THE IMPROVEMENT OF MULTI-MODAL FREIGHT TRANSPORT NETWORKS

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THE IMPROVEMENT OF MULTI-MODAL

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SUMMARY

Although the concept of multi-modal freight transport planning has been given little serious attention in the past, public sector transport planners are becoming increasingly concerned with the efficient development of all freight transport modes. Existing network improvement algorithms concentrate on the improvement of a single mode and, thus, are of little value in multi-modal freight transport planning. In this thesis a multi-modal freight transport improvement problem is formulated, and a heuristic solution methodology is developed for the solution of large scale problems. The problem is concerned with the modification of arcs on a multi-modal network so that total disutility associated with the network is minimized. Distinctive features of the formulation include a mode abstract multinomial logit modal split model and convex arc transport characteristic improvement functions. The heuristic solution methodology developed to solve the problem is based on the general Continuous Optimal Adjustment heuristic suggested by Steenbrink. The principal component of the methodology is the solution of a concave disutility transportation assignment problem. Two methods are developed to solve this problem. The first uses Dantzig-Wolfe decomposition to solve an arc-path formulation of the general multi-commodity fixed charge network flow problem. The second is a heuristic based on the local optimum seeking procedure developed by Yaged. The

Continuous Optimal Adjustment solution methodology was programmed using the latter solution procedure for the transportation assignment problem. Thirty test runs were made on a large scale test problem derived from the Multi-State Transportation Corridor research program. A number of conclusions are reached after analyzing the results, the most important being that the solution methodology is viable. Although solution times are long, this is not unusual for problems of this size or design construction projects of this scale. Solution times may be shortened considerably by proper selection of methodology parameters. As a final step, the solution methodology is extended to include multiple transport commodity classes.

CHAPTER I

INTRODUCTION AND REVIEW OF THE LITERATURE

The purpose of this research is to formulate a multi-modal freight transport network improvement problem, and to develop a practical methodology for its solution. The network improvement problem is concerned with the modification of arcs on a multi-modal network so that the total disutility associated with the network is minimized. The present research is motivated by needs encountered in a sponsored research program conducted by a consortium of nine universities of the U. S. Department of Transportation [Jones, 1977]. The primary objective of that program is the development of a general analytical procedure which jointly identifies economic development opportunities and the transportation services needed to assure their viability. An integral component of that procedure is a methodology which can be used to identify desirable improvements in a large-scale multi-modal freight transport network. A large-scale network is defined as one with about 300 nodes and 1000 arcs.

This particular problem has been formally addressed in the literature, although this fact represents more the historical development of current multi-modal freight transportation than the true importance of the problem. As background, intercity freight moves primarily on three independent transport modes: highway, rail, and water. From the inception of current multi-modal freight transportation, the three modes have been owned and operated independently. Fierce modal competition, mistrust, and national transportation policy encouraged and even legally required their independence. A natural consequence of modal independence was that, for all practical purposes, system improvements were performed independently for each mode. Waterway improvements, highway improvements, and railway improvements were planned by the U.S. Army Corps of Engineers, Federal and state highway agencies, and railroad companies, respectively. As a result, almost all network improvement models which were developed for or are adaptable to freight networks are oriented toward a specific mode. Since the single-mode problem has been widely studied and is closely related to the multi-mode problem, it is useful to review previous work in this area.

1. The Single-Mode Network Improvement Problem

Both the single-mode and multi-mode network improvement models can be classified in the following ways: [Dantzig, 1976]

(1) Whether the investment decision variables are discrete or continuous.

(2) Whether flow assignment is based on user equilibrium (first principle of Wardrop) or systems optimal (second principle of Wardrop) [Potts and Oliver, 1972].

(3) Whether congestion is allowed.

Dantzig classified some previous models using these factors. After analyzing the ability of the various models to solve large-scale network problems, Dantzig reached several important conclusions:

(1) For problems with discrete investment decision variables, integer programming techniques, such as branch and bound, will usually be necessary. As a result, such models will not have the ability to solve large-scale network problems.

(2) For problems utilizing user equilibrium traffic assignment and congestion, a more complex objective function is required. Again, the result is an inability to solve large-scale network problems. It should be noted that with no congestion effects a user equilibrium traffic assignment reduces to a systems optimal assignment.

Thus, Dantzig concluded that only a network improvement model utilizing continuous investment decision variables and systems optimal assignment could handle large-scale networks. This will be the approach followed in this research. Such a model may be formulated as:

Problem S:

$$\text{Min} \quad \sum_{j \varepsilon A} [H_j(f_j, I_j) + \lambda G_j(I_j)] \tag{1-1}$$

$$\text{s.t.} \quad \sum_{j \varepsilon W_i} f_j^r - \sum_{j \varepsilon V_i} f_j^r = h_i^r \qquad \forall \; i \; \varepsilon \; N, \; r \; \varepsilon \; O \tag{1-2}$$

$$f_j = \sum_{r \varepsilon O} f_j^r \qquad \forall \; j \; \varepsilon \; A \tag{1-3}$$

$$L_j \leq I_j \leq U_j \qquad \forall \; j \; \varepsilon \; A \tag{1-4}$$

$$f_j^r \geq 0 \qquad \forall \; j \; \varepsilon \; A, \; r \; \varepsilon \; O \tag{1-5}$$

where:

$N \equiv$ set of nodes

$A \equiv$ set of arcs

$O \equiv$ set of origins

$W_i \equiv$ set of arcs originating at node i

$V_i \equiv$ set of arcs terminating at node i

$D_i \equiv$ set of destinations for flow from origin i

$f_j^r \equiv$ flow on arc j from origin r

$f_j \equiv$ total flow on arc j

$I_j \equiv$ investment decision for arc j

$L_j \equiv$ lower bound on investment decision for arc j

$U_j \equiv$ upper bound on investment decision for arc j

$S_{ij} \equiv$ flow from origin i to destination j

$$h_i^r = \begin{cases} -S_{ri} & \text{if } i \text{ is a destination node} \\ \sum_{j\varepsilon D_i} S_{rj} & \text{if } i = r \\ 0 & \text{otherwise} \end{cases} \tag{1-6}$$

$\lambda \equiv$ conversion between investment dollars and travel time

$G_j(I_j) \equiv$ cost of making investment decision I_j on arc j

$H_j(f_j, I_j) \equiv$ total travel time on arc j assuming flow f_j and investment I_j

The objective (1-1) is to minimize the total social transportation cost, both travel time and investment. Constraint set (1-2) is the familiar conservation of flow equations. Constraint set (1-3) forces total flow on each arc to equal the sum of flow over all origins. Constraint set (1-4) sets lower and upper bounds on investment. Constraint set (1-5) forces nonnegative arc flows.

The actual form of the model and the resulting solution procedure depend upon the functions H_j. Dafermos assumed that total arc travel time was a quadratic function of arc flow and that investment only affected capacity [Dafermos, 1968]. Using a systems optimal assignment, she proved that a gradient technique could be used to reach optimality. Morlok assumed that total arc travel time was a piecewise linear function of arc flow and that investment only shifted the location of the breakpoints [Morlok, 1969]. Using systems optimal assignment, he solved the resulting problem using linear programming. Steenbrink assumed that total arc travel time was a nonlinear differentiable function of arc flow, capacity, and free flow travel time and that investment only affected capacity [Steenbrink, 1974]. The function was similar to the Federal Highway Administration (FHWA) travel time curve [COMSIS, 1973]. To solve the problem using systems optimal assignment, Steenbrink developed the following decomposition procedure: [Steenbrink, 1974]

(1) A subproblem is solved for each network arc.

(2) The results of these subproblems are substituted into the objective function to obtain a master problem equivalent to the nonlinear transportation assignment problem.

(3) The assignment problem is solved to obtain the optimal flows of the original problem.

Dantzig showed that Steenbrink's decomposition technique would work equally well on the total arc travel time functions proposed by Dafermos and Morlok and that the technique would be an efficient method for solving large-scale network problems [Dantzig, 1976]. He also showed that, in general, one cannot modify unit arc travel time by investment without introducing nonconvexities into the objective function. One exception to this finding is through the use of piecewise-linear total travel time functions where distinct new arcs are introduced to represent distinct unit travel time options [Dantzig, 1976, Ch. 4].

It is interesting to note that heuristic procedures have not been developed extensively for problem S. There are two primary reasons. The first is that most researchers have made assumptions which result in a linear or convex objective function. The second is that many researchers do not deal with large-scale networks. For example, Dafermos and Morlok each assumed that investment could only affect capacity and could not affect free flow travel time [Dafermos, 1968; Morlok, 1969]. On the other hand, Steenbrink used the FHWA travel time curve which assumes that travel time is a function of capacity and, thus, of investment [Steenbrink, 1974]. This assumption led to a nonconvex objective function in S. When Steenbrink performed the decomposition described previously, the resulting master problem took the form of a nonconvex transportation assignment problem. Had Steenbrink been modelling a smaller network, he might have used a branch and bound scheme such as that of Rech and Barton to solve the problem [Rech and Barton, 1970]. However, since he was modelling the entire Dutch highway system, Steenbrink concluded that a branch and bound procedure was not tractable. Thus, Steenbrink developed a heuristic transportation assignment routine called SALMOF (Stepwise Assignment according to Least Marginal Objective Function). Heuristics are commonly used on the single-mode network design problem with discrete variables for the same reason. These heuristics include those of Scott, Billheimer, O'Connor, Barbier, and Haubrich [Scott, 1969; Billheimer, 1970; O'Connor, 1970; Barbier, 1966; Haubrich, 1972].

LeBlanc proposed a rail network improvement model [LeBlanc, 1976]. The formulation was stated as:

$$\text{Min} \quad \sum_{j\varepsilon A} [H_j(f_j, I_j) + \lambda G_j(I_j)] \tag{1-7}$$

$$\text{s.t.} \quad \sum_{j\varepsilon W_i} f_j - \sum_{j\varepsilon V_i} f_j = h_i \qquad \forall\, i \varepsilon N \tag{1-8}$$

$$I_j \leq c_j \qquad \forall\, j\varepsilon A \tag{1-9}$$

$$0 \leq f_j \leq U_j \qquad \forall\, j\varepsilon A \tag{1-10}$$

where:

$$H_j(f_j, I_j) = I_j(f_j)^{1/2} \tag{1-11}$$

$$\lambda G_j(I_j) = b_j(I_j - c_j)^2 + d_j \tag{1-12}$$

$b_j, c_j, d_j \equiv$ parameters of model

$$h_i = \begin{cases} -D_i & \text{if } i \text{ is destination node} \\ S_i & \text{if } i \text{ is a source node} \\ 0 & \text{otherwise} \end{cases} \tag{1-13}$$

$D_i \equiv$ net demand at node i

$S_i \equiv$ net supply at node i

$U_j \equiv$ upper bound on flow on arc j

One should note that the objective function is neither convex nor concave. Several differences in the contraint set of the LeBlanc model (1-7)-(1-10) and the standard formulation (1-1)-(1-5) are discernible:

(1) The LeBlanc formulation is a single commodity transshipment problem with many origins, while the standard formulation is a multi-commodity problem with one distinct commodity per origin.

(2) The LeBlanc formulation has upper bounds (1-10) in arc flow while the standard formulation does not.

(3) The LeBlanc formulation has no lower bounds on the investment decision variables.

LeBlanc proved that constraint set (1-9) was redundant and, thus, the objective could be transformed into a strictly concave function over f_j. The resulting problem is a concave transshipment problem which LeBlanc solved by using a branch and bound procedure. Since the LeBlanc formulation does not allow one to preselect origin-destination (O-D) flows, it will not be considered further.

2. The Multi-Modal Network Improvement Problem

It is interesting to consider the multi-modal problem in terms of the single-mode problem. Consider three single-mode problems, one for each mode. Adding the objective functions and combining the constraint sets, one is left with a multi-modal design problem which is separable by mode. However, as it now stands, this problem is not a complete representation of the multi-modal problem. For example, O-D demands are fixed for each mode, which implies that investment in one mode will not affect demand on it or the other two modes. Recent results in freight demand and modal split modelling indicate that O-D demand for any mode is a function of the transport characteristics (cost, time, etc.) of the given mode as well as of those of competing modes [Creighton, 1977; Townsend, 1969; Sharp, 1977; Herendeen, 1969]. Thus, if one makes the reasonable assumption that investment in a mode will modify any of its transport characteristics, then one must also conclude that the model described above is inadequate. Furthermore, the model does not recognize the possibility of multi-modal shipments. To complete the model one would need to make two additions. The first is a set of modal demand or split relationships which relate O-D modal demand, the h_i^r, to the transport characteristics and, thus, to investment. The second is a set of intermodal transfer arcs which recognize the possibility of intermodal shipments. Theoretically, the current independent planning of modal subsystem improvements represents a single stage modal decomposition of the multi-mode problem. Such a decomposition procedure would yield a global optimal solution to the initial problem described above. However, there is no guarantee that this procedure will yield an optimal solution to the complete multi-modal problem.

While the formulation proposed for this research is developed principally for freight transport networks, this is not to say that the methodology could not be extended to passenger transport. In fact, the little research that has been done on multi-modal network design and improvement has been related to passenger transport. Since the passage of the Federal Aid Highway Act of 1962, urban passenger transportation planning has, at least superficially, been multi-modal. Two multi-modal, passenger network design and improvement models have been formulated for use in the urban setting. Creighton has developed a model to determine optimal aggregate investment in individual (highway) and group (mass transit) transportation systems in an urban area [Creighton, 1966]. The study area involved is a one-square mile section of urban land, for which investment decisions are to be made, surrounded by a large urban area of uniform density, trip lengths, transportation facilities, etc. The methodology is essentially to construct a response surface of total daily transportation cost, including user and investment cost, as a function of the spacing of individual and group transport facilities in the study area. Spacing and user costs on a mode are assumed to be simple functions of investment in the mode. Spacing is assumed uniform throughout the study area. Although the author does not explicitly state how trips are divided between individual and group transport, he apparently divides trips on the basis of spacing and, thus, investment. Having constructed the response surface, one selects the combination of spacings which yield the minimum point on the surface.

Morlok has developed a methodology utilizing linear programming (LP) for finding the optimal combination of modal services in an urban transportation corridor. The study area involved is a linear corridor radiating from the central business district (CBD). The corridor is assumed to be segmented into a finite number of linear zones with each zone spanned by a road segment. The decision variables of the problem are:

(1) Zone through which the rapid transit line should be completed from the CBD.

(2) Capacity of the rapid transit line on each completed segment of the corridor.

(3) Capacity of each road segment in the corridor.

(4) Slowness on each road segment in the corridor.

(5) Flow on each segment for each type of facility.

Variables 2 through 5 are assumed continuous. The objective of the analysis is to minimize the total annual out-of-pocket cost of transportation, including:

(1) Annual road capital cost =

$$\sum_{i=1}^{n} \left[C_i c_i + M_i \left(1 - \frac{m_i}{\bar{M}_i} \right) + S_i (m_i - s_i) \right] \tag{1-14}$$

where:

$C_i \equiv$ annual unit capacity cost

$c_i \equiv$ capacity

$S_i \equiv$ annual unit cost of additional non-peak period slowness, slowness is expressed in minutes per mile

$s_i \equiv$ non-peak period slowness

$n \equiv$ number of zones in corridor

$M_i \equiv$ annual unit peak period speed cost, normalized to be consistent with slowness units

$m_i \equiv$ peak period slowness

$\bar{M}_i \equiv$ maximum technological slowness

(2) Annual vehicle operating costs $= \sum_{i=1}^{n} V_i v_i$ (1-15)

where:

$V_i \equiv$ cost per vehicle on segment i

$v_i \equiv$ annual volume on segment i

(3) Annual rapid transit capital and operating cost =

$$C_{rt} + \sum_{i=1}^{n} P_i p_i \tag{1-16}$$

where:

$C_{rt} \equiv$ capital cost of completing rapid transit line (dependent on length of line)

$P_i \equiv$ annual cost per unit of peak capacity

$p_i \equiv$ peak period demand for rapid transit from zone i

It should be noted that the assumed time cost of travelers is not included in the objective. Optimization of the objective is subject to a number of constraints which include:

(1) Modal choice constraints:

$$P_j = Y_j \frac{\sum_{i=1}^{h} L_i m_i + H_h}{\sum_{i=1}^{h} L_i R + W_j} + B_j \qquad h = \begin{cases} j & \text{if } j \leq k \\ k & \text{if } j > k \end{cases} \tag{1-17}$$

where:

$L_i \equiv$ length of zone

$R \equiv$ uniform average slowness of transit

$k \equiv$ segment through which rapid transit line is completed

$H_h \equiv$ from the point of modal decision, the time required to get from decision point to corridor highway plus that required to get from corridor highway in CBD to job.

$Y_j, B_j \equiv$ parameters of the modal choice model

Several characteristics of the modal choice constraints should be noted:

(a) The transit travel demand from any origin is a simple linear function of the ratio of highway travel time to transit travel time at the point of modal decision.

(b) Once the rapid transit line is fixed, the denominator of the travel time ratio becomes a constant. This implies that the resulting constraint is now linear. This would not be the case if one could vary the speed or frequencies of service on the line.

(2) Capacity constraints stipulate that total capacity on any segment must equal total demand.

(3) Level of service constraints specify minimum levels of transport service in the corridor.

(4) Technological constraints place lower and upper bounds on decision variables.

Constraint sets 2, 3, and 4 are linear. The solution procedure suggested by Morlok is obvious, given the formulation. For each zone in the corridor, assume that the transit line extends from the CBD to this zone. This fixes C_{rt} in the objective function and the denominator of the modal choice constraints, leaving a linear program which can be easily solved. After solving all linear programs, one selects the optimal values of the decision variables associated with the linear program having the minimum value of the objective function.

There has been only one significant study of the more general case of intercity, multi-modal network design [Morlok, 1969]. This study concerned the passenger transport network in the Northeast corridor. Morlok and his associates developed a methodology utilizing dynamic programming (DP) and LP for solving a multi-period, multi-modal, intercity, passenger transport network design and improvement problem. The study area can be any region representable by a network rather than the limiting cases discussed previously. The decision variables include:

(1) New additions to the network for each time period.

(2) Level of service characteristics for existing arcs for each time period.

    (a) Capacity and travel time of highway arcs.

    (b) Capacity and frequency of movements on common carrier arcs.

(3) Arc flows for each time period.

The second and third sets of variables are assumed continuous. The objective of the analysis is to minimize the total discounted out-of-pocket costs of the network over the desired time span.

The solution methodology involved the use of DP in order to make choices concerning the integer variables, the addition of new arcs over time, and the use of LP to make choices regarding the values of the continuous variables. Each stage of the DP corresponds to one time period. Alternatives to be considered at each stage correspond to the different alternatives of fixed plant. For each such fixed network there remains the task of selecting the optimal values of the continuous variables. This can be accomplished by solving what Morlok calls the Optimal Multi-Modal Network Operation Model. This model essentially corresponds to the multi-modal network improvement problem.

The Optimal Multi-Modal network Operations Model has a set of decision variables which include all the continuous variables of the original problem for a single time period. The objective is to minimize the total, out-of-pocket costs associated with a given network configuration for a single time period. These costs includes:

(1) Improvement cost on highway arcs =

$$\sum_{j \in A_{HW}} (C_j c_j + T_j t^j + \hat{T}_j) \qquad (1\text{-}18)$$

where:

C_j = annual unit capacity cost on arc j

c_j = additional vehicle capacity on arc j

T_j ≡ annual unit cost of travel time on arc j, a negative number

t_j ≡ travel time on arc j

$\hat{T}_j$ ≡ annual cost associated with designing a highway for a designated maximum speed

A_{HW} ≡ set of Highway (HW) arcs

(2) Highway operating cost =

$$\sum_{j\varepsilon A_{HW}} F_j \sum_{r\varepsilon\emptyset} \sum_{d\varepsilon D_r} \sum_{p\varepsilon P^j_{rd}} d_{rd,HW,p}(1/E) \tag{1-19}$$

where:

F_j ≡ annual cost of daily auto trip on fixed arc j

$d_{rd,HW,p}$ ≡ daily demand from r to d via highway path p

E ≡ average auto occupancy

$\emptyset$ ≡ set of all origins

D_r ≡ set of all destinations corresponding to origin r

P^j_{rd} ≡ set of all paths between r and d containing arc j

(3) Common Carrier operating costs =

$$\sum_{m\varepsilon M} \sum_{j\varepsilon A_m} (F_j f_j + Y_j y_j) \tag{1-20}$$

where:

F_j ≡ annual unit cost of trip frequency on arc j

f_j ≡ daily vehicle trip frequency on arc j

Y_j ≡ annual unit cost of vehicle capacity on arc j

$y_j \equiv$ vehicle capacity on arc j

$M \equiv$ set of common carrier modes

$A_m \equiv$ set of arcs corresponding to mode m

Optimization of the operations model is subject to a set of constraints which include demand constraints for common carrier modes:

$$\sum_{p \varepsilon P_{rd,m}} d_{rd,m,p} \geq U_{rd} + F \sum_{p \varepsilon P_{rd,m}} f_{rd,m,p} + P(p_{tm} + D_{rd} p_{vm}) + Tt_{rd,m}$$

$$- P(p_{t,best} + D_{rd} P_{v,best}) - Tt_{rd,best} \geq 0 \qquad (1\text{-}21)$$

where:

$f_{rd,m,p} \equiv$ frequency of path p on mode m from r to d

$p_{t,m} \equiv$ threshold price associated with price of trip on mode m

$p_{t,best} \equiv$ threshold price associated with price of trip on minimum price mode

$D_{rd} \equiv$ distance between r and d

$P_{v,m} \equiv$ price per unit distance associated with price of trip on mode m

$p_{v,best} \equiv$ price per unit distance associated with price of trip on minimum price mode

$t_{rd,m} \equiv$ travel time of trip from r to d on mode m

$U_{rd}, F, P, T \equiv$ parameters of model

$P_{rd,m} \equiv$ set of paths by mode m between r and d

Several characteristics of the common carrier modal demand constraints should be noted:

(1) The constraints place a lower bound on total demand over all

mode m paths between r and d.

(2) This modal demand on the shortest path is a function of the sum of frequencies over all paths.

(3) Prices are set exogenously with auto assumed best.

(4) Time on common carrier paths is set exogenously. Time between a city pair by auto is equal to average time over all auto paths between a city pair.

For the auto mode the constraints are identical except for the frequency term being fixed. There are also included a series of other, linear constraints relating to technology, accessibility, social states, profit, and budgets.

In the same study, Morlok extended the operations model to include certain nonlinear relationships. In the objective function he removed highway travel time as a decision variable and, instead, made it a nonlinear function of free-flow arc travel time, arc capacity, and arc flow according to the FHWA travel time curve. In the demand constraints he replaced the linear demand functions with multiplicative, nonlinear functions based on a Mathematica study [Mathematica, 1967]. Both types of these new nonlinear constraints are log-linear, and, thus, the resulting problem must be solved by separable programming instead of LP.

Steenbrink, discussing possible extensions of his single-mode network design methodology, considered the multi-modal, intercity, passenger transport network improvement problem, but conceded that difficulties in formulation and solution have resulted in few attempts at solving the problem [Steenbrink, 1974]. He suggested that branch and bound solution procedures might be a fruitful area for very small networks, but that heuristics may be necessary for larger networks. To demonstrate the difficulties inherent in a heuristic procedure, he attempted to solve a very simple two node, two arc problem, where each arc corresponded to a different mode. He used a heuristic he termed continuous optimal adjustment. The procedure converged to several possible solutions depending upon the initial starting solution; however, this was due as much to the nonconvexity of the problem as to performance of the heuristic. Finally he suggested a decomposition of the problem into a number of modal subproblems with a controlling master problem defining overall policy.

3. Freight Modal Choice Models

As noted previously, an important component in a multi-modal, freight transport network improvement formulation is the freight modal choice model. While freight modal choice has not received the attention of passenger modal choice, a significant amount of research has been completed. The results of this research are thoroughly summarized by Creighton [Creighton, 1977]. Three of the most widely used freight modal choice models are the inventory theoretic model, the multiplicative freight modal choice model, and the multinomial logit model.

In 1969 Townsend, attempting to model freight modal choice in the Northeast corridor, developed an inventory theoretic freight modal choice model [Townsend, 1969]. The general form of the model, given a specific O-D pair, is as follows:

$$S_{m,n} = \frac{1}{1 + \left(\frac{d_m}{d_n}\right)^B} \tag{1-22}$$

where:

$$S_{m,n} = \frac{W_m}{W_m + W_n}$$

$W_m \equiv$ tonnage moved from O to D by mode m

$d_m \equiv$ unit disutility of shipping from O to D by mode m

$B \equiv$ parameter of model

$$d_m = a_1 c_m + a_2 t_m \tag{1-24}$$

$c_m \equiv$ unit cost of shipping from O to D by mode m

$t_m \equiv$ travel time from O to D by mode m

$a_1, a_2 \equiv$ parameters of disutility function

Herendeen used a multiplicative freight modal choice model of the form: [Herendeen, 1969]

$$S_m = B\left(\frac{C_{Best}}{C_m}\right)^{a_1} \cdot \left(\frac{t_{Best}}{t_m}\right)^{a_2} \cdot \left(\frac{r_{Best}}{r_m}\right)^{a_3} \tag{1-25}$$

where:

$C_m \equiv$ unit cost of shipping from O to D by mode m

$C_{Best} \equiv$ unit cost of shipping from O to D by mode having minimum cost

$t_m \equiv$ transport time from O to D by mode m

$t_{Best} \equiv$ transport time from O to D by mode with minimum travel time

$r_m \equiv$ reliability of shipping from O to D by mode m

$r_{Best} \equiv$ reliability of shipping from O to D by mode with best reliability

$B, a_1, a_2, a_3 \equiv$ parameters of model

Sharp, after testing a number of model forms, concluded that the multinomial logit modal split model provided a reasonable fit to commodity flow data [Sharp, 1977]. Given a specific O-D pair, the multinomial logit model may be expressed as:

$$S_m = \frac{U_m}{\sum_{n \in M} U_n} \tag{1-26}$$

where:

$S_m \equiv$ share of flow moving between O and D by mode m

$U_m \equiv$ unit utility associated with moving from O to D by mode m

$M \equiv$ set of all modes

$$U_m = \text{Exp } (a_1 c_m + a_2 t_m + a_3 v_m) \tag{1-27}$$

$c_m \equiv$ unit cost of shipping from O to D by mode m

$t_m \equiv$ transport time from O to D by mode m

$v_m \equiv$ transport time variability from O to D by mode m

$a_1, a_2, a_3 \equiv$ model parameters (theoretically nonpositive)

Each of these models were fit using data from a number of different commodities. Comparison of fit between the models is difficult if not impossible. This results from several factors:

(1) Models were calibrated using different data sets and different commodities.

(2) Different models required different calibration techniques. For example, Herendeen used linear regression, Sharp used linear regression and nonlinear search, and Townsend used iterative, nonlinear techniques.

4. The Need for a New Formulation

The need to develop a new formulation and solution procedure rather than adapt one of the existing passenger formulations is a result of several factors. First, Creighton's formulation and Morlok's first formulation model specific applicational settings rather than the more general problem. Creighton assumes a uniform spacing of transport facilities throughout a square parcel of urban land, while Morlok assumes a linear corridor. Morlok's recent formulation attempts to model the more general problem. However, he appears to have sacrificed much to achieve linearity or log-linearity, such as:

(1) All improvement cost functions are assumed linear over the region of interest.

(2) Travel time on existing common carrier arcs cannot be modified by investment.

(3) Operating costs on arcs cannot be modified by investment.

(4) Demand functions for a given mode do not realistically consider the level of service characteristics on competing modes.

  (a) Cost - although the actual form of the constraints implies that the costs of the given and competing modes are considered, costs are fixed exogenously (not as a function of investment in improvements) and the highway mode is assumed the minimum cost mode.

  (b) Time - time for common carrier modes is fixed exogenously.

5. Approach

The specific objectives of the proposed research are presented below together with its scope and limitations, and the methodology used and the expected nature of conclusions.

(1) Specific objectives

(a) Given the problem as formulated in Chapter II, investigate general approaches for its solution, This investigation will focus on the relationship between the single-mode and multi-modal improvement problems.

(b) Select the general approach appearing to have the most promise and use it to develop a functional solution procedure.

(2) Scope and limitations

(a) Only the formulation developed in Section II and those extensions which appear to be amenable to the selected procedure are considered.

(b) Only those approaches which are capable of solving realistic-sized problems are considered.

(c) Only the most promising general approach is extended into a functional solution procedure.

(3) Methodology

(a) Postulate several possible solution approaches to the problem.

(b) Select the approach that appears to have the most promise, that is, that approach which appears likely to achieve a good solution for a realistic-sized problem in a reasonable amount of CPU time.

(c) Develop the approach into a functional algorithm and program it in the FORTRAN language for use on the Cyber 74 computer system. This program will consist of a battery of subprograms, some of which, such as shortest path and assignment routines, may have already been programmed as part of the ongoing Multi-State University Research Program [Jones, 1977].

(d) Test the algorithm on a problem arising from the Multi-State University Research Program.

CHAPTER II

THE FORMULATION

1. Modelling Assumptions

Certain aspects of the proposed formulation are considered fixed. First, the study area is represented by a finite number of zones. Second, all flow on the network belongs to the same transport commodity class. By a transport commodity class, it is meant goods or products which have similar transport characteristics. This differs from the concept of a math programming commodity class which typically corresponds to flow from a specific origin. Third, origin-destination (O-D) flows of the commodity are known and fixed. This item has several implications:

(1) Theoretically, network improvements will not significantly upset existing or proposed production-market relationships.

(2) Practically, even though only one transport commodity class is being considered, a number of math programming commodity classes must be considered.

Fourth, zones and the intercity freight transport system are represented by a network composed of nodes and arcs. By the intercity freight transport system one means all transportation facilities used to move freight in the study area. This includes:

(1) Line-haul facilities such as highways, railways and waterways.

(2) Loading and unloading facilities for each mode.

(3) Forwarding facilities for each mode such as railroad classification yards.

(4) Intermodal transfer facilities.

The actual network is constructed in the following manner:

(1) Each zone is represented by a node.

(2) Associated with each zonal node is a set of artificial nodes, an inbound and an outbound artificial node for each mode serving the zone.

(3) Each one-way modal line-haul facility is represented by a one-way arc from the outbound node associated with the origin zone and mode to the inbound node associated with the destination zone and mode. Two-way facilities are represented as two one-way facilities.

(4) Each modal loading facility is represented by a one-way arc from the zonal node to the appropriate outbound node. Similarly, each modal unloading facility is represented by a one-way arc to the zonal node from the appropriate inbound node. A combined loading and unloading facility is represented as separate loading and unloading facilities.

(5) Each modal forwarding facility is represented by a one-way arc from the inbound node to the outbound node associated with the zone and mode. For convenience, such facilities are located at zonal nodes.

(6) Each one-way intermodal transfer facility is represented by a one-way arc from the inbound node associated with the unloading mode to the outbound node associated with the loading mode. Two-way intermodal transfer facilities are represented as two one-way facilities. Such facilities are located at zonal nodes.

A representation of this procedure at a zonal node for the two mode case is shown in Figure 2-1 below.

Fifth, each arc in the network has an associated set of arc transport characteristics (ATC). This set is quite large and includes length, unit cost of transport, transport time, transport time variability, capacity, and congestion effects, among others. In the formulation, however, it is sufficient to recognize only those ATC which describe the arc in sufficient detail to allow flow assignment and

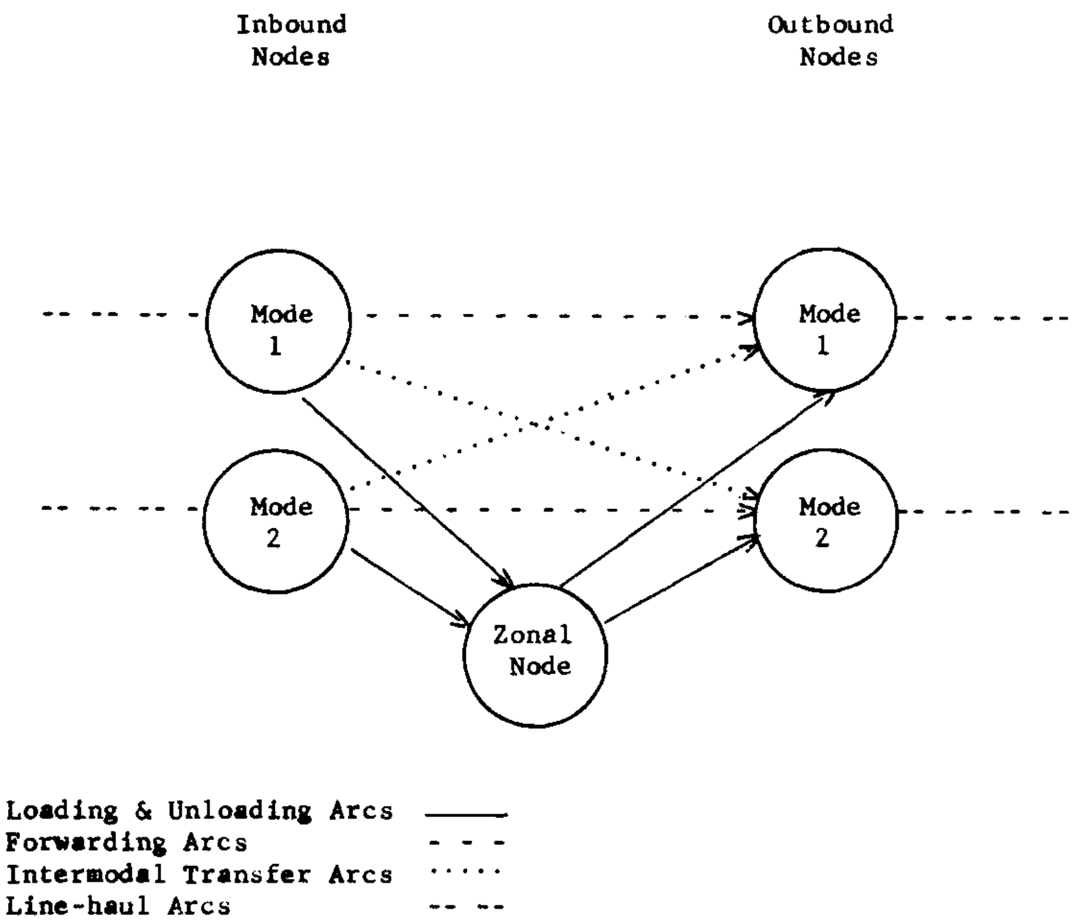


Fig. 2-1. Representation of a Network Zone

investment decisions to be made. Jones has identified all of the above mentioned ATC, except length, as being important [Jones, 1977]. In the interest of simplicity the formulation assumes arcs to be uncapacitated and to suffer no congestion effects due to freight traffic. Note that the latter assumption implies that the ATC are not functions of arc flow. Since intercity line-haul facilities typically operate at levels far below capacity, the assumption regarding congestion is probably valid for them. In the case of intercity highways, freight traffic represents such a small portion of total traffic that even a moderate change in freight traffic is unlikely to significantly increase transport time, cost, or time variability. However, this assumption does not necessarily accurately model terminal related facilities. The resulting set of ATC to be used in the formulation includes transport cost, transport time, and transport time variability.

The sixth assumption is actually a set of assumptions relating to the movement of O-D flows on the network. From basic micro-economic theory, one can assume that each shipper will transport his commodity to market via the unique modal path which he perceives as yielding the maximum utility. For purposes of this discussion, the multi-modal option is considered a distinct mode. However, each O-D commodity flow data point is the result of an aggregation over time of all shippers of a single class of commodities produced in an origin zone and marketed in a destination zone. Furthermore, shippers have different perceptions of utility; all commodities comprising a single class may not be homogeneous with respect to shipping characteristics; shippers may be located far from each other in the origin zone; and actual markets may be located for from each other in the destination zone. Thus, the aggregated flow represented by one O-D data point is often transported on more than one modal path instead of a unique "best" path. This is shown in the U.S. Census of Transportation Commodity Flow Data Base [Bureau of the Census, 1975].

In order to reasonably model this phenomenon without being forced to accept a much finer breakdown of commodities and zones, one can use the concept of modal split. Certain detailed assumptions aid in this analysis:

(1) Given an O-D commodity flow, for each mode there is a unique path on which all flow on the mode will occur.

(2) This unique modal path is defined as the one having maximum utility to the shipper over the set of all other paths for the mode.

To perform adequately in the proposed formulation, a modal split model should possess three properties:

(1) It should provide a reasonable fit to existing flow data.

(2) It should be mode abstract so that the multi-modal option may be considered.

(3) A monotonic transformation of the path utility function should be linear in the ATC along the path. This will facilitate use of a standard shortest path algorithm to determine maximum utility modal paths, as well as provide a convenient measure of path disutility to be used in the objective function.

Of the three modal split models discussed earlier, only Townsend's inventory-theoretic model and the multinomial logit model satisfy all the desired properties. Since the multinomial logit model is easier to calibrate, has already been calibrated for several commodities as part of the ongoing Multi-State University Research

Program, and is much simpler for the case of three or more modes, it will be used in the formulation.

Restating the model for the current purpose:

$$S_m = \frac{U_m}{\sum_{n \in M} U_n} \tag{2-1}$$

where:

$S_m \equiv$ share of flow moving between O and D by mode m

$U_m \equiv$ unit utility along maximum utility mode m path between O and D

$M \equiv$ set of all modes with paths connecting O and D

$$U_m = \mathrm{Exp}\,(a_1 c_m + a_2 t_m + a_3 v_m) \tag{2-2}$$

$c_m \equiv$ sum of arc unit transport costs on maximum utility mode m path connecting O-D

$t_m \equiv$ sum of arc transport times on maximum utility mode m path connecting O-D

$v_m \equiv$ sum of arc transport time variabilities on maximum utility mode m path connecting O-D

$a_1, a_2, a_3 \equiv$ parameters of modal split model

The seventh assumption formalizes the objective function. It states that minimizing the total disutility associated with the network is the proper objective of the improvement methodology. Total disutility is equal to the sum of total shipper disutility and total investment in the network. Unit shipper disutility along a path is expressed in terms of equivalent dollars and is derived from the argument of the path utility function. For example, for a particular O-D commodity flow:

$$U_m = \text{Exp } (a_1c_m + a_2t_m + a_3v_m) \tag{2-2}$$

where the variables have been previously defined.

$$d_m = \frac{a_1c_m + a_2t_m + a_3v_m}{a_1} \tag{2-3}$$

where:

$d_m \equiv$ unit disutility on maximum-utility mode m path expressed in equivalent dollars.

The eighth and last modelling assumption deals with the relationships between the three ATC and arc investment.

(1) For any level of investment on an arc, only one set of physical improvements will be considered. This implies that there will exist unique values of the three ATC for any level of investment.

This assumption can be justified as follows:

Let:

$T_j(I_j) \equiv$ the set of all possible physical improvements which can be effected on arc j given investment I_j

$c_{ij} \equiv$ unit transport cost on arc j given improvement $i \varepsilon T_j(I_j)$

$t_{ij} \equiv$ transport time on arc j given improvement $i \varepsilon T_j(I_j)$

$v_{ij} \equiv$ transport time variability on arc j given improvement $i \varepsilon T_j(I_j)$

$i^* \equiv$ that element of $T_j(I_j)$ which will be considered in the analysis

If one selects i* such that:

$$c_{i*j} + \frac{a_2}{a_1} t_{i*j} + \frac{a_3}{a_1} v_{i*j} = \min_{i \varepsilon T_j(I_j)} \left\{ c_{ij} + \frac{a_2}{a_1} t_{ij} + \frac{a_3}{a_1} v_{ij} \right\} \quad (2\text{-}4)$$

where:

$a_1, a_2, a_3 \equiv$ parameters of modal split model

then one cannot decrease total disutility in the network by selecting some other improvement project $i \varepsilon T_j(I_j)$. In other words, this assumption is based on a preliminary screening of improvement projects on each arc.

(2) Each ATC is a continuous, strictly decreasing function of the investment on the arc over the interval of interest. This is demonstrated in Figure 2-2 below.

A number of functional forms might be considered:

(a) LeBlanc suggests a function of the form: [LeBlanc, 1974]

$$I = b(c - \bar{c})^2 + d \quad (2\text{-}5)$$

where:

$I \equiv$ level of investment on arc

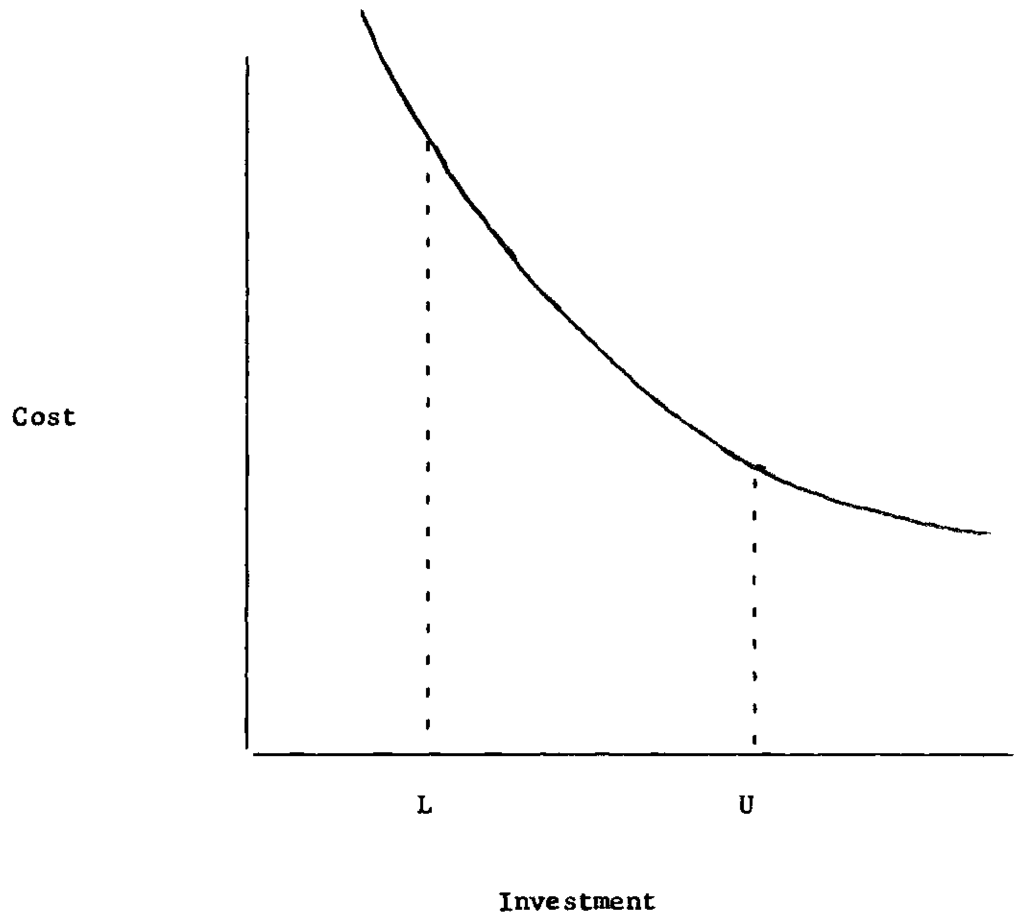
$c \equiv$ arc transport characteristic

$b, \bar{c}, d \equiv$ parameters of model (≥ 0)

Transformed for current purposes, this form yields:

$$c = \left(\frac{I - d}{b}\right)^{1/2} + \bar{c} \quad (2\text{-}6)$$

or



L lower bound on investment
U upper bound on investment

Fig. 2-2. Relationship Between Investment and an ATC

$$c = b'(I - d)^{1/2} + \bar{c} \tag{2-7}$$

where:

$$(I - d)^{1/2} \leq 0 \tag{2-8}$$

$$d \leq I \leq d + b\bar{c}^2 \tag{2-9}$$

Other possible forms are

(b)
$$c = b(I - \bar{c})^2 + d \tag{2-10}$$

where:

$$o \leq I \leq \min \{\bar{c}_c, \bar{c}_t, \bar{c}_v\} \tag{2-11}$$

(c)
$$c = d + \frac{b}{I - \bar{c}} \tag{2-12}$$

where:

$$\bar{c} < I \tag{2-13}$$

(d)
$$c = b \cdot d^{-\bar{c} \cdot I} \tag{2-14}$$

where:

$$I \geq 0 \tag{2-15}$$

2. The Formulation

The multi-modal freight transport network improvement problem can now be formulated as:

Problem P: $$\text{Min} \quad Z = \sum_{r\varepsilon O} \sum_{d\varepsilon D_r} \sum_{p\varepsilon P_{rd}} Y^p_{rd} \left[\sum_{j\varepsilon A^p_{rd}} c_j(I_j) + \frac{a_2}{a_1} \sum_{j\varepsilon A^p_{rd}} t_j(I_j) + \frac{a_3}{a_1} \sum_{j\varepsilon A^p_{rd}} v_j(I_j) \right] + \sum_{j\varepsilon A} I_j \tag{2-16}$$

s.t.

$$Y^p_{rd} = \frac{D_{rd} \cdot \text{Exp}\left[a_1 \sum_{j\varepsilon A^p_{rd}} c_j(I_j) + a_2 \sum_{j\varepsilon A^p_{rd}} t_j(I_j) + a_3 \sum_{j\varepsilon A^p_{rd}} v_j(I_j) \right]}{\sum_{q\varepsilon B_{rd}(I)} \text{Exp}\left[a_1 \sum_{j\varepsilon A^q_{rd}} c_j(I_j) + a_2 \sum_{j\varepsilon A^q_{rd}} t_j(I_j) + a_3 \sum_{j\varepsilon A^q_{rd}} v_j(I_j) \right]}$$

$$\forall\ r \ \varepsilon\ O,\ d\ \varepsilon\ D_r,\ p\ \varepsilon\ B_{rd}(I) \tag{2-17}$$

$$Y^p_{rd} = 0 \qquad \forall\ r\ \varepsilon\ O,\ d\ \varepsilon\ D_r,\ p \not\varepsilon\ B_{rd}(I) \tag{2-18}$$

$$L_j \le I_j \le U_j \qquad \forall\ j\ \varepsilon\ A \tag{2-19}$$

$$\sum_{p\varepsilon P_{rd}} Y^p_{rd} = D_{rd} \qquad \forall\ r\ \varepsilon\ O,\ d\ \varepsilon\ D_r \tag{2-20}$$

$$Y^p_{rd} \ge 0 \qquad \forall\ r\ \varepsilon\ O,\ d\ \varepsilon\ D_r,\ p\ \varepsilon\ P_{rd} \tag{2-21}$$

where:

$I_j \equiv$ investment on arc j

$Y^p_{rd} \equiv$ flow on path P from node r to d

D_{rd} ≡ total demand for commodity from node r at node d

L_j, U_j ≡ lower and upper bounds for investment on arc j

a_1, a_2, a_3 ≡ modal split model parameters

$c_j(I_j)$ ≡ unit transport cost on arc j as a function of I_j

$t_j(I_j)$ ≡ transport time on arc j as a function of I_j

$v_j(I_j)$ ≡ transport time variance on arc j as a function of I_j

O ≡ set of origin nodes

D_r ≡ set of destinations associated with origin r

P_{rd} ≡ set of paths connecting r and d

A^p_{rd} ≡ set of arcs comprising p^{th} path connecting r and d

$B_{rd}(I)$ ≡ set of maximum utility paths connecting r and d given an investment vector I. The set includes one path for each mode including the multi-modal option where it is distinct from a single-mode path

N ≡ set of network nodes

A ≡ set of network arcs

The objective function (2-16) is the total disutility associated with the network. It is the sum of shipper disutility and total investment in the network. Constraint sets (2-17) and (2-18) perform the modal split and drive flow assignment on the network. Constraint set (2-17) states that the flow on any maximum utility path connecting an O-D pair is equal to the total flow between the O-D pair times the share as determined by the modal split model. Constraint set (2-18) forces flow on any non-maximum utility path to zero. Constraint set (2-19) sets upper and lower bounds on arc investment.

Constraint set (2-20) states that the sum of the flow over all paths connecting an O-D pair equals the desired flow between the pair.

Constraint set (2-21) forces all flows to be nonnegative. It is obvious that constraint sets (2-17) and (2-18) imply constraint sets (2-20) and (2-21), and thus, after rearranging terms, P can be restated as:

$$\text{Problem P:} \quad \text{Min} \quad Z = \sum_{r\varepsilon O} \sum_{d\varepsilon D_r} \sum_{p\varepsilon P_{rd}} Y^p_{rd} \cdot \sum_{j\varepsilon A^p_{rd}} \left[c_j(I_j) + \frac{a_2}{a_1} t_j(I_j) + \frac{a_3}{a_1} v_j(I_j) \right] + \sum_{j\varepsilon A} I_j \qquad (2\text{-}16)$$

s.t.

$$Y^p_{rd} = \frac{D_{rd} \cdot \text{Exp}\left\{ \sum_{j\varepsilon A^p_{rd}} [a_1 c_j(I_j) + a_2 t_j(I_j) + a_3 v_j(I_j)] \right\}}{\sum_{q\varepsilon B_{rd}(I)} \text{Exp}\left\{ \sum_{j\varepsilon A^q_{rd}} [a_1 c_j(I_j) + a_2 t_j(I_j) + a_3 v_j(I_j)] \right\}}$$

$$\forall\, r\ \varepsilon\ O,\ d\ \varepsilon\ D_r,\ p\ \varepsilon\ B_{rd}(I) \qquad (2\text{-}17)$$

$$Y^p_{rd} = 0 \qquad \forall\, r\ \varepsilon\ O,\ d\ \varepsilon\ D_r,\ p \notin B_{rd}(I) \qquad (2\text{-}18)$$

$$L_j \le I_j \le U_j \qquad \forall\, j\ \varepsilon\ A \qquad (2\text{-}19)$$

CHAPTER III

A SOLUTION METHODOLOGY

The solution of the formulation defined in Chapter II presents serious difficulties. Although the decision variables are continuous, the problem has integer characteristics. This results from the modal split and assignment relationships, which divide all flow among maximum utility paths and which force flow on non-maximum utility paths to zero. To further complicate matters, the objective function is non-convex. At the present time, the only methods which can guarantee a global optimal solution to the formulation are integer procedures, such as branch and bound or cutting plane methods. However, as both Dantzig and Steenbrink have concluded, these methods cannot solve problems involving large-scale transport networks [Steenbrink, 1974; Dantzig, 1976]. Thus, this research will consider a heuristic methodology.

1. Continuous Optimal Adjustment: A Heuristic

1.1 The General Methodology

The general methodology developed in this research is based on the continuous optimal adjustment heuristic suggested by Steenbrink. The general procedure might be briefly summarized:

(1) Fix the initial modal split between each O-D pair.

(2) Determine the O-D demand by mode.

(3) Solve the resulting multi-modal network improvement with fixed modal split.

(4) Does the fixed set of modal splits agree with the set of modal splits feasible with respect to (2-17), (2-18) given the solution of (3) above?

   (a) If yes, go to (5) below.

   (b) Otherwise, adjust the assumed modal splits and go to (2) above.

(5) Is the solution satisfactory?

   (a) If yes, terminate.

   (b) Otherwise, determine a completely different set of initial modal splits and return to (2) above.

In the first step, a modal split for each O-D pair is assumed. Call this set of modal splits MS^0. For convenience, MS^0 might be the existing modal split. Given MS^0, one can uniquely define O-D demand by mode. Let the resulting multi-modal network improvement problem with fixed modal splits be $P1(MS^0)$. For the moment, assume that one can obtain a good solution to $P1(MS^0)$:

$I^*(MS^0) \equiv$ investment vector from $P1(MS^0)$

$f^*(MS^0) \equiv$ flow pattern from $P1(MS^0)$

Although the maximum utility paths implied by $f^*(MS^0)$ are identical to those identified by (2-17) and (2-18) given $I^*(MS^0)$, MS^0 may not be equivalent to the set of modal splits MS^F identified by (2-17) given $I^*(MS^0)$. Thus, the solution of $P1(MS^0)$ may not be feasible to problem P. If this is the case, then MS^0 can be modified using knowledge of MS^F, and problem $P1(MS^0)$ resolved.

Assume that at some iteration MS^0 is equivalent to MS^F. Then the current solution possesses several important properties:

(1) It is a good solution to $P1(MS^0)$.

(2) It is feasible to P.

While these two properties certainly do not guarantee that the current solution is optimal to P, they do suggest that the current solution may be an improvement over a randomly chosen solution. Note that the optimal solution to P must satisfy these properties. Thus, the methodology is capable of producing the optimal solution. An important issue, then, is the existence of other solutions, not optimal to P, which satisfy these properties. Obviously, if the optimal solution to P is the only solution with these properties, then if the methodology converges, the current solution is optimal to P. To resolve this issue, note that problem P is nonconvex and that it is being solved by an iterative heuristic. Thus, there is no guarantee that the current solution is unique and, thus, optimal to P. Steenbrink demonstrates this using a similar procedure on a simplified network consisting of two nodes and two arcs, one arc for each mode serving the pair. He found that his procedure produced two different solutions, depending on the set of initial modal splits [Steenbrink, 1974]. Since there is no guarantee of global optimality, the methodology can either be terminated upon convergence or reinitiated with a different set of parameters. The overall methodology is shown in Figure 3-1 below.

Throughout the discussion it has been assumed that at some finite iteration of the methodology MS^0 and MS^F converge. However, there is no inherent feature of the methodology which guarantees convergence. In practical applications, however, such procedures do seem to converge. In solving the simplified problem discussed earlier, Steenbrink demonstrated convergence of a similar procedure. In addition, Steenbrink and Ventker have suggested the use of similar heuristics in solving large

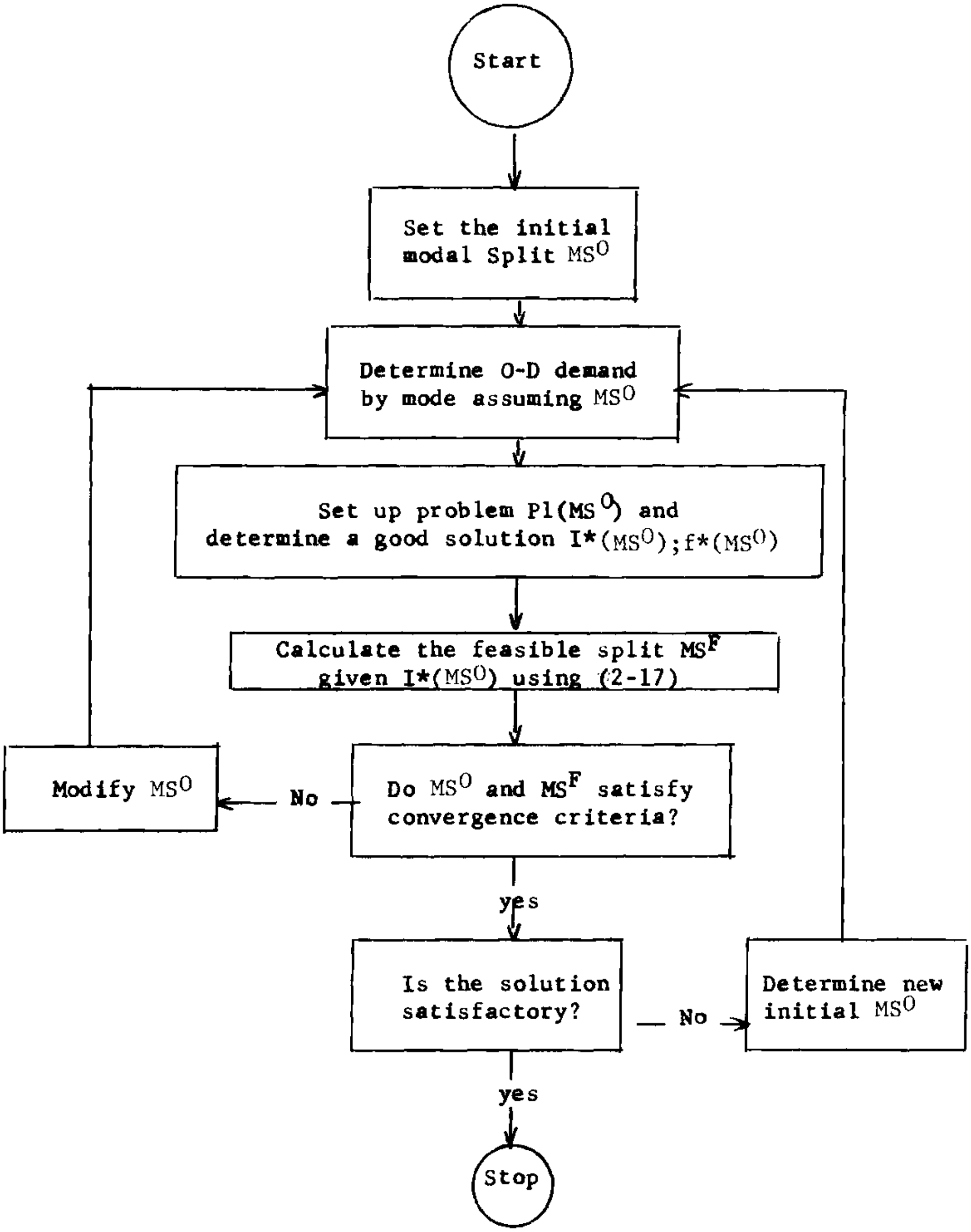


Fig. 3-1. Continuous Optimal Adjustment Applied to the Multi-Modal Improvement Problem.

traffic assignment problems [Steenbrink, 1970; Steenbrink, 1971; Ventker, 1970].

1.2 Modifiable Components and Parameters of the Methodology

The methodology proposed in Section 1.1 above has a number of modifiable components and parameters which might affect its performance. First, there is the solution of $P1(MS^0)$. This is discussed at length in Section 2 below. The second is the convergence criterion involving MS^0 and MS^F. The third is the manner in which MS^0 will be adjusted given knowledge of MS^F. A fourth is the overall termination criteria and the subsequent determination of an entirely different set of initial modal splits. Consider the second item. Assume that at the end of some iteration there exist a current set of initial modal splits MS^0 as well as a current set of modal splits MS^F as determined by (2-17) given $I^*(MS^0)$. There are a number of practical tests which might be used to test the convergence of MS^0 and MS^F.

Let:

MS^0_{im} ≡ the initial estimate of the mode m share for O-D pair i

MS^F_{im} ≡ the feasible mode m share for O-D pair i given investment $I^*(MS^0)$

$\varepsilon_{im} \equiv MS^F_{im} - MS^0_{im}$

R ≡ the set of O-D pairs

M ≡ the set of modes

Then, these tests include rejecting the convergence assumption if:

(1) $$|\varepsilon_{im}| > \varepsilon_1 \quad \text{for some } i \in R,\ m \in M \tag{3-1}$$

(2) $$\sum_{i \in R} \sum_{m \in M} |\varepsilon_{im}| > \varepsilon_2 \tag{3-2}$$

(3) $$\sum_{i\varepsilon R} \sum_{m\varepsilon M} \varepsilon_{im}^2 > \varepsilon_3 \qquad (3-3)$$

where:

ε_1, ε_2, ε_3, are convergence parameters

The advantage of the first convergence test relative to the second is that it will identify a single large error, event if the remaining errors are relatively small. A disadvantage of the first test relative to the second is that it will tolerate a large number of errors as large as ε_1. Somewhat of a compromise might be struck by utilizing the third test. Of course, multiple convergence criteria might be used to the same end. It should be noted that the convergence test is not only significant because of its role in identifying convergence, but also for its potential role in hastening convergence. If the methodology demonstrates convergent behavior, then increasing ε_i should hasten convergence. The practical effect of such action is to loosen modal split feasibility requirements. Practically, this may not significantly affect the validity of the results due to the primitive state-of-the-art in modal split modelling and the condition of the freight data bases [Creighton, 1977; Hartwig and Linton, 1974].

Dealing with the third item, assume that at the end of some iteration, MS^O and MS^F have not satisfied the convergence criteria. Then there is a need to update MS^O and iterate. In updating MS^O it would be useful to obtain a revised MS^O which will yield a new MS^F close to MS^O. A reasonable approach might be to let the new MS^O be the current MS^F. However, such an approach often results in an overcorrection and could possibly result in an undercorrection. Thus, define the new MS^O such that:

$$MS^0_{New} = MS^0_{Old} + \alpha\varepsilon \tag{3-4}$$

where:

$\alpha \equiv$ the correction factor, positive

$\varepsilon \equiv$ the error vector as calculated above

Note that this approach moves MS^0 in the direction of MS^F while not necessarily forcing MS^0 exactly equal to MS^F. In many nonlinear programming algorithms, when an improving direction is determined, a search is made along that direction until the best point is identified. Consider such an approach for the present case. Noting that problem $P1(MS^0)$ would need to be solved repeatedly, it should be obvious that solution times would quickly become excessive.

