# DATA-DRIVEN OPTIMIZATION FOR POLICE BEAT DESIGN IN SOUTH FULTON, GEORGIA

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**DATA-DRIVEN OPTIMIZATION FOR POLICE BEAT DESIGN IN SOUTH FULTON, GEORGIA**

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”We are certain that we’ll be able to reduce our beat times significantly and double the number of police officers we’re putting on the street.”

*Keith Meadows, South Fulton Police Chief*

This dissertation work is dedicated to my parents, many friends, church family and God.

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# SUMMARY

We redesign the police patrol beats in South Fulton, Georgia, in collaboration with the South Fulton Police Department (SFPD), using a predictive data-driven optimization approach. Due to rapid urban development and population growth, the original police beats arrangement designed in the 1970s was far from efficient, which leads to low policing efficiency and long 911 call response time. We balance the police workload among different regions in the city, improve operational efficiency, and reduce 911-call response time by redesigning beat boundaries for the SFPD. We discretize the city into small geographical atoms, which correspond to our decision variables; the decision is to map the atoms into "beats", which are the basic units of the police operation. We analyze workload and trend in each atom using the rich dataset for police incidents reports and U.S. census data and predict future police workload for each atom using spatial statistical regression models. Basing on this, we formulate the optimal beat design as a mixed-integer programming (MIP) program with contiguity and compactness constraints on the shape of the beats. The optimization problem is solved using simulated annealing due to its large-scale and non-convex nature. Our resulted beat design can reduce workload variance by over 90% according to our simulation. Our new optimal beat design has been approved by the City Council of South Fulton and implemented in January 2020.

# CHAPTER 1

# INTRODUCTION

The City of South Fulton, Georgia, was recently established in May 2017 from previously unincorporated land outside of Atlanta. It is now the third-largest city in Fulton County, Georgia, and serves a population of over 98,000, among which 91.4% are black or African American [1]. South Fulton is a historic area renowned for its art and activism. Despite this, the city has often faced the challenge of climbing crime rates and long police response times. In a 2019 survey, 46.48% of residents responded that they do not feel safe in South Fulton. In the same year, the South Fulton City Council made it clear that their number one priority was to make South Fulton safer [2].

The South Fulton Police Department (SFPD) is the main policing force in the city. From 2019 to early 2020, our team worked with the SFPD to improve their police operation efficiency. Our project specifically focused on redesigning beat configurations (by completely re-draw the boundaries and changing the number of beats), intending to balance the workload of SFPD officers. The initial analysis identified that workload unbalance among different areas of the city was caused by an outdated beat design that had not been changed for over five decades; the inefficient beat design, in turn, lead to long 911 call response time in some areas.

Previously, the South Fulton police operates according to seven police *beats*, which divide the city geographically as shown in Figure 1.1. 117 police personnel were allocated to the beats for patrolling and responding to the 911 calls [3]. Typically, for each beat, at each shift, there is one response unit, which is one police car with one to two officers, who respond to all 911 calls in that beat. If the response unit is handling another incident, nearby available response units may be dispatched to answer the call by the operator.

The most recent South Fulton police beat redesign occurred in the 1970s – almost five

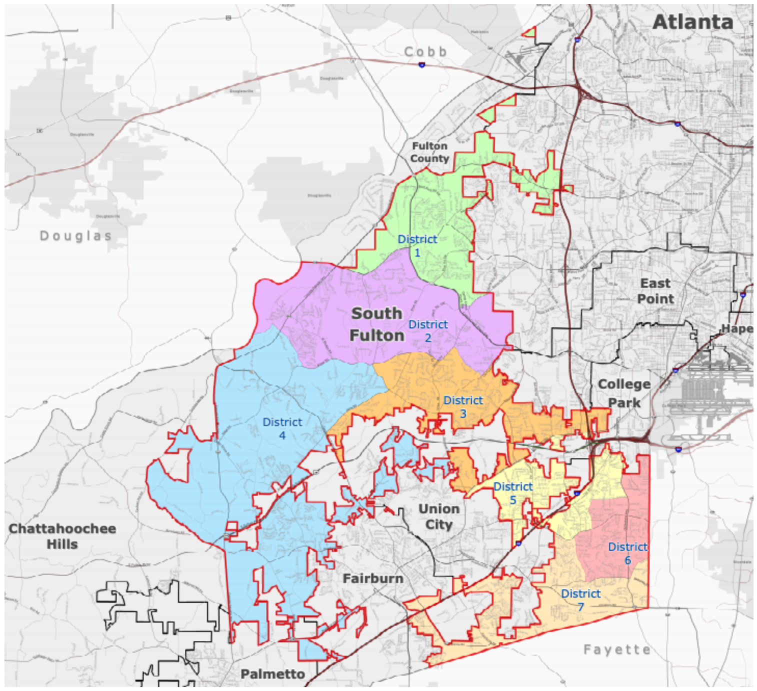



Figure 1.1: City-wide police district map of South Fulton, GA. There were 7 beats, which was initially designed in the 1970s. The city boundary is highly irregular which requires intricate design of police beats.

decades ago. Since then, the area (which eventually became the City of South Fulton) has undergone a tremendous urban growth that drastically changes its landscape. The U.S. Census Bureau estimated that the population of South Fulton has increased by 13.7% from 2010 to 2018 [1]. This has led to a significant increase in police workload, which is exacerbated by the difficulty in officer recruitment and retention faced by the SFPD. Moreover, demographic and traffic pattern changes also create an unbalanced workload among different regions. Figure 1.2 shows the distribution of 911 calls, which we estimated from 911-call data provided by SFPD from 2018 to 2019. It is evident from the figure that certain beats faced a significantly higher workload than others. For example, police officers in the southeastern area of the city respond to many more calls than those in the western area.

Since the seminal work by R. Larson and others [4, 5], researchers have recognized that beat configuration may significantly impact police response time to 911 calls and operational efficiency. In particular, the area and shape of beats determine the workload and

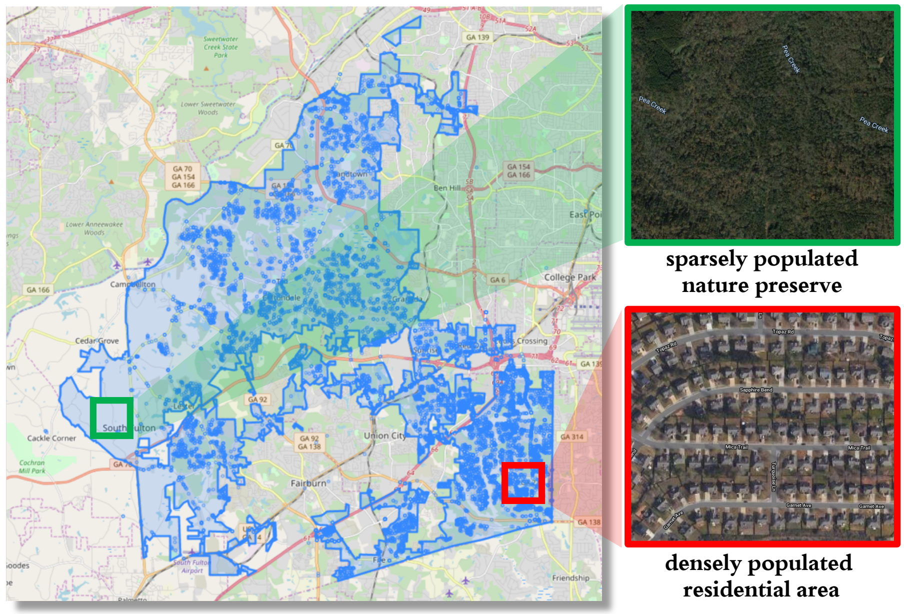



Figure 1.2: Distribution of 911 calls-for-services requests in South Fulton, GA. Blue shaded area indicates the city area of South Fulton. Blue dots are locations of requests. The requests are unevenly distributed among different regions.

travel time in that beat. Hence, it is critical to design the boundaries of beats to balance the workload.

