the printing, distribution, and use of manuals and regulations. The personnel pool from which trainees can be chosen is reduced, and the training itself must be given extra attention. This cost is certainly one which must not be overlooked.

The procedure used to determine the "per item" value of each device is, of course, the sum of all incurred costs divided by the number of devices received. This value will be denoted by d_j .

There is a hidden problem in this simple step. Suppose that for an initial order of 1,000 penails of a particular type, the total cost is \$1,000,000. This would imply that the individual cost is \$1,000 per item. Then, suppose a supplemental order for another 1,000 penails of the same type is placed, and the cost is only \$100,000. Thus, this latter set of penails would seem to cost only \$100 per item. The true cost is \$550 ($=\frac{1,100,000}{2,000}$) per item when considering all 2,000 items, and this is the figure which should be used in the evaluation.

Also note that an order placed at a time in the future, when retooling would be required, would be more expensive than an additional (or an increase in the original) order.

While these problems are simple and easy to overcome, the important thing is that they be recognized and steps taken to deal with them. These changes in d_j could very well affect the outcome of the decision

of which set of penails should be used in a particular situation.

(Since $E(C) = D + S$ and $E(C)$ is the criterion under which selection is made, a change in D would have an affect on the expected cost.)

CHAPTER III

THE ANALYSIS OF SINGLE MISSIONS

General

It is the purpose of this chapter to discuss the method which will be used to deal with the uncertainty involved in the actual use of expendable penetration aids. First the general model to be used will be introduced, with accompanying notation. Next, the feasibility problem will be discussed in the context of a mission involving only one aircraft. Then a mission involving several aircraft will be introduced and the concept of "minimum number of hits per target" will be discussed.

Table 2 defines the notation to be used throughout this thesis. A few general comments are in order. Since X_j and X_{ij} are, of necessity, integer-valued, linear programming techniques are eliminated as a possible solution procedure. Small individual units such as chaff could be under consideration and thus linear programming would be an acceptable solution procedure. However, we will assume that physically small penetration aids such as chaff are "baled" and bales may not be fractionated. In any case, mixed-integer programming computer programs are available and difficulty of solution should be minimized.

Obviously, there is a relationship between each k and each γ . Thus, for a given type of outcome, we weight the cost of that outcome by the conditional probability of that outcome, given in the specific mix. This leads to the model to be used:

Table 2. Definition of Symbols

$E(C)$	expected cost
T	the specific threat to be encountered during the mission
M	the specific mission to be run
j	penetration aid types, $j = 1, 2, \dots, m$
D	deterministic cost for all penaid's used for the mission
d_j	individual costs of penaid j
X_j	in a mission involving only one aircraft, the number of penaid's of type j
v_j	volume of (individual) penaid j
w_j	weight of (individual) penaid j
V	volume capacity for penaid's of the given aircraft
W	weight capacity for penaid's of the given aircraft
Γ	class of all mixes of penaid's considered
L	class of indices on the mixes and/or ratios under consideration
l	an element of L
γ_ℓ	an element of Γ
K	class of all possible considered outcomes of the mission
R	class of indices for the elements in K
r	an element of R
k_r	an element of K
F	class of the costs of outcomes given the mission M

Table 2. Definition of Symbols (Concluded)

f_k	an element of F denoting the cost of outcome K
$P_{\gamma_\ell k_r}$	probability the mix γ_ℓ gives result k_r given the threat and the mission $= \text{pr}(\gamma_\ell \rightarrow k_r M T)$

WHEN MORE THAN ONE AIRCRAFT IS INVOLVED:

i	index on aircraft, $i = 1, 2, \dots, n$
X_{ij}	the amount of payload of type j used on aircraft i ; integer valued
V_i	volume capacity of aircraft i
W_i	weight capacity of aircraft i
\bar{V}	volume capacity of all n aircraft
\bar{W}	weight capacity of all n aircraft

FOR MARKOV CHAINS:

E_i	states of the markov chain
ρ_{ij}	$\text{Pr} \{ \text{going from state } E_i \text{ to } E_j \text{ in one transition} \}$
$\rho_{ij}^{(n)}$	$\text{Pr} \{ \text{going from state } E_i \text{ to } E_j \text{ in } n \text{ transitions} \}$
P	the stochastic matrix of transition probabilities
$a_k^{(n)}$	the absolute probability of being in state E_k after n transitions
u_k	the invariant distribution of an ergodic chain
H	the class of transient states in a reducible chain
R	one of a class of closed sets of persistent states in a reducible chain
y_i	$\text{Pr} \{ \text{ultimate absorption in } R \text{the initial state is } E_i \}$

$$E(C) = D + S$$

$$= \sum_j X_j \cdot d_j + \sum_k P_{Y_{\ell} k_r} \cdot f_k$$

where these terms are defined in Table 2. The application of this model will be discussed and demonstrated throughout this chapter.

The generation of the $P_{Y_{\ell} k_r}$'s is an interesting question. In some cases, they will be easily obtained (e.g., a radar jammer designed to operate against S-band radar will have little or no effect against UHF-band radar), or may be obtained analytically. However, if they are unobtainable analytically, computer simulations may be used to give estimates of these probabilities. This research is not concerned with the method which is used to obtain the $P_{Y_{\ell} k_r}$'s. It will be assumed that the required probabilities are given or obtainable from some other source. When simulation is the method used, special problems arise, and these problems will be discussed in this thesis when they affect the cost-effectiveness resolution.

One Aircraft

Consider the situation where a mission is to be run, and only one aircraft is to be used. In this chapter, we assume that the mission envisioned is to be run once, and, if successful, need not be run again (e.g., photographing and/or bombing a particular installation). If it is required, the same mission could be run again. In any case, continued missions involving the same threat/target are not envisioned. This latter case involves several considerations not covered in this chapter and is discussed in Chapter IV.

In order to describe the problems to be encountered, they are discussed in the framework of a simple example. Assume that we are to evaluate two different jammers to be used by single aircraft against a specified target, with a known threat involved in the mission. The jammers are to be evaluated singly and in conjunction with each other via simulation. Let jammer type 1 be a cheap, effective, but short-lived penaid. Jammer 2 could be an expensive, long-lived jammer.

For purposes of this example, we will assume that target acquisition is identical to mission accomplishment with respect to the target. While this is not necessarily true, we wish to keep the example as simple as possible. Problems such as the aircraft successfully acquiring the target but not accomplishing its mission with respect to the target, partial mission success with respect to the target, and non-acquisition of the target can easily be handled by entering them in K and considering them as possible outcomes of the mission.

Suppose that the following jammer and aircraft data are given (note that the type of aircraft, one of the mission parameters, is specified):

j	w_j	v_j	d_j
1	20	10	100
2	300	70	5,000

$$V = 500$$

$$W = 2,000$$

Next, for the example, assume that there are three possible outcomes to the mission. These are listed below with their respective costs.

$$K = \begin{bmatrix} k_1 = \text{Mission Failure} \\ k_2 = \text{Partial Success} \\ k_3 = \text{Success} \end{bmatrix} \quad F = \begin{bmatrix} f_1 = \$100,000 \\ f_2 = \$ 1,000 \\ f_3 = \$ 100 \end{bmatrix}$$

The outcomes defined in K are intentionally nebulous, and the costs in F purely fictitious. This research is not intended to include an opportunity-cost-of-target investigation. Personal communications with Dr. H. M. Wadsworth, Dr. J. J. Talavage, Mr. R. P. Zimmer, et al., indicate that some work has been done in this area, but without too much success. Most of this work has been done with utility theory, and the difference in the utility of a target between, say, a field commander and the pilot flying the mission, can be quite large. Such possible outcomes as "aircraft destroyed before (after) acquiring target," "target acquired and partially (fully) destroyed," and "target not acquired" would necessarily imply that some cost be attached to the target in the evaluation of the cost of the outcome. The pen aids used obviously have an affect on the nature of the threat which is met by the aircraft in the area of the target. Hence they affect the outcome of the mission with respect to the target. For example, if pen aids inhibit the use of enemy ground-to-air missiles and/or interceptor aircraft, the probability of accurate camera work and/or bombing is higher; however, the primary purpose of this thesis is to aid in research dealing with the use of pen aids in the entry to and exit from the target area. Hence, we will leave our possible outcomes quite loosely defined.

It should also be noted that, if a dollar cost is attached to the completion of the mission with respect to the target, a negative expected

cost would be possible. If so, this negative expected cost could well be used as a criterion on which to base the resolution of whether or not the mission should be run. However, this thesis does not purport to propose a decision rule on whether or not a mission should be run. Given that the mission is going to be run, we wish to determine the best mix of pen aids to be used on the mission.

The mixes of jammers to be tested in the analysis must be defined. What mixes are to be tested should be decided by a person familiar with the radar and penetration aid fields. This thesis does not try to investigate how these γ 's should be determined. Rather, it assumes predetermined γ 's and discusses, and in some cases demonstrates, how one would obtain the amounts of the various pen aids to be used in testing for these γ 's.

Under normal circumstances, the expert who determines the mixes to be tested would probably have some information concerning the pen aid(s) to be tested before the actual selection of the mixes is accomplished. When this is the case, relatively few mixes may have to be investigated. This is true since the researcher may be able to use this prior information to eliminate many of the mixes theoretically possible. For example, assume that it is known that a certain minimum power requirement exists for active jammers to be effective against some specified threat. Further assume that for the first jammer under consideration, ten units of that jammer are required to meet this minimum power requirement. Thus the researcher could decide to make the smallest mix involving only that jammer one containing ten units. Additional mixes could be defined using more units of this jammer at the researcher's discretion.

To continue with this example, assume that a second jammer is to undergo analysis, and that it is desired to analyze this jammer singly and in conjunction with the first jammer. Assume that the power capabilities of this second jammer are not fully known, but that it is thought that approximately 1,000 units of this jammer would be sufficient to effectively jam the threat under consideration. With these considerations in mind, the researcher might decide to tentatively select the following mixes to be analyzed:

- γ_{11} = 10 units of jammer type 1
- γ_{12} = 12 units of jammer type 1
- γ_{13} = 14 units of jammer type 1
- γ_{14} = 20 units of jammer type 1
- γ_{21} = 800 units of jammer type 2
- γ_{22} = 1,000 units of jammer type 2
- γ_{23} = 1,200 units of jammer type 2
- γ_{31} = 3 units of jammer type 1 and 700 units of jammer type 2
- γ_{32} = 5 units of jammer type 1 and 500 units of jammer type 2
- γ_{33} = 7 units of jammer type 1 and 300 units of jammer type 2.

Note that these original mix selections are only tentative: information gained in the initial analyses could modify the mixes yet to be analyzed.

In the above example, no reference was made to aircraft which might be used to carry these jammer mixes. The researcher may wish to determine how many aircraft would be required to carry a particular mix. Alternatively, he may wish to determine if a particular mix can be carried by a certain number of aircraft. One technique for solving such problems is integer programming, which will be discussed below and in the appendices.

For example, if only one type of penaid is under consideration, it may be desirable to ascertain the maximum amount of this type of penaid capable of being carried by one aircraft. An integer programming problem would be solved to identify the maximum number of penaid that may be carried without violating the constraints imposed by the aircraft under consideration.

Next, consider the situation in which no prior information concerning the penaid is known. This may be the case when a proposed jammer is to be evaluated, and analytical analysis is not possible. (Thus, simulation must be used.) The researcher may wish to investigate a broad spectrum of mixes, using this jammer singly and in conjunction with others. Included in this investigation might be an attempt to ascertain the optimal mix to be used by one aircraft. In the example for which K , F , and the aircraft data were given above, it is assumed that this is the case. Note that the formulation for Γ given below would, in general, be applicable only in the case where no prior information is known. This type of mix determination would not usually be constructed when there is prior information and specified jammers and mixes are to be investigated.

If a jammer is being tested with which the investigator is unfamiliar, the ratios undergoing analysis may be far from the optimal ones. Personal communications with Dr. H. A. Ecker and Mr. R. P. Zimmer reveal that the situation normally encountered is one wherein a minimum amount of a jammer must be used in order to have any effect on an enemy radar (generally because of power requirements). If this is the case, a large difference in the probabilities obtained in the simulation should

be evident. This difference for the mixes tested could be the result of one mix containing not enough pen aids to reach, and the other a number at or above the critical point. If this situation occurs, more simulation may be in order to enable the critical point to be determined. Of course whether this would be worthwhile would depend upon factors such as the probabilities obtained, costs, and time. The initial mixes considered should be of a range wide enough to discover if and when the jump in probabilities occurs. If simulation is the investigation technique, relatively few initial points should be run (with the proviso that the range is sufficient) since the process of simulation is expensive. That is, the initial investigation should consider a few points which span the entire feasible region in order to determine if a critical point occurs.

In our example, assume that two jammers are being tested about which no information is known. We wish to determine the optimal mix of these two jammers which can be carried on one aircraft. Thus we decide to define our mixes as::

$$\Gamma = \left[\begin{array}{l} \gamma_0 = \text{no jammers used} \\ \gamma_{11} = \frac{1}{2} \text{ of the aircraft capacity used for jammer} \\ \quad \text{type 1 (other half empty)} \\ \gamma_{12} = \text{all of the aircraft capacity used for jammer} \\ \quad \text{type 1} \\ \gamma_{21} = \frac{1}{2} \text{ of the aircraft capacity used for jammer} \\ \quad \text{type 2 (other half empty)} \\ \gamma_{22} = \text{all of the aircraft capacity used for jammer} \\ \quad \text{type 2} \\ \gamma_{31} = \text{numerically, 2 of jammer type 1 to 1 of jammer} \\ \quad \text{type 2 } \pm 20\% \\ \gamma_{32} = \text{numerically, a 50:50 mix of jammers type 1 and 2 } \pm 20\% \\ \gamma_{33} = \text{numerically, 1 of jammer type 1 to 2 of jammer} \\ \quad \text{type 2 } \pm 20\% \end{array} \right.$$

In defining the subscripts of the γ 's, the following convention

was used and will be followed in this thesis:

- 0 → no jammers used
- 1x → jammer type 1 only used
- 2x → jammer type 2 only used
-
- mx → jammer type m only used
- (m+1)x → the various mixes containing more than one
type of jammer where the second subscript
denotes some arbitrary subgroup.

There is one important factor which has not been mentioned up to this point. This factor is the strategy to be employed in the use of these various mixes. The timing and location of dispersal of the pen-aids undergoing test is obviously a very important factor in their effectiveness. This consideration could be handled by adding a third subscript onto the γ 's to denote the strategy employed in that particular evaluation. Alternatively, we could specify the strategy used beforehand and consider it as an input parameter. These techniques are applicable to any other factors which are found to be important throughout the course of the research.

Using the aircraft parameters and γ 's as given, integer-programming problems are solved and the following results are obtained. The formulations of these problems are given in Appendix A in the context of a similar example.