Last, assume that MS^0 and MS^F have passed the convergence test. There is left the question of whether to search for better solutions or terminate the search. A tight lower bound for the value of the objective would greatly facilitate the decision. Unfortunately, there appears to be no reasonable method to obtain a tight lower bound. A loose lower bound on user disutility can be determined by setting arc investment at its upper bound and allowing flow to use any modal path. A loose lower bound on investment can be found by setting all arc investment at its lower bound. Thus, a very loose lower bound on overall network disutility is the sum of these lower bounds. Since the resulting lower bound is very poor, it will be of little value in the search termination decision. A better lower bound could be obtained by placing all flow on the multi-modal mode and determining the optimal solution to problem $P1(MS^0)$. However, this is a difficult task as demonstrated in the following section. Barring the determination of a good lower bound, a less desirable strategy will need to be developed. Such a strategy might be to perform a more exhaustive search until it appears that no significant improvement can be made.

Assume that MS^0 and MS^F have passed the convergence test, and it has been decided to continue the search. The final step is the identification of a new set of initial modal splits. One important factor in this step is the need to identify a new MS^0 sufficiently distant from all of the preceding MS^0's so that the methodology does not converge to a previously obtained solution. Sets which should probably always be investigated are the current set of modal splits and the extreme sets:

(1) Those sets with all flow assigned to a single mode.

(2) Those sets with all flow assigned equally to two modes.

(3) Those sets with all flow assigned equally to three modes.

(4) Those sets with all flow divided equally between all four modes.

Hartman reviewed a variety of procedures used in identifying new starting solutions for nonconvex programs. However, these procedures are precluded by the large number of decision variables in this problem [Hartman, 1972]. Finally, new sets might be generated randomly.

2. $P1(MS^0)$: The Multi-modal Network Improvement Problem with Fixed Modal Splits

In Section 1 above there was defined the multi-modal network improvement problem with fixed modal splits $P1(MS^0)$. If the set of modal splits MS^0 were such that all flow was assigned to pure modes, as opposed to the multi-modal option, then $P1(MS^0)$ would be separable into a number of smaller single-mode network improvement problems. However, the presence of the multi-modal option significantly complicates this simple decomposition procedure, since there is not an easy way to represent multi-modal flow on a single-mode network.

The approach taken here will be to transform the multi-modal problem into a single-mode problem with two additional constraint sets. This transformation is accomplished by the addition of nodes and arcs, shown in Figure 3-2 below, and can be described as follows. For each original origin node:

(1) Add a new origin node to the network for each single mode serving the original origin node.

(2) Remove all loading arcs. Place these arcs between the new origin nodes and the appropriate outbound nodes.

(3) Add a new origin node corresponding to the multi-modal option.

(4) From each new origin node corresponding to the multi-modal option, add an arc to each new single-mode origin node.

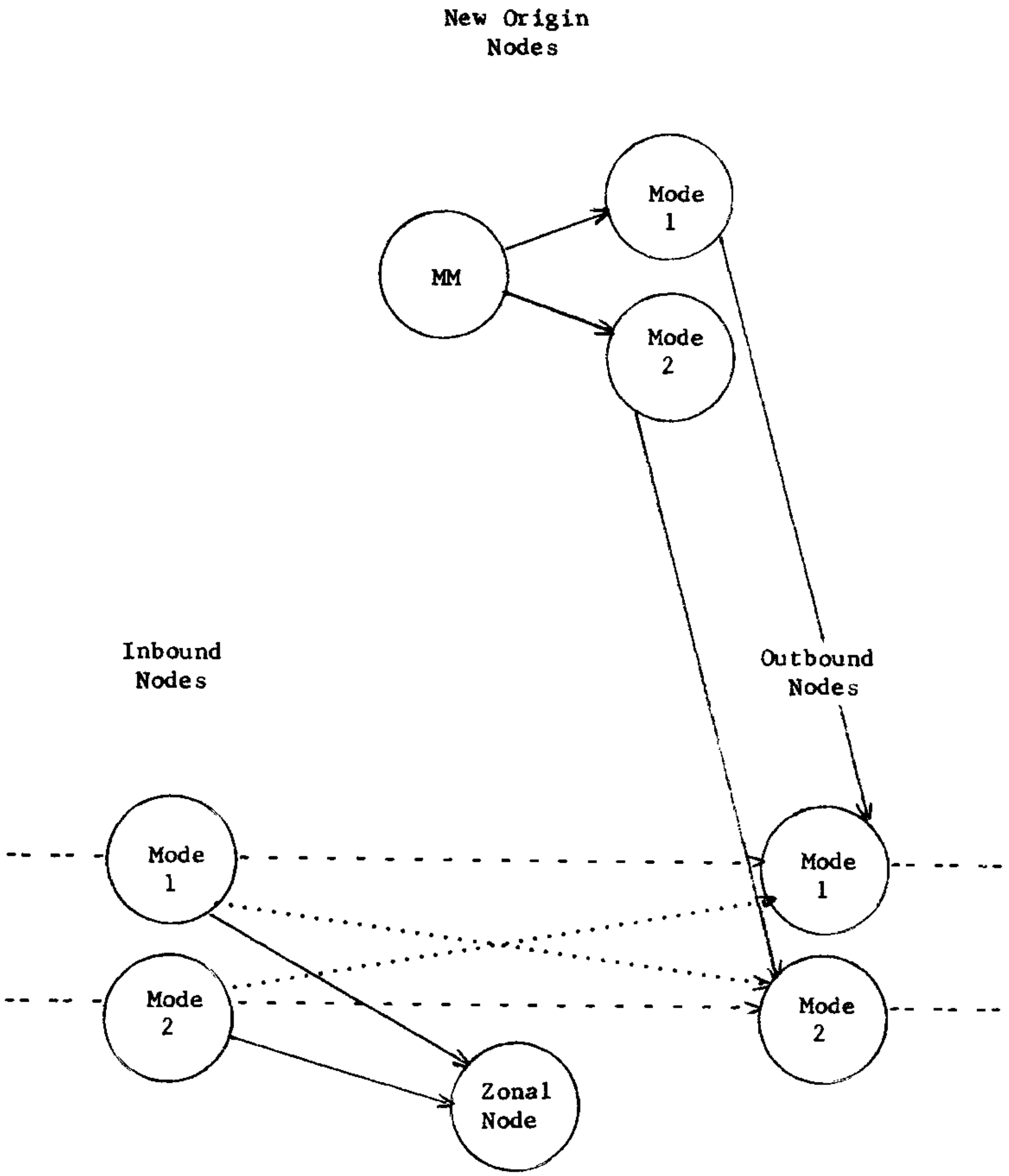
(5) Remove all flow originating at the original origin node.

(6) Place the O-D demand by mode determined by MS^0 at the appropriate new origin.

$P1(MS^0)$ can now be expressed in node-arc terms as:

$$P1(MS^0): \quad \min_{I_j, f_j, f_j^r} Z1 = \sum_{j \varepsilon A} f_j \left[c_j(I_j) + \frac{a_2}{a_1} t_j(I_j) + \frac{a_3}{a_1} v_j(I_j) \right] \qquad (3\text{-}5)$$

$$+ \sum_{j \varepsilon A} I_j$$



Loading and unloading arcs ____
Forwarding arcs - - -
Intermodal transfer arcs .....
Line-haul arcs -- --

Fig. 3-2. Addition of Nodes and Arcs to Network

$$\text{s.t.} \quad \sum_{j \varepsilon W_i} f_j^r - \sum_{j \varepsilon V_i} f_j^r = h_i^r \qquad \forall\ i\ \varepsilon\ N,\ r\ \varepsilon\ O \tag{3-6}$$

$$f_j = \sum_{r \varepsilon O} f_j^r \qquad \forall\ j\ \varepsilon\ A \tag{3-7}$$

$$f_j^r = 0 \qquad \forall\ j\ \varepsilon\ ITA,\ r\ \varepsilon\ SMO \tag{3-8}$$

$$f_j^r \geq 0 \qquad \forall\ j\ \varepsilon\ A,\ r\ \varepsilon\ O \tag{3-9}$$

$$L_j \leq I_j \leq U_j \qquad \forall\ j\ \varepsilon\ A \tag{3-10}$$

O-D flows must occur only on maximum utility paths. If there is more than one maximum utility path for any O-D pair and mode, it is assumed that all flow occurs on only one path. (3-11)

where:

f_j ≡ total flow on arc j

f_j^r ≡ flow on arc j from origin r

I_j ≡ investment on arc j

L_j ≡ lower bound on investment on arc j

U_j ≡ upper bound on investment on arc j

$$h_i^r \equiv \begin{cases} -x_{ri} & \text{if } i\ \varepsilon\ D_r \\ \sum_{j \varepsilon D_i} x_{rj} & \text{if } i = r \\ 0 & \text{otherwise} \end{cases} \tag{3-12}$$

x_{ij} ≡ flow from origin i to destination j

$a_1, a_2, a_3 \equiv$ modal split parameters

$c_j(I_j) \equiv$ unit transport cost on arc j given investment I_j

$t_j(I_j) \equiv$ transport time on arc j given investment I_j

$v_j(I_j) \equiv$ transport time variability on arc j given investment I_j

$N \equiv$ set of nodes

$A \equiv$ set of arcs

$O \equiv$ set of origins

$D_r \equiv$ set of destinations for origin r

$W_i \equiv$ set of arcs originating at node i

$V_i \equiv$ set of arcs terminating at node i

SMO $\equiv$ set of origins corresponding to single modes

ITA $\equiv$ set of intermodal transfer arcs

Constraint sets (3-6), (3-8), (3-9), and (3-11) assure that the selection of paths is in accordance with constraint sets (2-17) and (2-18) of P. Constraint set (3-8) insures that only flow from multimodal origins can use intermodal transfer arcs. The objective (3-5) coupled with constraint set (3-7) is equivalent to the objective (2-16) of P. Problem $P1(MS^0)$ is a single-mode network improvement problem with additional constraint sets (3-8) and (3-11). However, it can be shown that constraint set (3-11) is redundant.

<u>Theorem 3.1</u>

Constraint set (3-11) is redundant to problem $P1(MS^0)$.

Proof. To prove that (3-11) is redundant, it is sufficient to show that the optimal solution of problem $P1(MS^0)'$, problem $P1(MS^0)$ with (3-11) relaxed, always satisfies (3-11). This would imply that the optimal solution of $P1(MS^0)'$, whose objective by the relaxation theorem must be less than or equal to that of $P1(MS^0)$, is also feasible to $P1(MS^0)$ and, thus, is optimal to $P1(MS^0)$.

To show this, assume the optimal solution of $P1(MS^0)'$ does not satisfy (3-11), i.e.

Let:

$I_j^* \equiv$ optimal investment on arc j from $P1(MS^0)'$

$Z^* \equiv$ optimal value of objective from $P1(MS^0)'$

$d_P^* \equiv$ unit disutility along any path P for I_j^*

$$d_P^* \equiv \sum_{j \varepsilon P} \left[c_j(I_j^*) + \frac{a_2}{a_1} t_j(I_j^*) + \frac{a_3}{a_1} v_j I_j^*) \right]$$

Then, for some O-D pair and mode there exist paths P and Q such that:

(1) $d_P^* < d_Q^*$

and

(2) There is positive flow along path Q for the previously designated I_j^*.

or:

(3) $d_P^* = d_Q^*$

and

(4) There is positive flow on both paths P and Q.

To show that assumption (2) must be false if assumption (1) is true, consider the alternate feasible solution obtained by placing the flow currently on path Q onto path P. Since $d_P^* < d_Q^*$, the value of the objective of $P1(MS^0)'$ must decrease. Therefore, assumption (2) is false. If assumption (3) is true, then simply form an equivalent alternative optimal solution by placing all flow on either path P or Q. This makes assumption (4) false. Thus, the optimal solution of $P1(MS^0)'$ must satisfy (3-11), and (3-11) is redundant in $P1(MS^0)$.

Q.E.D.

Thus, problem $P1(MS^0)$ can be stated as a single-mode network improvement problem with additional constraint set (3-8).

Returning to the review of the single-mode network improvement problem in Chapter I, recall that Dantzig found that a Steenbrink type decomposition technique might be an efficient method for solving problems involving large-scale networks. In terms of problem $P1(MS^0)$ the Steenbrink technique requires the following steps:

(1) A subproblem is solved for each network arc.

$$\text{Let:} \quad H_j(f_j) = \underset{I_j}{\text{Min}}\ f_j \left[c_j(I_j) + \frac{a_2}{a_1} t_j(I_j) + \frac{a_3}{a_1} v_j(I_j) \right] + I_j \tag{3-13}$$

$$\text{s.t.} \quad L_j \le I_j \le U_j \tag{3-14}$$

(2) The results of these subproblems are substituted into the objective function of $P1(MS^0)$ to obtain a master problem $P2(MS^0)$ equivalent to the nonlinear transportation assignment problem with additional constraint set (3-8).

$$\text{Problem } P2(MS^0)\text{:} \quad \underset{f_j}{\text{Min}} \sum_{j \in A} H_j(f_j) \tag{3-15}$$

$$\text{s.t.} \quad \sum_{j \in W_i} f_j^r - \sum_{j \in V_i} f_j^r = h_i^r \qquad \forall\ i \in N,\ r \in O \tag{3-16}$$

$$f_j = \sum_{r \in O} f_j^r \qquad \forall\, j \in A \tag{3-17}$$

$$f_j^r = 0 \qquad \forall\, \begin{matrix} j \in ITA \\ r \in SMO \end{matrix} \tag{3-18}$$

$$f_j^r \geq 0 \qquad \forall\, \begin{matrix} j \in A \\ r \in O \end{matrix} \tag{3-19}$$

2.1 <u>Determination of Subproblem Objective Function $H_j(f_j)$</u>

For the moment, assume that the ATC and investment are linked by functions of the form:

$$c_j(I_j) = b_{1j}(I_j - \bar{c}_j)^2 + d_{1j} \tag{3-20}$$

$$t_j(I_j) = b_{2j}(I_j - \bar{t}_j)^2 + d_{2j} \tag{3-21}$$

$$v_j(I_j) = b_{3j}(I_j - \bar{v}_j)^2 + d_{3j} \tag{3-22}$$

A graph of $c_j(I_j)$ is shown in Figure 3-3 below. The reason for selecting this functional form will become apparent later. Thus, $H_j(f_j)$ can be determined by solving the problem $S1(MS^0)$:

Problem $S1(MS^0)$: $$H_j(f_j) = \underset{I_j}{\text{Min}}\ A_j f_j I_j^2 + B_j f_j I_j + c_j f_j + I_j \tag{3-23}$$

$$\text{s.t.} \quad L_j \leq I_j \leq U_j \tag{3-24}$$

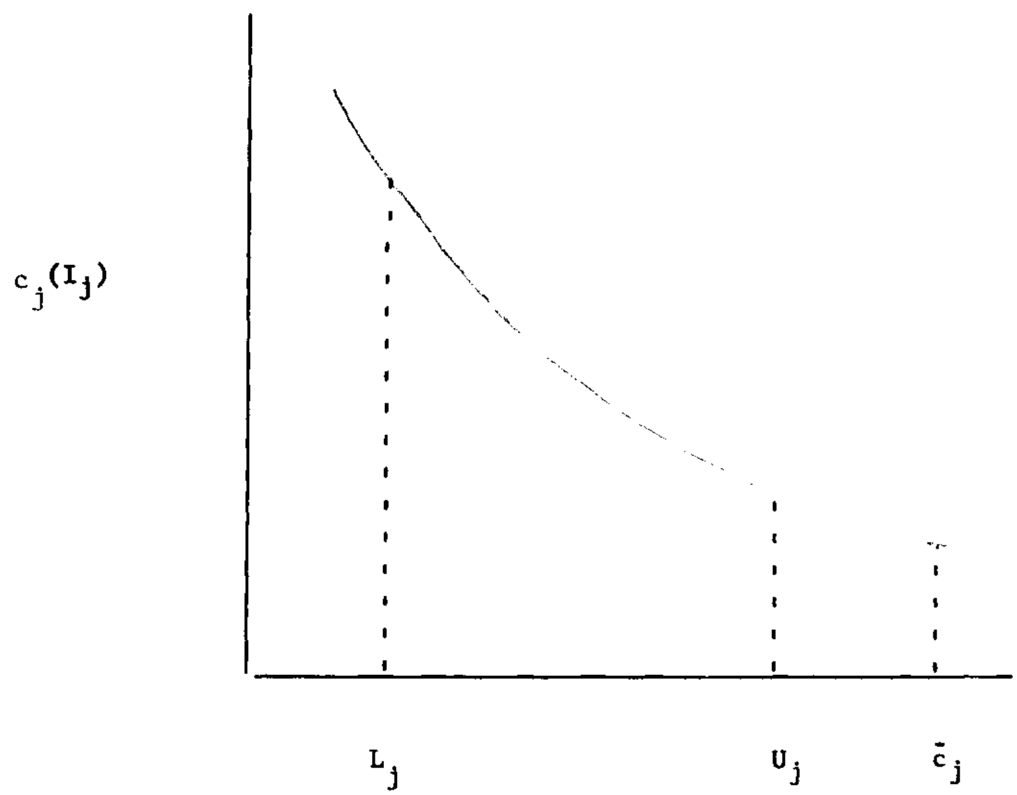


Fig. 3-3. $c_j(I_j)$ as a function of I_j

where:

$$A_j = b_{1j} + \frac{a_2}{a_1} b_{2j} + \frac{a_3}{a_1} b_{3j} \qquad (3\text{-}25)$$

$$B_j = -2 \left\{ b_{1j}\bar{c}_j + \frac{a_2}{a_1} b_{2j}\bar{t}_j + \frac{a_3}{a_1} b_{3j}\bar{v}_j \right\} \qquad (3\text{-}26)$$

$$C_j = b_{1j}\bar{c}_j^2 + d_{1j} + \frac{a_2}{a_1} (b_{2j}\bar{t}_j^2 + d_{2j}) + \frac{a_3}{a_1} (b_{3j}\bar{v}_j^2 + d_{3j}) \qquad (3\text{-}27)$$

Note that if f_j is fixed and nonnegative, the objective function (3-23) is the sum of a convex quadratic term, two linear terms, and a constant term. Thus, (3-23) is convex. $S1(MS^0)$ is a convex program, and the Kuhn-Tucker conditions apply:

<u>Theorem 3.2</u>

The optimal solution to $S1(MS^0)$ is defined by:

$$I_j^* = \begin{cases} L_j & \text{if } \hat{I}_j \le L_j \\ \hat{I}_j & \text{if } L_j \le \hat{I}_j \le U_j \\ U_j & \text{if } \hat{I}_j \ge U_j \end{cases} \qquad (3\text{-}28)$$

where:

$$\hat{I}_j = \frac{-B_j}{2A_j} - \frac{1}{2A_j f_j} \qquad (3\text{-}29)$$

Proof. The Kuhn-Tucker conditions for $S1(MS^0)$ are:

(1) $2A_j f_j I_j^* + B_j f_j + 1 - v_1 + v_2 = 0$

(2) $v_1(L_j - I_j^*) = 0$

(3) $v_2(I_j^* - U_j) = 0$

(4) $L_j - I_j \leq U_j$

(5) $v_1, v_2 \geq 0$

Case 1

If $\hat{I}_j \leq L_j$:

Let: $I_j^* = L_j$

$v_1 = 2A_jf_jL_j + B_jf_j + 1$

$v_2 = 0$

Clearly conditions 1, 2, 3, and 4 are satisfied.

Thus, if:

$$v_1 = 2A_jf_jL_j + B_jf_j + 1 \geq 0$$

then the K-T conditions are satisfied and Theorem 3.1 holds for Case 1.

Since:

$$\hat{I}_j \leq L_j, \quad A_j > 0, \quad \text{and} \quad f_j \geq 0$$

$$v_1 \geq 2A_jf_j\hat{I}_j + B_jf_j + 1$$

$$\geq 2A_jf_j\left(\frac{-B_j}{2A_j} - \frac{1}{2A_jf_j}\right) + B_jf_j + 1$$

$$\geq B_jf_j + 1 - B_jf_j - 1$$

$$\geq 0$$

$\therefore$ Theorem 3.1 holds for Case 1.

Case 2

$$L_j \leq \hat{I}_j \leq U_j$$

Let: $I^*_j = \hat{I}_j$

$v_1 = v_2 = 0$

Clearly conditions 2, 3, 4, and 5 are satisfied.

Regarding condition 1:

$$2A_j f_j I^*_j + B_j f_j + 1 - v_1 + v_2 = 2A_j f_j \left(\frac{-B_j}{2A_j} - \frac{1}{2A_j f_j} \right) + B_j f_j + 1$$

$$= 0$$

$\therefore$ All K-T conditions are satisfied and Theorem 3.1 holds for Case 2.

Case 3

If $\hat{I}_j \geq U_j$

Let: $I^*_j = U_j$

$v_1 = 0$

$v_2 = -2A_j f_j U_j - B_j f_j - 1$

Conditions 1, 2, 3, and 4 are clearly satisfied.

Thus, if:

$$v_2 = -2A_j f_j U_j - B_j f_j - 1 \geq 0$$

then the K-T conditions are satisfied and Theorem 3.1 holds for Case 3.

Since:

$$\hat{I}_j \geq U_j$$

$$\therefore \quad v_2 \geq -2A_j f_j \hat{I}_j - B_j f_j - 1$$

$$\geq -2A_j f_j \left(\frac{-B_j}{2A_j} - \frac{1}{2A_j f_j} \right) - B_j f_j - 1$$

$$\geq 0$$

$\therefore$ Theorem 3.1 holds for all cases.

Q.E.D.

Consider the form of I_j^*, (3-28), paying particular attention to its performance with respect to f_j, the flow on arc j. First note that the term $\frac{-B_j}{2A_j}$ forms an economic upper bound on investment (as opposed to a budget related bound). Note:

$$\frac{-B_j}{2A_j} = \frac{b_{1j}\bar{c}_j + \frac{a_2}{a_1} b_{2j}\bar{t}_j + \frac{a_3}{a_1} b_{3j}\bar{v}_j}{b_{1j} + \frac{a_2}{a_1} b_{2j} + \frac{a_3}{a_1} b_{3j}}$$

Then, every arc in the network will fall into one of three categories:

<u>Category I arcs</u>: $L_j \geq \frac{-B_j}{2A_j}$

For a Category I arc, there will never be sufficient flow to justify anything more than the lower bound on investment.

$$I_j^* = L_j \qquad \forall\ f_j \tag{3-30}$$

This is shown in Figure 3-4 below.

<u>Category II arcs</u>: $U_j < \frac{-B_j}{2A_j}$

For a Category II arc, the actual upper bound on arc investment will be a budget-related bound rather than an economic bound.

Let:

$f_j^L \equiv$ the minimum level of flow which will induce investment above the minimum level on arc j

$f_j^U \equiv$ the maximum level of flow which will induce additional investment in arc j

$$L_j = \frac{-B_j}{2A_j} - \frac{1}{2A_j f_j^L}$$

$$U_j = \frac{-B_j}{2A_j} - \frac{1}{2A_j f_j^U}$$

or:

$$f_j^L = -\frac{1}{2L_jA_j + B_j}$$

$$f_j^U = -\frac{1}{2U_jA_j + B_j} \tag{3-31}$$

Therefore, I_j^* can be restated as:

$$I_j^* = \begin{cases} L_j & \text{if } f_j \le f_j^L \\ \hat{I}_j & \text{if } f_j^L \le f_j \le f_j^U \\ U_j & \text{if } f_j \ge f_j^U \end{cases} \tag{3-32}$$

This is shown in Figure 3-5 below.

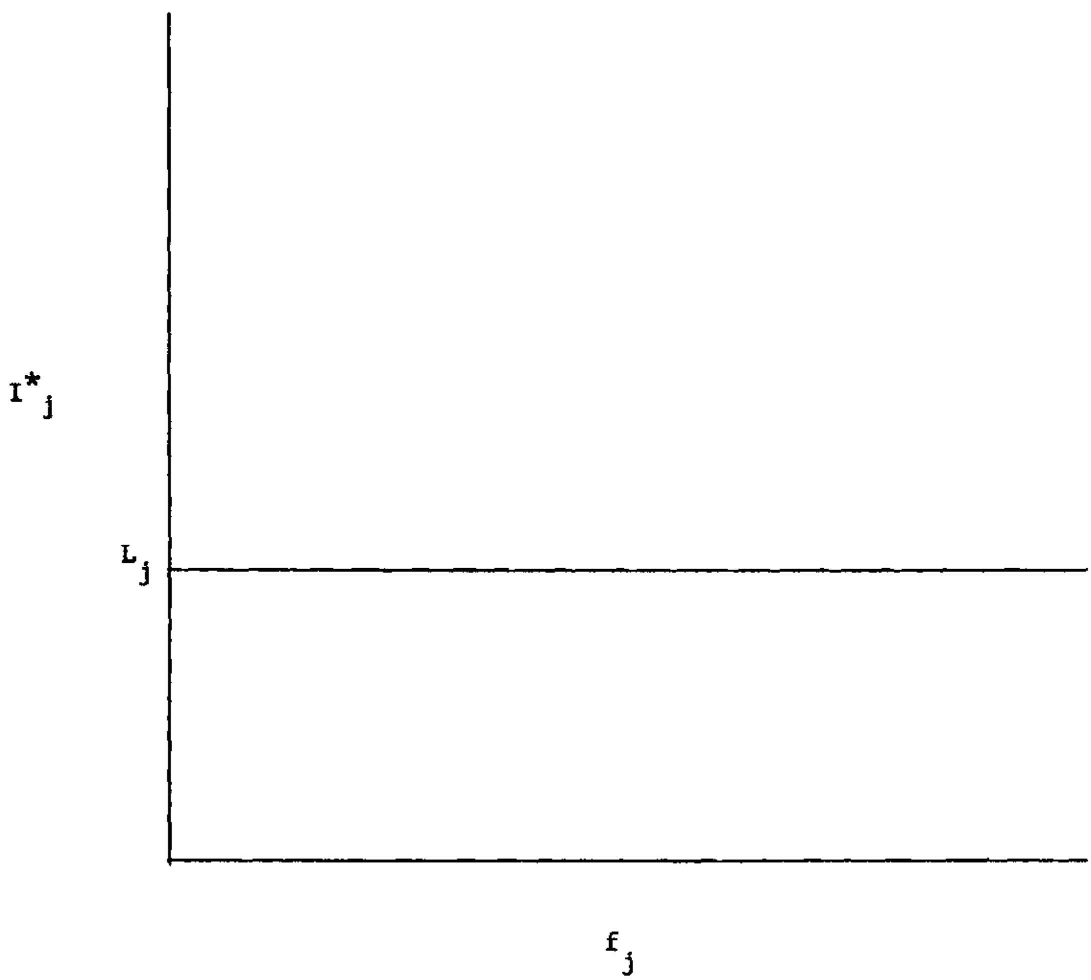


Fig. 3-4. I^*_j as a function of f_j for Category I arcs

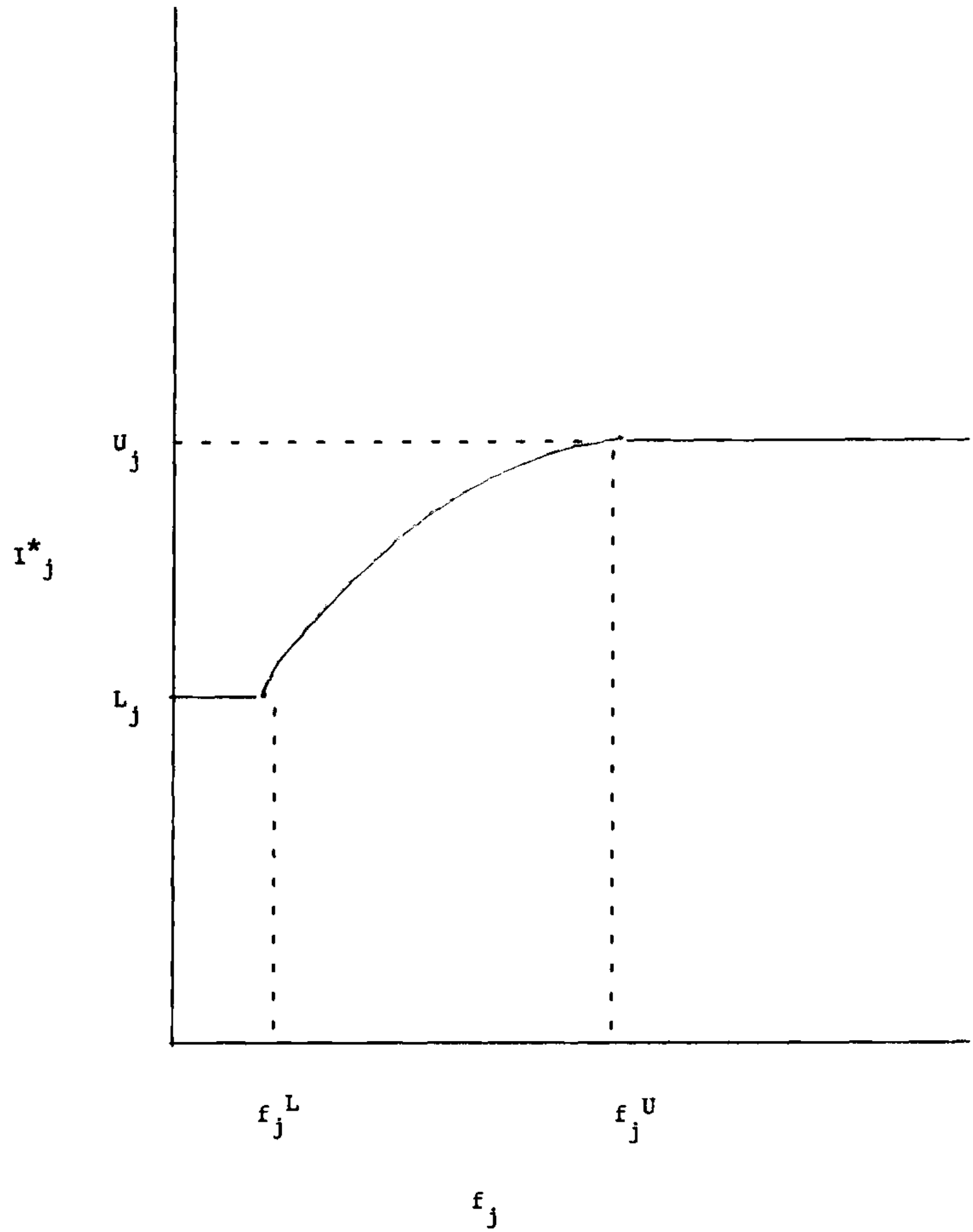


Fig. 3-5. I^*_j as a function of f_j

for Category II arcs

<u>Category III arcs</u>: $L_j < \frac{-B_j}{2A_j}$

$$U_j \geq \frac{-B_j}{2A_j}$$

For a Category III arc, $\frac{-B_j}{2A_j}$ will form the actual upper bound on arc investment.

$$I_j^* = \begin{cases} L_j & \text{if } f_j \leq f_j^L \\ \hat{I}_j & \text{if } f_j \geq f_j^L \end{cases} \tag{3-33}$$

This is shown in Figure 3-6 below.

Now, $H_j(f_j)$ can be expressed as a function of I_j^*:

<u>Category I arcs</u>: $L_j \geq \frac{-B_j}{2A_j}$

$$H_j(f_j) = A_j f_j L_j^2 + (B_j f_j + 1)L_j + c_j f_j \qquad \forall\ f_j \tag{3-34}$$

This is shown in Figure 3-7 below.

<u>Category II arcs</u>: $U_j < \frac{-B_j}{2A_j}$

$$H_j(f_j) = \begin{cases} A_j f_j L_j^2 + (B_j f_j + 1)L_j + c_j f_j & \text{if } f_j \leq f_j^L \\ \frac{4A_j c_j - B_j^2}{4A_j} \cdot f_j - \frac{1}{4A_j} \cdot \frac{1}{f_j} - \frac{b_j}{2A_j} & \text{if } f_j^L \leq f_j \leq f_j^U \\ A_j f_j U_j^2 + (B_j f_j + 1)U_j + c_j f_j & \text{if } f_j \geq f_j^U \end{cases} \tag{3-35}$$

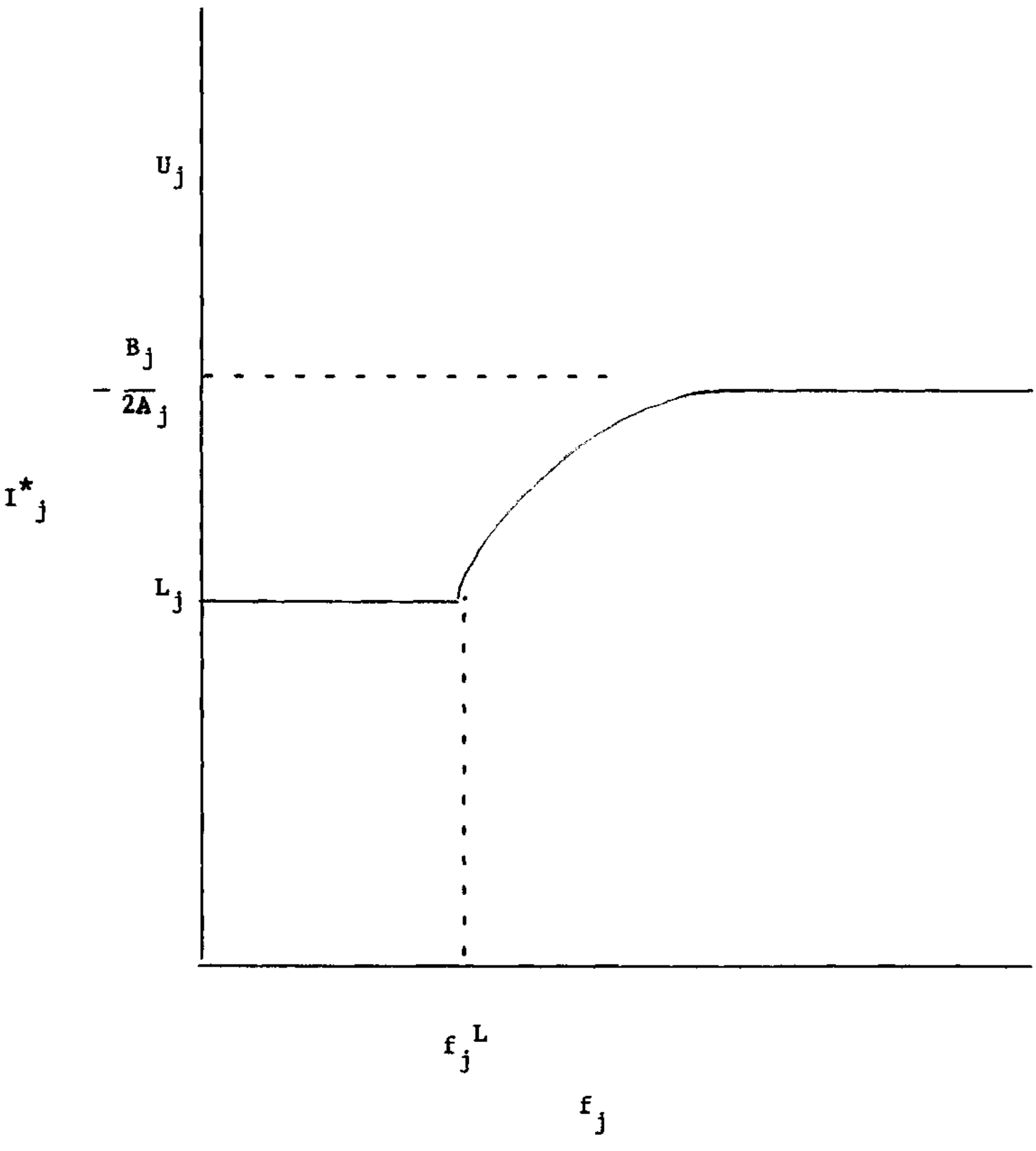


Fig. 3-6. I^*_j as a function of f_j for Category III arcs

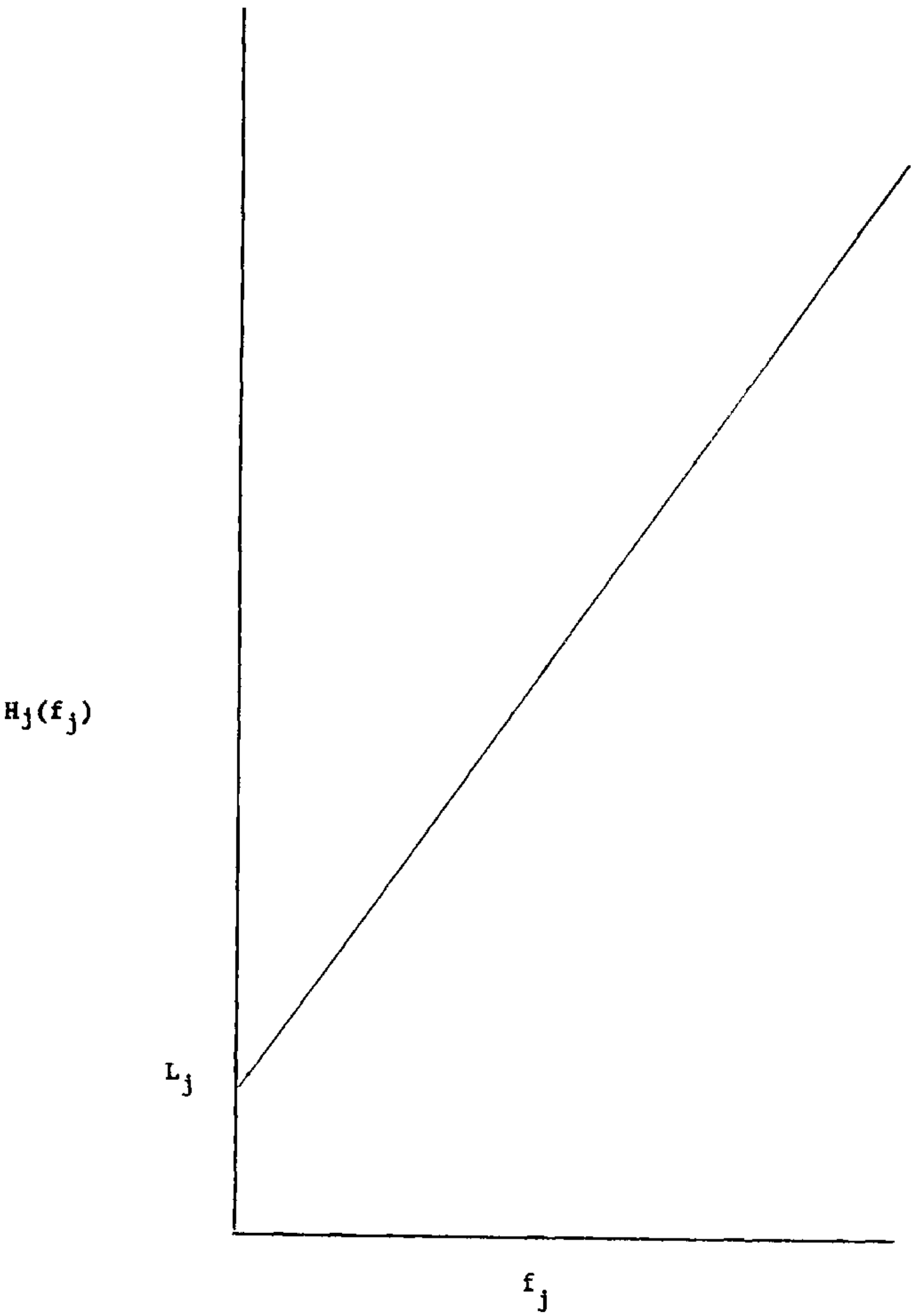


Fig. 3-7. $H_j(f_j)$ for Category I arcs

This is shown in Figure 3-8 below.

<u>Category III arcs</u>: $L_j < \frac{-B_j}{2A_j}$

$$U_j \geq \frac{-B_j}{2A_j}$$

$$H_j(f_j) = \begin{cases} A_j f_j L_j^2 + (B_j f_j + 1)L_j + c_j f_j & \text{if } f_j \leq f_j^L \\ \frac{4A_j c_j - B_j^2}{4A_j} \cdot f_j - \frac{1}{4A_j} \cdot \frac{1}{f_j} - \frac{b_j}{2A_j} & \text{if } f_j^L \leq f_j \leq f_j^U \end{cases} \tag{3-36}$$

This is shown in Figure 3-9 below.

2.2 <u>Properties of Subproblem Objective Function $H_j(f_j)$</u>

The expressions for $H_j(f_j)$ derived above can be shown to have a number of important properties.

<u>Theorem 3.3</u>

$H_j(f_j)$ is both continuous and differentiable for feasible f_j.

Proof.

<u>Category I arcs</u>: $H_j(f_j)$ is linear, continuous, and differentiable.

<u>Category II arcs</u>: Over the regions $0 \leq f_j < f_j^L$ and $f_j^U < f_j$, $H_j(f_j)$ is linear, continuous, and differentiable. Over the region $f_j^L < f_j < f_j^U$, $H_j(f_j)$ is concave, continuous, and differentiable, since

$\frac{\partial^2 H_j(f_j)}{\partial f_j^2} = -\frac{1}{4A_j} \cdot \frac{1}{f_j^4} \leq 0$. Therefore, $H_j(f_j)$ is continuous and differentiable at all points except possibly the two breakpoints f_j^L and f_j^U.

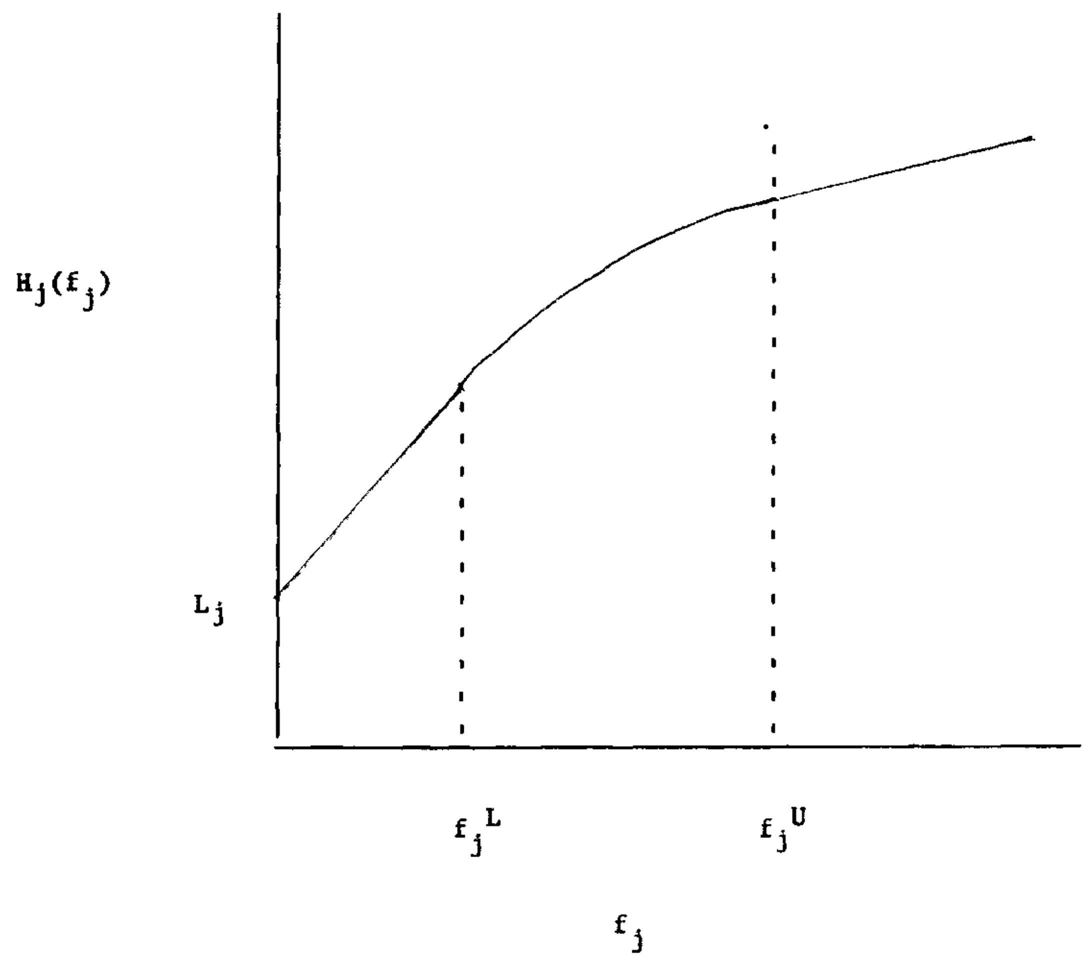


Fig. 3-8. $H_j(f_j)$ for Category II arcs

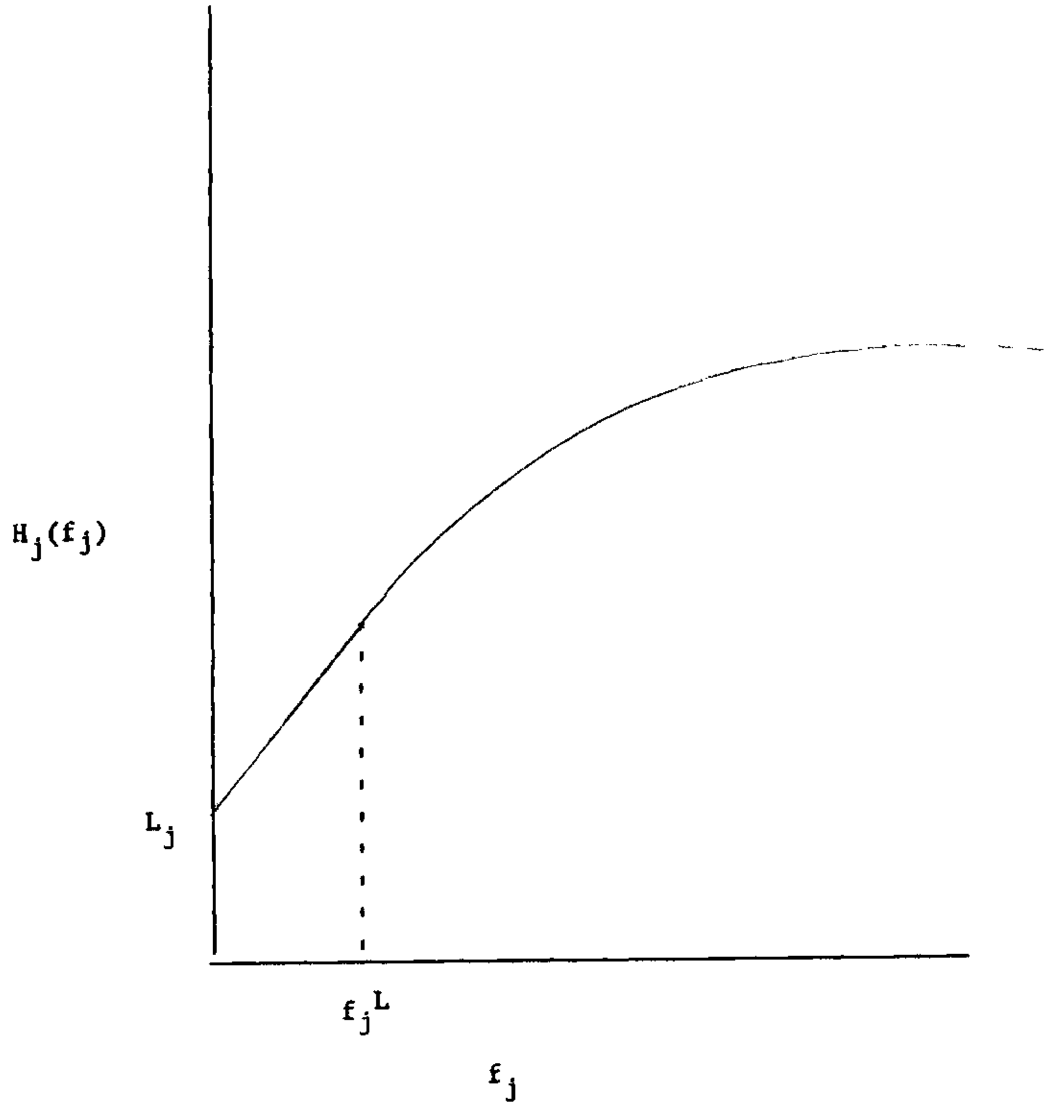


Fig. 3-9. $H_j(f_j)$ for Category III arcs

Consider f_j^L. For the linear segment:

$$H_j(f_j^L) = (A_jL_j^2 + B_jL_j + C_j)f_j^L + L_j$$

$$= (A_jL_j^2 + B_jL_j + C_j) \cdot \frac{-1}{2L_jA_j + B_j} + L_j$$

$$= \frac{L_j^2A_j - C_j}{2L_jA_j + B_j}$$

$$\frac{\partial H_j(f_j^L)}{\partial f_j} = A_jL_j^2 + B_jL_j + C_j$$

For the concave segment:

$$H_j(f_j^L) = \frac{4A_jC_j - B_j^2}{4A_j} f_j^L - \frac{1}{4A_j} \cdot \frac{1}{f_j^L} - \frac{B_j}{2A_j}$$

$$= \frac{4A_jC_j - B_j^2}{4A_j} \cdot \frac{-1}{2L_jA_j + B_j} - \frac{1}{4A_j} \cdot \frac{2L_jA_j + B_j}{-1} - \frac{B_j}{2A_j}$$

$$= \frac{L_j^2A_j - C_j}{2L_jA_j + B_j}$$

$$\frac{\partial H_j(f_j^L)}{\partial f_j} = \frac{4A_jC_j - B_j^2}{4A_j} + \frac{1}{4A_j} \cdot \left(\frac{1}{f_j^L}\right)^2$$

$$= \frac{4A_jC_j - B_j^2}{4A_j} + \frac{1}{4A_j} \cdot (4L_j^2A_j^2 + 4L_jA_jB_j + B_j^2)$$

$$= A_jL_j^2 + B_jL_j + C_j$$

Since the functional values and the derivatives are the same for both segments, $H_j(f_j)$ is both continuous and differentiable at f_j^L.

Similarly, consider f_j^U. For the linear segment:

$$H_j(f_j^U) = \frac{U_j^2A_j - C_j}{2U_jA_j + B_j}$$

$$\frac{\partial H_j(f_j^U)}{\partial f_j} = A_jU_j^2 + B_jU_j + C_j$$

For the concave segment:

$$H_j(f_j^U) = \frac{U_j^2A_j - C_j}{2U_jA_j + B_j}$$

$$\frac{\partial H_j(f_j^U)}{\partial f_j} = A_jU_j^2 + B_jU_j + C_j$$

Therefore, $H_j(f_j)$ is everywhere continuous and differentiable.

Category III arcs: Continuity and differentiability follow directly from that of Category II arcs.

Therefore, $H_j(f_j)$ is continuous and differentiable for all arcs.

Q.E.D.

Theorem 3-4

$H_j(f_j)$ is concave for feasible f_j.

Proof. Follows directly from fact that each segment is concave and $H_j(f_j)$ is continuous and differentiable for feasible f_j.

Q.E.D.

2.3 Problem P2(MS^0): The Uncapacitated, Concave Disutility Transportation Assignment Problem

The resulting master problem P2(MS^0) which becomes the key component in the heuristic methodology is an uncapacitated, concave disutility transportation assignment problem. This particular problem has not received the attention given other types of transportation assignment problems. The most simple type of transportation assignment problem is uncapacitated with linear arc disutilities. This problem is solved by constructing a minimum disutility tree for each origin and placing all flow on the minimum disutility path between each O-D pair. Next, there is the capacitated problem with linear arc disutilities. Here, all flow may not be assignable to a minimum disutility path, since there are capacities on arc flow. Solution procedures capable of solving problems involving large-scale networks include the out-of-kilter algorithm and the network simplex [Ford and Fulkerson, 1974; Jarvis and Bazaraa, 1977]. Third, there is the problem with convex arc disutilities. Here, unit arc disutility increases as flow increases. This raises the possibility of multiple paths serving the same O-D pair, even without binding capacity constraints. As is the case with other nonlinear convex programs, this implies the possibility of a non-extreme point local optimum. However, a local optimum is also a global optimum. A number of nonlinear algorithms have been developed specifically to solve this problem, but currently none are capable of solving problems involving large-scale networks [Dantzig, 1976; Le Blanc, 1973; Nguyen, 1974; Ruiter, 1974; Potts and Oliver, 1972; Peterson, 1975; Murchland, 1969].

The most difficult of the transportation assignment problems is that problem with concave arc disutilities. The difficulty lies in the fact that, although local optima lie at extreme points, local optimality does not imply global optimality. Thus, obtaining the global optimal solution to the problem necessitates the use of some sort of enumeration scheme. Rech and Barton have developed a branch and bound algorithm capable of obtaining a global optimum to the problem [Rech and Barton, 1970]. Their algorithm utilizes an increasingly more accurate linear approximation to the disutility surface. While theoretically capable of obtaining the global optimum, the Rech and Barton algorithm will not practically solve problems involving large-scale networks. Solving a very similar problem, Jarvis, Rardin, and Unger approximate the concave disutility surface with a piecewise, linear surface [Jarvis, Rardin, and Unger, 1976]. This results in a general fixed charge network flow problem. A node-arc formulation of the problem is solved using a branch and bound procedure. Again, the procedure will not practically solve problems involving large-scale networks.

Barring the use of a global optimum seeking procedure to solve $P2(MS^0)$, any of a number of heuristics might be used to obtain a solution. An obvious initial possibility is to approximate each concave arc disutility surface with its linear convex envelope. This will result in an easily solved uncapacitated transportation assignment problem with linear arc disutilities. The principal drawback to such a procedure is that the maximum flow which might use any arc is very large. This implies that the linear convex envelope will be a very poor approximation to the original concave disutility surface. This has several consequences:

(1) The solution obtained is probably far from optimal.

(2) The resulting lower bound on the value of the objective is probably very poor.

Finally, there is no guarantee that the solution is even a local optimum to $P2(MS^0)$. A second possibility is the use of Steenbrink's SALMOF (Stepwise Assignment according to Least Marginal Objective Function) heuristic [Steenbrink, 1974]. As the name implies, the heuristic assigns a fraction of the flow to the network at each step. Flow is placed on paths with the minimum marginal disutility, evaluated at current flow levels. When used to solve a problem with concave arc disutilities, the SALMOF heuristic has the undesirable characteristic of assigning all flow to those paths with minimum initial marginal disutility. Thus, the solution obtained is probably far from optimal. Again, there is no guarantee that the solution is even a local optimum to $P2(MS^0)$. An advantage of the previous two methods is that they can both solve problems involving large-scale networks. A third possible approach is to use one of the existing convex disutility techniques to yield a local optimal solution to $P2(MS^0)$. However, as stated previously, these techniques cannot currently accommodate problems involving large-scale networks.

A fourth possible approach to the solution of $P2(MS^0)$ is the use of an algorithm developed by Yaged [Yaged, 1971]. This algorithm exploits special properties of local optima to $P2(MS^0)$ in order to efficiently generate these solutions. This algorithm is capable of determining local optimal solutions to large-scale problems within a reasonable computation time. A fifth possible approach is motivated by the Jarvis, Rardin, and Unger approach described previously. In essence, the approach seeks to obtain a good solution to the fixed-charge problem using an arc-path formulation instead of the node-arc formulation. Performance of such a procedure on a large-scale problem is not known.

The solution procedures to receive further consideration are constrained by the necessity of working with a large-scale network. This immediately rules out the global optimum seeking procedures as well as the nonlinear algorithms designed to solve the problem with convex arc disutilities. A second criterion which is also desirable is the need to identify the best possible solutions to $P2(MS^0)$. Two approaches appear worthy of further consideration: the Yaged algorithm is explained in greater detail in the next section; the fifth heuristic approach, that involving the arc-path formulation of the general fixed-charge network flow problem, is developed in Chapter VI.

2.4 The Yaged Algorithm

The Yaged Algorithm was developed to aid in the design and improvement of the long distance telephone network spanning the United States. It seeks to determine local optimal solutions to the uncapacitated transportation assignment problem in which each link disutility function is a strict concave function of link flow. The efficiency of the algorithm stems from its exploitation of special properties of local optima to the problem. This efficiency allows the user to examine a number of local optima in hopes of obtaining a good solution.

The Yaged Algorithm requires the satisfaction of several assumptions concerning the link disutility functions:

(1) The functional values are nonnegative for feasible f_j.

(2) The first derivative of the function exists and is positive for feasible f_j.

(3) The second derivative of the function exists and is negative for feasible f_j, i.e., the function is strictly concave.

(4) The function is continuous for feasible f_j.

From the previous discussion, the only assumption which is not satisfied is the third. The $H_j(f_j)$ are concave, but not strictly concave. Fortunately, however, the algorithm can be modified slightly to take this into account.

Given the modified set of assumptions, certain properties of a local optimal solution to $P2(MS^0)$ can be ascertained:

<u>Property 1</u>. For any local optimal solution, either:

(a) All flow between any O-D pair occurs on a single path, or

(b) An equivalent (in terms of value of the objective) local optimal solution can be determined satisfying property 1.a.

Property 1 results from the fact that an equivalent extreme point solution can always be found for any local optimum of $P2(MS^0)$. Note that since $P2(MS^0)$ is uncapacitated, an extreme point solution is synonymous with Property 1.a. For the remainder of the discussion, it is assumed that a local optimum is the extreme point equivalent defined in Property 1.a.

<u>Property 2</u>. For any local optimal solution, if flow between an

O-D pair uses path p, and nodes a and b lie on path p, then the flow between a and b can also be made to lie on path p.

Property 3. A solution is a local optimum if and only if the path between each O-D pair is the shortest path between the pair where arc length is defined to be the first derivative of the disutility function evaluated at the current flow level.

From these properties follows Yaged's Algorithm:

Initialization: Determine an initial set of arc lengths. Set old arc flows to 0.

Step I. Construct shortest path trees from each origin using the current set of arc lengths. Assign all flow to the shortest path between each O-D pair.

Step II. If the newly generated arc flows are equivalent to the old arc flows, go to Step IV. Otherwise, go to Step III.

Step III. Set each arc length to equal the marginal arc disutility evaluated at the new arc flow. Let the new arc flows become the old arc flows. Go to Step I.

Step IV. Is the current local optimum satisfactory? If not, determine a new set of initial arc lengths and go to Step I. Otherwise, terminate.

Yaged has proven the convergence of the algorithm and has demonstrated its ability to solve problems involving large-scale networks in a reasonable computation time. The only modification to the basic algorithm needed to accommodate the non-strictly concave arc disutility functions is that trees must be constructed consistently at each iteration. By consistently, it is meant that given the same set of arc lengths, the tree-building algorithm will always construct the same set of trees. This will assure that some O-D flow will not cycle between two paths with equal linear disutilities.

2.5 A Global Optimum-Seeking Extension to the Yaged Algorithm

A significant undesirable property of the Yaged Algorithm is that a number of local optima must be generated in order to verify the quality of the final solution. It would therefore be useful to start in the vicinity of a good local optimum rather than at some point about which little is known. Rardin has suggested a procedure which could hopefully lead to a starting solution in the vicinity of a good local optimum [Rardin, 1978]. The procedure, which is patterned after a similar procedure developed by Tiplitz, is iterative in nature [Tiplitz, 1973]. The procedure is essentially the Yaged Algorithm substituting average disutility for marginal disutility in Step III. The reasoning behind such a substitution is that the Yaged Algorithm does not give sufficient consideration to the overall disutility associated with a given flow pattern. Instead, it assigns flow on the basis of marginal disutility. Thus, one could obtain a better solution in the global sense by substituting average disutility for marginal disutility. Several points concerning such an extension should be noted:

(1) Given that the extension converges, there is no guarantee of a local optimal solution to $P2(MS^0)$.

(2) There is no guarantee that the extension will converge.

Thus, an ideal role for the extension is to act as Phase I in the solution of $P2(MS^0)$. Given the solution provided by Phase I, Phase II, the Yaged Algorithm, would then determine a local optimum in the vicinity of the Phase I solution. Note also that since Phase I is used only to obtain an initial solution for Phase II, it need not be iterated to convergence. Thus, one might set a maximum number of iterations at which Phase I would terminate, given no previous convergence.

3. Extending the ATC To Be Functions of Flow

In Chapter II it was assumed that the arc transport characteristics (ATC) associated with an arc were not affected by flow on the arc. It is interesting to consider relaxations of this assumption with respect to the methodology developed in this chapter. First, consider problem $\hat{P}$ which corresponds to problem P (Chapter II) with $c_j(I_j)$ replaced by $\hat{c}_j(I_j, f_j)$ where

$$\hat{c}_j(I_j, f_j) = c_{1j}(I_j) + c_{2j}(I_j)/f_j \tag{3-37}$$

with $f_j > 0$ assumed

Here the cost attribute consists of a variable portion and a fixed portion, the latter being obtained from a fixed cost shared by all flow units on the arc. Both components of cost depend on the investment I_j.

Letting $\hat{Z}$ be the objective of $\hat{P}$, then:

$$\hat{Z} = Z + \sum_{j \varepsilon A} c_{2j}(I_j) \tag{3-38}$$

with the $c_{1j}(I_j)$ now replacing $c_j(I_j)$ in Z. The other difference between problems $\hat{P}$ and P occurs in constraint set (2-17), the modal split constraints, where $c_j\ (I_j,\ f_j)$ is substituted for $c_j(I_j)$.