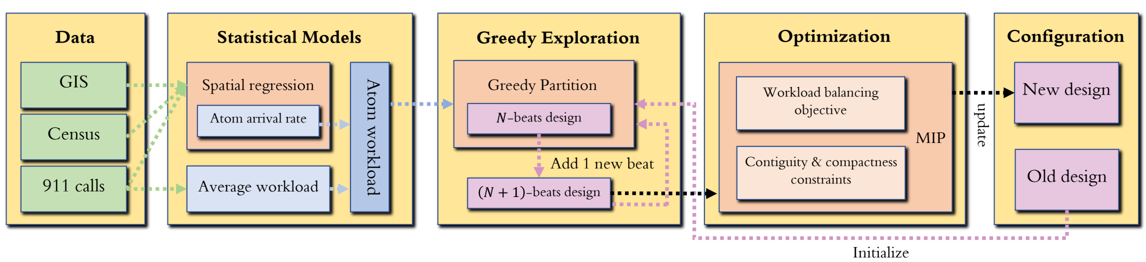



Figure 1.3: An illustration for data-driven optimization framework of police beat redesign.

**Outline.** We redesign the police patrol beats in the City of South Fulton using a data-driven optimization approach. The outline of our approach is summarized in Figure 1.3. Our objective is to balance police workload in each beat by redrawing beat boundaries. First, we divided the geographical areas of the city into a large number of "atoms". Then, we estimated the workload in each atom using police reports data as well as census data, including population and socio-economic factors. These steps are described in Chapter 2 and 3. Based on the workload estimation, we developed statistical models to predict police workload in the next few years (Chapter 4). We then formulate the beat redesign problem as a clustering problem: each beat is formed with a cluster of atoms. This clustering problem is formulated and solved using mixed-integer programming (MIP), where the objective function is a metric of workload unbalance (defined as the workload variance across all beats). We also impose constraints that require beats to be contiguous and compact so that they are not irregularly shaped. The problem formulation is described in Chapter 5. To tackle the computational complexity of solving a large-scale optimization problem, we developed a simulated annealing based approach with efficient solution exploration. We also study the effect of the different number of beats and find the optimal number of beats with the highest cost-effectiveness. Numerical results (Chapter 7) show that our proposed beat design can reduce workload variance among different regions by over 90%. In January 2020, together with the SFPD, we presented our final redesign plan to the South Fulton city council, which was officially approved for implementation.

**Contribution.** Our work proposes a new data-driven framework that integrates data, statistical prediction, and optimization in the context of police beat design. Previous works in the predictive policing literature tend to focus on only the prediction aspect. The operations research literature often studies police zone design based on analyzing stochastic models without explicitly considering data sources. We take advantage of the availability of abundant data and adopt a new data-driven approach - the workload and other important parameters for optimization are estimated and predicted from data. From a methodological perspective, we use geo-spatial atoms to define city boundary and police beat boundary. This approach enables accurate workload prediction by correlating historical police data with the census data, as well as beat design optimization.

Our project has also had a significant societal impact and directly improved the police operations of SFPD and the safety of residents in South Fulton. It is worth mentioning that although we focus on the study of police beat redesign in South Fulton, our method can be applied to other cities facing similar issues.

**Literature Review.** Police districting (designing beats or zones) is a classical problem studied in operations research dating back to the 1970s (see the seminal work [4] and the surveys by [6, 7] for reviews). [8] is one of the earliest works that study optimal beat allocation using integer programming. [9] considers the beat allocation problem to minimize response time to calls for police service. In particular, the paper considers overlapping beats, where multiple patrol officers share one patrol area. [10, 11] use queueing models to estimate travel time. In particular, our proposed data-driven model includes the travel time in the workload calculation. [12] introduces a heuristic approach to the design of beats with implementation in Boston. [13] considers fairness issues of police zone design. We remark that most classical works rely on analyzing stochastic models for police workload estimation, which usually requires stringent assumptions, e.g., calls arrive according to homogeneous Poisson processes (with the notable exception of [11]). Here, rather than obtained from stochastic models, we take advantage of the availability of abundant data and adopt a data-driven approach: the workload and other important parameters for optimization are estimated and predicted from data.

There is also a large body of works on other types of geographical districting problems, such as political districting. This includes the pioneering work [14] that studies political districting using integer programming. Their method is extended by [15] for other geographical districting problems. A few other works [16, 17, 18, 19, 20, 21] apply metaheuristics (e.g., genetic algorithms, simulated annealing) to geographical districting, which usually lack optimality guarantees. Geographic districting often includes criteria such as contiguity [22, 17, 14, 23, 24, 20] and compactness [14, 25, 26], which are also important in the police zone design context. However, political districting has different considerations than police districting.

For police staffing study, [27] proposes a linear programming approach using queueing models. There is also a number of papers in the queueing literature that has studied server staffing to meet time-varying demand [28, 29, 30]. We did not consider the queueing model in our paper and only focusing on analyzing staffing levels (the number of required police officers and how it is related to the number of beats), as this will simplify the problem and provide a practical guideline.

# CHAPTER 2

# DATA SOURCES FOR SOUTH FULTON

We start by describing the various sources of data used for South Fulton police beats reconfiguration, including 911 calls-for-service reports, geographical data of the city, and the socio-economic data collected by the American Community Survey (ACS) from the U.S. Census Bureau.

## 2.1 911 Calls-for-Service Data

The SFPD provides comprehensive 911-call reports between May 2018 to April 2019, which contains 69,170 calls in total (Figure 1.2). The recorded 911 calls cover more than 600 categories of incidents, including assaults, terrorist threats, domestic violence, robbery, burglary, larcenies, auto-thefts, etc. These reports are generated by mobile patrol units in the city, which handle 911 calls 24/7. Teams of *response units* (police cars and officers) are assigned to patrol city streets, and answer calls for service. When a 911 call for a traffic incident comes in at the *call time*, a new incident record will be created at the dispatch center, and the call location will be recorded. The operator assigns an officer to handle the call. The unit arrives at the scene and starts the investigation. Once the police complete the investigation and clear the incident, the police report will be closed and record the *clear time*. The time interval that it takes police to process the call between the call time and the clear time is called *processing time*.

The police workload is calculated using both the geolocation data and 911 call processing time data. (The calculation method which will be discussed in more detail in Sec. 3.3.) The geolocation consists of the longitude and latitude location of reported incidents. From the geographical data of South Fulton, we are also able to identify which beat each incident is located.