γ_0 : $X_1 = 0$ $X_2 = 0$	γ_{22} : $X_1 = 0$ $X_2 = 6$
γ_{11} : $X_1 = 25$ $X_2 = 0$	γ_{31} : $X_1 = 11$ $X_2 = 5$
γ_{12} : $X_1 = 50$ $X_2 = 0$	γ_{32} : $X_1 = 6$ $X_2 = 6$
γ_{21} : $X_1 = 0$ $X_2 = 3$	γ_{33} : $X_1 = 3$ $X_2 = 6$

This example uses only weight and volume constraints. This is not intended to imply that there may not be other constraints extant in a real-world situation, but rather that these are typical. A full explanation of the word "numerically" preceding and the "420%" following the definitions of the last three mixes in Γ is given in Appendix A.

Returning to our example, assume that a simulation study is run, and the probabilities are obtained as below. For convenience, we adopt the convention that $P_{\gamma_{\ell}^k r} = P_{\ell r}$. For example, $P_{\gamma_{00}^k o} = P_{oo}$.

<u>γ_o</u> :	$P_{o1} = 0.40$	<u>γ_{11}</u> :	$P_{111} = 0.37$	<u>γ_{12}</u> :	$P_{121} = 0.30$
	$P_{o2} = 0.40$		$P_{112} = 0.43$		$P_{122} = 0.45$
	$P_{o3} = 0.20$		$P_{113} = 0.20$		$P_{123} = 0.25$
<u>γ_{21}</u> :	$P_{211} = 0.30$	<u>γ_{22}</u> :	$P_{221} = 0.25$	<u>γ_{31}</u> :	$P_{311} = 0.20$
	$P_{212} = 0.30$		$P_{222} = 0.25$		$P_{312} = 0.25$
	$P_{213} = 0.40$		$P_{223} = 0.50$		$P_{313} = 0.55$
<u>γ_{32}</u> :	$P_{321} = 0.23$	<u>γ_{33}</u> :	$P_{331} = 0.25$		
	$P_{322} = 0.25$		$P_{332} = 0.25$		
	$P_{323} = 0.52$		$P_{333} = 0.50$		

The next step is to evaluate the expected cost for each γ_{ℓ} . Recall that

$$\begin{aligned}
 E(C) &= D + S \\
 &= \sum_j d_j X_j + \sum_k P_{\gamma_{\ell}^k r} \cdot f_k
 \end{aligned}$$

In the following, the subscript on the expected cost, $E(C)$, will be the

same as on γ_ℓ (i.e., an ℓ) in order to readily distinguish the different mixes.

$$\begin{aligned} E_0(C) &= [0] + [(0.40)(100,000) + (0.40)(1,000) + (0.20)(100)] \\ &= \$40,420 \end{aligned}$$

$$\begin{aligned} E_{11}(C) &= [(25)(100)] + [(0.37)(100,000) + (0.43)(1,000) + (0.20)(100)] \\ &= \$39,950 \end{aligned}$$

$$\begin{aligned} E_{12}(C) &= [(50)(100)] + [(0.30)(100,000) + (0.45)(1,000) + (0.25)(100)] \\ &= \$35,475 \end{aligned}$$

$$\begin{aligned} E_{21}(C) &= [(3)(5,000)] + [(0.30)(100,000) + (0.30)(1,000) + (0.40)(100)] \\ &= \$45,340 \end{aligned}$$

$$\begin{aligned} E_{22}(C) &= [(6)(5,000)] + [(0.25)(100,000) + (0.25)(1,000) + (0.50)(100)] \\ &= \$55,300 \end{aligned}$$

$$\begin{aligned} E_{31}(C) &= [(11)(100) + (5)(5,000)] \\ &\quad + [(0.20)(100,000) + (0.25)(1,000) + (0.55)(100)] \\ &= \$46,405 \end{aligned}$$

$$\begin{aligned} E_{32}(C) &= [(6)(100) + (6)(5,000)] \\ &\quad + [(0.23)(100,000) + (0.25)(1,000) + (0.52)(100)] \\ &= \$53,902 \end{aligned}$$

$$\begin{aligned} E_{33}(C) &= [(3)(100) + (6)(5,000)] \\ &\quad + [(0.25)(100,000) + (0.25)(1,000) + (0.50)(100)] \\ &= \$55,600 \end{aligned}$$

Using the figures as given, the optimal ratio and mix is γ_{12} . If there is confidence in the probabilities used, this is the mix to be chosen in a situation in the real-world which closely matches the mission, threat, and other parametric stipulations used in the probability evaluations.

However, if the researcher does not have complete confidence in the probabilities generated (e.g., they were generated via simulation and there are inherent inaccuracies), a sensitivity analysis is indicated. A sensitivity analysis is accomplished by holding constant the deterministic cost and some subset of the probabilities. Then other probabilities (or sets of probabilities) are varied and the results graphed. For example, since the cost of failure is much higher than the other costs, it is decided that it is important that these probabilities should be varied in order to determine the effect of changes in this particular probability. Since the cost of Success is relatively small, hold the associated probabilities constant and vary those probabilities associated with Partial Success. The resultant graph is depicted in Figure 2. The circles indicate the points which were first evaluated. For brevity, a graph of $E(C)$ versus P_{11} when P_{12} was held constant and P_{13} was varied concurrently with P_{11} was not drawn. Because of the artificiality of the figures in the example, this graph would be much the same as the one depicted. A situation where both graphs are drawn is in Appendix A.

The next procedure is to evaluate the accuracy of the probability estimation procedure used (e.g., simulation). If the researcher is confident that the probabilities generated are within, say, 0.1 of being correct, the min-max table of costs given in Table 3 could be constructed. Table 4 shows the same type of data for a difference of 0.05 in the probabilities.

Using Table 3 (i.e., $\Delta = 0.1$), note that γ_{22} , γ_{32} and γ_{33} could be eliminated from further consideration since their best possibilities are approximately equal to the cost of the worst possibilities of γ_0 ,

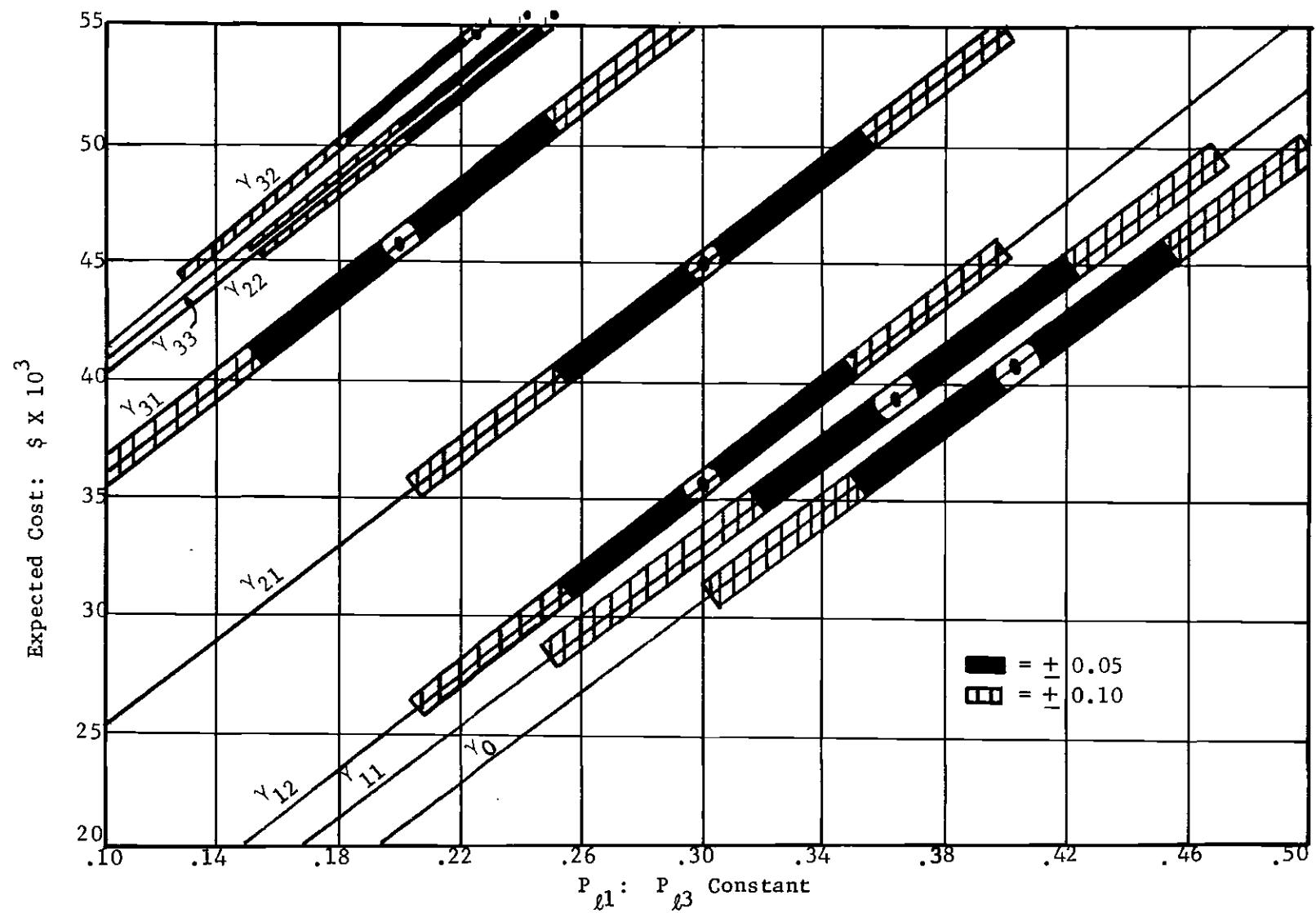


Figure 2. Sample Sensitivity Graph

Table 3. Expected Costs for a Sensitivity Analysis, $\Delta = 0.10$

γ_ℓ	Original Estimate of $P_{\ell 1}$	+0.10	Cost x 10^3	-10.0	Cost x 10^3
γ_0	0.40	0.50	50.2	0.30	30.4
γ_{11}	0.37	0.47	49.8	0.27	27.7
γ_{12}	0.30	0.40	45.5	0.20	25.4
γ_{21}	0.30	0.40	55.1	0.20	35.3
γ_{22}	0.25	0.35	65.7	0.15	45.4
γ_{31}	0.20	0.30	56.3	0.10	36.4
γ_{32}	0.23	0.33	63.1	0.13	44.2
γ_{33}	0.25	0.35	66.0	0.15	45.7

Table 4. Expected Costs for a Sensitivity Analysis, $\Delta = 0.50$

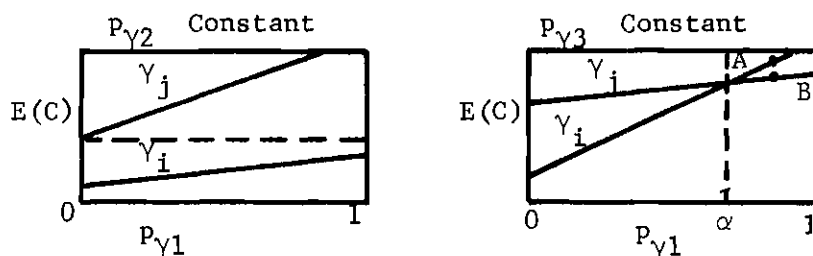
γ_{ℓ}	Original Estimate of $P_{\ell 1}$	+0.05	Cost $\times 10^3$	-0.05	Cost $\times 10^3$
γ_0	0.40	0.45	45.3	0.35	35.8
γ_{11}	0.37	0.42	44.9	0.32	34.9
γ_{12}	0.30	0.35	40.6	0.25	30.5
γ_{21}	0.30	0.35	50.3	0.25	40.4
γ_{22}	0.25	0.30	60.3	0.20	50.3
γ_{31}	0.20	0.25	51.0	0.15	41.3
γ_{32}	0.23	0.28	59.1	0.18	49.2
γ_{33}	0.25	0.30	60.4	0.20	50.7

γ_{11} , and γ_{12} . It may be desirable to keep γ_{21} and γ_{31} under consideration to meet the possibility that P_{01} , P_{11} , and P_{12} have all been rated too low and this fact is ascertained. In such a case, of course, P_{21} and P_{31} would also have to be revalidated. However, the main interest should be centered around the probabilities for γ_0 , γ_{11} , and γ_{12} . Since there is a great deal of overlap in the costs for these mixes, a slight change in the relevant probabilities could well change the mix to be chosen. If the probabilities are obtained via expensive simulation and the only method to revalidate the probabilities is simulation, then, based upon the graphs, the number of mixes chosen for further study should be as small as is logically feasible. In the example, only γ_0 , γ_{12} and γ_{21} might be selected for further simulation study. This would eliminate γ_{11} and γ_{31} which could possibly be optimal if the situation described above were to occur. The decision, of course, is up to experimenter, and would depend on factors such as time and money.

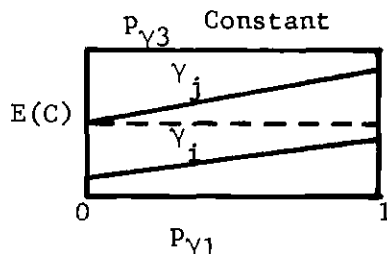
Using Table 4 (i.e., $\Delta = 0.05$), one can again immediately eliminate γ_{22} , γ_{32} and γ_{33} . In this case, however, one can also immediately eliminate γ_{21} and γ_{31} from further consideration. In this situation, both of these are dominated by γ_{12} . Thus, only γ_0 , γ_{11} and γ_{12} remain to be further investigated. As before, it might be decided to eliminate γ_{11} from further study.

Once the feasibility solutions are available, and before simulation, this type of graph could have been plotted. However, the operating ranges would be unknown. By plotting before simulation, the situation in which one particular mix is completely dominated by or dominates

another mix(es) at all probabilities (i.e., from zero to one) can be discovered and the dominated mixes can be eliminated before the simulation is run. Note however that all subsets to be varied must reveal dominance before a mix can be eliminated. For example, with three mixes being considered, the graphs might be of the form:



in which case Y_j could not be eliminated before the simulation since the probabilities could be such that p_{Y1} is in the interval $[\alpha, 1]$ and p_{Y2} and p_{Y3} such that that portion of the graph were applicable (e.g., points A and B as shown above). However, if holding p_{Y3} constant yielded



we could eliminate Y_j before the simulation since it is completely dominated by Y_i .

Notice that the graphs of Expected Cost versus Probabilities are linear. However, there is a possible nonlinearity demonstrated by these figures. For the mixes involving jammer type 1 only, a lower expected cost using 50 units was found than when using 25 units. Obviously, there is a critical point of some sort between 25 and 50 units. (Note that this did not occur using jammer type 2). From the data available, it is

impossible to decide whether this increase in effectiveness is a smooth increase, or if there is a minimum number at which there is a discontinuity and a jump in effectiveness. In this example there is only a difference of approximately \$5,000 in the expected cost of using no jammers and 50 jammers of type 1. (Remember that these figures are (purposely) completely artificial.) The differences in a true situation could be quite distinct. The deterministic cost of the penails and the differences in effectiveness (in the probabilities) could well yield such large differences in the expected cost that a further investigation to determine a graph of Number of Penails versus $E(C)$ becomes mandatory.