Consider now the corresponding multi-modal network improvement problem with fixed modal splits, problem $\hat{P}1(MS^o)$. The objective of this problem, $\hat{Z}1$ can be stated as:

$$\hat{Z}1 = Z1 + \sum_{j \in A} c_{2j}(I_j) \tag{3-39}$$

The constraints are identical to those of problem $P1(MS^o)$ which include (3-6),(3-7), (3-8), (3-9), (3-10), and (3-11). In problem $P1(MS^o)$ constraint set (3-11) was shown to be redundant at optimality; i.e., O-D flows would occur only on maximum utility modal paths. For problem $\hat{P}1(MS^o)$ this need not always be true. For example, it is not difficult to find constant $c_{2j}(I_j)$'s which foil this simplifying property. Assume, however, that this property holds for problem $\hat{P}1(MS^o)$. Then, the associated Steenbrink type subproblem $\hat{H}_j(f_j)$ can be stated as:

$$\hat{H}_j\ (f_j) = \min_{I_j}\ f_j[c_{1j}(I_j) + c_{2j}\ (I_j)/f_j + \frac{a_2}{a_1} t_j\ (I_j) + \frac{a_3}{a_1} v_j\ (I_j)] + I_j \tag{3-40}$$

s.t. (3-14)

or:

$$\hat{H}_j(f_j) = \min_{I_j}\ \hat{Z}S = f_j\ [c_{1j}\ (I_j) + \frac{a_2}{a_1}\ t_j(I_j) + \frac{a_3}{a_1} v_j(I_j)] + I_j + c_{2j}(I_j) \tag{3-41}$$

s.t. (3-14)

Now, note that if the objective $\hat{Z}S$ of the subproblem is convex, one might still use Steenbrink decomposition to solve problem $\hat{P}1(MS^0)$.

Two applications immediately suggest themselves. First, the $c_{2j}(I_j)$ may represent a fixed operating and maintenance cost to be shared by the users, perhaps for a rail or waterway arc. Second, the $c_{2j}(I_j)$ may simply be the investment I_j which is to be recovered from the users. In the case of investment, the $c_{2j}(I_j)$ is linear, and the $\hat{Z}S$ remains convex. In the case of operating and maintenance costs, the $c_{2j}(I_j)$ is likely to be concave, but probably not enough to render $\hat{Z}S$ nonconvex. Even $\hat{Z}S$ should become nonconvex, it should probably remain pseudo-convex, enabling one to find the optimal solution $\hat{H}_j(f_j)$. Finally, if the $\hat{H}_j(f_j)$ remain concave, then either the Yaged or the two phase algorithm might be used to determine a local optimum to problem $\hat{P}1(MS^0)$.

Now, consider problem $\bar{P}$ which differs from problem P with the ATC suffering congestion effects; i.e., the ATC are increasing functions of flow. It should be noted that problem P is inconsistent. In the objective the ATC deteriorate as flow increases, implying that more than one path per mode may carry flow. In constraint sets (2-17) and (2-18) flow is constrained to only one path per mode, the maximum utility modal path. Thus, some basic modifications in the assumptions underlying the problem would need to be made in order to make problem P consistent.

4. Summary

In this chapter a solution methodology was developed for problem P. The general methodology was based on the continuous optimal adjustment heuristic proposed by Steenbrink [Steenbrink, 1974]. The modifiable components and parameters of the methodology were identified and their roles in the methodology analyzed. The major component in the methodology was found to be a solution procedure for problem $P1(MS^0)$, the multi-modal network improvement problem with fixed modal splits. Problem $P1(MS^0)$ was translated into an equivalent single-mode network improvement problem with an additional constraint set. Using a Steenbrink type decomposition procedure, problem $P1(MS^0)$ was transformed into master problem $P2(MS^0)$, an uncapacitated, concave disutility transportation assignment problem. A two phase algorithm was developed to determine a good local optimal solution to $P2(MS^0)$. In the first phase a modification of the Yaged Algorithm was used to locate a point in the vicinity of a good local optimum. In the second phase the Yaged Algorithm was used to pinpoint this local optimum [Yaged, 1971].

CHAPTER IV

IMPLEMENTATION AND RESULTS

1. Implementation

The methodology developed in Chapter III was implemented on the Georgia Tech CDC Cyber 74 computing system. All programming was done in the FORTRAN programming language. The basic flow of the methodology is shown in the flowchart in Figure 4-1 below. The first component of the methodology determines a set of initial modal splits for each original O-D pair. Several programs have been developed for this purpose:

(1) Program PREPF7 generates the current modal splits based on actual flow data.

(2) Program SMS generates extreme modal splits as discussed in Chapter III, Section 1.2 above.

(3) Program INMS generates random modal splits.

The second component calculates the O-D demand for the expanded network using the original O-D flows and the initial set of modal splits MS^0. Program INFOD is the functional version of this component. The third component determines an initial set of arc lengths for the expanded network. Program INAC was developed to randomly generate these lengths. The fourth component solves problem $P2(MS^0)$ using the two-phase algorithm developed in Chapter III. Program CNCASNB is the functional version of this component. Program CNCASNB is discussed at length in Section 1.1 below. The fifth component of the methodology performs several functions.

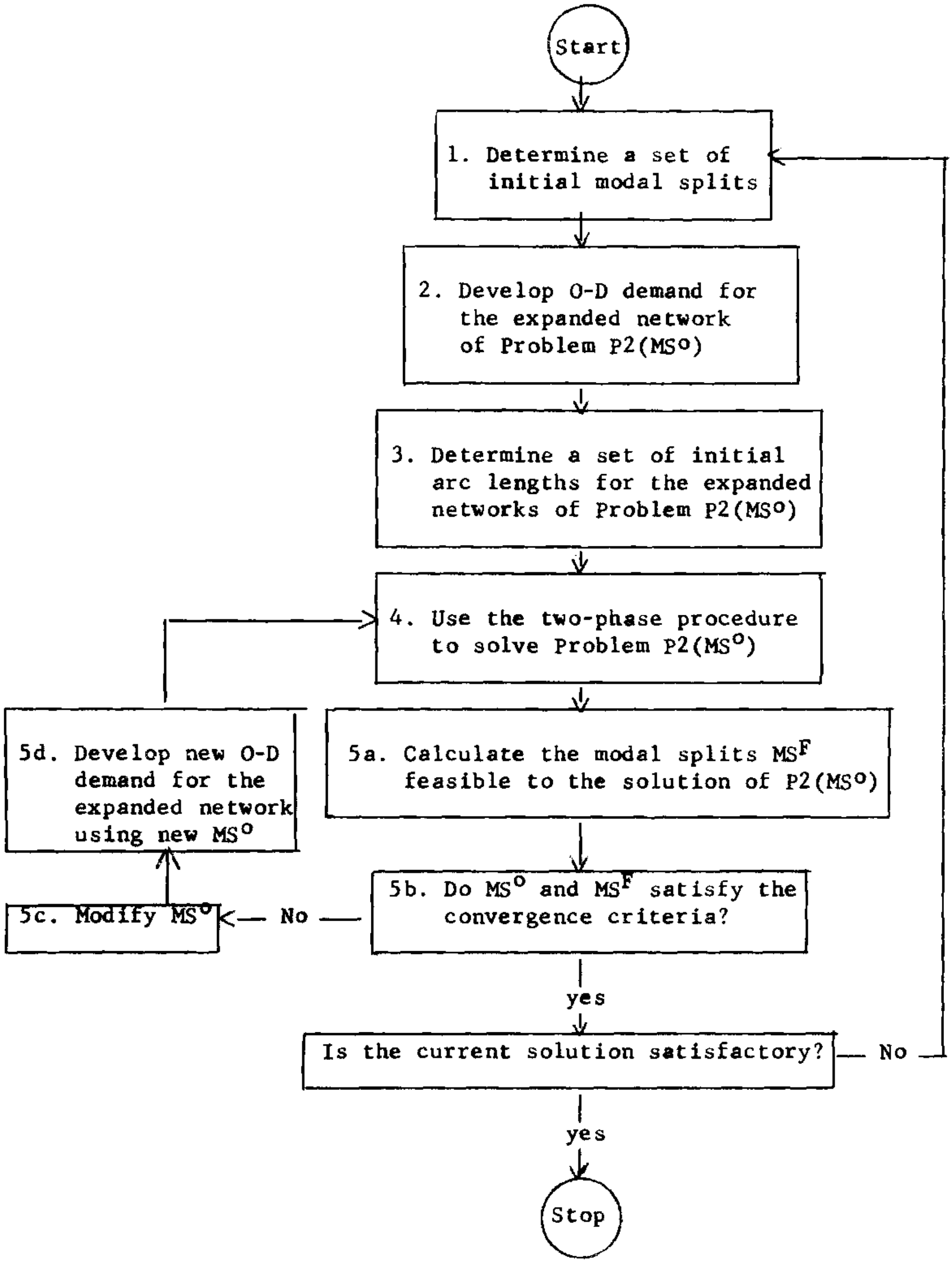


Fig. 4-1. Macro-Flowchart of Algorithm

(a) It calculates the set of feasible modal splits MS^F using the solution of $P2(MS^0)$.

(b) It tests the convergence of MS^0 and MS^F.

(c) It modifies MS^0 if there is not convergence.

(d) It develops the new set of O-D demands for the expanded network using the revised MS^0.

Program TESTMS is the functional version of this component.

The convergence test used in Program TESTMS is:

$$|\varepsilon_{sm}| \leq \varepsilon_1 \qquad \forall\ s \in R,\ m \in M \tag{4-1}$$

where:

$\varepsilon_{sm} = MS^F_{sm} - MS^0_{sm}$

$MS^0_{sm} \equiv$ the initial estimate of the mode m share for O-D pair s

$MS^F_{sm} \equiv$ the feasible mode m share for O-D pair s

$R \equiv$ the set of O-D pairs

$M \equiv$ the set of modes

$\varepsilon_1 \equiv$ the convergence parameter

The convergence parameter ε_1 is an input variable to TESTMS. MS^0 is updated according to the equation:

$$MS^0_{New} = MS^0_{Old} + \alpha\varepsilon \tag{4-2}$$

$\alpha \equiv$ the correction factor, positive

$\varepsilon \equiv$ the error vector, as calculated above

The correction factor α is also an input variable to TESTMS. The sixth and final component of the methodology is the analyst's decision to terminate the methodology with an existing solution or to reinitiate it with another set MS^O. All of the programs are listed in Appendix A.

1.1 Program CNCASNB: A Two-Phase Solution Procedure for $P2(MS^O)$

Program CNCASNB is the functional version of the two-phase algorithm developed in Chapter III. A basic flowchart of CNCASNB is shown in Figure 4-2. Inputs to the program include:

(1) The expanded network including the expanded set of nodes and arcs.

(2) The expanded set of O-D pairs.

(3) The initial arc lengths.

(4) The maximum number of Phase I iterations.

Program CNCASNB is always either in Phase I or Phase II. The basic structure of the program is the same for both phases. A shortest path tree is constructed for each expanded origin using the current set of arc lengths. The tree-building algorithm used is a Dijkstra type labeling algorithm with several special features: [Dijkstra, 1959]

(1) The algorithm does not consider intermodal transfer arcs when constructing a tree from a single-mode origin; i.e., an origin corresponding to the highway, rail, or water mode.

(2) For a given origin, the tree-building process terminates when all destinations for that origin have been labeled.

Only the most current tree is stored, with each tree occupying the same core storage block, approximately 3,000 decimal words. This is necessitated by the relatively small core storage capacity of the Cyber 74. approximately 80,000 decimal words, coupled with the large size of the

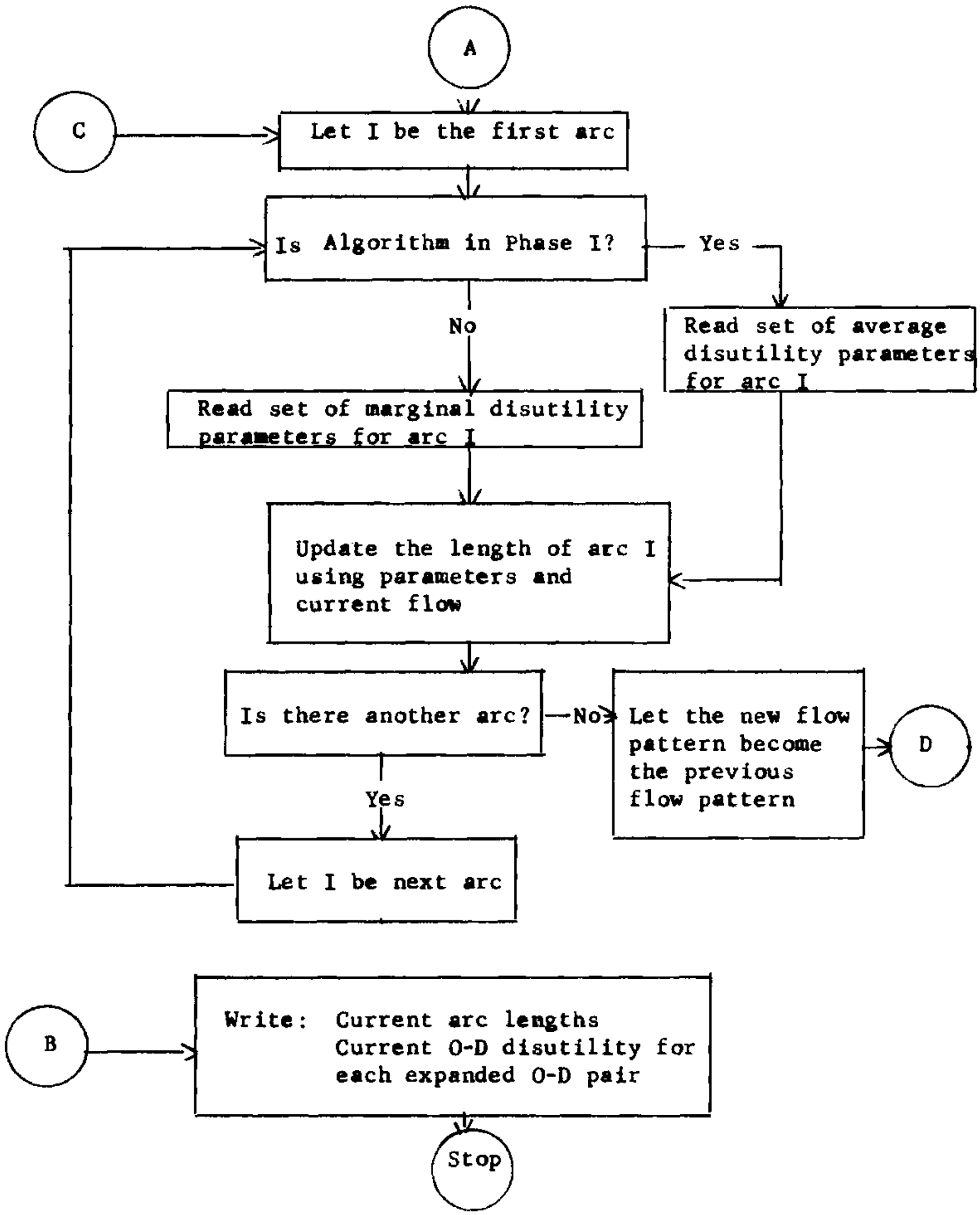
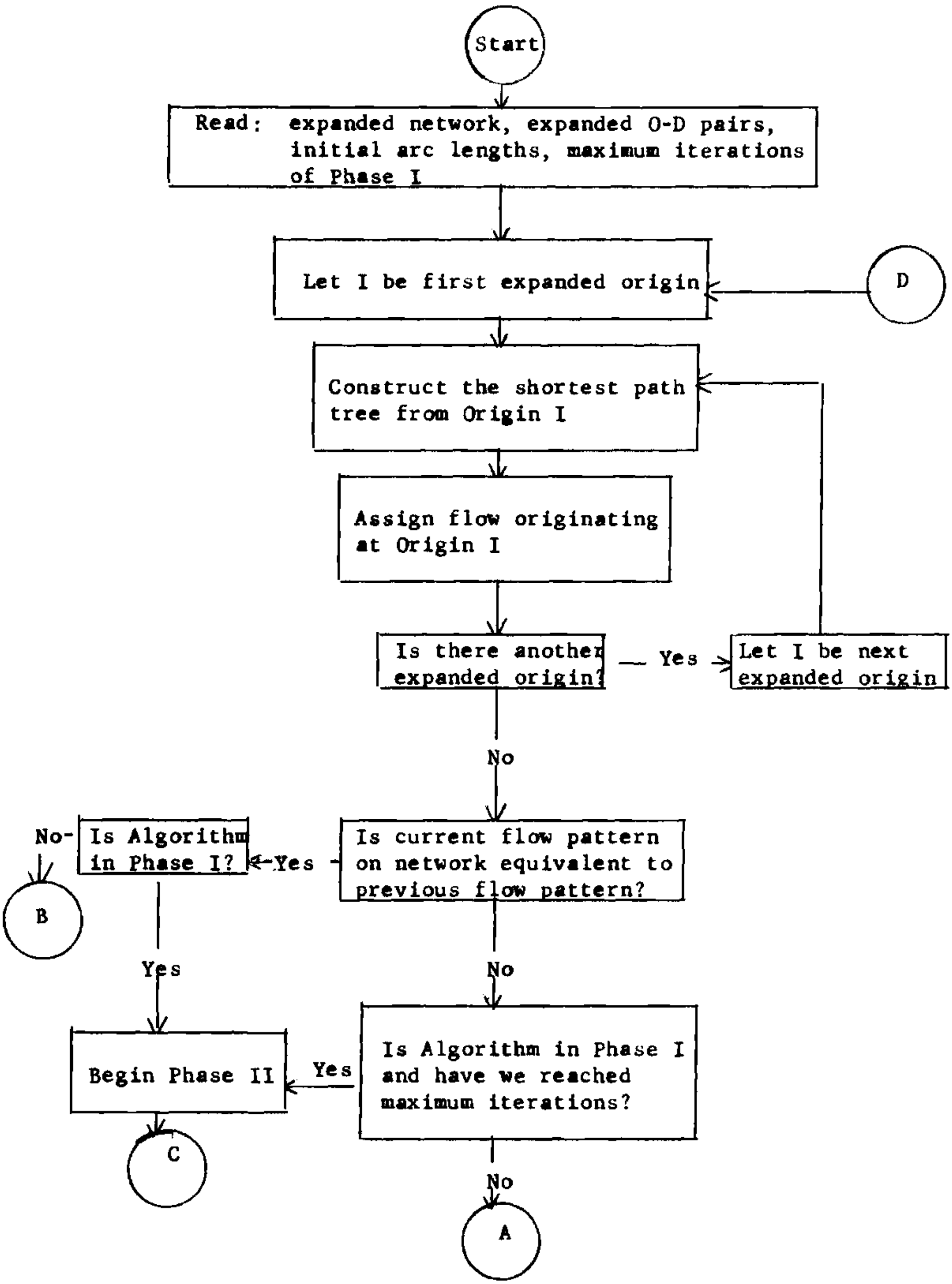


Fig. 4-2. Flowchart for CNCASNB


Start
Read: expanded network, expanded O-D pairs, initial arc lengths, maximum iterations of Phase I
Let I be first expanded origin
D
Construct the shortest path tree from Origin I
Assign flow originating at Origin I
Is there another expanded origin?
Yes
Let I be next expanded origin
No
Is current flow pattern on network equivalent to previous flow pattern?
Yes
Is Algorithm in Phase I?
No
B
Yes
No
Is Algorithm in Phase I and have we reached maximum iterations?
Yes
Begin Phase II
C
No
A

(continued)

network. When a tree has been constructed for an origin node, all flow originating from that node is assigned to the shortest paths as determined by the tree.

After a tree has been constructed for each expanded origin node, and all flow has been assigned to the network, the current iteration is completed. It must now be determined whether the algorithm is to terminate, shift phases, or merely iterate in the same phase. The only way the algorithm may terminate is to be in Phase II and to have converged to a local optimum. Convergence has occurred when arc flows are identical on two successive iterations. The algorithm may shift from Phase I to Phase II in two ways.

(1) The algorithm is in Phase I and converges.

(2) The algorithm is in Phase I and has reached its maximum number of iterations.

Finally, the algorithm will remain in its current phase and iterate when convergence has not occurred and when Case 2 above does not hold.

The only effect of phase on the operation of the algorithm is in the calculation of arc length at the beginning of an iteration. When the algorithm is in Phase I it is seeking a point which is good globally. Thus, it bases arc length on average arc disutility. When in Phase II, the algorithm defines arc length as marginal arc disutility. Note that the parameters used to calculate these lengths are input from the appropriate data file at each iteration. Again, this is necessitated by the limited core storage available and the large number of arc parameters, approximately 33,000.

1.2 Data Requirements

The network used in the implementation of the methodology was the network developed by Jones for use in the Multi-State Corridor Research Project [Jones, 1977]. The continental United States is divided into 120 separate zones as shown in Figure 4-3 below. A complete description of these zones is given in Appendix B below. Note that zone size is smallest in the Multi-State Corridor area, roughly corresponding to an area planning and development district, and grows larger with distance from the corridor. A node representing each zone is placed at the major population/industrial center in the zone.

Line-haul arcs representing actual line-haul transport facilities connect the various nodes. For a complete listing of the line-haul arcs by mode, see Appendix C below. Additional nodes and arcs were added to the network in order to represent other transport facilities located within the zones. For a review of this procedure, see Chapter II, Section 1 above. The size of the resulting expanded network was approximately 1,000 nodes and 3,000 arcs.

The functions relating the arc transport characteristics (ATC) to arc investment were developed as follows:

(1) The general form used:

$$c_j(I_j) = b_{1j}(I_j - \bar{c}_j)^2 + d_{1j} \qquad (4\text{-}3)$$

where:

$c_j(I_j) \equiv$ the arc transport characteristic for arc j given I_j

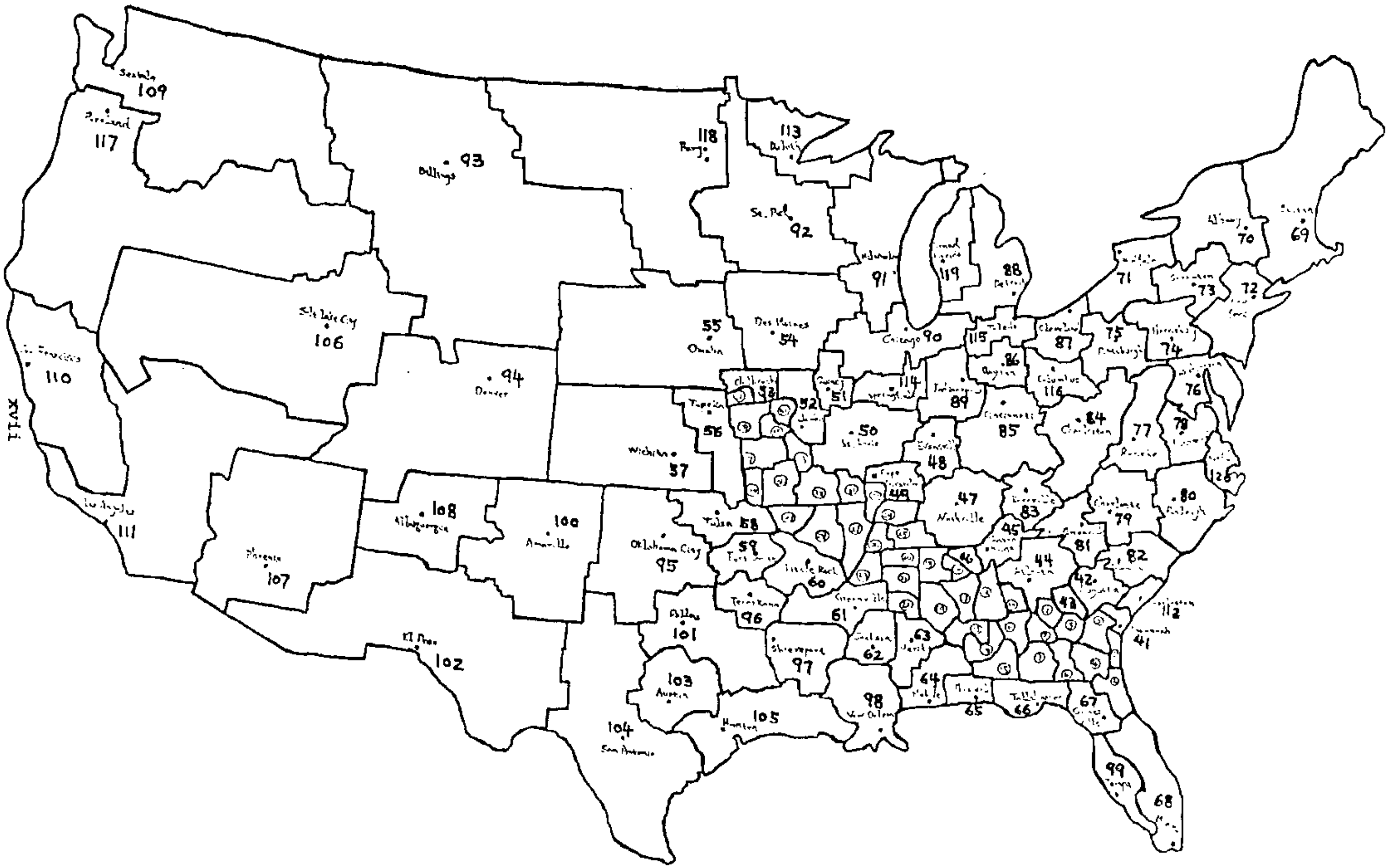


Figure 4-3
Multi-State Zone Structure

$I_j \equiv$ investment in arc j

$b_{1j}, \bar{c}_j, d_{1j} \equiv$ parameters of the model

(2) Select the current value of the ATC averaged over all commodities for use as base value. Jones has estimated these base values for unit cost, time, and time variability for all transport facilities [Jones, 1977].

Let: $\hat{c}_j \equiv$ the current value of the ATC on arc j

(3) Divide the current value into two parts, one susceptible to improvement by investment and the other fixed.

Let: $\hat{c}_j^1 \equiv$ the part susceptible to improvement

$\hat{c}_j^2 \equiv$ the fixed part

Note:

$$\hat{c}_j = \hat{c}_j^1 + \hat{c}_j^2$$

$$d_{1j} = \hat{c}_j^2$$

(4) Let: $L_j \equiv$ the lower bound on investment

$U_j \equiv$ the upper bound on investment

Then:

$$\bar{c}_j = U_j$$

$$b_{1j} = \hat{c}_j^1/(L_j - \bar{c}_j)^2$$

The factors by which the base values were divided as well as the lower and upper bounds on investment are given in Appendix D below. It should be noted that only those arcs wtihin the Multi-State Corridor, approximately 1,000 arcs, were allowed to be improved. This was accomplished by setting $L_j = U_j$ for all arcs outside of the Multi-State Corridor.

The O-D flow data set used in the implementation, listed in Appendix E below, was derived from a data base prepared by Mullens and Sharp for use in the Multi-State Corridor Research Project [Mullens and Sharp, 1978]. Three commodities were selected for inclusion:

(1) SIC 22 - Textile Mill Products

(2) SIC 24 - Lumber and Wood

(3) SIC 287 - Agricultural Chemicals

Flows of these commodities were combined to form a single macro-commodity. The final flow data set was obtained by selecting 63 different O-D flow data points of the macro-commodity and multiplying their flows by a factor of 25. Only those flows most likely to use the corridor were selected. Flows were multiplied by an expansion factor in order to increase them to realistic flow levels roughly equivalent to the sum of all commodity flows included in the original data base [Mullens and Sharp, 1978].

It should be noted that for the special case where only a portion of the overall network is allowed to improve, some O-D pairs might be excluded from the analysis without affecting the results. For the Multi-State Corridor example, consider a flow from Seattle to Los Angeles. It is highly unlikely that this flow would ever be routed through the Multi-State Corridor, even given maximum improvement in the Corridor. A simple criterion for O-D pair elimination is the following:

(1) Route all O-D flows assuming maximum investment in the network.

(2) If the path for some O-D pair does not include an arc subject to improvement, then the O-D flow can be eliminated.

The proof of this criterion follows from the fact that even with maximum improvement on the network, the identified O-D flow uses a better non-improvable path. Thus, no other path could possibly divert the flow, and it can have no effect on the analysis.

A final data input to the implementation is the set of modal split model parameters. The multinomial logit modal split model is used. The three parameters needed to use the logit model were taken from the Multi-State Corridor Research Project [Jones, 1977]. Values are representative for the three commodities selected.

$$a_1 = -.01$$

$$a_2 = -.00033$$

$$a_3 = -.0004$$

Note:

$$\frac{a_2}{a_1} = .033 \text{ equivalent dollars/min.}$$

$$\frac{a_3}{a_1} = .04 \text{ equivalent dollars/hr.}^2$$

Aggregate statistics relating to the size of the resulting problem are shown in Table 4.1 below.

Table 4-1. Approximate Aggregate Statistics of Problem

Actual transport facilities	2600
Arcs (after network expansion)	3000
Zones	120
Nodes (prior to expansion)	840
Nodes (after expansion)	1000
O-D pairs (prior to expansion)	63
O-D pairs (after expansion)	227
Transport commodity classes	1
Math programming commodity classes* (prior to expansion)	14
Math programming commodity classes* (after expansion)	56

*In the mathematical programming literature a commodity is usually defined by origin and by a set of arc costs (alternatively, by destination and by arc costs). The term commodity as used in this research is defined by arc costs only.

2. Results

2.1 Description of Test Runs

The battery of programs described in Section 1.1 above was used to obtain solutions to the problem described in Section 1.2. Solutions were obtained for a variety of different methodological configurations, initial solutions, and parameters. Specific factors which were varied for the test runs included:

(1) The set of initial modal splits MS^0.

(2) The modal split updating parameter ALPHA.

(3) The modal split convergence parameter EPSILON.

(4) The use of a Phase I type procedure in the two-phase algorithm.

(5) The set of initial arc lengths used in the first iteration of the two-phase algorithm. These arc lengths correspond to an initial set of arc investments.

Although a full or fractional factorial experimental design could have been used to estimate the effects of these factors, the lengthy computation times and subsequent high cost of solution made this alternative impractical. Accordingly, since several factors, such as the use of a Phase I procedure, the set of initial modal splits, and the set of initial arc lengths, were thought to be more critical than others, these factors were given more careful consideration in what can best be termed a "guided" experiment. The resulting set of test runs are described in detail in Table 4-2 below. These runs are placed in the format of a full factorial experimental design in Appendix F.

The first six runs were designed to demonstrate the effect of using a Phase I type procedure. The presence of a Phase I procedure was crossed with three sets of initial arc lengths (sets A, B, and C) while the initial modal split was held at the current split. It should be noted that each set of initial arc lengths was identified by a descriptor representing the seed used to randomly generate the set. These seeds and their descriptors were:

Table 4-2. Test Runs

Run	Modal Split	Alpha	Epsilon	Phase I Used	Arc Length
1	Current	1	.02	No	A
2	Current	1	.02	No	B
3	Current	1	.02	No	C
4	Current	1	.02	Yes	A
5	Current	1	.02	Yes	B
6	Current	1	.02	Yes	C
7	1000	1	.02	Yes	A
8	1000	1	.02	Yes	B
9	1000	1	.02	Yes	C
10	0100	1	.02	Yes	A
11	0100	1	.02	Yes	B
12	0100	1	.02	Yes	C
13	0010	1	.02	Yes	A
14	0010	1	.02	Yes	B
15	0010	1	.02	Yes	C
16	0001	1	.02	Yes	A
17	0001	1	.02	Yes	B
18	0001	1	.02	Yes	C
19	1001	1	.02	Yes	A
20	1001	1	.02	Yes	B
21	1001	1	.02	Yes	C
22	1110	1	.02	Yes	A
23	1110	1	.02	Yes	B
24	1110	1	.02	Yes	C
25	1110	.8	.02	Yes	A
26	1110	.8	.02	Yes	B
27	1110	.8	.02	Yes	C
28	0110	1	.02	Yes	A
29	0110	1	.01	Yes	A
30	0100	1	.01	Yes	B

Notes:

1. Modal split: $(C_{MM}, C_{HW}, C_{RR}, C_W)$

   $C_m = 0 \Rightarrow$ mode m receives no share

   $C_m = 1 \Rightarrow$ mode m receives equal share

2. Arc Lengths: A $\rightarrow$ Seed 767676767676
   B $\rightarrow$ Seed 765676567656
   C $\rightarrow$ Seed 656565656565

Descriptor	Seed
A	7676767676
B	765676567656
C	656565656565

The next factor examined was the initial set of modal splits. To demonstrate the effect of this factor seven modal splits were crossed with the same three sets of initial arc lengths while all other factors were held constant, resulting in runs 4 through 24. An additional run, number 28, used an eighth set of initial modal splits. Except for the current modal split, all sets of initial modal splits were identified by a descriptor of the form:

$$(C_{MM}, C_{HW}, C_{RR}, C_{W})$$

where:

$C_m = 0 \Rightarrow$ mode m receives no share

$C_m = 1 \Rightarrow$ mode m receives an equal share for each O-D pair it connects

Note: $C_{MM} = 1$ if no other designated mode connects the O-D pair.

The third factor examined was ALPHA, the modal split updating parameter. To demonstrate the effect of this factor, two values of ALPHA were crossed with the three sets of initial arc lengths, resulting in runs 22 through 27. The initial modal split for these runs was 1110. The final factor examined was EPSILON, the modal split convergence parameter.

To demonstrate the effect of this factor, four runs were made with two different values of ALPHA. Runs 28 and 29 were made with a 0110 modal split and arc length set A, and runs 11 and 30 were made with a 0100 modal split and arc length set B. Finally, with the exception of runs 1 through 3, all test runs used a Phase I type procedure in the two-phase algorithm. This was done because it was thought that utilization would result in better, more uniform solutions to Problem $P2(MS^0)$, the concave transportation assignment problem. Selected results from the test runs are given in Appendix F.

2.2 <u>Convergence of the Methodology</u>

In Chapter III it was noted that convergence of the methodology could not be guaranteed. Specific areas of concern included:

(1) Convergence of MS^0 and MS^F in the general methodology.

(2) Convergence of the first phase of the two-phase algorithm.

From the results in Appendix F it appears that the methodology converged for all runs. The minimum CPU time required for convergence was 632 seconds, the maximum 1213 seconds, and the average 1064 seconds. The overall computation time was the result of two factors:

(1) The number of macro-iterations of the general methodology required for convergence.

(2) The CPU time for each macro-iteration.

The minimum number of macro-iterations was two and maximum was three. The minimum CPU time required for a macro-iteration was 164 seconds, the maximum 466 seconds, and the average 364 seconds. The time required for a macro-iteration was determined by two factors:

(1) The number of micro-iterations constituting the macro-iteration. A micro-iteration is defined to be one Phase I or Phase II iteration of the two-phase algorithm.

(2) The time required for each micro-iteration.

The minimum number of micro-iterations in a macro-iteration was two and the maximum was eight. It is interesting to note that as the general methodology approached convergence, the number of micro-iterations per macro-iteration decreased. For example, the average number of micro-iterations required for the first macro-iteration was 6.6, the average for the second was 6.3, and the average for the third was 4.6.

The average CPU time required to complete a micro-iteration was 61 seconds. Each micro-iteration consisted of two components:

(1) Reading a set of 11 arc disutility parameters for each of the 3080 network arcs and calculating the current arc length.

(2) The construction of 56 shortest path trees, for 227 O-D pairs. Assigning the flow to the network.

Thus, the average CPU time required to construct one tree was something less than 1.1 second. For a given origin, a shortest path tree was completed when all destination for that origin were attached to the tree. Thus, the actual CPU time required to construct a tree depended upon the relative arc lengths for that iteration.

Now, consider the effects of the factors listed in Table 4-1 on the rate of convergence. Since the number of micro-iterations required for convergence is roughly proportional to the CPU time, this number can be used as a measure of rate of convergence. Consider the effect of the first factor, the set of initial modal splits MS^0. Micro-iterations are plotted against initial modal splits in Figure 4-4 below. Holding all other factors constant, the methodology converged at approximately the same rate, 3 macro-iterations and 18 to 20 micro-iterations, for <u>most</u> initial modal splits. Obvious exceptions involved the splits 1110 and 0110 which converged in two macro-iterations and 12 to 13 micro-iterations.

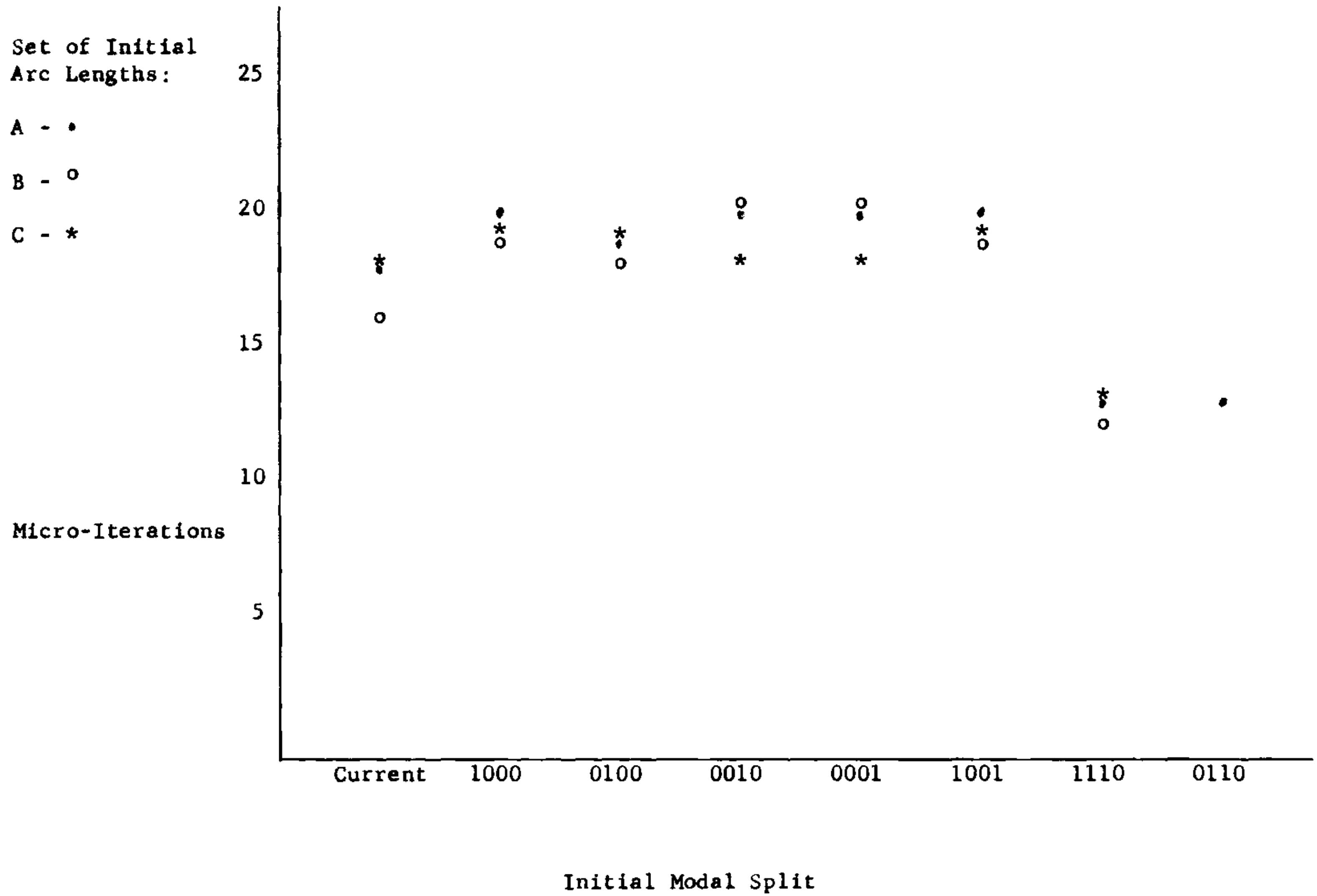


Fig. 4.4. Initial Modal Split Versus Rate of Convergence

Second, consider the effect of the modal split updating parameter ALPHA. Results are shown in Table 4-3 below.

Table 4-3. Effect of ALPHA on Rate of Convergence

Run	Set of Initial Arc Lengths	ALPHA	Macro-Iterations	Micro-Iterations
22	A	1.0	2	13
25	A	.8	3	18
23	B	1.0	2	12
26	B	.8	3	17
24	C	1.0	2	13
27	C	.8	3	18

From preliminary program battery verification runs with a much smaller problem, it was demonstrated that deviation of ALPHA from 1.0 resulted in a significant decrease in the rate of convergence. The same effect was demonstrated by the test runs. Changing ALPHA from 1.0 to 0.8 resulted in increases in the number of macro-iterations from two to three and in the number of micro-iterations by five. Next, consider the effect of the third factor, the modal split convergence parameter EPSILON. It was postulated previously that as EPSILON was decreased, the time required for convergence might increase substantially. This was verified by runs 28 and 29. As EPSILON was decreased from .02 to .01, the number of macro-iterations increased from two to three and the number of micro-iterations increased from 13 to 18. Runs 11 and 30 showed little difference in convergence as EPSILON was decreased from

.02 to .01.

Next, consider the effect of using a Phase I procedure in the two-phase algorithm. Since this factor is internal to a single component of the general methodology, it might not be expected to affect the number of macro-iterations. Runs 1 through 6 demonstrated this, each requiring three macro-iterations. However, the use of a Phase I procedure had a substantial effect on the number of micro-iterations required for convergence. The results are shown in Table 4.4. Several points should be noted:

(1) The use of a Phase I procedure increased the number of micro-iterations by approximately 100%.

(2) Although the use of the Phase I procedure decreased the number of Yaged type Phase II iterations required, the Phase I iterations more than made up for this decrease.

Finally, consider the effect of the initial set of arc lengths. Returning to Figure 4.4, note that this factor did not cause the rate of convergence to change greatly. The maximum change was 2 micro-iterations. While no set of initial arc lengths uniformly hastened convergence, set A appeared to slightly impede convergence.

2.3 Aggregate Characteristics of Solutions

In this section the solutions obtained from the test runs are analyzed with respect to three characteristics: total savings over current total disutility associated with the network (the objective), investment over the current minimum level, and user savings over current level. The current values were obtained by setting all arc investments at their lower bounds and allowing the flow assignment constraints (2-17) and (2-18) to assign flow to the network. All values are given in terms of millions of equivalent annual dollars. The current values were estimated as:

Table 4-4. Effect of Phase I Procedure on Rate of Convergence (Micro-iterations)

Macro Iteration/		I			II			III			Totals		
Run	Initial Set of Arc Lengths	Phase I	Phase II	Total	Phase I	Phase II	Total	Phase I	Phase II	Total	Phase I	Phase II	Total
1	A	0	4	4	0	3	3	0	2	2	0	9	9
4	A	4	2	6	5	2	7	3	2	5	12	6	18
2	B	0	5	5	0	4	4	0	2	2	0	11	11
5	B	4	2	6	3	2	5	3	2	5	10	6	16
3	C	0	4	4	0	4	4	0	2	2	0	10	10
6	C	4	2	6	5	2	7	3	2	5	12	6	18

Total disutility - 16,626

Investment - 269

User disutility - 16,357

The total savings associated with the solutions ranged from a minimum of 1140 for run 21 to a maximum of 1318 for run 7. Investment associated with solutions ranged from a minimum of 187 for run 4 to a maximum of 220 for run 3. User savings associated with solutions ranged from a minimum of 1351 for run 21 to a maximum of 1537 for run 7. The objective, total savings, is plotted against investment in Figure 4-5 below. Although the best solutions occurred at high levels of investment, very poor solutions also occurred at these levels. In general, there appears to be little relationship between investment and total savings. Solutions tended to congregate around certain combinations of investment and total savings. This tendency could indicate the presence of local optimal solutions. Investment is plotted against user savings in Figure 4-6 below. As might be expected, the solutions having the highest user savings also had some of the highest levels of investment. However, high investment did not assure a high level of user savings. In general, the relationship between investment and user savings was not strong. Finally, total savings are plotted against user savings in Figure 4-7 below. The strong relationship between total savings and user savings results from the fact that investment is relatively small compared to these two values. Thus, one closely approximates the other. This is also the reason for the similarity of Figures 4-5 and 4-6 above.

Total savings are plotted against the total number of

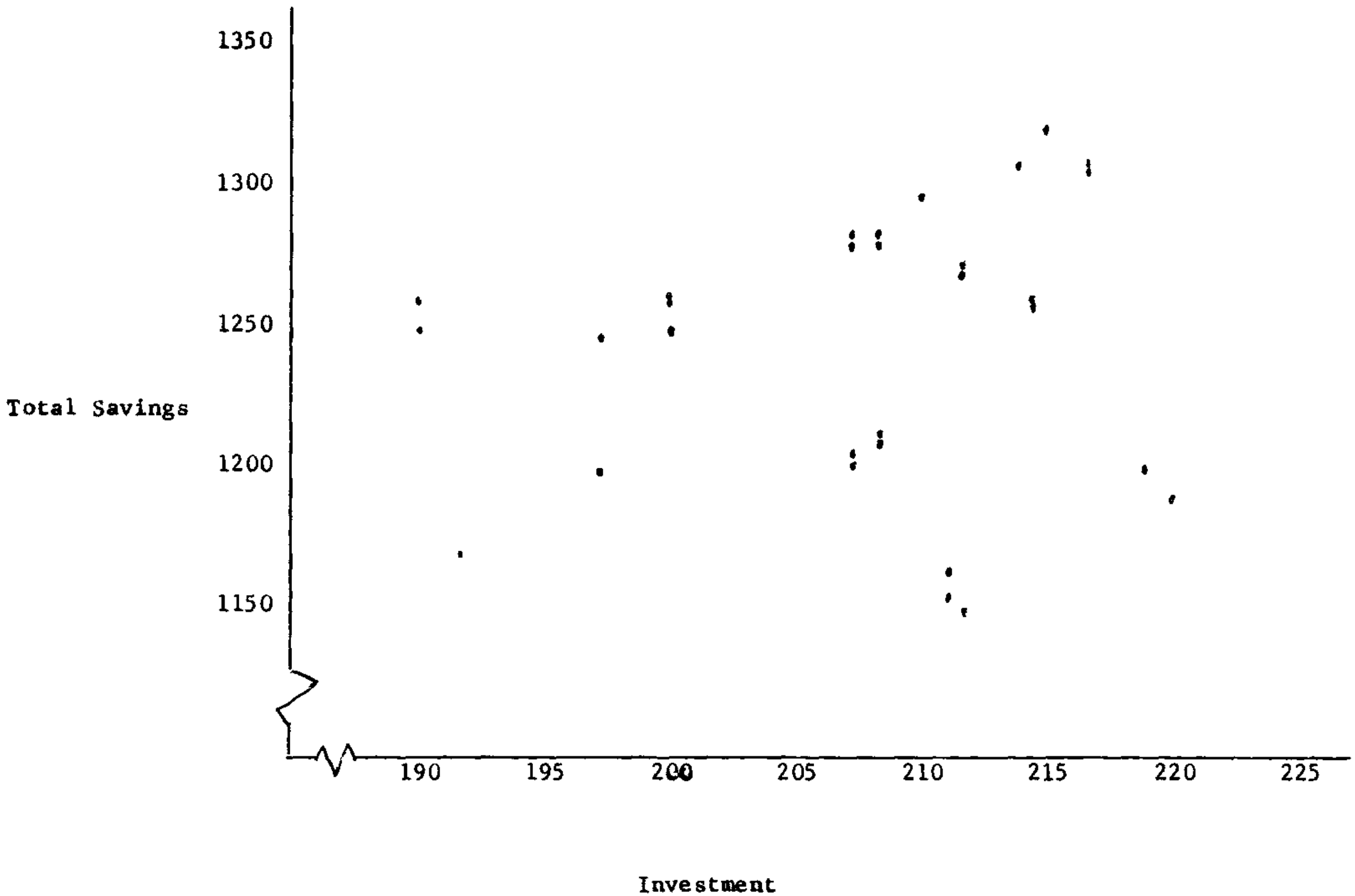


Fig. 4.5. Total Savings Versus Investment, All Runs

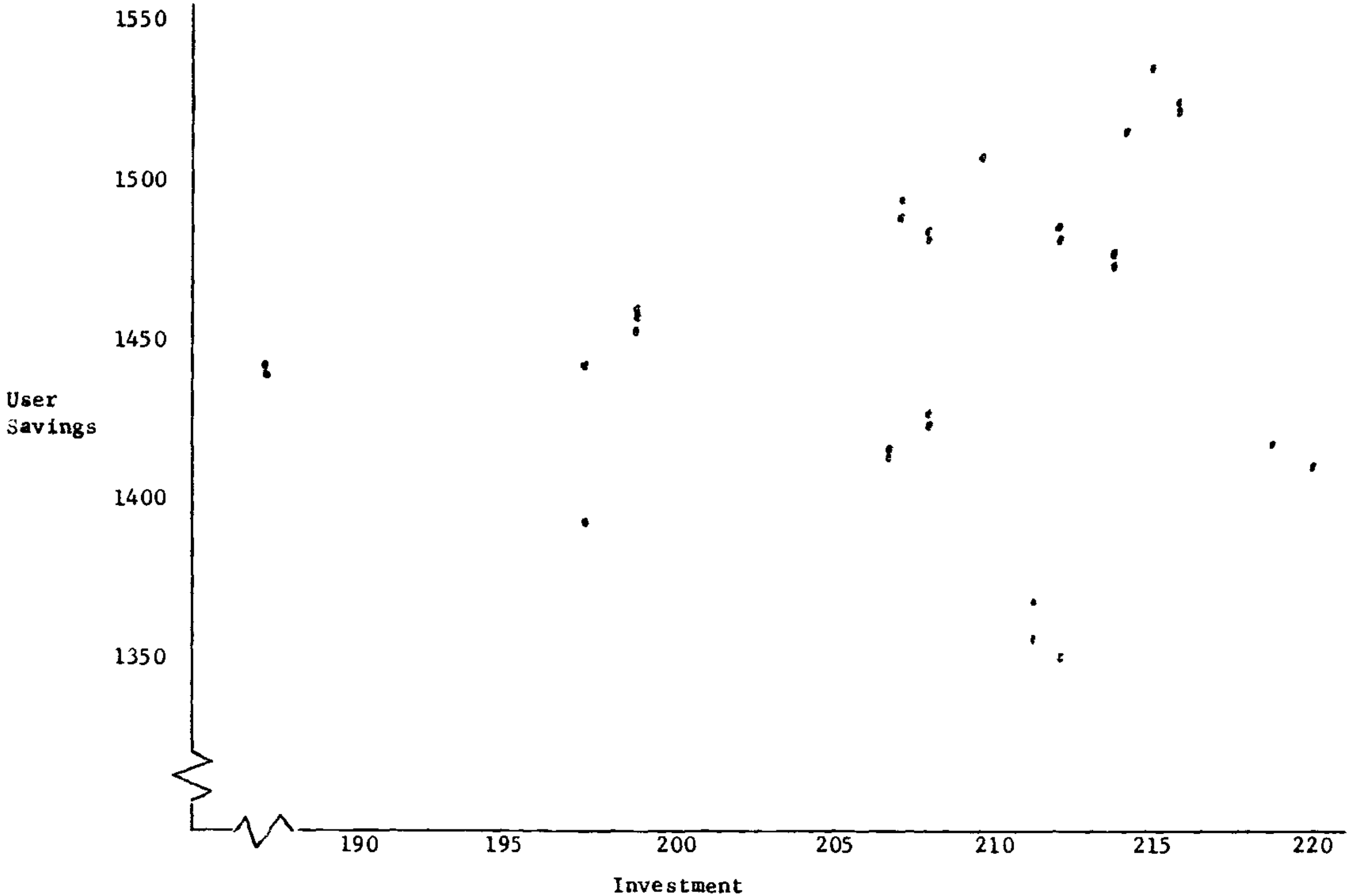


Fig. 4.6. User Savings Versus Investment, All Runs

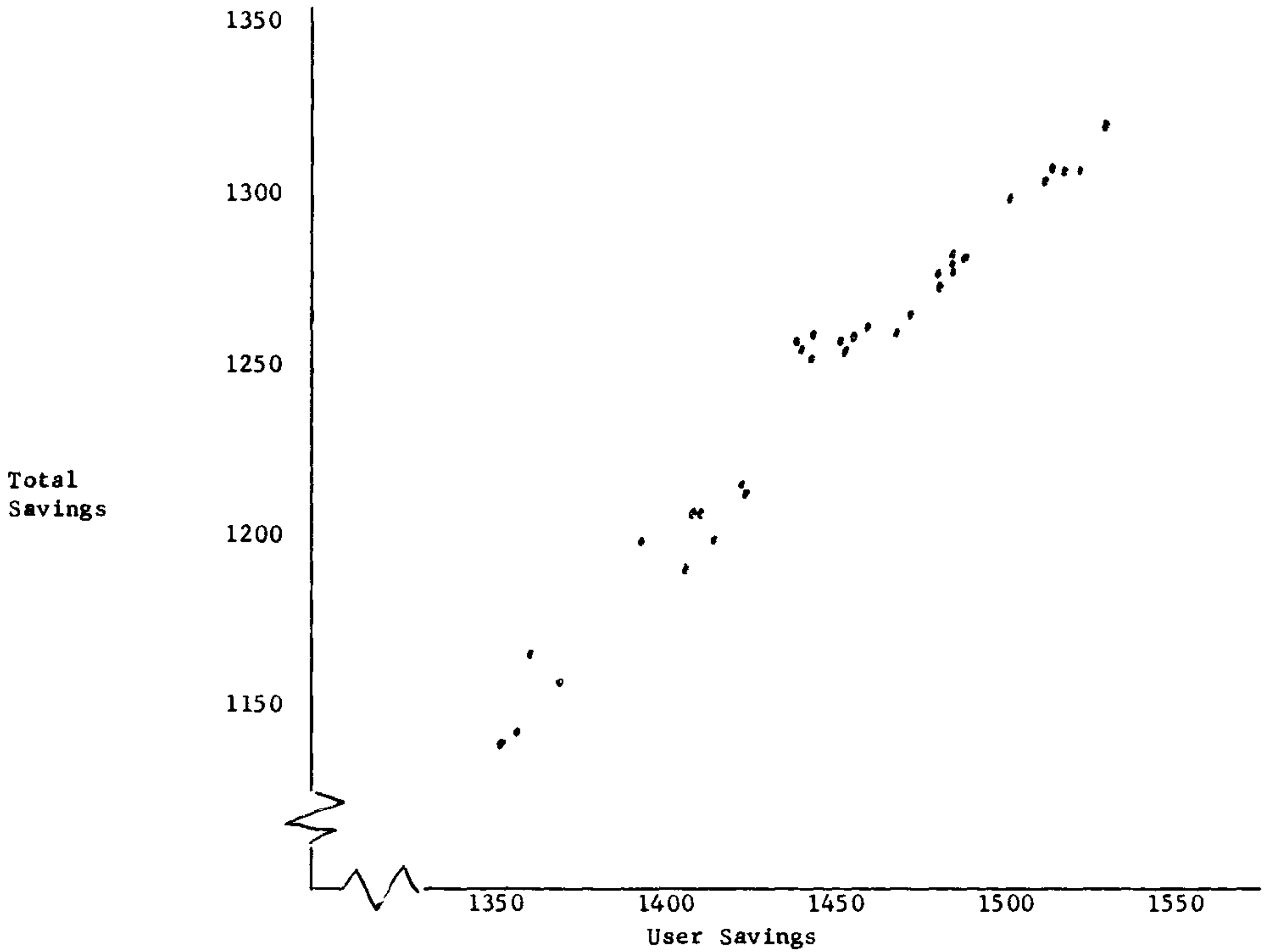


Fig. 4.7. Total Savings Versus User Savings, All Runs

micro-iterations in Figure 4-8 below. There appears to be little relationship between the two. Investment and user savings are plotted against the number of micro-iterations in Figures 4-9 and 4-10 respectively. Again, there appears to be little relationship between the characteristics being considered and the rate of convergence.

Now, consider the effects of the factors listed in Table 4-1 on the characteristics' total savings, investment, and user savings. First, consider the effect of the set of initial modal splits MS^0 and the set of initial arc lengths. Total savings is plotted against these factors in Figure 4-11 below. Several points should be noted:

(1) The set of initial modal splits apparently had a strong effect on total savings for each set of initial arc lengths.

(2) No set of initial modal splits was superior or inferior for all sets of initial arc lengths.

(3) The set of initial arc lengths had a strong effect on total savings for each of the sets of initial modal splits, with the possible exception of 0010.

(4) The set of initial arc lengths A was consistently superior to sets B and C. Set C was consistently inferior to sets A and B.

Investment is plotted against these factors in Figure 4-12 below. Several points should be noted:

(1) While most sets of initial modal splits and arc lengths resulted in approximately the same level of investment, several sets of initial modal splits resulted in substantially lower levels of investment.

(2) The current set of modal splits and the 0010 set resulted in much lower levels of investment for all sets of initial arc lengths.

(3) The set of initial arc lengths did not have a strong effect on the level of investment given a set of initial modal splits.