## 2.2 GIS Data & Beat Configuration

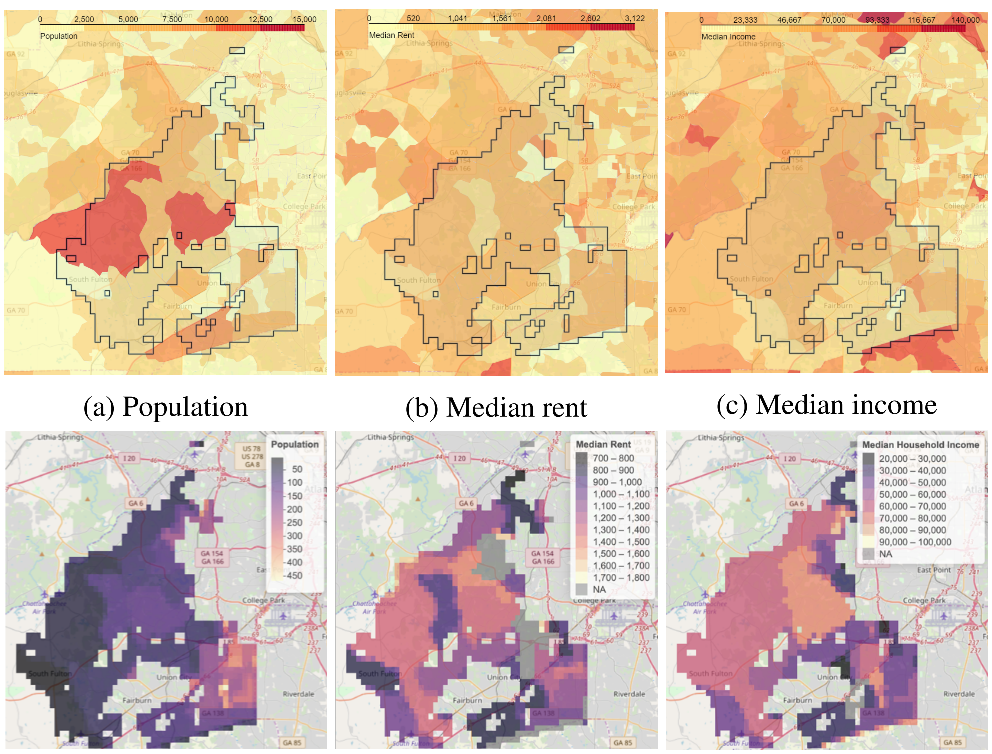
In this section, we describe the geographic information system (GIS) data that we used to reconfigure South Fulton's beats. To predict demand for 911 calls, we estimated demographic and socioeconomic features of South Fulton using data from the American Community Survey (ACS). To get these estimates, we used GIS data to define the area based on the Fulton County Special Services District digest parcel data [31]. Geographically, the city boundary of South Fulton is quite irregular with jagged edges, holes, and disconnected segments (Figure 1.1). This irregularity is due to the formation of the City of South Fulton, with the city being a new combination of all the unincorporated land in southwest Fulton County.

The city is bordered by several current municipalities, including Atlanta, College Park, Union City, Palmetto, Hapeville, East Point, and Fairburn. The City of South Fulton neighbors a relatively new city on its western border, the City of Chattahoochee Hill Country. The City of South Fulton also shares borders with four counties, Cobb, Coweta, Douglas, and Fayette. The municipalities in the east, such as Atlanta and College Park, have a much denser population than those municipalities in the west, like the City of Chattahoochee Hill Country. This leads to the City of South Fulton having a much denser population in the east versus the west.

Prior to this project, there are seven beats in the City of South Fulton, and GIS information of these beats was provided to us by the SFPD. As shown in figure 1.1, beats 1, 2, 3, and 4 include larger areas that are relatively compact, while remaining beats contain smaller scattered areas. The irregular shape of the city brings difficulty to police officers while reaching locations of requests and patrolling. In addition, there are four airports located to the east, north, east, and southeast of the city, which may affect the workload of police officers in such areas. The airport to the east is the Hartsfield-Jackson Atlanta International Airport, which is the busiest airport in the world. This only adds to the workload disparity in the city.

## 2.3 Census Data

Data from the American Community Survey (ACS) provided by the U.S. Census Bureau provides comprehensive information about the population, demographic, and economic status of different areas of Georgia. Unlike the census, which takes place every ten years, the ACS is conducted once per year. The latest year available is 2018. Some census factors are particularly useful for us in making a prediction of future workload (by correlating city's socio-economic profile with the workload), and these factors contain essential information about the development and economic growth of the city. More specifically, we aggregate the 911-call data by month to perform a more accurate workload prediction for the future.




(a) Population (b) Median rent (c) Median income

(d) Population (e) Median rent (f) Median income

Figure 2.1: (a-c): Raw data for census factors (population, median rent, median income) of South Fulton, GA in 2019, from American Community Survey, organized by census blocks (projected from a sequence of five-year census data). (d-f): Corresponding atomized census data of South Fulton, GA, in 2019.

To match this time resolution with the census data, we need to estimate the census data for each month from May 2018 to June 2019 following the resolution of the 911-call dataset. In addition, census data is organized by census blocks, as shown in Figure 2.1 (a - c), which is also different from the geographical atoms we consider for our study.

The census data we retrieved contains more than 75,000 tables. Many of these tables contain no data, or the data has no correlation with the police workload. Some tables include as many as 20 different socio-economic factors. Additionally, many of the tables are highly correlated with other tables, for example, median income and household income. Because of the inconsistent availability and format of such data, we need to perform variable selection over the data set to select the most important factors. In our discussions with the SFPD, we learned from their experience regarding which factors may affect the police workload the most, such as population and median rent, school enrollment, and the average age structures were built. The full list of census factors we are considering is shown in Table 4.1.

# CHAPTER 3

# DATA PREPROCESSING

In this section, we describe three key steps in data preprocessing before performing the beat design. In particular, we need to address the following two challenges in using the data: how to align (1) time resolution and (2) spatial resolution from the raw data with what we need in the design.

## 3.1 Geographical Atoms for the Beat Design

To accurately capture changing demographics and determine the new boundaries for each police beat, we must define high-resolution geographical atoms describing each area of the city. However, the size of the census blocks is determined by the population. In South Fulton, many low population density suburbs may be included in the same large census block. This results in very low-resolution geographical atoms that are not suitable to use in the beat design. We address this by creating artificial polygons of identical size as our geographical atoms to break up the city, where the optimal beat design can be found by aggregating multiple adjacent polygons.

The size of geographical atoms is essential to the performance of our optimization algorithm since it determines the number of variables and the precision of the workload estimation. There is a trade-off between computational efficiency and model accuracy in determining the size of geographical atoms. If the size of each atom is too large, we are unable to capture community demographics accurately; if the size of each atom is too small, the problem will become computationally intractable.

We found that using atoms with a side length of 0.345 miles allows us to estimate the local workload accurately while maintaining a reasonable number of decision variables in our optimization problem. We also found empirically that these atoms are roughly the size of a city block. The atomized map of the city was generated by intersecting the city boundary with a grid of atoms, resulting in a new grid of 1,187 geographical atoms, as shown in Figure 3.1. The police workload estimation and prediction will be performed based on these predefined geographical atoms.

Let $i \in \mathscr{I} = \{1, \dots, I\}$ denote the $i$-th atom and $k \in \mathscr{K} = \{1, \dots, K\}$ denote the $k$-th beat in our design. Let the binary decision variable $d_{ik} \in \{0, 1\}$ denote whether or not atom $i$ is assigned to beat $k$. A particular beat design is a unique graph partition determined by a matrix $D = \{d_{ik}\} \in \{0, 1\}^{I \times K}$. For each $i$, it satisfies $\sum_{k=1}^{K} d_{ik} = 1$. Given the beat design $D$, the set of atoms assigned to beat $k$ is denoted by $\mathscr{I}_k(D) = \{i : d_{ik} = 1\} \subseteq \mathscr{I}$. This leads to 1,187 $\times K$ decision variables in the optimization model. Figure 3.1 also shows the discretization of the existing beat configuration, where atoms with the same color represent a police beat. This configuration will also be used as an initial design in our algorithm, which will be discussed in Sec. 5.3.