To further the understanding of the reader concerning sensitivity analyses, another example is given in Appendix A to this thesis. The same input costs, mixes, and determined probabilities are used, but the possible outcomes and their associated costs are different. Since the mixes to be used are the same, the integer programming feasibility problems will be the same as in our example. An opportunity cost is associated with the target, and thus negative expected costs are possible. Because of the differences in the f_k 's, it is necessary to alter both P_{12} and P_{13} , leaving the other constant, while varying P_{11} .

The discussion up to this point assumes that essentially no prior information concerning the probabilities is known, and an analytical analysis or a simulation is run in order to obtain this knowledge. If there is partial knowledge concerning the relevant mixes, say from experience with the penails involved, it is possible that the probabilities could be roughly estimated. In this case, one technique to obtain some idea of the sensitivity of the problem is to estimate min-max range on

the probabilities. These min-max figures could be entered into the expected cost equation and evaluated. This would yield not only some estimation of the sensitivity of the expected cost to the probabilities, but could indicate the areas into which further study is required. As an example, the expected cost is little sensitive to the probabilities associated with the third outcome (success) in the sample problem in this section. A pre-simulation examination would show that the value of the probabilities used in conjunction with the first outcome (i.e., Mission Failure) has a large effect on the expected cost and consequently the simulation study could be slanted to give the most precise accuracy in evaluating the P_{11} , and less accuracy in the P_{12} and P_{13} probabilities.

In the above, the only mission parameter discussed was that of the choice of aircraft, since the capacity of the aircraft affected the constraints. Not discussed were mission parameters such as terrain, timing, load, enemy threat capabilities, strategy, and the purpose of the mission. It is assumed that these were used in the generation of the probabilities. The only restriction indicated is that the mission is not to be repeated, since this would affect the probabilities (see Chapter IV). While parameters such as those mentioned are vitally important to the use and effectiveness of penetration aids, it is not the purpose of this thesis to delve into them. Rather, it is assumed that these facts will be accounted for in the analysis and/or simulation studies. While different input parameters will undoubtedly lead to different optimal mixes and ratios, the results of the simulation studies to be used in this cost-effectiveness model are the probabilities. From these probabilities, a cost-effectiveness study can be made using the

model presented here.

More Than One Aircraft

This situation is very similar to the one in the previous section. The problem is larger and slightly more complicated, but the situation in which a particular threat is to be encountered only once or twice is still under consideration. The notation will be slightly different (see Table 2) in order to handle the fact that a flight of aircraft is being used instead of just one.

When considering the feasibility problem, there are four different possibilities, and each must be handled slightly differently.

(1) The deployment of the pen aids among the aircraft is unimportant. The feasibility problem is:

$$\begin{aligned}
 & \text{Max } X_{11} \\
 & \text{s.t.} \quad - \text{ratio constraints} - \\
 & * \sum_i \sum_j w_j X_{ij} \leq \bar{W} \\
 & * \sum_i \sum_j v_j X_{ij} \leq \bar{V} \\
 & \sum_j w_j X_{ij} \leq W_i \quad \forall i \\
 & \sum_j v_j X_{ij} \leq V_i \quad \forall i \\
 & X_{ij} \geq 0, \text{ Integer } \forall i, j.
 \end{aligned}$$

A simple example of a ratio constraint would be:

$m = 2$; a 50:50 $\pm 10\%$ ratio

$$0.9 \sum_i X_{i1} \leq \sum_i X_{i2} \leq 1.1 \sum_i X_{i1}$$

Notice that

$$\sum_i W_i = \bar{W} \text{ and } \sum_i V_i = \bar{V}$$

and consequently constraints marked * are redundant in the formulation and need not be used in the actual solution procedure. They are included here simply for clarification. It is also true that if all the aircraft are identical, $W_i = W$ and $V_i = V$.

(2) In the case in which it is desired that one aircraft carry all the penaid, the problem is identical to the single aircraft problem given in the "Single Aircraft" section of this chapter.

(3) When it is desired that one aircraft carry at least a certain amount of some type of penaid, simply add another constraint. For example, aircraft #1 carries at least 10 units of penaid type 2. Add the constraint $X_{12} \geq 10$ (assuming that this is feasible).

Note that the capacity constraints will not allow this aircraft to be overloaded, and the ratio constraints will maintain the correct overall ratio.

(4) The most intricate case is one in which certain aircraft in the flight must maintain a given ratio of certain jammers, and the overall flight must maintain a different overall ratio of jammers.

For example, in a flight of five aircraft and three penaid types ($n = 5$, $m = 3$), a strict ratio of two jammers of type 1 to one jammer

of type 3 must be maintained in both aircraft #1 and #2. The overall flight ratio of jammers must be $\{1:1:1\} \pm 6\%$. Assume that the specific mix of the other three aircraft is unimportant (if not, more constraints of a similar nature are required). The feasibility problem is:

$$\begin{aligned}
 & \text{Max } X_{11} \\
 & \text{s.t. } \sum_{j=1}^3 w_j X_{ij} \leq W_i, \quad i = 1, \dots, 5 \\
 & \quad \sum_{j=1}^3 v_j X_{ij} \leq V_i, \quad i = 1, \dots, 5 \\
 & \left. \begin{aligned}
 0.97 \sum_{i=1}^5 X_{i1} &\leq \sum_{i=1}^5 X_{i2} \leq 1.03 \sum_{i=1}^5 X_{i1} \\
 0.97 \sum_{i=1}^5 X_{i1} &\leq \sum_{i=1}^5 X_{i3} \leq 1.03 \sum_{i=1}^5 X_{i1}
 \end{aligned} \right\} \text{Overall Ratio} \\
 & X_{11} = 2X_{13} \\
 & X_{21} = 2X_{23} \\
 & X_{ij} \geq 0, \text{ Integers } \forall i, j
 \end{aligned}$$

Several comments concerning the feasibility problem for multiple aircraft and multiple pen aids are in order. The first thing to note is the size of the problem. In the case of a single aircraft, the problem was fairly easy to solve, and possibly could be done mentally. It is obvious that with just a few aircraft and pen aids, the problem becomes so large that it would be quite laborious to solve by hand. Consequently, it is recommended that a mixed-integer programming computer program be

obtained and used.

The second comment concerns the constraints themselves. When forming the constraints, the size of the problem often makes it difficult to keep track of the implication of each constraint. If one is not careful, one could restrict the problem so much as to eliminate any feasible solution. Sometimes, this will not at all be obvious by inspection. Hence, it is recommended that all formulations be double checked, preferably by a person other than the originator, to eliminate the waste involved in having to redo the whole problem.

In this connection, it should also be noted that the greater the number of constraints, the smaller is the feasible space. This fact could lead to a legitimately optimal solution to the integer programming problem which is far from the optimal solution to the practical problem being considered. This is especially true when an equality constraint(s), such as in (4) above, is entered into the problem. This type of constraint, when coupled with the fact that integer solutions are the only acceptable ones, greatly restricts the solution space of the integer programming problem, and may eliminate it entirely. Additionally, since the solutions of these integer programming may be used in simulation, with its inherent limitations, the author feels that an exact equality constraint would be inadvisable in this investigation. The $x:y+z\%$ would seem to be much more practical. (The $x:y+z\%$ is discussed more fully in Appendix A).

Once the feasibility problems have been solved, the procedure for evaluating the expected costs and the sensitivity analyses are the same as in the single aircraft case. The primary difference will be

the size of the problem. The possible outcome states, K , as well as the mixes and ratios to be considered, Γ , will contain many more elements in order to account for the increased number of aircraft and pen aids. It is recommended that a computer program be written in order to handle most of the manipulations.

Up to this point, the discussion has concentrated purely on non-expendable penetration aids. Ignored has been the fact that, in some cases, the aircraft is configured such that expendable pen aids, non-expendable pen aids, and ordnance are all carried by the aircraft in the same manner and thus one is used at the expense of the others. An example might be an aircraft which carries all such items in pods mounted below its wings and fuselage, and has only a fixed number of locations at which these pods can be added. Thus we must decide upon the allocations to be used.

The usual condition will be one in which a minimum and/or fixed amount of ordnance will be required, and the rest of the available capacity would be available for penetration aids. In the case where it is certain that only a fixed number of aircraft are to be used, the problem is as covered earlier in this section, since the total capacity can be reduced by ordnance requirements and the rest of the capacity is available for penetration aids. Even in this case, however, the researcher must still ascertain the precise allocation of the various types of pen aids to be used among the aircraft. The optimal allocation amongst the aircraft is certainly not a trivial problem. However, this is a problem in defining the elements in Γ and their evaluation, as has been discussed above.

When the number of aircraft is not fixed, an interesting situation develops. If a certain amount of ordnance is to be delivered, what is the optimal number of aircraft to be used? Should the minimum number of aircraft capable of carrying the ordnance be used, or should additional aircraft, with the added capacity for penails, be added to the flight? It is recommended that a cost-effectiveness marginal analysis be used to solve this problem. For example, suppose that some number, say t , of aircraft are necessary to carry the ordnance. First, a standard cost-effectiveness study on t aircraft is run, using any excess capacity for penails. Next, a cost-effectiveness study on $(t + 1)$ aircraft is run, using the extra capacity obtained for penails. Continue until the expected cost for the optimum mix for the last case investigated increases. Note that, for each number of aircraft investigated, the entire cost-effectiveness study, including feasibility, allocation, and sensitivity, must be accomplished in order to find the expected cost.

This process, at first glance, would seem to be a long one. If the research were dealing with only effectiveness, it possibly could be. However, this research is interested in cost-effectiveness. The addition of another aircraft will undoubtedly increase effectiveness, though there is probably an upper limit on this in any given situation. But the addition of extra aircraft and penails increases the deterministic cost, as well as adding elements to K which would affect the probabilistic cost (e.g., there are more aircraft which could be lost). These factors would tend to limit the number of investigations required. In addition, the researcher may have prior knowledge concerning the penails to be used. If this is the case, he could use this knowledge not only in the

evaluation of the cost-effectiveness for particular mixes, but it could be an aid in deciding the number of aircraft to be used. (For example, should he investigate t aircraft and $(t + 2)$ aircraft and not bother to investigate $(t + 1)$ aircraft?)

Note that the same type of investigation is applicable if certain types of non-expendable penetration aids are used, either without or in conjunction with expendables. Instead of a d_j as we have defined it, a "usage cost per use" would be appropriate.

The same question can be observed from another viewpoint. Say that a particular aircraft has three pod mounts to carry whatever is desired. It is known that a particular mix of penetration aids (either expendables, non-expendables, or both) is optimal. This mix requires the use of two of the pod mounts, leaving one for ordnance. But the minimum ordnance requirement necessitates the use of two of the pod mounts. Thus, if it is to be used, this particular mix requires the addition of another aircraft to the mission. When this situation occurs, it is evident that the researcher must accomplish two evaluations: the first being one aircraft using the best mix which can be mounted on one pod; the other being the use of two aircraft, utilizing the two pods not specifically taken to the best advantage.

Thus it is readily apparent that multiple aircraft usage involves a larger size problem, more numerous investigations, and questions arise not found in the single aircraft case for which solutions must be found. Techniques for solution to some of these problems have been presented in this section. Admittedly the solution procedures recommended here are quite long, time-consuming, and possibly very expensive. However, the

real-world problem being investigated is not a simple one by any means. If several million dollars are lost when an aircraft is shot down, it would seem quite cost-effective to spend several million dollars in an investigation which would yield results which could well save dozens or hundreds of aircraft.

CHAPTER IV

THE EFFECT OF REPEATED MISSIONS

Up to now, this thesis has been concerned with missions which are planned to be run over the same terrain, enemy threat, etc., only once, and will be run more than once only if required by mission failure. In the generation of the corresponding probabilities, the mathematical independence of these probabilities has been implicitly assumed. This chapter will deal with the generation of these probabilities without the assumption of independence. Such is the case when the same basic mission is to be run many times over the same threat and locale. A typical example from the current world situation would be the bombing runs across the Suez Canal by the Israelis.

There are two major reasons why, in this type of situation, the probabilities are not independent. The first concerns the effect of the prior missions on the enemy threat itself. Assume that the original (i.e., before any missions are run) threat consists of some number of, say, a radars and b anti-aircraft artillery (AAA). On the first mission, we destroy one radar set and no AAA. There is no destruction of threat on the second mission, but on the third we destroy two AAA. Thus the second and third missions were run against a different threat than the first, and the fourth will face yet another threat. Hence, the destruction, and possible erection, of enemy threat capabilities should be considered in the determination of the probabilities.

The second major reason for the dependence of the probabilities

is the effect that the continued missions will have on the radar and AAA operators; they will undoubtedly learn from their experience. This increase in experience on the part of the operators should be taken into account when determining the probabilities.

This problem can be dealt with through the theory and application of Markov chains with stationary transition probabilities. In such Markov chains, the outcome of a particular mission depends not only on that mission itself, but on the mission directly preceding it, and only on it. Thus, to continue with our example, the outcome of the fourth mission is directly dependent on the threat and learning state left by the third mission only, as well as the number of aircraft remaining.

In what follows, the theory of Markov chains will not be discussed. Rather, discussion and demonstration of the use of such concepts in this situation will be presented. An excellent reference on Markov chains is Feller (6), whose terminology and notation will be used here.

As noted in Table 2, we let E_i denote the possible states.

$$\begin{aligned}\rho_{ij} &= \text{Pr (going from state } E_i \text{ to } E_j \text{ in one transition)}^* \\ \rho_{ij}^{(n)} &= \text{Pr (going from state } E_i \text{ to } E_j \text{ in } n \text{ transitions)}^* \\ &= \sum_v \rho_{iv} \rho_{vj}^{(n-1)}\end{aligned}$$

P = the stochastic matrix of transition probabilities

* Feller uses p_{ij} and $p_{ij}^{(n)}$. We will use ρ (rho) to avoid confusion with the p_{yk} 's.