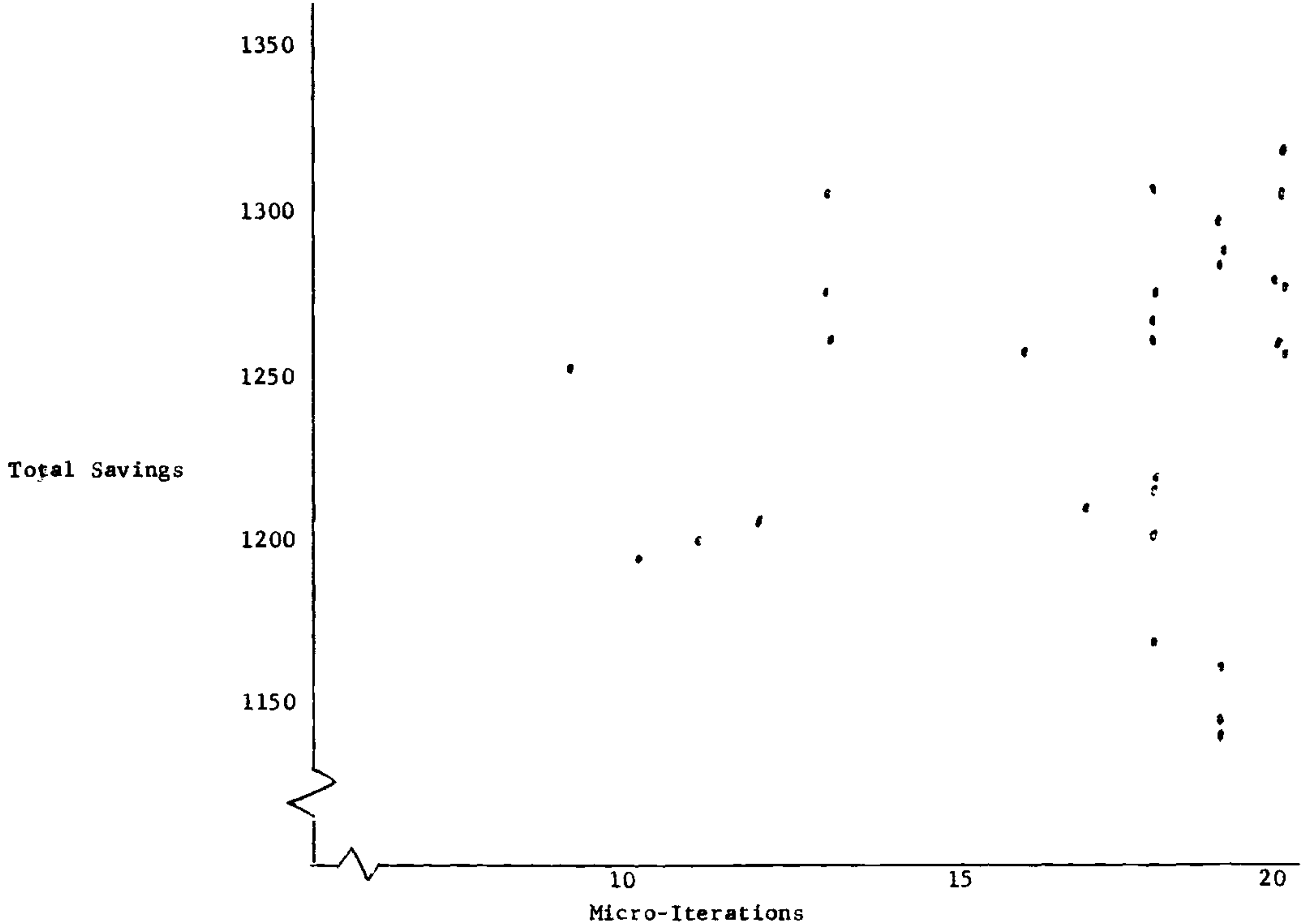


Fig. 4.8. Micro-Iterations Versus Total Savings, All Runs

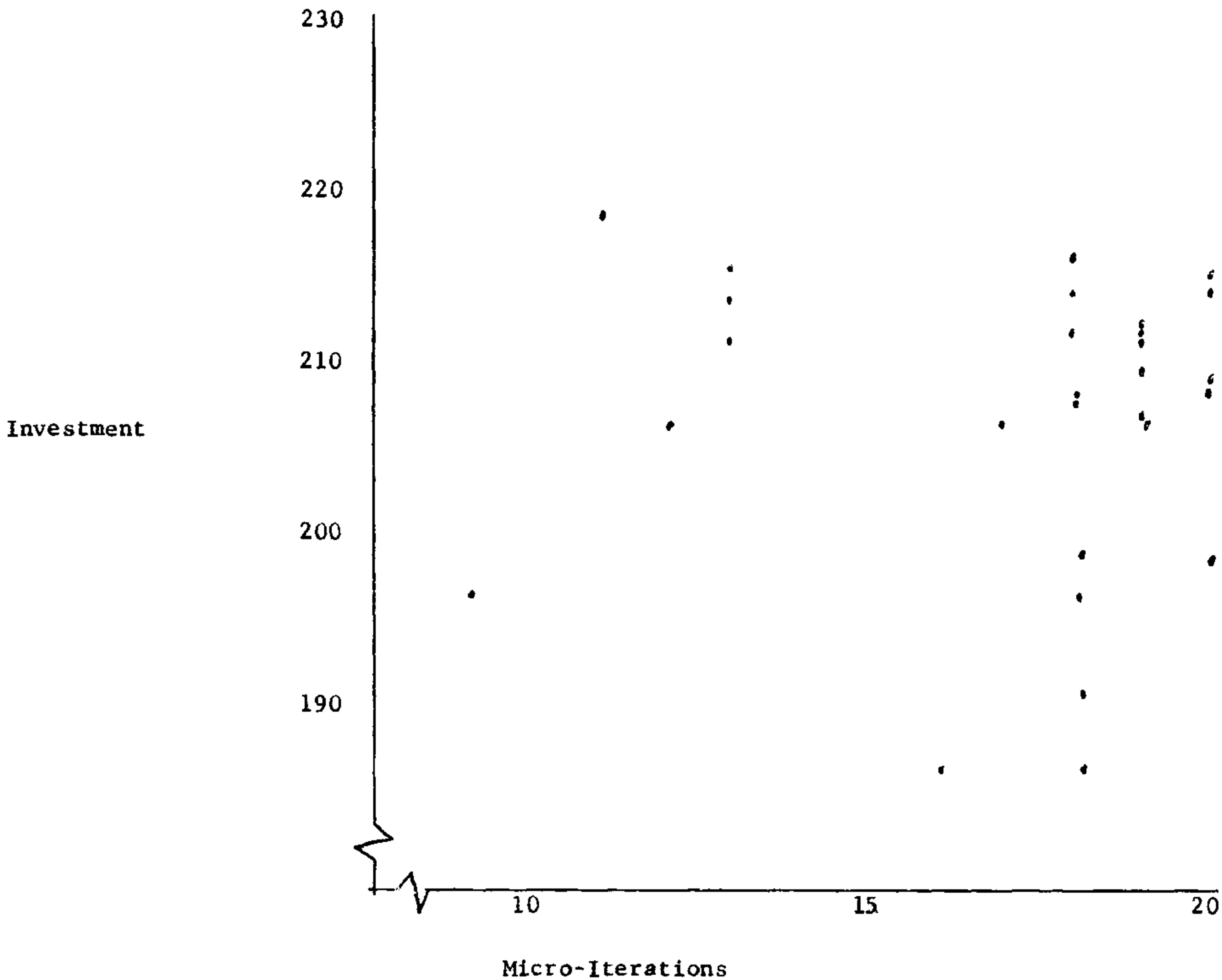


Fig. 4.9. Micro-Iterations Versus Investment, All Runs

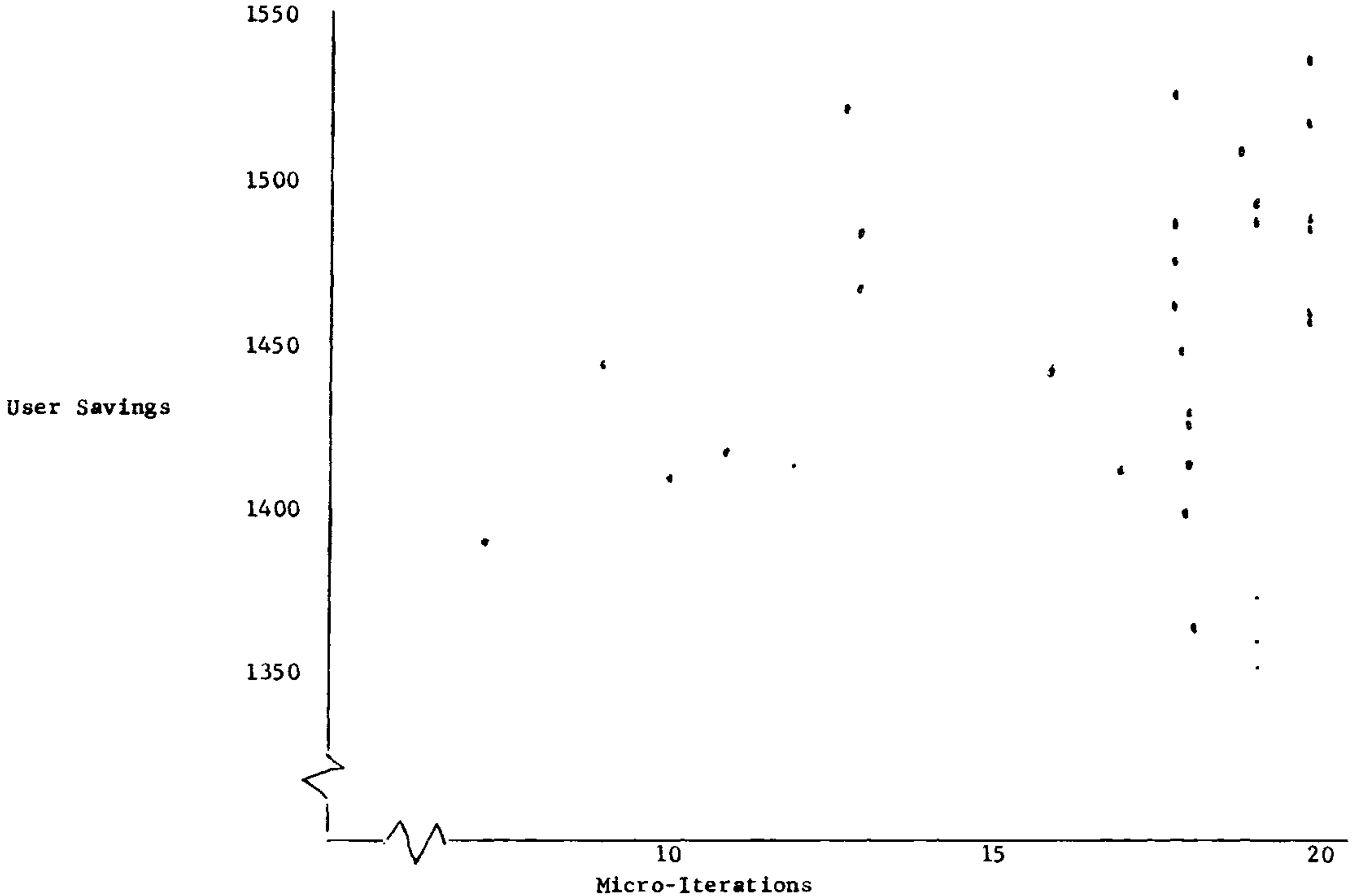


Fig. 4.10. Micro-Iterations Versus User Savings

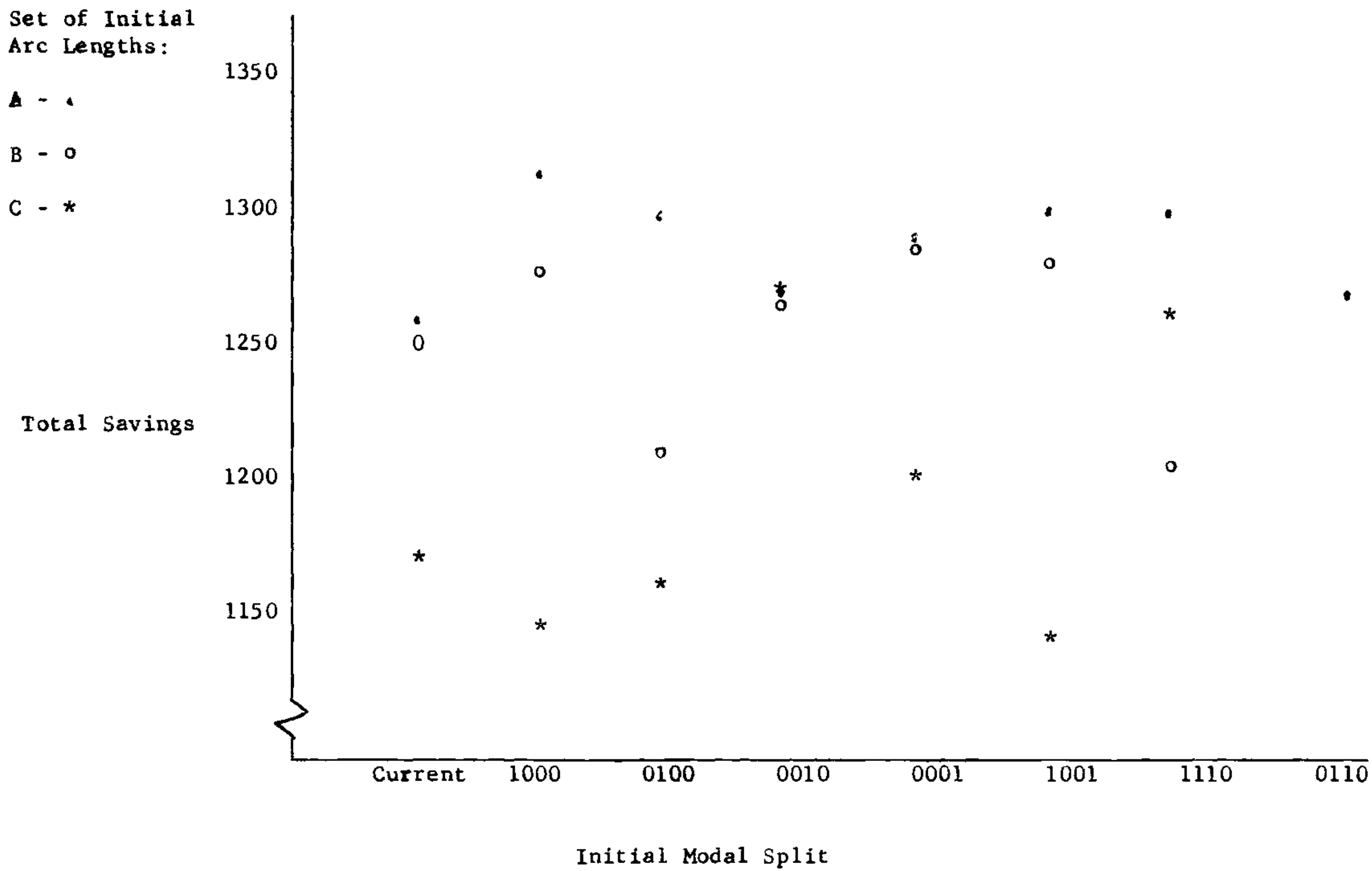


Fig. 4.11. Total Savings Versus Initial Modal Split

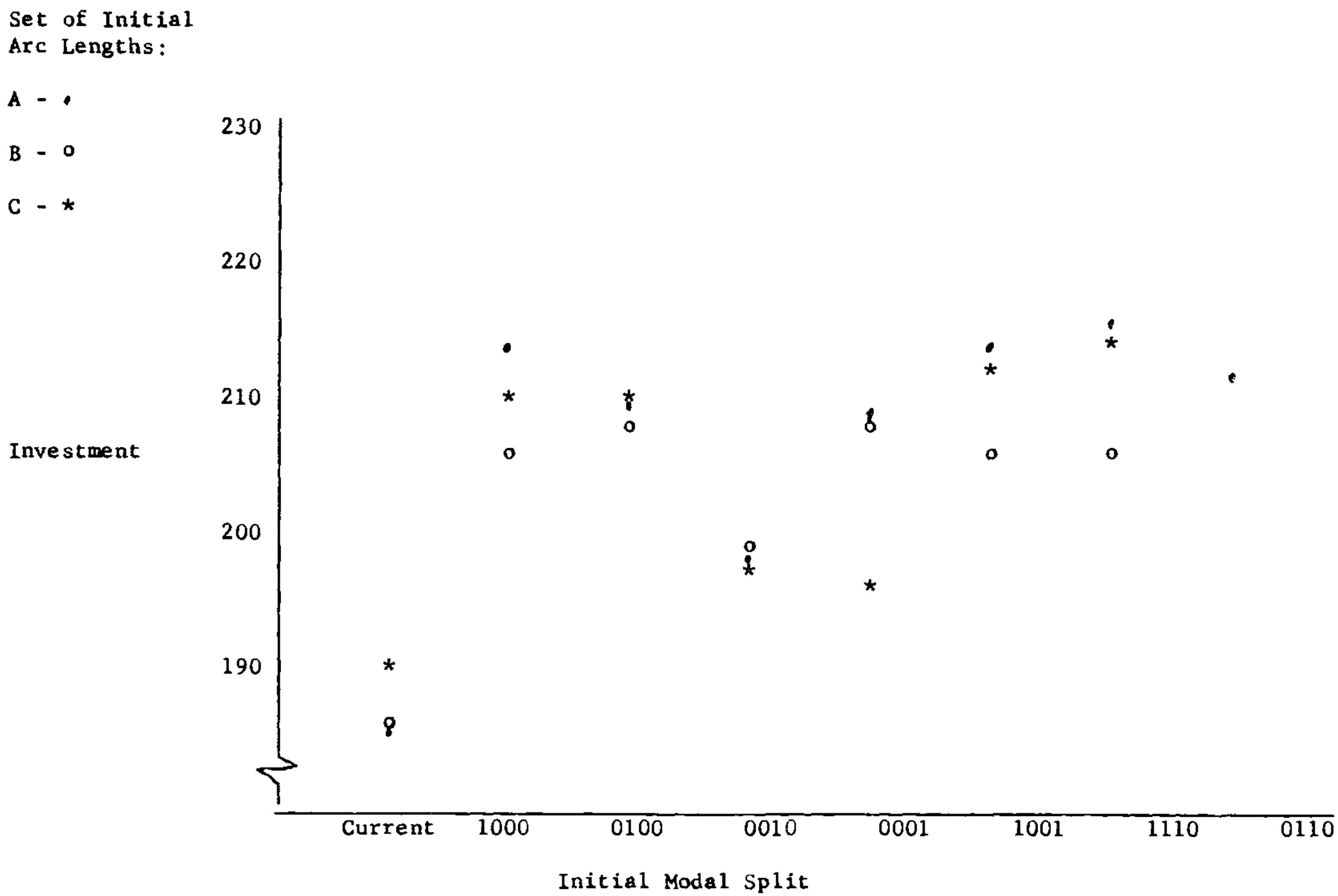


Fig. 4.12. Investment Versus Initial Modal Split

User savings are plotted against these factors in Figure 4-13 below. Since user savings is closely related to total savings, the same observations can be made.

Next, consider the effect of the modal split updating parameter ALPHA. Comparing the characteristics of runs 22-24 with those of runs 25-27, note that for each set of initial arc lengths the methodology converged to the same solution, regardless of the value of ALPHA. Next, consider the effect of the modal split convergence parameter EPSILON. Comparing the characteristics of runs 28 and 11 to those of runs 29 and 30 respectively, it should be noted that the methodology converged to the same solutions. Finally consider the effect of the use of a Phase I procedure in the two-phase algorithm. The results are shown in Table 4-5 below.

Table 4-5. Solution Characteristics and Use of a Phase I Procedure

Run	Set of Initial Arc Lengths	Phase I Used	Total Savings	Investment	User Savings
1	A	No	1250	197	1446
4	A	Yes	1260	187	1447
2	B	No	1199	219	1418
5	B	Yes	1255	187	1442
3	C	No	1190	220	1410
6	C	Yes	1170	191	1361

For the sets of initial arc lengths A and B, use of the Phase I procedure resulted in an improved objective, lower investment, and higher user

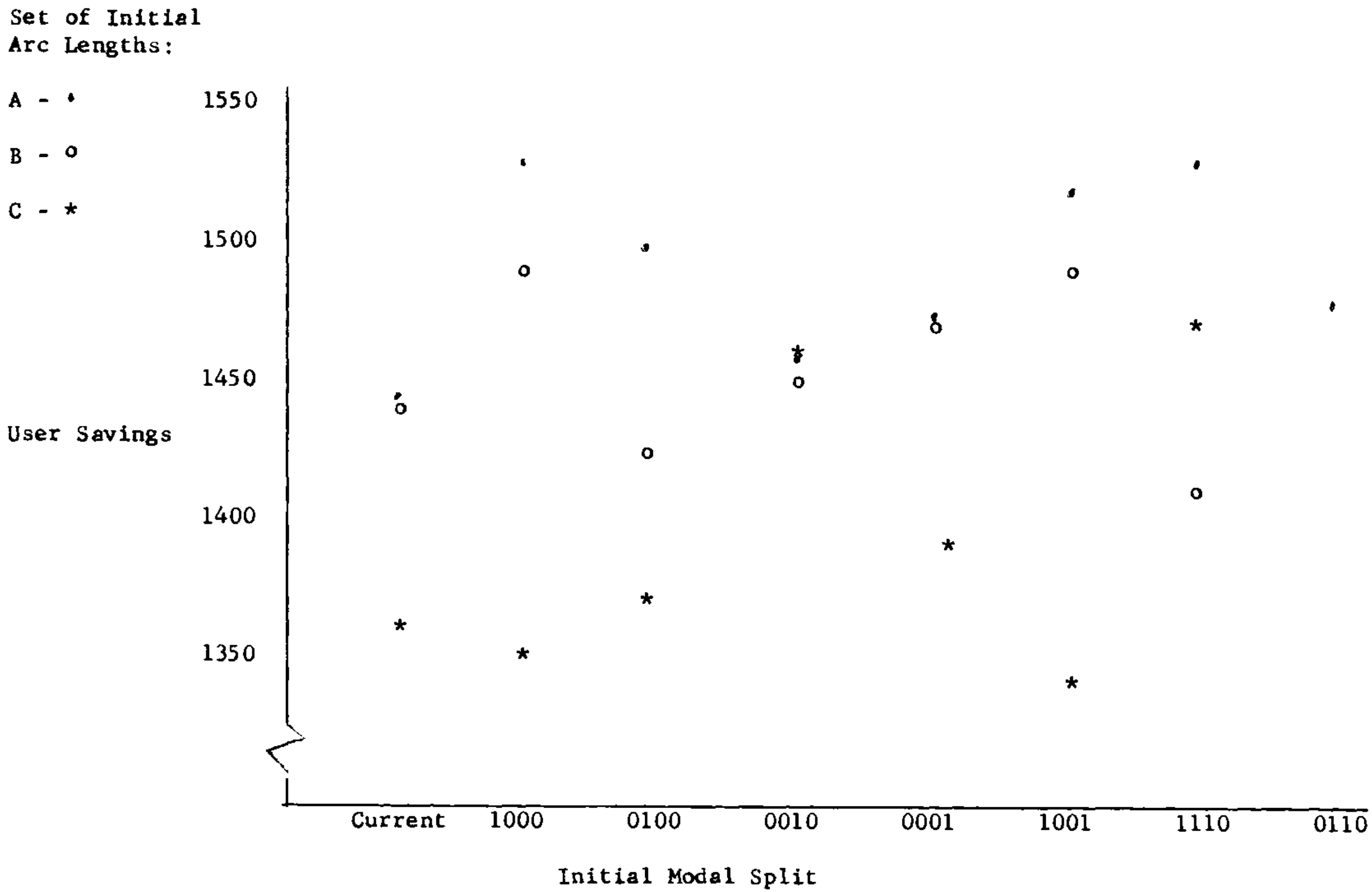


Fig. 4.13. User Savings Versus Initial Modal Split

savings. For the set C, use of Phase I resulted in an increase in the objective, a decrease in investment, and a decrease in user savings. Thus, no specific conclusions can be drawn regarding this factor.

2.4 Detailed Characteristics of Solutions

In the previous sections the solutions were analyzed with respect to some aggregate characteristics. Although the solutions appeared to differ with respect to these characteristics, there was still some question as to whether the differences in solutions represented true investment and flow pattern differences. The purpose of this section is to examine this question in greater detail. The solutions of 3 test runs were analyzed:

(1) Run 7 - the best solution

(2) Run 21 - the worst solution

(3) Run 6 - a low investment solution

First, consider investment over minimum for each solution. Investment was 215 for run 7, 212 for run 21, and 191 for run 6. This may be separated by mode as shown in Table 4-6 below.

Table 4-6. Investment by Mode

	Highway	Railroad	Water	Intermodal Transfer	Total
Run 7	120	95	.08	0	215
Run 21	116	96	.08	0	212
Run 6	94	96	.08	0	191

Several points should be noted:

(1) Investment in water and intermodal transfer facilities was negligible for all runs.

(2) Investment in railroad facilities remained virtually constant.

(3) Investment in highways decreased from run 7 to 21 and from run 21 to 6.

Examining highway and railway facility investment in greater detail, investment can be further subdivided by type of facility as shown in Table 4-7 below. Several points should be noted:

(1) The loss in highway investment across runs occurred in line-haul facilities.

(2) A slight gain in rail investment occurred.

Thus, the differences between solutions noted in the previous section appear to be real and to be concentrated primarily on highway line-haul facilities. To examine these differences in still greater detail, the first 50 highway line-haul arcs in the arc list were selected for further examination. All 50 arcs were corridor arcs and thus subject to improvement. Investment and flow on these arcs for the 3 runs are shown in Table 4-8 below. Although this is but a small sample of the over 700 corridor arcs, it should be clear that investment and flow patterns did differ from solution to solution.

2.5 Summary of Results and Conclusions

The battery of programs developed to implement the methodology was used to solve the problem described in Section 1.2. Solutions were obtained for a variety of different methodological configurations, initial solutions and parameters. Specific factors which were varied included:

(1) The set of initial modal splits MS^0.

Table 4-7. Investment by Mode and Type of Facility

	Highway				Rail			
	Loading	Line-Haul	Transfer	Unloading	Loading	Line-Haul	Transfer	Unloading
Run 7	.3	119	0	.4	.3	93	.8	.4
Run 21	.3	115	0	.4	.3	94	.8	.4
Run 6	.3	94	0	.4	.3	95	.8	.4

Table 4-8. Investment and Flow on HW Line-Haul Arcs

		Investment			Flow (÷ 1000)		
Origin	Dest.	Run 7	Run 21	Run 6	Run 7	Run 21	Run 6
1	2	0	0	0	0	0	0
1	4	0	0	0	0	0	0
1	5	0	0	0	0	0	0
2	1	2.8	2.8	.3	3,346	3,346	633
2	4	0	0	3.7	0	0	13,764
2	6	0	0	0	0	0	0
3	4	0	0	0	0	0	0
3	5	0	0	0	0	0	0
3	8	0	0	0	0	0	0
4	1	0	0	0	0	0	0
4	2	0	0	0	0	0	0
4	3	0	0	5.0	0	0	7,185
4	5	0	0	0	0	0	0
4	6	0	0	0	0	0	0
4	8	0	0	5.1	0	0	6,579
4	9	0	0	0	0	0	0
5	1	0	0	0	0	0	0
5	3	0	0	0	0	0	0
5	4	0	0	0	0	0	0
5	7	0	0	0	0	0	0
5	8	0	0	0	0	0	0
6	2	3.3	3.3	3.3	4,316	4,342	4,342
6	4	0	0	0	0	0	0
6	8	4.2	0	0	11,092	0	0
6	9	4.3	4.4	0	16,530	27,891	0
7	5	0	0	0	0	0	0
7	8	0	0	0	0	0	0
7	11	0	0	0	0	0	0
8	3	0	0	0	0	0	0
8	4	0	0	0	0	0	0
8	5	0	0	0	0	0	0
8	6	0	0	0	0	0	0
8	7	2.7	0	2.6	11,092	0	6,579
8	9	0	0	0	0	0	0
8	11	0	0	0	0	0	0
9	4	0	0	0	0	0	0
9	6	3.5	3.5	3.5	2,461	2,474	2,474
9	8	0	0	0	0	0	0
9	11	3.7	3.8	0	14,558	25,912	0
9	13	0	0	0	0	0	0
9	15	3.6	3.6	3.3	4,611	4,623	2,643
10	11	0	0	0	0	0	0

(continued)

Table 4-8 (cont'd)

		Investment			Flow (÷ 1000)		
Origin	Dest.	Run 7	Run 21	Run 6	Run 7	Run 21	Run 6
10	12	0	0	0	0	0	0
10	13	0	0	0	0	0	0
11	7	0	1.8	1.8	0	838	838
11	8	0	0	0	0	0	0
11	9	0	0	0	0	0	0
11	10	1.9	1.5	1.5	2,323	1,492	1,492
11	13	4.1	0	1.4	15,387	0	838
11	15	0	0	0	0	0	0

(2) The modal split updating parameter ALPHA.

(3) The modal split convergence parameter EPSILON.

(4) The use of a Phase I procedure in the two-phase algorithm.

(5) The set of initial arc lengths used in the first macro-iteration.

The methodology converged for all runs which were attempted. Convergence occurred after two to three macro-iterations or 9 to 20 micro-iterations. The corresponding solution times ranged from 632 to 1213 seconds. The total savings ranged from 1140 to 1318, with investment ranging from 187 to 220. Although the best solutions occurred at high levels of investment, a number of poorer solutions did also. In general, there was little relationship between total savings and investment. In addition, there was little relationship between the rate of convergence and total savings, investment, or user savings.

Regarding the factors which were varied, it was demonstrated that the set of initial modal splits could affect the rate of convergence, the total savings, investment, and user savings. It was also demonstrated that the set of initial arc lengths could affect the rate of convergence, the total savings, and user savings. Regarding the modal split updating parameter ALPHA, it was demonstrated that convergence deteriorated as ALPHA was varied from 1. . As for the modal split convergence parameter EPSILON, it was demonstrated that convergence deteriorated as EPSILON decreased. Holding all other factors constant, modifying ALPHA and EPSILON did not change the final solution. Finally, it was demonstrated that a Phase I procedure could be utilized in the two-phase algorithm to improve the solution objective. However, this improvement will be accompanied by a substantial increase in computation time.

A number of conclusions might be drawn from the results presented in this chapter. First, the methodology developed appears to be a viable method of generating solutions for the multi-modal network improvement problem. Although solution times are long, this is certainly not unusual for problems of this size or design-construction projects of this magnitude. Solution times might be shortened considerably by selecting an ALPHA of 1.0 and an EPSILON as large as modal split error tolerances will permit. Solution times might be further shortened by as much as 50% by eliminating the Phase I portion of the two-phase algorithm. However, the latter effort may cause an appreciable deterioration in the value of the objective.

Second, any reasonable search procedure developed for this methodology must consider varying both the set of initial modal splits and the set of initial arc lengths. One such reasonable search procedure might be as follows:

(1) Select a group of initial modal split sets to be investigated. This group might contain the sets of extreme modal splits as defined in Chapter III.

(2) For some initial modal split set in this group, randomly generate a set of initial arc lengths and use the methodology to obtain the solution. Continue generating sets of initial arc lengths and solving until n consecutive solutions are obtained which are not better by X% than the best previous solution. At this point, terminate efforts on this initial modal split set, select a new set from the group, and iterate.

The results of any search procedure can be plotted on a total savings-investment graph similar to that of Figure 4-5 above. After defining the maximum savings envelope over investment, the decision maker might then select the preferred combination of investment and total savings.

Finally, the tendency of solutions generated from many starting points to cluster around certain points on all graphs tends to indicate the presence of local optimal solutions to the multi-modal network improvement problem. This finding would not be inconsistent given the nonconvex nature of this problem.

CHAPTER V

AN EXTENSION OF THE METHODOLOGY TO INCLUDE MULTIPLE COMMODITIES

In Chapter II, Section 1, an assumption was made that all flows on the network belonged to the same transport commodity class. This is a tenuous assumption at best. In real problems there are thousands of transport commodity classes, each with its very own transport characteristics [Bureau of the Census, 1975]. Jones has identified 53 different transport commodity classes for use in the Multi-State Corridor Research Project [Jones, 1977]. Thus, it would be highly desirable to extend the methodology developed in Chapter III to the multi-commodity case. Note that in this chapter multi-commodity refers to multi-transport commodity classes. The purpose of this chapter is to extend the methodology developed in Chapter III to the multi-commodity case. The development will follow directly from that of Chapter III.

1. Modelling Assumptions and the Formulation

1.1 Modelling Assumptions

Consider the set of modelling assumptions made in Chapter II, Section 1. A number of modifications and additions must be made. First, each transport facility represented by an arc can carry flow of any commodity. Second, each such arc has a set of arc transport characteristics (ATC) for each commodity. This implies that the cost, time, and time variability may not be uniform for all commodities using a given arc. A direct result of the second assumption is that, while the function relating an arc's ATC and investment retains its general form, the parameters of the function become commodity specific. Third, although the general form of the modal split model remains the same, the parameters of the model become commodity specific. Finally, consider the first part of the eighth assumption given in Chapter II, Section 1. It states that for any level of investment on an arc, only one set of physical improvements will be considered. Justification for this assumption came from the fact that for each level of arc investment, preliminary screening had identified the best physical improvement. Now, consider the multi-commodity case. For line-haul arcs the assumption may still be reasonable. However, for loading, unloading, and intermodal transfer arcs, the physical improvement may not only depend upon level of investment, but also upon the specific commodity using the facility. The reason is that equipment used in this type facility is commodity dependent. For example, if one were to spend $100,000 to improve the loading of lumber, he would probably not spend it in the same way were he to improve the loading of grain. One possible method of dealing with this problem is to expand the number of arcs, replacing each loading, unloading, or intermodal transfer arc with an arc for each commodity. Of course, flow of commodity x would not be permitted on an arc corresponding to commodity y. However, for the purposes of the following discussion, assume that the initial assumption is still valid.

1.2 The Formulation

The formulation of the multi-commodity problem can be stated as:

Problem MP:
$$\text{Min } Z = \sum_{r\varepsilon O} \sum_{c\varepsilon C} \sum_{d\varepsilon D_r^c} \sum_{p\varepsilon P_{rd}} y_{rd}^{pc} \sum_{j\varepsilon A_{rd}^p} \left[c_j^c(I_j) + \frac{a_2^c}{a_1^c} t_j^c(I_j) + \frac{a_3^c}{a_1^c} v_j^c(I_j) \right] + \sum_{j\varepsilon A} I_j \qquad (5\text{-}1)$$

s.t.

$$y_{rd}^{pc} = \frac{D_{rd}^c \ \text{Exp} \left\{ \sum_{j\varepsilon A_{rd}^p} [a_1^c c_j^c(I_j) + a_2^c t_j^c(I_j) + a_3^c v_j^c(I_j)] \right\}}{\sum_{q\varepsilon B_{rd}^c(I)} \text{Exp} \left\{ \sum_{j\varepsilon A_{rd}^q} [a_1^c c_j^c(I_j) + a_2^c t_j^c(I_j) + a_3^c v_j^c(I_j)] \right\}}$$

$$\forall\ r \varepsilon O,\ c \varepsilon C,\ d \varepsilon D_r^c,\ p \varepsilon B_{rd}^c(I) \qquad (5\text{-}2)$$

$$y_{rd}^{pc} = 0 \qquad \forall\ r \varepsilon O,\ c \varepsilon C,\ d \varepsilon D_r^c,\ p \not\varepsilon B_{rd}^c(I) \qquad (5\text{-}3)$$

$$L_j \le I_j \le U_j \qquad \forall\ j \varepsilon A \qquad (5\text{-}4)$$

where:

I_j ≡ investment on arc j

y_{rd}^{pc} ≡ flow of commodity c on path p from node r to d

D_{rd}^c ≡ total demand for commodity c from node r at node d

L_j, U_j ≡ lower and upper bounds for investment on arc j

a_1^c, a_2^c, a_3^c ≡ modal split parameters for commodity c

$c_j^c(I_j)$ ≡ unit transport cost of commodity c on arc j as a function of I_j

$t_j^c(I_j)$ ≡ transport time of commodity c on arc j as a function of I_j

$v_j^c(I_j)$ ≡ transport time variance of commodity c on arc j as a function of I_j

O ≡ set of origin nodes

D_r^c ≡ set of destinations for commodity c associated with origin r

P_{rd} ≡ set of paths connecting r and d

A_{rd}^p ≡ set of arcs comprising p^{th} path connecting r and d

$B_{rd}^c(I)$ ≡ set of maximum utility paths connecting r and d for commodity c given an investment vector I. The set includes one path for each mode including the multi-modal option where it is distinct from a single-mode path.

N ≡ set of network nodes

A ≡ set of network arcs

C ≡ set of commodities

2. The Solution Methodology

2.1 Continuous Optimal Adjustment Extended to Multiple Commodities

The general methodology of continuous optimal adjustment can be easily extended to multiple commodities:

(1) Fix the initial modal split for each commodity between each O-D pair.

(2) Determine the O-D demand by commodity and mode.

(3) Solve the resulting multi-commodity, multi-modal network improvement problem with fixed modal splits.

(4) For each commodity, does the fixed set of modal splits agree with the set of modal splits feasible with respect to (5-2), (5-3) given the solution of (3) above?

  (a) If yes, go to (5) below.

  (b) Otherwise, adjust the assumed modal splits and go to (2) above.

(5) Is the solution satisfactory?

  (a) If yes, terminate.

(b) Otherwise, determine a completely different set of initial modal splits and return to (2) above.

The general multi-commodity methodology is almost identical to that of the single-commodity. The only difference is the need to consider a set of modal splits for each commodity for an O-D pair. Let MMS^0 be the initial set of modal splits and let MMS^F be the feasible set. Let $MP1(MMS^0)$ be the multi-commodity, multi-modal network improvement problem with fixed modal splits.

2.2 $MP1(MMS^0)$: The Multi-commodity, Multi-modal Network Improvement Problem with Fixed Modal Splits

After adding nodes and arcs exactly as was done for Problem $P1(MS^0)$, $MP1(MMS^0)$ can be stated in node-arc terms as:

Problem $MP1(MMS^0)$:

$$\min_{I_j, f_j^c, f_j^{rc}} \sum_{j \varepsilon A} \sum_{c \varepsilon C} f_j^c \left[c_j^c(I_j) + \frac{a_2^c}{a_1^c} t_j^c(I_j) + \frac{a_3^c}{a_1^c} v_j^c(I_j) \right] + \sum_{j \varepsilon A} I_j \qquad (5\text{-}5)$$

s.t.

$$\sum_{j \varepsilon W_i} f_j^{rc} - \sum_{j \varepsilon V_i} f_j^{rc} = h_i^{rc} \qquad \forall\ i \ \varepsilon\ N,\ r\ \varepsilon\ O,\ c\ \varepsilon\ C \qquad (5\text{-}6)$$

$$f_j^c = \sum_{r \varepsilon O} f_j^{rc} \qquad \forall\ j\ \varepsilon\ A,\ c\ \varepsilon\ C \qquad (5\text{-}7)$$

$$f_j^{rc} = 0 \qquad \forall\ j\ \varepsilon\ ITA,\ r\ \varepsilon\ SMO,\ c\ \varepsilon\ C \qquad (5\text{-}8)$$

$$f_j^{rc} \geq 0 \qquad \forall\ j\ \varepsilon\ A,\ r\ \varepsilon\ O,\ c\ \varepsilon\ C \qquad (5\text{-}9)$$

$$L_j \leq I_j \leq U_j \qquad \forall\ j\ \varepsilon\ A \tag{5-10}$$

For each commodity, O-D flows must occur only on maximum utility modal paths. If there is more than one maximum utility path for any commodity, O-D pair, and mode, it is assumed that all flow occurs on only one path. (5-11)

where:

$f_j^c \equiv$ total flow of commodity c on arc j

$f_j^{rc} \equiv$ total flow of commodity c from node r on arc j

$$h_i^{rc} = \begin{cases} -x_{ri}^c & \text{if } i\ \varepsilon\ D_r^c \\ \sum_{j \varepsilon D_i^c} x_{rj}^c & \text{if } i = r \\ 0 & \text{otherwise} \end{cases}$$

$x_{ij}^c \equiv$ flow of commodity c from origin i to destination j

$W_i \equiv$ set of arcs originating at node i

$V_i \equiv$ set of arcs terminating at node i

SMO $\equiv$ set of single-mode origins

ITA $\equiv$ set of intermodal transfer arcs

As was the case with problem P1(MS^0), constraint set (5-11) is redundant and can be eliminated. The proof follows directly from the previous proof.

Proceeding as before, assume that the ATC for any commodity c and arc j are related to the arc investment I_j by functions of the form:

$$c_j^c(I_j) = b_{1j}^c(I_j - \bar{c}_j^c)^2 + d_{1j}^c \tag{5-12}$$

$$t_j^c(I_j) = b_{2j}^c(I_j - \bar{t}_j^c)^2 + d_{2j}^c \tag{5-13}$$

$$v_j^c(I_j) = b_{3j}^c(I_j - \bar{v}_j^c)^2 + d_{3j}^c \tag{5-14}$$

where:

$b_{ij}^c, \bar{c}_j^c, \bar{t}_j^c, \bar{v}_j^c, d_{ij}^c$ are commodity c specific parameters for arc j

Again using the Steenbrink decomposition procedure to solve problem $MP1(MMS^0)$, a subproblem is solved for each arc:

Problem $MS1_j(MMS^0)$: $$H_j(\bar{f}_j) = \underset{I_j}{\text{Min}} \sum_{c\varepsilon C} (A_j^c f_j^c I_j^2 + B_j^c f_j^c I_j + c_j^c f_j^c) + I_j \tag{5-15}$$

$$\text{s.t.} \quad L_j \le I_j \le U_j \tag{5-16}$$

where:

$$\bar{f}_j = [f_j^c] \tag{5-17}$$

$$A_j^c = b_{1j}^c + \frac{a_2^c}{a_1^c} b_{2j}^c + \frac{a_3^c}{a_1^c} b_{3j}^c \tag{5-18}$$

$$B_j^c = -2 \left(b_{1j}^c \bar{c}_j^c + \frac{a_2^c}{a_1^c} b_{2j}^c \bar{t}_j^c + \frac{a_3^c}{a_1^c} b_{3j}^c \bar{v}_j^c \right) \tag{5-19}$$

$$C_j^c = b_{1j}^c (\bar{c}_j^c)^2 + d_{1j}^c + \frac{a_2^c}{a_1^c} [b_{2j}^c (\bar{t}_j^c)^2 + d_{2j}^c] + \frac{a_3^c}{a_1^c} [b_{3j}^c (\bar{v}_j^c)^2 + d_{3j}^c] \tag{5-20}$$

or, after rearranging terms:

$$\text{Problem } MS1_j(MMS^0): \quad H_j(\bar{f}_j) = \min_{I_j} I_j^2 \sum_{c\epsilon C} A_j^c f_j^c + I_j \left(\sum_{c\epsilon C} B_j^c f_j^c + 1 \right)$$

$$+ \sum_{c\epsilon C} c_j^c f_j^c \tag{5-21}$$

$$\text{s.t.} \quad L_j \leq I_j \leq U_j \tag{5-22}$$

If the vector $\bar{f}_j$ is fixed, i.e., flow is fixed on arc j for all commodities, then the objective (5-21) is the sum of a convex quadratic, a linear term, and a constant. Thus, (5-21) is convex, $MS1_j(MMS^0)$ is a convex program, and the Kuhn-Tucker conditions apply:

<u>Theorem 5.1</u>

The optimal solution to $MS1_j(MMS^0)$ is defined by:

$$I_j^* = \begin{cases} L_j & \text{if } \hat{I}_j \leq L_j \\ \hat{I}_j & \text{if } L_j \leq \hat{I}_j \leq U_j \\ U_j & \text{if } \hat{I}_j \geq U_j \end{cases} \tag{5-23}$$

where:

$$\hat{I}_j = \frac{-\sum_{c\epsilon C} B_j^c f_j^c - 1}{2 \sum_{c\epsilon C} A_j^c f_j^c} \tag{5-24}$$

Proof. The Kuhn-Tucker conditions for $MS1_j(MMS^0)$ are:

$$(1)\quad 2I_j^* \sum_{c\varepsilon C} A_j^c f_j^c + \sum_{c\varepsilon C} B_j^c f_j^c + 1 - v_1 + v_2 = 0$$

$$(2)\quad v_1(L_j - I_j^*) = 0$$

$$(3)\quad v_2(I_j^* - U_j) = 0$$

$$(4)\quad L_j \leq I_j \leq U_j$$

$$(5)\quad v_1,\ v_2 \geq 0$$

Case 1

If $\hat{I}_j \leq L_j$

Let: $I_j^* = L_j$

$$v_1 = 2L_j \sum_{c\varepsilon C} A_j^c f_j^c + \sum_{c\varepsilon C} B_j^c f_j^c + 1$$

$$v_2 = 0$$

Clearly conditions 1, 2, 3, and 4 are satisfied.

Thus, if $v_1 \geq 0$, then the K-T conditions are satisfied.

Note: $\hat{I}_j \leq L_j$

$$\therefore\quad v_1 \geq 2\hat{I}_j \sum_{c\varepsilon C} A_j^c f_j^c + \sum_{c\varepsilon C} B_j^c f_j^c + 1 = 0$$

Thus, the K-T conditions are satisfied for Case 1.

Case 2

$L_j \leq \hat{I}_j \leq U_j$

Let: $I_j^* = \hat{I}_j$

$$v_1 = v_2 = 0$$

Clearly conditions 2, 3, 4, and 5 are satisfied.

Regarding condition 1:

$$2\hat{I}_j \sum_{c\varepsilon C} A_j^c f_j^c + \sum_{c\varepsilon C} B_j^c f_j^c + 1 - v_1 + v_2 = 0$$

Therefore, all K-T conditions are satisfied for Case 2.

Case 3

$\hat{I}_j \geq U_j$

Let: $I_j^* = U_j$

$v_1 = 0$

$v_2 = -2U_j \sum_{c \in C} A_j^c f_j^c - \sum_{c \in C} B_j^c f_j^c + 1$

Conditions 1, 2, 3, and 4 are satisfied. Thus, if $v_2 \geq 0$, then the K-T conditions are satisfied.

Note: $\hat{I}_j \geq U_j$

$\therefore \quad v_2 \geq -2\hat{I}_j \sum_{c \in C} A_j^c f_j^c - \sum_{c \in C} B_j^c f_j^c + 1 = 0$

Thus, the K-T conditions are satisfied for all three cases.

Q.E.D.

Substituting the results of Theorem 5.1 into (5-15):

$$H_j(\bar{f}_j) = \begin{cases} L_j^2 \sum_{c \in C} A_j^c f_j^c + L_j \left(\sum_{c \in C} B_j^c f_j^c + 1 \right) + \sum_{c \in C} c_j^c f_j^c & \text{if } \hat{I}_j \leq L_j \\ \dfrac{-\left(\sum_{c \in C} B_j^c f_j^c + 1 \right)^2}{4 \sum_{c \in C} A_j^c f_j^c} + \sum_{c \in C} c_j^c f_j^c & \text{if } L_j \leq \hat{I}_j \leq U_j \\ U_j^2 \sum_{c \in C} A_j^c f_j^c + U_j \left(\sum_{c \in C} B_j^c f_j^c + 1 \right) + \sum_{c \in C} c_j^c f_j^c & \text{if } \hat{I}_j \geq U_j \end{cases} \qquad (5\text{-}25)$$

As for the properties of $H_j(\bar{f}_j)$:

Theorem 5.2

$H_j(\bar{f}_j)$ is both continuous and differentiable for feasible $\bar{f}_j$.

Proof. Over the region where $\hat{I}_j < L_j$ and $\hat{I}_j > U_j$, $H_j(\bar{f}_j)$ is linear, continuous, and differentiable.

Over the region where $L_j < \hat{I}_j < U_j$, $H_j(\bar{f}_j)$ is pseudo-concave, continuous, and differentiable [Bazaraa and Shetty, 1975]. Thus, the only regions in question are those where:

(1) $\hat{I}_j = L_j$

(2) $\hat{I}_j = U_j$

$H_j(\bar{f}_j)$ is trivially continuous over both regions. As for differentiability:

(1) Consider the first region in question: $\hat{I}_j = L_j$. For the linear segment:

$$\frac{\partial H_j(\bar{f}_j)}{\partial f_j^c} = L_j^2 A_j^c + L_j B_j^c + c_j^c$$

For the pseudo-concave segment:

$$\frac{\partial H_j(\bar{f}_j)}{\partial f_j^c} = L_j^2 A_j^c + L_j B_j^c + c_j^c$$

Thus, $H_j(\bar{f}_j)$ is differentiable over this region.

(2) Consider the second region: $\hat{I}_j = U_j$. For the pseudo-concave segment:

$$\frac{\partial H_j(\bar{f}_j)}{\partial f_j^c} = U_j^2 A_j^c + U_j B_j^c + c_j^c$$

For the linear segment:

$$\frac{\partial H_j(\bar{f}_j)}{\partial f_j^c} = U_j^2 A_j^c + U_j B_j^c + c_j^c$$

Thus, $H_j(\bar{f}_j)$ is continuous and differentiable over feasible $(\bar{f}_j)$.

Q.E.D.

Theorem 5.3

$H_j(\bar{f}_j)$ is pseudo-concave for feasible f_j.

Proof. Follows directly from fact that each segment is pseudo-concave and $H_j(f_j)$ is continuous and differentiable.

Q.E.D.

The Steenbrink type master problem can now be stated as:

Problem MP2(MMS^0): $\quad \text{Min} \sum_{j \varepsilon A} H_j(\bar{f}_j)$ (5-26)

s.t.

$$\sum_{j \varepsilon W_i} f_j^{rc} - \sum_{j \varepsilon V_i} f_j^{rc} = h_i^{rc} \qquad \forall\ i \ \varepsilon\ N,\ r\ \varepsilon\ O,\ c\ \varepsilon\ C \qquad (5\text{-}27)$$

$$f_j^c = \sum_{r \varepsilon O} f_j^{rc} \qquad \forall\ j\ \varepsilon\ A,\ c\ \varepsilon\ C \qquad (5\text{-}28)$$

$$f_j^{rc} = 0 \qquad \forall\ j\ \varepsilon\ ITA,\ r\ \varepsilon\ SMO,\ c\ \varepsilon\ C \qquad (5\text{-}29)$$

$$f_j^{rc} \geq 0 \qquad \forall\ j\ \varepsilon\ A,\ r\ \varepsilon\ O,\ c\ \varepsilon\ C \qquad (5\text{-}30)$$

where $H_j(\bar{f}_j)$ is defined in (5-25). Note that problem MP2(MMS^0) is a multi-commodity, uncapacitated transportation assignment problem with pseudo-concave arc disutilities. Several properties of $MP2(MMS^0)$ should be noted:

(1) $MP2(MMS^0)$ is a true multi-commodity problem. Problem $P2(MS^0)$ was multi-commodity in the sense that flow from each origin was considered a separate commodity. However, arc disutility was based only on total arc flow irrespective of where the flow originated. For Problem $MP2(MMS^0)$ several commodities may originate at the same origin. Furthermore, flows of these commodities must maintain their identity in order to calculate arc disutility, since identical flows of different commodities may have different effects on an arc's cumulative disutility.

(2) The sum of the pseudo-concave $H_j(\bar{f}_j)$ is not necessarily pseudo-concave. Thus, there is a question of whether an optimal solution must lie at an extreme point of the solution space. However, it should be noted that since the problem is uncapacitated, an extreme point solution corresponds to one in which each O-D flow of a commodity occurs on a single path. This property has already been established for a local optimal solution of $MP1(MMS^0)$. Thus, since the optimal flow pattern of $MP2(MMS^0)$ must correspond to that of $MP1(MMS^0)$, it can be inferred that an optimal solution to $MP1(MMS^0)$ can always be assumed to lie at an extreme point.

(3) A local optimal solution of $MP2(MMS^0)$, lying at an extreme point, need not be a global optimum. This follows from the fact that $MP2(MMS^0)$ is not a convex program.

A reasonable solution procedure for problem $MP2(MMS^0)$ might be an extension of the Yaged Algorithm. A local optimal solution to the problem has several properties similar to those of Problem $P2(MS^0)$:

Property 1. For any local optimal solution, either:

(a) All flow of any commodity between any O-D pair occurs on a single path, or

(b) An equivalent (in terms of value of the objective) local optimum can be determined satisfying Property 1.a.

Property 2. For any local optimal solution, if flow of any commodity between any O-D pair uses path p, and nodes a and b lie on path p, then the flow of the same commodity between a and b can also be assumed to lie on path p.

Property 3. A solution is a local optimum if and only if for any commodity k the flow-carrying path between each O-D pair is the shortest path between the pair where arc length is defined as the first partial derivative of the arc disutility function with respect to the flow of commodity k on the arc evaluated at this current flow.

The proof of Property 3 is a direct extension of Yaged's proof.

From these properties is developed the multi-commodity extension of the Yaged Algorithm:

Initialization: Determine an initial set of arc lengths for each commodity being considered. Set old arc flows to zero.

Step I. For each commodity construct shortest path trees from each origin using the current set of arc lengths for that commodity. Assign all flow of the commodity to the shortest path between each O-D pair.

Step II. If the newly generated arc flows are equivalent to the old arc flows for each commodity, go to Step IV. Otherwise, go to Step III.

Step III. For each commodity, determine a new set of arc lengths. For some arc j and commodity k, this new length is defined as the first partial derivative of the arc disutility function with respect to the flow of commodity k on the arc, evaluated at this current flow.

Let the new arc flows become the old arc flows for each commodity.

Go to Step I.

Step IV. Is the current solution satisfactory? If not, determine a new set of arc lengths for each commodity and go to Step I. Otherwise, terminate.

Finally, it may again prove useful to use a Phase I type procedure prior to using the Yaged extension. For any arc j the average unit disutility of shipping commodity k is defined to be:

$$a_{jk} = U_{jk} + \frac{I_j - L_j}{f_j}$$

where:

a_{jk} ≡ average unit disutility of shipping commodity k on arc j

U_{jk} ≡ unit user disutility of shipping commodity k on arc j

I_j ≡ investment on arc j

L_j ≡ minimum investment on arc j

f_j ≡ total flow over all commodities on arc j

3. Implementation and Results

3.1 Implementation

The methodology developed in this chapter was implemented on the Georgia Tech CDC Cyber 74 computing system. All programming was done in the FORTRAN programming language. The basic flow of the methodology is shown in the flowchart in Figure 5-1 below. The programs which comprise the methodology are, for the most part, simple extensions of those prepared for the single-commodity methodology. These programs include MMS,

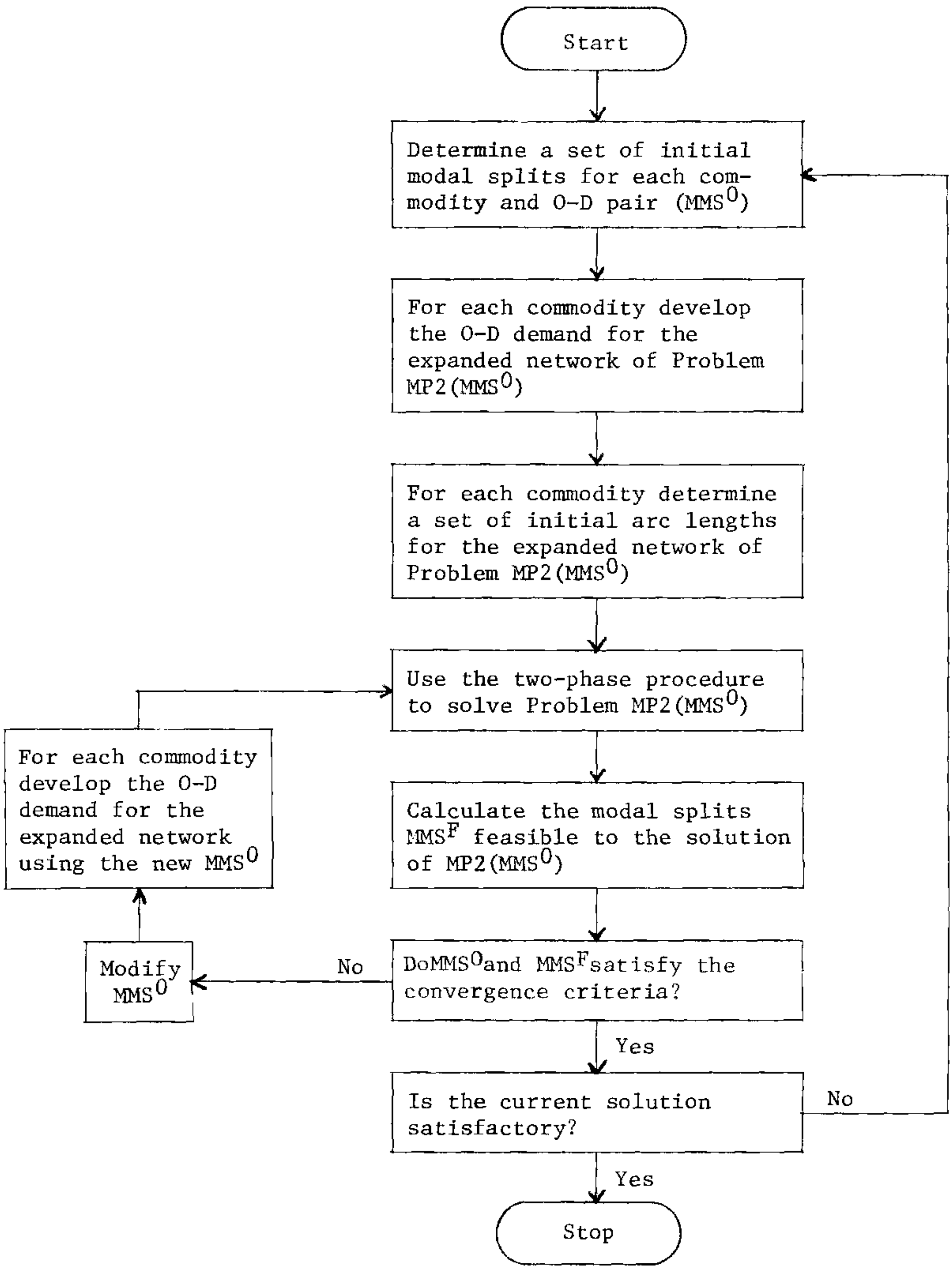
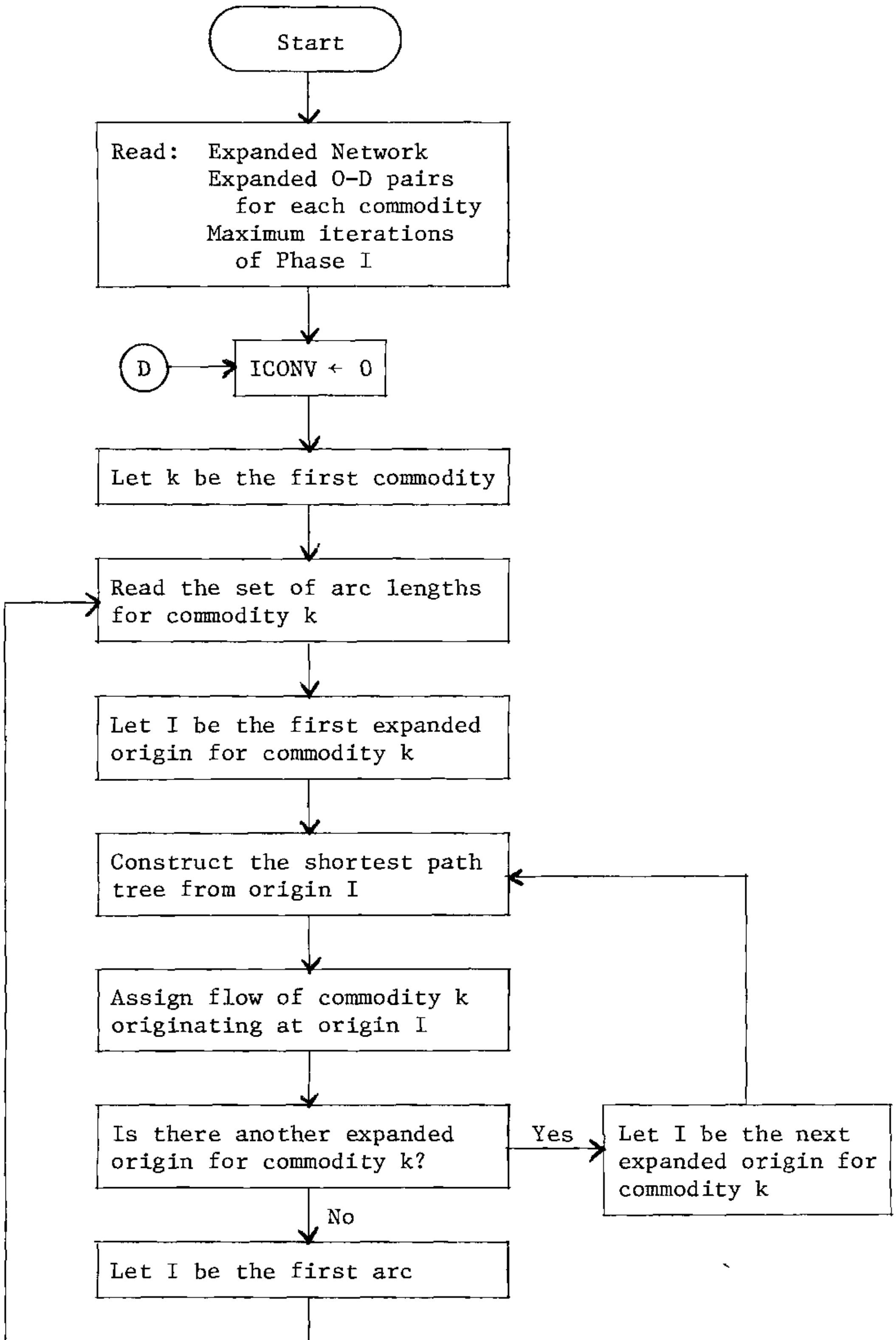


Figure 5-1. Macro-Flowchart for the Multi-Commodity Algorithm

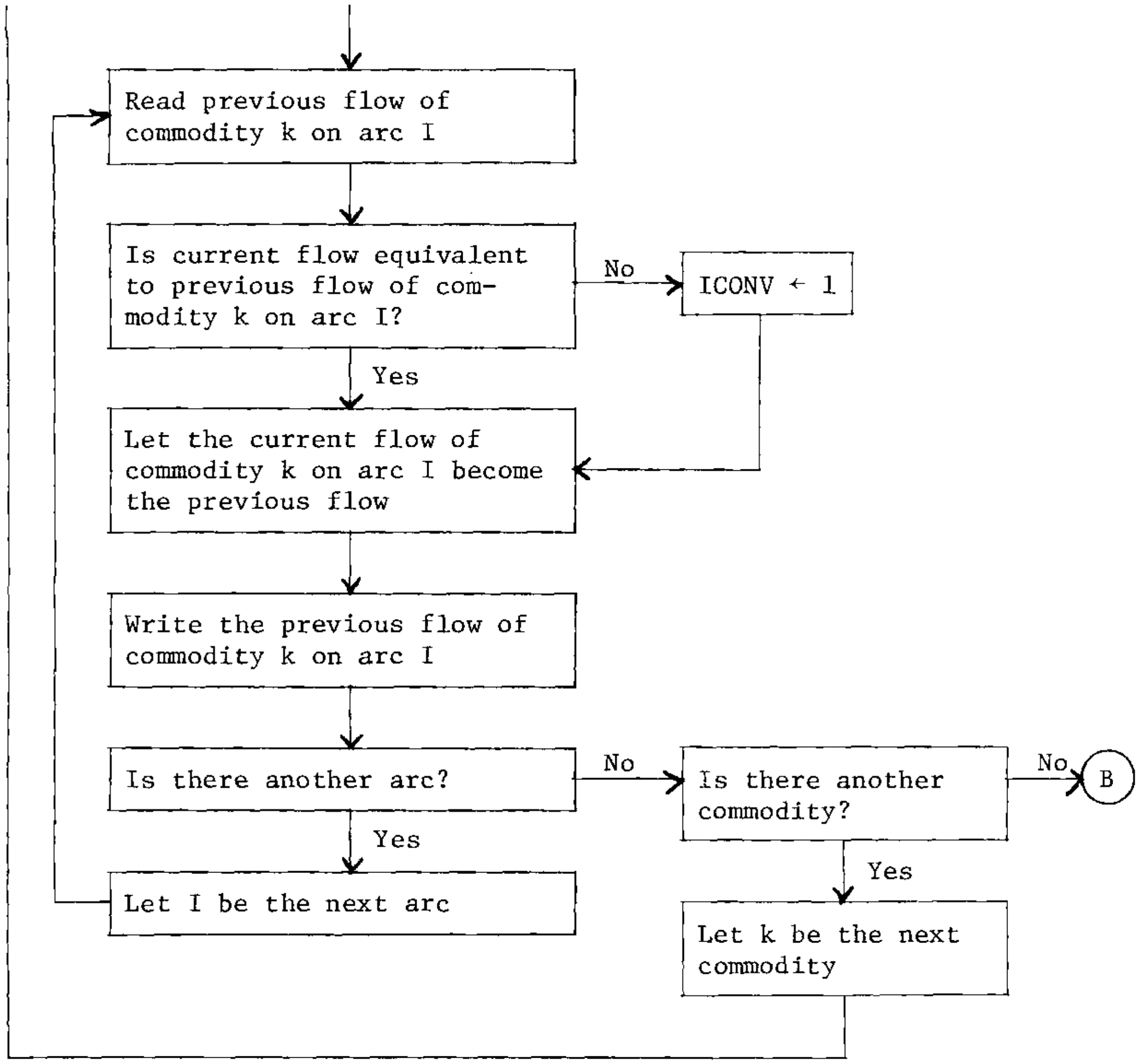
MINMS, MINFOD, MINAC, MCNCASB, and MTESTMS. The programs are listed in Appendix H below.