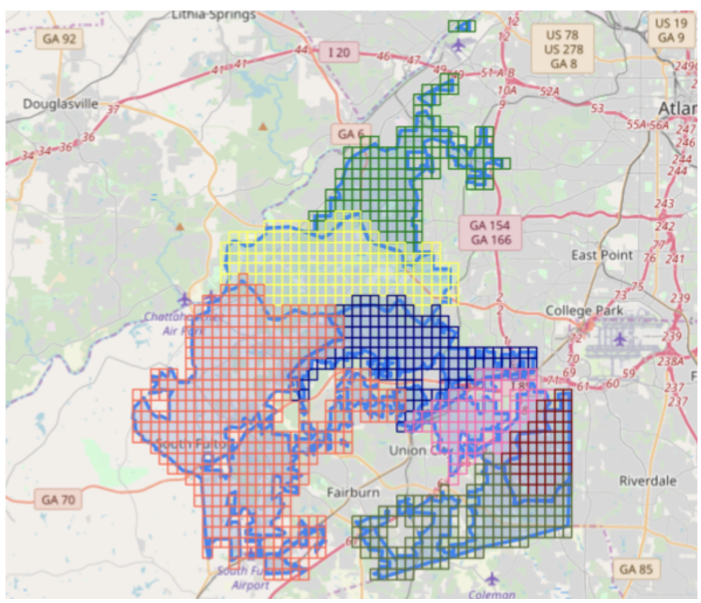



Figure 3.1: South Fulton region is partitioned into 1,187 square geographical atoms.

### 3.2 Census Data Atomization

A major challenge for estimating the socio-economic data for each geographical atom using census data is the inconsistency between census blocks and geographical atoms, where, as shown in Figure 2.1 (a-c), census blocks usually have a much larger area than geographical atoms. Here we need to perform a spatial interpolation to align the census data with our geographical atoms.

Specifically, we assume the census data, such as population are evenly distributed within the same census block. The data of each census factor in a geographical atom can be estimated by proportionally dividing the value in the census block where the atom falls into. The weight of the portion that an atom takes from a census block can be measured by the proportion between their areas. As shown in Figure 2.1 (d - f), the census data collected by census blocks have been discretized into geographical atoms. Given historical census data in the month $\ell \in [L - L_0, L]$, where $L$ and $L_0$ denote the last month and the time span of the historical data, respectively. The preprocessed census data is denoted as a tensor $X = \{x_{i\ell m}\} \in \mathbb{R}^{I \times L_0 \times M}$, as shown in Figure 3.2, where each entry $x_{i\ell m}$ indicates the value of the census factor $m \in \mathscr{M} = \{1, \ldots, M\}$ in atom $i$ and month $\ell$.

### 3.3 911 Calls-for-Service Data Preprocessing

We estimate the police workload for each geographical atom using the 911 calls-for-service dataset. The workload of each 911 call is evaluated by its processing time, i.e., the total time that the police spend on traveling and the investigation.

We calculate the workload by two steps: (1) count the number of 911 calls occurred in the $i$-th atom in $\ell$-th month, denoted as $N_{i\ell}$; (2) estimate the total workload for the $i$-th atom in the $\ell$-th month by multiplying $N_{i\ell}$ by the average processing time, denoted as $w_{i\ell}$. As shown in Figure 3.2, the count of 911 calls will be further used as the predictor in our spatial regression model, which will be discussed in Chapter 4.

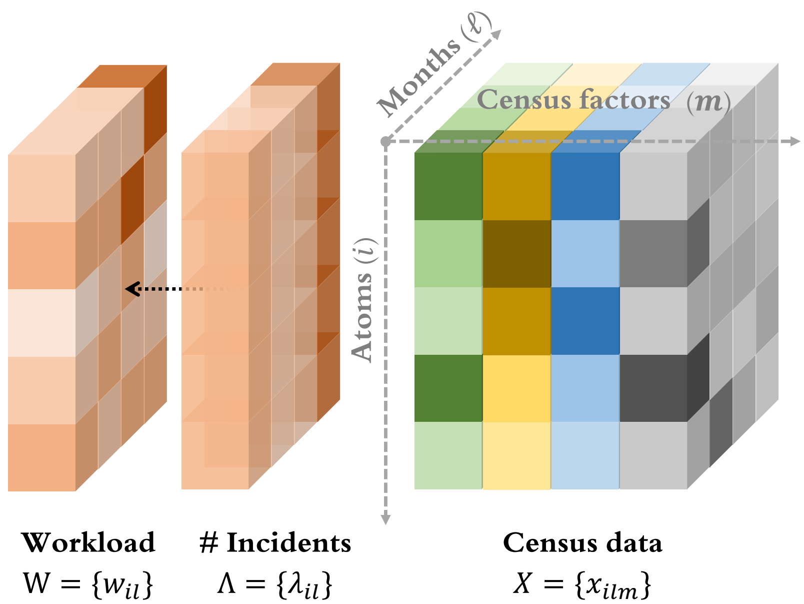



Figure 3.2: An illustration for the result of data preprocessing.

# CHAPTER 4
# POLICE WORKLOAD PREDICTION

We model the 911 calls-to-service incidents using non-homogeneous Poisson processes. To obtain higher resolution of workload prediction, we chose the month as a unit of time for the prediction. As a piece-wise approximation, we assume that the intensity $\lambda_{i\ell}$ for atom $i$ in a month $\ell$ is a constant. Thus, each beat is a homogeneous Poisson process with intensity of $\lambda_{i\ell}$. The arrival rates $\Lambda = \{\lambda_{i\ell}\} \in \mathbb{R}_+^{I \times L_0}$ can be approximated by $N_{i\ell}$, where $L_0 = 12$. We learned from South Fulton Police that, according to their experience, the occurrence of 911 calls is highly correlated with population and economic status of the beat and its neighborhood. We predict the future arrival rate $\lambda_{i\ell}$ in the future month $\ell = L + t, t = 1, 2, \ldots$ by a linear model that regresses the arrival rate to other endogenous variables (arrival rates in other beats) and exogenous factors (census factors). As shown in Table 4.1, we consider $M = 8$ census factors, which are statistically verified to be good predictors, including population, education level, and household income. Specifically, we use the spatially lagged endogenous regressors [32] defined as

$$\lambda_{i\ell} = \sum_{(i,j) \in \mathscr{A}} \alpha_{ij} \lambda_{j\ell} + \beta_0 \lambda_{i,\ell-1} + \sum_{t=1}^{p} \boldsymbol{\beta}_t^\intercal X_{i,\ell-t} + \epsilon_i, \quad \forall \ell \in [L - L_0, L],$$

where $p$ is the total number of past months of data that we consider for fitting the regressor, which in our case was 1. The adjacency matrix $A = \{\alpha_{ij}\} \in \mathbb{R}^{I \times I}$ specifies adjacency relationships between atoms. The temporal coefficient $\beta_0 \in \mathbb{R}$ specifies the influence of the last month. The coefficient $\boldsymbol{\beta}_t \in \mathbb{R}^M, \forall 1 \leq t \leq p$ specifies correlations with census factors and error term $\epsilon_i$ are spatially correlated. The set of adjacency pairs is defined by $\mathscr{A} = \{(i,j) : i, j \text{ are adjacent in } \mathcal{G};\ i, j \in \mathscr{I}\}$. The graph $\mathcal{G}$ is given by associating a node with every atom and connecting two nodes by an edge whenever the corresponding atoms are geographically adjacent. Here, we capture the spatial correlation between data using the standard spatial statistics approach, by assuming $\epsilon_i$ to be spatially correlated with correlation depending distance between two locations [33].