For purposes of illustration, let

$$\begin{aligned}
 E_1 &= x \text{ radars, } y \text{ AAA, learning state } z, u \text{ aircraft} \\
 E_2 &= (x-1) \text{ radars, } y \text{ AAA, learning state } z, u \text{ aircraft} \\
 E_3 &= x \text{ radars, } (y-1) \text{ AAA, learning state } z, u \text{ aircraft} \\
 E_4 &= (x-1) \text{ radars, } (y-1) \text{ AAA, learning state } z, u \text{ aircraft} \\
 E_5 &= x \text{ radars, } y \text{ AAA, learning state } (z+1), u \text{ aircraft} \\
 E_6 &= x \text{ radars, } (y-1) \text{ AAA, learning state } (z+1), u \text{ aircraft} \\
 E_7 &= (x-1) \text{ radars, } y \text{ AAA, learning state } (z+1), u \text{ aircraft} \\
 E_8 &= (x-1) \text{ radars, } (y-1) \text{ AAA, learning state } (z+1), u \text{ aircraft} \\
 E_9 &= x \text{ radars, } y \text{ AAA, learning state } z, (u-1) \text{ aircraft} \\
 E_{10} &= (x-1) \text{ radars, } y \text{ AAA, learning state } z, (u-1) \text{ aircraft} \\
 E_{11} &= x \text{ radars, } (y-1) \text{ AAA, learning state } z, (u-1) \text{ aircraft} \\
 E_{12} &= (x-1) \text{ radars, } (y-1) \text{ AAA, learning state } z, (u-1) \text{ aircraft} \\
 E_{13} &= x \text{ radars, } y \text{ AAA, learning state } (z+1), (u-1) \text{ aircraft} \\
 E_{14} &= x \text{ radars, } (y-1) \text{ AAA, learning state } (z+1), (u-1) \text{ aircraft} \\
 E_{15} &= (x-1) \text{ radars, } y \text{ AAA, learning state } (z+1), (u-1) \text{ aircraft} \\
 E_{16} &= (x-1) \text{ radars, } (y-1) \text{ AAA, learning state } (z+1), (u-1) \text{ aircraft}
 \end{aligned}$$

The corresponding P matrix, with corresponding states shown, is

$$P = \begin{array}{c} \begin{array}{cccccc} E_1 & E_2 & E_3 & \cdot & \cdot & \cdot & E_{16} \end{array} \\ \left[\begin{array}{cccccc} p_{11} & p_{12} & p_{13} & \cdot & \cdot & \cdot & p_{1 \ 16} \\ p_{21} & p_{22} & p_{23} & \cdot & \cdot & \cdot & p_{2 \ 16} \\ p_{31} & p_{32} & p_{33} & \cdot & \cdot & \cdot & p_{3 \ 16} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ p_{16 \ 1} & p_{16 \ 2} & p_{16 \ 3} & \cdot & \cdot & \cdot & p_{16 \ 16} \end{array} \right] \begin{array}{c} E_1 \\ E_2 \\ E_3 \\ \cdot \\ \cdot \\ \cdot \\ E_{16} \end{array} \end{array}$$

In general, the number of states defined should be as numerous as is possible. The limitation which should be of prime importance is the number of and accuracy with which the p 's can be generated or estimated. The x , y , and z used in defining the E_i could be viewed as individual, specific units, or as sets or classes of units. This would depend on the effect each of these units has on the outcome of the mission. For example, there might be no noticable difference between 27 and 26 radar units, but a great deal of difference between two and one radar units. How the different possible outcomes are defined is, of course, left to the discretion and experience of the experimenter. However, some simple examples should help to clarify what is meant.

Assume that the penaid under consideration is packages of chaff, cut to be effective against one particular frequency. If the enemy radar consists solely of units carrying that frequency, and the chaff packages are properly dropped (i.e., with relationship to the location of the radars and the route the aircraft are to fly), the physical number of radar units might be unimportant due to the nature of chaff jamming. Thus the researcher might decide to define only two different possibilities:

- (1) There are radar units on this frequency, and
- (2) There are not radar units on this frequency.

An entirely different situation would be met if the penaid under consideration were an active "repeater" type of jammer with a unidirectional antenna. This type of jammer accepts the radar signal, delays it, and returns the same signal to the sending radar set in order to create erroneous range and/or altitude indications. As in all work of this type,

the location of the enemy radar relative to the planned flight path is important. However, assume that this jammer has the capability to effectively jam, for example, five different radars located in one central area. Thus, the researcher must be concerned with more than five radars in the central area, and radars located outside of this central area, since these are the radars which will not be jammed by this penaid. Further assume that there are three basic states of enemy effectiveness possible when considering only radar. These are:

- (1) No non-jammed radar,
- (2) One or more non-jammed radar, but their locations such that triangulation is not possible, and
- (3) Two or more non-jammed radars located such that triangulation is possible.

With this consideration in mind, it might be decided to establish the following sets of radar classes to be used in defining our Markov states:

- (1) Five or less radars in one location; no other radar.
- (2) Five or less radars in one location; other radar exist, but located such that triangulation is impossible.
- (3) Five or less radars in one location; other radar exist, and triangulation is possible.
- (4) Six or more radar in one location; no other radar.
- (5) Six or more radar in one location; at least one other radar (which implies that triangulation is possible).

Note that we may wish to condense these five possible states into three possible states: (1) would be one state; combine (2) and (4) into one state; and (3) and (5) would be the last state. This particular set would be aligned with the definition of the states of enemy radar effectiveness.

Possible sets of AAA for use in defining Markov states must include the type of weapons to be met: for example, ground fire, recoilless rifle, enemy fighter aircraft, or ground-to-air missiles. Note that the effectiveness of radar jamming may have a large effect on the AAA available for use by the enemy. Such things as recoilless rifles and ground-to-air missiles are often controlled directly by radar and/or radar operators.

Next, a brief discussion of possible "learning" states is in order. The continual flying of the same mission over the same target/threat will give the enemy operators experience. The interest of this research is centered in two capabilities on the part of the radar operator and his equipment which will directly affect the outcome of our missions. The first of these is the increase in the probability of detection which will result when the same type of jamming is used in a day-after-day situation. Personal communication with Dr. T. L. Sadosky reveals that the general nature of this type of situation is as depicted in Figure 3. N is the number of times which the same type of jamming is used over the same target/threat. We do not specify units on N because this will depend on the type of jamming, the skill of the operator, the terrain, weather, etc. Note that the probability of detection increases until it eventually reaches unity. The slope of the line, of course, represents the rate with which probability of detection increases. The graph as depicted is continuous. However, for use in the Markov chain, discrete possibilities must be defined. One possible breakdown is into three conditions as indicated on the graph. Thus, there would be a slow learning state, a fast learning state, and another slow learning

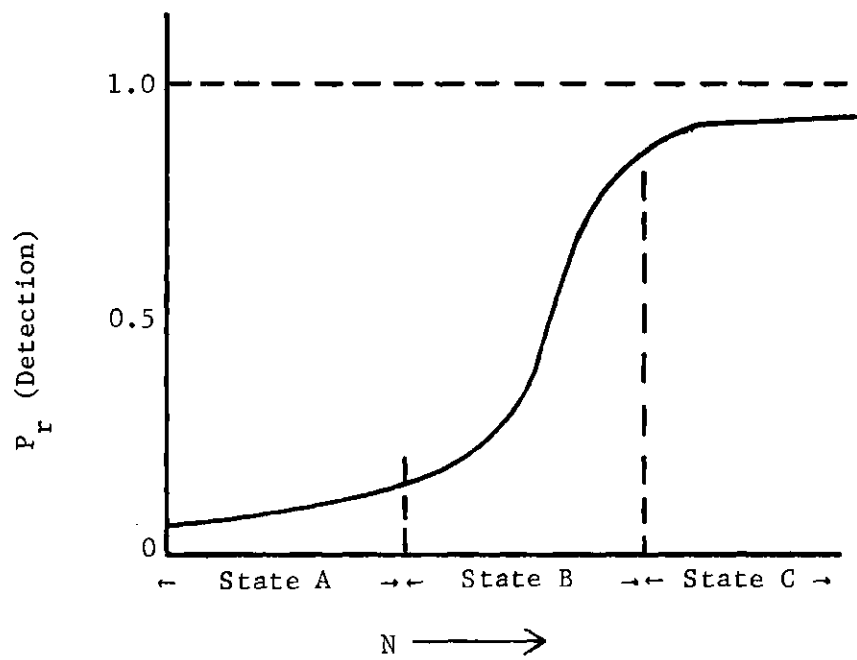


Figure 3. Probability of Detection Curve.

state.

Figure 4 represents the other important facet of the experience gained by the enemy. Given that the aircraft are detected, how effective are the actions of the enemy in shooting them down? The curve is of the same general shape as the detection curve, but ends at some unspecified probability less than one. As mentioned before, it is felt that even with perfect knowledge as to the flight profile, e.g., speed, range, altitude, of the aircraft, the enemy will not always be able to destroy their targets. As before, this continuous curve must be broken into discrete states.

The "number of aircraft" portion of state definition would obviously depend on the number lost and replacements. As before, this category may be divided into sets.

It should be obvious that the categories listed are to be taken only as examples of possible categories. In fact, in research of this type, it may be desirable to eliminate the AAA, aircraft, and the curve depicting the probability of effective action given detection categories. That is, only in the number and location of radar sites and the probability of detection by these sites may be of interest. This would enable the researcher to obtain a relative measure of the effectiveness of various penetration aids. The depth and detail used in defining the states would be up to the experimenter, and would depend on the exact purpose of the research.

To return to the discussion of the matrix, notice that the size of P will increase greatly with the number of states defined. The number of states is in turn dependent on the number of sets (or units)

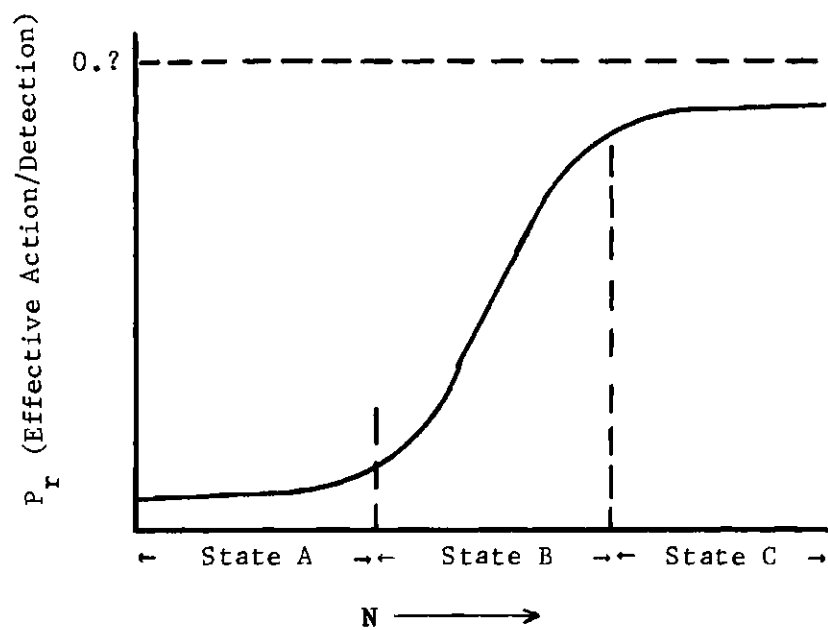


Figure 4. Probability of Effective Action Curve.

differentiated amongst in each category defined. The total number of states is found by multiplication of the number of classes in each category. For example, five classes within the "radars" category, four within "AAA," and six different learning states yield $5 \cdot 4 \cdot 6 = 120$ different states. (Above, we had $2 \cdot 2 \cdot 2 \cdot 2 = 16$ different states.) The point to be made is that size itself should not be a factor in determining how many states are defined. Electronic computer programs are available which are capable of effectively dealing with the procedures to be introduced here.

Theoretically, it would be possible to have two individual absorbing states in the chain. One such state would be perfect knowledge and capability on the part of the enemy operators. The other would be the complete destruction of the enemy radars. Practically, however, neither of these could occur. In the first case, it is felt that learning will reach a certain maximum and level off (reference personal communications with Dr. T. L. Sadosky). In any case, one would not wish to run missions against perfect opponents.

If total destruction of enemy radar capabilities occurred, jammers would not need to be carried. Hence, this case does not enter into the realm of interest to which this thesis is directed. Thus we will assume that no individual absorbing states exist in the chain. This does not rule out the possibility that a set of states is absorbing. For example, the mission may be such that our aircraft may be able to keep the number of enemy radar to five or less, but could not control the learning state of the operators. Thus, the set containing five or less radars and all learning states would be absorbing.

If it is known that the mission in this situation is to be run a fixed, finite number of times, say n , one can easily calculate the (absolute) probability that the chain will be in some particular state, say E_k , at the end of the n missions. Using the fact that the state in which the chain started is known, say E_j , the absolute probability of being in state E_k after n missions, $a_k^{(n)}$, is just

$$a_k^{(n)} = \rho_{jk}^{(n)} .$$

Several comments concerning the nature of this chain are in order. The first thing to notice is the fact that the ρ_{ij} 's will depend greatly on the purpose of the mission. If the objective is to destroy radar and AAA sites, the ρ_{ij} 's for $i > j$ in these categories may be relatively large as compared to the ρ_{ii} 's and ρ_{ji} 's. The ρ_{ij} 's for learning categories may tend to be smaller for $i < j$ and larger for $i \geq j$ due to the fact that operators will be lost and their replacements will not be as experienced. However, if the purpose of the missions is, for example, to destroy industry and transportation facilities, just the opposite will probably be true.

The chain is, by definition, closed. By considering the nature of the events it describes, it can also safely be stated that it is aperiodic.

Whether or not the chain is reducible depends on several factors. For example, in the radar and AAA categories, it would depend on the purpose of the mission. If the purpose is to destroy AAA and radar, how effective is the job done, and what is the reaction of the enemy to the destruction? If the enemy does not attempt to rebuild destroyed radar

and/or AAA sites, it is obvious that the states containing the destroyed radar are transient. Similarly, if the enemy does attempt to rebuild, but our aircraft intend to and are effective enough to keep him from reaching the previously high levels, these states would still be transient. Because of the learning the enemy operators obtain, and their casualties inherent in the process, it is felt that this category will not tend to make a state either transient or persistent. It is unlikely that the absorbing state of no enemy radars will be reached, which directly implies that there must always be some old radar sites remaining, or new ones built, no matter what action we take. With the situation as defined in this paragraph, the chain would be reducible, containing both persistent and transient states. Note that several closed sets are possible.

It is also possible that the conditions above do not apply, and that the chain is irreducible and all states are persistent. In this case, the entire chain is ergodic. We also note that there are no null states possible in this chain. The method of dealing with this type of chain will be discussed next.

We define the limits in an irreducible ergodic chain as

$$u_k = \lim_{n \rightarrow \infty} \rho_{jk}^{(n)}$$

which are independent of the initial state j . The u_k satisfy the properties:

- (1) $u_k > 0$
- (2) $\sum u_k = 1$

$$(3) \ u_j = \sum_i u_i \rho_{ij} \text{ for all } j.$$

The probability distribution defined by these limits, called the invariant distribution, is the one in which the researcher is interested since the assumption has been made that the mission is to be run an unknown, but large, number of times.

Rewriting property (3) in vector notation, we get

$$\underline{u} = \underline{u}P$$

$$\underline{u} (P - I) = \underline{0}$$

That we want the largest such solution is proved by Feller (pp 393-394). This is a typical eigenvector problem with $\lambda = 1$, and hence no example will be given. Note that if an ergodic chain which is not irreducible is under consideration, then $u_k = 0$ for all states which are transient.