Program MCNCASB, a two-phase solution procedure for problem MP2(MMS^0), is the only multi-commodity program that differs significantly from its single-commodity counterpart. A basic flowchart of MCNCASB is shown in Figure 5-2 below. The primary difference between the single-commodity and multi-commodity programs is the need to maintain commodity identity throughout the latter. This includes the expanded O-D pairs, arc length, arc average and marginal disutility parameters, and arc flows. This need to maintain commodity identity for arc length and arc flow creates a significant core storage problem on the Cyber 74. In program CNCASNB these variables were maintained in core. However, in MCNCASB, they must be kept in mass (disk) storage. This, in turn, creates a new problem, that of excessive time spent in input-output (I-O) operations.

The network, including both nodes and arcs, used in the single-commodity implementation is also used in the multi-commodity implementation. Arc transport characteristic (ATC) functions are derived in the same manner with the actual commodity specific values of the ATC used as the base values. Time and time variance are assumed the same for all commodities. Note that this will yield a set of ATC functions for each arc and commodity. The O-D flow data set used in the multi-commodity implementation is the same as that used previously. However, the commodities, textile mill products, lumber and wood, and agricultural chemicals, must now retain their identity as well as their origin and destination. Finally, the commodity-specific modal split model


Start
Read: Expanded Network
Expanded O-D pairs
for each commodity
Maximum iterations
of Phase I
D
ICONV ← 0
Let k be the first commodity
Read the set of arc lengths
for commodity k
Let I be the first expanded
origin for commodity k
Construct the shortest path
tree from origin I
Assign flow of commodity k
originating at origin I
Is there another expanded
origin for commodity k?
Yes
Let I be the next
expanded origin for
commodity k
No
Let I be the first arc

(continued)


Read previous flow of commodity k on arc I
Is current flow equivalent to previous flow of commodity k on arc I?
No
ICONV ← 1
Yes
Let the current flow of commodity k on arc I become the previous flow
Write the previous flow of commodity k on arc I
Is there another arc?
No
Is there another commodity?
No
B
Yes
Let I be the next arc
Yes
Let k be the next commodity

(continued)

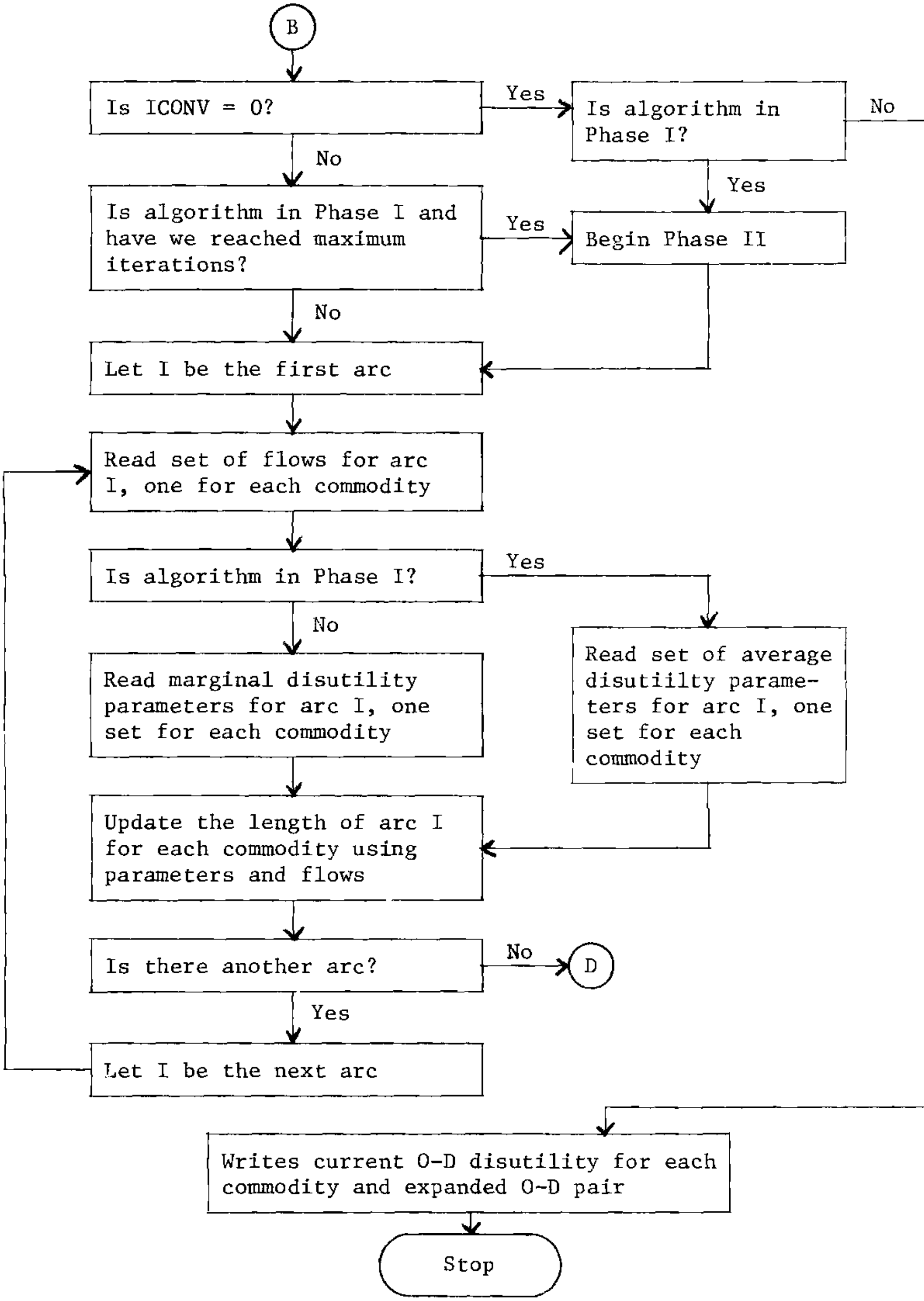


Figure 5-2. Flowchart for MCNCASB

parameters used are those given by Jones: [Jones, 1977]

$$\text{SIC 22:} \quad a_1 = -.0287$$
$$a_2 = -.00012$$
$$a_3 = -.0013$$

$$\text{SIC 24:} \quad a_1 = -.01$$
$$a_2 = -.0000016$$
$$a_3 = -.0003$$

$$\text{SIC 287:} \quad a_1 = -.01$$
$$a_2 = -.00033$$
$$a_3 = -.0004$$

Aggregate statistics relating to the size of the resulting problem are shown in Table 5-1 below.

3.2 Results

The battery of programs described in Section 3.1 above was used to obtain solutions to the problem described in the same section. The test runs were made in the same manner as those of the single-commodity problem. For a review of those test runs see Chapter IV, Section 2.1. Selected results from the test runs are given in Appendix F.

The multi-commodity methodology converged for all test runs. The minimum CPU time required for convergence was 1394 seconds, the maximum 2504 seconds, and the average 2092 seconds. The overall computation time

Table 5-1. Approximate Aggregate Statistics of Problem

Actual transport facilities	2600
Arcs (after network expansion)	3000
Zones	120
Nodes (prior to expansion)	840
Nodes (after expansion)	1000
O-D pairs (prior to expansion)	63
O-D pairs (after expansion)	227
Transport commodity classes	3
Math programming commodity classes* (prior to expansion)	14
Math programming commodity classes* (after expansion)	56

*In the mathematical programming literature a commodity is usually defined by origin and by a set of arc costs (alternatively, by destination and by arc costs). The term commodity as used in this research is defined by arc costs only.

was the result of two factors:

(1) The number of macro-iterations of the methodology.

(2) The CPU time for each macro-iteration.

The minimum number of macro-iterations was two and the maximum three. The minimum CPU time required for a macro-iteration was 238 seconds, the maximum 1008 seconds, and the average 730 seconds. The time required for a macro-iteration was determined by two factors:

(1) The number of micro-iterations constituting the macro-iteration.

(2) The time required for each micro-iteration.

The minimum number of micro-iterations in a macro-iteration was two and the maximum was eight. As the methodology approached convergence, the number of micro-iterations per macro-iteration decreased. The average number of micro-iterations required for the first macro-iteration was 6.5, the average for the second was 5.8, and the average for the third was 4.6.

The average CPU time for a micro-iteration was 126 seconds. Each micro-iteration consisted of a number of components:

(1) For each commodity:

 (a) Reading the current set of arc lengths for the commodity.

 (b) Constructing the set of shortest path trees for the commodity and assigning commodity flow to the network.

 (c) Writing the commodity flow for each arc.

(2) Reading a set of 12 arc disutility parameters for each arc. Reading the latest set of commodity flows for each arc. Calculating the new sets of arc lengths, one set for each commodity. Writing the new sets of arc lengths.

A total of 56 trees were constructed in each micro-iteration.

Now, consider the effects of the factors listed in Table 4-1 on the rate of convergence. Note that since the number of micro-iterations required for convergence is roughly proportional to time, this number can be used as a measure of rate of convergence. Consider the effect of the first factor, the set of initial modal splits MS^0. Micro-iterations are plotted against initial modal splits in Figure 5-3 below. Holding all other factors constant, the methodology converged at approximately the same rate, 3 macro-iterations and 16 to 20 micro-iterations,

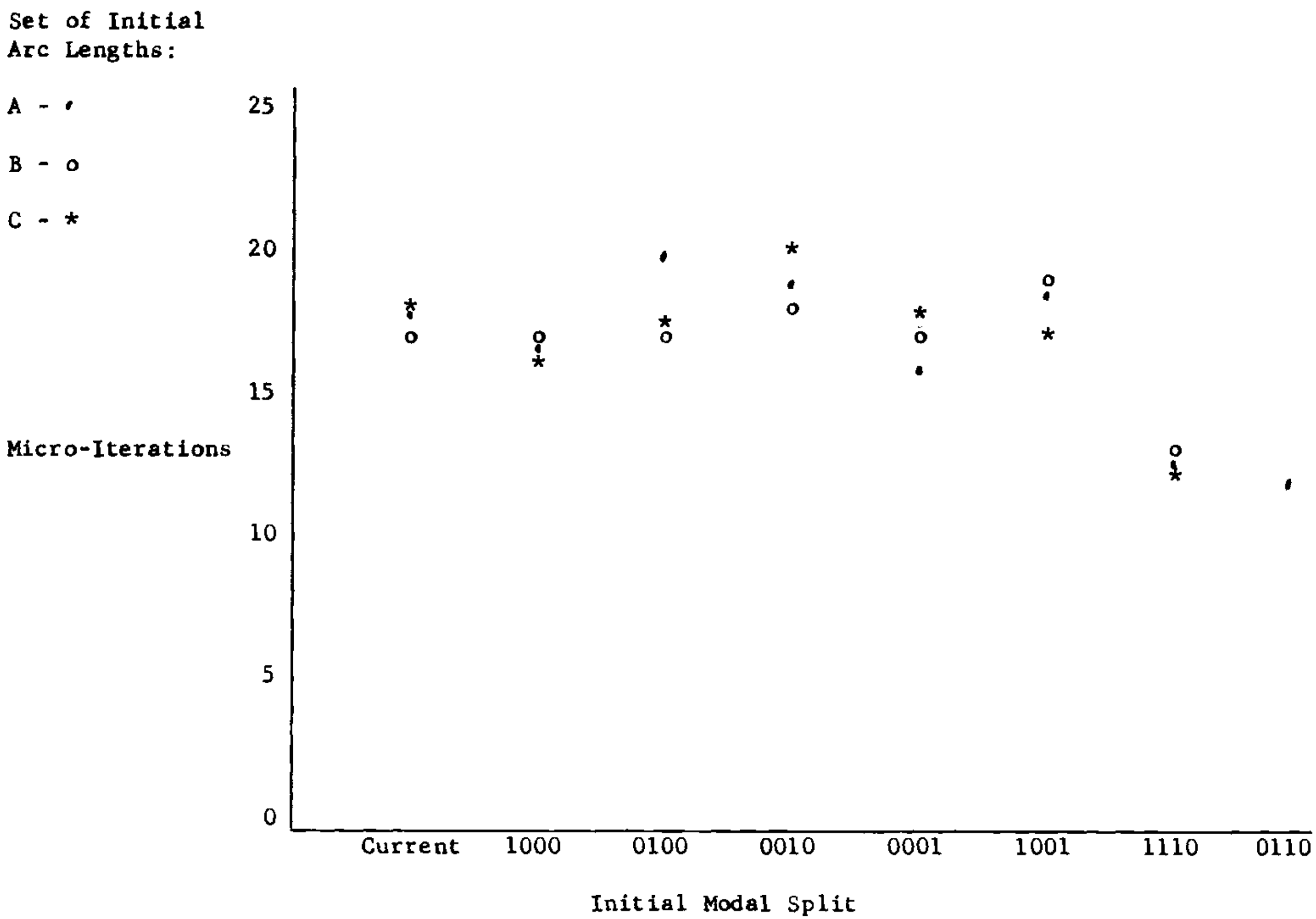


Fig. 5.3. Initial Modal Split Versus Rate of Convergence

for most initial modal splits. Obvious exceptions involved the splits 1110 and 0110 which converged in 2 macro-iterations and 12 to 13 micro-iterations.

Second, consider the effect of the modal split updating parameter ALPHA. Results are shown in Table 5-2 below.

Table 5-2. Effect of ALPHA on Rate of Convergence

Run	Set of Initial Arc Lengths	ALPHA	Macro-Iterations	Micro-Iterations
22	A	1.0	2	13
25	A	.8	3	19
23	B	1.0	2	13
26	B	.8	3	19
24	C	1.0	2	13
27	C	.8	3	19

Note that changing ALPHA from 1.0 to .8 resulted in increases in the number of macro-iterations from two to three and in the number of micro-iterations from 13 to 19. Next, consider the effect of the third factor, the modal split convergence parameter EPSILON. From runs 28 and 29, it should be noted that as EPSILON was decreased from .02 to .01, the number of macro-iterations increased from 2 to 3 and the number of micro-iterations increased from 12 to 18. Runs 11 and 30 demonstrated little difference in convergence as EPSILON was decreased from .02 to .01.

Next, consider the effect of utilizing a Phase I type procedure in the two-phase algorithm. From runs 1 through 6 it should be noted that its use had no effect on the number of macro-iterations required for convergence. All runs required 3 macro-iterations. The number of micro-iterations required for convergence are shown in Table 5-3 below. Several points should be noted:

(1) The use of a Phase I procedure increased the number of micro-iterations by approximately 50%.

(2) Although the use of the Phase I procedure decreased the number of Yaged type Phase II iterations required, the Phase I iterations more than made up for this decrease.

Finally, consider the effect of the initial set of arc lengths. Returning to Figure 5-3 above, it should be noted that this factor did not cause the rate of convergence to change greatly. The maximum change was 3 micro-iterations. Note also that no set of initial arc lengths uniformly hastened or impeded convergence.

Next, the solutions obtained from the test runs are analyzed with respect to three characteristics: total savings over current total disutility associated with the network (the objective), investment over the current minimum level, and user savings over current level. All values are given in terms of millions of equivalent annual dollars. The current values were estimated as:

Total disutility - 9,463

Investment - 269

User disutility - 9,194

The total savings associated with the solutions ranged from a minimum of 481 for run 3 to a maximum of 644 for run 22. Investment associated with solutions ranged from a minimum of 153 for run 14 to a

Table 5-3. Effect of Phase I Procedure on Rate of Convergence (Micro-Iterations)

Macro-Iteration/		I			II			III			Totals		
Run	Initial Set of Arc Lengths	Phase I	Phase II	Total	Phase I	Phase II	Total	Phase I	Phase II	Total	Phase I	Phase II	Total
1	A	0	4	4	0	5	5	0	2	2	0	11	11
4	A	5	2	7	4	2	6	3	2	5	12	6	18
2	B	0	5	5	0	5	5	0	2	2	0	12	12
5	B	5	2	7	4	2	6	3	2	5	12	6	18
3	C	0	5	5	0	6	6	0	2	2	0	13	13
6	C	5	2	7	4	2	6	3	2	5	12	6	18

maximum of 169 for runs 11, 22, 25, and 30. User savings associated with solutions ranged from a minimum of 640 for run 3 to a maximum of 814 for run 22. The objective, total savings, is plotted against investment in Figure 5-4 below. Note that although the best solutions occurred in the upper half of the investment range, a number of poorer solutions did also. In general, there appears to be little relationship between investment and total savings. Investment is plotted against user savings in Figure 5-5 below. Again, solutions having the highest user savings occurred in the upper half of the investment range. However, high investment did not assure a high level of user savings. In general, the relationship between investment and user savings was not strong. Finally, total savings is plotted against user savings in Figure 5-6 below. The strong relationship between these two characteristics results from the fact that investment is relatively small compared to them. Thus, one closely approximates the other.

Total savings is plotted against the total number of micro-iterations in Figure 5-7 below. There appears to be little relationship between the two. Investment and user savings are plotted against the number of micro-iterations in Figures 5-8 and 5-9 respectively. Again, there appears to be little relationship between the characteristics and the rate of convergence.

Now, consider the effects of the factors listed in Table 4-1 on the characteristics' total savings, investment, and user savings. First, consider the effect of the set of initial modal splits MMS^0 and the set of initial arc lengths. Total savings is plotted against these factors in Figure 5-10 below. Several points should be noted:

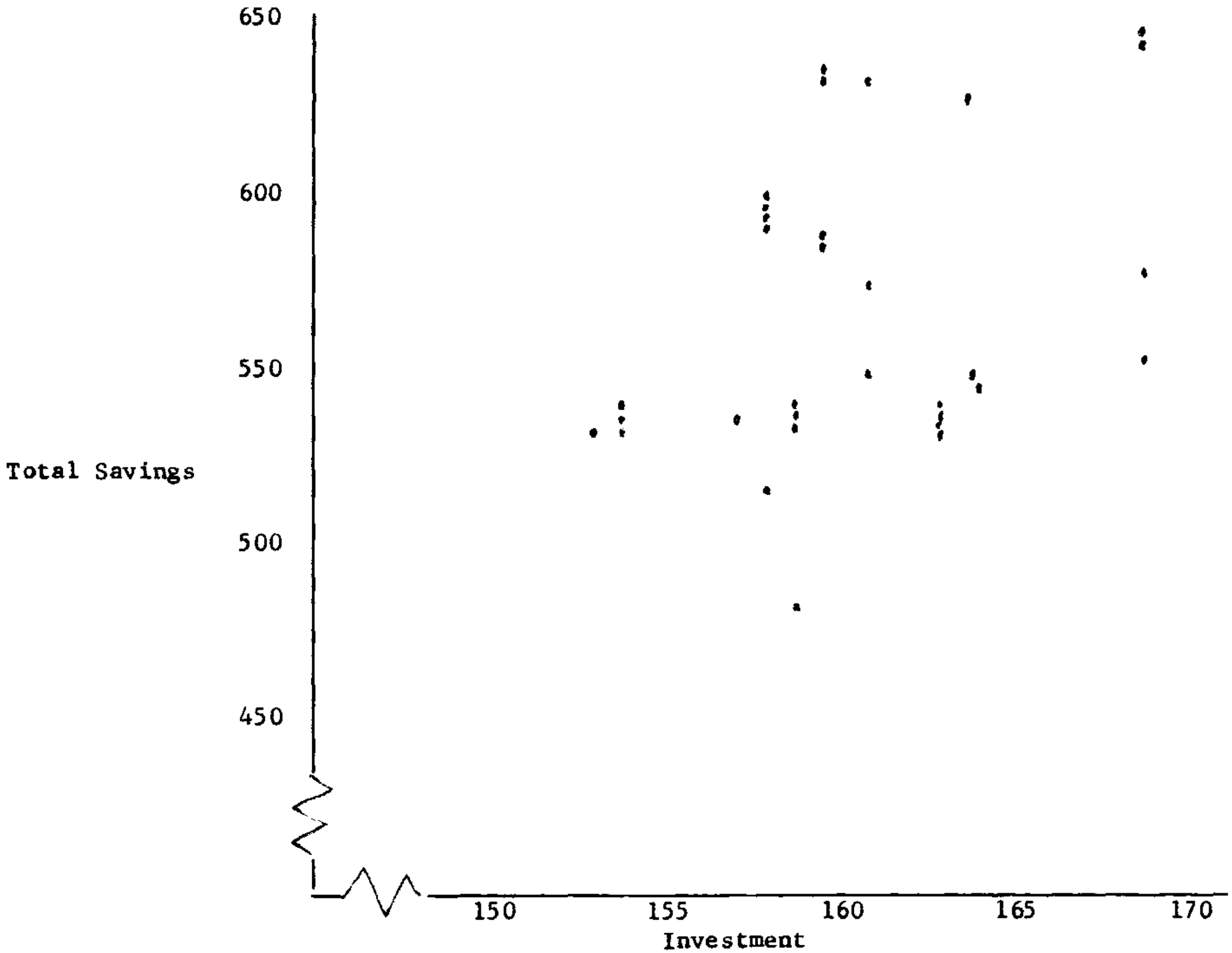


Fig. 5.4. Total Savings Versus Investment, All Runs

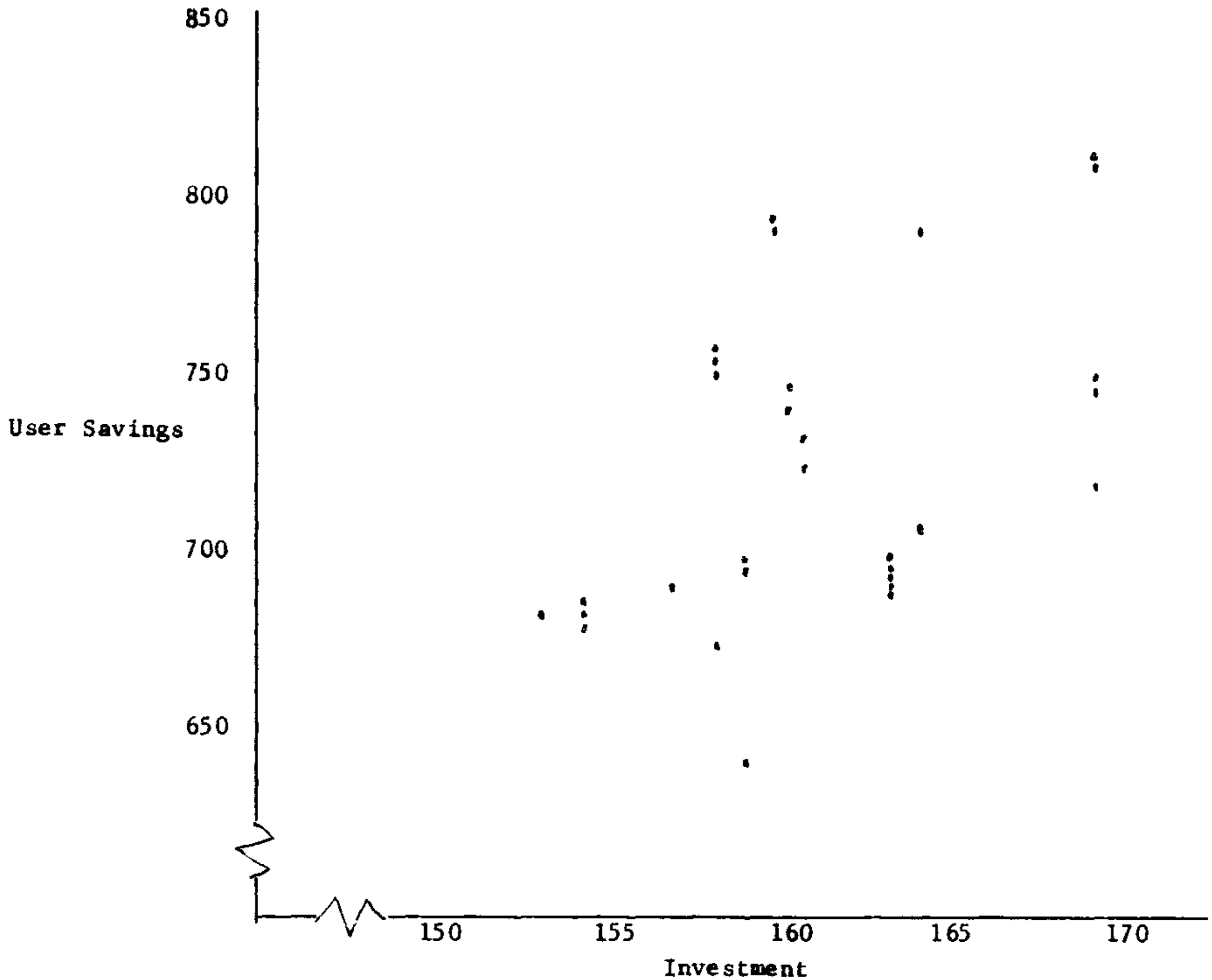


Fig. 5.5. User Savings Versus Investment, All Runs

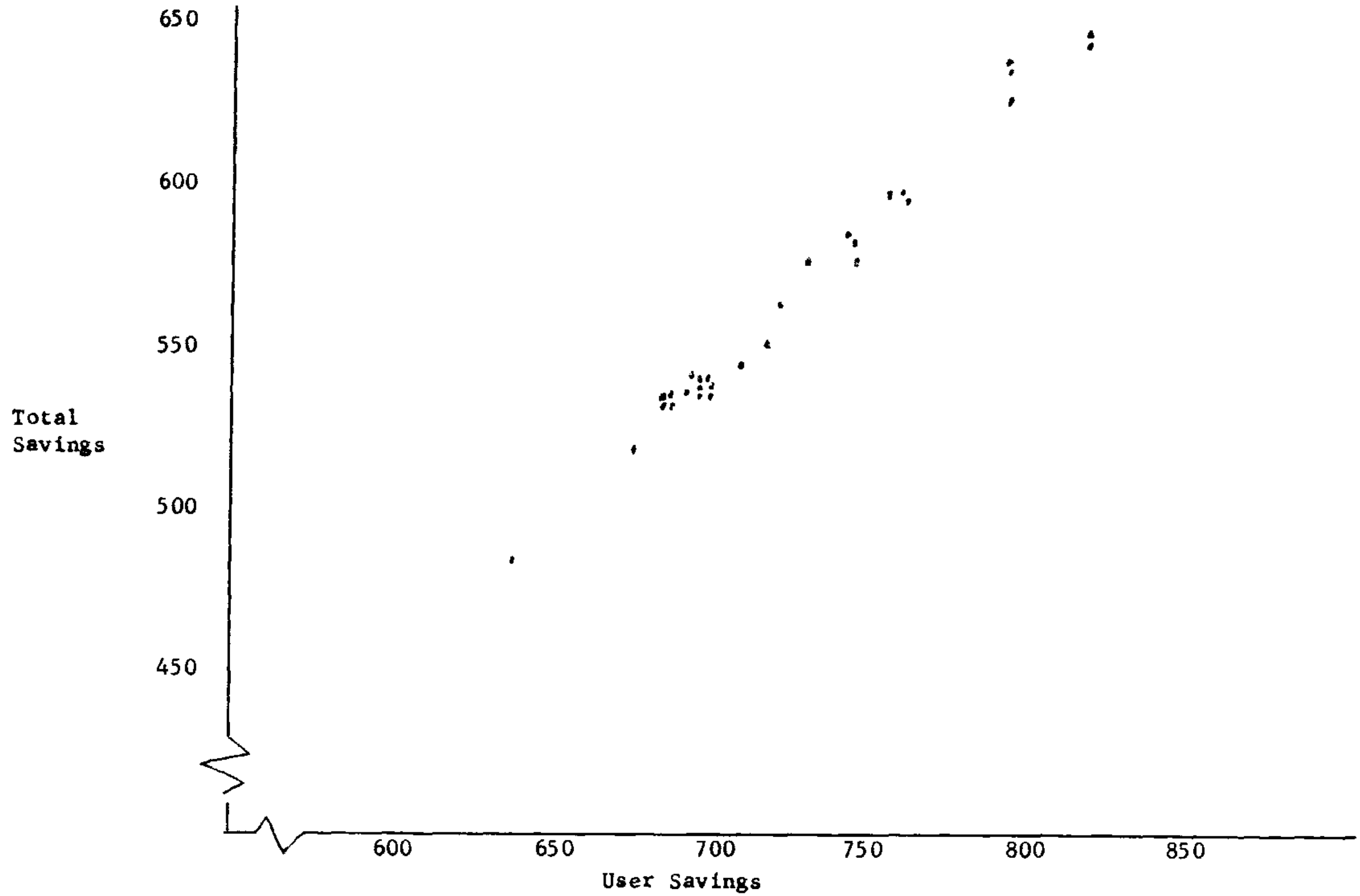


Fig. 5.6. Total Savings Versus User Savings, All Runs

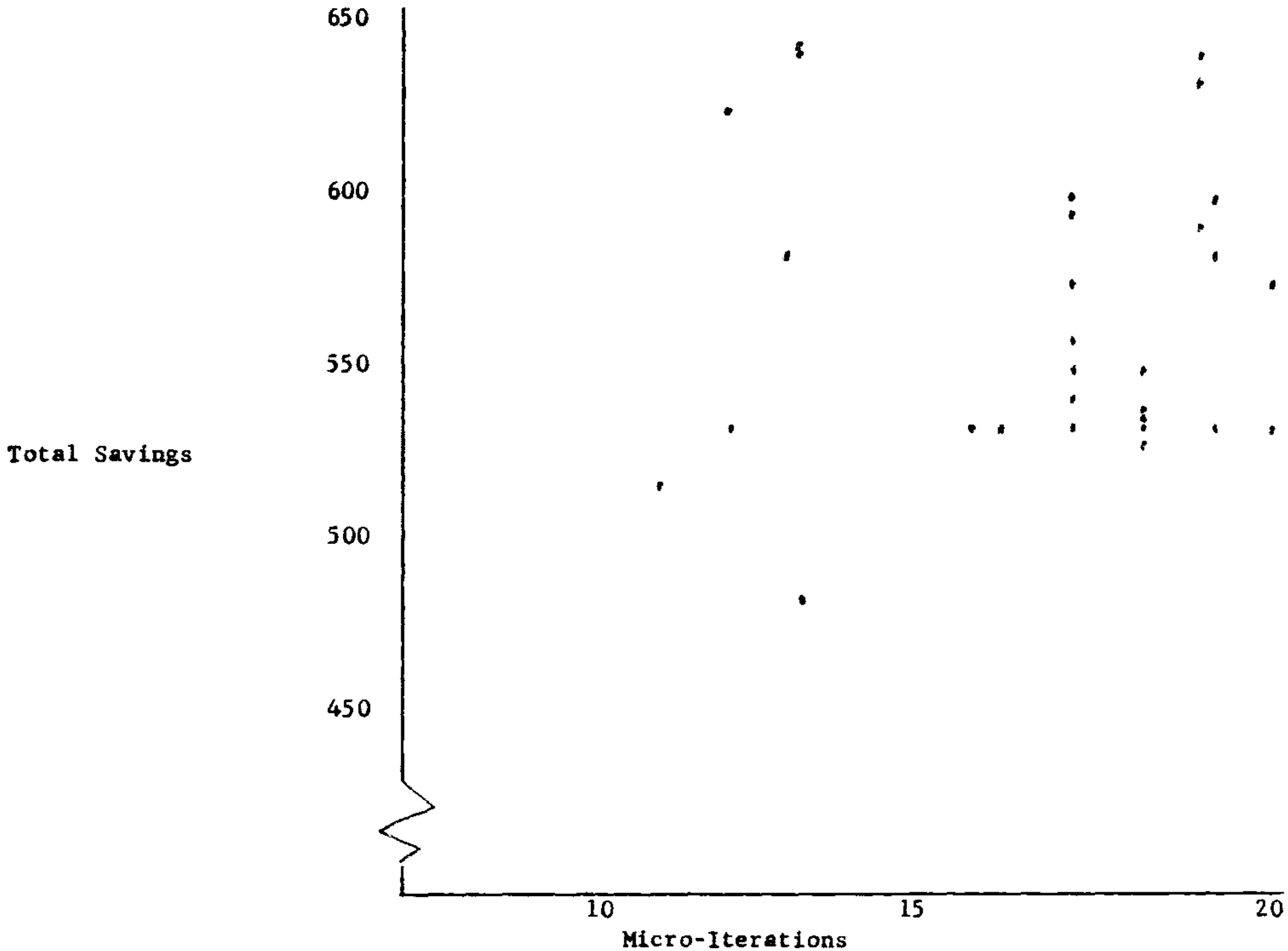


Fig. 5.7. Micro-Iterations Versus Total Savings, All Runs

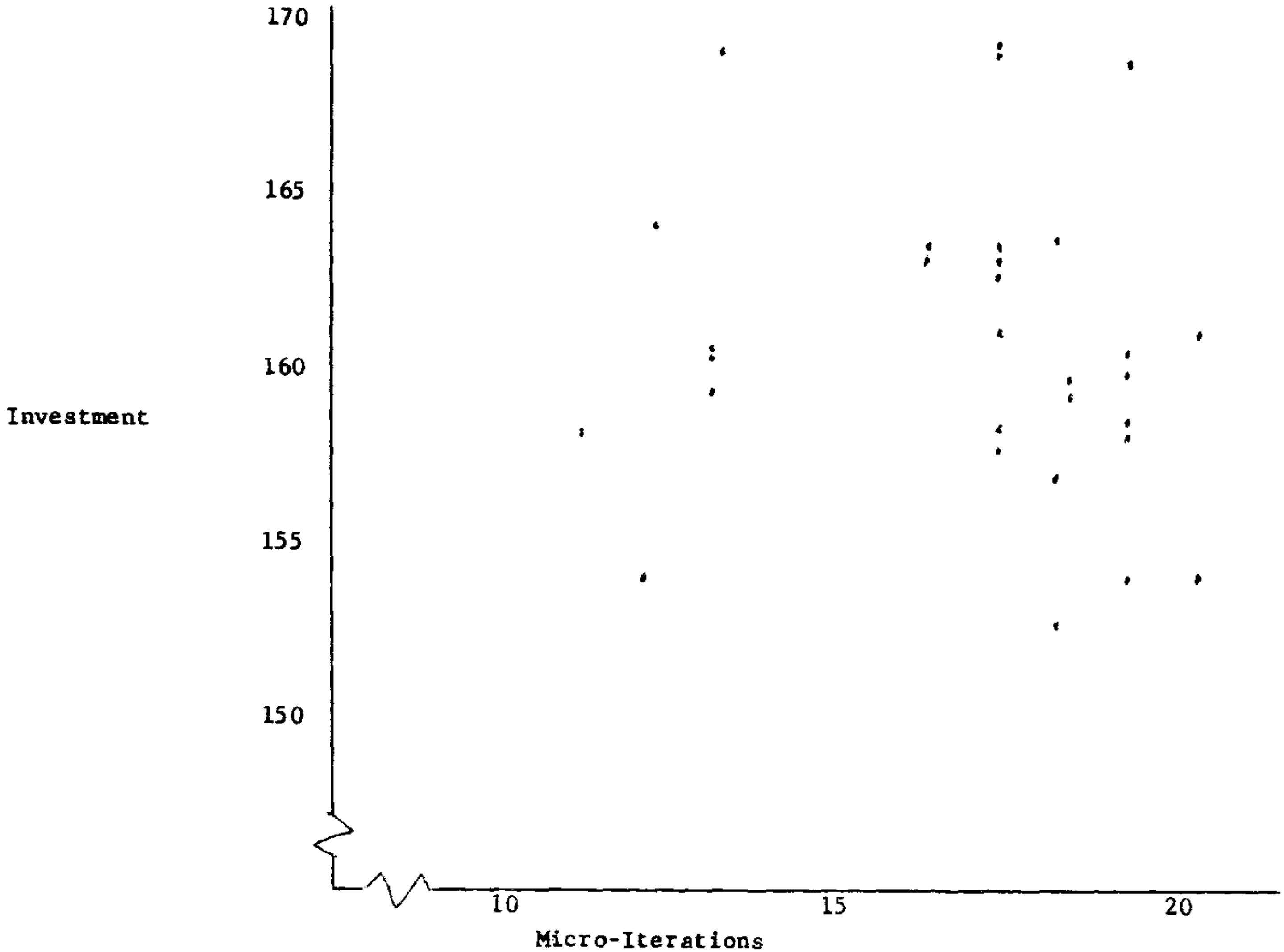


Fig. 5.8. Micro-Iterations Versus Investment

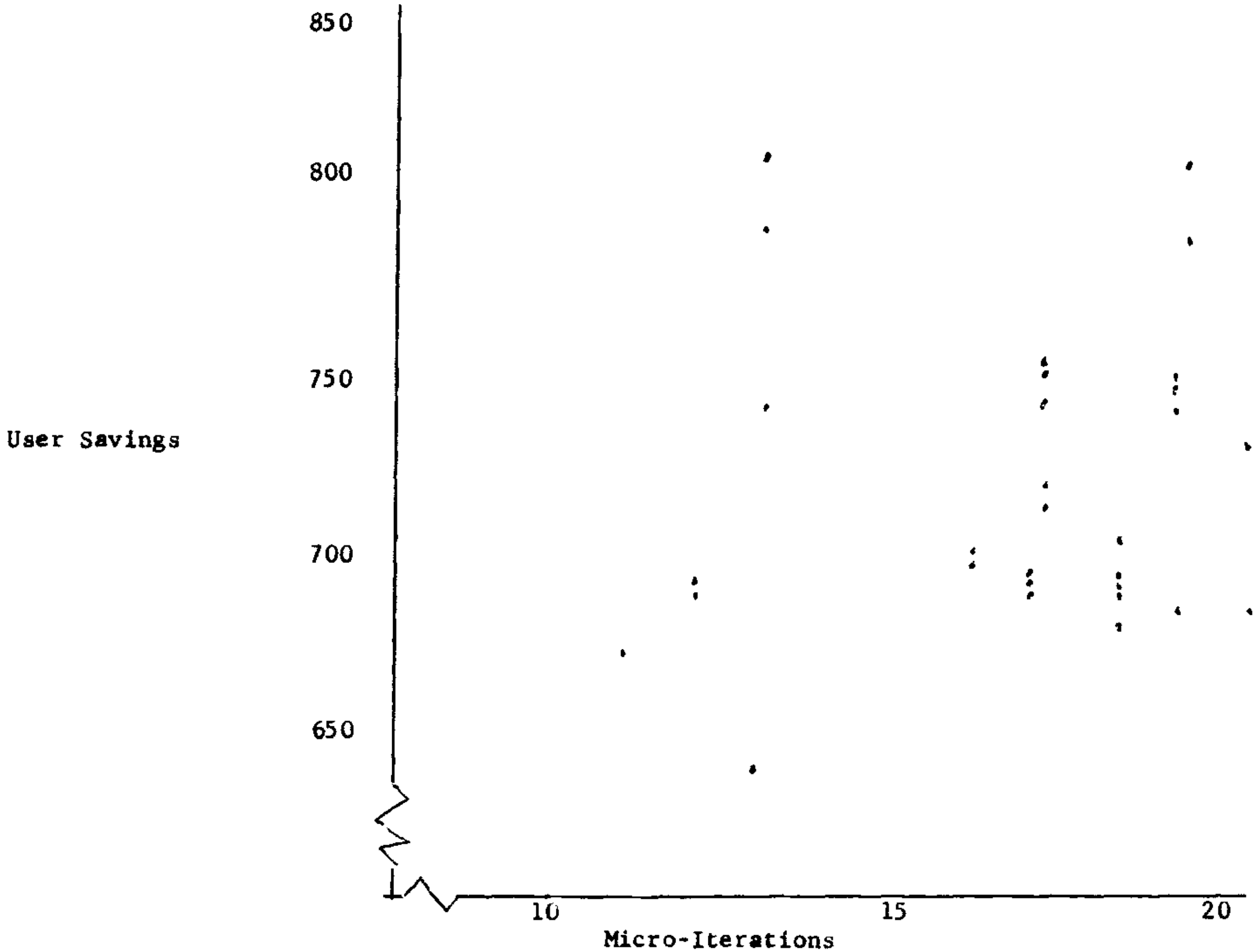


Fig. 5.9. Micro-Iterations Versus User Savings

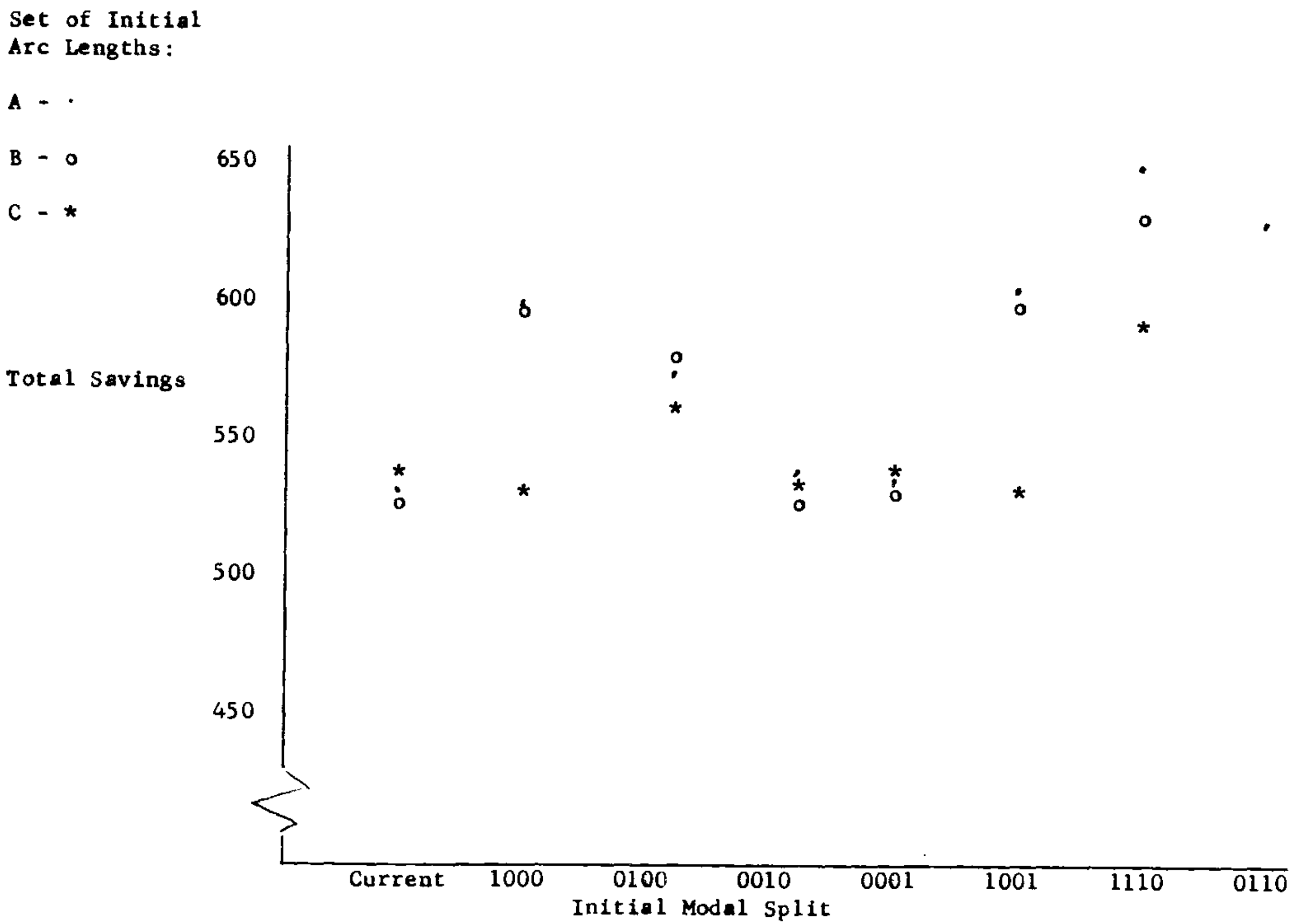


Fig. 5.10. Total Savings Versus Initial Modal Split

(1) The set of initial modal splits apparently had a strong effect on total savings for each set of initial arc lengths.

(2) No set of initial modal splits was superior or inferior for all sets of initial arc lengths.

(3) Several sets of modal splits resulted in approximately the same total savings for each set of initial arc lengths. These included:

  (a) Current, 0010, 0001

  (b) 1000, 1001

(4) The set of initial arc lengths had a strong effect on total savings for approximately half of the sets of initial modal splits.

(5) The set of initial arc lengths C was consistently inferior to sets A and B. Sets A and B resulted in approximately the same quality of solutions for each set of initial modal splits.

Investment is plotted against these factors in Figure 5-11 below.

Several points should be noted:

(1) Most sets of initial modal splits and arc lengths resulted in approximately the same level of investment.

(2) The 0010 set of initial modal splits resulted in lower investment levels for all sets of initial arc lengths.

(3) The set of initial arc lengths did not have a strong effect on the level of investment given a set of initial modal splits. The maximum range was 9.

User savings is plotted against these factors in Figure 5-12 below. Since user savings is closely related to total savings, the same observations can be made.

Next, consider the effect of the modal split updating parameter ALPHA. Comparing the characteristics of runs 22-24 with those of runs 25-27, it should be noted that for each set of initial arc lengths the methodology converged to the same solution, regardless of the value of

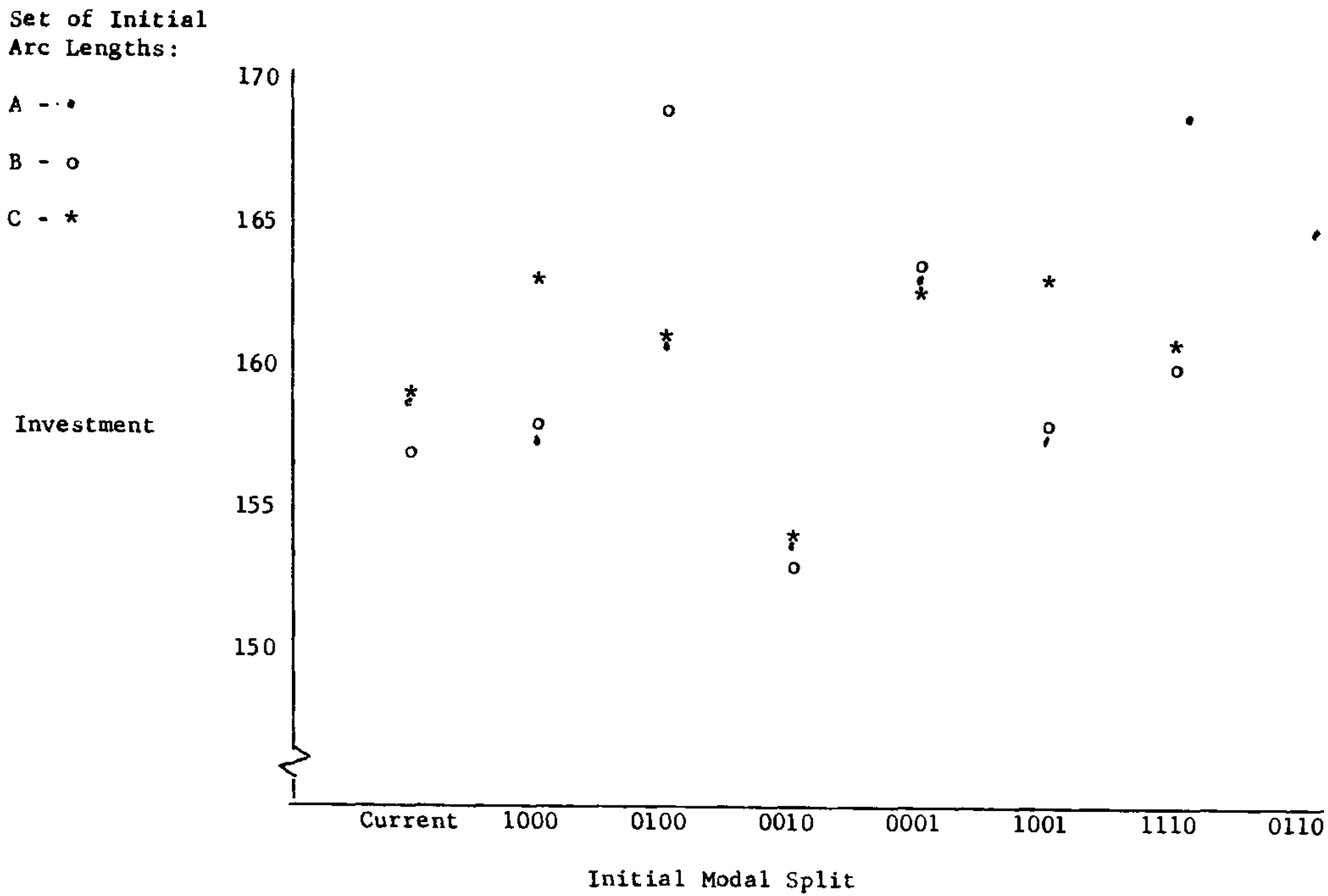


Fig. 5.11. Investment Versus Initial Modal Split

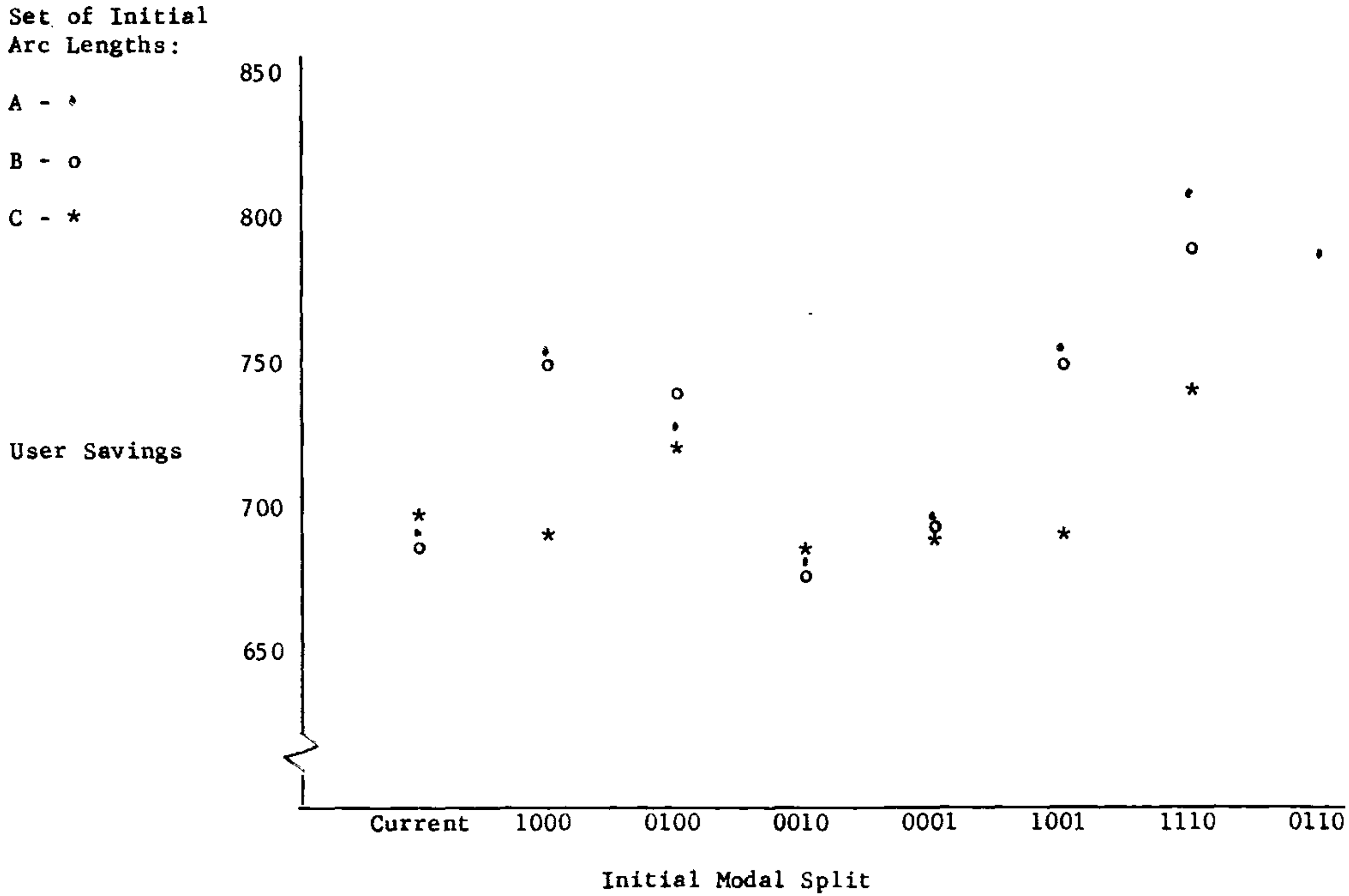


Fig. 5.12. User Savings Versus Initial Modal Split

ALPHA. Next, consider the effect of the modal split convergence parameter EPSILON. The investments associated with runs 28 and 29 were equal, as were those of runs 11 and 30. However, as EPSILON decreased from .02 to .01, the user savings and total savings (and disutility) decreased substantially for both sets of runs. This phenomenon is an example of how the modal split constraints affect the value of the objective function.

Finally, consider the effect of the use of a Phase I procedure in the two-phase algorithm. The results are shown in Table 5-4 below.

Table 5-4. Solution Characteristics and Use of a Phase I Procedure

Run	Set of Initial Arc Lengths	Phase I Used	Total Savings	Investment	User Savings
1	A	No	515	158	673
4	A	Yes	536	159	695
2	B	No	535	154	689
5	B	Yes	535	157	691
3	C	No	481	159	640
6	C	Yes	536	159	695

For the sets of initial arc lengths A and C, use of the Phase I procedure resulted in an improved objective. In general, use of the Phase I procedure resulted in a more uniform solution, higher investment, and higher user savings.

Previously, the solutions were analyzed with respect to some aggregate characteristics. Although the solutions appeared to differ with respect to these characteristics, there was still some question as to whether the differences in solutions represented true investment and flow pattern differences. The purpose of this section is to examine this question in greater detail. The solutions of 2 test runs were analyzed:

(1) Run 22 - the best solution, relatively high investment

(2) Run 6 - a poor solution, relatively low investment

First, consider investment over minimum for each solution. Investment was 169 for run 22 and 159 for run 6. This may be separated by mode as shown in Table 5-5 below.

Table 5-5. Investment by Mode

	Highway	Railroad	Water	Intermodal Transfer	Total
Run 22	97	62	10	0	169
Run 6	86	62	10	0	159

Several points should be noted:

(1) Investment in intermodal transfer facilities was negligible for both runs.

(2) Investment in water facilities and rail facilities remained constant.

(3) Investment in highway facilities decreased substantially.

The investment in highway and rail facilities can be examined in greater detail as shown in Table 5-6 below.

Table 5-6. Investment by Mode and Type of Facility

	Highway				Rail			
	Load-ing	Line-Haul	Trans-fer	Unload-ing	Load-ing	Line-Haul	Trans-fer	Unload-ing
Run 22	.3	96	0	.4	.3	61	.9	.4
Run 6	.3	85	0	.4	.3	61	1	.4

It should be noted that the loss of highway investment occurred in line-haul facilities. Thus, the differences between solutions noted previously appear to be real and to be concentrated primarily on highway line-haul arcs. To examine these differences in still greater detail, the first 50 highway line-haul arcs in the arc list were selected for further examination. All 50 arcs were corridor arcs and thus subject to improvement. Investment and flow on these arcs are shown in Table 5-7 below. It should be noted that investment and flow patterns differ substantially between the two solutions.

With one major exception, the conclusions which can be drawn from the multi-commodity results parallel those of the single-commodity problem and, thus, will not be repeated. The one exception pertains to the CPU time required to generate a solution and its effect on the practicality of the methodology. While a solution for the single-commodity problem could be generated in an average of 1064 seconds, the equivalent time for the multi-commodity problem was 2092 seconds. This was an increase of 100 percent for a problem of the same size. The increase in CPU time was directly attributable to the large number of I-O operations required by the programmed version of the multi-commodity methodology.

Table 5-7. Investment and Flow on HW Line-Haul Arcs

		Investment		Flow (÷ 1000) Comm. 1		Comm. 2		Comm. 3	
Origin	Dest.	Run 22	Run 6	Run 22	Run 6	Run 22	Run 6	Run 22	Run 6
1	2	0	0	0	0	0	0	0	0
1	4	0	0	0	0	0	0	0	0
1	5	0	0	0	0	0	0	0	0
2	1	2.3	2.3	0	0	0	0	1,921	1,921
2	4	0	0	0	0	0	0	0	0
2	6	0	0	0	0	0	0	0	0
3	4	0	0	0	0	0	0	0	0
3	5	0	0	0	0	0	0	0	0
3	8	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0	0
4	2	0	0	0	0	0	0	0	0
4	3	0	0	0	0	0	0	0	0
4	5	0	0	0	0	0	0	0	0
4	6	0	0	0	0	0	0	0	0
4	8	0	0	0	0	0	0	0	0
4	9	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0
5	3	0	0	0	0	0	0	0	0
5	4	0	0	0	0	0	0	0	0
5	7	0	0	0	0	0	0	0	0
5	8	0	0	0	0	0	0	0	0
6	2	2.7	2.7	0	0	3,457	3,460	0	0
6	4	0	0	0	0	0	0	0	0
6	8	0	3.9	0	0	0	0	0	5,886
6	9	4.3	4.1	0	0	1,386	1,387	14,140	7,755
7	5	0	0	0	0	0	0	0	0
7	8	0	0	0	0	0	0	0	0
7	11	0	0	0	0	0	0	0	0
8	3	0	0	0	0	0	0	0	0
8	4	0	0	0	0	0	0	0	0
8	5	0	0	0	0	0	0	0	0
8	6	0	0	0	0	0	0	0	0
8	7	0	2.5	0	0	0	0	0	5,886
8	9	0	0	0	0	0	0	0	0
8	11	0	0	0	0	0	0	0	0
9	4	0	0	0	0	0	0	0	0
9	6	2.3	2.3	0	0	1,968	1,970	0	0
9	8	0	0	0	0	0	0	0	0
9	11	3.7	3.5	0	0	0	0	14,140	7,755
9	13	0	0	0	0	0	0	0	0
9	15	2.9	2.9	0	0	3,226	3,230	0	0

(continued)

Table 5-7 (cont'd)

		Investment		Flow (÷ 1000) Comm. 1		Comm. 2		Comm. 3	
Origin	Dest.	Run 22	Run 6	Run 22	Run 6	Run 22	Run 6	Run 22	Run 6
10	11	0	0	0	0	0	0	0	0
10	12	0	3.2	0	585	0	0	0	7,755
10	13	0	0	0	0	0	0	0	0
11	7	0	0	660	653	0	0	0	0
11	8	0	0	0	0	0	0	0	0
11	9	0	0	0	0	0	0	0	0
11	10	.3	2.3	1,314	1,909	0	0	0	7,755
11	13	0	0	0	0	0	0	0	0
11	15	0	0	0	0	0	0	0	0

The net result of this 100 percent increase in computation time is that it is not clear whether the methodology is an acceptable means of generating solutions to the multi-commodity problem. If computation times can be decreased substantially by adoption of additional simplifying assumptions and/or by programming the methodology in a more time-efficient manner, then the methodology may be acceptable. Otherwise, it remains highly suspect.

Continuing along the same line, there is some question as to whether the solutions obtained from the multi-commodity problem are sufficiently different from the corresponding solutions to the single-commodity problem to warrant the additional computation expense. In attempting to address this question, a pairwise comparison was made between the single-commodity and multi-commodity results for two different test runs. The two test runs selected for the comparison were run 22, which resulted in very good solutions, and run 6, which resulted in poor solutions. Investment and commodity flow for each of the same 50 highway arcs examined earlier are given for these test runs in Table 5-8 below. Several points should be noted:

(1) For run 22 and the 50 arcs examined, although there were some deviations in investment and flow patterns, the basic investment and flow pattern remained the same.

(2) For run 6 and the 50 arcs examined, the investment and flow patterns differed substantially between the single-commodity and multi-commodity solutions.

Thus, the results were inconclusive. One pair of solutions appeared similar while another pair differed substantially. One final speculation might be advanced on this subject: as the parameters of the ATC and modal split functions differ to a greater and greater degree between commodities, the investment and flow pattern differences between the single and multi-commodity solutions will increase.