Table 4.1: Variables used for workload prediction

| PREDICTOR | REGRESSION COEFFICIENT | P-VALUE |
|---|---|---|
| POPULATION | 439.558 | 0.007 |
| NUMBER OF HOUSING UNITS | 158.440 | 0.019 |
| SCHOOL ENROLLMENT | 79.236 | 0.008 |
| MEDIAN HOUSEHOLD INCOME | 59.420 | 0.000 |
| MEDIAN NUMBER OF ROOMS | -10.560 | 0.006 |
| MEDIAN AGE | -7.421 | 0.001 |
| MEDIAN HOUSE PRICE | -16620 | 0.000 |
| AVERAGE YEAR BUILT | 170.140 | 0.003 |

# CHAPTER 5
# BEAT REDESIGN OPTIMIZATION

In this section, we introduce our objective and solution methods to the beat redesign optimization problem. We develop an optimization framework to shift beat boundaries, where artificial geographical atoms were assigned to some specific beats while balancing the workload. We formulate this problem as minimizing the workload variance by reconfiguring the beat plan with constraints, including the continuity and compactness of beats.

## 5.1 Objective

Based on the current situation of the imbalanced workload among existing police beats, our goal is to shift beat boundaries and make inter-beat workload distribution even. Hence, we introduced workload variance for evaluation. The ultimate objective of this problem is to minimize the inter-beat workload variance $Z(D)$ given a beat design $D$. The variance is a quadratic function of the workload in each beat, which implies that the objective function is convex with respect to the decision variables. A smaller variance indicates a more balanced inter-beat police workload.

The beat redesign problem can be formulated as:

$$\begin{aligned}
\underset{D}{\text{minimize}} \quad & Z(D) := \sum_{k=1}^{K} \left( w_{k\ell}(D) - \frac{\sum_{\kappa=1}^{K} w_{\kappa\ell}(D)}{K} \right)^2 \\
\text{subject to} \quad & \sum_{k=1}^{K} d_{ik} = 1, \quad \forall i \\
& \text{contiguity and compactness for each beat.}
\end{aligned} \tag{5.1}$$

Recall that the matrix $D = \{d_{ik}\} \in \{0,1\}^{I \times K}$ represents decision variables, where binary variable $d_{ik} \in \{0,1\}$ indicates whether or not geographical atom $i$ is assigned to beat $k$;

and $w_{k\ell}(D) = \sum_{i \in \mathscr{I}_k(D)} w_{i\ell}$ represents the total workload in beat $k = 1, \cdots, K$ in month $\ell$. The constraints will be explicitly defined in Sec. 5.2.

### 5.2 Compactness and Contiguity Constraints

To model the beat contiguity and beat compactness as constraints that the optimization problem subjects to, we introduced additional variables: $f_{ijk}$ is the flow from atom $i$ to atom $j$ in beat $k$; $h_{ik}$ equals to 1 if atom $i \in \mathscr{I}$ is selected as a sink in beat $k \in \mathscr{K}$, otherwise 0; $q$ is the maximum beat capacity. We follow [14, 25, 34, 26] to model the contiguity and compactness as linear constraints. Hence, there are 21,170,145 variables with 63,421,410 constraints in total.

**Contiguity constraints.** Contiguity constraints are imposed on each beat using the flow method [34]. For each beat $k$, there is a flow $f_{ijk}$ on the graph, where $f_{ijk}$ denotes flow from $i$ to $j$. Each beat has a hub vertex whose net flow is at most the number of vertices in the beat, less one. Each other vertex in the beat has a net flow of at most $-1$. This ensures that there is a path of positive flow from any vertex in the beat to the hub, implying contiguity.

Specifically, constraints (5.2a) represent the net outflow from each beat. The two terms on the left indicate, respectively, the total outflow and total inflow of atom $i$. If atom $i$ is included in beat $k$ but is not a sink, then we have $d_{ik} = 1$, $h_{ik} = 0$, and thus atom $i$ must have supply $\geq 1$. If atom $i$ is included in beat $k$ and is a sink, then we have $d_{ik} = 1$, $h_{ik} = 1$, and thus atom $i$ can have demand (negative net outflow) $\leq q - 1$. If atom $i$ is not included in beat $k$ and is not a sink, then we have $d_{ik} = 0$, $h_{ik} = 0$, and thus atom $i$ must have supply 0. If atom $i$ is not included in beat $k$ but is a sink, then we have $d_{ik} = 0$, $h_{ik} = 1$, and the rest of $d_{\cdot k}$ are forced to be 0, that is, no atoms are selected. Constraints (5.2b) specify the number of atoms that can be used as sinks. Constraints (5.2c) ensure that each beat must have only one sink. Constraints (5.2d) ensure that there is no flow into any atom $i$ from outside of beat $k$ (where $d_{ik} = 0$), and that the total inflow of any atom in beat

$k$ (where $d_{ik} = 1$) does not exceed $q-1$. Constraints (5.2e) make sure unless a atom $i$ is included in beat $k$, the atom $i$ cannot be a sink in beat $k$. Constraints (5.2f) and (5.2g) ensure that there are no flows (inflows and outflows) between different beats which forces eligible contiguity.

$$\sum_{(i,j)\in\mathscr{A}} f_{ijk} - \sum_{(i,j)\in\mathscr{A}} f_{jik} \geq d_{ik} - qh_{ik}, \qquad \forall i,k, \tag{5.2a}$$

$$\sum_{k}^{K}\sum_{i}^{N} h_{ik} = K, \tag{5.2b}$$

$$\sum_{i}^{N} h_{ik} = 1, \qquad \forall k, \tag{5.2c}$$

$$\sum_{(i,j)\in\mathscr{A}} f_{jik} \leq (q-1)d_{ik}, \qquad \forall k, \tag{5.2d}$$

$$h_{ik} - d_{ik} \leq 0, \qquad \forall i,k, \tag{5.2e}$$

$$f_{ijk} + f_{jik} \leq (q-1)d_{ik}, \qquad \forall i,k, \tag{5.2f}$$

$$f_{ijk} + f_{jik} \leq (q-1)d_{jk}, \qquad \forall j,k, \tag{5.2g}$$

$$d_{ik}, h_{ik} \in \{0,1\}, \qquad \forall i,k, \tag{5.2h}$$

$$f_{ijk} \geq 0, \qquad \forall i,j,k, \tag{5.2i}$$

**Compactness constraints.** Compactness is defined as geographical compactness with distance compactness and shape compactness [25, 26]. For distance compactness, a district is feasible only if the distance between population units must be less than a specified upper bound. For shape compactness, a district is feasible only if the square of the distance's maximum diameter divided by the district's area must be less than another upper bound [14].