Next, the case where a reducible chain may be divided into a class of transient states and a class of persistent irreducible states, will be considered. Given that the system starts from a transient state, there are two possibilities: either the system ultimately passes into one of the closed sets and stays there forever, or else the system remains forever in the transient set. Our problem consists in determining the corresponding probabilities.

The last question is easily answered. Since any chains encountered in this research will be strictly finite, the probability of remaining among the transient states forever is zero. In the situation in which there is only one (irreducible) closed set in the chain, the first question is also simply answered: the probability sought is unity.

Next, consider the case where there are several closed sets, and a set of transient states. Given that the chain starts from the transient set, the researcher wishes to know the respective probabilities of entering each of these closed sets. (Note that this is the same as the probability the chain will be in each of these sets after an infinite number of missions.) Denote by H the class of transient states and let R be any one of the closed sets of persistent states. Denote by y_i the probability of ultimate absorption in R given that the initial state is E_i . Feller (p. 403) proves that the probabilities y_i are given by the minimal non-negative solution of

$$y_i = \sum_H p_{iv} y_v + \sum_R p_{iv}$$

where E_v is some arbitrary state. The above summations extend over those v for which $E_v \in H$ and $E_v \in R$, respectively. Note that there may be several independent solutions, but we seek the minimal one.

As an example, consider

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 1/8 & 3/4 & 1/8 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that there is no loss of generality by assigning the value of 1 to an entire closed set. Our original P above may have been 30×30 , and rearranging and simplifying has been accomplished.

Obviously, $H = (E_2, E_3, E_4)$; $R_1 = (E_1)$; $R_2 = (E_5)$. For the probability of being absorbed into R_1 :

$$\left[\begin{pmatrix} 0 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/8 & 3/4 \end{pmatrix} - \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \right] \begin{pmatrix} y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow y_2 = 3/4, y_3 = 1/2, y_4 = 1/4$$

Thus, the probabilities of absorption into R_1 from the various states are:

$$\begin{aligned} E_1: & 1.00 \\ E_2: & 0.75 \\ E_3: & 0.50 \\ E_4: & 0.25 \\ E_5: & 0.00 \end{aligned}$$

Since there are only two absorbing states, the probabilities of absorption into R_2 are just $\{1 - \text{Pr}(\text{Absorbed in } R_1)\}$. Thus,

$$\begin{aligned} E_1: & 0.00 \\ E_2: & 0.25 \\ E_3: & 0.50 \\ E_4: & 0.75 \\ E_5: & 1.00 \end{aligned}$$

In general, further calculations will be required.

Finally, the relationship of these u_k 's and y_i 's to our original $p_{y_k^k}$'s must be discussed. This actually quite straightforward. Earlier, a specific mix was evaluated which resulted in a probability being generated for a specific outcome. In that situation, such factors as the level of learning, number of radar and number of AAA played no part in the determination of the probabilities except that they were fixed input

parameters.

In this situation, a specific mix is still under evaluation. Here, the above-mentioned factors are not stable, but are subject to change since the same mission is to be run many times. As before, possible outcomes are defined, but they are outcomes of the entire series of forays, not just the one raid. As before, these outcomes will have costs associated with them, and our objective is to minimize the expected cost. Depending on which is applicable, the costs of outcomes are multiplied by either the u_k 's or y_i 's in order to obtain the S portion, which is added to the D portion in order to obtain the expected costs.

It is envisioned that this type of analysis would be used as the final effort in a study of this sort. Thus, the experimenter would have all the figures generated by past research. This would enable the definition of the states, as well as be a great aid in the estimation of the ρ 's. Note that the ρ 's depend greatly on the definitions of the Markov states. With a judicious choice of state definitions, some, if not all, of the $p_{\gamma k}$'s found from previous work could be used as ρ 's in the matrix. The other ρ 's would be estimated by a worker familiar with the results of all previous work. In this way, the estimates of the ρ 's would not be far from their actual value. A sensitivity analysis on the ρ_{ij} 's could be accomplished as was done on the $p_{\gamma k}$'s. Note that there will be many permutations which must be considered, and hence the problem is possibly quite large. However, the results to be used in the expected cost evaluation are not the ρ 's, but the y_i 's and the u_k 's. Fractional changes in the ρ 's would not have a great effect on these

figures, and hence the expected cost figures would not change a great deal. This implies, of course, that the expected cost is not too sensitive to the ρ 's.

The following example should demonstrate the use of these concepts. In order to keep the example of a reasonable size, we will define three learning states, A, B, and C, to correspond with those shown in Figure 3. Two arbitrary classes of radar sites will be used, M and N, N containing more radar than M. Thus there will be $3 \cdot 2 = 6$ different Markov states. We define

E_1 = learning state A, radar class M

E_2 = learning state A, radar class N

E_3 = learning state B, radar class M

E_4 = learning state B, radar class N

E_5 = learning state C, radar class M

E_6 = learning state C, radar class N.

Since the type of mission and reaction of the enemy is critical to the ρ 's, let us assume that missions are being run in which the destruction of radar is incidental, and that the enemy is not building radar sites at a rapid pace. The first specification implies that the probabilities of destruction of radar and of decrease in learning states are low. The second implies that the probability of going from M to N is also fairly low. With these facts in mind, and observing the effect that this particular mix of pen aids has had on enemy operators (judging from previous experience), the researcher decides that the P matrix is as follows:

$$\underline{P} = \begin{array}{c} \begin{array}{cccccc} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 \end{array} \\ \left[\begin{array}{cccccc} 1/2 & 0 & 3/8 & 1/16 & 1/16 & 0 \\ 1/32 & 1/2 & 1/32 & 3/8 & 0 & 1/16 \\ 1/16 & 0 & 1/2 & 1/16 & 3/8 & 0 \\ 0 & 1/16 & 0 & 1/2 & 1/16 & 3/8 \\ 0 & 0 & 1/8 & 0 & 1/2 & 3/8 \\ 0 & 0 & 0 & 1/16 & 1/16 & 7/8 \end{array} \right] \begin{array}{c} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \\ E_6 \end{array} \end{array}$$

The chain is irreducible and ergodic, and hence the researcher is interested in obtaining the u_k 's. Thus, the following problem is formulated:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}^t \begin{bmatrix} -1/2 & 0 & 3/8 & 1/16 & 1/16 & 0 \\ 1/32 & -1/2 & 1/32 & 3/8 & 0 & 1/16 \\ 1/16 & 0 & -1/2 & 1/16 & 3/8 & 0 \\ 0 & 1/16 & 0 & -1/2 & 1/16 & 3/8 \\ 0 & 0 & 1/8 & 0 & -1/2 & 3/8 \\ 0 & 0 & 0 & 1/16 & 1/16 & -1/8 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}^t$$

A solution to this problem is:

$$u_1' = 0.0430$$

$$u_2' = 1.000$$

$$u_3' = 0.5316$$

$$u_4' = 24.0000$$

$$u_5' = 1.4910$$

$$u_6' = 8.5526$$

where the " ' " indicates that the solution has yet to be normalized.

The normalized solution is:

$$u_1 = 0.0012$$

$$u_2 = 0.0281$$

$$u_3 = 0.0149$$

$$u_4 = 0.6738$$

$$u_5 = 0.0419$$

$$u_6 = 0.2401$$

Assume that the deterministic cost for the mix under consideration is \$100. Further, assume that the following costs for the Markov states are applicable:

$$\text{cost of } E_1 = \$30,000$$

$$\text{cost of } E_2 = \$50,000$$

$$\text{cost of } E_3 = \$50,000$$

$$\text{cost of } E_4 = \$80,000$$

$$\text{cost of } E_5 = \$90,000$$

$$\text{cost of } E_6 = \$100,000$$

Therefore, the expected cost for this mix, when used in a situation in which the Markov analysis is applicable, is

$$\begin{aligned} E(C) &= D + S \\ &= [100] + [(0.0012)(3) + (0.0281)(5) + (0.0149)(5) \\ &\quad + (0.6738)(8) + (0.0418)(9) + (0.9401)(10)] \times 10^4 \\ &= \$83,962 \end{aligned}$$

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this thesis has been to present an analytical technique to enable a researcher to evaluate the cost-effectiveness of various penetration aids and mixes of penetration aids. The real-world situation analyzed is one of uncertainty since little is known about the probabilities of the possible outcomes of the mission(s) to be run. The effectiveness of the various mixes of pen aids is assumed to be expressed as probabilities, thus arriving at a (discrete) probability distribution for the possible outcomes. Hence the situation is no longer one of uncertainty, but of risk. Using these probabilities as weights, the expected cost of the stochastic portion of the model is found. To this stochastic portion, the deterministic costs involved in the use of that particular mix of pen aids is added, thus arriving at an expected cost figure associated with the use of that set of penetration aids. By comparing the expected cost figures and choosing the minimal one, one can select the most cost-effective mix of penetration aids to be used for the particular situation under investigation.

This particular procedure is not necessarily unique. Since it was desired to keep this thesis unclassified, discussion and demonstration of real-world examples was impossible. Due to the specific characteristics involved, the general techniques presented in this thesis may require minor modifications when working on real-world pen aids. It is felt that these modifications, if required, will be obvious when a

specific penetration aid is studied.

One effect of this research into a method of evaluating cost-effectiveness was to raise many questions which must be satisfactorily answered in order to conduct a thorough investigation of the cost-effectiveness of expendable penetration aids. These will be briefly discussed below.

In our model, we assumed that certain types of costs were deterministic. In the real world, these may not, in fact, be deterministic at all. This would obviously be the case when a newly developed penaid was being tested. Most of the costs involved in the deterministic portion of the model would not have occurred as yet, and thus would have to be estimated. If it were possible to estimate the individual cost of each unit to within, say, 20%, the investigator could use a minimum, maximum, and "most likely" cost as d_j in the evaluations. Note that, using these three different costs, different mixes could be chosen as optimal. However, the researcher should be able to make a fairly accurate guess as to the advisability of developing and manufacturing a proposed penaid. Even when a developed, manufactured penetration aid is being investigated, these "deterministic" costs may not lend themselves to exact costing, and the same technique may be applied. However, in this latter case, costs should, of course, be much more accurate.

Perhaps the most important questions which must be answered are how the mixes to be tested are chosen and the techniques and strategies to be used in the testing of the various mixes of pen aids. Some information can be obtained from the specific characteristics belonging to

the penaid. Is it an active or passive penaid? If active, what are its power requirements and/or power capabilities? If it is designed to work against only certain frequencies, we may be able to eliminate its use against radar threats of different frequencies. If it is to be used in conjunction with other penaid, are there any characteristics of the penaid which would interfere with the effectiveness of any of the others (and vice versa)? Is there anything inherent in the characteristics of this penaid which would give us insight into the number of these penaid required to reach a critical point of effectiveness? Can we infer what the optimum altitude, distance from threat radar, formation of the drop of these penaid, etc., should be most effective from the penaid characteristics? If it is an active penaid, is it unidirectional, omnidirectional, or somewhere in between?

Questions like the above should be answered as fully as possible through analytical techniques. It is assumed that those questions which are not answered analytically will be answered as well as possible through the technique of simulation. Since a simulation of this type could easily cost thousands of dollars per computer hour, it obviously would be economical to obtain as much information as is possible before a simulation study is run. Even if quantitative answers cannot be obtained analytically, a presimulation analysis could well indicate the direction in which the emphasis in the simulation should be placed. For example, the minimum number of penaid which would have any effect on a radar of certain (fixed) capabilities may not be able to be determined exactly by analytical techniques, but the general area which must be investigated could be found.

One major source of information about the effectiveness and use of pen aids is data from the field: i.e., real data. While this type of data is notorious for incompleteness and usually contains some error, this can be a source of invaluable information to the researcher. Naturally this type of data could contain information superfluous to the study being undertaken, and may not contain certain information which is vital to the research. Nevertheless, a critical analysis of any data available would impart some important information to the investigator. If such data is available, it should be thoroughly reviewed.

Assuming that all possible information has been obtained from pre-simulation investigations, we next come to the simulation itself. Before a simulation can be run, it must be decided exactly what is to be obtained from the simulation. Once this question has been settled, the input parameters can be decided upon.

One of the first things which must be decided is the nature of the threat itself. It is anticipated that the first type of investigations will be one-radar-to-one-aircraft solutions. (It was this type of situation which was envisioned when the examples in the second section of Chapter III and the example in Appendix A were presented. With a specified aircraft and no information about the pen aids, we may wish to define the mixes to be tested in terms of the aircraft itself.) This type of investigation should yield such information as the optimum altitude, timing, distance, location, and types of mixes of pen aids which are most effective against that particular radar for that aircraft. The number of possible strategies (with respect to the just mentioned type of data) that may be used in this situation is quite large. Hence it is readily

apparent that any information which can be gained before the simulation is run is quite valuable, and can save much simulation time.

Given that the one-on-one situation has been analyzed and the capabilities of the various mixes of penaid has been evaluated, the next step would be to increase the number of threat radar and/or the number of aircraft. This obviously increases the number of possible penaid mixes, the number of possible outcomes, and hence the possible costs. We may wish to increase first one (e.g., the number of radars) keeping the other constant at unity, then increase the number of the other, keeping the first constant at unity. Finally, testing would be accomplished in the situation where both multiple threat and multiple aircraft are being considered.

Consider the situation in which the threat consists of multiple radar. The decision might be made to group the radar into classes, as mentioned in Chapter IV. These classifications could be made on the basis of number, location, capabilities (e.g., frequency), etc., of the radar, depending on the particular mixes under consideration. Note that these classes could be different for different mixes of penaid.

When dealing with multiple aircraft, not only do the various mixes have to be considered (with all the strategies involved therein), but the strategies for the use of the aircraft themselves must be taken into account. For example, if an optimal mix (vs. a particular threat) is known, how should this mix be distributed amongst the aircraft? For a particular distribution of this mix, what is the optimal aircraft formation to be flown? The investigation to find the solution of these last two questions will be quite involved. It is therefore obvious that in situations for

which no optimal mix is known, and many various mixes must be tested under various strategies, meaningful evaluation will be quite large, time-consuming, and expensive.

The ideal situation would be one in which an analytical, quantitative method could be found which uses the information obtained in the one-on-one situation and applies it to the multiple radar and aircraft case. As a simple example, assume that the probability of loss of the aircraft in the one aircraft situation (versus some specified threat) is distributed according to a binomial distribution when plotted against, say, the number of pen aids of a particular type used. If we could say the probability of kills in the multiple aircraft case (against the same threat) when plotted using the same pen aid was distributed according to a multinomial distribution, we would have a simple analytical technique with which to evaluate the multiple aircraft case. However, the author feels that such a method will, unfortunately, not be found. The reason for this pessimism is the fact that there are too many variables involved in the problem under investigation. For example, if a formula for one particular situation could be found, a change in one of the input parameters, e.g., terrain, could easily invalidate the formula. Thus the relationship would possibly have to be modified for each situation, and the determination of the required modification would necessitate an evaluation. It would be possible, at the conclusion of the research, to examine all data obtained in order to investigate the possibility that such a relationship exists. However, if found, there is no guarantee that the relationship would be solvable in an actual case. Due to the many factors involved in the problem, any such relationship would

undoubtedly involve many nonlinearities and dependencies which could well make the relationship impossible to use. Thus the author concludes that, as undesirable as it is, simulation may be the only technique which is available to deal with the multiple threat/aircraft situations.