Table 5-8. Investment and Flow on HW Line-Haul Arcs

		Run 22				Run 6			
		Invest.		Flow (÷ 1000)		Invest.		Flow (÷ 1000)	
Origin	Dest.	S.C.	M.C.	S.C.	M.C.	S.C.	M.C.	S.C.	M.C.
1	2								
1	4								
1	5								
2	1	2.8	2.3	3,345	1,921	.3	2.3	633	1,921
2	4					3.7		13,764	
2	6								
3	4								
3	5								
3	8								
4	1								
4	2								
4	3					5.0		7,185	
4	5								
4	6								
4	8					5.1		6,579	
4	9								
5	1								
5	3								
5	4								
5	7								
5	8								
6	2	3.3	2.7	4,323	3,457	3.3	2.7	4,342	3,460
6	4								
6	8						3.9		5,886
6	9	4.4	4.3	27,849	15,526		4.1		9,142
7	5								
7	8								
7	11								
8	3								
8	4								
8	5								
8	6								
8	7		2.5		5,886	2.6	2.5	6,579	5,886
8	9								
8	11								
9	4								
9	6	3.5	2.3	2,464	1,968	3.5	2.3	2,474	1,970
9	8								
9	11	3.8	3.7	25,879	14,140		3.5		7,755
9	13								

(continued)

Table 5-8 (cont'd)

		Run 22				Run 6			
		Invest.		Flow (÷ 1000)		Invest.		Flow (÷ 1000)	
Origin	Dest.	S.C.	M.C.	S.C.	M.C.	S.C.	M.C.	S.C.	M.C.
9	15	3.6	2.9	4,594	3,226	3.3	2.9	2,643	3,230
10	11								
10	12						3.2		8,340
10	13								
11	7	1.8	0	838	660	1.8	0	838	653
11	8								
11	9								
11	10	1.5	.3	1,482	1,314	1.5	2.3	1,492	9,664
11	13					1.4	0	838	0
11	15								

CHAPTER VI

THE APPROXIMATION OF PROBLEM P2(MS^o) BY A GENERAL MULTI-COMMODITY FIXED CHARGE NETWORK FLOW PROBLEM: PROBLEM AP(MS^o)

In Chapter III, Section 2.3 Problem P2(MS^o) was formulated and identified as the principal component in a single-commodity, multi-modal transport network improvement methodology. Problem P2(MS^o) is an uncapacitated, concave disutility transportation assignment problem. In Chapter II, sections 2.4 and 2.5 a two-phase solution methodology was developed for P2(MS^o). In Phase I the methodology attempts to locate the general vicinity of a good local optimum. In Phase II the Yaged Method is used to locate this local optimum. A significant objection to this solution approach is that there is no basis on which to judge the quality of the resulting solution, at least with respect to the global optimum. Since no good lower bound was identified, one must repeat the algorithm a number of times in order to increase confidence in the final solution accepted. The purpose of this chapter is to describe a solution procedure which appears capable of providing a very good initial solution of the Yaged Algorithm. It would also provide a practical approximation to the lower bound of P2(MS^o). The basic strategy of the procedure is to approximate problem P2(MS^o) with a general multi-commodity fixed charge network flow problem and then solve a relaxation of the fixed charge problem. Note that in this chapter multi-commodity refers to multi-math programming commodity classes, which vary by origin.

1. Initial Formulations

Consider approximating the concave arc disutility functions (3-34), (3-35), and (3-36) shown Figure 3-7, 3-8, and 3-9 with a piecewise-linear underestimate. It should be noted that:

1. Category I arcs are represented exactly by a single linear segment.
2. Category II arcs can be naturally represented by three linear segments.
3. Category III arcs can be naturally represented by two linear segments.

Consider the most general case, that of the Category II arc. The natural underestimate of the concave arc disutility function by a piecewise linear function is shown in Figure 6-1 below. Note that the disutility axis has been shifted so that L_j, the lower bound on investment in arc j, is now zero.

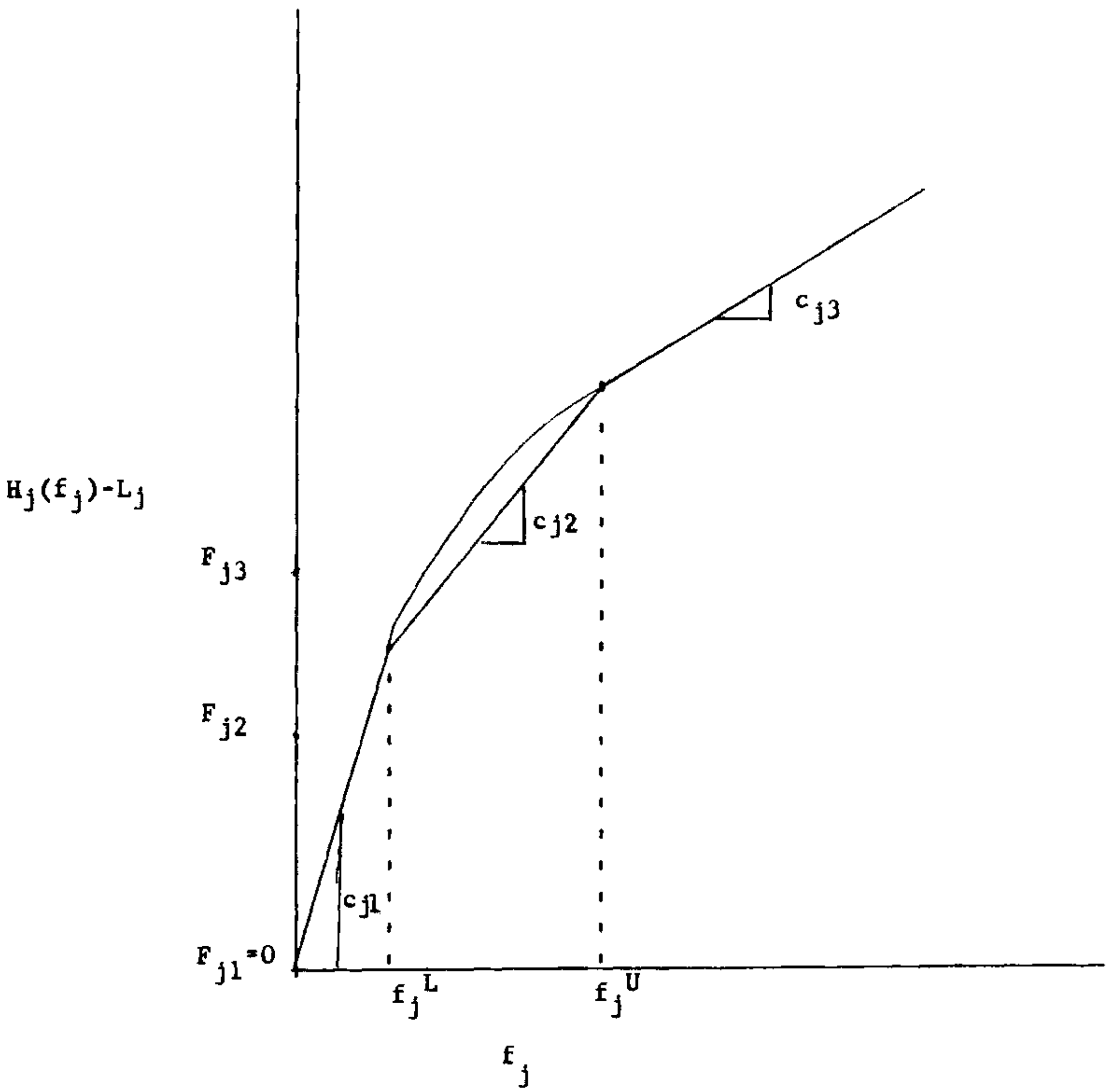


Fig. 6-1. Piecewise Linear Underestimate

to the Category II Arc Disutility Function

1.1 The Node-Arc Formulation

The node-arc formulation of the general multi-commodity fixed charge network flow problem now provides an approximation to Problem P2(MS°):

$$\text{NA(MS°)}: \min_{f^r_{jk}, f_{jk}, y_{jk}} \sum_{j\varepsilon A} \left[\sum_{k=1}^{3} (c_{jk} \cdot f_{jk} + F_{jk} \cdot y_{jk})\right] \qquad (6\text{-}1)$$

$$\text{s.t.} \sum_{j\varepsilon W_i} \left(\sum_{k=1}^{3} f^r_{jk}\right) - \sum_{j\varepsilon V_i} \left(\sum_{k=1}^{3} f^r_{jk}\right) = h^r_i \qquad \forall\ i\varepsilon N,\ r\varepsilon O \qquad (6\text{-}2)$$

$$f_{jk} = \sum_{r\varepsilon O} f^r_{jk} \qquad \forall\ j \ \varepsilon\ A,\ k\ \varepsilon\ \{1,2,3\} \qquad (6\text{-}3)$$

$$f^r_{jk} = 0 \qquad \forall\ j\ \varepsilon\ ITA,\ r\ \varepsilon\ SMO,\ k\ \varepsilon\ \{1,2,3\} \qquad (6\text{-}4)$$

$$f^r_{jk} \geq 0 \qquad \forall\ j\ \varepsilon\ A,\ r\ \varepsilon\ O,\ k\ \varepsilon\ \{1,2,3\} \qquad (6\text{-}5)$$

$$f_{jk} \leq My_{jk} \qquad \forall\ j\ \varepsilon\ A,\ k\ \varepsilon\ \{1,2,3\} \qquad (6\text{-}6)$$

$$y_{jk}\ \varepsilon\ \{0,1\} \qquad \forall\ j\ \varepsilon\ A,\ k\ \varepsilon\ \{1,2,3\} \qquad (6\text{-}7)$$

$$\sum_{k=1}^{3} y_{jk} \leq 1 \qquad \forall\ j\ \varepsilon\ A \qquad (6\text{-}8)$$

where: f^r_{jk} = flow of commodity from origin r on arc j, segment k

f_{jk} = flow on arc j, segment k

y_{jk} = logical variable corresponding to arc j, segment k

$$y_{jk} = \begin{cases} 1 & \text{if segment is used} \\ 0 & \text{otherwise} \end{cases}$$

c_{jk} = variable cost on arc j, segment k

F_{jk} = fixed cost on arc j, segment k

$$h_i^r = \begin{cases} -x_{ri} & \text{if } i \in D_r \\ \sum_{j \in D_i} x_{rj} & \text{if } i=r \\ 0 & \text{otherwise} \end{cases}$$

x_{ij} = flow from origin i to destination j

M = large constant, >>>0

A = set of arcs

W_i = set of arcs originating at node i

V_i = set of arcs terminating at node i

N = set of nodes

O = set of origin nodes

ITA = set of intermodal transfer arcs

SMO = set of single mode origins

D_r = set of destinations for origin r

Note that problem NA(MS°) is a general multi-commodity fixed charge network flow problem with an effective network consisting of three times as many arcs as that of problem P2(MS°). This results from the need to represent each of the three linear segments for each arc. The objective (6-1) is an approximation to (3-5). Constraint sets (6-2), (6-3), (6-4), and (6-5) represent constraint sets (3-6), (3-7), (3-8), and (3-9) respectively. Constraint sets (6-6) and (6-7) force the payment of a segment's fixed cost whenever the segment carries positive flow. Constraint set (6-8) allows flow on at most one segment for any arc.

Solution procedure have not been developed for the general multi-commodity fixed charge network flow problem, [Rardin, 1978]. However, solution procedures have been developed for the node-arc formulation of the general single commodity fixed-charge network flow problem [Rardin, 1974; Rardin and Unger, 1976; Jarvis, Rardin, and Unger, 1977]. These integer programming procedures are not, however, capable of solving problems involving large-scale networks. The formulation NA(MS°) might still be useful in obtaining a good solution and lower bound for problem P2(MS°). A strategy often used in branch-and-bound solution procedures is to relax the integrality constraints (6-7) and solve the resulting problem NAR(MS°). Thus, a reasonable heuristic solution strategy might be to solve problem NAR(MS°), which is now tractable, and translate the resulting solution into one feasible to problem P2(MS°). Thus, one would have a lower bound to P2(MS°) as well as a feasible solution. However, Choe has shown that the relaxation of the node-arc formulation of the general single commodity fixed charge network flow problem did not provide a very good lower bound for the problem [Choe, 1978]. Instead, he suggested an arc-path formulation. Since the lower bound obtained from a relaxation of the node-arc formulation of the single commodity problem is poor, there is every reason to believe that this will also be true for the multi-commodity problem. Furthermore, a poor lower bound would tend to indicate that the corresponding solution feasible to problem P2(MS°) would also be poor. Thus, it is useful to consider the arc-path formulation.

1.2 The Arc-Path Formulation

The arc-path formulation of the general multi-commodity fixed charge network flow problem provides the following approximation to problem P2(MS°):

$$\text{AP(MS°)}: \quad \text{Min} \sum_{p\varepsilon P} c_p f_p + \sum_{j\varepsilon A} \left[\sum_{k=1}^{3} F_{jk} \cdot y_{jk} \right] \tag{6-9}$$

$$\text{s.t.} \quad \sum_{p\varepsilon P_s} f_p = t_s \qquad \forall\ s\ \varepsilon\ R \tag{6-10}$$

$$f_p \le t_s\, y_{jk} \qquad \forall\ j\ \varepsilon\ A,\ k\ \varepsilon\ \{1,2,3\},\quad p\ \varepsilon\ P^{j,k},\ p\ \varepsilon\ P_s \tag{6-11}$$

$$f_p \ge 0 \qquad \forall\ p\ \varepsilon\ P \tag{6-12}$$

$$y_{jk} \varepsilon \{0,1\} \qquad \forall\ j\ \varepsilon\ A,\ k\ \varepsilon\ \{1,2,3\} \tag{6-13}$$

$$\sum_{k=1}^{3} y_{jk} \le 1 \qquad \forall\ j\ \varepsilon\ A \tag{6-14}$$

where:

f_p = flow on path p

y_{jk} = a logical variable corresponding to arc j segment k

$$y_{jk} = \begin{cases} 1 & \text{if there is positive flow on the segment} \\ 0 & \text{otherwise} \end{cases}$$

c_p $\equiv$ variable cost on path p

F_{jk} $\equiv$ fixed cost on arc j, segment k

t_s $\equiv$ total flow between O-D pair s

P $\equiv$ set of all paths; this includes paths created by the different segments on a single arc.

A $\equiv$ set of arcs

P_s $\equiv$ set of paths connecting O-D pair s

$P^{j,k}$ $\equiv$ set of paths utilizing arc j, segment k

As in the node-arc formulation, problem AP(MS°) effectively has three times the number of arcs as P2(MS°). The objective (6-9) is an approximation of (3-5). Constraint set (6-10) forces all O-D flow to be allocated to paths. Constraint sets (6-11) and (6-13) force the payment of a segment's fixed cost whenever some path utilizing the segment has positive flow. Constraint set (6-12) requires the non-negativity of all path flows, and set (6-14) allows flow on at most one segment for any arc.

Choe reviews the literature relating to this problem and develops a branch-and-bound solution procedure [Choe, 1977]. The branch-and-bound solution procedure is not capable of solving problems involving large-scale networks. Thus, consider the same heuristic solution strategy as before:

1. Relax constraints until the resulting problem becomes tractable.
2. Solve the resulting problem yielding a lower bound on P2(MS°).
3. Translate the solution to one feasible to P2(MS°).

As a first step, consider relaxing the integrality constraint set (6-13) to:

$$0 \leq y_{jk} \leq 1 \qquad \forall\ j \in A,\ k \in \{1,2,3\} \qquad (6\text{-}15)$$

Note that t_s is the maximum flow that can use some path p containing arc j segment k. Thus, y_{jk} must be less than or equal to one at optimality. Otherwise, one could reduce y_{jk} to one, reducing the objective, and not affect the optimal flow pattern. Thus, (6-15) can be restated.

$$0 \leq y_{jk} \qquad \forall\ j \in A,\ k \in \{1,2,3\} \qquad (6\text{-}16)$$

As a second step, consider relaxing constraint set (6-14). Choe has shown that this set of constraints is redundant for problem AP(MS°). However, he has also demonstrated that when the integrality constraints (6-13) are relaxed, a member of (6-14) may be binding at optimality. Nevertheless, relax (6-14). The resulting problem APR(MS°) can be stated as:

$$\text{APR(MS°)}: \quad \text{Min} \sum_{p \in P} c_p\, f_p + \sum_{j \in A} \left[\sum_{k=1}^{3} F_{jk} \cdot y_{jk} \right] \qquad (6\text{-}17)$$

$$\text{s.t.} \sum_{p \in P_s} f_p = t_s \qquad \forall\ s \in R \qquad (6\text{-}18)$$

$$f_p \leq t_s \cdot y_{jk} \qquad \forall\ j \in A,\ k \in \{1,2,3\}\ \ p \in \ \ p \in P^{j,k},\ p \in P_s \qquad (6\text{-}19)$$

$$f_p,\ y_{jk} \geq 0$$

Problem APR(MS°) has a very simple structure. However, the size of the problem is formidable. Consider a single physical path connecting an O-D pair. Suppose the path is comprised of n arcs. Then there are 3^n parallel paths in problem APR(MS°) representing the one physical path. Each parallel path represents one distinct disutility structure for the physical path. When one considers that some physical paths may contain in excess of 100 arcs, it becomes immediately obvious that problem APR(MS°) is intractable. It is also obvious that the number of paths must be reduced if the problem is to be solved. This might be accomplished in several ways:

1. Select certain physical paths to be considered between each O-D pair. Develop the set of parallel paths for each physical path, and use only these paths in the formulation.
2. For each O-D pair select a subset of the entire set of parallel paths for inclusion in the analysis.

The first strategy has the advantage of reducing the total number of paths without destroying the disutility surfaces of the physical paths being considered. Remembering that some physical paths may contain in excess of 100 arcs, one should note that the first strategy still appears intractable. The second strategy has the advantage of reducing the total number of paths to the range of tractability. However, it has the disadvantage of arbitrarily modifying the disutility surfaces of each of the corresponding physical paths. An additional disadvantage of the first two strategies is that the resulting problem would no longer produce a lower bound to problem P2(MS°). Since neither strategy appears particularly appealing, a completely new approach to the solution of problem P2(MS°) might be considered.

2. An Approximation to Problem AP(MS°)

In the preceding section it was found that Problem APR(MS°) might not be an adequate means for the solution of problem AP(MS°) and, thus, problem P2(MS°). A major problem with this formulation was the segmentation of each arc to represent the disutility surface. This resulted in a very large number of parallel paths for each physical path. To eliminate this excessive number of parallel paths, the disutility surface implied by objective (6-9) and constraint sets (6-11), (6-13), and (6-14) must be modified.

2.1 A Simplified Case

To better understand the arc disutility surface implied by the objective (6-9) and constraint sets (6-11), (6-13), and (6-14), it is useful to analyze a simplified example. For some arc j:

1. Assume only two paths p_1 and p_2 use arc j.
2. Assume arc j is represented by only two segments instead of three.

First, consider Case I.

Case I. $t_1, t_2 > B_{j1}$

where:

$t_i \equiv$ maximum possible flow on path i

$B_{j1} \equiv$ breakpoint on arc j

$$B_{j1} = \frac{F_{j2} - F_{j1}}{c_{j1} - c_{j2}}$$

The disutility surface for a Case I arc is shown in Figure 6.2 below. The feasible space S_j consists of the convex hull formed by the extreme points:

$$A = (0,0)$$

$$G = (0,t_2)$$

$$H = (t_1, 0)$$

$$I = (t_1,t_2)$$

The disutility surface over S_j consists of two planes. The first corresponds to the lower linear segment and is formed by the points A, E, and F.

$$A = (0,0,0)$$

$$E = (0, B_{j1}, D_j(B_{j1}))$$

$$F = (B_{j1},0,D_j(B_{j1}))$$

where: $D_j(x) \equiv$ the disutility associated with flow x

The second plane, corresponding to the upper linear segment, intersects the first plane along the line from E to F which corresponds to the breakpoint B_{j1}. The second plane is formed by the points B,D,C,F, and E.

$$B = (0,t_2,D_j(t_2))$$

$$C = (t_1,0,D_j(t_1))$$

$$D = (t_1,t_2,D_j(t_1 + t_2))$$

In considering any approximation to this disutility surface, several properties are desirable:

1. A reasonable fit to the existing surface.
2. Convexity, so that the approximation can be solved to optimality without resorting to arc segmentation.
3. An underestimate so that a lower bound to P2(MS°) can be maintained.

$f_i \equiv$ flow on i^{th} path

$D_j \equiv$ disutility on arc j

• - point on disutility surface

o - point in solution space S_j

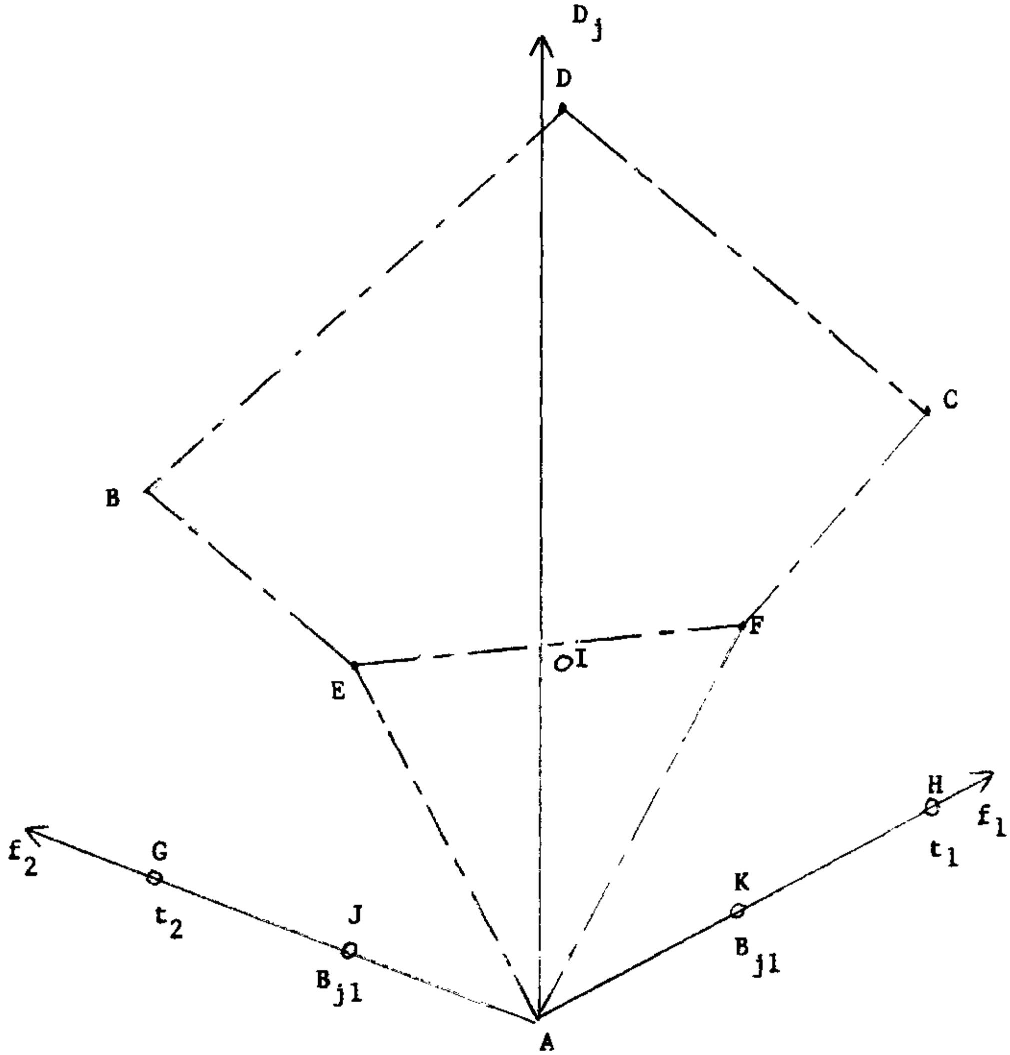


Fig. 6-2. Case I arc Disutility Surface

Several convex underestimators to this surface come immediately to mind.

1. A plane passing through points A and D and making an equal angle to the f_1 and f_2 axes.
2. A plane passing through points A and D and also touching either point B or C.

Consider the performance of these estimators over the feasible space S_j.

Let:

$D_j \equiv$ disutility on arc j shown in Figure 6-2.

$\hat{D}_j^i \equiv$ the i^{th} estimator of D_j

Concerning the first estimator, $\hat{D}_j^1 = D_j$ only at the extreme points A and I. For more distant points, $|\hat{D}_j^1 - D_j|$ increases with maximum underestimate occurring along the line from point J to k.

$J = (0, B_{j1})$

$k = (B_{j1}, 0)$

Concerning the second estimator, $\hat{D}_j^2 = D_j$ at extreme points A, I, and either G or H. Assume $\hat{D}_j^2 = D_j$ at point G. Consider the two subspaces of S_j formed by dividing S_j along the line from point A to I.

Then:

$$S_j^1 = \{(f_1, f_2): \quad \frac{f_1}{t_1} \geq \frac{f_2}{t_2}\}$$

$$S_j^2 = \{(f_1, f_2): \quad \frac{f_2}{t_2} \geq \frac{f_1}{t_1}\}$$

and:

$$\hat{D}_j^1 \leq \hat{D}_j^2 \qquad \forall\ (f_1, f_2) \varepsilon S_j^2$$

$$\hat{D}_j^2 \leq \hat{D}_j^1 \qquad \forall\ (f_1, f_2) \varepsilon S_j^1$$

In fact, it is evident that $\hat{D}_j^2$ is the best linear underestimator for D_j over the space S_j^2 and a very poor linear underestimator over the space S_j^1. The opposite can be shown when $\hat{D}_j^2 = D_j$ at point H. The first estimator provides somewhat of a compromise between the two extremes available with the second estimator.

It would be highly desirable to identify a third type of convex underestimator which possessed all of the desirable properties of the second estimator, but none of the undesirable properties. Call this estimator a Type III estimator. The primary properties of a Type III estimator D_j would be:

1. Convexity
2. $\hat{D}_j \leq D_j$ at all points in S_j.
3. Over each subspace of the form S_j^i, $\hat{D}_j$ is the best linear underestimator of D_j.

$$S_j^1 = \{\bar{f}: \ \frac{f_i}{t_i} = \underset{p \varepsilon P^j}{\text{Max}} \ \{\frac{f_p}{t_p}\}\}$$

$$\bar{f} = (f_1, \ldots, f_{|P^j|})$$

P^j = set of paths using arc j.

Several points should be noted concerning the third property:

1. For this property to be achievable, all points on the surface of D_j corresponding to the extreme points of S_j^i must lie in the same plane. Otherwise, there could be no best linear underestimator for D_j over S_j^i.
2. The surface of $\hat{D}_j$ must form a hyperplane over the space S_j^i, touching all points on the surface of D_j corresponding to the extreme points of S_j^i. Otherwise, $\hat{D}_j$ could be made a better linear underestimator by doing this.

Fortunately, a Type III estimator exists for the case being considered. The surface of this estimator is superimposed on the original disutility surface in Figure 6-3 below. The surface of the estimator is formed by two planes. The first plane represents the surface over the region S_j^1 and is defined by the points A, C, and D. The second plane represents the surface over the region S_j^2 and is defined by the points A,B, and D. The two planes intersect along the line from A to D which corresponds to the region of the feasible space where:

$$S_j^1 \cap S_j^2 \quad \text{or} \quad \frac{f_1}{t_1} = \frac{f_2}{t_2}$$

The estimator can be expressed mathematically as:

$$\hat{D}_j = c_{j2}(f_1 + f_2) + F_{j2} \cdot \text{Max}\ \{\frac{f_1}{t_1}, \frac{f_2}{t_2}\} \tag{6-21}$$

Theorem 6.1: $\hat{D}_j$ is a Type III estimator for D_j for Case I arcs.

Proof:

It is well known that $\hat{D}_j$ is convex.

Thus, it only remains to be shown that:

$$\hat{D}_j \leq D_j \qquad \forall\ 0 \leq f \ \leq t_1,\ 0 \leq f_2 \leq t_2$$

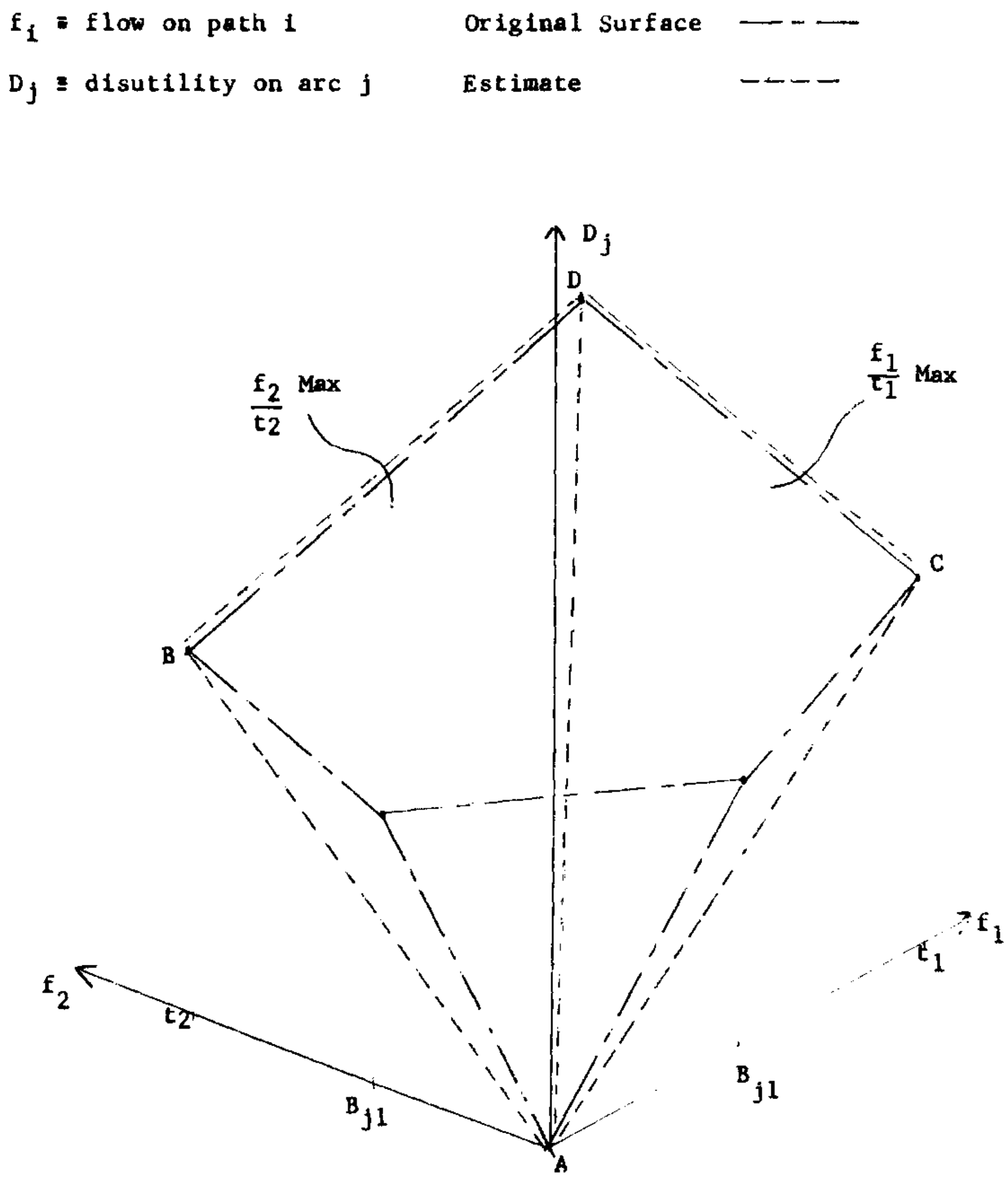


Fig. 6-3. Case I Arc Disutility Surface and

Type III Estimator

and: $\hat{D}_j = D_j$ for all extreme points of S_j^1 and S_j^2

Now consider the math program:

$$\text{T:} \qquad \underset{f_1,f_2}{\text{Min}} \quad D_j\,(f_1,f_2) - \hat{D}_j(f_1,f_2) \tag{6-22}$$

$$\text{s.t.} \quad 0 \le f_1 \le t_1 \tag{6-23}$$

$$0 \le f_2 \le t_2 \tag{6-24}$$

The objective of problem T is concave, sine D_j is concave and $\hat{D}_j$ is convex. The optimal solution of problem T must lie at an extreme point (EP) of the solution space (6-23) - (6-24). Note that an EP of this solution space corresponds to any combination of f_1 and f_2 at their upper and lower bounds.

This includes the points:

$(0,0)$

$(0,t_2)$

$(t_1,0)$

(t_1,t_2)

Thus, if $D_j < \hat{D}_j$ for some point in the solution space, it will also be true for some EP. Also, if it is not true for the EP, i.e., if $D_j \ge \hat{D}_j$ for all EP then $D_j \ge \hat{D}_j$ for all (f_1,f_2) in the solution space.

Point $(0,0)$

$$\hat{D}_j = 0 = D_j$$

Point $(0,t_2)$

$$\hat{D}_j = c_{j2}\,t_2 + f_{j2} = D_j$$

Point $(t_1, 0)$

$$\hat{D}_j = c_{j2}t_1 + F_{j2} = D_j$$

Point (t_1, t_2)

$$\hat{D}_j = c_{j2}\ (t_1 + t_2) + F_{j2} = D_j$$

Q.E.D.

There are three additional simplified cases which must be considered:

Case II: $t_2 \leq B_{j1}$

$t_1 > B_{j1}$

Case III: $t_1, t_2 \leq B_{j1}$

$t_1 + t_2 > B_{j1}$

Case IV: $t_1 + t_2 \leq B_{j1}$

A similar Type III estimator can be found for each of these cases:

$$D_j^{II} = [\,F_{j2} - F_{j1} - c_{j1}t_2 + c_{j2}\ (t_1 + t_2)]\cdot \frac{1}{t_1}\cdot f_1 + c_{j2}\cdot f_2 + [\,F_{j1} + t_2\ c_{j1} - c_{j2})]\cdot \text{Max}\ \{\frac{f_1}{t_1}, \frac{f_2}{t_2}\} \tag{6-25}$$

$$D_j^{III} = [\,F_{j2} - F_{j1} + c_{j2}\ (t_1 + t_2) - c_{j1}t_2]\cdot \frac{1}{t_1}\cdot f_1 + [\,F_{j2} - F_{j1} + c_{j2}\ (t_1 + t_2) - c_{j1}t_1]\cdot \frac{1}{t_2}\cdot f_2 + [\,2F_{j1} - F_{j2} + c_{j1}\ (t_1 + t_2) - c_{j2}\ (t_1 + t_2)]\cdot \text{Max}\ \{\frac{f_1}{t_1}, \frac{f_2}{t_2}\} \tag{6-26}$$

$$\hat{D}_j^{IV} = D_j^{IV} = c_{j1}(t_1 + t_2) \qquad (6\text{-}27)$$

The proofs follow that of Case I above. The original disutility surfaces and their Type III estimators are shown in Figures (6-4), (6-5), and (6-6) below.

2.2 The General Case

In the preceding section a Type III estimator was identified for a simplified arc disutility surface. However, there is some question as to whether the strategy will generalize. Consider the same situation as that given above, except let $|pi|$ paths use arc j.

Case I: $t_p \geq B_{j1} \qquad \forall\ p \in P^j$

Let: $$\hat{D}_j = c_{j2} \sum_{p\in P^j} f_p + F_{j2} \cdot \underset{p\in P^j}{\mathrm{Max}} \{\frac{f_p}{t_p}\} \qquad (6\text{-}28)$$

Theorem 6.2 $\hat{D}_j$ is a Type III estimator for Case I.

Proof:

$\hat{D}_j$ is convex

Thus, it remains to show that:

1. $\hat{D}_j \leq D_j \qquad \forall \qquad 0 \leq f_p \leq t_p$
2. $\hat{D}_j = D_j \qquad \forall \qquad$ extreme points of S_j^i, $i \in P^j$

But 1 is true when:

$\hat{D}_j \leq D_j \qquad \forall \qquad E\ P$

For the 0 case:

$\hat{D}_j = 0 = D_j$

f_i = flow on path i — Original Surface —— - ——

D_j = disutility on arc j — Estimate — — — —

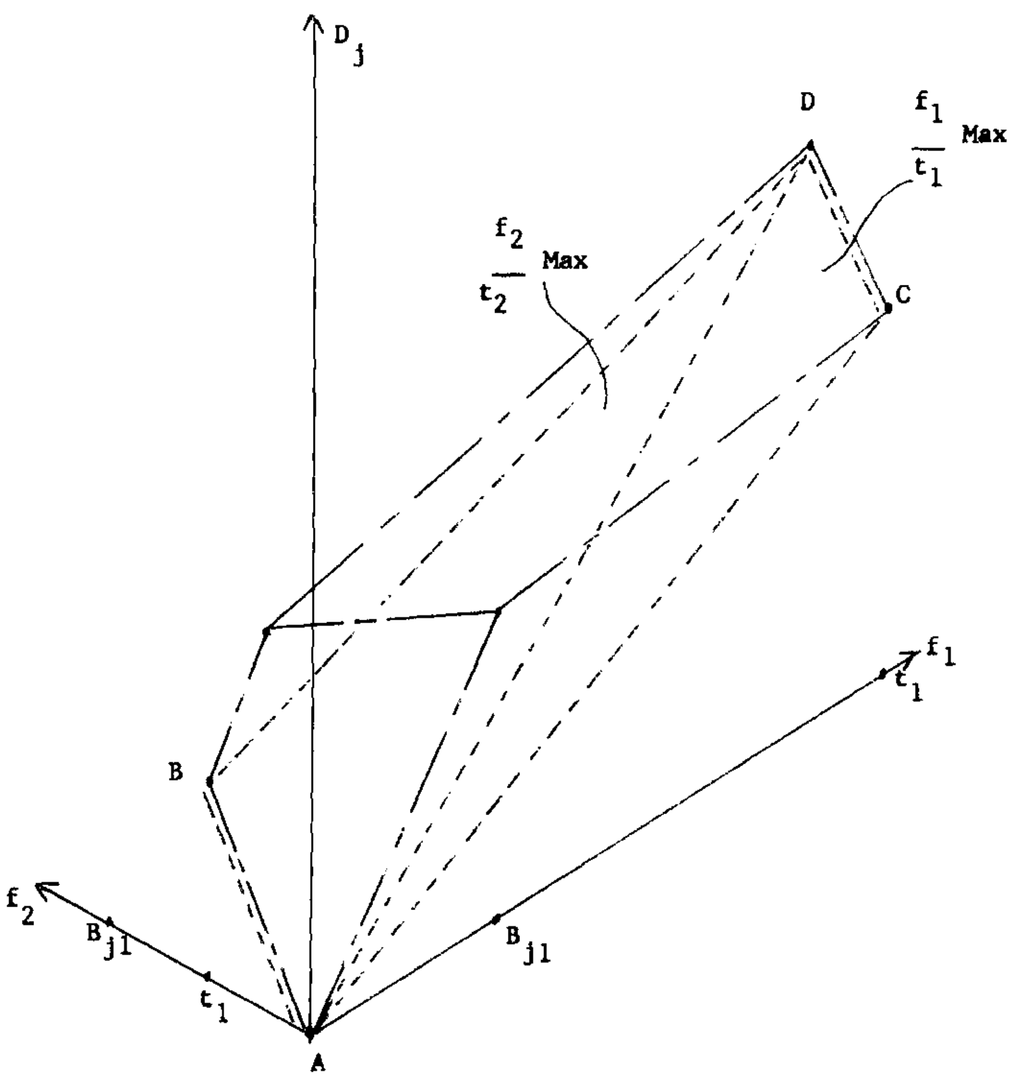


Fig. 6-4. Case II Arc Disutility Surface and

Type III Estimator

$f_i \equiv$ flow on path i — Original Surface ——— – ———

$D_j \equiv$ disutility on arc j — Estimate – – – – –

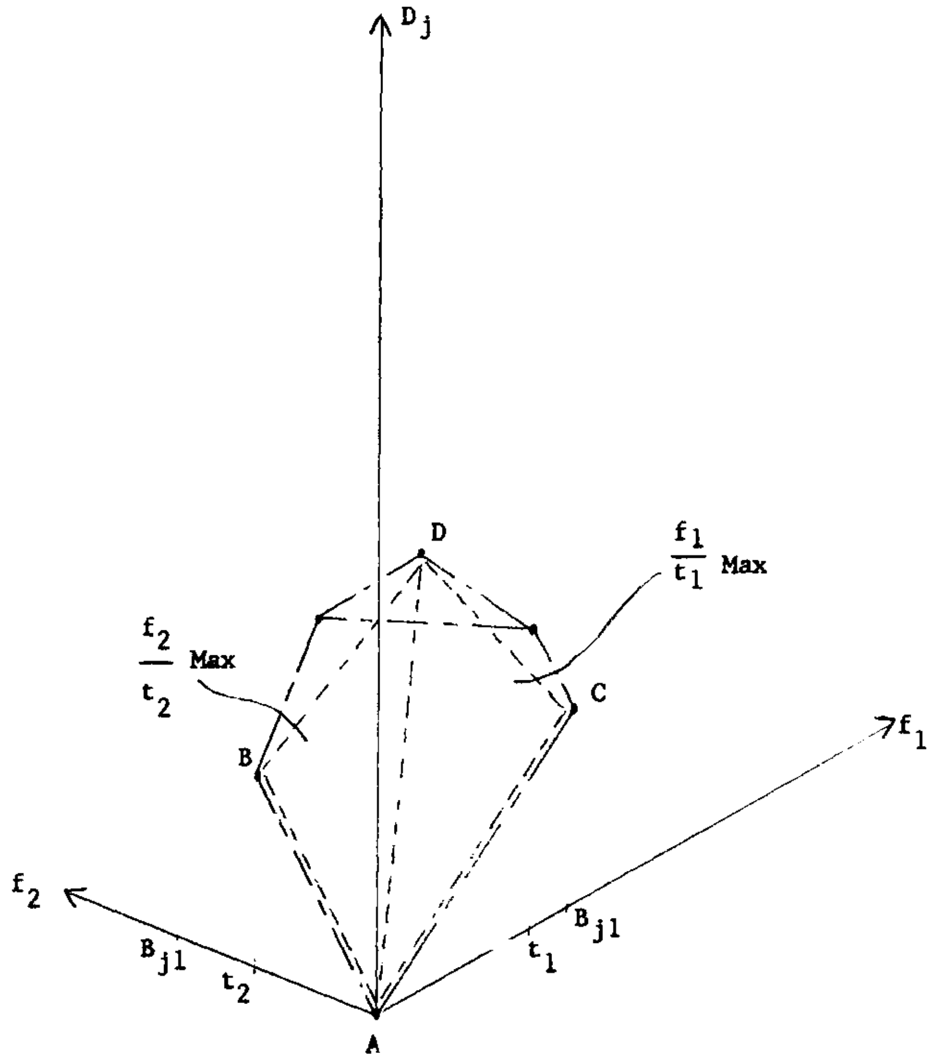


Fig. 6-5. Case III Arc Disutility Surface and Type III Estimator

$f_i \equiv$ flow on path i

$D_j \equiv$ disutility on arc j

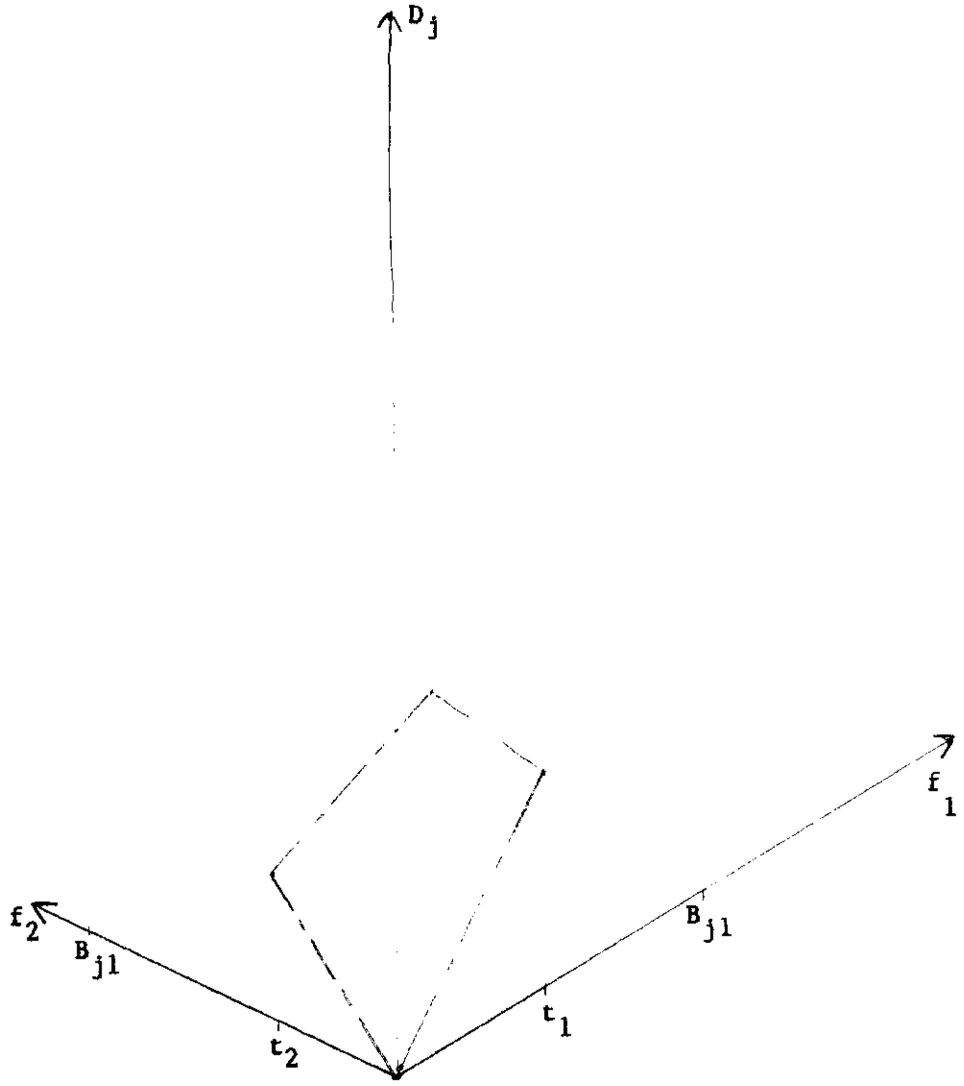


Fig. 6-6. Case IV Arc Disutility Surface and Type III Estimator

For every other EP:

$$\hat{D}_j = c_{j2} \sum_{p \varepsilon P j} \hat{f}_p + F_{j2} = D_j$$

where:

$$\hat{f}_p = \begin{cases} 0 & \text{if } f_p = 0 \text{ in EP} \\ t_p & \text{if } f_p = t_p \text{ in EP} \end{cases}$$

Q.E.D.

For the remaining general cases a counterexample can be provided to demonstrate that a Type III estimator cannot always be found. Consider the case where:

$$t_1 = 10$$
$$t_2 = 3$$
$$t_3 = 2$$
$$c_{j1} = 2$$
$$c_{j2} = \tfrac{1}{2}$$
$$F_{j1} = 0$$
$$F_{j2} = 6$$
$$B_{j1} = 4$$

Consider the region S_j^3 with EP:

(0,0,0), (10,3,2), (0,0,2), (10,0,2), (0,3,2)

The corresponding points on the disutility surface are:

(0,0,0,0), (10,3,2,13½), (0,0,2,4), (10,0,2,12), (0,3,2,8½)

It can easily be shown that these points do not lie on the same hyperplane. Thus, a Type III estimator does not exist.

2.3 The Development of an Alternative Convex Underestimator

Section 2.1 developed a Type III estimator for a simplified arc disutility function. Section 2.2 noted that a Type III estimator could not be determined for the general case. In this section an alternative convex underestimator for the general case is presented and its properties are proven. Let:

$t_s \equiv$ demand between O-D pair S

$O_j \equiv$ the set of all O-D pairs with at least one path on arc j

$B_{j1} \equiv$ first break point on arc j

$B_{j2} \equiv$ second break point on arc j

$c_{ji} \equiv$ slope of i^{th} segment on arc j

$F_{ji} \equiv$ fixed cost of i^{th} segment on arc j

$p^j \equiv$ set of paths using arc j

$A \equiv \{S: \ t_s < B_{j1}\}$

$B \equiv \{S: \ B_{j1} \leq t_s < B_{j2}\}$

$C \equiv \{S: \ t_s \geq B_{j2}\}$

Consider the following underestimators:

$$\hat{D}_j = d_j \sum_{p \varepsilon p^j} f_p + \operatorname*{Max}_{p \varepsilon p^j} \{e_{jp} \ f_p\} \tag{6-29}$$

Where for some arc j:

Case I: $t_s \leq B_{ji}$ $\forall\ s\ \varepsilon\ 0_j$

and $\sum_{s\varepsilon 0_j} t_s \leq B_{j1}$

$d_j = c_{j1}$

$$e_{jp} = \frac{F_{j1}}{t_p} \qquad (6\text{-}30)$$

Case II: $B_{j1} \leq t_s \leq B_{j2}$ $\forall\ s\ \varepsilon\ 0_j$

and $\sum_{s\varepsilon 0_j} t_s \leq B_{j2}$

$d_j = c_{j2}$

$$e_{jp} = \frac{F_{j2}}{t_p} \qquad (6\text{-}31)$$

Case III: $t_s \geq B_{j2}$ $\forall\ s\ \varepsilon\ 0_j$

$d_j = c_{j3}$

$$e_{jp} = \frac{F_{j3}}{t_p} \qquad (6\text{-}32)$$

Case IV: either $t_s \leq B_{j1}$ $\forall\ s\ \varepsilon\ 0_j$

and

$B_{j1} < \sum_{s\varepsilon 0_j} t_s \leq B_{j2}$

or $\quad t_s \le B_{jk}$ $\qquad \forall\ s \in O_j \cap A$

and

$B_{j1} \le t_s \quad B_{j2}$ $\qquad \forall\ s \in O_j \cap B$

and

$\sum_{S \in O_j} t_s \quad B_{j2}$

$d_j = c_{j2}$

$$e_{jp} = \begin{cases} \dfrac{F_{j2}}{t_p} & \text{if } t_p \ge B_{j1} \\ \dfrac{F_{j1}}{t_p} + c_{j1} - c_{j2} & \text{if } t_p < B_j 1 \end{cases} \tag{6-33}$$

Case V: either $B_{j1} \le t_s \le B_{j2}$ $\qquad \forall\ s \in O_j$

and

$\sum_{s \in O_j} t_s > B_{j2}$

or $\quad t_s \ge B_{j2}$ $\qquad \forall\ s \in O_j \cap C$

and

$B_{j1} \le t_s \le B_{j2}$ $\qquad \forall\ s \in O_j \cap B$

and

$B \cup C = O_j$

$$d_j = c_{j3}$$

$$e_{jp} = \begin{cases} \dfrac{F_{j3}}{t_p} & \text{if } t_p \geq B_{j2} \\[2ex] \dfrac{F_{j2}}{t_p} + c_{j2} - c_{j3} & \text{if } t_p < B_{j2} \end{cases} \tag{6-34}$$

Case VI For all other arcs j:

$$d_j = c_{j3}$$

$$e_{jp} = \begin{cases} \dfrac{F_{j3}}{t_p} & \text{if } t_p \geq B_{j2} \\[2ex] \dfrac{F_{j1}}{t_p} + c_{j1} - c_{j3} & \text{if } t_p < B_{j1} \\[2ex] \dfrac{F_{j2}}{t_p} + c_{j2} - c_{j3} & \text{if } B_{j1} \leq t_p \leq B_{j2} \end{cases} \tag{6-35}$$

Theorem 6.3 $\hat{D}_j$ is a convex underestimator of D_j.

The proof of Theorem 6.3 is lengthy and is given in Appendix G. Several characteristics of this estimator can be determined from the proof:

1. For Case I, $\hat{D}_j = D_j$ over the entire feasible space S_j.
2. For Cases II and III, $\hat{D}_j = D_j$ for the extreme points of the subspaces S_j^i, $i \in P^j$. Thus, the estimators are Type III estimators.

3. $\hat{D}_j = D_j$ at the zero flow point and at all other extreme points of S_j where one path flow is at its maximum and the rest are at their minimum.

4. $\hat{D}_j$ may not equal D_j at the maximum flow point. This will depend on the arc and its path flow configuration.

5. $\hat{D}_j$ may equal D_j at a number of other extreme points of S_j. The actual number and type of these extreme points will depend on the arc and its path flow configuration.

2.4 Problem Q(MS°): An Approximation to Problem AP(MS°)

In the preceding section a good convex underestimator was developed for the original disutility surface of problem AP(MS°). Problem AP(MS°) can now be approximated by:

$$\text{APA(MS°)}: \quad \text{Min} \sum_{j \varepsilon A} [d_j \sum_{p \varepsilon P^j} f_p + \operatorname*{Max}_{p \varepsilon P^j} \{e_{jp} \; f_p\}] \qquad (6\text{-}36)$$

$$\text{s.t.} \sum_{p \varepsilon P_s} f_p = t_s \qquad \forall \; s \; \varepsilon \; R \qquad (6\text{-}37)$$

$$f_p \geq 0 \qquad \forall \; p \; \varepsilon \; P \qquad (6\text{-}38)$$

where:

f_p ≡ flow on path p

t_s ≡ required flow between O-D pair s

d_j, e_{jp} ≡ parameters of convex underestimate

A ≡ set of arcs

P ≡ set of paths

P^j ≡ set of paths passing through arc j

P_s ≡ set of paths connecting O-D pair s

R ≡ set of O-D pairs

Problem APA(MS°) can be formulated in the more traditional arc-path form as:

$$Q(MS°): \quad \text{Min} \sum_{p \varepsilon P} c_p f_p + \sum_{j \varepsilon A} y_j \tag{6-39}$$

$$[u_s] \quad \text{s.t.} \sum_{p \varepsilon P_s} f_p = t_s \qquad \forall\, s \;\varepsilon\; R \tag{6-40}$$

$$[v_{jp}] \quad f_p \leq \frac{y_j}{e_{jp}} \qquad \forall\, j \;\varepsilon\; A,\; p \;\varepsilon\; P^j \tag{6-41}$$

$$f_p,\; y_j \geq 0 \qquad \forall\, p \;\varepsilon\; P,\; j \;\varepsilon\; A \tag{6-42}$$

where:

$$c_p = \sum_{j \varepsilon A^p} d_j$$

A^p ≡ the set of arcs comprising path p

y_j ≡ an arc j capacity related variable

u_s, v_{jp} ≡ the related dual variables

In the following section a decomposition approach is proposed for the solution of Q(MS°). As a preliminary to the section, note that constraint set (6-40) can be rewritten as:

$$\sum_{p \varepsilon P_s} f_p \geq t_s \qquad \forall\, s \;\varepsilon\; R \tag{6-43}$$

This is true since each constraint in (6-43) would have to be binding at optimality. Finally, constraint set (6-41) can be rewritten as:

$$e_{jp} f_p - y_j \leq 0 \qquad \forall\ j \in A,\ p \in P^j \tag{6-44}$$

3. A Proposed Solution Procedure for Problem Q(MS°)

3.1 Problem DQ(MS°): The Dual of Problem Q(MS°)

In the previous section problem Q(MS°) has a large number of columns, but a much larger number of rows. This large number of rows constitutes the most serious difficulty in solving the problem. As is often done with problems of many rows, consider the dual of problem Q(MS°):

$$\text{DQ(MS°):} \quad \text{Max} \sum_{s \in R} t_s\, u_s \tag{6-45}$$

$$[y_j] \quad \text{s.t.} \quad \sum_{p \in P^j} v_{jp} \leq 1 \qquad \forall\ j \in A \tag{6-46}$$

$$[f_p] \qquad u_s - \sum_{j \in A^p} e_{jp}\, v_{jp} \leq c_p \qquad \forall\ s \in R,\ p \in P_s \tag{6-47}$$

$$u_s,\ v_{jp} \geq 0 \qquad \forall\ s \in R,\ p \in P_s,\ j \in A^p \tag{6-48}$$

Problem DQ(MS°) has a very large number of columns and a large number of rows. Furthermore, constraint sets (6-47) and (6-48) are block diagonal by O-D pair. This is evident by noting that u_s is unique to O-D pair s and the v_{jp} are unique to the set of paths P_s which connect O-D pair s.

The Dantzig-Wolfe decomposition procedure can be used for problems of this type.

3.2 Dantzig-Wolfe Decomposition of DQ(MS°)

Consider the use of Dantzig-Wolfe decomposition in the solution of problem DQ(MS°). First, let the constraint set (6-46) form the set of linking constraints. Note that there will be one row in the master problem for each linking constraint. For problem DQ(MS°) there are $|A|$ rows. Second, because of the block diagonal structure of constraint sets (6-47) and (6-48), one can use between one and $|R|$ subproblems. Note that for each subproblem used, there will be one additional convexity constraint in the master problem. Since the number of rows in the master problem is already large $|A|$, it might be useful to utilize a single subproblem.

Let:

$$S = \{(\bar{u},\bar{v}): \text{ (6-47) and (6-48) are satisfied}\}$$

$$w = (w_0, w_1) \geq 0$$

$$w_0 = (w_{01},\ldots,w_{0|A|})$$

$w_{0i} \equiv$ the dual variable of the master problem corresponding to arc i

$w_1 \equiv$ the dual variable of the master problem corresponding to the convexity constraint

The subproblem can now be stated as:

$$S(MS°): \text{ Max } \sum_{s\in R} (t_s u_s - \sum_{p\in P_s} \sum_{j\in A^p} w_{0j} v_{jp}) - w_1 \qquad (6\text{-}49)$$

$$\text{s.t.} \qquad (6\text{-}47),\ (6\text{-}48)$$

Constraint sets (6-47) and (6-48) are block diagonal by O-D pair. Thus, since the objective (6-49) is linear, problem S(MS°) is separable into $|R|$ subproblems of the form:

$$S_s(MS°): \quad \text{Max } t_s u_s - \sum_{p \varepsilon P_s} \sum_{j \varepsilon A^p} w_{0j} v_{jp} \tag{6-50}$$

$$\text{s.t. } u_s - \sum_{j \varepsilon A^p} e_{jp} v_{jp} \leq c_p \qquad \forall\ p \ \varepsilon\ P_s \tag{6-51}$$

$$u_s,\ v_{jp} \geq 0 \qquad \forall\ p \ \varepsilon\ P_s,\ j \ \varepsilon\ A^p \tag{6-52}$$

Given the solutions to the $|R|$ subproblems $S_s(MS°)$, the column which is a candidate to enter the basis of the master problem as well as its reduced cost coefficient can be determined.

Let:

rcc reduced cost coefficient of column entering master problem

Z^*_s the optimal value of the objective for the s^{th} sub-problem

$$rcc = \sum_{s \varepsilon R} Z^*_s - w_1 \tag{6-53}$$

If $rcc \leq 0$, then the current solution to the problem is optimal. If $rcc > 0$, then column $\hat{a}$ must enter the current basis of the master problem where:

$$\hat{a} = \begin{matrix} a \\ \cdots \\ 1 \end{matrix} \tag{6-54}$$

$$a_j = \sum_{p \varepsilon P^j} v_{jp} \tag{6-55}$$

Given the solution of DQ(MS°), one must still construct the solution of Q(MS°). From duality theory relating to the Dantzig-Wolfe decomposition procedure, it is well known that:

$$y_j^* = w_{0j}^* \tag{6-56}$$

$$f_p^* = \bar{f}_p^* \tag{6-57}$$

where: f_p^*, y_j^* ≡ the optimal solution of f_p, y_j from problem Q(MS°)

w_{0j}^* ≡ the optimal value of w_{0j} in the master problem

$\bar{f}_p^*$ ≡ the optimal value of $\bar{f}_p$, a dual variable of problem S_s(MS°), (to be further defined in the following section)

3.3 The Solution of the Subproblem S_s(MS°)

While problem S_s(MS°) could be solved by a linear programming algorithm, the simple structure indicates that a less sophisticated (and less time and storage consuming) algorithm might be developed. Consider the dual of problem S_s(MS°)

$$DS_s(MS°): \quad \underset{\bar{f}_p}{\mathrm{Min}} \sum_{p \varepsilon P_s} c_p \bar{f}_p \tag{6-58}$$

$$\text{s.t.} \quad \sum_{p \varepsilon P_s} \bar{f}_p \geq t_s \tag{6-59}$$

$$\bar{f}_p \leq \frac{w_{0j}}{e_{jp}} \qquad \forall\, p \in P_s,\ j \in A^p \tag{6-60}$$

$$\bar{f}_p \geq 0 \qquad \forall\, p \in P_s \tag{6-61}$$

It appears that $DS_s(MS°)$ could be solved by inspection if it were feasible. However, infeasibility may be a problem. Note that the maximum flow on any path p is simply:

$$\bar{f}_p^{Max} = \underset{j \varepsilon A^p}{Min} \quad \frac{w_{0j}}{e_{jp}}$$

Thus, $DS_s(MS°)$ is infeasible whenever:

$$t_s \quad > \quad \sum_{p \varepsilon P_s} \bar{f}_p^{Max}$$

When $DS_s(MS°)$ is infeasible, $S_s(MS°)$ is unbounded. When this occurs, an extreme ray (ER) from problem $S_s(MS°)$ must be returned to the master problem. Obtaining the ER would not be difficult if one were using LP to solve $S_s(MS°)$, however, the use of LP is not desirable.