Following the existing literatures, we add two additional linear constraints (5.3a), (5.3b) to ensure the compactness of beats. For each atom $i$, let $A_i$ be the area of $i$, and for each pair of atoms $i$ and $j$, let $l_{ij}$ be the square of the distance between the centroids of the beats. We also have a parameter $c_1, c_2 > 0$ controlling the degree of compactness.

$$l_{ij} e_{ijk} \leq c_1, \qquad \forall i, j, k, \tag{5.3a}$$

$$l_{ij} e_{ijk} \leq c_2 \sum_{i=1}^{K} d_{ik} A_i, \qquad \forall i, j, k, \tag{5.3b}$$

## 5.3 Solution Methods

Three methods were discussed in our experiments to search for optimal police beat design. The greedy algorithm serves to generate new beats iteratively and confirms the optimal number of beats for the future redesign. Following the Greedy redesign, we employ a heuristic optimization approach to help with optimizing beat design in contrast to the mixed-integer programming (MIP) approach.

### 5.3.1 Greedy Exploration

To determine the optimal number of beats in the final design, we perform an iterative greedy algorithm, which attempts to generate new beat greedily for the design for each iteration while preserving the original structure of the existing beat as much as possible. Intuitively, more beats may result in a more balanced workload distribution. However, the manpower of the SFPD and resources of the South Fulton City Council are limited. It is unrealistic to deploy such a design with a large number of beats. Hence, we adopt the Greedy algorithm to explore the optimal number of beats in our design. The procedure for "Greedily" creating new beat designs is demonstrated as follows.

For the $n$-th iteration, we define $D_n$ as the beat design, and $K$ is the number of beats at the last iteration. For the predicted workload in month $\ell$, the greedy algorithm can be performed by selecting the beat $k$ in $D_n$ with the largest workload, i.e., $\arg\max_k \{w_{k\ell}(D_n)\}_{k \in \mathscr{K}}$.

Then we split up the beat $k$ evenly into two beats using the K-means algorithm, where each atom in the beat is considered as a point. This will lead to generating a new beat, i.e., $K := K + 1$ and $\mathscr{K} =: \mathscr{K} \cup K$. The above process can be carried out iteratively until we find the design with the optimal number of beats.

We visualize our greedy design with different number of beats in Figures 7.3. As seen from the result, the beat with the highest workload, shown in red, is split in each iteration as a result. We also examine the variance of beat workload versus number of beats, and find the optimal number of beats, which will be discussed in Sec. 7.2.

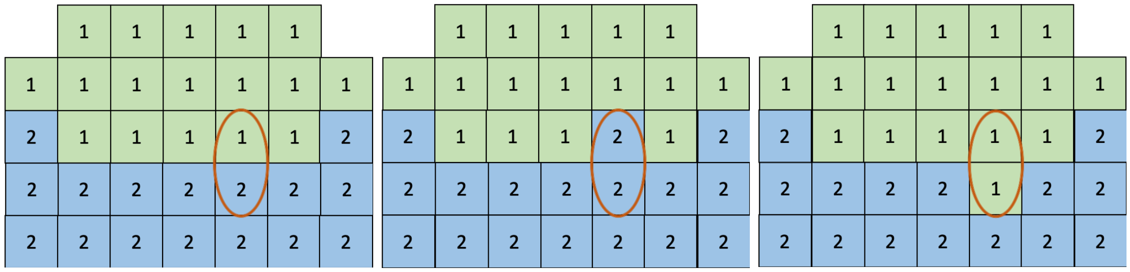
### 5.3.2 Mixed-Integer Programming

Once we determine the optimal number of beats in the design, we can consider this problem as a mixed-integer programming (MIP), where the global optimal design can be found by conventional techniques in convex optimization. However, as shown in Sec. 5.2, we have 21,134,535 continuous variables and 35,610 binary variables, as well as a set of additional linear constraints needed to be satisfied. In practice, the problem itself of searching for the global optimal design is computationally intractable and hard to be implemented on a large scale.

### 5.3.3 Heuristic Search

To tackle the computational issue in MIP, we propose a heuristic method to search for local optimal design at a reasonable computational cost [35]. In addition, the SFPD expects a design with a reasonable small adjustment on the existing beat configuration to avoid unnecessary deployment costs. Therefore, we use the beat design with the optimal number of beats as the initial design, and explore the "adjacent" design via swapping the beat assignments on the board of two beats, as shown in Figure 5.1.

Specifically, we adopt the simulated annealing algorithm to implement our heuristic search method. For $n$-th iteration of the search, we consider the adjacent designs which



(a) original (b) adjacent 1 (c) adjacent 2

Figure 5.1: An example of a design and its adjacent designs. (5.1a): original design; (5.1b): a adjacent design; (5.1c): another adjacent design.

satisfy the contiguity and compactness constraints as “candidate”. Then we choose one of these candidate designs with higher probability if it returns a lower workload variance and vice versa. The simulated annealing mechanism tends to less likely to accept a design with a higher workload variance as time goes on. We accept the candidate design when $P(D_{n+1}, D_n, T) \geq U(0,1)$, where $U(0,1)$ is the uniform distribution between $0$ and $1$, $D_n$ is the last candidate design, $D_{n+1}$ is the current candidate design, $T$ is a preset temperature that determines the speed of convergence, and the $P$ is the acceptance probability defined as

$$P(D_{n+1}, D_n, T) = \begin{cases} 1, & Z(D_{n+1}) < Z(D_n) \\ \exp\{|Z(D_{n+1}) - Z(D_n)|/T\}, & \text{otherwise.} \end{cases}$$

The last accepted candidate design will be considered as the refined beat design based on the greedy design. Figure 7.4 shows the refined 15-beat design based on 15-beat greedy design using simulated annealing.

# CHAPTER 6

# NUMERICAL EXPERIMENTS

In order to obtain the best result, we performed several numerical experiments with different solution methods to explore which method is suitable for our problem and performs the best.

At the first phase of the experiment, we apply the simulated annealing algorithm as our heuristic optimization method to the original police beat design (with 7 beats in total), satisfying the contiguous and compactness constraints. On average, the variance is reduced by about 6% after 7 iterations; about 15% after 15 iterations; about 60% after 127 iterations. The SFPD requires to check the redesign effect of plans with different number of beats, so this experiment has the disadvantage of the fixed beat number. Then, we move to the next experiment.

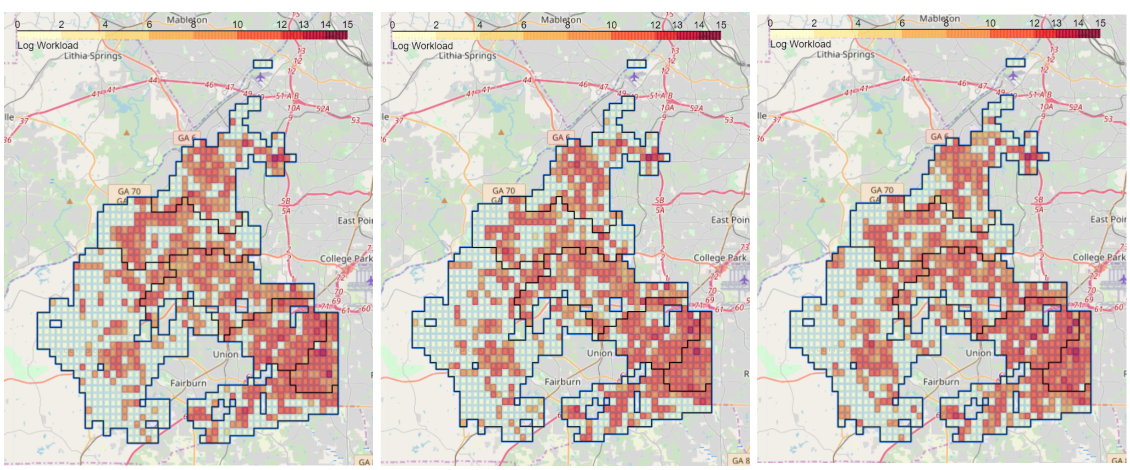
At the second phase, with the demand of checking the optimal number of beats, we generate the new beat greedily. Firstly, we generate the new beat manually by choosing beat boundaries on natural rivers and roads. Grids within the boundary are selected by breadth-first-search. This experiment shows a good result that over 90% of the original workload variance has been reduced after the new beats generation and heuristic optimization. However, this method is not efficient enough and in practice it is unnecessary to reduce the workload variance that much. Then, we tried the K-means algorithm as substitution for generating the new beat iteratively as discussed in Sec. 5.3.1, which is much faster than the manual work, while shows a relatively ideal result. Then the simulated annealing algorithm is applied to each greedy design for further variance reduction. We will discuss its performance in Chapter 7.