When dealing with the Markov analysis, it should be emphasized that any factor not explicitly used in the definition of the states is to be considered a fixed input parameter. Thus, for example, the strategies employed could be made either a category to be used in state definition, or could be considered fixed and several chains evaluated for different strategies.

When dealing with Markov analyses, another type of strategy needs to be considered. In addition to learning and increasing in capabilities, operators have a tendency to forget and decrease in capability when they do not see the effect of a particular penetration aid mix on their screens. This implies that a strategy of varying the mixes of pen aids when flying continually over the same threat/target should be employed. The timing of the usage of the mixes should, of course, be on an irregular basis so that the radar operators will not be able to anticipate the picture which they will see on their screens.

The exact curves of learning, capabilities, and "forgetfulness" will have to be determined by competent researchers. Personal communications with Dr. T. L. Sadosky reveal that work has been done in the area of the learning ability of radar operators. However, most of this work has been done with inexperienced operators. It is also noted that the speed of learning may vary with the particular mix used, and thus

learning curves may have to be developed for each mix investigated. This holds true for both the capability and "forgetfulness" curves.

From the above discussions, it is noted that further work is required in the following areas:

- (a) The "deterministic" portion of the model (i.e., d_j).
- (b) Methods of analysis of pre-simulation data, including
 - (i) characteristics inherent in the penetration aid,
 - (ii) "live" data from the field.
- (c) A computer program capable of simulation of all the parameters and variables involved in the estimation of the probabilities used as the measure of effectiveness.
- (d) The determination of an analytical relationship between the use of information obtained in the one-on-one situation as applied to the multiple aircraft, multiple threat cases.
- (e) Investigation of the learning and capability curves of radar operators.
- (f) "Forgetfulness" curves of radar operators, to include the effect of the strategy of changing the mixes used when flying a mission many times over the same threat/target.
- (g) The method of combining the curves in (e) and (f) to get the overall effect of whatever strategy is employed.

In conclusion, we note that the techniques presented in this thesis are very general, and give a method for the determination of the most cost-effective mix of penetration aids. Several of the problems which were met in the course of the investigation were discussed, and their resolution presented. Finally, some of the problems which have yet to be solved, but must be adequately dealt with in order to accomplish a thorough investigation of expendable penetration aids were discussed. Research into the cost-effectiveness of expendable penetration

aids will be long, arduous, and expensive. However, the real-world situation which it is designed to investigate is an expensive, and possible vital, one, and the time, effort, and expense should be well worth their expenditure.

APPENDIX A

A FULL EXAMPLE FOR A SINGLE AIRCRAFT

In this example, it is assumed that there are one aircraft and two types of penetration aids to be evaluated. Further, assume that there is an opportunity cost of approximately \$600,000 associated with the accomplishment of the mission with respect to the target. In addition, assume that if the target is acquired, the probability of the mission accomplishment with respect to the target is unity. Three possible mission outcomes are defined:

$$K = \begin{cases} k_1 = \text{aircraft destroyed before acquiring target} \\ k_2 = \text{aircraft destroyed after acquiring target} \\ k_3 = \text{aircraft not destroyed, target acquired.} \end{cases}$$

Table 2 is repeated to avoid notational confusion.

There are other elements of K possible, but we include only these three in order to keep the example small.

The associated costs are:

$$F = \begin{cases} f_1 = \$1,000,000 \\ f_2 = \$400,000 \\ f_3 = -\$600,000 \end{cases}$$

The same parameters will be used as were used in the section on single aircraft in Chapter III. These are:

Table 2. Definition of Symbols

$E(C)$	expected cost
T	the specific threat to be encountered during the mission
M	the specific mission to be run
j	penetration aid types, $j = 1, 2, \dots, m$
D	deterministic cost for all penaid s used for the mission
d_j	individual costs of penaid j
X_j	in a mission involving only one aircraft, the number of penaid s of type j
v_j	volume of (individual) penaid j
w_j	weight of (individual) penaid j
V	volume capacity for penaid s of the given aircraft
W	weight capacity for penaid s of the given aircraft
Γ	class of all mixes of penaid s considered
L	class of indices on the mixes and/or ratios under consideration
l	an element of L
γ_l	an element of Γ
K	class of all possible considered outcomes of the mission
R	class of indices for the elements in K
r	an element of R
k_r	an element of K
F	class of the costs of outcomes given the mission M

Table 2. Definition of Symbols (Concluded)

f_k	an element of F denoting the cost of outcome K
$P_{Y_\ell^k r}$	probability the mix Y_ℓ gives result k_r given the threat and the mission $= \text{pr}(Y_\ell \rightarrow k M T)$
WHEN MORE THAN ONE AIRCRAFT IS INVOLVED:	
i	index on aircraft, $i = 1, 2, \dots, n$
X_{ij}	the amount of payload of type j used on aircraft i ; integer valued
V_i	volume capacity of aircraft i
W_i	weight capacity of aircraft i
\bar{V}	volume capacity of all n aircraft
\bar{W}	weight capacity of all n aircraft
FOR MARKOV CHAINS:	
E_i	states of the markov chain
ρ_{ij}	$\text{Pr} \{ \text{going from state } E_i \text{ to } E_j \text{ in one transition} \}$
$\rho_{ij}^{(n)}$	$\text{Pr} \{ \text{going from state } E_i \text{ to } E_j \text{ in } n \text{ transitions} \}$
P	the stochastic matrix of transition probabilities
$a_k^{(n)}$	the absolute probability of being in state E_k after n transitions
u_k	the invariant distribution of an ergodic chain
H	the class of transient states in a reducible chain
R	one of a class of closed sets of persistent states in a reducible chain
y_i	$\text{Pr} \{ \text{ultimate absorption in } R \text{the initial state is } E_i \}$

j	w_j	v_j	d_j	
1	20	10	100	$W = 2,000$
2	300	70	5,000	$V = 500$

Similarly, the same mixes will be tested as were in Chapter III. In this Appendix, the integer programming problems will be formulated in order to show the technique used. As mentioned before, the method of solution will not be discussed. Next the definitions of the mixes to be used is given.

$$\Gamma = \begin{cases} \gamma_0 & = \text{no jammers used} \\ \gamma_{11} & = \frac{1}{2} \text{ full of jammer type 1 (other half empty)} \\ \gamma_{12} & = \text{full of jammer type 1} \\ \gamma_{21} & = \frac{1}{2} \text{ full of jammer} \\ \gamma_{22} & = \text{full of jammer type 2} \\ \gamma_{31} & = \text{numerically, 2 of jammer type 1 to 1 of jammer type 2} \\ & \quad \pm 20\% \\ \gamma_{32} & = \text{numerically, a 50:50 mix of jammers type 1 and 2} \\ & \quad \pm 20\% \\ \gamma_{33} & = \text{numerically, 1 of jammer type 1 to 2 of jammer type 2} \\ & \quad \pm 20\% \end{cases}$$

The formulation of the integer programming problems to be solved for feasibility will now be presented. The problems given here are quite simple, and could be solved mentally. However, it is the technique which is important since problems dealing with multiple aircraft are much more complicated.

for γ_0 : none required

for γ_{11} : divide the results of γ_{12} by two

$$\begin{aligned}
&\underline{\text{for } \gamma_{12}:} \quad \text{Max. } X_1 \\
&\quad \text{s.t. } 20 X_1 \leq 2,000 \text{ (weight constraint)} \\
&\quad \quad 10 X_1 \leq 500 \text{ (volume constraint)} \\
&\quad \quad X_1 \geq 0, \text{ integer (non-negativity and integer constraint)} \\
&\quad \Rightarrow X_1 = 50
\end{aligned}$$

Therefore, use 50 jammers for the γ_{12} simulation, and 25 jammers for the γ_{11} simulation.

Note that, had the answer to the γ_{12} feasibility problem been odd, the solution to γ_{11} would have been non-integer. Whether to go up or down to arrive at an integer-valued solution for γ_{11} is up to the experimenter.

for γ_{21} : same procedure as in γ_{11} .

$$\begin{aligned}
&\underline{\text{for } \gamma_{22}:} \quad \text{Max. } X_2 \\
&\quad \text{s.t. } 300 X_2 \leq 2,000 \\
&\quad \quad 70 X_2 \leq 500 \\
&\quad \quad X_2 \geq 0, \text{ integer} \\
&\quad \Rightarrow X_2 = 6 \text{ for } \gamma_{22} \\
&\quad \Rightarrow X_1 = 3 \text{ for } \gamma_{21}
\end{aligned}$$

$$\begin{aligned}
&\underline{\text{for } \gamma_{31}:} \quad \text{Max. } X_1 \\
&\quad \text{s.t. } 20 X_1 + 300 X_2 \leq 2,000 \\
&\quad \quad 10 X_1 + 70 X_2 \leq 500 \\
&\quad \quad 1.8 X_2 \leq X_1 \leq 2.2 X_2 \\
&\quad \quad X_1, X_2 \geq 0, \text{ integer} \\
&\quad \Rightarrow X_1 = 11, X_2 = 5
\end{aligned}$$

Note that the objective function could be either {Max. X_1 } or {Max. X_2 }. The only care that needs to be taken is that the ratio constraints must be formulated such that the correct ratio is obtained.

$$\begin{array}{ll}
 \underline{\text{for } \gamma_{32}:} & \text{Max. } X_1 \\
 & \text{s.t. } 20 X_1 + 300 X_2 \leq 2,000 \\
 & \quad 10 X_1 + 70 X_2 \leq 500 \\
 & \quad 0.9 X_1 \leq X_2 \leq 1.1 X_2 \\
 & \quad X_1, X_2 \geq 0, \text{ integer} \\
 & \implies X_1 = 6, X_2 = 6
 \end{array}$$

$$\begin{array}{ll}
 \underline{\text{for } \gamma_{33}:} & \text{Max. } X_1 \\
 & \text{s.t. } 20 X_1 + 300 X_2 \leq 2,000 \\
 & \quad 10 X_1 + 70 X_2 \leq 500 \\
 & \quad 1.8 X_1 \leq X_2 \leq 2.2 X_1 \\
 & \quad X_1, X_2 \geq 0, \text{ integer} \\
 & \implies X_1 = 3, X_2 = 6
 \end{array}$$

Before we continue with the example, a few explanations must be made.

First, note the word "numerically" preceding the description of the last three mixes to be tested. One must specify the basis for the ratios since it would be possible to find a ratio based on every constraint entered in the problem (except the integer, non-negativity, and minimum requirements constraints). In this example, there could be an x:y mix by weight, by volume, or numerically. Which basis is to be used must obviously be specified. In this thesis, numerical ratios are

being used since it is felt that this is the most relevant basis for the problem under investigation.

The " $\pm 20\%$ " entered at the end of the descriptions of the last three mixes is necessitated by the fact that X_j is restricted to integer values. The $x:y \pm z\%$ capability of the formulation should be a great boon to the evaluation. Since the X_j are integers, it is possible that an exact ratio would come nowhere close to filling the capacity of the aircraft, and thus the simulation investigation could well be in the wrong area of the feasible space. In addition, it is felt that the probabilities generated from the simulation would not be sufficiently precise to differentiate between limits of an appropriate $\pm z\%$.

The question of how to choose an appropriate $z\%$ is not an easy one. Each situation would have to be evaluated, and the z decided upon would depend on the characteristics of the penaid itself. However, there are two considerations which would be applicable in all cases. The first thing to note is that, if there is prior knowledge of the critical point of some type of penaid, one obviously would not want to go below that point. If the researcher can tell that the $z\%$ chosen would allow some number below this critical point to be reached, he should add a constraint so that this will not happen. (This type of constraint will be covered below.) If the mixes under consideration are not in the area of a critical point or this point is unknown, and simulation is being used, the sensitivity of the simulation to the number of jammers could be a criterion on which to base the selection of the proper z .

Since the capability to deal with this type of ratio is available, any number of different penaid types and mixes could be considered. However, the number of constraints will increase, and must be modified by the ratios to be investigated. For example, if one were considering three jammer types and wanted the ratio to be $\{X_1 = X_2 = X_3\} \pm 10\%$, two constraints would be required:

$$\begin{aligned} 0.95 X_2 &\leq X_1 \leq 1.05 X_2 \\ 0.95 X_2 &\leq X_3 \leq 1.05 X_2 . \end{aligned}$$

Next, consider the case where the critical minimum number of some type of penaid is known, and it is desired to include this number of this type of penaid in the aircraft load. There are two possible sub-cases:

- (1) This minimum number of this penaid is required, and the rest of the capacity is to be filled by other types. In this case, simply modify the capacity by the appropriate amount and formulate as in our present example, using the reduced capacity in the constraints.
- (2) Say the minimum number of jammer type 3 required is q , yet it is wished to investigate what happens when an additional amount of this type is used in conjunction with others. This would be formulated, using the original capacities, and adding the constraint

$$X_3 \geq q$$

to the problem.

The formulation of the feasibility integer programming problems is quite simple once the basic concepts are understood. Consequently, formulation to satisfy different constraints of the same type by personnel unfamiliar with integer programming could be accomplished. It is anticipated that a mixed-integer integer programming computer program

will be available if work of this type is contemplated, and hence solution procedures for these problems will not be discussed in this thesis. A word of warning: while linear programming is a commonly known technique, integer programming can be quite different. If an objective function or constraints of a form different from those presented here is contemplated, an expert in integer programming should be consulted.