To avoid this problem, consider a redundant path A_s in addition to each set P_s in problem Q(MS°). Call the resulting problem $\bar{Q}(MS°)$.

Let:

$$c_{A_s} \geq \underset{p \varepsilon P_s}{Max} \quad \{c_p + \sum_{j \varepsilon A^p} \quad e_{jp}\} \qquad (6\text{-}62)$$

Since the cost of this path is higher than that of all other paths serving the O-D pair s, then no path A_s can carry positive flow in the optimal solution of $\bar{Q}(MS°)$. Thus, $\bar{Q}(MS°)$ is equivalent to Q(MS°). Let the dual of $\bar{Q}(MS°)$ be $\overline{DQ}(MS°)$. Then $\overline{DQ}(MS°)$ is equivalent to DQ(MS°) with the following redundant constraint set:

$$U_s \leq C_{A_s} \qquad \forall\, s \in R \qquad (6\text{-}63)$$

The subproblem for $\overline{DQ}(MS°)$, $\bar{S}(MS°)$, is then equivalent to $S(MS°)$ with the added constraint set (6-63). The $|R|$ subproblems separated from $\bar{S}(MS°)$, $\bar{S}_s(MS°)$, are equivalent to $S_s(MS°)$ with the added constraints:

$$u_s \leq c_{A_s}$$

The dual of subproblem $\bar{S}_s(MS°)$ can be stated as

$$DS_s(MS°): \qquad \text{Min} \sum_{p \varepsilon P_s} c_p \bar{f}_p + c_{A_s} \bar{f}_{A_s} \qquad (6\text{-}64)$$

$$\text{s.t.} \qquad \sum_{p \varepsilon P_s} \bar{f}_p + \bar{f}_{A_s} \geq t_s \qquad (6\text{-}65)$$

$$\bar{f}_p \leq \frac{w_{0j}}{e_{jp}} \qquad \forall\, p \varepsilon P_s,\ j \varepsilon A^p \qquad (6\text{-}66)$$

$$\bar{f}_p \geq 0, \quad f_{A_s} \geq 0 \quad \forall\, p \varepsilon P_s \qquad (6\text{-}67)$$

Note that $\overline{DS}_s(MS°)$ is now feasible and subproblem $\bar{S}_s(MS°)$ is bounded.

The following algorithm can be used to solve problem $\overline{DS}_s(MS°)$

1. Arrange the paths in order of increasing c_p.

   Let: $i = 1$
   $RF = t_s$

2. Determine $\bar{f}_i^{Max} = \underset{j \varepsilon A^i}{\text{Min}} \dfrac{w_{0j}}{e_{jp}}$

3. Let: $\bar{f}^*_i = \text{Min}\ \{RF,\ f_i^{Max}\}$

4. If $\bar{f}^*_i = RF$, let: $\bar{f}^*_{i+1}, \ldots, \bar{f}^*_{|P_s|} = 0$ and $f^*_{A_s} = 0$

   STOP

   Otherwise, go to step 5.

5. Let: $RF = RF - \bar{f}^*_i$

6. If $i = |P_s|$, let: $f^*_{A_s} = RF$

   STOP

   Otherwise, go to step 7.

7. $i = i+1$, Go to step 2.

The flow pattern produced by the algorithm is feasible since all flow is assigned to paths. It is optimal since flow is assigned in order of increasing unit cost. The algorithm will terminate in a maximum of $|P_s|$ iterations.

Finally, the optimal solution to problem $\overline{DS}_s(MS°)$ must be used to develop the optimal solution to subproblem $\bar{S}_s(MS°)$. This can be accomplished in the following manner:

1. If $\bar{f}_{A_s} = 0$, let path q be the last path assigned flow in the ordered arc list. Let $u^*_s = c_q$

   a. For any path $p \ni c_p < c_q$:

      i. For any one (possibly the first) constraining arc j, let:

$$v^*_{jp} = \frac{c_q - c_p}{e_{jp}} \qquad (6\text{-}68)$$

ii. For all other arcs j, let:

$$v^*_{jp} = 0 \qquad (6\text{-}69)$$

b. For any path $p \ni c_p \geq c_q$, let:

$$v^*_{jp} = 0 \qquad (6\text{-}70)$$

2. If $\bar{f}^*_{A_s} > 0$, let $u^*_s = c_{A_s}$, for each path p:

a. For any one (possibly the first) constraining arc j, let:

$$v^*_{jp} = \frac{c_{A_s} - c_p}{e_{jp}}$$

b. For all other arcs j, let:

$$v^*_{jp} = 0$$

This solution is feasible to problem $\bar{S}_s(MS°)$ and the complementary primal and dual solutions to the subproblem satisfy the complementary slackness conditions. Thus, the Kuhn-Tucker conditions are satisified, and the proposed solution must be optimal.

4. Problems Associated With the Proposed Solution Procedure

The solution procedure proposed for problem Q(MS°) has a number of significant drawbacks. First, even after eliminating all parallel paths, there remain a very large number of physical paths connecting each O-D pair. Thus, it may still be necessary to eliminate a large number of physical paths to assure tractability.

This might be accomplished analytically by the following procedure:

1. Find the shortest path between each O-D pair where arc length is defined as arc disutility given minimum investment.

   Let: $d_s \equiv$ length of shortest path between O-D pair s.

2. Eliminate all paths between O-D pair s whose length is greater than d_s, where arc length is defined as arc disutility given maximum investment.

This procedure will still require a great deal of computation. Perhaps an equally satisfactory method of path elimination is the "eyeball" procedure. Since one is dealing with a physical network with few or no special peculiarities, the "eyeball" method is particularly attractive. However, one should note that when using an "eyeball" method, one is no longer theoretically guaranteed a lower bound on the original problem. Thus, one would have a practical lower bound rather than a theoretical lower bound.

A second drawback of the procedure is the need to maintain a master problem basis of size $|A| + 1$. A complicating factor is that the master problem basis will probably be dense. Third, when using the Dantzig-Wolfe decomposition procedure on the dual, feasibility of the primal is not reached until optimality. Each iteration of the master problem provides an increasing lower bound on the optimal value of the primal objective. Thus, if convergence were slow near optimality, then one could iterate many times before obtaining a feasible solution to Q(MS°). There are two possible approaches to the solution of this problem:

1. Cease iteration of the decomposition procedure.
   A lower bound exists for the solution.
   Take the current primal solution, which is infeasible, and translate it into a feasible solution.
   he quality of the feasible solution can be verified by comparison with the lower bound.

2. Use a dual-simplex method to solve the master problem [Lasdon, 1978], this will assure primal feasibility at each iteration of the decomposition procedure. Thus, the algorithm can be terminated at any time yeilding a primal feasible solution, but no lower bound. A significant drawback of this approach is that the resulting subproblem becomes a linear fractional program. Using a typical formulation to solve the fractional program, the special structure of the subproblem would be lost, and one might need to resort to LP. Jacobsen has recommended an algorithm developed by Abadie and Williams for the solution of linear fractional programs with special structure [Jacobsen, 1967; Abadie and Williams, 1963]. The algorithm solves a sequence of LP's, each having the same constraint structure, but with different objectives. Thus, the price one

would pay for primal feasibility is the solution of a number of subproblems of the form $\bar{S}_s(MS°)$ at each iteration of the decomposition procedure.

5. Summary

In this chapter problem P2(MS°) was approximated by a general multi-commodity fixed charge network flow problem. The approach was similar to that used by Jarvis, Rardin, and Unger in dealing with a single commodity problem [Jarvis, Rardin, Unger, 1977]. From the work of Choe it was inferred that the solution of a relaxed arc-path formulation would provide a lower bound superior to that of the node-arc formulation [Choe, 1977]. However, it was also determined that the relaxed arc-path formulation was intractable due to the immense number of parallel paths representing a single physical path. To reduce this number to one, it was determined that another approximation to the disutility surface was required. A number of potential estimators were analyzed, and a new type of estimator, Type III, was defined and found to be desirable. Type III estimators were identified for arcs with certain simplified structures. However, it was found that Type III estimators did not exist for the general case. Lacking a Type III estimator, another less desirable estimator was identified and its properties proven. Finally, a solution procedure based on the Dantzig-Wolfe Decomposition Principle was proposed for the resulting problem. The subproblem for this procedure was found to possess a simple structure amenable to a very simple solution technique.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

1. Conclusions

1.1. The Single Commodity Problem

A number of conclusions can be drawn from the research relating to the single-commodity multi-modal freight transport network improvement problem. These include:

1. Existing multi-modal improvement formulations and solution procedures are inadequate for the specific purposes of this research. Specific weaknesses include limited applicational settings, inability to handle large-scale problems, and questionable simplifying assumptions.
2. The new formulation models the general network improvement problem while making few questionable simplifying assumptions. Distinctive features of the formulation include a mode abstract multinomial logit modal split model and convex arc transport characteristic improvement functions. Two of the most questionable assumptions are that there is only one transport commodity class and that the arc transport characteristics are not affected by arc flows.
3. The proposed new formulation is very difficult to solve. Since the objective is non-convex, the only methods which can guarantee a global optimal solution are integer

procedures such as branch and bound. Since integer procedures cannot solve problems involving large-scale networks, a heuristic solution methodology is developed.

4. A solution methodology is developed based on Steenbrink's continuous optimal adjustment heuristic and the solution of a concave disutility transportation assignment problem [Steenbrink, 1974]. In very test run the methodology converges to a solution, which is guaranteed to be feasible to the original problem and "good" in the sense that it is the result of a suboptimization process. The solution need not be unique. Finally, the methodology cannot be guaranteed to converge to a global optimal solution.
5. The general solution methdology can be used with any mode-abstract modal split model.
6. The proposed methodology is easily extendible to some specific cases where the arc transport characteristics are functions of arc flow.
7. The algorithm which is developed and tested for the solution of the concave disutility transportation assignment problem is based on the heuristic local optimum-seeking procedure proposed by Yaged [Yaged, 1971]. A second algorithm is proposed, which uses Dantzig-Wolfe decomposition to solve an Arc-Path formulation of the general multi-commodity fixed charge network flow problem.

8. The solution methodology is viable. Although solution times are long, this is not unusual for problems of this size or design-construction projects of this scale. Solution times might be shortened considerably by proper selection of methodology parameters. Solution times, might be further shortened by as much as 50% by eliminating Phase I portion of the two phase algorithm. However, this may cause an appreciable deterioration in solution quality.
9. Any reasonable search procedure developed for this methodology must consider varying both the set of initial modal splits and the set of initial arc lengths. The results of any search procedure can be plotted on a total savings - investment graph, and, after defining the maximum savings envelope over investment, the decision maker might then select the preferred combination of investment and total savings.
10. The tendency of solutions to cluster around certain points tends to indicate the presence of local optimal solutions to the problem.

1.2 The Multi-Commodity Problem

A number of conclusions can be drawn from the research relating to the multi-commodity extension of the problem:

11. No significant research results have been published concerning the improvement of multi-modal networks with more than one transport commodity class.
12. The proposed multi-commodity formulation eliminates one of the questionable assumptions discussed in conclusion 2 above and, thus, is a more realistic model of the problem. The rest of conclusion 2 and conclusion 3 also hold for the multi-commodity formulation.
13. The methodology for the single commodity problem is extended to generate solutions to the multi-commodity problem. The resulting multi-commodity methodology has characteristics similar to those discussed in conclusions 4, 5, and 6 above.
14. The Yaged Algorithm is extended to generate local optimal solutions to the multi-commodity, concave disutility transportation assignment problem.
15. With one major exception, conclusions drawn from the multi-commodity results parallel those of the single commodity problem, conclusions 8, 9, and 10 above. The single exception is that the multi-commodity methodology requires a substantial amount of CPU time to generate a single solution, approximately twice that required for the corresponding single commodity problem. Thus, there

is a valid question as to whether the proposed multi-commodity methodology is a viable vehicle for generating solutions to the multi-commodity problem.

16. Limited comparisons between single-commodity and multi-commodity solutions to the same general problem suggest that, at least for some starting solutions, the final solutions may not differ enough to warrant the additional expense required to solve the multi-commodity problem. However, this may be a function of the uniformity of the transport commodity classes being considered as well as the initial starting solutions.

2. Recommendations for Future Research

Prior to concluding this research effort, it should be noted that a number of important questions remain unanswered and a number of important issues remain unaddressed. The purpose of this section is to identify these questions and issues and indicate potential directions for future research efforts.

In Chapter II a number of assumptions were made in order to simplify the problem. However, several of these assumptions are known to be questionable. First, it was assumed that there was only one transport commodity class using the network. This was later relaxed to include multiple commodities, and a solution methodology was developed. Results obtained from test runs were not encouraging however, with the principal difficulty being lengthy computation times. In general, these times were twice those required to generate comparable single commodity solutions. Lengthy CPU times were directly attributable to the large number of I-O generations necessitated by the large storage requirements of the solution methodology coupled with the relatively small core storage capacity of the CYBER 74 computing system. Potential research relating to this problem includes:

1. Revise the computer programs assuming additional core storage availability and evaluate the effect on solution times. Note that most variables would be maintained in core storage.
2. Revise the computer programs to make them more time and storage efficient and evaluate the effect on solution times. In particular, the network algorithms can be made much more efficient and to provide for restart capability [Rardin, 1978].
3. Identify additional simplifying assumptions, implement these assumptions, and evaluate their effect on solution times. One such assumption might be that all commodities use the same paths.
4. Evaluate the use of the single commodity problem solution as an approximation to that of the multi-commodity problem.

A second questionable assumption was that the arc transport characteristics (ATC) were not affected by arc flow. Subsequent analysis showed that if the proposed methodology were used to solve a problem where an initial arc fixed cost and/or arc investment were shared proportionately by users, the decomposition procedure might still be useful in solving the resulting problem. Analysis also showed that if the assumption negating ATC congestion effects were relaxed, the resulting problem would be inconsistent. Potential research relating to this problem includes: investigate the development of a consistent formulation which would include congestion. Note that such a formulation must extend the concept of maximum utility modal paths and modal split.

A third questionable assumption was that the various transport facilities could be separated into a number of functional independent, uni-directional facilities. Potential research relating to this assumption includes:

1. Examine the various types of physical transport facilities and determine which are separable into independent functional uni-directional facilities and which must be considered as a unit.
2. Revise the methodology to include these interdependencies. Note that this should cause little problem since the Yaged Algorithm was originally intended for use with undirected arcs.

In the development of the general methodology, a number of modifiable components and parameters were identified, Potential research relating to these include:

1. Revise the computer programs so that they may use the

alternate modal split convergence criteria proposed in Chapter III, Section 1.2. Evaluate the performance of the methodology using these criteria.

2. Investigate the development of lower bounds on the objective of the problem which are tighter than those proposed in Chapter III, Section 1.2. Note that a tight lower bound would greatly facilitate the decision to terminate the overall methodology.
3. Lacking a good lower bound, identify reasonable non-lower bound-based decision rules which could be used to terminate the overall methodology. Evaluate their performance. Note that one such rule is postulated in Chapter IV, Section 2.4.
4. Evaluate the performance of the two phase algorithm versus the Yaged algorithm in solving the concave disutility transporation assignment problem.
5. Investigate methods which could be used to generate sets of initial arc lengths for use in the two phase (or Yaged) algorithm. Evaluate the performance of the methods. One reasonable method might be to let the initial length associated with a line haul arc be proportional to the physical length of the arc.
6. Implement the procedure developed in Chapter VI to solve the concave disutility transportation assignment problem. Evaluate its performance with respect to that of the two phase algorithm.

APPENDICES

APPENDIX A

SINGLE COMMODITY PROGRAMS

1. NETED
2. INFOD
3. INMS
4. INAC
5. SMS
6. CNCASNB
7. TESTMS

```
C       PROGRAM NETED DEVELOPS THE EXPANDED NETWORK USED IN THE CONCAVE
C       ASSIGNMENT ROUTINE.
        PROGRAM NETED(OUTPUT=400B,TAPE1=400B,TAPE2=400B,TAPE7=400B,
       &TAPE8=400B,TAPE6=OUTPUT)
        INTEGER Z,PZ(121),POD(1152,2),O,P(1152),A(3080,3),D(50),
       &OD(1600),ODC(400,4),AP(50),ARC
        DIMENSION MZ(120,3),M(120),FD(50),FODC(400),LA(3080)
        READ(7,100) Z,MD,MA
        READ(7,101)((MZ(I,J),J=1,3),I=1,Z)
        IN=1
        DO 1 I=1,Z
        M(I)=0
        DO 2 J=1,3
        M(I)=M(I)+MZ(I,J)
  2     CONTINUE
        PZ(I)=IN
        IN=IN+3*M(I)+2
  1     CONTINUE
        PZ(Z+1)=IN
        N=IN-1
        DO 3 I=1,N
        POD(I,2)=0
  3     CONTINUE
        ND=0
        NOD=1
        NA=1
        NN=1
        DO 4 I=1,Z
        POD(NN,2)=1
        O=M(I)
        P(NN)=NA
        DO 5 J=1,O
        A(NA,1)=NN
        A(NA,2)=NN+J
        A(NA,3)=0
        IZ=0
        DO 30 KM=1,3
        IF(MZ(I,KM).EQ.0)GO TO 30
        IZ=IZ+1
        IF(IZ.EQ.J)GO TO 31
        GO TO 30
 31     MODE=KM
        GO TO 32
 30     CONTINUE
 32     LA(NA)=MODE
        NA=NA+1
  5     CONTINUE
        READ(7,100)(D(J),J=1,MD)
        READ(7,102)(FD(J),J=1,MD)
        POD(NN,1)=NOD
        DO 6 J=1,MD
        IF(D(J).EQ.0) GO TO 7
        OD(NOD)=PZ(D(J)+1)-1
        ODC(ND+J,1)=NOD
        FODC(ND+J)=FD(J)
        NOD=NOD+1
  6     CONTINUE
        NDZ=MD
        GO TO 8
  7     NDZ=J-1
  8     NN=NN+1
        DO 9 J=1,3
        IF(MZ(I,J).EQ.0)GO TO 10
        P(NN)=NA
        A(NA,1)=NN
```

```
      A(NA,2)=NN+0
      A(NA,3)=0
      LA(NA)=3+J
      NA=NA+1
      POD(NN,1)=NOD
      DO 11 K=1,MD
      IF(D(K).EQ.0)GO TO 12
      IF(MZ(D(K),J).EQ.0)  GO TO 13
      OD(NOD)=PZ(D(K)+1)-1
      ODC(ND+K,J+1)=NOD
      NOD=NOD+1
      GO TO 11
13    ODC(ND+K,J+1)=0
11    CONTINUE
12    NN=NN+1
      GO TO 9
10    DO 14 L=1,MD
      IF(D(L).EQ.0)GO TO 9
      ODC(ND+L,J+1)=0
14    CONTINUE
9     CONTINUE
      ND=ND+NDZ
      DO 15 J=1,3
      READ(7,100)(AR(K),K=1,MA)
      IF(MZ(I,J).EQ.0)GO TO 15
      P(NN)=NA
      DO 16 K=1,MA
      IF(AR(K).EQ.0)GO TO 17
      A(NA,1)=NN
      IC=0
      DO 18 L=1,J
      IC=IC+MZ(AR(K),L)
18    CONTINUE
      A(NA,2)=PZ(AR(K))+2*M(AR(K))+IC
      A(NA,3)=0
      LA(NA)=6+J
      NA=NA+1
16    CONTINUE
17    POD(NN,1)=NOD
      NN=NN+1
15    CONTINUE
      K1=NN-O
      K2=NN-1
      DO 19 J=1,O
      P(NN)=NA
      IZ=0
      DO 33 KM=1,3
      IF(MZ(I,KM).EQ.0)GO TO 33
      IZ=IZ+1
      IF(IZ.EQ.J)GO TO 34
      GO TO 33
34    MODE=KM
      GO TO 35
33    CONTINUE
35    DO 20 K=K1,K2
      A(NA,1)=NN
      A(NA,2)=K
      IF(K.EQ.NN-O)GO TO 21
      A(NA,3)=1
      IZ=0
      DO 36 KM=1,3
      IF(MZ(I,KM).EQ.0)GO TO 36
      IZ=IZ+1
      IF(IZ.EQ.K-K1+1)GO TO 37
      GO TO 36
```

```
37      MODE1=KM
        GO TO 38
36      CONTINUE
38      IF(MODE.EQ.1.AND.MODE1.EQ.2)GO TO 39
        IF(MODE.EQ.2.AND.MODE1.EQ.1)GO TO 39
        IF(MODE.EQ.1.AND.MODE1.EQ.3)GO TO 40
        IF(MODE.EQ.3.AND.MODE1.EQ.1)GO TO 40
        LA(NA)=15
        GO TO 22
39      LA(NA)=13
        GO TO 22
40      LA(NA)=14
        GO TO 22
21      A(NA,3)=0
        LA(NA)=9+MODE
22      NA=NA+1
20      CONTINUE
        A(NA,1)=NN
        A(NA,2)=P7(I+1)-1
        A(NA,3)=0
        LA(NA)=15+MODE
        NA=NA+1
        POD(NN,1)=NOD
        NN=NN+1
19      CONTINUE
        P(NN)=NA
        POD(NN,1)=NOD
        NN=NN+1
4       CONTINUE
        P(NN)=NA
        POD(NN,1)=NOD
        POD(NN,2)=0
        ARC=NA-1
        IP=NOD-1
        WRITE(1,100)ND,IP
        WRITE(1,100)((ODC(I,J),J=1,4),I=1,ND)
        WRITE(1,102)(FODC(I),I=1,ND)
        WRITE(2,100)N,ARC,Z,IP
        WRITE(2,100)((A(I,J),J=1,3),I=1,ARC)
        WRITE(2,100)(P(I),I=1,IN)
        WRITE(2,100)(OD(I),I=1,IP)
        WRITE(2,100)((POD(I,J),J=1,2),I=1,IN)
        WRITE(8,100)ARC
        WRITE(8,100)(LA(I),I=1,ARC)
        STOP
100     FORMAT(20I4)
101     FORMAT(80I1)
102     FORMAT(8F10.0)
        END
```

```
C     PROGRAM INFODC SETS UP THE INITIAL FOD VECTOR, O-D DEMAND
C     BY MODE, USING AN INITIAL ESTIMATE OF MODAL SPLIT.
      PROGRAM INFOD(OUTPUT,TAPE1,TAPE4,TAPE14,TAPE6=OUTPUT)
      INTEGER ODC(400,4)
      REAL MS(4)
      DIMENSION FODC(400),FOD(1600)
      READ(1,100)ND,IP
      READ(1,100)((ODC(I,J),J=1,4),I=1,ND)
      READ(1,102)(FODC(I),I=1,ND)
      DO 1 I=1,ND
      READ(14,105)(MS(J),J=1,4)
      DO 2 J=1,4
      IF(ODC(I,J).EQ.0)GO TO 2
      FOD(ODC(I,J))=FODC(I)*MS(J)
    2 CONTINUE
    1 CONTINUE
      WRITE(4,102)(FOD(I),I=1,IP)
      STOP
  100 FORMAT(20I4)
  102 FORMAT(8F10.0)
  105 FORMAT(4F4.2)
      END
```

```
C     PROGRAM INMS RANDOMLY DETERMINES AN INITIAL SET OF MODAL SPLITS.
      PROGRAM INMS(INPUT,OUTPUT,TAPE7,TAPE14,TAPE18,TAPE5=INPUT,
     &TAPE6=OUTPUT)
      DIMENSION MZ(120,3),R(4)
      INTEGER O,D
      REAL MS(4)
      READ(5,1000)ISEED
      CALL RANSET(ISEED)
      READ(7,100)I
      READ(7,101)((MZ(I,J),J=1,3),I=1,120)
      DO 1 I=1,376
      READ(18,100)O,D
      R(1)=RANF(N)
      DO 2 J=1,3
      IF(MZ(O,J).NE.1)GO TO 3
      IF(MZ(D,J).NE.1)GO TO 3
      R(J+1)=RANF(N)
      GO TO 2
    3 R(J+1)=0.
    2 CONTINUE
      T=0.
      DO 4 J=1,4
      T=T+R(J)
    4 CONTINUE
      DO 5 J=1,4
      MS(J)=R(J)/T
    5 CONTINUE
      WRITE(14,105)(MS(J),J=1,4)
    1 CONTINUE
      STOP
  100 FORMAT(20I4)
  101 FORMAT(30I1)
  105 FORMAT(4F4.2)
 1000 FORMAT(I15)
      END
```

```
C         PROGRAM INAC RANDOMLY GENERATES AN INITIAL SET OF ARC COSTS.
          PROGRAM INAC(INPUT,OUTPUT,TAPE13,TAPE5=INPUT,TAPE6=OUTPUT)
          DIMENSION A(3080)
          READ(5,120)N,ISEED
          CALL RANSET(ISEED)
          DO 1 I=1,N
          A(I)=RANF(IX)
 1        CONTINUE
          WRITE(13,103)(A(I),I=1,N)
          STOP
 120      FORMAT(I6,I15)
 103      FORMAT(8F10.5)
          END
```

```
C         PROGRAM SMS DEVELOPS A SET OF EXTREME MODAL SPLITS
C         ACCORDING TO THE INPUT VECTOR M(J).  IF M(J)=1,
C         THEN MODE J RECIEVES AN EQUAL MODAL SHARE.
          PROGRAM SMS(INPUT,OUTPUT,TAPE7,TAPE14,TAPE18,TAPE5=INPUT,
         &TAPE6=OUTPUT)
          INTEGER O,D
          DIMENSION MZ(120,3),M(4)
          REAL MS(4)
          READ(5,101)(M(I),I=1,4)
          READ(7,100)N
          READ(7,101)((MZ(I,J),J=1,3),I=1,120)
          DO 1 I=1,63
          READ(18,100)O,D
          IC=0
          DO 6 J=1,4
          MS(J)=0.
 6        CONTINUE
          IF(M(1).EQ.1)IC=1
          DO 2 J=2,4
          IF(M(J).EQ.0)GO TO 2
          IF(MZ(O,J-1).EQ.0.OR.MZ(D,J-1).EQ.0)GO TO 2
          IC=IC+1
 2        CONTINUE
          IF(IC.EQ.0)GO TO 4
          RIC=IC
          IF(M(1).EQ.1)MS(1)=1./RIC
          DO 3 J=2,4
          IF(M(J).EQ.0)GO TO 3
          IF(MZ(O,J-1).EQ.0.OR.MZ(D,J-1).EQ.0)GO TO 3
          MS(J)=1./RIC
 3        CONTINUE
          GO TO 5
 4        MS(1)=1.
 5        WRITE(14,105)(MS(J),J=1,4)
 1        CONTINUE
          STOP
 100      FORMAT(20I4)
 101      FORMAT(80I1)
 105      FORMAT(4F4.2)
          END
```

```
C     PROGRAM CNCASNB IS A CONCAVE ASSIGNMENT ROUTINE BASED ON THE
C     YAGED METHOD OF DETERMINING A LOCAL OPTIMAL SOLUTION.
      PROGRAM CNCASNB(OUTPUT=400B,TAPE2=400B,TAPE3=400B,TAPE4=400B,
     &TAPE9=400B,TAPE12=400B,TAPE13=400B,TAPE22=400B,INPUT=400B,
     &TAPE5=INPUT,TAPE6=OUTPUT)
      REAL IPM
      INTEGER ARC,Z,A,P,OD,POD,C,DE,T
      COMMON FOD(230),AC(3080),F(3080),A(3080,3),P(1152),
     &OD(230),POD(1152,2),COD(230),N,TC(1152),T(1152),D(1152)
      DIMENSION F1(3080),C(11),R(12)
      READ(2,100)N,ARC,Z,IP
      READ(2,100)((A(I,J),J=1,3),I=1,ARC)
      N1=N+1
      READ(2,100)(P(I),I=1,N1)
      READ(2,100)(OD(I),I=1,IP)
      READ(2,100)((POD(I,J),J=1,2),I=1,N1)
      READ(4,102)(FOD(I),I=1,IF)
      READ(13,103)(AC(I),I=1,ARC)
      READ(5,100)ICOU
      ICOUN=1
      IY=0
      IF(ICOU.LT.1)IY=IY+1
      DO 1 I=1,ARC
      F1(I)=0.
      F(I)=0.
1     CONTINUE
17    DO 2 I=1,N
      I1=POD(I,1)
      I2=POD(I+1,1)-1
      IF(I1.GT.I2)GO TO 2
      DO 3 J=1,N
      D(J)=0
3     CONTINUE
      DE=0
      DO 4 J=I1,I2
      D(OD(J))=J
      DE=DE+1
4     CONTINUE
      IF(POD(I,2).EQ.1)GO TO 5
      IQ=0
      GO TO 6
5     IQ=1
6     CALL SPTR(I,DE,IQ)
2     CONTINUE
      IF(IY.EQ.1)GO TO 45
      WRITE(6,2000)
2000  FORMAT(" ","*")
      GO TO 46
45    WRITE(6,2001)
2001  FORMAT(" ","$")
46    DO 7 I=1,ARC
      IF(ABS(F1(I)-F(I)).GE.10.)GO TO 8
7     CONTINUE
      IF(IY.EQ.1)GO TO 9
      IY=1
8     IF(IY.EQ.0.AND.ICOUN.EQ.ICOU)IY=1
      DO 10 I=1,ARC
      F1(I)=F(I)
      F(I)=0.
10    CONTINUE
      IF(IY.EQ.0)GO TO 50
      DO 51 I=1,ARC
      READ(12,111)NS,(C(J),J=1,8)
      IF(NS.EQ.1)GO TO 52
      IF(NS.EQ.3)GO TO 53
```

```
      IF(F1(I).LE.C(1))GO TO 54
      IF(F1(I).GE.C(2))GO TO 55
      AC(I)=C(5)+C(6)/F1(I)**2
      GC TO 51
52    AC(I)=C(1)
      GO TO 51
53    IF(F1(I).LE.C(1))GO TO 56
      AC(I)=C(3)+C(4)/F1(I)**2
      GC TO 51
56    AC(I)=C(2)
      GC TO 51
54    AC(I)=C(3)
      GC TO 51
55    AC(I)=C(4)
51    CONTINUE
      GO TO 57
50    DC 11 I=1,ARC
      READ(12,111)NS,(C(J),J=1,11)
      IF(NS.EQ.1)GC TO 12
      IF(NS.EQ.3)GO TO 13
      IF(F1(I).LE.C(1))GC TO 14
      IF(F1(I).GE.C(2))GO TO 15
      AC(I)=(C(5)*F1(I)-C(6)/F1(I)+C(7)*C(10)-C(9)*C(10))/F1(I)
      GC TO 11
12    AC(I)=C(1)
      GO TO 11
13    IF(F1(I).LE.C(1))GO TO 16
      AC(I)=(C(3)*F1(I)-C(4)/F1(I)+C(5)*C(10)-C(9)*C(10))/F1(I)
      GC TO 11
16    AC(I)=C(2)
      GO TO 11
14    AC(I)=C(3)
      GO TO 11
15    AC(I)=(C(4)*F1(I)+(C(11)-C(9))*C(10))/F1(I)
11    CCNTINUE
57    REWIND 12
      GC TO 17
9     READ(9,103)A2,A3
      TCTDIS=0.
      TCTI=0.
      TCTU=0.
      DC 20 I=1,ARC
      READ(9,100)J
      READ(9,110)(R(J),J=1,12)
      READ(12,111)NS,(C(J),J=1,8)
      IF(NS.EQ.1)GO TO 21
      IF(NS.EQ.3)GC TO 22
      IF(F(I).LE.C(1))GO TO 23
      IF(F(I).GE.C(2))GO TO 24
      IPM=C(7)+C(8)/F(I)
      GC TO 25
21    IPM=R(10)
      GC TO 25
22    IF(F(I).LE.C(1))GO TO 26
      IPM=C(5)+C(6)/F(I)
      GC TO 25
23    IPM=R(10)
      GC TO 25
24    IPM=R(11)
      GC TO 25
26    IPM=R(10)
25    AC(I)=R(12)*(R(1)*(IPM-R(4))**2+R(7)+A2*(R(2)*(IPM-R(5))**2+R(8)
     &+A3*(R(3)*(IPM-R(6))**2+R(9)))
      TCTDIS=TOTDIS+AC(I)*F(I)+IPM*R(12)
      AI=IPM*R(12)
```

```
      TOTI=TOTI+AI
      TOTU=TOTU+AC(I)*F(I)
      WRITE(22,102)AI,F(I)
20    CONTINUE
      DO 27 I=1,ARC
      F(I)=0.
27    CONTINUE
      DO 28 I=1,N
      I1=POD(I,1)
      I2=POD(I+1,1)-1
      IF(I1.GT.I2)GO TO 28
      DO 29 J=1,N
      D(J)=0
29    CONTINUE
      DE=0
      DO 30 J=I1,I2
      D(OD(J))=J
      DE=DE+1
30    CONTINUE
      IF(POD(I,2).EQ.1)GO TO 31
      IQ=0
      GO TO 32
31    IQ=1
32    CALL SPTR(I,DE,IQ)
28    CONTINUE
      WRITE(3,103)(COD(I),I=1,IP)
      WRITE(3,102)(FCD(I),I=1,IP)
      WRITE(6,112)TOTDIS,TOTI,TOTU
      REWIND 13
      WRITE(13,103)(AC(I),I=1,ARC)
      STOP
100   FORMAT(20I4)
102   FORMAT(8F10.0)
103   FORMAT(8F10.5)
110   FORMAT(12E10.3)
111   FORMAT(I1,11E10.3)
112   FORMAT(3F14.0)
      END
C     SUBROUTINE SPTR IS A SHORTEST PATH TREE ROUTINE
      SUBROUTINE SPTR(O,DE,IQ)
      INTEGER O,DE,T,B,FS,P,S(3080,2),D,A,OD,POD
      COMMON FOD(230),AC(3080),F(3080),A(3080,3),P(1152),
     &OD(230),POD(1152,2),COD(230),N,TC(1152),T(1152),D(1152)
      DIMENSION SC(3080)
      DO 1 I=1,N
      TC(I)=-1.
1     CONTINUE
      ITOT=0
      T(1)=0
      TC(O)=0.
      B=O
      IT=2
      FS=1
      L=2
      IS=1
14    IA=P(B)
      IB=P(B+1)-1
      IF(IB+1-IA.EQ.0)GO TO 2
      DO 3 I=IA,IB
      IF(IQ.EQ.0)GO TO 4
5     S(IS,1)=I
      S(IS,2)=0
      SC(IS)=TC(B)+AC(I)
      S(L,2)=IS
      L=IS
```

```
      IS=IS+1
      GO TO 3
4     IF(A(I,3).EQ.1)GO TO 3
      GO TO 5
3     CONTINUE
2     K=0
      J=FS
      CM=10.**50
8     IF(TC(A(S(J,1),2)).GE.0.)GO TO 6
      IF(SC(J).GE.CM)GO TO 7
      MJ=J
      CM=SC(J)
7     K=J
      J=S(J,2)
      IF(J.NE.0)GO TO 8
      GO TO 9
6     IF(J.EQ.FS)GO TO 10
      IF(J.NE.L)GO TO 11
      L=K
      S(K,2)=0
      GO TO 9
10    FS=S(J,2)
      J=FS
      GO TO 8
11    S(K,2)=S(J,2)
      J=S(J,2)
      GO TO 8
9     ID=A(S(MJ,1),2)
      T(IT)=S(MJ,1)
      TC(ID)=SC(MJ)
      IF(D(ID).EQ.0)GO TO 12
      ITOT=ITOT+1
      IF(ITOT.EQ.DE)GO TO 13
12    B=ID
      IT=IT+1
      GO TO 14
13    CALL ASNFLW(O,IT)
      RETURN
      END
C     SUBROUTINE ASNFLW ASSIGNS THE FLOW TO THE NETWORK
C     ACCORDING TO THE TREES DETERMINED IN SPTF.
      SUBROUTINE ASNFLW(O,IT)
      INTEGER O,POD,CD,T,A,D,P
      COMMON FOD(230),AC(3080),F(3080),A(3080,3),P(1152),
     &OD(230),POD(1152,2),COD(230),N,TC(1152),T(1152),D(1152)
      DIMENSION FL(1152)
      I1=POD(O,1)
      I2=POD(O+1,1)-1
      DO 1 I=1,N
      FL(I)=0.
1     CONTINUE
      DO 2 I=I1,I2
      FL(OD(I))=FOD(I)
2     CONTINUE
      I2=IT-1
      DO 3 I1=1,I2
      I=IT-I1+1
      L=T(I)
      J=A(L,1)
      K=A(L,2)
      F(L)=F(L)+FL(K)
      FL(J)=FL(J)+FL(K)
      IF(D(K).EQ.0)GO TO 3
      COD(D(K))=TC(K)
3     CONTINUE


      RETURN
      END
```

```
C       PROGRAM TESTMS TESTS THE ASSUMED MODAL SPLIT AGAINST THE
C       ESTIMATED MODAL SPLIT.
        PROGRAM TESTMS(INPUT,OUTPUT,TAPE1,TAPE3,TAPE4,TAPE15,TAPE5=INPUT,
       &TAPE6=OUTPUT)
        INTEGER ODC(400,4)
        REAL MSO,MSE,MS(400,4)
        DIMENSION FODC(400),COD(1600),FOD(1600)
        READ(5,106)EPS,AL,A1
        READ(1,100)ND,IP
        READ(1,100)((ODC(I,J),J=1,4),I=1,ND)
        READ(1,102)(FODC(I),I=1,ND)
        READ(3,103)(COD(I),I=1,IP)
        READ(3,102)(FOD(I),I=1,IP)
        M=0
        DO 1 I=1,ND
        MI=0
        DO 2 J=2,4
        IF(ODC(I,J).EQ.0)GO TO 2
        IF(ABS(COD(ODC(I,1))-COD(ODC(I,J))).LE..001)GO TO 3
  2     CONTINUE
        DU=EXP(A1*COD(ODC(I,1)))
        GO TO 4
  3     DU=0.
        MI=1
  4     DO 5 J=2,4
        IF(ODC(I,J).EQ.0)GO TO 5
        DU=DU+EXP(A1*COD(ODC(I,J)))
  5     CONTINUE
        DO 6 J=1,4
        IF(ODC(I,J).EQ.0)GO TO 7
        MSO=FOD(ODC(I,J))/FODC(I)
        IF(J.NE.1)GO TO 8
        IF(MI.EQ.0)GO TO 8
        MSE=0.
        GO TO 9
  8     MSE=EXP(A1*COD(ODC(I,J)))/DU
  9     E=MSE-MSO
        IF(ABS(E).GT.EPS)M=1
        MS(I,J)=MSO+AL*E
        FOD(ODC(I,J))=FODC(I)*MS(I,J)
        GO TO 6
  7     MS(I,J)=0.
  6     CONTINUE
  1     CONTINUE
        WRITE(4,102)(FOD(I),I=1,IP)
        WRITE(15,100)M
        WRITE(15,103)((MS(I,J),J=1,4),I=1,ND)
        STOP
  100   FORMAT(20I4)
  102   FORMAT(8F10.0)
  103   FORMAT(8F10.5)
  106   FORMAT(2F3.1,F10.8)
        END
```

APPENDIX B

ZONE STRUCTURE

1. Corridor Zones
2. Non-BEA External Zones
3. Zones Comprised of Integral BEAs

CORRIDOR ZONES

Zone No.	Nodal City	APDC*	Included Counties
1.	Brunswick, Ga.	-------------	Liberty, Long, McIntosh, Glynn, Camden Co., Ga.
2.	Jacksonville, FL	APDC 1, Fl.	Baker, Clay, Duval, Nassau, Putnam, St. Johns
3.	Statesboro, Ga.	Southern	Appling, Bullock, Candler, Evans, Jeff Davis, Tattnall, Toombs, Wayne
4.	Waycross, Ga.	Slash Pine	Atkinson, Bacon, Brantley, Charlton, Clinch, Coffee, Pierce, Ware
5.	Dublin, Ga.	Heart of Ga.	Bleckley, Dodge, Laurens, Montgomery, Pulaski, Telfair, Treutlen, Wheeler, Wilcox
6.	Valdosta, Ga.	Coastal Plain	Ben Hill, Berrier, Brooks, Cook, Echols, Irwin, Lanier, Lowndes, Tift, Turner
7.	Macon, Ga.	Middle Ga.	Bibb, Crawford, Houston, Jones, Monroe, Peach, Twiggs
8.	Cordele, Ga.	Middle Flint	Crisp, Dooly, Marion, Macon, Schley, Sumter, Taylor, Webster,
9.	Albany, Ga.	S.W. Ga.	Baker, Calhoun, Colquitt, Decatur, Dougherty, Early, Grady, Lee, Miller, Mitchell, Seminole, Terrell, Thomas, Worth
10.	Lagrange, Ga.	Chattahoochee-Flint	Carroll, Coweta, Heard, Meriwether, Troup
11.	Columbus, Ga.	Lower Chattahoochee Valley APDC 10, Al.	Chattahoochee, Clay, Harris, Muscogee, Quitman, Randolph Stewart, Talbot, Ga., Lee, Russell, Al.
12.	Anniston, Al.	APDC-4	Calhoun, Chambers, Cherokee, Clay, Cleburne, Cosa, Etowah, Randolph, Talladega, Tallapoosa
13.	Montgomery, Al.	APDC-9+	Autauga, Dallas, Elmore, Montgomery, Perry
14.	Troy, Al.	APDC-5	Bullock, Butler, Crenshaw, Lowndes, Macon, Pike
15.	Dothan, Al.	APDC-7	Barbour, Coffee, Covington, Dade, Geneva, Henry, Houston
16.	Decatur, Al.	APDC-11	Cullman, Lawrence, Morgan
17.	Birmingham, Al.	APDC-1	Blount, Chilton, Jefferson, St. Clair, Shelby, Walker
18.	Florence, Al.	APDC-1	Colbert, Franklin, Lauderdale, Marion, Winston
19.	Tuscaloosa, Al.	APDC-2	Bibb, Greene, Fayette, Hale, Lamar, Pickens, Tuscaloosa
20.	Corinth, Ms.	N.E. Ms.	Alcorn, Benton, Marhsall, Prentiss, Tippah, Tishomingo

*Area Planning and Development Commission or equivalent comprehensive planning agency.

Zone No.	Nodal City	APDC*	Included Counties
21.	Tupelo, Ms.	3 Rivers	Calhoun, Chickasaw, Itawanba, Lafayette, Lee, Monroe, Pontotac, Union
22.	Columbus, Ms.	Golden Triangle	Clay, Choctaw, Lowndes, Noxubee, Ortibbeh, Webster
23.	Clarksdale, Ms.	No. Delta	Coahoma, DeSoto, Quitman, Panola, Tate, Tunica
24.	Dyersburg, Ten.	N.W. APDC-	Carroll, Crockett, Dyer, Gibson, Henry, Lake, Obion, Weakley
25.	Jackson, Tn.	SW APDC+	Chester, Decatur, Hardeman, Hardin, Haywood, Henderson, McNairy, Madison, Wayne
26.	Memphis, Tn.	Memphis Delta	Fayette, Lauderdale, Shelby, Tipton
27.	Jonesboro, Ak.	East	Clay, Craighead, Crittenden, Cross, Greene, Lawrence, Lee, Ms. Phillips, Poinsett, Randolph, St. Francis
28.	Searcy, Ak.	White River	Cleburne, Fulton, Independence, Izard, Jackson, Sharp, Stone, Van Buren, White, Woodruff
29.	Harrison, Ak.	--------	Baxter, Boone, Carroll, Marion, Newton, Searcy
30.	Sikeston, Mo.	Bootheel	Bunklin, Mississippi, New Madrid, Plemescot, Scott, Stoddard
31.	Poplar Bluff, Mo.	Ozark Foothills	Butler, Carter, Reynolds, Ripley, Wayne
32.	West Plains, Mo.	So. Cent. Ozark	Douglas, Howell, Oregon, Ozark, Shannon, Texas, Wright
33.	Lebanon, Mo.	Lake of the Ozarks	Camden, Laclede, Miller, Morgan, Pulaski
34.	Marshall, Mo.	Mo. Valley	Carroll, Chariton, Saline
35.	Sedalia, Mo.	Show-Me	Johnson, Lafayette, Pettis
36.	Springfield, Mo.	Lakes Country	Barry, Christian, Dade, Dallas, Greene, Lawrence, Polk, Stone, Taney, Webster
37.	St. Joseph, Mo.	Bi State	Andrew, Buchanon, Clinton, DeKalb, Mo., Doniphan, Ks.
38.	Kansas City, Mo.	Mid America Reg. Council	Cass, Clay, Jackson, Platte, Ray, Mo., Johnson, Leavenworth, Wyandatte Ks.
39.	Nevada, Mo.	Kaysinger Basin	Bates, Benton, Cedar, Henry, Hickory, St. Clair, Vernon
40.	Joplin, Mo.	Ozark Gateway	Barton, Jasper, McDonald, Newton

2. NON BEA EXTERNAL ZONES

BEAs Disrupted: 33, 34, 40, 41, 42, 43, 44 45, 46, 47, 111, 112, 114, 115, 116, 117

Zone No.	Nodal City	BEA	Included Counties
41	Savannah, Ga.		Bryan, Chatham, Effingham, Screven, Ga.; Jasper, S.C.
43	Milledgeville, Ga.		Oconee APDC, Ga: Baldwin, Hancock, Jasper, Putnam, Washington, Wilkerson
44	Atlanta, Ga.	BEA 44 minus:	Cleburne Co., Ala.; Carroll, Coweta Co., Ga.
46	Huntsville, Al.		Limestone, Madison, Marshall Co., Ala.; Lincoln, Franklin Co., Tenn.
49	Cape Girardeau, Mo.		Bolinger, Cape Girardeau, Mo.; Alexander, Hardin, Johnson, Massac, Pope, Pulaski, Union, Ill.; Ballard, Carlisle, Calloway, Fulton, Graves, Hickman, Livingston, Lyon, Marshall, McCracken, Ky.
50	St. Louis, Mo.	BEA 114 minus:	Laclede, Pulaski, Reynolds, Texas, Mo.
52	Columbia, Mo.	BEA 112 minus:	Putnam, Sullivan, Linn, Chariton, Morgan, Camden, Miller Co., Mo.
53	Chillicothe, Mo.		Northwest, Mo., Green Hills APCD, Mo., Atchison, Caldwell, Daviess, Gentry, Grundy, Harrison, Holt, Linn, Livingston, Mercer, Nodaway, Putnam, Sullivan, Worth
56	Topeka, Ks.		Allen, Anderson, Atchison, Bourbon, Brown, Cherokee, Craig, Crawford, Douglas, Franklin, Geary, Jackson, Jefferson, Labette, Linn, Lyon, Marshall, Miami, Montgomery, Nemaha, Neosho, Osage, Ottawa, Pottawatomie, Riley, Shawnee, Wabaunsee, Washington, Wilson, Woodson, Ks.
60	Little Rock, Ak.	BEA 117 minus:	White River APDC, Ak. (See zone 28 for omitted counties)
67	Gainesville, Fl.		Alachua, Bradford, Columbia, Dixie, Gilchrist, Hamilton, Lafayette, Levy, Marion, Sewannee, Union, Fl.

3. ZONES COMPRISED OF INTEGRAL BEAS

Zone No.	Nodal City	BEAs
42	Augusta, Ga.	32
45	Chattanooga, Tn.	48
47	Nashville, Tn.	49
48	Evansville, In.	55
51	Quincy, Il.	113
54	Des Moines, Ia.	80,81, 104, 105, 106
55	Omaha, Ne.	102, 103, 107,108
57	Wichita, Ks.	109, 110
58	Tulsa, Ok.	119
59	Ft. Smith, Ok.	118
61	Greenville, Ms.	134
62	Jackson, Ms.	135
63	Meridian, Ms.	136
64	Mobile, Al.	137
65	Pensacola, Fl.	39
66	Tallahassee, Fl.	38
68	Miami, Fl.	35, 36
69	Boston, Ma.	1, 2, 3, 4, 5
70	Albany, N.Y.	6, 7
71	Buffalo, N.Y.	8, 9, 10
72	New York, N.Y.	14, 15
73	Scranton, Pa.	12, 13
74	Harrisburg, Pa.	11, 16
75	Pittsburgh, Pa.	66, 67
76	Washington, D. C.	17, 18
77	Roanoke, Va.	19, 20
78	Richmond, Va.	21
79	Charlotte, N.C.	25, 26
80	Raleigh, N.C.	23, 24
81	Greenville, S.C.	27, 28
82	Columbia, S.C.	29, 30
83	Knoxville, Tn.	50
84	Charleston, W.V.	51, 52, 65
85	Cincinnati, Oh.	53, 54, 62
86	Dayton, Oh.	61, 63, 69
87	Cleveland, Oh.	68
88	Detroit, Mi.	71, 72, 74
89	Indianapolis, In.	56, 59, 60
90	Chicago, Il.	76, 77, 78, 79
91	Milwaukee, Wi.	82, 83, 84, 85, 86
92	St. Paul, Mn.	88, 89, 90, 91
93	Billings, Mn.	94, 95, 100, 101, 150
94	Denver, Co.	147, 148, 149
95	Oklahoma City, Ok.	120, 121
96	Texarkana, Tx.	131
97	Shreveport, La.	132, 133
98	New Orleans, La.	138
99	Tampa, Fl.	37
100	Amarillo, Tx.	122, 123

Zone No.	Nodal City	BEAs
101	Dallas, Tx.	127, 130
102	El Paso, Tx.	124, 145, 163
103	Austin, Tx.	128, 129
104	San Antonio, Tx.	125, 126, 142, 143, 144
105	Houston, Tx.	139, 140, 141
106	Salt Lake City, Ut.	151, 160
107	Phoenix, Ar.	162
108	Albuqurque, NM	146
109	Seattle, Wa.	153, 154, 155, 156
110	San Francisco, Ca.	166, 167, 168, 171
111	Los Angeles, Ca.	161, 164, 165
112	Charleston, S.C.	31
113	Duluth, Mn.	87
114	Springfield, Il.	57, 58
115	Toledo, Oh.	70, 75
116	Columbus, Oh.	65
117	Portland, Or.	152, 157, 158, 159, 169, 1[illegible]
118	Fargo, ND	92, 93, 96, 97, 98, 99
119	Grand Rapids, Mi.	73
120	Norfolk, Va.	22

APPENDIX C

LINE-HAUL ARCS

1. Highway Arcs
2. Rail Arcs
3. Water Arcs

HIGHWAY ARCS

Arc	Orig.	Dest.	Dist.	Time	Ln. Routes
1	1	2	68	74	4 I-95
2	1	4	49	65	2US-84
3	1	41	70	76	4 I-95
4	2	66	163	177	4 I-10
5	2	67	49	53	4US301 S-24
6	2	68	349	379	4 I-95
7	3	4	108	144	2US-25US-82
8	3	5	72	96	2US-80
9	3	8	130	170	2 I-16 US-1US280
10	3	41	53	58	4 I-16
11	3	42	47	63	2US-25
12	4	2	78	104	2US-23
13	4	6	61	81	2US-84
14	4	8	111	139	2US-82 I-75
15	4	9	113	151	2US-82
16	4	41	94	120	2US-82 I-95
17	5	1	146	195	2US441US341
18	5	4	121	153	2 I-16 US-1
19	5	7	52	57	4 I-16
20	5	8	92	123	2US441US280
21	5	42	85	113	2US319 US-1
22	5	43	47	63	2US441
23	6	2	75	82	4 I-75 I-10
24	6	8	88	96	4 I-75
25	6	9	89	107	2 I-75US-82
26	6	66	71	82	2US-84US221 I-10
27	6	67	93	101	4 I-75
28	7	43	31	46	2 S-49
29	7	44	78	85	4 I-75
30	8	7	56	61	4 I-75
31	8	11	87	116	2US280
32	9	8	34	51	2S-257
33	9	11	77	104	2US-82 S-55US280
34	9	66	98	131	2US-19US319
35	10	44	49	53	4 I-85
36	11	7	98	131	2US-80
37	11	10	50	54	4I-185
38	12	10	69	95	2US431S-244
39	12	13	88	96	2US231
40	12	17	61	66	4 I-20
41	12	44	86	93	4 I-20
42	12	45	111	129	2US431 I-59
43	12	46	98	131	2US431
44	13	9	150	200	2US-82
45	13	10	88	96	4 I-85
46	13	11	86	100	2 I-85US280
47	13	14	44	48	4US231
48	13	17	94	102	4 I-65
49	13	19	105	140	2US-82
50	13	63	153	204	2US-80
51	13	64	179	195	4 I-65
52	13	65	154	187	2 I-65US-31US-29
53	14	64	159	185	2US-29 S-10 I-65
54	14	65	162	216	2US-29
55	15	9	82	122	2 S-62
56	15	11	105	140	2US431
57	15	14	56	61	4US231
58	15	65	141	161	2US231 I-10
59	15	66	101	117	2US231 I-10
60	16	18	41	45	4US-72
61	16	46	23	25	4US-72
62	16	47	116	126	4 I-65
63	17	11	148	197	2US280
64	17	16	81	88	4 I-65
65	17	19	56	61	4 I-59
66	17	21	165	220	2US-78
67	17	45	150	163	4 I-59
68	18	20	54	72	2US-43US-72
69	18	22	127	169	2US-43US-78 S-12
70	18	47	104	131	2US-43 I-65
71	19	18	116	155	2US-43
72	19	22	61	81	2US-82
73	19	63	75	82	4 I-59
74	19	64	197	262	2US-43
75	20	25	54	72	2US-45
76	20	26	94	125	2US-72
77	21	20	50	67	2US-45
78	21	23	110	164	2 S-6
79	21	26	97	129	2US-78
80	21	62	215	263	2 S-6 I-55
81	22	21	68	91	2US-45
82	22	23	165	220	2US-82US49E
83	22	61	160	213	2US-82
84	22	62	168	203	2US-82 I-55
85	22	63	89	118	2US-45
86	23	26	78	104	2US-61
87	23	28	136	181	2US-61US-49 S-1 S-64
88	23	60	140	173	2US-61US-49 I-40

Arc.	Orig.	Dest.	Dist.	Time	La. Routes
89	23	61	70	93	2US-61
90	23	62	186	217	2 S-6 I-55
91	24	27	100	121	2I-155 I-55 S-18
92	24	30	78	85	4I-155 I-55
93	24	48	203	232	2US-51PTNPKTPKWYPPKWY
94	25	24	41	61	2 S-20
95	25	47	150	163	4 U-40
96	25	48	235	271	2 I-40 S-13
97	26	24	74	80	4US-51
98	26	25	75	82	4 I-40
99	26	27	65	81	2 I-55US-63
100	26	28	92	123	2US-64
101	26	30	145	157	4 I-55
102	26	60	138	150	4 I-40
103	27	28	79	105	2 S-39US-64US-67
104	27	29	166	221	2US-63US-62
105	27	30	120	149	2 S-18 I-55
106	27	31	91	121	2US-63US-67
107	27	32	104	139	2US-63
108	28	29	165	220	2US167US-64US-65
109	28	32	142	189	2US167US-63
110	28	60	43	47	4US-67
111	29	32	109	145	2US-62 S-5US-160
112	29	36	65	87	2US-65
113	29	40	148	197	2US-62US-71
114	29	58	186	248	2US-62 S-33
115	29	59	132	176	2US-62US-71
116	29	60	134	171	2US-65 I-40
117	30	31	47	63	2US-60
118	30	47	190	223	2US-60 I-24
119	30	48	227	267	2 I-57 S-13US-60
120	30	49	38	41	4 I-55
121	31	32	100	133	2US160
122	31	50	202	231	2US-60 I-55
123	32	33	111	148	2US-60 S-5
124	32	36	110	146	2US-60
125	32	50	210	254	2US-63 I-44
126	32	52	205	273	2US-63
127	33	35	99	132	2 S-64US-65
128	33	39	123	164	2 S-5US-54
129	33	49	181	230	2 I-44 S-8US-67US-72
130	33	50	165	179	4 I-44
131	33	52	151	187	2 I-44US-63
132	34	51	75	100	2 S-41US-24
133	34	52	61	69	2US-65 I-70
134	34	53	65	87	2US-65

Arc	Orig.	Dest.	Dist.	Time	La. Routes
135	35	34	24	32	2US-65
136	35	38	83	95	2US-65 I-70
137	35	52	68	78	2US-65 I-70
138	36	33	53	58	4 I-44
139	36	35	120	160	2US-65
140	36	39	91	121	2 S-13US-54
141	36	40	69	75	4 I-44
142	37	53	74	99	2US-36
143	37	54	181	204	2US-36 I-35
144	37	55	152	165	4 I-29
145	37	56	85	113	2US-59
146	38	34	76	85	2 I-70US-65
147	38	37	52	57	4 I-29
148	38	53	100	118	2I-35US-36
149	38	54	195	212	4 I-35
150	38	56	65	71	4 I-70
151	38	57	200	217	4 I-35
152	39	35	127	169	2US-54US-65
153	39	38	98	131	2US-71
154	39	57	170	226	2US-54
155	40	39	64	85	2US-71
156	40	57	218	279	2US166 I-35
157	40	58	95	103	4 I-44
158	40	59	149	199	2US-71
159	41	82	142	154	4 I-95 I-26
160	42	82	69	75	4 I-20
161	43	42	80	100	2 S-22 S-16US278 I-2
162	43	81	158	211	2US441
163	44	42	150	120	4 I-20
164	44	81	119	129	4 I-85
165	45	44	114	124	4 I-75
166	45	83	112	122	4 I-75
167	46	45	75	100	2US-72
168	47	45	128	139	4 I-24
169	47	46	187	209	2 I-65US-72
170	47	83	177	192	4 I-40
171	47	84	384	417	4 I-64
172	47	85	269	315	4 I-65 I-71
173	48	47	159	173	4US-41 I-24
174	48	84	392	426	4 I-64
175	48	85	224	243	4 I-64 I-71
176	49	47	171	194	2US-60 I-24
177	50	47	328	357	4 I-64 I-57 I-24
178	50	48	172	187	4 I-64
179	50	49	148	161	4 I-55
180	50	89	235	255	4 I-70