# CHAPTER 7
# RESULTS

In this chapter, we present our numerical results and final beat redesign for the City of South Fulton.

## 7.1 Workload Analysis and Prediction

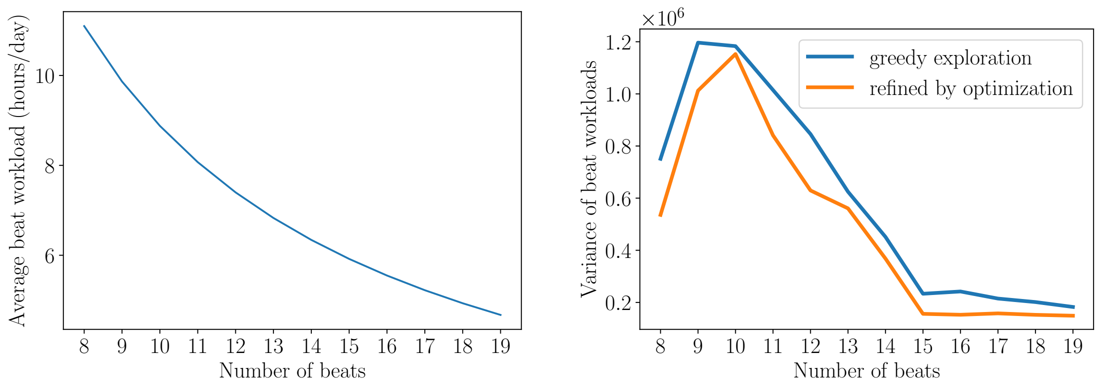
As mentioned in Chapter 1, under the existing beat design, the disparity of workload over different beats in the City of South Fulton has deteriorated rapidly in the past decade due to population growth and increasing traffic congestion. The most important metric for evaluating imbalance we considered is workload variance over beats. As we defined in Chapter 5, the variance is the sum of the squared deviation of the beat workload from its mean. To fully understand the workload imbalance situation, it is necessary to show how the existing configuration exacerbates the unbalance of workload over beats in the past and how the existing configuration will impact the future.




(a) 2020 prediction (b) 2021 prediction (c) 2022 prediction

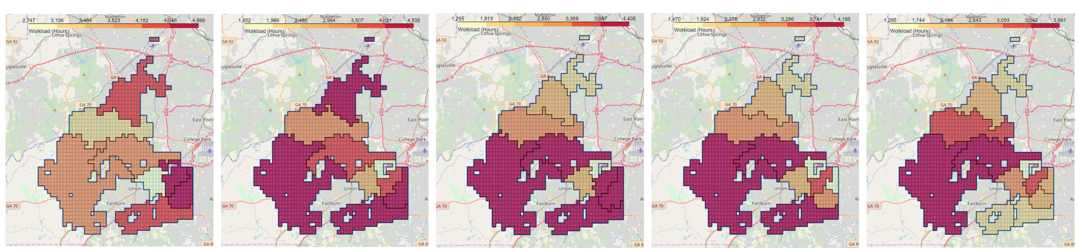
Figure 7.1: Workload prediction where dark lines outline boundaries of beats and the color shade indicates the level of the atom workload in each year.

Figures 7.1 summarizes the predicted workload distribution over the entire city for the next three years from 2020 to 2022. As we can see from the map, there is a clear trend that the general workload level continues to increase, and the major workload concentrates on particular areas (such as College Park in the east of the city and I-285 & I-20 in beat 4). Due to the increasing growth of South Fulton and urban sprawl, this trend is leading to a police workload imbalance.

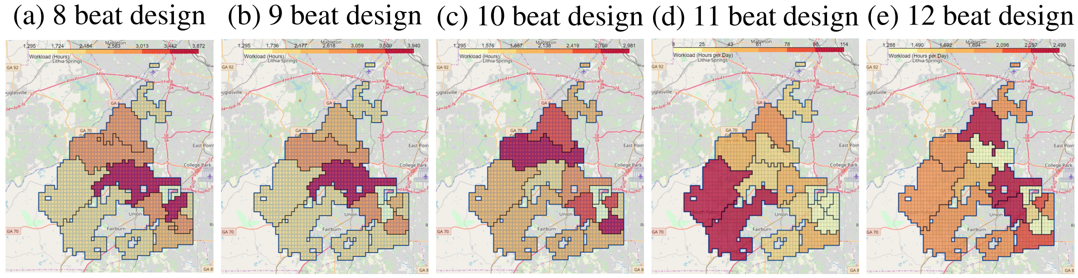



(a) Mean of workload (b) Variance of workload

Figure 7.2: Variance (right) and mean (left) of beat workloads versus number of beats.

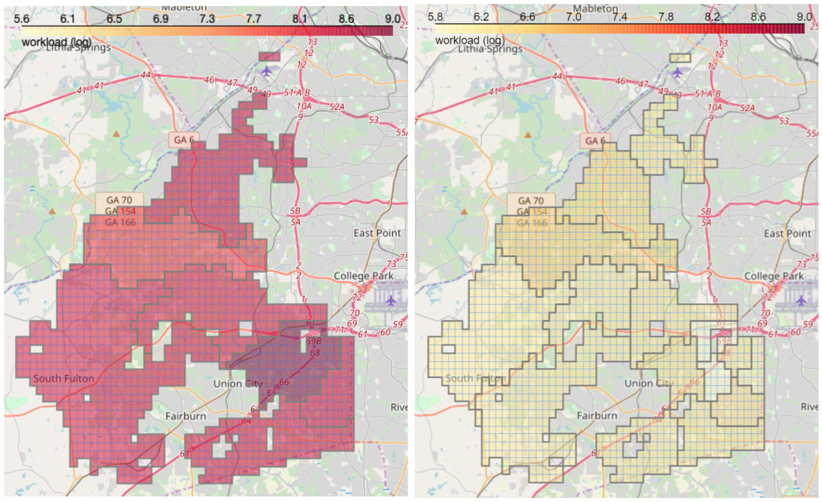


(a) 8 beat design (b) 9 beat design (c) 10 beat design (d) 11 beat design (e) 12 beat design



(f) 13 beat design (g) 14 beat design (h) 15 beat design (i) 16 beat design (j) 17 beat design

Figure 7.3: Greedy beat designs where dark lines outline boundaries of beats and the color depth represents the level of the beat workload. The scale is adjusted in each image.




(a) Origin (b) Refined

Figure 7.4: Beat redesign where dark lines outlines boundaries of beats and the color shade indicates the predicted level of the beat workload in 2021. (7.4a): Original beat design before 2020; (7.4b): Refined beat redesign with 15 beats.