We assume the same probabilities as in the example in Chapter III. These probabilities are given, along with the evaluated expected cost equations.

$$\begin{aligned} \underline{Y_0}: \quad & p_{01} = 0.40 & E_0(C) &= [0] + [(0.40)(1,000,000) + (0.40)(400,000) \\ & p_{02} = 0.40 & &+ (0.20)(-600,000)] = + \$440,000 \\ & p_{03} = 0.20 \end{aligned}$$

$$\begin{aligned} \underline{Y_{11}}: \quad & p_{11\ 1} = 0.37 & E_{11}(C) &= [(25)(100)] + [(0.37)(1,000) + (0.43)(400) \\ & p_{11\ 2} = 0.43 & &+ (0.20)(-600)] \times 10^3 = + \$424,500 \\ & p_{11\ 3} = 0.20 \end{aligned}$$

$$\begin{aligned} \underline{Y_{12}}: \quad & p_{12\ 1} = 0.30 & E_{12}(C) &= [(50)(100)] + [(0.30)(1,000) + (0.45)(400) \\ & p_{12\ 2} = 0.45 & &+ (0.25)(-600)] \times 10^3 = + \$270,000 \\ & p_{12\ 3} = 0.25 \end{aligned}$$

$$\begin{aligned} \underline{Y_{21}}: \quad & p_{21\ 1} = 0.30 & E_{21}(C) &= [(3)(5,000)] + [(0.30)(1,000) + (0.30)(400) \\ & p_{21\ 2} = 0.30 & &+ (0.40)(-600)] \times 10^3 = + \$195,000 \\ & p_{21\ 3} = 0.40 \end{aligned}$$

$$\begin{aligned} \underline{Y_{22}}: \quad & p_{22\ 1} = 0.25 & E_{22}(C) &= [(6)(5,000)] + [(0.25)(1,000) + (0.25)(400) \\ & p_{22\ 2} = 0.25 & &+ (0.50)(-600)] \times 10^3 = \$80,000 \\ & p_{22\ 3} = 0.50 \end{aligned}$$

$$\begin{aligned}
 \underline{\gamma_{31}}: \quad p_{31\ 1} &= 0.20 & E_{31}(C) &= [(11)(100) + (5)(5,000)] + [(0.20)(1,000) \\
 p_{31\ 2} &= 0.25 & &+ (0.25)(400) + (0.55)(-600)] \times 10^3 \\
 p_{31\ 3} &= 0.55 & &= - \$3,900
 \end{aligned}$$

$$\begin{aligned}
 \underline{\gamma_{32}}: \quad p_{32\ 1} &= 0.23 & E_{32}(C) &= [(6)(100 + 5,000)] + [(0.23)(1,000) \\
 p_{32\ 2} &= 0.25 & &+ (0.25)(400) + (0.25)(-600)] \times 10^3 \\
 p_{32\ 3} &= 0.52 & &= + \$48,600
 \end{aligned}$$

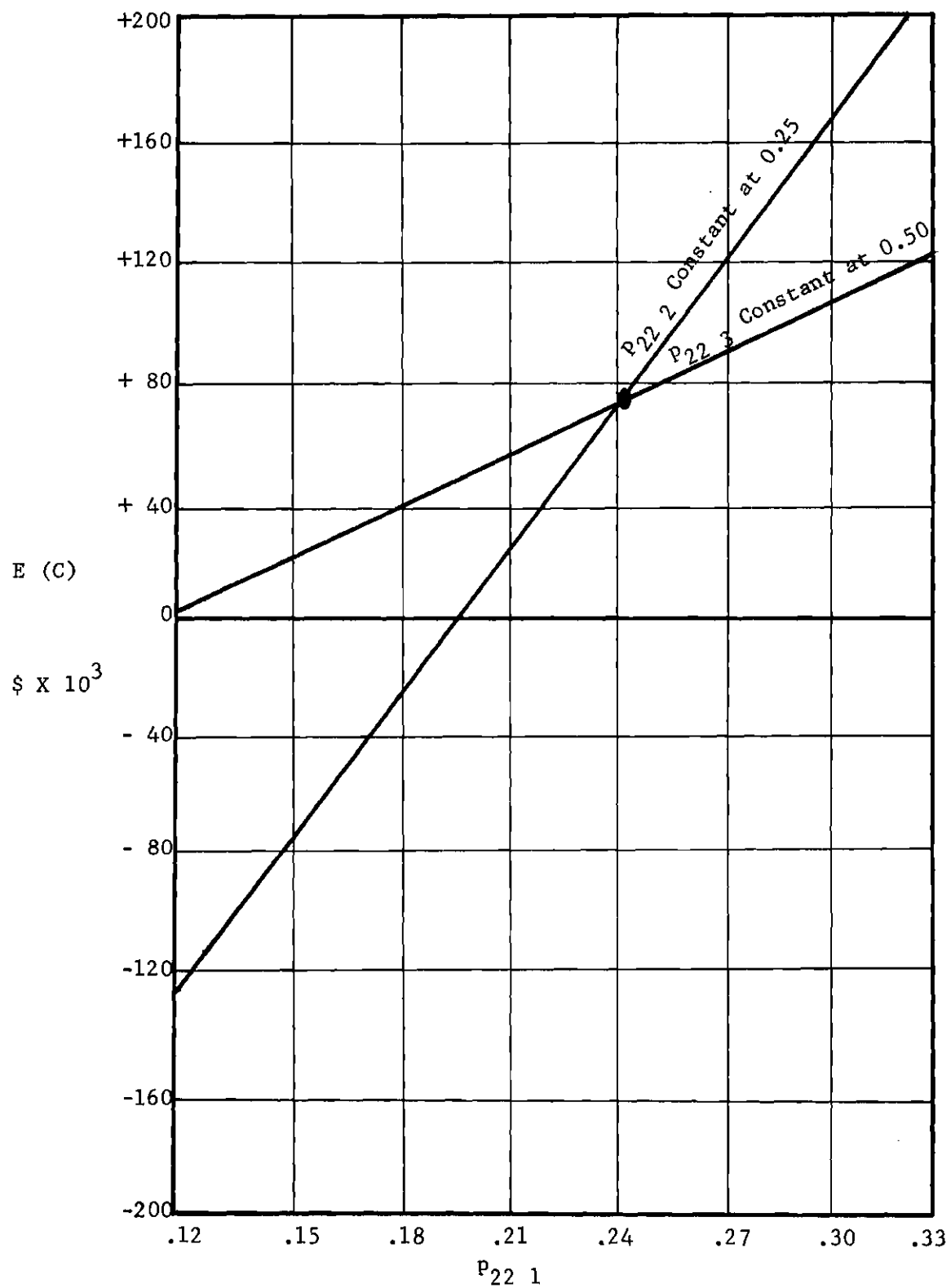
$$\begin{aligned}
 \underline{\gamma_{33}}: \quad p_{33\ 1} &= 0.25 & E_{33}(C) &= [(3)(100) + (6)(5,000)] + [(0.25)(1,000) \\
 p_{33\ 2} &= 0.25 & &+ (0.25)(400) + (0.50)(-600)] \times 10^3 \\
 p_{33\ 3} &= 0.50 & &= + \$80,300
 \end{aligned}$$

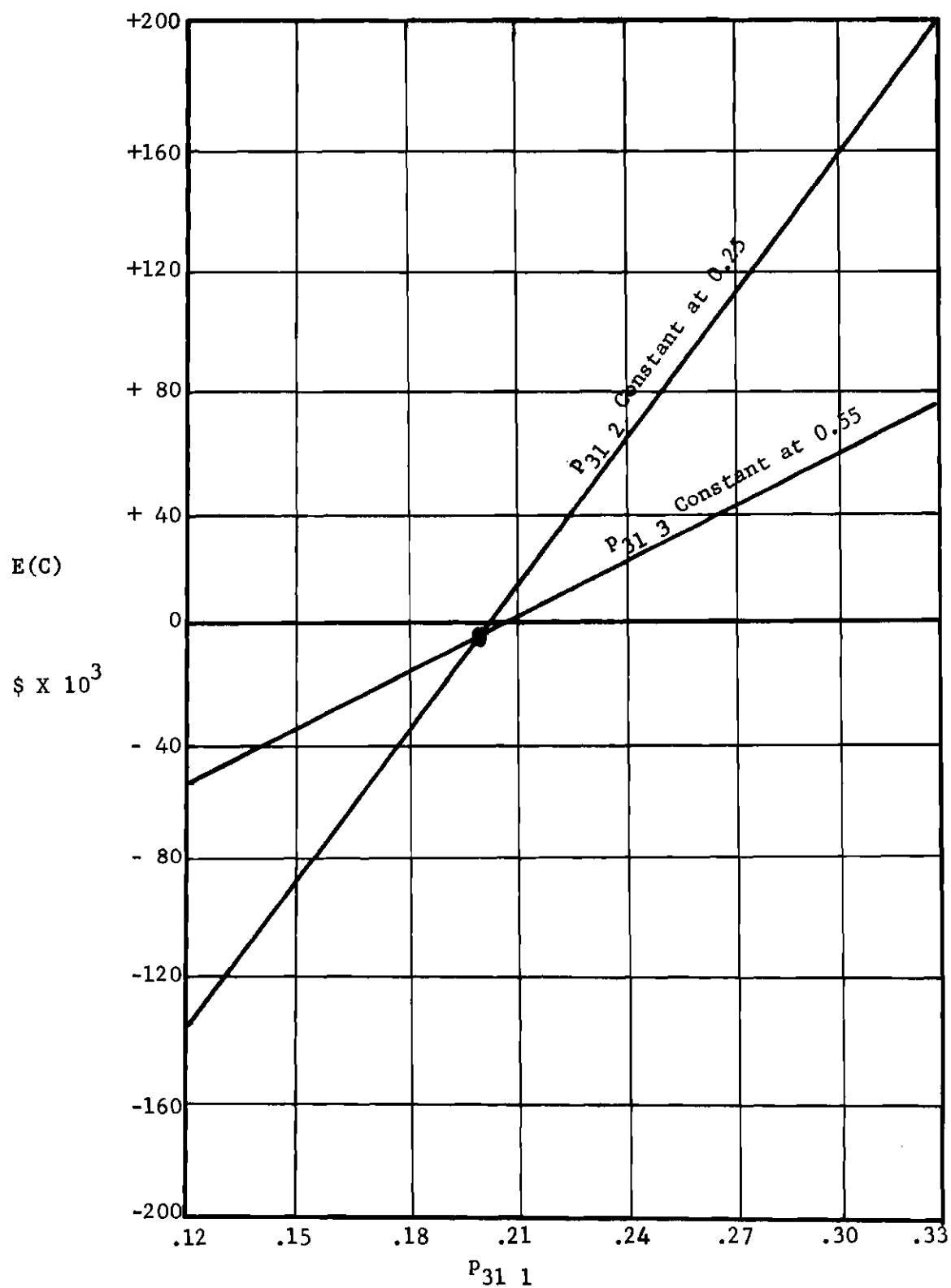
Using the figures as given, the optimal ratio and mix is obviously γ_{31} . However, a sensitivity analysis is in order. Note that in γ_{22} , γ_{31} , γ_{32} , and γ_{33} , costs are relatively close to zero (or negativity). Thus, further investigation should be directed toward achieving a better estimation of the respective probabilities. For example, if the probabilities in γ_{22} were changed only by 0.05, to $p_{22\ 1} = 0.20$, $p_{22\ 2} = 0.25$, $p_{22\ 3} = 0.55$, we would get $E_{22}(C) = 0$.

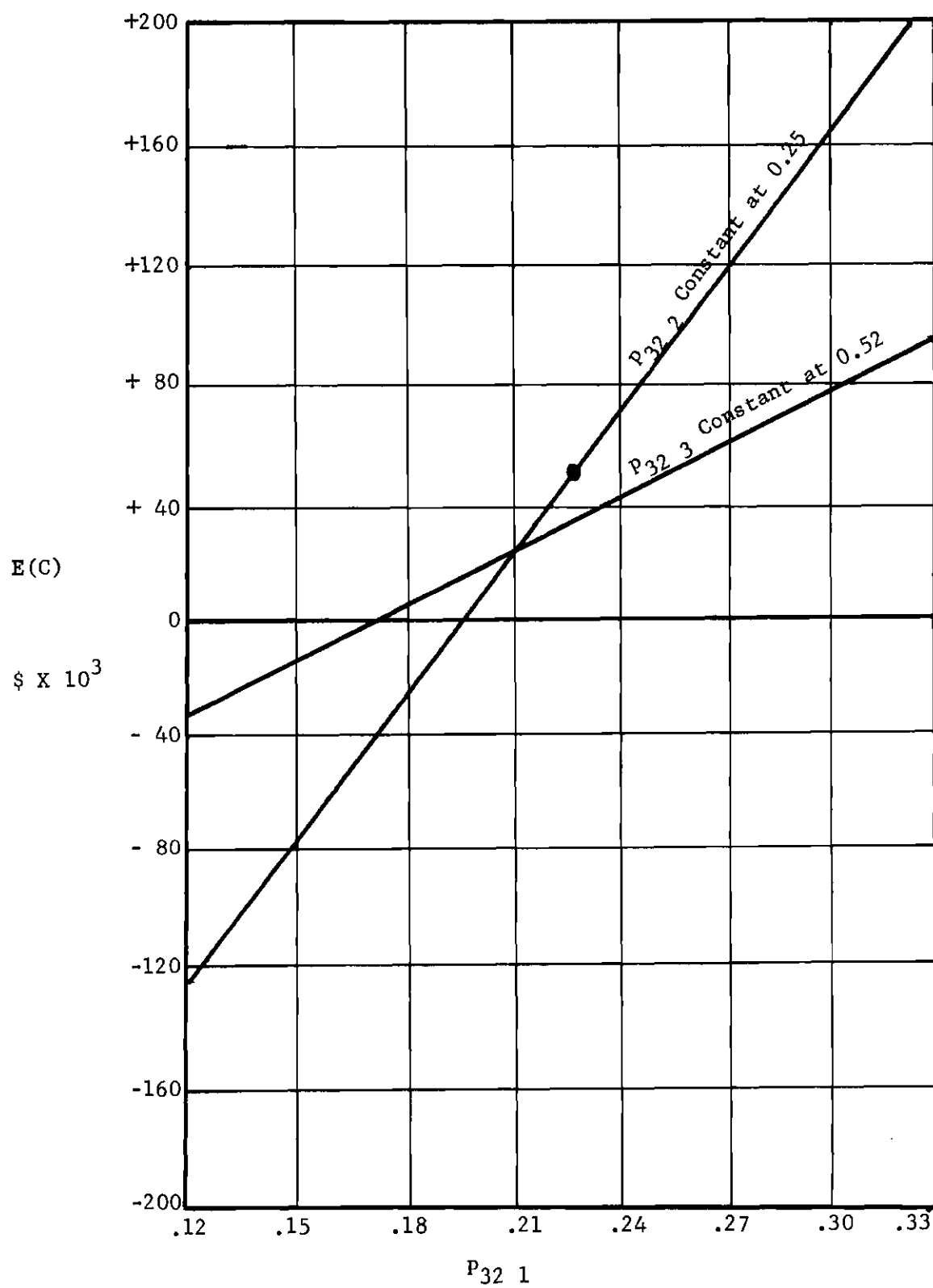
In order to analyze the effect of the probabilities, graph the expected cost vs. possible values of $p_{\gamma 1}$, as before. In this situation, it is necessary to hold first $p_{\gamma 2}$, and then $p_{\gamma 3}$, constant and vary the other. This is a requirement because of the nature of the costs: i.e., all three costs of outcomes are important and have a noticeable effect on the expected cost. Note that all other possible combinations of changes of probabilities would be represented by straight lines between these two extremes (e.g., as $p_{\gamma 1}$ is changed, a 50:50 change between $p_{\gamma 2}$