Arc	Orig.	Dest.	Dist.	Time	La. Routes
181	51	50	116	155	2US-61
182	51	52	119	159	2US-61US-54
183	52	50	106	115	4 I-70
184	53	51	130	172	2US-36US-61
185	53	54	149	171	2US-36 I-35
186	54	89	465	521	4 I-80 I-74
187	54	90	327	355	4 I-80
188	54	92	252	274	4 I-35
189	55	54	132	143	4 I-80
190	56	55	159	212	2US-75
191	57	56	127	138	4KTNPK
192	58	56	195	260	2US-75
193	59	58	117	127	4 I-40
194	59	101	243	264	4US-69 I-40
195	60	59	154	167	4 I-40
196	61	60	151	201	2US-65
197	62	61	120	113	2 I-20US-61
198	62	97	219	238	4 I-20
199	63	62	93	101	4 I-20
200	63	98	194	211	4 I-59
201	64	62	182	198	2US-49US-98
202	64	63	133	146	2US-45
203	64	98	144	157	4 I-10
204	65	64	62	67	4 I-10
205	66	65	186	202	4 I-10
206	67	66	133	145	4 I-75 I-10
207	68	99	268	291	4 I-75
208	70	69	163	177	4 I-90
209	71	70	283	308	4 I-90
210	71	73	246	330	4 I-90 I-81
211	72	69	206	224	4 I-84
212	72	70	154	167	4 I-87
213	72	74	180	196	4 I-78
214	72	76	233	253	4 I-95
215	73	70	173	188	4 I-81 I-88
216	73	72	138	150	4 I-84
217	74	71	278	365	4 2-79 I-90
218	74	73	118	128	4 I-81
219	74	75	189	205	4 I-76
220	75	71	216	236	4 I-79 I-90
221	76	74	107	116	4 I-83
222	76	75	221	240	4 I-76 I-70
223	76	84	344	374	4 I-81 I-64
224	77	74	289	314	4 I-81
225	77	76	225	245	4 I-81 I-66
226	77	78	164	180	4 I-64
227	78	76	106	115	4 I-95
228	79	77	189	205	4 I-77 I-81
229	79	80	167	182	4 I-85 I-40
230	80	77	163	177	4US220 I-85
231	80	78	173	188	4 I-85
232	81	42	104	139	2US-25
233	81	79	90	98	4 I-85
234	81	82	95	103	4 I-26
235	82	79	94	102	4 I-77
236	82	80	205	223	4 I-20 I-95
237	83	77	263	286	4 I-81
238	83	80	359	405	4 I-40
239	83	81	150	163	4 I-40 I-26
240	83	84	335	364	4 I-81 I-77
241	83	85	253	275	4 I-75
242	84	75	213	231	4 I-79
243	84	77	181	197	4 I-77 I-81
244	84	78	306	330	4 I-64
245	84	79	287	312	4 I-77
246	85	84	208	226	4 I-75 I-64
247	85	86	52	56	4 I-75
248	87	71	187	203	4 I-90
249	87	73	310	337	4 2-80 I-84
250	87	75	129	140	4I-80S
251	87	84	243	264	4 I-77
252	89	47	279	320	4 I-65
253	89	48	167	210	2 I-70US-41
254	89	85	106	115	4 I-74
255	89	86	107	116	4 I-70
256	90	48	296	322	4 I-57 I-64
257	90	49	376	409	4 I-57
258	90	51	308	363	2 I-55 S-125US-24
259	90	88	266	289	4 I-94
260	90	89	181	197	4 I-65
261	91	90	87	95	4 I-94
262	92	90	405	440	4 I-94 I-90
263	92	91	349	379	4 I-94
264	93	55	897	975	4 I-90 I-29
265	93	94	559	608	4 I-90 I-25
266	94	55	537	584	4I-80S I-80
267	94	56	540	587	4 I-70
268	94	57	509	553	4 I-70I-35W
269	94	100	423	510	2 I-25US-87
270	94	108	456	496	4 I-25
271	95	57	159	173	4 I-25
272	95	58	105	114	4 I-44
273	95	59	184	200	4 I-40
274	96	59	181	241	2US-71
275	96	60	140	152	4 I-20
276	96	61	206	275	2US-82
277	97	61	210	257	2 I-20US165US-82
278	97	96	70	76	4 I-71
279	98	62	178	293	4 I-55
280	98	97	313	396	2 I-10US-71
281	99	67	127	138	4 I-75
282	100	95	258	280	4 I-40
283	100	101	358	390	4US287
284	100	102	419	559	2US-70US-54
285	100	104	516	688	2US-87
286	101	95	206	224	4 I-35
287	101	96	175	190	4 I-30
288	101	97	185	201	4 I-20
289	102	101	620	674	4 I-20
290	102	104	574	624	4 I-10
291	103	97	309	388	2 I-35 S-31 I-20
292	103	101	193	210	4 I-35
293	103	105	164	201	2US183 I-10
294	104	103	77	83	4 I-35
295	104	105	197	214	4 I-10
296	105	97	234	262	2US-59US-79
297	105	98	356	387	4 I-10
298	105	101	243	264	4 I-45
299	106	93	551	654	4 I-15 I-90
300	106	94	504	548	4 I-80 I-25
301	106	117	780	848	4I-80N
302	107	108	432	490	4 I-17 I-40
303	107	102	443	482	4 I-10
304	108	100	284	308	4 I-40
305	108	102	266	289	4 I-25
306	109	93	845	918	4 I-90
307	109	106	871	947	4 I-90 I-82I-80N
308	110	106	752	817	4 I-80
309	110	111	379	412	4 I-5
310	111	94	1059	1151	4 I-15 I-70
311	111	106	715	777	4 I-15
312	111	107	389	423	4 I-10
313	112	41	106	115	2US-17 I-95
314	112	42	139	185	2US-78US-28
315	112	80	255	277	2US-52 I-95
316	112	82	113	123	4 I-26
317	113	92	153	166	4 I-35
318	113	118	251	334	2 US-2ST200ST-34US-
319	114	50	100	109	4 I-55
320	114	51	127	169	4US-36
321	114	54	326	354	4 I-55 I-74 I-80
322	114	89	193	260	2US-36
323	114	90	189	205	4 I-55
324	115	86	155	168	4 I-75
325	115	87	111	120	4 I-90
326	115	88	61	66	4 I-75
327	115	89	219	245	4 I-69US-24
328	115	90	232	252	4I-90
329	115	116	133	180	2US-23
330	116	75	182	198	4 I-70
331	116	84	164	219	2US-33
332	116	85	108	117	4 I-71
333	116	86	65	71	4 I-70
334	116	87	139	151	4 I-71
335	117	119	172	187	4 I-5
336	117	110	640	695	4 I-5
337	118	92	234	254	4 I-94
338	118	93	611	664	4 I-94
339	119	88	147	160	4 I-96
340	119	89	241	321	4US131US-31
341	119	90	168	183	4I-196 I-94
342	120	78	90	98	4 I-64
343	120	80	168	225	2US-58 I-95

RAIL ARCS

Arc.	Orig.	Dest.	Dist.	Speed	Cap.	RRCo.				
345	1	2	87	35	40	23				
346	1	4	48	12	10	23				
347	1	7	176	12	10	23				
348	2	67	70	45	100	23				
349	2	68	366	45	40	11				
350	2	99	210	35	24	23				
351	3	6	181	12	10	24				
352	3	7	112	12	10	24				
353	3	42	54	12	10	24				
354	4	2	76	35	40	23				
355	4	6	61	12	10	23				
356	4	8	108	35	40	23				
357	4	9	112	12	10	23				
358	5	7	54	12	10	23				
359	6	2	110	28	24	24				
360	6	8	86	28	24	24				
361	6	15	134	12	10	23				
362	6	67	108	12	10	24				
363	7	8	62	28	24	24				
364	7	11	101	28	24	24				
365	7	43	33	12	10	12				
366	7	44	88	35	40	24	24			
367	8	9	36	12	10	24				
368	8	10	123	35	20	23				
369	8	11	95	12	10	23				
370	8	13	170	12	10	23				
371	8	41	168	12	10	23				
372	8	44	138	35	20	23				
373	9	7	106	28	24	24				
374	9	11	77	28	24	24				
375	9	15	72	12	10	24				
376	9	66	99	12	10	23				
377	10	12	114	35	40	23				
378	10	44	69	12	10	2				
379	11	17	171	28	24	24				
380	11	44	120	12	20	24	24			
381	11	66	163	12	10	23				
382	12	17	64	35	80	16	24(0	23	24	23
383	12	44	99	35	40	24	24			
384	12	45	122	12	10	24				
385	13	10	104	12	10	31				
386	13	14	51	12	10	23				
387	13	17	97	35	40	16	24			
388	13	19	104	12	10	13				

Arc	Orig.	Dest.	Dist.	Speed	Cap.	RRCo.			
389	13	63	171	12	10	16			
390	13	64	178	35	40	16	24		
391	13	65	158	12	10	16			
392	14	11	84	12	10	24			
393	14	15	68	12	10	23			
394	16	17	85	35	40	16			
395	16	18	43	28	24	24			
396	16	46	24	28	24	24			
397	16	47	121	35	40	16	16		
398	17	18	129	12	10	24			
399	17	19	56	35	40	24	16		
400	17	20	148	28	24	13			
401	17	21	138	35	40	27			
402	17	22	118	12	10	24			
403	17	45	143	28	24	24			
404	18	20	54	28	24	24			
405	18	47	126	12	10	16			
406	19	22	60	12	10	13			
407	19	63	96	35	40	24			
408	20	21	50	12	20	13	13		
409	20	25	57	35	40	13			
410	20	26	94	28	24	24	13		
411	21	22	65	12	20	13	13		
412	21	63	104	35	40	27			
413	22	61	169	12	20	13	13		
414	22	63	99	12	20	13	13		
415	22	65	60	12	10	27			
416	23	26	76	12	20	13	13		
417	23	61	63	12	30	13	13	13	
418	24	49	124	35	80	13	13	13	13
419	25	24	48	28	24	13	13	13	
420	25	47	153	35	40	16	16		
421	26	24	78	28	24	13			
422	26	25	89	12	10	16			
423	26	60	135	28	24	28			
424	26	62	214	28	24	13			
425	27	26	68	35	40	27			
426	27	28	90	35	40	27	19		
427	27	31	82	35	40	19			
428	27	32	105	35	40	27			
429	27	96	304	35	40	28			
430	28	26	90	28	24	19			
431	28	60	51	35	40	19			
432	30	26	142	35	40	27			

Arc.	Orig.	Dest.	Dist.	Speed	Cap.	RRCo.		
433	30	49	29	35	80	28	27	
434	31	30	44	35	40	19		
435	31	50	130	35	40	19		
436	32	36	113	35	40	27		
437	33	50	182	35	40	27		
438	34	51	155	35	40	1		
439	34	52	55	35	40	19	20	13
440	35	52	60	28	48	18	19	8
441	36	33	57	35	40	27		
442	36	39	83	28	24	27		
443	36	40	65	35	40	27		
444	37	54	170	28	24	3	7	
445	37	55	127	12	10	3		
446	38	34	80	45	50	13	19	3,20, 1
447	38	35	94	28	24	19	8	
448	38	36	184	12	10	27		
449	38	37	60	35	80	3	19	
450	38	39	103	35	40	27	14	19
451	38	53	87	45	100	17	8	
452	38	56	65	35	72	30	1	
453	38	57	227	45	100	1		
454	38	58	195	28	72	18	1	19
455	39	35	92	12	10	18		
456	40	28	310	12	10	19		
457	40	39	63	35	40	27	14	19
458	40	58	115	35	40	27		
459	40	59	175	28	24	14	27	
460	41	1	78	35	40	23		
461	41	3	75	12	10	24		
462	41	4	97	35	40	23		
463	41	5	118	12	10	23		
464	41	80	361	35	40	23		
465	41	82	141	35	40	23		
466	42	43	93	12	5	12		
467	42	44	159	28	24	12		
468	42	81	128	12	10	23		
469	42	82	82	12	10	24		
470	44	45	136	35	80	16	16	24
471	45	46	98	28	24	16	24	
472	47	45	151	35	40	16		
473	47	48	160	35	40	16		
474	48	50	166	28	24	16		
475	48	90	289	35	40	16		
476	49	50	130	40	72	27	19	
477	51	50	129	28	24	3		
478	52	50	130	45	100	8,3, 19,14,18 , 20		
479	52	51	88	35	80	20	3	
480	53	37	75	28	24	3	1	
481	53	52	83	28	24	3	20	
482	53	54	161	28	24	8		
483	53	90	412	35	40	8		
484	54	55	135	35	192	7	17	8
485	55	93	896	12	10	3		
486	55	94	560	35	40	3	30	
487	56	57	160	28	24	8		
488	57	94	580	28	24	1		
489	57	100	348	28	48	1	8	
490	58	59	124	28	24	19		
491	58	95	119	28	24	27		
492	58	101	318	28	24	18		
493	59	60	160	28	24	19		
494	59	95	210	12	10	8		
495	59	96	190	35	40	14		
496	60	96	144	35	40	19		
497	60	98	484	35	40	19		
498	62	61	138	12	10	13	13	
499	62	63	97	12	20	13	13	
500	62	64	179	12	10	13		
501	62	98	183	35	72	13	13	13
502	63	64	137	12	20	13	27	
503	63	98	202	28	24	24		
504	64	65	96	12	10	16		
505	64	98	140	35	40	16		
506	65	66	202	12	10	16		
507	66	67	160	12	10	23		
508	67	99	141	35	40	23		
509	69	70	201	35	112	6	6	
510	69	72	230	35	72	6		
511	70	71	298	45	100	6		
512	70	73	190	35	40	6		
513	71	87	104	45	100	20	21	
514	71	88	252	35	40	6		
515	72	70	142	35	112	6	6	
516	72	73	134	35	72	6		
517	72	74	183	35	144	6	6	
518	72	76	225	35	112	6	5	
519	73	71	262	35	40	6		
520	73	74	136	35	40	6		
521	73	75	310	12	10	6		
522	74	75	245	45	100	21		
523	74	76	112	35	72	6		
524	75	76	296	35	72	5		

Arc	Orig.	Dest.	Dist.	Speed	Cap.	RRCo.		
525	75	87	131	35	216	21	5	6
526	75	116	191	35	72	21		
527	76	77	227	35	40	20		
528	76	78	117	45	200	24	22	
529	77	84	225	35	72	20		
530	78	77	174	35	80	20		
531	78	79	279	35	40	24		
532	78	80	159	35	40	23		
533	78	84	369	35	40	5		
534	79	80	156	35	40	23		
535	79	81	98	45	100	23	24	
536	80	41	375	35	40	23		
537	81	44	154	145	100	24	23	
538	82	79	108	28	24	24		
539	82	80	203	35	40	23		
540	82	81	111	12	20	24	23	
541	83	44	197	28	24	16		
542	83	45	111	35	80	24	24	16
543	83	47	216	12	10	16	24	
544	83	79	269	28	24	24		
545	84	85	204	45	100	5		
546	85	48	229	28	48	16	24	
547	85	50	338	28	24	5		
548	85	83	292	35	80	24	16	
549	85	90	281	35	40	5		
550	86	85	55	45	100	21		
551	86	87	109	35	144	21	21	
552	86	90	248	35	40	5		
553	87	90	340	35	256	20	5	21	6
554	88	90	272	35	72	21	20	
555	89	47	298	12	10	6	16	
556	89	50	240	35	40	6		
557	89	85	109	35	40	5		
558	89	90	184	35	40	21	16	
559	89	114	197	35	40	20		
560	90	49	364	35	72	13		
561	90	50	284	35	40	33		
562	90	51	272	45	100	3	1	
563	90	54	358	35	184	7	17	8	3
564	90	91	86	45	100	7	7	17
565	90	92	396	35	40	3		
566	92	91	327	45	72	25	17	
567	93	109	903	35	40	3	3	17
568	94	106	570	35	112	9	30	
569	95	57	172	35	80	8	1	
570	95	100	274	12	10	8		
571	96	101	182	45	30	29	26	
572	96	103	460	28	24	19		
573	97	62	218	12	10	13		

Arc	Orig.	Dest.	Dist.	Speed	Cap.	RRCo.			
574	97	96	73	28	24	14	26		
575	97	101	194	45	20	29			
576	97	105	232	35	40	26	14		
577	98	97	315	12	10	15			
578	98	105	363	12	10	26			
579	99	68	261	35	40	23			
580	100	108	374	35	40	1			
581	101	103	209	28	24	18			
582	101	105	264	35	40	1			
583	101	95	236	35	40	22	1		
584	102	100	446	28	24	26			
585	102	101	646	35	40	29			
586	102	104	610	35	40	26			
587	102	107	434	35	40	26			
588	102	108	255	28	24	1			
589	103	104	82	24	19				
590	105	103	174	35	40	1	26		
591	105	104	210	28	24	26			
592	106	110	821	35	72	26	32		
593	106	111	783	35	40	30			
594	107	111	425	35	112	1	26		
595	108	107	576	35	72	1			
596	110	117	742	35	40	26	32		
597	111	110	470	35	80	26	26	1	
598	112	41	111	35	40	23			
599	112	80	204	35	40	23			
600	112	82	129	12	10	24			
601	113	92	145	28	24	3			
602	114	50	99	35	72	13	13	20	
603	114	51	123	35	40	20			
604	114	90	185	35	40	13	13		
605	115	75	261	35	40	21			
606	115	86	160	35	40	5			
607	115	87	107	45	100	21			
608	115	88	56	35	184	10	21	20	
609	115	90	243	45	100	21			
610	115	116	135	45	100	20	21		
611	116	86	71	35	224	21	21	21	21
612	116	87	138	45	100	21			
613	116	84	204	45	100	20	5		
614	117	106	836	35	40	30			
615	117	109	183	45	100	3			
616	118	92	231	35	72	3	3		
617	118	93	640	35	40	3	3		
618	119	88	152	35	40	6			
619	119	90	184	35	40	6			
620	120	77	258	35	40	20			
621	120	78	109	45	100	6	20		

WATER ARCS

Arc	Orig.	Dest.	Dist.	Speed	Lock	Chan	Sys.
623	62	98	337	7	0	11	12
624	61	62	101	7	0	11	12
625	23	61	80	7	0	11	12
626	26	23	120	7	0	11	12
627	24	26	115	7	0	11	12
628	49	24	168	7	0	11	12
629	50	49	128	7	2	9	12
630	51	50	147	7	7	9	12
631	92	51	526	7	22	9	12
632	58	59	182	7	5		2
633	59	60	230	7	6		2
634	60	61	154	7	6		2
635	52	50	179	7	0	8	13
636	34	52	78	7	0	8	13
637	38	34	109	7	0	8	13
638	37	38	82	7	0	8	13
639	55	37	168	7	0	8	13
640	25	49	222	7	6	11	18
641	20	25	60	7	1	11	18
642	18	20	50	7	0	11	18
643	16	18	48	7	4	11	18
644	46	16	19	7	0	11	18
645	45	46	141	7	2	11	18
646	83	45	184	7	3	11	18
647	47	49	304	7	7	11	6
648	48	49	241	7	9	11	15
649	85	48	322	7	4	11	15
650	75	85	470	7	6	11	15
651	84	85	263	7	4		11
652	90	50	365	7	9		10
653	19	64	215	7	4		19
654	22	19	125	7	2		19
655	21	22	75	7	4		19

Arc	Orig.	Dest.	Dist.	Speed	Lock	Chan	Sys.
656	20	21	55	7	4		19
657	17	19	224	7	2		4
658	13	64	334	7	3		1
659	15	65	100	7	1		5
660	11	15	200	7	2		5
661	42	41	150	7	0		17
662	70	72	180	7	0	12	9
663	71	70	342	7	35	20	14
664	69	72	265	10	-1	-1	3
665	72	120	440	10	-1	-1	3
666	120	76	197	10	-1	-1	3
667	120	112	460	10	-1	-1	3
668	112	41	121	10	-1	-1	3
669	41	1	90	10	-1	-1	3
670	1	2	90	10	-1	-1	3
671	2	68	371	10	-1	-1	3
672	68	99	369	10	-1	-1	3
673	99	66	220	10	-1	-1	8
674	66	65	253	10	-1	-1	8
675	65	64	81	10	-1	-1	8
676	64	98	166	10	-1	-1	8
677	98	105	417	10	-1	-1	8
678	111	110	351	10	-1	-1	16
679	110	117	635	10	-1	-1	16
680	117	109	361	10	-1	-1	16
681	71	87	176	10	-1	-1	7
682	87	88	108	10	-1	-1	7
683	88	91	568	10	-1	-1	7
684	88	113	726	10	-1	-1	7
685	113	91	743	10	-1	-1	7
686	91	90	85	10	-1	-1	7
687	115	88	54	10	-1	-1	7
688	87	115	96	10	-1	-1	7
689	72	76	270	10	-1	-1	3

APPENDIX D

TEST RUN ARC PARAMETERS

TEST RUN ARC PARAMETERS

Arc Type

	Loading-Unloading	Line-Haul	Inter-modal Transfer
$\bar{c}_j$	100,000	50,000	100,000
$\bar{t}_j$	100,000	50,000	100,000
$\bar{v}_j$	100,000	50,000	100,000
f_c	.5	.6	.4
f_t	.5	.6	.4
f_v	.5	.6	.4
L_j	1,000	1,000	1,000
U_j	95,000	49,000	95,000

where:

f_c fraction of cost subject to improvement

f_t fraction of time subject to improvement

f_v fraction of variability subject to improvement

APPENDIX E

O-D FLOW DATA SET

O-D FLOW DATA SET

Origin	Destination	Mode	Tons, annual
11	44	1	26416.
11	44	2	6641.
11	81	1	54439.
11	94	1	49941.
11	112	1	7642.
11	112	2	43269.
44	90	1	47019.
44	90	2	25.
45	76	1	16703.
45	76	2	29322.
45	84	1	60981.
45	88	1	34242.
45	90	1	84436.
45	101	1	136358.
45	101	2	1905.
61	50	1	7761.
61	50	2	32853.
6	2	1	26522.
6	2	2	92668.
6	65	2	111927.
9	2	1	35157.
9	2	2	122839.
9	65	2	146368.
60	47	1	149242.
60	47	2	7825.
61	60	1	91641.
61	60	2	39569.
63	17	1	33541.
63	17	2	67043.
63	89	1	86217.
63	89	2	30859.
63	90	1	15845.
63	90	2	89422.
65	81	2	36216.
84	105	2	5411.
84	105	3	39854.
98	45	2	1551.
98	45	3	38899.
98	47	2	2326.
98	47	3	44361.
98	48	2	7356.
98	48	3	152144.
98	49	2	5651.
98	49	3	60418.
98	50	2	14102.
98	50	3	63563.
98	51	2	7754.
98	51	3	40389.
98	54	2	21205.
98	54	3	17169.
98	75	2	3574.
98	75	3	115044.
98	84	2	26485.
98	84	3	55288.
98	85	2	8411.
98	85	3	43535.
98	90	2	53112.
98	90	3	227603.
98	92	2	3392.
98	92	3	59757.
98	99	2	35473.
98	116	2	8866.
98	116	3	26633.
99	38	2	53170.
99	38	2	45652.

99	48	2	3[illegible]401.
99	50	2	123810.
99	51	2	168995.
99	55	2	101195.
99	57	2	67725.
99	76	2	68547.
99	79	2	54726.
99	85	2	59401.
99	88	2	60771.
99	89	2	32245.
99	90	2	190420.
99	91	2	75927.
99	92	2	79024.
99	101	2	54126.
99	112	2	37222.
99	114	2	67563.
99	115	2	72042.
99	116	2	712[illegible]4.
99	118	2	51067.
99	120	2	37301.
105	42	2	46367.
105	50	2	6[illegible]4.
105	50	3	45039.
105	65	2	1371.
105	65	3	41173.
105	75	3	92422.
105	85	3	61342.

APPENDIX F

ADDITIONAL RESULTS OF TEST RUNS

1. Layout of runs in terms of a factorial design
2. Test run results for single-commodity problem
3. Test run results for multi-commodity problem

Table F-1. Layout of Runs in Terms of a Factorial Design

Initial Arc Lengths/		A								B			
Phase I Used/		Yes				No				Yes			
EPSILON/		.02		.01		.02		.01		.02		.01	
ALPHA/		1.	.8	1.	.8	1.	.8	1.	.8	1.	.8	1.	.8
	Current	4	-	-	-	1	-	-	-	5	-	-	-
	1000	7	-	-	-	-	-	-	-	8	-	-	-
	0100	10	-	-	-	-	-	-	-	11	-	30	-
	0010	13	-	-	-	-	-	-	-	14	-	-	-
Initial Modal Splits	0001	16	-	-	-	-	-	-	-	17	-	-	-
	1001	19	-	-	-	-	-	-	-	20	-	-	-
	1110	22	25	-	-	-	-	-	-	23	26	-	-
	0110	28	-	29	-	-	-	-	-	-	-	-	-

(continued)

Table F-1 (cont'd)

Initial Arc Lengths/		B				C							
Phase I Used/		No				Yes				No			
EPSILON/		.02		.01		.02		.01		.02		.01	
ALPHA/		1.	.8	1.	.8	1.	.8	1.	.8	1.	.8	1.	.8
Initial Modal Splits	Current	2	-	-	-	6	-	-	-	3	-	-	-
	1000	-	-	-	-	9	-	-	-	-	-	-	-
	0100	-	-	-	-	12	-	-	-	-	-	-	-
	0010	-	-	-	-	15	-	-	-	-	-	-	-
	0001	-	-	-	-	18	-	-	-	-	-	-	-
	1001	-	-	-	-	21	-	-	-	-	-	-	-
	1110	-	-	-	-	24	27	-	-	-	-	-	-
	0110	-	-	-	-	-	-	-	-	-	-	-	-

Table F-2. Test Run Results for Single-Commodity Problem

Run #	Run time (sec.)	Macro-Iter.	Micro-Iter.	Total Dis.	Invest.	User Dis.	Total Savings	Invest. Over Min.	User Savings
1	632	3	9	15,376	466	14,911	1,250	197	1,446
2	736	3	11	15,427	488	14,939	1,199	219	1,418
3	679	3	10	15,436	489	14,947	1,190	220	1,410
4	1,102	3	18	15,366	456	14,910	1,260	187	1,447
5	982	3	16	15,371	456	14,915	1,255	187	1,442
6	1,089	3	18	15,456	460	14,996	1,170	191	1,361
7	1,182	3	20	15,308	484	14,823	1,318	215	1,537
8	1,130	3	19	15,342	476	14,866	1,284	207	1,491
9	1,147	3	19	15,483	480	15,004	1,143	211	1,353
10	1,162	3	19	15,330	479	14,851	1,296	210	1,506
11	1,126	3	18	15,412	477	14,934	1,214	208	1,423
12	1,173	3	19	15,467	480	14,987	1,159	211	1,370
13	1,204	3	20	15,366	468	14,898	1,260	199	1,459
14	1,209	3	20	15,371	468	14,903	1,255	199	1,454
15	1,108	3	18	15,365	468	14,897	1,261	199	1,460
16	1,209	3	20	15,346	477	14,869	1,280	208	1,488
17	1,213	3	20	15,346	477	14,869	1,280	208	1,488
18	1,106	3	18	15,427	466	14,960	1,199	197	1,397
19	1,210	3	20	15,323	483	14,841	1,303	214	1,516
20	1,155	3	19	15,345	476	14,869	1,281	207	1,488
21	1,152	3	19	15,486	481	15,006	1,140	212	1,351
22	792	2	13	15,321	485	14,836	1,305	216	1,521
23	742	2	12	15,422	476	14,946	1,204	207	1,411
24	788	2	13	15,367	483	14,885	1,259	214	1,472
25	1,111	3	18	15,319	485	14,834	1,307	216	1,523
26	1,061	3	17	15,422	476	14,945	1,204	207	1,412
27	1,115	3	18	15,365	483	14,882	1,261	214	1,475
28	785	2	13	15,354	481	14,873	1,272	212	1,484
29	1,107	3	18	15,354	481	14,873	1,272	212	1,484
30	1,086	3	18	15,412	477	14,934	1,214	208	1,423

Table F-3. Test Run Results for Multi-Commodity Problem

Run	CPU Sec.	Macro-Iter.	Micro-Iter.	Total Dis.	Invest.	User Dis.	Total Savings	Invest. Over Min.	User Savings
1	1,394	3	11	8,948	427	8,521	515	158	673
2	1,515	3	12	8,928	423	8,505	535	154	689
3	1,636	3	13	8,982	428	8,554	481	159	640
4	2,233	3	18	8,927	428	8,499	536	159	695
5	2,243	3	18	8,928	426	8,503	535	157	691
6	2,295	3	18	8,927	428	8,499	536	159	695
7	2,132	3	17	8,863	427	8,436	600	158	758
8	2,145	3	17	8,864	427	8,436	599	158	758
9	2,028	3	16	8,927	432	8,495	536	163	699
10	2,504	3	20	8,892	430	8,462	571	161	732
11	2,129	3	17	8,885	438	8,447	578	169	747
12	2,061	3	17	8,903	430	8,472	560	161	722
13	2,377	3	19	8,929	423	8,506	534	154	688
14	2,203	3	18	8,931	422	8,509	532	153	685
15	2,459	3	20	8,930	423	8,507	533	154	687
16	2,013	3	16	8,927	432	8,495	536	163	699
17	2,145	3	17	8,927	432	8,495	536	163	699
18	2,132	3	17	8,927	432	8,495	536	163	695
19	2,383	3	19	8,864	427	8,437	599	158	757
20	2,403	3	19	8,864	427	8,437	599	158	757
21	2,144	3	17	8,927	432	8,495	536	163	695
22	1,649	2	13	8,819	438	8,380	644	169	814
23	1,626	2	13	8,830	429	8,401	633	160	793
24	1,624	2	13	8,878	429	8,450	585	160	744
25	2,432	3	19	8,819	438	8,381	644	169	813
26	2,409	3	19	8,830	429	8,402	633	160	792
27	2,425	3	19	8,879	429	8,451	585	160	743
28	1,543	2	12	8,836	433	8,403	627	164	791
29	2,335	3	18	8,919	433	8,486	544	164	708
30	2,131	3	17	8,912	438	8,474	551	169	720

APPENDIX G

APPENDIX G

<u>Theorem 6.3</u>

$\hat{D}_j$ is a convex underestimator of D_j.

Proof. A number of cases and subcases must be considered:

Let $\phi_j = O_j$

(1) $t_s \leq B_j, \quad \forall\, s \in \phi_j$

(a) $\sum_{s \in \phi_j} t_s \leq B_{j1}$ (Case I)

(b) $B_{j1} < \sum_{s \in \phi_j} t_s \leq B_{j2}$ (Case IV)

(c) $\sum_{s \in \phi_j} t_s > B_{j2}$ (Case VI)

(2) $B_{j1} \leq t_s \leq B_{j2} \quad \forall\, s \in \phi_j$

(a) $\sum_{s \in \phi} t_s \leq B_{j2}$ (Case II)

(b) $\sum_{s \in \phi_j} t_s > B_{j2}$ (Case V)

(3) $t_s \geq B_{j2} \quad \forall\, s \in \phi_j$ (Case III)

(4) $t_s < B_{j1} \quad V\, s \in \phi_j \cap A$

$$B_{j1} \leq t_s \leq B_{j2} \qquad \forall\ s \in \phi_j \cap B$$

(a) $\sum_{s \in \phi_j} t_s \leq B_{j2}$ (Case IV)

(b) $\sum_{s \in \phi_j} t_s > B_{j2}$ (Case VI)

(5) $t_s \leq B_{j1} \qquad \forall\ s \in \phi_j \cap A$

$t_s > B_{j2} \qquad \forall\ s \in \phi_j \cap C$ (Case VI)

(6) $t_s \leq B_{j1} \qquad \forall\ s \in \phi_j \cap A$

$B_{j1} < t_s \leq B_{j2} \qquad \forall\ s \in \phi_j \cap B$

$t_s > B_{j2} \qquad \forall\ s \in \phi_j \cap C$ (Case VI)

(7) $B_{j1} \leq t_s \leq B_{j2} \qquad \forall\ s \in \phi_j \cap B$

$t_s > B_{j2} \qquad \forall\ s \in \phi_j \cap C$ (Case V)

<u>Case I</u>

For some arc j:

$$t_s \leq B_{j1} \qquad \forall\ s \in \phi_j$$

and

$$\sum_{s \in \phi_j} t_s \leq B_{j1}$$

Let: $\hat{D}_j = c_{j1} \sum_{p \varepsilon P^j} f_p + F_{j1} \underset{p \varepsilon P^j}{\text{Max}} \{\frac{f_p}{t_p}\}$

Proof. $\hat{D}_j$ is convex.

Consider the math program:

T:

$$\underset{\bar{f}}{\text{Min}}\ D_j(\bar{f}) - \hat{D}_j(\bar{f})$$

$$\text{s.t.} \quad 0 \leq f_p \leq t_p \qquad \forall\ p \ \varepsilon\ P^j$$

The objective is concave since D_j is concave and $\hat{D}_j$ is convex. The optimal solution of T must lie at an EP of the solution space. Note that an EP is any point: $(a_{j1}, \ldots, a_{j/P^j/})$

where:

$$a_{jp} = \begin{cases} 0 \\ \text{or} \\ t_p \end{cases}$$

Thus, if $D_j < \hat{D}_j$ for some point in the solution space, it will also hold for some EP. Also, if it is not true for the EP; i.e., if $D_j \geq \hat{D}_j$ for all EP, then $D_j \geq \hat{D}_j$ for all $\bar{f}\ \varepsilon$ s.

For the $\bar{0}$ point:

$$\hat{D}_j = 0 = D_j$$

For all other EP, the following holds:

$$\hat{D}_j = c_{j1} \sum_{p \varepsilon Pj} a_{jp} + F_{j1} = D_j$$

$\therefore$ Theorem 6.3 holds for Case 1.

Q.E.D.

Cases II and III are proven in a similar manner.

Case IV

For some arc j:

either $$t_s \leq B_{j1} \qquad \forall\ s\ \varepsilon\ \phi_j$$

and

$$B_{j1} < \sum_{s \varepsilon \phi_j} t_s \leq B_{j2}$$

or $$t_s < B_{j1} \qquad \forall\ s\ \varepsilon\ \phi_j \cap A$$

and

$$B_{j1} \leq t_s \leq B_{j2} \qquad \forall\ s\ \varepsilon\ \phi_j \cap B$$

and

$$\sum_{s \varepsilon \phi_j} t_s \leq B_{j2}$$

Let: $\hat{D}_j = c_{j2} \sum_{p \varepsilon Pj} f_p + \underset{p \varepsilon Pj}{\text{Max}} \{e_{jp} f_p\}$

where:

$$e_{jp} = \begin{cases} \dfrac{F_{j2}}{t_p} & \text{if } t_p \geq B_{j1} \\ \dfrac{F_{j1}}{t_p} + c_{j1} - c_{j2} & \text{if } t_p < B_{j1} \end{cases}$$

Proof. For the $\bar{0}$ point:

$$\hat{D}_j = 0 = D_j$$

The remaining EP can be subdivided:

<u>Set 1</u>. Those EP $\ni \sum_{p \varepsilon Pj} a_{jp} < B_{j1}$

<u>Set 2</u>. Those EP $\ni \sum_{p \varepsilon Pj} a_{jp} \geq B_{j1}$

Regarding Set 1, the following must hold:

$$D_j = c_{j1} \sum_{p \varepsilon Pj} a_{jp} + F_{j1}$$

$$\hat{D}_j = c_{j2} \sum_{p \varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j2}) a_{jMax}$$

where:

$$e_{jMax} a_{jMax} = \operatorname*{Max}_{p \varepsilon Pj} \{e_{jp} f_p\}$$

But $$a_{jMax} \leq \sum_{p \varepsilon Pj} a_{jp}$$

$$\therefore \quad \hat{D}_j \leq c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j2}) \sum_{p\varepsilon Pj} a_{jp}$$

$$\leq c_{j1} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} = D_j$$

Regarding Set 2, the following must hold:

$$D_j = c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j2}$$

$$\hat{D}_j = c_{j2} \sum_{p\varepsilon Pj} a_{jp} + \underset{p\varepsilon Pj}{\mathrm{Max}} \{e_{jp}a_{jp}\}$$

There are two possibilities regarding $\hat{D}_j$:

<u>Possibility 1</u>. The maximum involves some path $q \varepsilon B$

$$\hat{D}_j = c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j2} = D_j$$

<u>Possibility 2</u>. The maximum involves some path $q \varepsilon A$

$$D_j = c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j2})a_{jq}$$

But

$$a_{jq} \leq B_{j1} = \frac{F_{j2} - F_{j1}}{c_{j1} - c_{j2}}$$

$$\therefore \quad \hat{D}_j \leq c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + F_{j2} - F_{j1} = D_j$$

Q.E.D.

Case V is proven in a similar fashion.

Case VI

For all other arcs j:

Let: $\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} f_p + \underset{p\varepsilon Pj}{\text{Max}} \{e_{jp} f_p\}$

where:

$$e_{jp} = \begin{cases} \dfrac{F_j}{t_j} & \text{if } t_p \geq B_{j2} \\ \dfrac{F_{j1}}{t_p} + c_{j1} - c_{j3} & \text{if } t_p < B_{j1} \\ \dfrac{F_{j2}}{t_p} + c_{j2} - c_{j3} & \text{if } B_{j1} \leq t_p \leq B_{j2} \end{cases}$$

Proof. For the $\bar{0}$ point:

$$\hat{D}_j = 0 = D_j$$

The remaining EP can be subdivided:

Set 1. Those EP $\ni \sum_{p\varepsilon Pj} a_{jp} < B_{j1}$

Set 2. Those EP $\ni B_{j1} \leq \sum_{p\varepsilon Pj} a_{jp} < B_{j2}$

Set 3. Those EP $\ni \sum_{p\varepsilon Pj} a_{jp} \geq B_{j2}$

Regarding Set 1, the following must hold:

$$D_j = c_{j1} \sum_{p\varepsilon Pj} a_{jp} + F_{j1}$$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j3})a_{jMax}$$

But, $$a_{jMax} \leq \sum_{p\varepsilon Pj} a_{jp}$$

$$\therefore \quad \hat{D}_j \leq c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j3}) \sum_{p\varepsilon Pj} a_{jp}$$

$$\leq c_{j1} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} = D_j$$

Regarding Set 2, the following must hold:

$$D_j = c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j2}$$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + \underset{p\varepsilon Pj}{\mathrm{Max}} \{e_{jp}a_{jp}\}$$

There are two possibilities regarding $\hat{D}_j$:

<u>Possibility 1</u>. The maximum involves some path $q \in A$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j3})a_{jq}$$

But, $$a_{jq} \leq B_{j1} = \frac{F_{j2} - F_{j1}}{c_{j1} - c_{j2}}$$

$$= \frac{c_{j2} - c_{j3}}{c_{j1} - c_{j3}} \cdot \frac{F_{j2} - F_{j1}}{c_{j1} - c_{j2}} + \frac{F_{j2} - F_{j1}}{c_{j1} - c_{j3}} \cdot \frac{c_{j1} - c_{j2}}{c_{j1} - c_{j2}}$$

$$\leq \frac{c_{j2} - c_{j3}}{c_{j1} - c_{j3}} \sum_{p\varepsilon Pj} a_{jp} + \frac{F_{j2} - F_{j1}}{c_{j1} - c_{j3}} = U$$

since $$B_{j1} \leq \sum_{p\varepsilon Pj} a_{jp}$$

Thus, $$\hat{D}_j \leq c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j3})U$$

$$\leq c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j2} = D_j$$

<u>Possibility 2</u>. The maximum involves some path $q \varepsilon B$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j2} + (c_{j2} - c_{j3})a_{jq}$$

But $$a_{jq} \leq \sum_{p\varepsilon Pj} a_{jp}$$

$$\therefore \quad D_j \leq c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j2} + (c_{j2} - c_{j3}) \sum_{p\varepsilon Pj} a_{jp}$$

$$\leq c_{j2} \sum_{p\varepsilon Pj} a_{jp} + F_{j2} = D_j$$

Regarding Set 3, the following must hold:

$$D_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j3}$$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + \underset{p\varepsilon Pj}{\mathrm{Max}} \{e_{jp} a_{jp}\}$$

There are three possibilities regarding $\hat{D}_j$:

Possibility 1. The maximum involves some path $q \varepsilon A$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j3}) a_{jq}$$

But, $$a_{jq} \le B_{j1} \le B_{j2} \le B_{j3} = \frac{F_{j3} - F_{j1}}{c_{j1} - c_{j3}}$$

$$\therefore \quad \hat{D}_j \le c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j1} + (c_{j1} - c_{j3}) \frac{F_{j3} - F_{j1}}{c_{j1} - c_{j3}}$$

$$\le c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j3} = D_j$$

Possibility 2. The maximum involves some path $q \varepsilon B$

$$\hat{D}_j = c_{j3} \sum_{p\varepsilon Pj} a_{jp} + F_{j2} + (c_{j2} - c_{j3}) a_{jq}$$

But, $$a_{jq} \leq B_{j2} = \frac{F_{j3} - F_{j2}}{c_{j2} - c_{j3}}$$

$$\therefore \quad \hat{D}_j \leq c_{j3} \sum_{p \varepsilon Pj} a_{jp} + F_{j2} + (c_{j2} - c_{j3}) \frac{F_{j3} - F_{j2}}{c_{j2} - c_{j3}}$$

$$\leq c_{j3} \sum_{p \varepsilon Pj} a_{jp} + F_{j3} = D_j$$

Possibility 3. The maximum involves some path $q \varepsilon C$

$$\hat{D}_j = c_{j3} \sum_{p \varepsilon Pj} a_{jp} + F_{j3} = D_j$$

Q.E.D.

APPENDIX H

MULTI-COMMODITY LISTINGS

1. MNETED
2. MMS
3. MINFOD
4. MINAC
5. MTESTMS
6. MCNCASB

```
C       PROGRAM MNETED DEVELOPS THE EXPANDED NETWORK USED IN THE CONCAVE
C       MULTI-COMMODITY ASSIGNMENT ROUTINE.
        PROGRAM MNETED(OUTPUT=400B,TAPE1=400B,TAPE2=400B,TAPE7=400B,
      &TAPE8=400B,TAPE6=OUTPUT)
        INTEGER Z,PZ(121),POD(1152,2),O,P(1152),A(3080,3),D(50),
      &OD(2000,2),ODC(600,5),AR(50),ARC,CD(50)
        DIMENSION MZ(120,3),M(120),FD(50),FODC(600),LA(3080)
        READ(7,100) Z,MD,MA,NC1
        READ(7,101) ((MZ(I,J),J=1,3),I=1,Z)
        IN=1
        DO 1 I=1,Z
        M(I)=0
        DO 2 J=1,3
        M(I)=M(I)+MZ(I,J)
  2     CONTINUE
        PZ(I)=IN
        IN=IN+3*M(I)+2
  1     CONTINUE
        PZ(Z+1)=IN
        N=IN-1
        DO 3 I=1,N
        POD(I,2)=0
  3     CONTINUE
        ND=0
        NOD=1
        NA=1
        NN=1
        DO 4 I=1,Z
        POD(NN,2)=1
        O=M(I)
        P(NN)=NA
        DO 5 J=1,O
        A(NA,1)=NN
        A(NA,2)=NN+J
        A(NA,3)=0
        IZ=0
        DO 30 KM=1,3
        IF(MZ(I,KM).EQ.0)GO TO 30
        IZ=IZ+1
        IF(IZ.EQ.J)GO TO 31
        GO TO 30
 31     MODE=KM
        GO TO 32
 30     CONTINUE
 32     LA(NA)=MODE
        NA=NA+1
  5     CONTINUE
        READ(7,100) (D(J),J=1,MD)
        READ(7,102) (FD(J),J=1,MD)
        READ(7,100) (CD(J),J=1,MD)
        POD(NN,1)=NOD
        DO 6 J=1,MD
        IF(D(J).EQ.0) GO TO 7
        OD(NOD,1)=PZ(D(J)+1)-1
        OD(NOD,2)=CD(J)
        ODC(ND+J,2)=CD(J)
        ODC(ND+J,1)=NOD
        FODC(ND+J)=FD(J)
        NOD=NOD+1
  6     CONTINUE
        NDZ=MD
        GO TO 8
  7     NDZ=J-1
  8     NN=NN+1
        DO 9 J=1,3
```

```
      IF(MZ(I,J).EQ.0)GO TO 10
      P(NN)=NA
      A(NA,1)=NN
      A(NA,2)=NN+0
      A(NA,3)=0
      LA(NA)=3+J
      NA=NA+1
      POD(NN,1)=NOD
      DO 11 K=1,MD
      IF(D(K).EQ.0)GO TO 12
      IF(MZ(D(K),J).EQ.0) GO TO 13
      OD(NOD,1)=PZ(D(K)+1)-1
      OD(NOD,2)=CD(K)
      ODC(ND+K,J+1)=NOD
      NOD=NOD+1
      GO TO 11
13    ODC(ND+K,J+1)=0
11    CONTINUE
12    NN=NN+1
      GO TO 9
10    DO 14 L=1,MD
      IF(D(L).EQ.0)GO TO 9
      ODC(ND+L,J+1)=0
14    CONTINUE
9     CONTINUE
      ND=ND+NDZ
      DO 15 J=1,3
      READ(7,100)(AR(K),K=1,MA)
      IF(MZ(I,J).EQ.0)GO TO 15
      P(NN)=NA
      DO 16 K=1,MA
      IF(AR(K).EQ.0)GO TO 17
      A(NA,1)=NN
      IC=0
      DO 18 L=1,J
      IC=IC+MZ(AR(K),L)
18    CONTINUE
      A(NA,2)=PZ(AR(K))+2*M(AR(K))+IC
      A(NA,3)=0
      LA(NA)=6+J
      NA=NA+1
16    CONTINUE
17    POD(NN,1)=NOD
      NN=NN+1
15    CONTINUE
      K1=NN-0
      K2=NN-1
      DO 19 J=1,3
      P(NN)=NA
      IZ=0
      DO 33 KM=1,3
      IF(MZ(I,KM).EQ.0)GO TO 33
      IZ=IZ+1
      IF(IZ.EQ.J)GO TO 34
      GO TO 33
34    MODE=KM
      GO TO 35
33    CONTINUE
35    DO 20 K=K1,K2
      A(NA,1)=NN
      A(NA,2)=K
      IF(K.EQ.NN-3)GO TO 21
      A(NA,3)=1
      IZ=0
      DO 36 KM=1,3
```

```
      IF(MZ(I,KM).EQ.0)GO TO 36
      IZ=IZ+1
      IF(IZ.EQ.K-K1+1)GO TO 37
      GO TO 36
37    MODE1=KM
      GO TO 38
36    CONTINUE
38    IF(MODE.EQ.1.AND.MODE1.EQ.2)GO TO 39
      IF(MODE.EQ.2.AND.MODE1.EQ.1)GO TO 39
      IF(MODE.EQ.1.AND.MODE1.EQ.3)GO TO 40
      IF(MODE.EQ.3.AND.MODE1.EQ.1)GO TO 40
      LA(NA)=15
      GO TO 22
39    LA(NA)=13
      GO TO 22
40    LA(NA)=14
      GO TO 22
21    A(NA,3)=0
      LA(NA)=9+MODE
22    NA=NA+1
20    CONTINUE
      A(NA,1)=NN
      A(NA,2)=PZ(I+1)-1
      A(NA,3)=0
      LA(NA)=15+MODE
      NA=NA+1
      POD(NN,1)=NOD
      NN=NN+1
19    CONTINUE
      P(NN)=NA
      POD(NN,1)=NOD
      NN=NN+1
4     CONTINUE
      P(NN)=NA
      POD(NN,1)=NOD
      POD(NN,2)=0
      ARC=NA-1
      IP=NOD-1
      WRITE(1,100)ND,IP,NC1
      WRITE(1,100)((ODC(I,J),J=1,5),I=1,ND)
      WRITE(1,102)(FODC(I),I=1,ND)
      WRITE(2,100)N,ARC,Z,IP,NC1
      WRITE(2,100)((A(I,J),J=1,3),I=1,ARC)
      WRITE(2,100)(P(I),I=1,IN)
      WRITE(2,100)((OD(I,J),J=1,2),I=1,IP)
      WRITE(2,100)((POD(I,J),J=1,2),I=1,IN)
      WRITE(8,100)ARC
      WRITE(8,100)(LA(I),I=1,ARC)
      STOP
100   FORMAT(20I4)
101   FORMAT(80I1)
102   FORMAT(8F10.0)
      END
```

```
C     PROGRAM MMS DEVELOPS A SET OF EXTREME MODAL SPLITS
C     ACCORDING TO THE INPUT VECTOR M(J).  IF M(J)=1,
C     THEN MODE J RECIEVES AN EQUAL MODAL SHARE.
      PROGRAM MMS(INPUT,OUTPUT,TAPE7,TAPE14,TAPE18,TAPE5=INPUT,
     &TAPE6=OUTPUT)
      INTEGER O,D
      DIMENSION MZ(120,3),M(4)
      REAL MS(4)
      READ(5,101)(M(I),I=1,4)
      READ(7,100)N
      READ(7,101)((MZ(I,J),J=1,3),I=1,120)
      DO 1 I=1,E3
      READ(18,100)N,O,D
      IC=0
      DO 6 J=1,4
      MS(J)=0.
 6    CONTINUE
      IF(M(1).EQ.1)IC=1
      DO 2 J=2,4
      IF(M(J).EQ.0)GO TO 2
      IF(MZ(O,J-1).EQ.0.OR.MZ(D,J-1).EQ.0)GO TO 2
      IC=IC+1
 2    CONTINUE
      IF(IC.EQ.0)GO TO 4
      RIC=IC
      IF(M(1).EQ.1)MS(1)=1./RIC
      DO 3 J=2,4
      IF(M(J).EQ.0)GO TO 3
      IF(MZ(O,J-1).EQ.0.OR.MZ(D,J-1).EQ.0)GO TO 3
      MS(J)=1./RIC
 3    CONTINUE
      GO TO 5
 4    MS(1)=1.
 5    WRITE(14,105)(MS(J),J=1,4)
 1    CONTINUE
      STOP
 100  FORMAT(20I4)
 101  FORMAT(80I1)
 105  FORMAT(4F4.2)
      END
```

```
C     PROGRAM MINFOD SETS UP THE INITIAL FOD VECTOR, O-D DEMAND
C     BY MODE, USING AN INITIAL ESTIMATE OF MODAL SPLIT.
      PROGRAM MINFOD(OUTPUT,TAPE1,TAPE4,TAPE14,TAPE6=OUTPUT)
      INTEGER ODC(600,5)
      REAL MS(4)
      DIMENSION FODC(600),FOD(2000)
      READ(1,100)NC,IP
      READ(1,100)((ODC(I,J),J=1,5),I=1,ND)
      READ(1,102)(FODC(I),I=1,ND)
      DO 1 I=1,ND
      READ(14,105)(MS(J),J=1,4)
      DO 2 J=1,4
      IF(ODC(I,J).EQ.0)GO TO 2
      FOD(ODC(I,J))=FODC(I)*MS(J)
 2    CONTINUE
 1    CONTINUE
      WRITE(4,102)(FOD(I),I=1,IP)
      STOP
 100  FORMAT(20I4)
 102  FORMAT(8F10.0)
 105  FORMAT(4F4.2)
      END
```

```
C       PROGRAM MINAC RANDOMLY GENERATES AN INITIAL SET OF ARC COSTS.
        PROGRAM MINAC(INPUT,OUTPUT,TAPE13,TAPE5=INPUT,TAPE6=OUTPUT)
        DIMENSION AC(10)
        READ(5,122)NC,N,ISEED
        CALL RANSET(ISEED)
        DO 1 I=1,N
        DO 2 J=1,NC
        AC(J)=RANF(IX)
  2     CONTINUE
        WRITE(13,121)(AC(K),K=1,NC)
  1     CONTINUE
        STOP
  122   FORMAT(2I4,I15)
  121   FORMAT(10F10.5)
        END
```

```
C       PROGRAM MTESTMS TESTS THE ASSUMED MODAL SPLIT AGAINST THE
C       ESTIMATED MODAL SPLIT.
        PROGRAM TESTMS(INPUT,OUTPUT,TAPE1,TAPE3,TAPE4,TAPE15,TAPE5=INPUT,
       &TAPE6=OUTPUT)
        INTEGER ODC(600,5)
        REAL MSO,MSE,MS(600,4)
        DIMENSION FODC(600),COD(2000),FOD(2000),A1Z(10)
        READ(1,100)ND,IP,NC
        READ(5,106)EPS,AL,(A1Z(I),I=1,NC)
        READ(1,100)((ODC(I,J),J=1,5),I=1,ND)
        READ(1,102)(FODC(I),I=1,ND)
        READ(3,102)(COD(I),I=1,IP)
        READ(3,102)(FOD(I),I=1,IP)
        M=0
        DO 1 I=1,ND
        A1=A1Z(ODC(I,5))
        MI=0
        DO 2 J=2,4
        IF(ODC(I,J).EQ.0)GO TO 2
        IF(ABS(COD(ODC(I,1))-COD(ODC(I,J))).LE..001)GO TO 3
  2     CONTINUE
        DU=EXP(A1*COD(ODC(I,1)))
        GO TO 4
  3     DU=0.
        MI=1
  4     DO 5 J=2,4
        IF(ODC(I,J).EQ.0)GO TO 5
        DU=DU+EXP(A1*COD(ODC(I,J)))
  5     CONTINUE
        DO 6 J=1,4
        IF(ODC(I,J).EQ.0)GO TO 7
        MSO=FOD(ODC(I,J))/FODC(I)
        IF(J.NE.1)GO TO 8
        IF(MI.EQ.0)GO TO 8
        MSE=0.
        GO TO 9
  8     MSE=EXP(A1*COD(ODC(I,J)))/DU
  9     E=MSE-MSO
        IF(ABS(E).GT.EPS)M=1
        MS(I,J)=MSO+AL*E
        FOD(ODC(I,J))=FODC(I)*MS(I,J)
        GO TO 6
  7     MS(I,J)=0.
  6     CONTINUE
  1     CONTINUE
        WRITE(4,102)(FOD(I),I=1,IP)
        WRITE(15,100)M
        WRITE(15,103)((MS(I,J),J=1,4),I=1,ND)
        STOP
  100   FORMAT(20I4)
  102   FORMAT(8F10.0)
  103   FORMAT(8F10.5)
  106   FORMAT(2F5.1,10F8.6)
        END
```

```
C     PROGRAM MCNCASP IS A MULTI-COMMODITY CONCAVE ASSIGNMENT ROUTINE
C     BASED ON THE YAGED METHOD OF DETERMINING A LOCAL OPTIMAL SOLUTION.
      PROGRAM MCNCASE(OUTPUT=400B,TAPE2=400B,TAPE3=400B,TAPE4=400B,
     &TAPE12=400B,TAPE13=400B,TAPE19=400B,TAPE21=400B,TAPE22=400B,
     &TAPE6=OUTPUT,INPUT=400B,TAPE5=INPUT)
      REAL IPM
      INTEGER ARC,Z,A,P,OD,POD,C,DE,T
      COMMON FOD(230),AC(3080),F(3080),A(3080,3),P(1152),
     &OD(230,2),POD(1152,2),COD(230),N,TC(1152),T(1152),D(1152),IU
      DIMENSION C(3),R(10,3),F1(10),ACZ(10)
      READ(2,100)N,ARC,Z,IP,NC
      READ(2,100)((A(I,J),J=1,3),I=1,ARC)
      N1=N+1
      READ(2,100)(P(I),I=1,N1)
      READ(2,100)((OD(I,J),J=1,2),I=1,IP)
      READ(2,100)((POD(I,J),J=1,2),I=1,N1)
      READ(4,102)(FOD(I),I=1,IP)
      READ(5,100)ICOU
      ICOUN=1
      IY=0
      IF(ICOU.LT.1)IY=IY+1
      DO 60 I=1,NC
      F1(I)=0.
   60 CONTINUE
      DO 61 I=1,ARC
      WRITE(19,120)(F1(J),J=1,NC)
   61 CONTINUE
   62 JTE=0
      DO 63 IU1=1,NC
      IU=IU1
      REWIND 13
      REWIND 19
      DO 64 I=1,ARC
      F(I)=0.
      READ(13,121)(ACZ(J),J=1,NC)
      AC(I)=ACZ(IU)
   64 CONTINUE
      DO 2 I=1,N
      I1=POD(I,1)
      I2=POD(I+1,1)-1
      IF(I1.GT.I2)GO TO 2
      DO 3 J=1,N
      D(J)=0
    3 CONTINUE
      DE=0
      DO 4 J=I1,I2
      IF(OD(J,2).NE.IU)GO TO 4
      D(OD(J,1))=J
      DE=DE+1
    4 CONTINUE
      IF(DE.EQ.0)GO TO 2
      IF(POD(I,2).EQ.1)GO TO 5
      IC=0
      GO TO 6
    5 IC=1
    6 CALL SPTR(I,DE,IC)
    2 CONTINUE
      DO 7 I=1,ARC
      READ(19,120)(F1(J),J=1,NC)
      IF(ABS(F1(IU)-F(I)).LT..01)GO TO 7
      GO TO 8
    7 CONTINUE
      JTE=JTE+1
      GO TO 63
    8 REWIND 19
```

```
      REWIND 21
      DO 9 I=1,ARC
      READ(19,120)(F1(J),J=1,NC)
      F1(IU)=F(I)
      WRITE(21,120)(F1(J),J=1,NC)
 9    CONTINUE
      REWIND 19
      REWIND 21
      DO 99 I=1,ARC
      READ(21,120)(F1(J),J=1,NC)
      WRITE(19,120)(F1(J),J=1,NC)
 99   CONTINUE
 63   CONTINUE
      IF(IY.EQ.1)GO TO 45
      WRITE(6,2000)
 2000 FORMAT(" ","*")
      GO TO 46
 45   WRITE(6,2001)
 2001 FORMAT(" ","$")
 46   IF(JTE.EQ.NC)GO TO 10
 13   REWIND 12
      REWIND 13
      REWIND 19
      TOTI=0.
      GO TO 11
 10   IF(IY.EQ.1)GO TO 12
      IY=1
      GO TO 13
 11   IF(IY.EQ.1)GO TO 98
      IF(ICOUN.EQ.ICCU) IY=1
      ICOUN=ICOUN+1
 98   REWIND 22
      DO 14 I=1,ARC
      AS=0.
      BS=0.
      CS=0.
      FS=0.
      READ(12,110)(C(J),J=1,3),((R(J,K),K=1,3),J=1,NC)
      READ(19,120)(F1(J),J=1,NC)
      DO 15 J=1,NC
      AS=AS+R(J,1)*F1(J)
      BS=BS+R(J,2)*F1(J)
      CS=CS+R(J,3)*F1(J)
      FS=FS+F1(J)
 15   CONTINUE
      KAS=0
      IF(FS.LT.1.)GO TO 16
      IF(AS.LT..0000000000001)GO TO 16
      HI=(-BS-1.)/(2.*AS)
      IF(HI.LE.C(1))GO TO 16
      IF(HI.GE.C(2))GO TO 17
      IPM=HI
      GO TO 18
 16   IPM=C(1)
      KAS=1
      GO TO 18
 17   IPM=C(2)
 18   IF(IY.EQ.1)GO TO 19
      DO 20 J=1,NC
      ACZ(J)=C(3)*(R(J,1)*IPM**2+R(J,2)*IPM+R(J,3))
      IF(KAS.EQ.1)GO TO 20
      ACZ(J)=ACZ(J)+C(3)*(IPM-C(1))/FS
 20   CONTINUE
      GO TO 21
 19   DO 22 J=1,NC
```

```
      ACZ(J)=C(3)*(R(J,1)*IPM**2+R(J,2)*IPM+R(J,3))
22    CONTINUE
21    WRITE(13,121)(ACZ(J),J=1,NC)
      TOTI=TOTI+IPM*C(3)
      AINV=IPM*C(3)
      WRITE(22,102)AINV
14    CONTINUE
      GO TO 62
12    TOTU=0.
      REWIND 13
      REWIND 19
      DO 23 I=1,ARC
      READ(13,121)(ACZ(J),J=1,NC)
      READ(19,120)(F1(J),J=1,NC)
      DO 24 J=1,NC
      TOTU=TOTU+ACZ(J)*F1(J)
24    CONTINUE
      TOTDIS=TOTU+TOTI
23    CONTINUE
      WRITE(3,103)(COD(I),I=1,IP)
      WRITE(3,102)(FOD(I),I=1,IP)
      WRITE(6,112)TOTDIS,TOTI,TOTU
      STOP
100   FORMAT(20I4)
102   FORMAT(8F10.0)
103   FORMAT(8F10.5)
110   FORMAT(12E10.3)
111   FORMAT(I1,11E10.3)
112   FORMAT(3F14.0)
120   FORMAT(10F10.0)
121   FORMAT(10F10.5)
      END
C     SUBROUTINE SPTR IS A SHORTEST PATH TREE ROUTINE
      SUBROUTINE SPTR(O,DE,IQ)
      INTEGER O,DE,T,B,FS,P,S(3080,2),D,A,OD,POD
      COMMON FOD(230),AC(3080),F(3080),A(3080,3),P(1152),
     &OD(230,2),POD(1152,2),COD(230),N,TC(1152),T(1152),D(1152),IU
      DIMENSION SC(3080)
      DO 1 I=1,N
      TC(I)=-1.
1     CONTINUE
      ITOT=0
      T(1)=0
      TC(O)=0.
      B=O
      IT=2
      FS=1
      L=2
      IS=1
14    IA=P(B)
      IB=P(B+1)-1
      IF(IB+1-IA.EQ.0)GO TO 2
      DO 3 I=IA,IB
      IF(IQ.EQ.0)GO TO 4
5     S(IS,1)=I
      S(IS,2)=0
      SC(IS)=TC(B)+AC(I)
      S(L,2)=IS
      L=IS
      IS=IS+1
      GO TO 3
4     IF(A(I,3).EQ.1)GO TO 3
      GO TO 5
3     CONTINUE
2     K=0
```

```
      J=FS
5     CM=10.**50
8     IF(TC(A(S(J,1),2)).GE.0.)GO TC 6
      IF(SC(J).GE.CM)GO TO 7
      MJ=J
      CM=SC(J)
7     K=J
      J=S(J,2)
      IF(J.NE.0)GO TC 8
      GC TO 9
6     IF(J.EQ.FS)GO TO 10
      IF(J.NE.L)GO TC 11
      L=K
      S(K,2)=0
      GO TO 9
10    FS=S(J,2)
      J=FS
      GC TO 8
11    S(K,2)=S(J,2)
      J=S(J,2)
      GC TO 8
9     ID=A(S(MJ,1),2)
      T(IT)=S(MJ,1)
      TC(ID)=SC(MJ)
      IF(D(ID).EQ.0)GO TO 12
      ITOT=ITOT+1
      IF(ITOT.EG.DE)GO TO 13
12    B=ID
      IT=IT+1
      GC TO 14
13    CALL ASNFLW(O,IT)
      RETURN
      END
C     SUBROUTINE ASNFLW ASSIGNS THE FLOW TO THE NETWORK
C     ACCORDING TO THE TREES DETERMINED IN SPTR.
      SUBROUTINE ASNFLW(O,IT)
      INTEGER O,POD,OD,T,A,D,P
      COMMON FOD(230),AC(3080),F(3080),A(3080,3),P(1152),
     &OD(230,2),POD(1152,2),COD(230),N,TC(1152),T(1152),D(1152),IU
      DIMENSION FL(1152)
      I1=POD(O,1)
      I2=POD(O+1,1)-1
      DC 1 I=1,N
      FL(I)=0.
1     CONTINUE
      DC 2 I=I1,I2
      IF(OD(I,2).NE.IU)GO TO 2
      FL(OD(I,1))=FOD(I)
2     CONTINUE
      I2=IT-1
      DC 3 I1=1,I2
      I=IT-I1+1
      L=T(I)
      J=A(L,1)
      K=A(L,2)
      F(L)=F(L)+FL(K)
      FL(J)=FL(J)+FL(K)
      IF(D(K).EQ.0)GC TO 3
      COD(D(K))=TC(K)
3     CONTINUE
      RETURN
      END
```

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