## 7.2 Optimal Number of Beats

As we discussed in Sec. 5.3.1, when creating a beat design, the most important metric for evaluating imbalance is the workload variance over beats. However, for determining the optimal number of beats in the design, we also need to consider the cost associated with adding more beats, which includes the cost of additional training, hiring new officers and so on. Therefore, there is a trade-off to minimize the workload variance while avoiding unnecessary costs of adding new beats.

We find that as we first begin to increase the number of beats, the workload variance decreases sharply before 15 beats. Then as we further increase the number of beats beyond our initial increase, the workload variance reduction is much smaller. This is further demonstrated in Figure 7.2, where we can see that there are diminishing returns as we increase the number of beats. Therefore, we call 15 the optimal number of beats, and the corresponding 15-beat greedy design will be used as an initial design for further refinement. In addition, Figure 7.3 also shows the comparison between the workload variance achieved by the greedy exploration and the original design.

### 7.3 Beat Design Result

We present the existing beat design and the beat redesign for the year 2021 in Figure 7.4. We can see that the beat workload distribution with the beat redesign (Figure 7.4b) is more uniform than the existing beat plan (Figure 7.4a) for the year 2021.

We want to remark that, because of our effective solution exploration scheme, and constraints on the shapes of the beats (contiguity and compactness), the shapes of the beats are quite reasonable. We also confirmed our beat boundaries with the SFPD, and the solution is deemed feasible.

We also compare our proposed redesign with the existing beat plan analytically. As Figure 7.2 shows, the workload variance has been drastically reduced by 89% $\sim$ 92%. More concrete details are presented in Tables 7.1 and 7.2 where we compare the workload in the original design to the new refined design and the greedy beat design. Comparing to the existing configuration, we see the proposed beat reconfiguration achieves a lower level of workload variance as well as small variance increment in the future year 2021. Last but not least, there are only 11 beats being created in our proposed redesigned beat plan, which delivers significant expenses savings in terms of implementation of the redesigned beat plan.

### 7.4 Staffing Level Analysis

We quantify our potential police response workload by converting the workload in each beat into hours per day for each police officer. Table 7.1 shows workload in 2019 with the original design and Table 7.2 shows workload 2019 with the new design and predicted workload for 2021 with the new design. These tables tell us how many hours per day, a police officer expects to be responding to 911 calls. As we can see when comparing table 7.1 and table 7.2, our beat design drastically reduces the workload per day in each beat. Most importantly, our new beat design results in a decrease in workload variance of

Table 7.1: Workload per beat in the original beat design based on 2019 crime statistics

| BEAT NUMBER | WORKLOAD 2019 (HOURS/DAY) |
|---|---|
| 1 | 38.59 |
| 2 | 24.84 |
| 3 | 32.84 |
| 4 | 34.44 |
| 5 | 65.94 |
| 6 | 38.44 |
| 7 | 34.96 |
| VARIANCE | 142.91 |

over 85%, making policing more equal in the city.

In the City of South Fulton council meeting, the city council emphasizes the importance of community engagement from the police force. Thanks to our beat design, the police workload per day in each beat can be reduced drastically; this will allow police officers to participate in community events and start pro-active patrols. This is a huge difference from the past 50 years, where police officers have been going from call to call on their entire shift. Additionally, the staffing level prediction gives the SFPD how many officers they need to handle the 911 calls in a beat. They then can recruit more officers for the sole purpose of community engagement and pro-active patrolling if they desire.

Table 7.2: Workload per beat in the greedy beat design and refined beat design based on 2019 crime data and 2021 workload prediction

| BEAT NUMBER | WORKLOAD 2019 (HOURS/DAY) | | WORKLOAD 2021 (HOURS/DAY) | |
|---|---|---|---|---|
| | GREEDY | REFINED | GREEDY | REFINED |
| 1 | 17.15 | 17.15 | 18.05 | 18.05 |
| 2 | 24.84 | 23.56 | 27.09 | 25.61 |
| 3 | 18.78 | 20.08 | 17.91 | 19.91 |
| 4 | 17.45 | 17.08 | 16.83 | 16.14 |
| 5 | 22.10 | 20.31 | 21.40 | 19.32 |
| 6 | 14.69 | 18.30 | 14.54 | 16.73 |
| 7 | 17.55 | 19.99 | 17.67 | 20.01 |
| 8 | 12.51 | 12.51 | 11.66 | 11.66 |
| 9 | 10.79 | 10.79 | 11.10 | 11.10 |
| 10 | 21.45 | 21.87 | 21.45 | 21.87 |
| 11 | 23.75 | 19.33 | 22.2 | 22.62 |
| 12 | 17.41 | 17.41 | 23.40 | 21.60 |
| 13 | 17.00 | 16.82 | 16.70 | 15.87 |
| 14 | 20.53 | 18.89 | 19.99 | 17.81 |
| 15 | 14.06 | 15.94 | 13.18 | 15.93 |
| VARIANCE | 15.12 | 10.13 | 18.269 | 13.15 |

# CHAPTER 8
# IMPLEMENTATION

In January 2020, we submitted the final report to the South Fulton Police Department and the South Fulton City Council. The report was reviewed by Police Chief Meadows, Deputy Police Chief Rogers, and Mayor Bill Edwards. Our report analyzed the police workload and proposed a detailed redistricting plan. Our redistricting plan mainly changed in four areas (Figure 7.4b): We add three new beats in the southeast of the city near College Park, the area with the highest workload. The biggest beat in the west of the city is split into two beats. We add a beat in the north of the city near the airport. The southern beat is also split into two. In total, the redistricting plan has reduced the response time throughout the city and rebalanced the police workload between the fifteen beats.

Later that month, the South Fulton City Council approved the new beat design (Figure 8.1). The South Fulton Police Department plan to implement the new beat design in early 2020. The new beat design was praised by the city council, as some council members said that our beat design and study has been long needed and that it sets an example for other cities in the southeast. Residents of South Fulton acclaimed about the change on social media and thanked the City of South Fulton Police Department and our team for contributing to the communities. The new beat design also received coverage from several news sources, including Fox 5 Atlanta [36].


Figure 8.1: The City of South Fulton Council work session on January 14, 2020.

# CHAPTER 9
# CONCLUSIONS AND FUTURE WORK

In this paper, we presented our work on the City of South Fulton police beat redesign. We propose an optimization framework with the spatial regression model as well as large-scale data analytics. We construct an operational model to predict beat workload using an accurate and tractable linear approximation. The proposed method yields a redesigned beat plan with lower workload variance by only changing eight beats. Currently, we are continuing our partnership with the SFPD to observe the police workload in the City of South Fulton as the city and workload grow. If the workload becomes unbalanced once more, we can quickly suggest a new beat design using our already existing methods. As the SFPD continues to grow, they will also hire an information officer that will assist in workload analytics and carry on our workload prediction.

Our future work is improving our model to reduce cost and the time consuming. Currently, our MIP solution method is computationally intractable and hard to be implemented. Although our heuristic search method works efficiently in this problem, we are expecting to shorten the time for searching the large scale of designs. [37] introduces the Lagrangian-based variable fixing for Hess model and contiguity models. In H. Validi's experiment, the method has been implemented to 43 states and their experiment setting for the Georgia is similar to ours. With the setting of 1969 tracks (which is similar to our grids) and 14 districts (beats in our setting), the Lagrangian variable fixing method can return the result in 11.43 seconds, by which we can refer it to our problem and reduce the execution time for the MIP model.

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