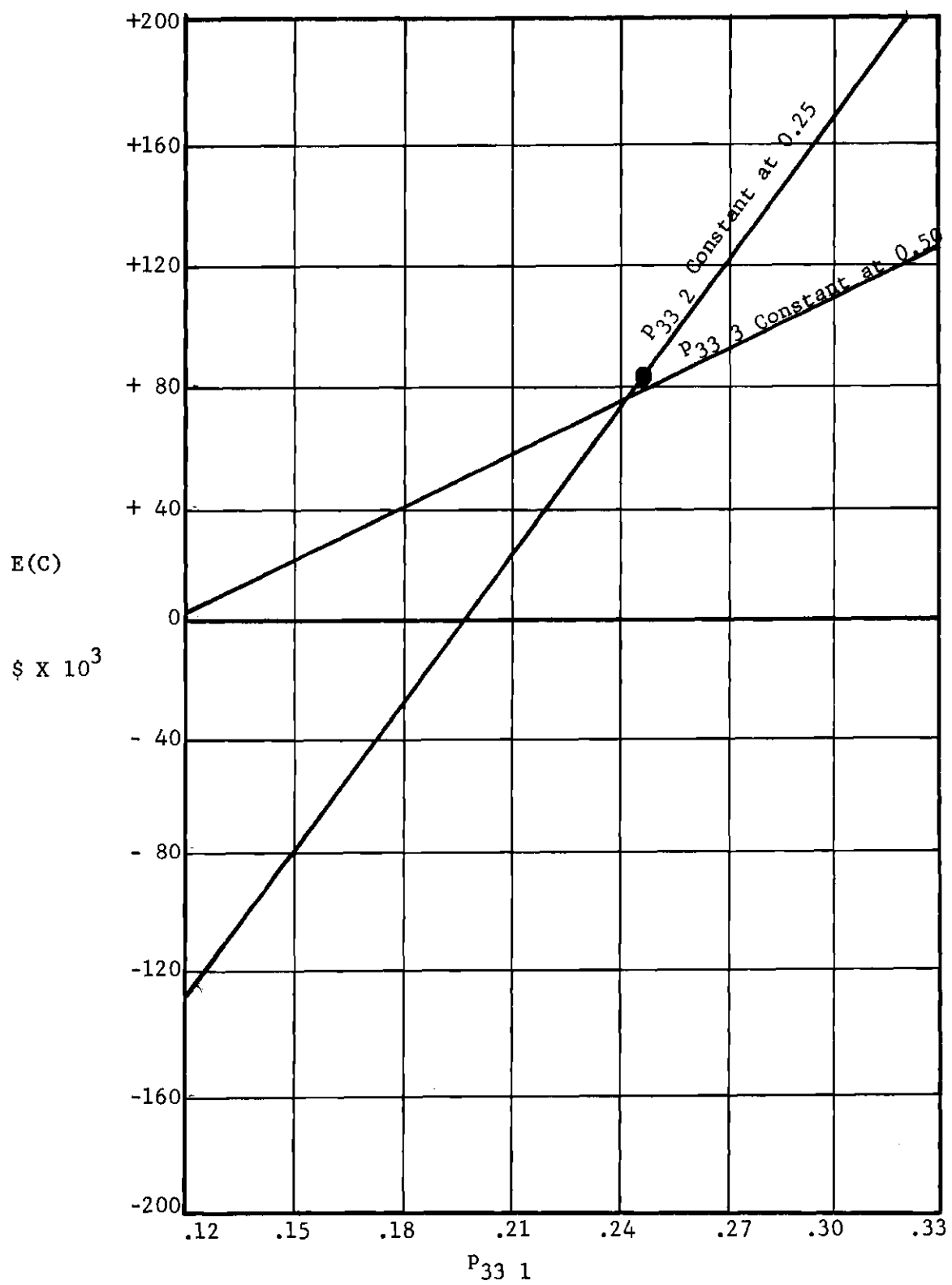
and p_{γ_3} would be exactly halfway between these two extrema lines). Figures I-1 through I-4 depict the possibilities for the four mixes of interest. As before, dots imply the original estimates.

Notice that in all four cases being considered, the deterministic cost of the penetration aids plays a major part of the final expected cost. This factor in itself may be a valid criteria for determining whether or not further investigation into the probabilities is warranted: i.e., if the deterministic cost is of the same decimal power as the probabilistic cost, further precision should be sought. (Of course this is applicable only when a negative cost is associated with successful accomplishment of the mission with respect to the target.)

Figure A-1. γ_{22} Sensitivity

Figure A-2. γ_{31} Sensitivity

Figure A-3. γ_{32} Sensitivity

Figure A-4. γ_{33} Sensitivity

APPENDIX B

INTEGER PROGRAMMING FORMULATIONS

This appendix will demonstrate integer-programming formulation procedures to be followed in the multiple aircraft and multiple penaid case. The example will be kept small, using only three aircraft of the same type and two different types of penaid. Even though this is a relatively small example, it will be noticed that the size of the problem is large enough to make hand calculations quite tedious. As mentioned before, a computer program should be written to do the calculations and keep track of the data in an actual investigation.

We will present only the mixes to be investigated and the formulation of the problems designed to specify the amounts of penaid to be used in each of the mixes. The expected cost evaluations and sensitivity analyses are accomplished in the same manner as those presented earlier, the only difference being the size and number of calculations required. Since nothing new would be added by including these analyses, they will not be included.

Let us assume that the following basic data are given:

j	v_j	w_j
1	15	200
2	10	50

$$V_1 = V = 300$$

$$W_1 = W = 2,000$$

The following mixes are the ones of interest:

$\Gamma =$	γ_0	= no jammers used
	γ_{10}	= aircraft #1 filled with jammer type 1 and the other aircraft carrying no jammers
	γ_{11}	= aircraft #1 and #2 filled with jammer type 1, and aircraft #3 with no jammers
	γ_{12}	= all three aircraft filled with jammer type 1
	γ_{13}	= all three aircraft each one-half full of jammer type 1
	γ_{20}	= aircraft #1 filled with jammer type 2
	γ_{21}	= two of the three aircraft filled with jammer type 2
	γ_{22}	= all three aircraft filled with jammer type 2
	γ_{23}	= all three aircraft one-half full of jammer type 2
	γ_{30}	= aircraft #1 full of jammer type 1; aircraft #2 full of jammer type 2; aircraft #3 with no jammers
	γ_{31}	= aircraft #1 full of jammer type 1; aircraft #2 full of jammer type 2; aircraft #3 has a load of ($\frac{1}{2}$ of jammer type 1 and $\frac{1}{2}$ of jammer type 2) $\pm 20\%$
	γ_{32}	= all three aircraft carrying loads of ($\frac{1}{2}$ jammer type 1 and $\frac{1}{2}$ jammer type 2) $\pm 20\%$
	γ_{33}	= a minimum of 10 units of jammer type 2 on each aircraft; the rest of the capacity to be a 50:50 $\pm 10\%$ mix of the two jammers per aircraft
	γ_{34}	= a minimum of 10 units of jammer type 2 on each aircraft, and a ratio of (2 of jammer type 1 to 1 of jammer type 2) $\pm 10\%$ per aircraft
	γ_{35}	= three units of jammer type 1 on each aircraft and the rest of the capacity to be filled with jammer type 2
	γ_{36}	= a minimum of three units of jammer type 1 per aircraft, and a (2 of jammer type 2 to 1 of jammer type 1) $\pm 20\%$ <u>overall</u> mix filling the rest of the capacity

γ_{37} = a minimum of three units of jammer type 1 per aircraft, and a (2 of jammer type 2 to 1 of jammer type 1) $\pm 10\%$ overall
 γ_{38} = aircraft #1 must have at least 5 units of jammer type 1 and the rest of its capacity is immaterial. Aircraft #2 must have a 50:50 $\pm 10\%$ mix and filled to capacity. Aircraft #3 must have a (2 of jammer type 2 to 1 of jammer type 1) $\pm 6\%$. Overall, we must have a 50:50 $\pm 20\%$ ratio.

As before, these ratios will be determined on a numerical basis.

From these data the following feasibility problems must be solved in order to determine the loads to be studied in the simulation.

for γ_0 : none required

for γ_{10} : Max. X_1

s.t. $15 X_1 \leq 300$

$200 X_1 \leq 2,000$

$X_1 \geq 0$, integer

for γ_{11} , γ_{12} , and γ_{13} : use the results obtained in γ_{10}

for γ_{20} : Max. X_2

s.t. $10 X_2 \leq 300$

$50 X_2 \leq 2,000$

$X_2 \geq 0$, integer

for γ_{21} through γ_{23} : use the results obtained in γ_{20}

for γ_{30} : use the results from γ_{10} and γ_{20}

for γ_{31} : use above results for first two aircraft; result from γ_{32} for the third aircraft

for γ_{32} : here we need only concern ourselves with one aircraft, and apply the results to all three aircraft

$$\begin{aligned}
 &\text{Max. } X_1 \\
 &\text{s.t. } 15 X_1 + 10 X_2 \leq 300 \\
 &\quad 200 X_1 + 50 X_2 \leq 2,000 \\
 &\quad 0.8 X_2 \leq X_1 \leq 1.2 X_2 \\
 &\quad X_1, X_2 \geq 0, \text{ integer}
 \end{aligned}$$

for γ_{33} : here, simply reduce the capacity by the amount required by the 10 units of jammer type 2 per aircraft. Again, one need only solve the problem for one aircraft and apply it to all.

$$\begin{aligned}
 &\text{Max. } X_1 \\
 &\text{s.t. } 15 X_1 + 10 X_2 \leq 200 \\
 &\quad 200 X_1 + 50 X_2 \leq 1,500 \\
 &\quad 0.9 X_2 \leq X_1 \leq 1.1 X_2 \\
 &\quad X_1, X_2 \geq 0, \text{ integer}
 \end{aligned}$$

for γ_{34} : one need only solve for one aircraft, after correcting the capacity for the required amounts

$$\begin{aligned}
 &\text{Max. } X_1 \\
 &\text{s.t. } 15 X_1 + 10 X_2 \leq 200 \\
 &\quad 200 X_1 + 50 X_2 \leq 1,500 \\
 &\quad 0.475 X_1 \leq X_2 \leq 0.525 X_1 \\
 &\quad X_1, X_2 \geq 0, \text{ integer}
 \end{aligned}$$

for γ_{35} : as before, reduce the capacity by the requirements. The thing to notice in this situation is that it would be wrong to use the total remaining capacity of all three aircraft, since this could well lead to infeasible solutions. As before, solve for one aircraft and apply the solution to all.

$$\begin{aligned}
 &\text{Max. } X_2 \\
 &\text{s.t. } 10 X_2 \leq 255 \\
 &\quad 50 X_2 \leq 1,400 \\
 &\quad X_2 \geq 0, \text{ integer}
 \end{aligned}$$

for γ_{36} : as before, reduce the capacity by the requirements

$$\text{Max. } X_{11}$$

$$\text{s.t. } 15 X_{11} + 10 X_{12} \leq 255$$

$$15 X_{21} + 10 X_{22} \leq 255$$

$$15 X_{31} + 10 X_{32} \leq 255$$

$$200 X_{11} + 50 X_{12} \leq 1,400$$

$$200 X_{21} + 50 X_{22} \leq 1,400$$

$$200 X_{31} + 50 X_{32} \leq 1,400$$

$$0.4(X_{11} + X_{21} + X_{31}) \leq (X_{12} + X_{22} + X_{32}) \leq 0.6 (X_{11} + X_{21} + X_{31})$$

$$X_{ij} \geq 0, \text{ integer, } i = 1,2,3; j = 1,2$$

Using the symbolism used in the text, these constraints for γ_{36} would be, of course,

$$\text{Max. } X_{11}$$

$$\text{s.t. } \sum_{j=1}^2 v_j X_{ij} \leq 255 \quad i = 1,2,3$$

$$\sum_{j=1}^2 w_j X_{ij} \leq 1,400 \quad i = 1,2,3$$

$$0.4 \sum_{i=1}^3 X_{i1} \sum_{i=1}^3 X_{i2} \leq 0.6 \sum_{i=1}^3 X_{i1}$$

$$X_{ij} \geq 0, \text{ integer } \forall i,j$$

The reason that one can write these constraints in this shorthand is obviously the fact that the aircraft are all identical. If the aircraft were different, with different capacities, one would be forced to write

all constraints explicitly. In what follows, the shorthand notation will be used.

for γ_{37} : Note that this is a completely different type of problem from γ_{36} . In γ_{36} , a particular mix filling the capacity remaining after the requirements had been met was desired. Here, an overall mix of certain proportions is desired.

$$\begin{aligned}
 &\text{Max. } X_{11} \\
 &\text{s.t. } \sum_{j=1}^2 v_j X_{ij} \leq 300, \quad i = 1, 2, 3 \\
 &\quad \sum_{j=1}^2 w_j X_{ij} \leq 2,000, \quad i = 1, 2, 3 \\
 &\quad X_{i1} \geq 3, \quad i = 1, 2, 3 \\
 &0.475 \sum_{i=1}^3 X_{i1} \leq \sum_{i=1}^3 X_{i2} \leq 0.525 \sum_{i=1}^3 X_{i1}
 \end{aligned}$$

for γ_{38} :

$$\begin{aligned}
 &\text{Max. } X_{11} \\
 &\text{s.t. } \sum_{j=1}^2 w_j X_{ij} \leq 2,000, \quad i = 1, 2, 3 \\
 &\quad \sum_{j=1}^2 v_j X_{ij} \leq 300, \quad i = 1, 2, 3 \\
 &0.8 \sum_{i=1}^3 X_{i2} \leq \sum_{i=1}^3 X_{i2} \leq 1.2 \sum_{i=1}^3 X_{i1} \\
 &\quad X_{11} \geq 5 \\
 &0.9 X_{22} \leq X_{21} \leq 1.1 X_{22}
 \end{aligned}$$

$$0.485 X_{31} \leq X_{32} \leq 0.515 X_{31}$$

$$X_{ij} \geq 0, \text{ integer, } \forall i, j$$

As mentioned before, solutions to these formulations, evaluations of expected cost, and a sensitivity analysis will not be presented because they have nothing new to offer.

BIBLIOGRAPHY

1. Fawcett, C. D., "Application of Dynamic Programming to Select Tactics for Air-to-Ground Attack Under Uncertainty," Ph.D. Dissertation, Ohio State University, 1969.
2. Dantzig, G. B., "Linear Programming Under Uncertainty," Management Science, Volume 1, Nos. 3 and 4, April-July, 1955, pp. 197-206.
3. Dantzig, G. B., and Ferguson, A. R., "The Allocation of Aircraft to Routes--An Example of Linear Programming Under Uncertain Demand," Management Science, Volume 3, October, 1965, pp. 45-73.
4. Reisman, A. and Buffa, E. S., "A General Model for Investment Policy," Management Science, Volume 8, No. 3, April, 1962, pp. 304-310.
5. Terry, H., "Comparative Evaluation of Performance Using Multiple Criteria," Management Science, Volume 9, No. 3, April, 1963, pp. 431-443.
6. Feller, William, An Introduction to Probability Theory and Its Applications, Volume 1, 3rd Edition, J. Wiley & Sons, Inc., New York, 1967.